Computer Algebra Independent Integration Tests

Summer 2024

2-Exponentials/2.4/159-2.4.3

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**399**]. This is test number [159].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	96.49 (385)	3.51 (14)
Mathematica	95.99 (383)	4.01 (16)
Fricas	74.94 (299)	25.06 (100)
Maple	51.13 (204)	48.87 (195)
Mupad	37.09 (148)	62.91 (251)
Giac	36.84 (147)	63.16 (252)
Maxima	36.59 (146)	63.41 (253)
Reduce	28.07 (112)	71.93 (287)
Sympy	24.56 (98)	75.44 (301)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

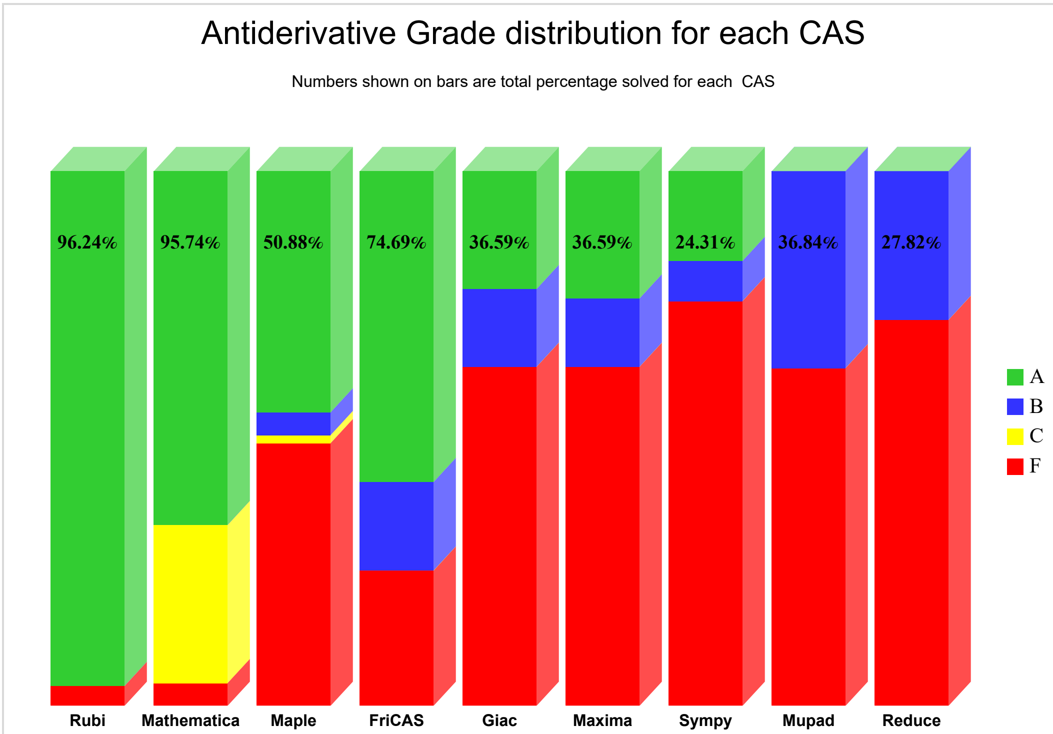
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

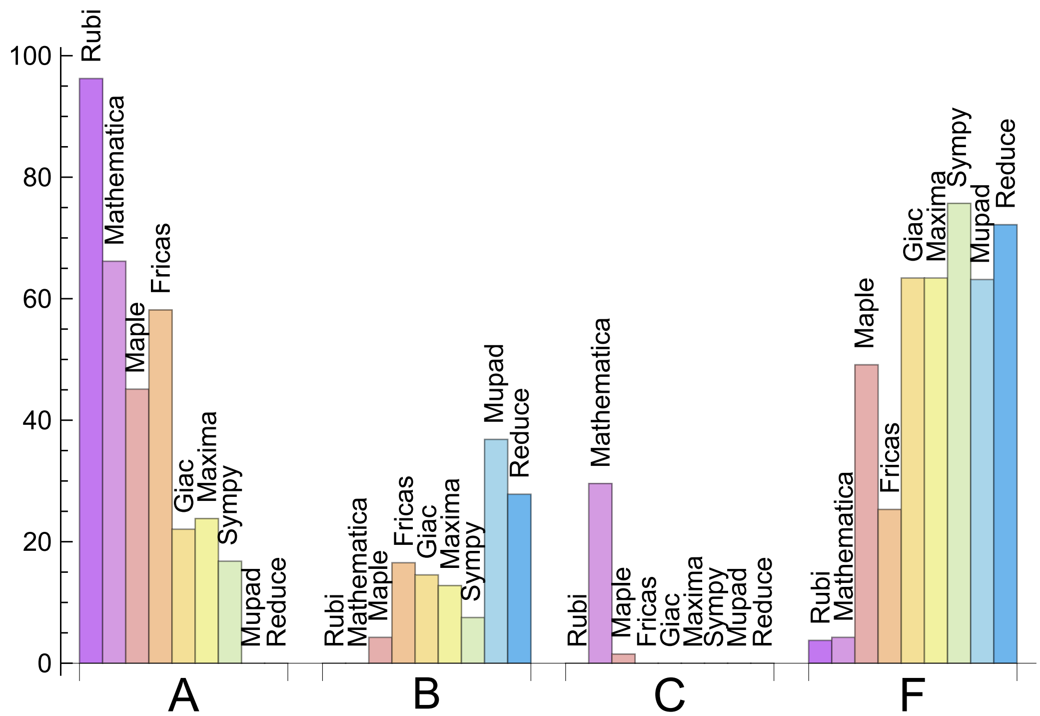
System	% A grade	% B grade	% C grade	% F grade
Rubi	96.241	0.000	0.000	3.759
Mathematica	66.165	0.000	29.574	4.261
Fricas	58.145	16.541	0.000	25.313
Maple	45.113	4.261	1.504	49.123
Maxima	23.810	12.782	0.000	63.409
Giac	22.055	14.536	0.000	63.409
Sympy	16.792	7.519	0.000	75.689
Mupad	0.000	36.842	0.000	63.158
Reduce	0.000	27.820	0.000	72.180

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	14	100.00	0.00	0.00
Mathematica	16	100.00	0.00	0.00
Fricas	100	100.00	0.00	0.00
Maple	195	100.00	0.00	0.00
Mupad	251	0.00	100.00	0.00
Giac	252	45.63	8.33	46.03
Maxima	253	95.26	0.00	4.74
Reduce	287	100.00	0.00	0.00
Sympy	301	84.05	15.95	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.07
Mathematica	0.10
Fricas	0.11
Giac	0.15
Reduce	0.24
Rubi	0.55
Maple	2.17
Sympy	4.33
Mupad	16.60

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	68.61	1.10	51.50	0.94
Mathematica	83.65	0.81	73.00	0.82
Maple	114.67	1.37	67.50	0.98
Sympy	118.27	1.68	51.00	1.13
Giac	134.65	1.58	80.00	1.35
Rubi	141.16	1.05	98.00	1.00
Reduce	168.14	1.85	86.50	1.47
Fricas	172.43	1.35	111.00	1.06
Maxima	211.64	1.74	67.00	1.26

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

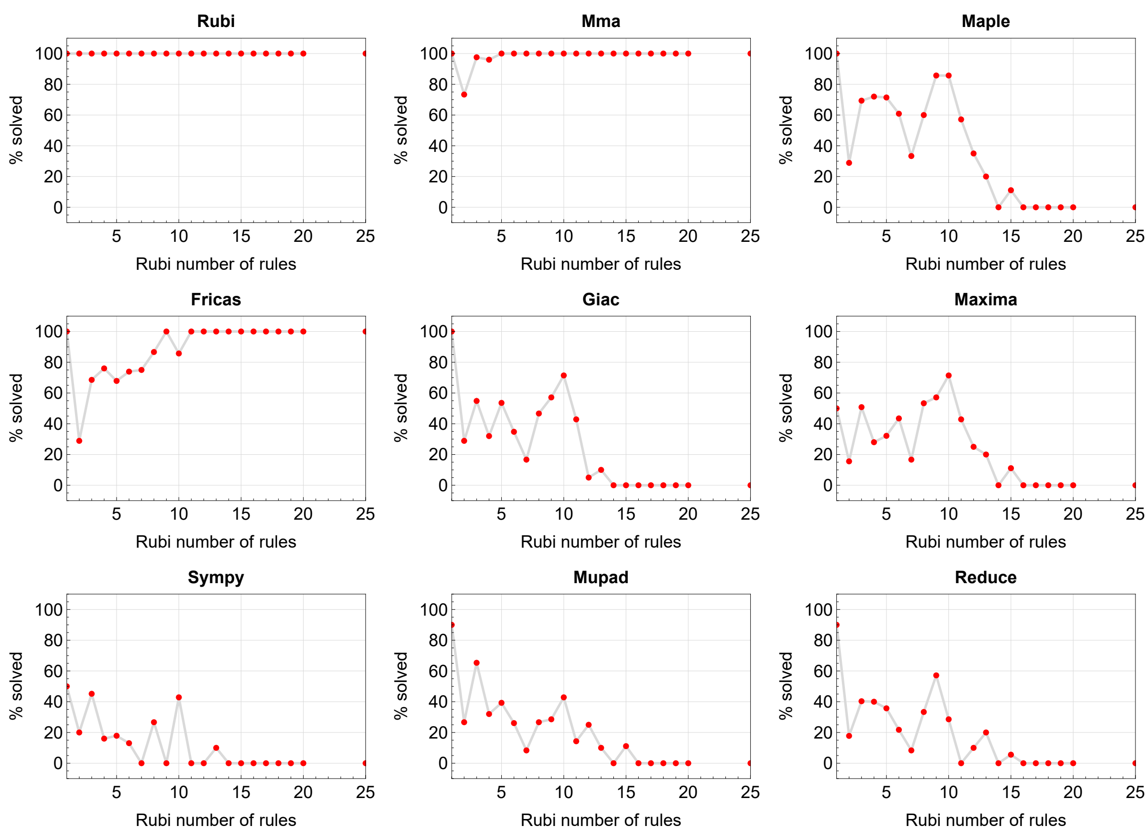


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

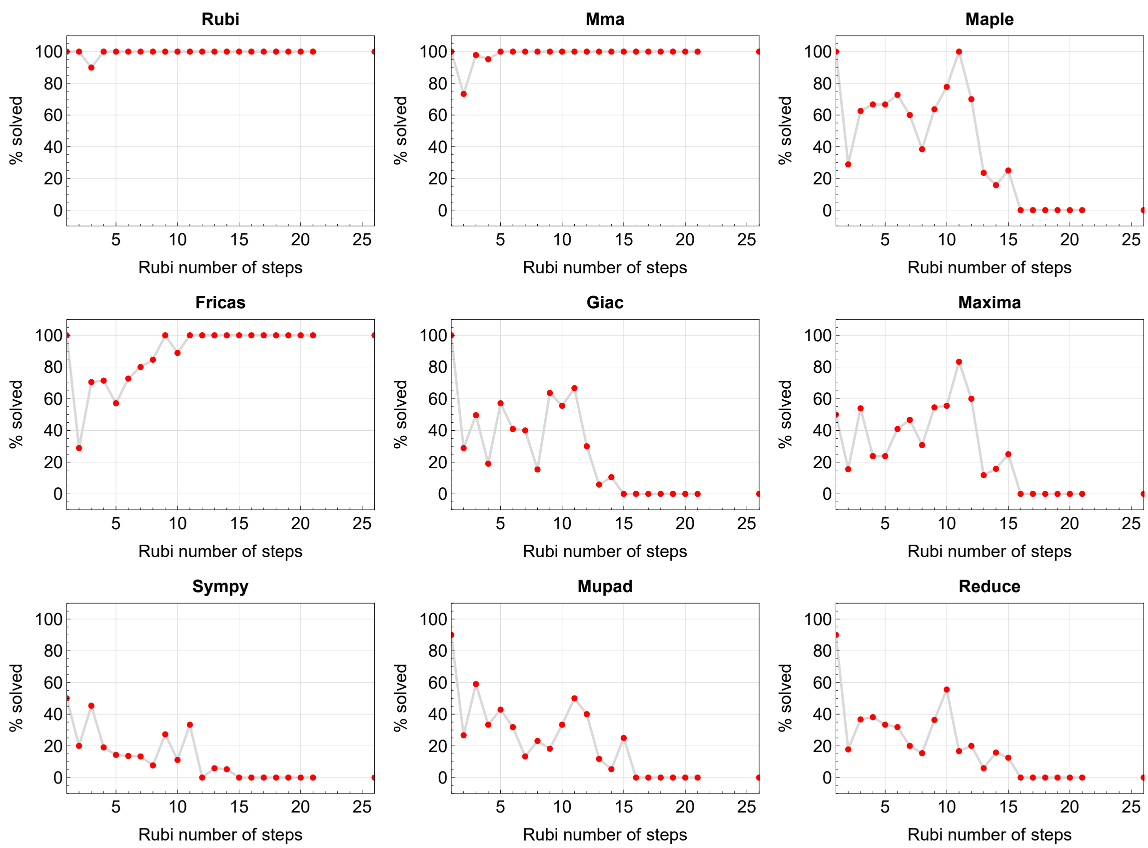


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

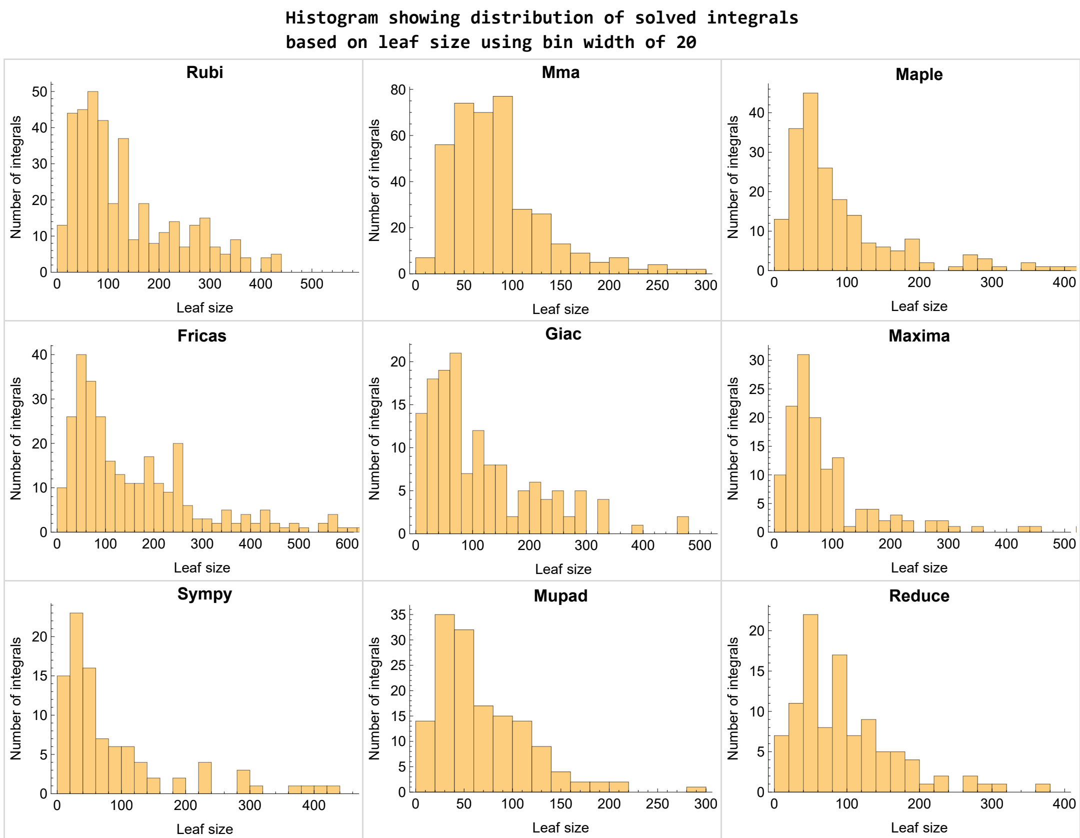


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

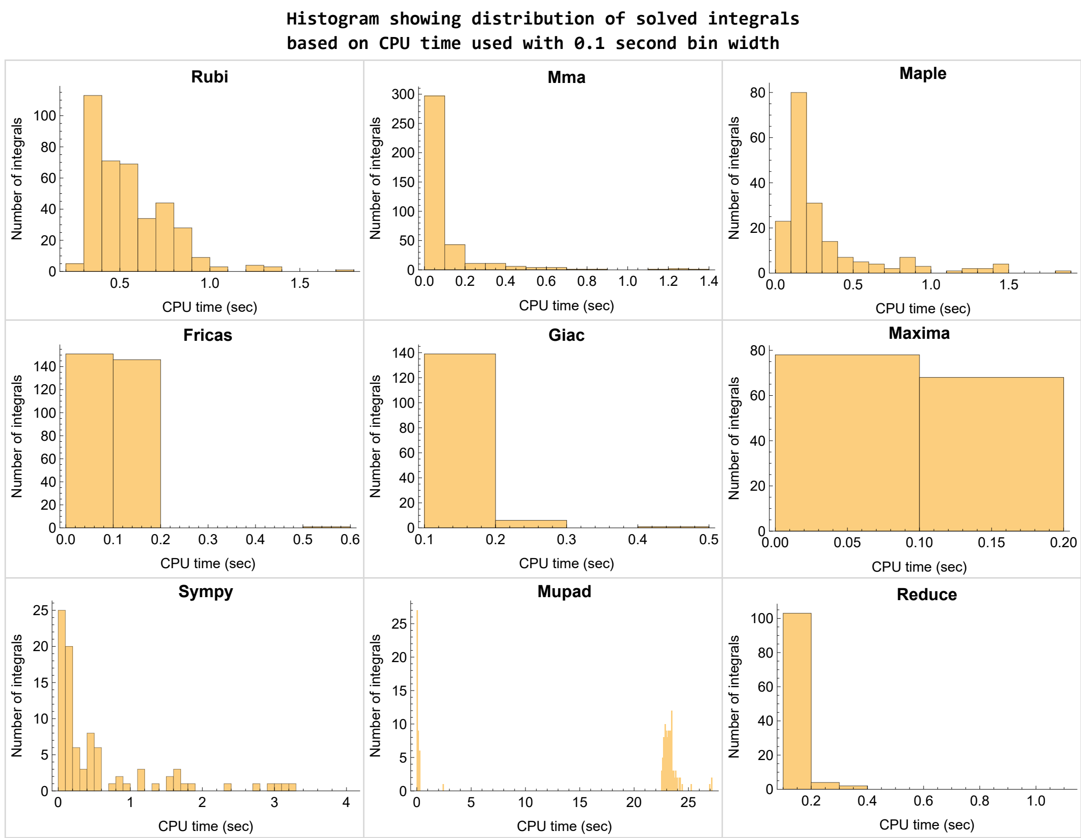


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

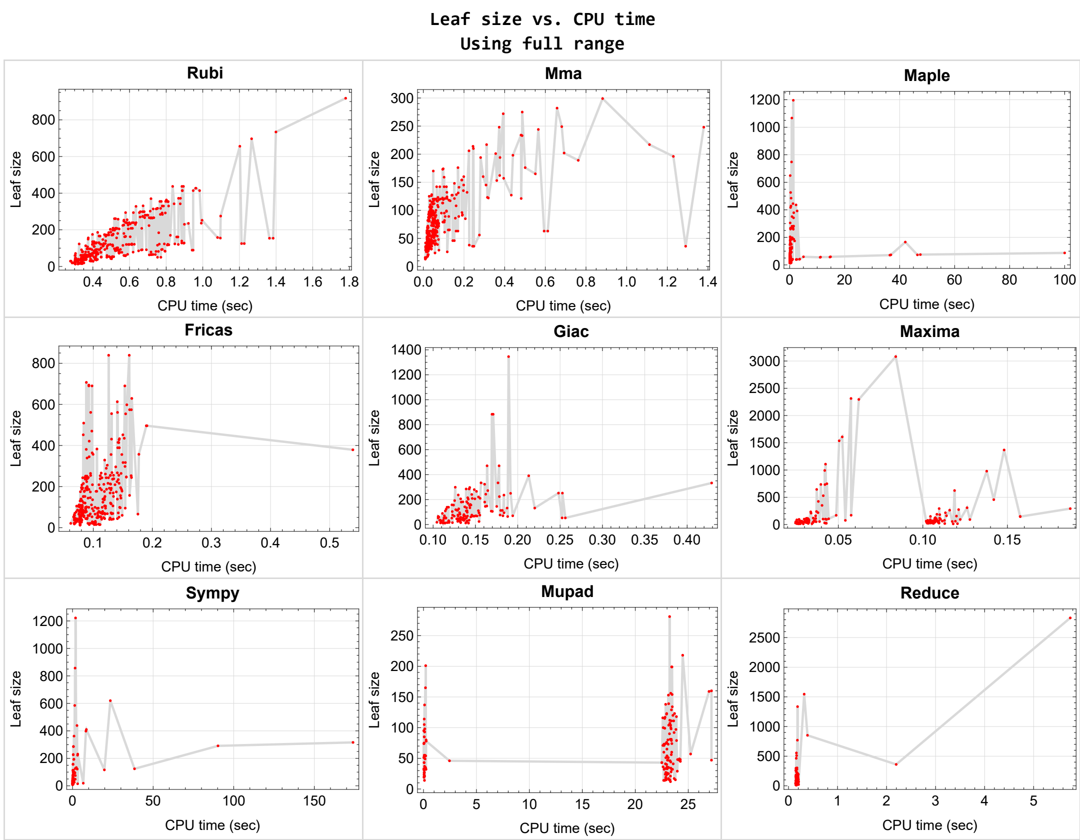


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{13}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {76, 77, 78, 79, 85, 86, 87, 88, 89, 94, 95, 96, 97, 98, 103, 104, 105, 106, 107, 112, 113, 114, 115, 116, 121, 122, 123, 124, 125, 130, 131, 132, 133, 143, 144, 145, 146, 230, 231, 232, 235, 236, 237, 238, 240, 241, 242, 245, 246, 247, 248, 316, 318, 321, 323}

Mathematica {155, 156, 157, 158, 176, 221, 222, 316, 367}

Maple {226}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

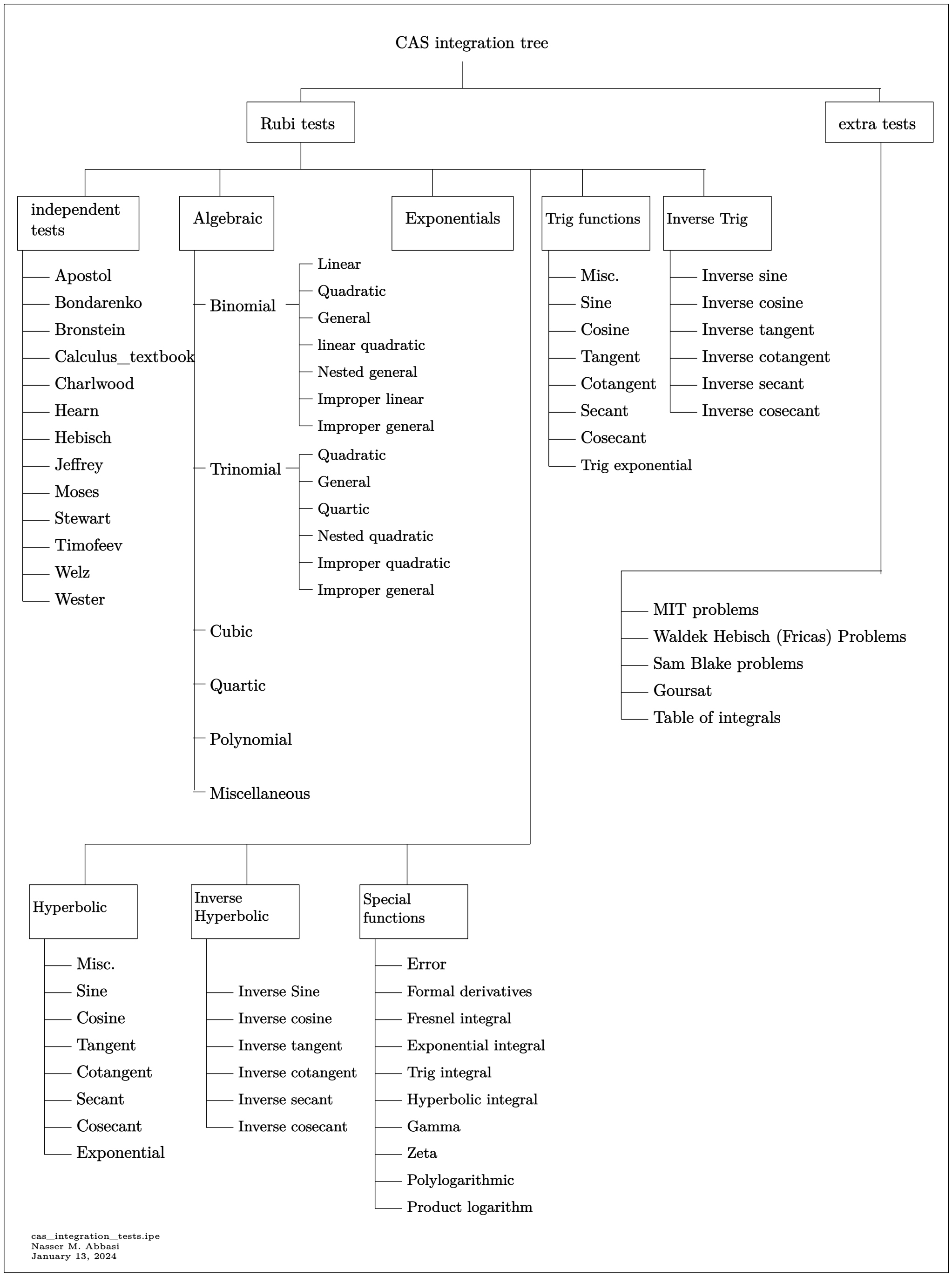
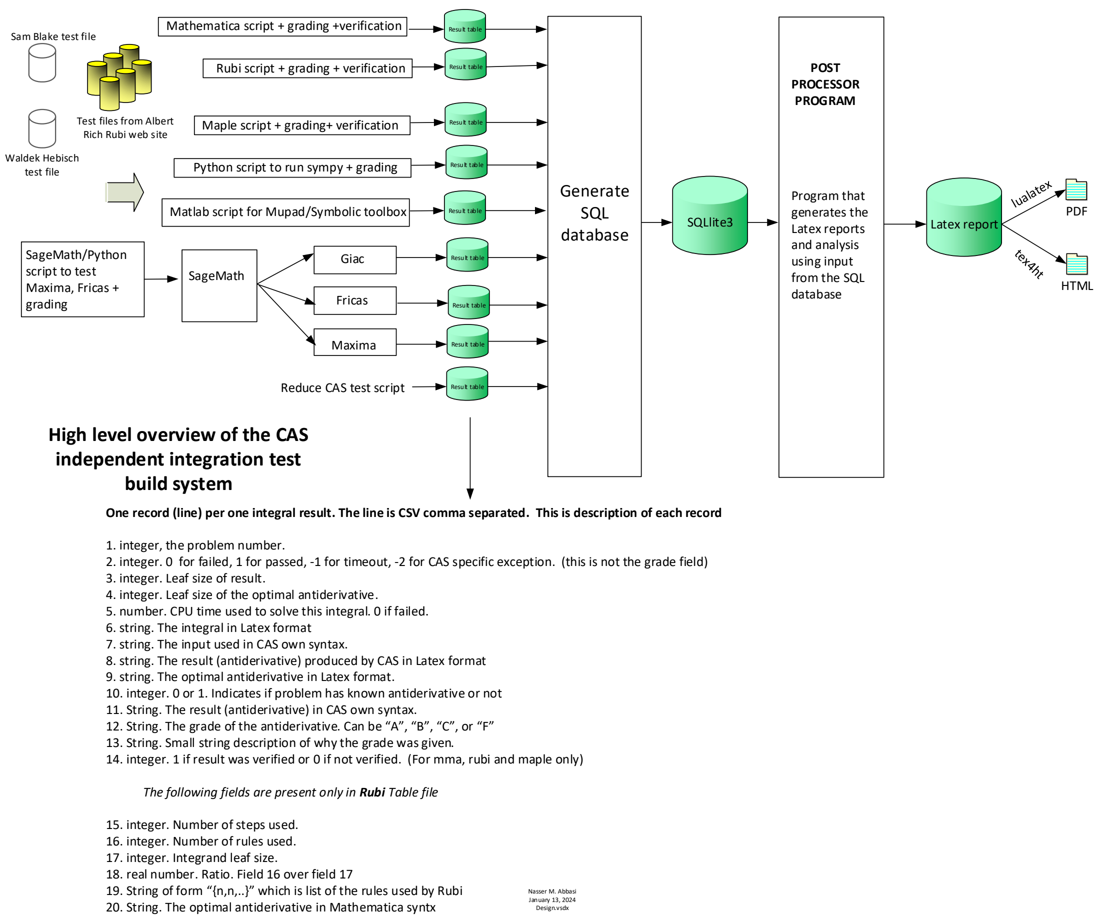


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 149, 150, 151, 152, 153, 154, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 309, 310, 311, 312, 313, 314, 315, 317, 318, 319, 320, 321, 322, 323, 324, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 358, 359, 360, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 379, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399 }

B grade { }

C grade { 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 155, 156, 157, 158, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 262, 263, 264, 265, 266, 277, 278, 279, 280, 291, 292, 293, 294, 305, 306, 307, 308, 316, 325, 357, 361 }

F normal fail { 159, 160, 161, 162, 163, 164, 165, 166, 167, 250, 378, 380, 381, 382, 383, 384 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 155, 156, 176, 177, 178, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 191, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 205, 206, 207, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 228, 229, 262, 263, 264, 265, 266, 270, 271, 272, 277, 278, 279, 280, 284, 285, 286, 291, 292, 293, 294, 298, 299, 300, 305, 306, 307, 308, 312, 313, 314, 315, 317, 319, 320, 322, 324, 326, 328, 329, 330, 331, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 357, 361, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399 }

B grade { 21, 38, 54, 71, 185, 192, 193, 199, 208, 226, 316, 318, 321, 323, 325, 327, 332 }

C grade { 149, 150, 151, 152, 153, 154 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 267, 268, 269, 273, 274, 275, 276, 281, 282, 283, 287, 288, 289, 290, 295, 296, 297, 301, 302, 303, 304, 309, 310, 311, 351, 352, 353, 354, 355, 356, 358, 359, 360, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 16, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 121, 122, 123, 124, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 148, 176, 177, 178, 179, 180, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 203, 204, 205, 206, 207, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 231, 232, 233, 237, 241, 242, 247, 262, 263, 264, 265, 266, 270, 271, 272, 277, 278, 279, 280, 284, 285, 286, 291, 292, 293, 294, 298, 299, 300, 305, 306, 307, 308, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 333, 334, 335, 336, 339, 340, 341, 342, 343, 344, 345, 348, 349, 350, 357, 361, 386, 387, 388, 391, 392 }

B grade { 1, 21, 22, 38, 39, 54, 55, 71, 72, 80, 90, 108, 117, 125, 126, 127, 147, 181, 182, 183, 184, 199, 200, 201, 202, 208, 209, 210, 211, 226, 227, 228, 229, 230, 234, 235, 236, 238, 239, 240, 243, 244, 245, 246, 248, 249, 324, 325, 326, 327, 328, 329, 330, 331, 332, 337, 338, 346, 347, 389, 390, 393, 394, 395, 398, 399 }

C grade { }

F normal fail { 14, 15, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 267, 268, 269, 273, 274, 275, 276, 281, 282, 283, 287, 288, 289, 290, 295, 296, 297, 301, 302, 303, 304, 309, 310, 311, 351, 352, 353, 354, 355, 356, 358, 359, 360, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 396, 397 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 4, 7, 8, 9, 10, 11, 12, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 59, 60, 61, 62, 63, 64, 65, 66, 69, 70, 180, 204, 205, 206, 207, 212, 213, 214, 215, 216, 217, 225, 262, 277, 291, 305, 315, 317, 318, 320, 321, 322, 323, 329, 330, 331, 332, 334, 336, 337, 339, 340, 347, 348, 349, 357, 386, 397, 398 }

B grade { 1, 2, 3, 5, 6, 67, 68, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 218, 219, 220, 221, 222, 223, 224, 316, 319, 333, 335, 341, 350, 389, 390, 391, 399 }

C grade { }

F normal fail { 14, 15, 55, 56, 57, 58, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 209, 210, 211, 226, 227, 228, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 306, 307, 308, 309, 310, 311, 312, 313, 314, 324, 325, 326, 327, 342, 343, 345, 351, 352, 353, 354, 355, 356, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 387, 388 }

F(-1) timedout fail { }

F(-2) exception fail { 13, 208, 229, 328, 338, 344, 346, 392, 393, 394, 395, 396 }

Giac

A grade { 17, 19, 20, 26, 27, 28, 29, 30, 31, 32, 33, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 59, 60, 64, 65, 66, 176, 177, 178, 179, 180, 181, 182, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 203, 204, 205, 206, 207, 208, 209, 216, 221, 222, 223, 224, 262, 271, 277, 291, 299, 305, 315, 316, 317, 319, 320, 322, 323, 325, 327, 330, 332, 333, 335, 336, 337, 338, 339, 340, 345, 348, 357, 386, 387, 391 }

B grade { 21, 22, 23, 24, 25, 54, 61, 62, 63, 183, 184, 198, 199, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 225, 226, 263, 264, 265, 266, 270, 272, 278, 279, 280, 284, 285, 286, 292, 293, 294, 298, 300, 306, 307, 308, 312, 313, 314, 334, 341, 343, 350, 361, 388, 389, 390, 392, 393 }

C grade { }

F normal fail { 35, 36, 37, 38, 39, 40, 41, 56, 58, 68, 69, 70, 71, 72, 73, 74, 75, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 149, 150, 151, 152, 153, 154, 156, 157, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 175, 200, 201, 202, 227, 228, 229, 258, 259, 260, 261, 269, 273, 274, 275, 276, 283, 287, 288, 289, 290, 297, 301, 302, 303, 304, 311, 318, 321, 324, 326, 328, 331, 342, 344, 351, 352, 353, 354, 355, 356, 358, 359, 360, 364, 365, 366, 368, 369, 370, 371, 372, 373, 375, 376, 377, 378, 379, 382, 383, 384, 385, 394, 395, 396, 397, 398 }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 251, 252, 253, 254, 255, 256, 257 }

F(-2) exception fail { 16, 18, 34, 50, 51, 55, 57, 67, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 143, 144, 145, 146, 147, 148, 155, 158, 159, 160, 161, 162, 163, 166, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 267, 268, 281, 282, 295, 296, 309, 310, 329, 346, 347, 349, 362, 363, 367, 374, 380, 381, 399 }

Mupad

A grade { }

B grade { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 180, 181, 182, 185, 186, 187, 188, 189, 190, 191, 192, 193, 212, 213, 214, 215, 216, 217, 218, 219, 220, 262, 263, 264, 265, 266, 270, 271, 272, 277, 278, 279, 280, 284, 285, 286, 291, 292, 293, 294, 298, 299, 300, 305, 306, 307, 308, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 348, 349, 350, 357, 361, 386, 387, 390, 391, 392, 393, 394, 395, 398, 399 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 183, 184, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 267, 268, 269, 273, 274, 275, 276, 281, 282, 283, 287, 288, 289, 290, 295, 296, 297, 301, 302, 303, 304, 309, 310, 311, 324, 325, 326, 327, 328, 329, 330, 331, 332, 346, 347, 351, 352, 353, 354, 355, 356, 358, 359, 360, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 388, 389, 396, 397 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 42, 43, 44, 45, 46, 47, 48, 49, 59, 60, 61, 62, 63, 64, 65, 66, 156, 180, 185, 186, 187, 188, 189, 212, 213, 214, 215, 216, 262, 277, 291, 305, 315, 317, 319, 320, 322, 333, 335, 337, 338, 340, 391, 392 }

B grade { 20, 151, 152, 176, 177, 178, 179, 190, 191, 192, 193, 217, 218, 219, 220, 263, 264, 265, 266, 278, 279, 280, 292, 293, 306, 307, 357, 389, 390, 393 }

C grade { }

F normal fail { 7, 8, 9, 10, 11, 12, 14, 15, 34, 35, 36, 37, 38, 39, 40, 41, 50, 51, 52, 53, 54, 55, 56, 57, 58, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 98, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 130, 131, 132, 133, 134, 135, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 153, 154, 155, 157, 158, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 181, 182, 183, 184, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 224, 225, 226, 230, 231, 232, 233, 234, 240, 241, 242, 243, 244, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 267, 268, 269, 270, 271, 273, 274, 275, 276, 281, 282, 283, 284, 285, 287, 288, 289, 290, 296, 297, 298, 301, 302, 303, 304, 310, 311, 312, 316, 318, 321, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 334, 336, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 358, 359, 360, 362, 363, 364, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 385, 386, 387, 395, 396, 397 }

F(-1) timedout fail { 94, 95, 96, 97, 99, 100, 101, 102, 128, 129, 136, 137, 138, 142, 159, 160, 221, 222, 223, 227, 228, 229, 235, 236, 237, 238, 239, 245, 246, 248, 249, 272, 286, 294, 295, 299, 300, 308, 309, 313, 314, 361, 365, 384, 388, 394, 398, 399 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 144, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 196, 197, 198, 199, 200, 202, 262, 263, 264, 265, 266, 270, 271, 272, 277, 278, 279, 280, 284, 285, 286, 291, 292, 298, 305, 306, 312, 315, 316, 317, 318, 319, 320, 324, 325, 326, 327, 328, 329, 333, 334, 335, 336, 337, 342, 343, 344, 345, 346, 357, 361, 386, 387, 389, 390, 391, 394, 395, 396 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 194, 195, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 267, 268, 269, 273, 274, 275, 276, 281, 282, 283, 287, 288, 289, 290, 293, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 307, 308, 309, 310, 311, 313, 314, 321, 322, 323, 330, 331, 332, 338, 339, 340, 341, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 358, 359, 360, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 388, 392, 393, 397, 398, 399 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	B	B	A	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	0	63	0	170	111	78	0	17	0
N.S.	1	0.00	0.91	0.00	2.46	1.61	1.13	0.00	0.25	0.00
time (sec)	N/A	0.000	0.065	0.000	0.058	0.076	0.348	0.000	0.177	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	B	A	A	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	46	0	100	59	37	0	17	0
N.S.	1	0.00	1.07	0.00	2.33	1.37	0.86	0.00	0.40	0.00
time (sec)	N/A	0.000	0.056	0.000	0.041	0.068	0.184	0.000	0.164	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	B	A	A	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	35	0	44	26	20	0	17	0
N.S.	1	0.00	1.35	0.00	1.69	1.00	0.77	0.00	0.65	0.00
time (sec)	N/A	0.000	0.030	0.000	0.035	0.103	0.089	0.000	0.177	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	A	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	35	0	43	26	22	0	19	0
N.S.	1	0.00	1.21	0.00	1.48	0.90	0.76	0.00	0.66	0.00
time (sec)	N/A	0.000	0.031	0.000	0.033	0.101	0.087	0.000	0.189	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	B	A	A	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	0	46	0	100	59	37	0	19	0
N.S.	1	0.00	0.94	0.00	2.04	1.20	0.76	0.00	0.39	0.00
time (sec)	N/A	0.000	0.059	0.000	0.042	0.091	0.185	0.000	0.164	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	B	A	A	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	0	63	0	169	111	80	0	19	0
N.S.	1	0.00	0.84	0.00	2.25	1.48	1.07	0.00	0.25	0.00
time (sec)	N/A	0.000	0.060	0.000	0.049	0.097	0.346	0.000	0.209	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	0	63	0	148	193	0	0	17	0
N.S.	1	0.00	0.37	0.00	0.87	1.13	0.00	0.00	0.10	0.00
time (sec)	N/A	0.000	0.161	0.000	0.158	0.082	0.000	0.000	0.166	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	0	46	0	93	122	0	0	17	0
N.S.	1	0.00	0.37	0.00	0.76	0.99	0.00	0.00	0.14	0.00
time (sec)	N/A	0.000	0.145	0.000	0.122	0.084	0.000	0.000	0.197	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	0	31	0	60	81	0	0	16	0
N.S.	1	0.00	0.39	0.00	0.76	1.03	0.00	0.00	0.20	0.00
time (sec)	N/A	0.000	0.040	0.000	0.119	0.088	0.000	0.000	0.173	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	0	31	0	56	81	0	0	18	0
N.S.	1	0.00	0.39	0.00	0.71	1.03	0.00	0.00	0.23	0.00
time (sec)	N/A	0.000	0.039	0.000	0.117	0.085	0.000	0.000	0.183	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	0	46	0	91	82	0	0	19	0
N.S.	1	0.00	0.37	0.00	0.74	0.67	0.00	0.00	0.15	0.00
time (sec)	N/A	0.000	0.152	0.000	0.128	0.100	0.000	0.000	0.178	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	0	63	0	145	124	0	0	19	0
N.S.	1	0.00	0.37	0.00	0.85	0.73	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	0.170	0.000	0.158	0.116	0.000	0.000	0.194	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	0	16	14	16	18	16
N.S.	1	1.00	1.14	1.00	0.00	1.14	1.00	1.14	1.29	1.14
time (sec)	N/A	0.440	0.709	0.089	0.000	0.087	3.554	1.404	0.185	22.716

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	0	77	0	0	0	0	0	16	0
N.S.	1	0.00	0.74	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.000	0.059	0.000	0.000	0.000	0.000	0.000	0.189	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	59	0	0	0	0	0	17	0
N.S.	1	0.00	0.72	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.000	0.060	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	156	64	84	100	67	114	0	106	98
N.S.	1	1.22	0.50	0.66	0.78	0.52	0.89	0.00	0.83	0.77
time (sec)	N/A	0.529	0.064	0.201	0.030	0.095	0.542	0.000	0.202	0.122

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	125	56	77	81	59	104	70	88	85
N.S.	1	1.19	0.53	0.73	0.77	0.56	0.99	0.67	0.84	0.81
time (sec)	N/A	0.468	0.047	0.141	0.031	0.081	0.544	0.113	0.169	0.077

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	92	46	67	62	51	90	0	69	71
N.S.	1	1.15	0.58	0.84	0.78	0.64	1.12	0.00	0.86	0.89
time (sec)	N/A	0.415	0.038	0.127	0.028	0.077	0.511	0.000	0.207	22.608

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	61	38	59	42	43	78	53	50	51
N.S.	1	1.13	0.70	1.09	0.78	0.80	1.44	0.98	0.93	0.94
time (sec)	N/A	0.335	0.032	0.110	0.028	0.078	0.518	0.139	0.170	0.045

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	41	25	37	61	41	32	32
N.S.	1	1.00	0.90	1.41	0.86	1.28	2.10	1.41	1.10	1.10
time (sec)	N/A	0.281	0.021	0.083	0.029	0.079	0.431	0.135	0.194	0.040

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	29	48	18	58	48	68	52	32
N.S.	1	1.00	1.16	1.92	0.72	2.32	1.92	2.72	2.08	1.28
time (sec)	N/A	0.342	0.019	0.133	0.028	0.078	1.610	0.152	0.176	0.038

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	47	34	29	66	26	75	62	33
N.S.	1	1.00	1.24	0.89	0.76	1.74	0.68	1.97	1.63	0.87
time (sec)	N/A	0.346	0.040	0.135	0.029	0.076	1.116	0.155	0.178	0.038

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	57	53	48	83	48	153	80	52
N.S.	1	1.00	0.90	0.84	0.76	1.32	0.76	2.43	1.27	0.83
time (sec)	N/A	0.396	0.054	0.145	0.031	0.076	1.621	0.143	0.187	0.043

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	92	70	68	67	92	75	161	109	74
N.S.	1	1.02	0.78	0.76	0.74	1.02	0.83	1.79	1.21	0.82
time (sec)	N/A	0.437	0.062	0.154	0.030	0.080	1.731	0.144	0.192	0.039

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	118	76	77	86	101	122	237	127	95
N.S.	1	1.04	0.67	0.68	0.76	0.89	1.08	2.10	1.12	0.84
time (sec)	N/A	0.507	0.069	0.165	0.025	0.082	2.921	0.130	0.182	0.034

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	46	56	46	41	46	55	43
N.S.	1	1.00	1.00	0.96	1.17	0.96	0.85	0.96	1.15	0.90
time (sec)	N/A	0.355	0.026	0.165	0.107	0.068	0.058	0.129	0.187	22.509

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	37	47	37	31	37	46	36
N.S.	1	1.00	1.00	0.95	1.21	0.95	0.79	0.95	1.18	0.92
time (sec)	N/A	0.342	0.017	0.142	0.104	0.068	0.063	0.135	0.161	22.625

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	29	38	29	22	29	38	27
N.S.	1	1.00	1.00	1.00	1.31	1.00	0.76	1.00	1.31	0.93
time (sec)	N/A	0.311	0.015	0.114	0.102	0.098	0.049	0.140	0.195	0.063

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	30	20	28	21	12	15	27	19
N.S.	1	1.00	1.58	1.05	1.47	1.11	0.63	0.79	1.42	1.00
time (sec)	N/A	0.287	0.015	0.085	0.104	0.099	0.057	0.123	0.169	0.047

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	13	21	15	17	12	22	14
N.S.	1	1.00	1.00	1.00	1.62	1.15	1.31	0.92	1.69	1.08
time (sec)	N/A	0.304	0.010	0.138	0.106	0.105	0.074	0.139	0.205	22.679

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	26	31	26	32	21	36	17
N.S.	1	1.00	1.00	1.00	1.19	1.00	1.23	0.81	1.38	0.65
time (sec)	N/A	0.311	0.015	0.163	0.103	0.077	0.084	0.119	0.172	23.673

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	38	42	39	42	31	53	27
N.S.	1	1.00	1.00	1.06	1.17	1.08	1.17	0.86	1.47	0.75
time (sec)	N/A	0.331	0.016	0.153	0.103	0.077	0.097	0.118	0.189	0.084

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	48	51	47	54	39	62	34
N.S.	1	1.00	1.00	1.00	1.06	0.98	1.12	0.81	1.29	0.71
time (sec)	N/A	0.347	0.019	0.167	0.103	0.078	0.112	0.135	0.177	22.946

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	154	80	124	114	88	0	0	172	137
N.S.	1	1.22	0.63	0.98	0.90	0.70	0.00	0.00	1.37	1.09
time (sec)	N/A	1.384	0.069	0.302	0.031	0.079	0.000	0.000	0.184	0.101

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	125	63	111	95	80	0	0	149	114
N.S.	1	1.19	0.60	1.06	0.90	0.76	0.00	0.00	1.42	1.09
time (sec)	N/A	1.226	0.063	0.282	0.032	0.077	0.000	0.000	0.194	0.070

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	89	54	106	76	72	0	0	135	104
N.S.	1	1.07	0.65	1.28	0.92	0.87	0.00	0.00	1.63	1.25
time (sec)	N/A	0.942	0.069	0.233	0.033	0.080	0.000	0.000	0.189	23.162

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	42	93	57	60	0	0	111	72
N.S.	1	1.00	0.70	1.55	0.95	1.00	0.00	0.00	1.85	1.20
time (sec)	N/A	0.378	0.041	0.200	0.036	0.080	0.000	0.000	0.170	22.736

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	55	101	46	100	0	0	175	73
N.S.	1	1.00	1.10	2.02	0.92	2.00	0.00	0.00	3.50	1.46
time (sec)	N/A	0.753	0.050	0.181	0.025	0.081	0.000	0.000	0.193	23.108

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	63	61	80	60	109	0	0	165	75
N.S.	1	0.95	0.92	1.21	0.91	1.65	0.00	0.00	2.50	1.14
time (sec)	N/A	0.708	0.067	0.232	0.031	0.078	0.000	0.000	0.167	0.061

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	92	79	105	81	130	0	0	195	99
N.S.	1	1.01	0.87	1.15	0.89	1.43	0.00	0.00	2.14	1.09
time (sec)	N/A	0.775	0.096	0.216	0.027	0.081	0.000	0.000	0.200	22.787

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	117	89	116	100	139	0	0	215	116
N.S.	1	0.98	0.74	0.97	0.83	1.16	0.00	0.00	1.79	0.97
time (sec)	N/A	0.806	0.125	0.263	0.032	0.079	0.000	0.000	0.181	22.596

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	67	77	70	56	60	114	60
N.S.	1	1.00	1.00	1.03	1.18	1.08	0.86	0.92	1.75	0.92
time (sec)	N/A	0.384	0.058	0.181	0.103	0.069	0.106	0.107	0.194	22.697

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	58	67	62	44	51	104	51
N.S.	1	1.00	1.00	1.09	1.26	1.17	0.83	0.96	1.96	0.96
time (sec)	N/A	0.369	0.043	0.168	0.109	0.068	0.109	0.115	0.164	0.064

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	50	60	53	36	43	96	43
N.S.	1	1.00	1.00	1.11	1.33	1.18	0.80	0.96	2.13	0.96
time (sec)	N/A	0.336	0.033	0.143	0.109	0.098	0.091	0.136	0.171	0.066

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	42	33	44	43	22	25	86	32
N.S.	1	1.00	1.35	1.06	1.42	1.39	0.71	0.81	2.77	1.03
time (sec)	N/A	0.302	0.032	0.109	0.106	0.072	0.096	0.129	0.172	22.721

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	22	18	10	13	37	14
N.S.	1	1.00	1.00	0.94	1.38	1.12	0.62	0.81	2.31	0.88
time (sec)	N/A	0.301	0.015	0.151	0.106	0.068	0.105	0.109	0.174	22.784

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	44	53	60	44	35	106	37
N.S.	1	1.00	1.00	1.16	1.39	1.58	1.16	0.92	2.79	0.97
time (sec)	N/A	0.335	0.033	0.175	0.106	0.078	0.156	0.112	0.179	0.100

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	53	69	77	58	46	122	43
N.S.	1	1.00	1.00	1.02	1.33	1.48	1.12	0.88	2.35	0.83
time (sec)	N/A	0.350	0.053	0.180	0.106	0.076	0.170	0.132	0.196	22.504

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	61	77	86	70	54	132	55
N.S.	1	1.00	1.00	0.98	1.24	1.39	1.13	0.87	2.13	0.89
time (sec)	N/A	0.369	0.053	0.178	0.108	0.073	0.201	0.136	0.155	0.142

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	125	56	77	76	59	0	0	24	85
N.S.	1	1.19	0.53	0.73	0.72	0.56	0.00	0.00	0.23	0.81
time (sec)	N/A	0.460	0.061	0.214	0.108	0.093	0.000	0.000	0.194	0.068

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	92	46	67	59	51	0	0	24	71
N.S.	1	1.15	0.58	0.84	0.74	0.64	0.00	0.00	0.30	0.89
time (sec)	N/A	0.395	0.041	0.143	0.106	0.079	0.000	0.000	0.159	22.713

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	61	38	59	42	43	0	53	22	51
N.S.	1	1.13	0.70	1.09	0.78	0.80	0.00	0.98	0.41	0.94
time (sec)	N/A	0.334	0.226	0.123	0.107	0.089	0.000	0.115	0.190	22.886

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	48	25	37	0	41	21	32
N.S.	1	1.00	0.90	1.66	0.86	1.28	0.00	1.41	0.72	1.10
time (sec)	N/A	0.278	0.118	0.090	0.103	0.081	0.000	0.133	0.185	0.047

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	29	121	26	58	0	68	23	32
N.S.	1	1.00	1.16	4.84	1.04	2.32	0.00	2.72	0.92	1.28
time (sec)	N/A	0.336	0.077	0.132	0.110	0.084	0.000	0.141	0.177	0.039

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	47	34	0	66	0	0	25	33
N.S.	1	1.00	1.24	0.89	0.00	1.74	0.00	0.00	0.66	0.87
time (sec)	N/A	0.332	0.037	0.146	0.000	0.082	0.000	0.000	0.173	22.797

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	64	57	60	0	83	0	0	25	52
N.S.	1	1.02	0.90	0.95	0.00	1.32	0.00	0.00	0.40	0.83
time (sec)	N/A	0.382	0.057	0.154	0.000	0.083	0.000	0.000	0.180	23.018

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	92	70	68	0	92	0	0	25	74
N.S.	1	1.02	0.78	0.76	0.00	1.02	0.00	0.00	0.28	0.82
time (sec)	N/A	0.432	0.061	0.168	0.000	0.087	0.000	0.000	0.171	0.042

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	118	76	77	0	101	0	0	25	95
N.S.	1	1.04	0.67	0.68	0.00	0.89	0.00	0.00	0.22	0.84
time (sec)	N/A	0.484	0.076	0.188	0.000	0.089	0.000	0.000	0.183	0.034

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	48	44	46	41	68	52	43
N.S.	1	1.00	1.00	0.98	0.90	0.94	0.84	1.39	1.06	0.88
time (sec)	N/A	0.348	0.028	0.136	0.029	0.069	0.062	0.129	0.172	0.060

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	40	35	37	31	58	52	36
N.S.	1	1.00	1.00	1.00	0.88	0.92	0.78	1.45	1.30	0.90
time (sec)	N/A	0.336	0.019	0.135	0.033	0.070	0.067	0.133	0.174	23.005

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	31	28	29	22	52	50	27
N.S.	1	1.00	1.00	1.03	0.93	0.97	0.73	1.73	1.67	0.90
time (sec)	N/A	0.320	0.016	0.110	0.025	0.072	0.054	0.113	0.163	22.887

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	30	19	16	21	14	65	48	19
N.S.	1	1.00	1.50	0.95	0.80	1.05	0.70	3.25	2.40	0.95
time (sec)	N/A	0.287	0.016	0.077	0.025	0.095	0.060	0.106	0.157	22.882

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	14	12	15	17	44	50	14
N.S.	1	1.00	1.00	1.00	0.86	1.07	1.21	3.14	3.57	1.00
time (sec)	N/A	0.304	0.011	0.125	0.026	0.078	0.072	0.137	0.190	0.092

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	25	34	26	32	34	50	17
N.S.	1	1.00	1.00	0.93	1.26	0.96	1.19	1.26	1.85	0.63
time (sec)	N/A	0.319	0.016	0.140	0.031	0.104	0.085	0.108	0.173	23.130

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	50	39	42	54	54	26
N.S.	1	1.00	1.00	0.92	1.35	1.05	1.14	1.46	1.46	0.70
time (sec)	N/A	0.328	0.017	0.145	0.025	0.076	0.103	0.115	0.195	23.015

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	57	47	54	67	56	33
N.S.	1	1.00	1.00	0.90	1.16	0.96	1.10	1.37	1.14	0.67
time (sec)	N/A	0.347	0.020	0.159	0.031	0.078	0.116	0.149	0.193	23.009

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	154	80	124	216	88	0	0	90	138
N.S.	1	1.24	0.65	1.00	1.74	0.71	0.00	0.00	0.73	1.11
time (sec)	N/A	1.364	0.071	0.249	0.113	0.082	0.000	0.000	0.249	22.868

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	125	63	111	181	80	0	0	90	115
N.S.	1	1.21	0.61	1.08	1.76	0.78	0.00	0.00	0.87	1.12
time (sec)	N/A	1.211	0.065	0.248	0.110	0.081	0.000	0.000	0.219	22.775

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	89	60	106	112	72	0	0	88	105
N.S.	1	1.07	0.72	1.28	1.35	0.87	0.00	0.00	1.06	1.27
time (sec)	N/A	0.947	0.055	0.227	0.107	0.112	0.000	0.000	0.210	0.071

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	60	42	93	65	60	0	0	87	73
N.S.	1	1.03	0.72	1.60	1.12	1.03	0.00	0.00	1.50	1.26
time (sec)	N/A	0.384	0.044	0.197	0.106	0.077	0.000	0.000	0.224	22.707

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	50	55	649	0	100	0	0	334	74
N.S.	1	1.04	1.15	13.52	0.00	2.08	0.00	0.00	6.96	1.54
time (sec)	N/A	0.771	0.058	0.191	0.000	0.088	0.000	0.000	0.312	22.876

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	61	82	0	109	0	0	90	76
N.S.	1	1.00	0.95	1.28	0.00	1.70	0.00	0.00	1.41	1.19
time (sec)	N/A	0.718	0.105	0.262	0.000	0.081	0.000	0.000	0.233	22.967

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	93	79	108	0	130	0	0	622	100
N.S.	1	1.04	0.89	1.21	0.00	1.46	0.00	0.00	6.99	1.12
time (sec)	N/A	0.779	0.103	0.294	0.000	0.078	0.000	0.000	0.531	22.641

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	89	116	0	139	0	0	96	117
N.S.	1	1.00	0.75	0.98	0.00	1.18	0.00	0.00	0.81	0.99
time (sec)	N/A	0.815	0.107	0.342	0.000	0.080	0.000	0.000	0.330	22.816

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	139	95	125	0	146	0	0	799	139
N.S.	1	1.03	0.70	0.93	0.00	1.08	0.00	0.00	5.92	1.03
time (sec)	N/A	0.885	0.103	0.339	0.000	0.095	0.000	0.000	0.631	23.154

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	317	82	0	0	244	0	0	27	0
N.S.	1	1.18	0.31	0.00	0.00	0.91	0.00	0.00	0.10	0.00
time (sec)	N/A	0.752	0.052	0.000	0.000	0.092	0.000	0.000	200.037	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	226	280	63	0	0	236	0	0	25	0
N.S.	1	1.24	0.28	0.00	0.00	1.04	0.00	0.00	0.11	0.00
time (sec)	N/A	0.697	0.028	0.000	0.000	0.088	0.000	0.000	200.040	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	195	239	41	0	0	209	0	0	23	0
N.S.	1	1.23	0.21	0.00	0.00	1.07	0.00	0.00	0.12	0.00
time (sec)	N/A	0.622	0.042	0.000	0.000	0.099	0.000	0.000	200.032	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	203	266	97	0	0	243	0	0	32	0
N.S.	1	1.31	0.48	0.00	0.00	1.20	0.00	0.00	0.16	0.00
time (sec)	N/A	0.708	0.049	0.000	0.000	0.085	0.000	0.000	0.235	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	96	71	0	0	151	0	0	27	0
N.S.	1	1.04	0.77	0.00	0.00	1.64	0.00	0.00	0.29	0.00
time (sec)	N/A	0.368	0.021	0.000	0.000	0.083	0.000	0.000	200.033	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	135	81	0	0	175	0	0	27	0
N.S.	1	1.02	0.61	0.00	0.00	1.33	0.00	0.00	0.20	0.00
time (sec)	N/A	0.399	0.026	0.000	0.000	0.078	0.000	0.000	200.028	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	170	172	93	0	0	184	0	0	27	0
N.S.	1	1.01	0.55	0.00	0.00	1.08	0.00	0.00	0.16	0.00
time (sec)	N/A	0.468	0.034	0.000	0.000	0.089	0.000	0.000	200.044	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	202	209	99	0	0	192	0	0	172	0
N.S.	1	1.03	0.49	0.00	0.00	0.95	0.00	0.00	0.85	0.00
time (sec)	N/A	0.510	0.038	0.000	0.000	0.087	0.000	0.000	0.316	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	240	248	111	0	0	200	0	0	227	0
N.S.	1	1.03	0.46	0.00	0.00	0.83	0.00	0.00	0.95	0.00
time (sec)	N/A	0.569	0.047	0.000	0.000	0.090	0.000	0.000	0.335	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	291	330	148	0	0	254	0	0	27	0
N.S.	1	1.13	0.51	0.00	0.00	0.87	0.00	0.00	0.09	0.00
time (sec)	N/A	0.785	0.153	0.000	0.000	0.096	0.000	0.000	200.032	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	317	82	0	0	247	0	0	27	0
N.S.	1	1.18	0.31	0.00	0.00	0.92	0.00	0.00	0.10	0.00
time (sec)	N/A	0.752	0.071	0.000	0.000	0.088	0.000	0.000	200.040	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	226	280	61	0	0	239	0	0	25	0
N.S.	1	1.24	0.27	0.00	0.00	1.06	0.00	0.00	0.11	0.00
time (sec)	N/A	0.700	0.025	0.000	0.000	0.090	0.000	0.000	200.030	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	195	239	41	0	0	215	0	0	23	0
N.S.	1	1.23	0.21	0.00	0.00	1.10	0.00	0.00	0.12	0.00
time (sec)	N/A	0.614	0.054	0.000	0.000	0.083	0.000	0.000	200.034	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	204	266	96	0	0	243	0	0	27	0
N.S.	1	1.30	0.47	0.00	0.00	1.19	0.00	0.00	0.13	0.00
time (sec)	N/A	0.730	0.043	0.000	0.000	0.082	0.000	0.000	200.027	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	96	68	0	0	157	0	0	63	0
N.S.	1	1.04	0.74	0.00	0.00	1.71	0.00	0.00	0.68	0.00
time (sec)	N/A	0.382	0.021	0.000	0.000	0.161	0.000	0.000	0.397	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	135	81	0	0	179	0	0	89	0
N.S.	1	1.02	0.61	0.00	0.00	1.36	0.00	0.00	0.67	0.00
time (sec)	N/A	0.407	0.062	0.000	0.000	0.085	0.000	0.000	0.521	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	170	172	93	0	0	187	0	0	116	0
N.S.	1	1.01	0.55	0.00	0.00	1.10	0.00	0.00	0.68	0.00
time (sec)	N/A	0.480	0.043	0.000	0.000	0.081	0.000	0.000	0.503	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	202	207	99	0	0	195	0	0	145	0
N.S.	1	1.02	0.49	0.00	0.00	0.97	0.00	0.00	0.72	0.00
time (sec)	N/A	0.533	0.044	0.000	0.000	0.081	0.000	0.000	0.568	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	325	372	96	0	0	251	0	0	27	0
N.S.	1	1.14	0.30	0.00	0.00	0.77	0.00	0.00	0.08	0.00
time (sec)	N/A	0.867	0.059	0.000	0.000	0.089	0.000	0.000	200.030	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	300	354	86	0	0	244	0	0	27	0
N.S.	1	1.18	0.29	0.00	0.00	0.81	0.00	0.00	0.09	0.00
time (sec)	N/A	0.811	0.046	0.000	0.000	0.165	0.000	0.000	200.026	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	255	314	72	0	0	236	0	0	25	0
N.S.	1	1.23	0.28	0.00	0.00	0.93	0.00	0.00	0.10	0.00
time (sec)	N/A	0.745	0.049	0.000	0.000	0.133	0.000	0.000	200.028	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	226	273	41	0	0	209	0	0	23	0
N.S.	1	1.21	0.18	0.00	0.00	0.92	0.00	0.00	0.10	0.00
time (sec)	N/A	0.671	0.062	0.000	0.000	0.137	0.000	0.000	200.035	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	230	292	112	0	0	267	0	0	27	0
N.S.	1	1.27	0.49	0.00	0.00	1.16	0.00	0.00	0.12	0.00
time (sec)	N/A	0.778	0.053	0.000	0.000	0.141	0.000	0.000	200.034	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	127	87	0	0	152	0	0	154	0
N.S.	1	1.05	0.72	0.00	0.00	1.26	0.00	0.00	1.27	0.00
time (sec)	N/A	0.400	0.030	0.000	0.000	0.134	0.000	0.000	0.391	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	166	99	0	0	176	0	0	182	0
N.S.	1	1.02	0.61	0.00	0.00	1.08	0.00	0.00	1.12	0.00
time (sec)	N/A	0.434	0.033	0.000	0.000	0.119	0.000	0.000	0.523	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	203	205	106	0	0	184	0	0	211	0
N.S.	1	1.01	0.52	0.00	0.00	0.91	0.00	0.00	1.04	0.00
time (sec)	N/A	0.524	0.042	0.000	0.000	0.114	0.000	0.000	0.522	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	233	240	118	0	0	192	0	0	241	0
N.S.	1	1.03	0.51	0.00	0.00	0.82	0.00	0.00	1.03	0.00
time (sec)	N/A	0.578	0.051	0.000	0.000	0.112	0.000	0.000	0.613	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	291	330	127	0	0	255	0	0	27	0
N.S.	1	1.13	0.44	0.00	0.00	0.88	0.00	0.00	0.09	0.00
time (sec)	N/A	0.779	0.140	0.000	0.000	0.134	0.000	0.000	200.024	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	317	73	0	0	247	0	0	27	0
N.S.	1	1.18	0.27	0.00	0.00	0.92	0.00	0.00	0.10	0.00
time (sec)	N/A	0.753	0.043	0.000	0.000	0.156	0.000	0.000	200.021	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	226	280	63	0	0	238	0	0	25	0
N.S.	1	1.24	0.28	0.00	0.00	1.05	0.00	0.00	0.11	0.00
time (sec)	N/A	0.685	0.022	0.000	0.000	0.135	0.000	0.000	200.032	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	195	239	41	0	0	213	0	0	23	0
N.S.	1	1.23	0.21	0.00	0.00	1.09	0.00	0.00	0.12	0.00
time (sec)	N/A	0.599	0.044	0.000	0.000	0.129	0.000	0.000	200.025	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	203	266	96	0	0	243	0	0	27	0
N.S.	1	1.31	0.47	0.00	0.00	1.20	0.00	0.00	0.13	0.00
time (sec)	N/A	0.733	0.035	0.000	0.000	0.126	0.000	0.000	200.034	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	96	69	0	0	156	0	0	63	0
N.S.	1	1.04	0.75	0.00	0.00	1.70	0.00	0.00	0.68	0.00
time (sec)	N/A	0.386	0.022	0.000	0.000	0.146	0.000	0.000	0.307	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	135	81	0	0	178	0	0	89	0
N.S.	1	1.02	0.61	0.00	0.00	1.35	0.00	0.00	0.67	0.00
time (sec)	N/A	0.403	0.026	0.000	0.000	0.149	0.000	0.000	0.347	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	170	172	92	0	0	187	0	0	116	0
N.S.	1	1.01	0.54	0.00	0.00	1.10	0.00	0.00	0.68	0.00
time (sec)	N/A	0.487	0.032	0.000	0.000	0.122	0.000	0.000	0.354	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	202	209	99	0	0	195	0	0	145	0
N.S.	1	1.03	0.49	0.00	0.00	0.97	0.00	0.00	0.72	0.00
time (sec)	N/A	0.526	0.043	0.000	0.000	0.146	0.000	0.000	0.375	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	291	330	127	0	0	251	0	0	27	0
N.S.	1	1.13	0.44	0.00	0.00	0.86	0.00	0.00	0.09	0.00
time (sec)	N/A	0.813	0.143	0.000	0.000	0.087	0.000	0.000	200.027	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	317	73	0	0	243	0	0	27	0
N.S.	1	1.18	0.27	0.00	0.00	0.91	0.00	0.00	0.10	0.00
time (sec)	N/A	0.770	0.040	0.000	0.000	0.115	0.000	0.000	200.033	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	226	280	63	0	0	236	0	0	25	0
N.S.	1	1.24	0.28	0.00	0.00	1.04	0.00	0.00	0.11	0.00
time (sec)	N/A	0.693	0.023	0.000	0.000	0.086	0.000	0.000	200.026	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	195	239	39	0	0	209	0	0	23	0
N.S.	1	1.23	0.20	0.00	0.00	1.07	0.00	0.00	0.12	0.00
time (sec)	N/A	0.636	0.057	0.000	0.000	0.110	0.000	0.000	200.027	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	204	266	97	0	0	243	0	0	27	0
N.S.	1	1.30	0.48	0.00	0.00	1.19	0.00	0.00	0.13	0.00
time (sec)	N/A	0.718	0.032	0.000	0.000	0.088	0.000	0.000	200.028	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	96	69	0	0	152	0	0	27	0
N.S.	1	1.04	0.75	0.00	0.00	1.65	0.00	0.00	0.29	0.00
time (sec)	N/A	0.371	0.020	0.000	0.000	0.083	0.000	0.000	200.025	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	135	81	0	0	176	0	0	101	0
N.S.	1	1.02	0.61	0.00	0.00	1.33	0.00	0.00	0.77	0.00
time (sec)	N/A	0.398	0.025	0.000	0.000	0.085	0.000	0.000	1.618	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	170	172	93	0	0	184	0	0	113	0
N.S.	1	1.01	0.55	0.00	0.00	1.08	0.00	0.00	0.66	0.00
time (sec)	N/A	0.472	0.031	0.000	0.000	0.130	0.000	0.000	2.019	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	202	207	99	0	0	192	0	0	101	0
N.S.	1	1.02	0.49	0.00	0.00	0.95	0.00	0.00	0.50	0.00
time (sec)	N/A	0.522	0.048	0.000	0.000	0.127	0.000	0.000	2.364	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	325	372	100	0	0	304	0	0	118	0
N.S.	1	1.14	0.31	0.00	0.00	0.94	0.00	0.00	0.36	0.00
time (sec)	N/A	0.878	0.062	0.000	0.000	0.124	0.000	0.000	0.782	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	300	354	91	0	0	296	0	0	118	0
N.S.	1	1.18	0.30	0.00	0.00	0.99	0.00	0.00	0.39	0.00
time (sec)	N/A	0.810	0.044	0.000	0.000	0.138	0.000	0.000	0.995	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	255	314	63	0	0	289	0	0	116	0
N.S.	1	1.23	0.25	0.00	0.00	1.13	0.00	0.00	0.45	0.00
time (sec)	N/A	0.729	0.037	0.000	0.000	0.122	0.000	0.000	0.746	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	226	273	39	0	0	261	0	0	115	0
N.S.	1	1.21	0.17	0.00	0.00	1.15	0.00	0.00	0.51	0.00
time (sec)	N/A	0.673	0.070	0.000	0.000	0.121	0.000	0.000	0.714	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	230	292	106	0	0	329	0	0	116	0
N.S.	1	1.27	0.46	0.00	0.00	1.43	0.00	0.00	0.50	0.00
time (sec)	N/A	0.796	0.073	0.000	0.000	0.119	0.000	0.000	0.790	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	127	69	0	0	212	0	0	117	0
N.S.	1	1.05	0.57	0.00	0.00	1.75	0.00	0.00	0.97	0.00
time (sec)	N/A	0.416	0.024	0.000	0.000	0.129	0.000	0.000	0.574	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	166	81	0	0	238	0	0	120	0
N.S.	1	1.02	0.50	0.00	0.00	1.46	0.00	0.00	0.74	0.00
time (sec)	N/A	0.439	0.030	0.000	0.000	0.112	0.000	0.000	1.213	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	203	205	93	0	0	246	0	0	122	0
N.S.	1	1.01	0.46	0.00	0.00	1.21	0.00	0.00	0.60	0.00
time (sec)	N/A	0.543	0.036	0.000	0.000	0.156	0.000	0.000	15.842	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	233	240	99	0	0	254	0	0	122	0
N.S.	1	1.03	0.42	0.00	0.00	1.09	0.00	0.00	0.52	0.00
time (sec)	N/A	0.583	0.045	0.000	0.000	0.129	0.000	0.000	0.705	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	255	302	73	0	0	208	0	0	20	0
N.S.	1	1.18	0.29	0.00	0.00	0.82	0.00	0.00	0.08	0.00
time (sec)	N/A	0.713	0.052	0.000	0.000	0.081	0.000	0.000	0.219	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	216	270	57	0	0	195	0	0	18	0
N.S.	1	1.25	0.26	0.00	0.00	0.90	0.00	0.00	0.08	0.00
time (sec)	N/A	0.658	0.028	0.000	0.000	0.080	0.000	0.000	0.216	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	196	237	34	0	0	195	0	0	17	0
N.S.	1	1.21	0.17	0.00	0.00	0.99	0.00	0.00	0.09	0.00
time (sec)	N/A	0.587	0.027	0.000	0.000	0.090	0.000	0.000	0.180	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	311	437	90	0	0	339	0	0	20	0
N.S.	1	1.41	0.29	0.00	0.00	1.09	0.00	0.00	0.06	0.00
time (sec)	N/A	0.835	0.037	0.000	0.000	0.090	0.000	0.000	0.248	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	199	258	64	0	0	211	0	0	20	0
N.S.	1	1.30	0.32	0.00	0.00	1.06	0.00	0.00	0.10	0.00
time (sec)	N/A	0.538	0.018	0.000	0.000	0.118	0.000	0.000	0.292	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	228	294	72	0	0	234	0	0	20	0
N.S.	1	1.29	0.32	0.00	0.00	1.03	0.00	0.00	0.09	0.00
time (sec)	N/A	0.578	0.022	0.000	0.000	0.142	0.000	0.000	0.307	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	265	328	81	0	0	243	0	0	20	0
N.S.	1	1.24	0.31	0.00	0.00	0.92	0.00	0.00	0.08	0.00
time (sec)	N/A	0.635	0.026	0.000	0.000	0.122	0.000	0.000	0.384	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	177	186	73	0	0	117	0	0	20	0
N.S.	1	1.05	0.41	0.00	0.00	0.66	0.00	0.00	0.11	0.00
time (sec)	N/A	0.426	0.044	0.000	0.000	0.130	0.000	0.000	0.152	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	156	54	0	0	116	0	0	18	0
N.S.	1	1.11	0.39	0.00	0.00	0.83	0.00	0.00	0.13	0.00
time (sec)	N/A	0.375	0.028	0.000	0.000	0.130	0.000	0.000	0.164	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	123	34	0	0	104	0	0	17	0
N.S.	1	1.06	0.29	0.00	0.00	0.90	0.00	0.00	0.15	0.00
time (sec)	N/A	0.325	0.032	0.000	0.000	0.075	0.000	0.000	0.167	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	175	90	0	0	145	0	0	20	0
N.S.	1	1.07	0.55	0.00	0.00	0.89	0.00	0.00	0.12	0.00
time (sec)	N/A	0.398	0.038	0.000	0.000	0.119	0.000	0.000	0.157	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	113	59	0	0	120	0	0	20	0
N.S.	1	1.02	0.53	0.00	0.00	1.08	0.00	0.00	0.18	0.00
time (sec)	N/A	0.361	0.018	0.000	0.000	0.129	0.000	0.000	0.185	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	149	69	0	0	138	0	0	20	0
N.S.	1	1.05	0.49	0.00	0.00	0.97	0.00	0.00	0.14	0.00
time (sec)	N/A	0.374	0.023	0.000	0.000	0.117	0.000	0.000	0.178	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	571	734	83	0	0	435	0	0	25	0
N.S.	1	1.29	0.15	0.00	0.00	0.76	0.00	0.00	0.04	0.00
time (sec)	N/A	1.399	0.055	0.000	0.000	0.152	0.000	0.000	0.191	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	523	697	63	0	0	428	0	0	31	0
N.S.	1	1.33	0.12	0.00	0.00	0.82	0.00	0.00	0.06	0.00
time (sec)	N/A	1.266	0.028	0.000	0.000	0.144	0.000	0.000	0.166	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	504	656	41	0	0	383	0	0	22	0
N.S.	1	1.30	0.08	0.00	0.00	0.76	0.00	0.00	0.04	0.00
time (sec)	N/A	1.202	0.035	0.000	0.000	0.106	0.000	0.000	0.168	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	630	918	97	0	0	509	0	0	25	0
N.S.	1	1.46	0.15	0.00	0.00	0.81	0.00	0.00	0.04	0.00
time (sec)	N/A	1.780	0.047	0.000	0.000	0.084	0.000	0.000	0.217	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	259	331	71	0	0	345	0	0	27	0
N.S.	1	1.28	0.27	0.00	0.00	1.33	0.00	0.00	0.10	0.00
time (sec)	N/A	0.674	0.022	0.000	0.000	0.093	0.000	0.000	200.026	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	295	370	84	0	0	381	0	0	27	0
N.S.	1	1.25	0.28	0.00	0.00	1.29	0.00	0.00	0.09	0.00
time (sec)	N/A	0.717	0.027	0.000	0.000	0.088	0.000	0.000	200.022	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	114	94	748	0	0	0	0	0	0
N.S.	1	0.92	0.76	6.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.456	0.052	0.679	0.000	0.000	0.000	0.000	0.179	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	58	417	0	0	0	0	0	0
N.S.	1	1.00	1.16	8.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.350	0.030	0.331	0.000	0.000	0.000	0.000	0.179	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	175	0	0	126	0	122	0
N.S.	1	1.00	0.74	4.49	0.00	0.00	3.23	0.00	3.13	0.00
time (sec)	N/A	0.303	0.011	0.177	0.000	0.000	1.660	0.000	0.169	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	158	0	0	133	0	55	0
N.S.	1	1.00	0.74	4.05	0.00	0.00	3.41	0.00	1.41	0.00
time (sec)	N/A	0.306	0.013	0.193	0.000	0.000	2.364	0.000	0.178	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	58	428	0	0	0	0	411	0
N.S.	1	1.00	1.16	8.56	0.00	0.00	0.00	0.00	8.22	0.00
time (sec)	N/A	0.350	0.026	0.474	0.000	0.000	0.000	0.000	0.204	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	115	94	1196	0	0	0	0	838	0
N.S.	1	0.93	0.76	9.65	0.00	0.00	0.00	0.00	6.76	0.00
time (sec)	N/A	0.458	0.047	1.326	0.000	0.000	0.000	0.000	0.199	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	161	113	146	0	0	0	0	167	0
N.S.	1	1.36	0.96	1.24	0.00	0.00	0.00	0.00	1.42	0.00
time (sec)	N/A	0.858	0.096	0.111	0.000	0.000	0.000	0.000	0.227	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	85	71	0	0	94	0	41	0
N.S.	1	1.00	1.08	0.90	0.00	0.00	1.19	0.00	0.52	0.00
time (sec)	N/A	0.376	0.055	0.091	0.000	0.000	1.504	0.000	0.171	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	85	0	0	0	0	0	24	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.361	0.054	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	161	113	0	0	0	0	0	93	0
N.S.	1	1.36	0.96	0.00	0.00	0.00	0.00	0.00	0.79	0.00
time (sec)	N/A	0.864	0.094	0.000	0.000	0.000	0.000	0.000	0.310	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0	1098	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	30.50	0.00
time (sec)	N/A	0.321	0.000	0.000	0.000	0.000	0.000	0.000	1.454	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0	128	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	3.56	0.00
time (sec)	N/A	0.311	0.000	0.000	0.000	0.000	0.000	0.000	0.753	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0	24	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.67	0.00
time (sec)	N/A	0.311	0.000	0.000	0.000	0.000	0.000	0.000	0.375	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0	127	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	3.53	0.00
time (sec)	N/A	0.316	0.000	0.000	0.000	0.000	0.000	0.000	0.549	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0	27	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.75	0.00
time (sec)	N/A	0.312	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	0	0	0	0	0	0	12	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.319	0.000	0.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	0	0	0	0	0	0	12	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.310	0.000	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	0	0	0	0	0	0	25	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.308	0.000	0.000	0.000	0.000	0.000	0.000	0.266	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	0	0	0	0	0	0	15	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.320	0.000	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	207	177	210	0	0	0	0	0	15	0
N.S.	1	0.86	1.01	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.511	0.246	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	165	116	0	0	0	0	0	15	0
N.S.	1	1.04	0.73	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.470	0.070	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	105	0	0	0	0	0	13	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.371	0.037	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	53	0	0	0	0	0	11	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.305	0.048	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	106	0	0	0	0	0	15	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.408	0.032	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	82	0	0	0	0	0	44	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.342	0.022	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	114	0	0	0	0	0	83	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.397	0.042	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	174	119	0	0	0	0	0	138	0
N.S.	1	1.02	0.70	0.00	0.00	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	0.463	0.077	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	284	217	197	749	177	1222	206	27	0
N.S.	1	0.90	0.69	0.63	2.38	0.56	3.88	0.65	0.09	0.00
time (sec)	N/A	0.780	1.112	0.852	0.043	0.105	1.865	0.149	200.017	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	208	176	150	529	139	857	156	361	0
N.S.	1	0.87	0.74	0.63	2.22	0.58	3.60	0.66	1.52	0.00
time (sec)	N/A	0.608	0.501	0.510	0.041	0.116	1.581	0.145	2.197	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	180	135	113	351	106	585	114	228	0
N.S.	1	1.05	0.79	0.66	2.05	0.62	3.42	0.67	1.33	0.00
time (sec)	N/A	0.535	0.197	0.445	0.040	0.119	1.348	0.128	0.169	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	121	108	87	209	79	362	76	124	0
N.S.	1	1.10	0.98	0.79	1.90	0.72	3.29	0.69	1.13	0.00
time (sec)	N/A	0.445	0.161	0.310	0.034	0.116	0.915	0.129	0.169	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	65	28	69	62	60	36	52	49	97
N.S.	1	1.25	0.54	1.33	1.19	1.15	0.69	1.00	0.94	1.87
time (sec)	N/A	0.354	0.025	0.225	0.028	0.108	0.801	0.132	0.161	23.840

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	111	142	107	233	144	0	113	152	118
N.S.	1	1.25	1.60	1.20	2.62	1.62	0.00	1.27	1.71	1.33
time (sec)	N/A	0.478	0.114	0.221	0.034	0.091	0.000	0.180	0.150	23.887

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	120	93	239	224	0	145	198	218
N.S.	1	1.00	0.92	0.72	1.84	1.72	0.00	1.12	1.52	1.68
time (sec)	N/A	0.420	0.096	0.661	0.037	0.089	0.000	0.162	0.187	24.478

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	198	154	187	424	452	0	471	510	0
N.S.	1	0.99	0.77	0.93	2.11	2.25	0.00	2.34	2.54	0.00
time (sec)	N/A	0.509	0.188	0.355	0.038	0.083	0.000	0.164	0.166	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	294	234	281	644	690	0	884	852	0
N.S.	1	1.04	0.83	0.99	2.28	2.44	0.00	3.12	3.01	0.00
time (sec)	N/A	0.621	0.480	0.822	0.037	0.093	0.000	0.171	0.387	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	160	149	105	110	123	261	201
N.S.	1	1.00	1.00	1.74	1.62	1.14	1.20	1.34	2.84	2.18
time (sec)	N/A	0.464	0.193	0.272	0.104	0.072	0.250	0.115	0.148	0.207

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	109	116	77	75	83	189	153
N.S.	1	1.00	1.00	1.51	1.61	1.07	1.04	1.15	2.62	2.12
time (sec)	N/A	0.416	0.075	0.245	0.110	0.071	0.184	0.132	0.163	23.131

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	70	87	53	46	53	128	107
N.S.	1	1.00	1.00	1.30	1.61	0.98	0.85	0.98	2.37	1.98
time (sec)	N/A	0.384	0.045	0.194	0.104	0.067	0.153	0.106	0.172	23.266

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	41	64	35	29	35	79	60
N.S.	1	1.00	1.00	1.11	1.73	0.95	0.78	0.95	2.14	1.62
time (sec)	N/A	0.341	0.029	0.180	0.107	0.070	0.118	0.143	0.155	23.440

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	32	21	46	22	14	16	37	21
N.S.	1	1.00	1.60	1.05	2.30	1.10	0.70	0.80	1.85	1.05
time (sec)	N/A	0.293	0.013	0.121	0.107	0.068	0.076	0.119	0.165	23.323

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	31	47	78	27	100	33	84	32
N.S.	1	1.00	0.82	1.24	2.05	0.71	2.63	0.87	2.21	0.84
time (sec)	N/A	0.351	0.028	0.229	0.108	0.080	0.414	0.136	0.153	23.517

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	39	96	126	40	156	61	161	98
N.S.	1	1.00	0.71	1.75	2.29	0.73	2.84	1.11	2.93	1.78
time (sec)	N/A	0.377	0.036	0.231	0.107	0.128	0.315	0.145	0.149	23.381

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	63	201	188	69	228	89	303	154
N.S.	1	1.00	0.83	2.64	2.47	0.91	3.00	1.17	3.99	2.03
time (sec)	N/A	0.407	0.045	0.271	0.108	0.133	0.444	0.111	0.173	23.482

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	88	267	263	94	286	126	464	199
N.S.	1	1.00	0.95	2.87	2.83	1.01	3.08	1.35	4.99	2.14
time (sec)	N/A	0.422	0.059	0.279	0.116	0.107	0.556	0.130	0.158	23.464

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	342	249	436	3081	264	0	334	29	0
N.S.	1	0.95	0.69	1.21	8.58	0.74	0.00	0.93	0.08	0.00
time (sec)	N/A	0.877	0.681	2.319	0.084	0.147	0.000	0.157	200.015	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	264	201	342	2295	216	0	285	29	0
N.S.	1	0.94	0.71	1.21	8.14	0.77	0.00	1.01	0.10	0.00
time (sec)	N/A	0.724	0.356	1.414	0.062	0.125	0.000	0.142	200.013	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	229	160	259	1608	174	0	243	770	0
N.S.	1	1.01	0.70	1.14	7.08	0.77	0.00	1.07	3.39	0.00
time (sec)	N/A	0.632	0.294	1.323	0.052	0.139	0.000	0.162	0.181	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	171	132	186	1108	136	0	209	555	0
N.S.	1	1.05	0.81	1.14	6.80	0.83	0.00	1.28	3.40	0.00
time (sec)	N/A	0.503	0.215	0.825	0.042	0.119	0.000	0.146	0.161	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	109	45	120	736	99	0	180	298	0
N.S.	1	1.16	0.48	1.28	7.83	1.05	0.00	1.91	3.17	0.00
time (sec)	N/A	0.406	0.049	0.782	0.040	0.115	0.000	0.161	0.152	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	162	196	485	733	356	0	252	1336	0
N.S.	1	1.21	1.46	3.62	5.47	2.66	0.00	1.88	9.97	0.00
time (sec)	N/A	0.573	1.230	0.516	0.042	0.130	0.000	0.248	0.184	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	185	145	174	992	389	0	0	1546	0
N.S.	1	1.05	0.82	0.99	5.64	2.21	0.00	0.00	8.78	0.00
time (sec)	N/A	0.486	0.308	1.803	0.042	0.141	0.000	0.000	0.320	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	253	194	269	1536	574	0	0	29	0
N.S.	1	0.96	0.73	1.02	5.82	2.17	0.00	0.00	0.11	0.00
time (sec)	N/A	0.570	0.377	1.234	0.051	0.161	0.000	0.000	200.024	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	356	282	379	2313	839	0	0	2831	0
N.S.	1	1.05	0.83	1.12	6.84	2.48	0.00	0.00	8.38	0.00
time (sec)	N/A	0.801	0.658	1.477	0.057	0.126	0.000	0.000	5.747	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	284	248	197	456	177	0	206	35	0
N.S.	1	0.90	0.79	0.63	1.45	0.56	0.00	0.65	0.11	0.00
time (sec)	N/A	0.768	1.379	0.964	0.142	0.106	0.000	0.133	0.148	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	208	202	150	308	139	0	156	35	0
N.S.	1	0.89	0.87	0.64	1.32	0.60	0.00	0.67	0.15	0.00
time (sec)	N/A	0.616	0.692	0.626	0.126	0.141	0.000	0.124	0.165	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	180	162	113	161	106	0	114	35	0
N.S.	1	1.05	0.95	0.66	0.94	0.62	0.00	0.67	0.20	0.00
time (sec)	N/A	0.569	0.376	0.478	0.114	0.133	0.000	0.129	0.163	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	121	131	87	97	79	0	76	33	0
N.S.	1	1.10	1.19	0.79	0.88	0.72	0.00	0.69	0.30	0.00
time (sec)	N/A	0.442	0.171	0.386	0.110	0.109	0.000	0.123	0.151	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	65	28	69	35	60	0	52	32	0
N.S.	1	1.25	0.54	1.33	0.67	1.15	0.00	1.00	0.62	0.00
time (sec)	N/A	0.358	0.023	0.306	0.103	0.149	0.000	0.122	0.173	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	111	142	260	0	144	0	113	35	0
N.S.	1	1.25	1.60	2.92	0.00	1.62	0.00	1.27	0.39	0.00
time (sec)	N/A	0.456	0.102	0.310	0.000	0.147	0.000	0.182	0.147	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	119	93	0	224	0	145	39	0
N.S.	1	1.00	0.92	0.72	0.00	1.72	0.00	1.12	0.30	0.00
time (sec)	N/A	0.433	0.098	0.464	0.000	0.140	0.000	0.175	0.176	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	198	154	187	0	452	0	471	29	0
N.S.	1	0.99	0.77	0.93	0.00	2.25	0.00	2.34	0.14	0.00
time (sec)	N/A	0.518	0.190	0.478	0.000	0.150	0.000	0.178	200.022	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	294	233	281	0	690	0	884	29	0
N.S.	1	1.04	0.82	0.99	0.00	2.44	0.00	3.12	0.10	0.00
time (sec)	N/A	0.654	0.486	0.938	0.000	0.153	0.000	0.170	200.017	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	95	125	105	105	114	215	153	165
N.S.	1	1.00	0.96	1.26	1.06	1.06	1.15	2.17	1.55	1.67
time (sec)	N/A	0.476	0.089	0.219	0.044	0.121	0.221	0.141	0.171	0.187

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	85	73	77	76	158	153	129
N.S.	1	1.00	1.00	1.10	0.95	1.00	0.99	2.05	1.99	1.68
time (sec)	N/A	0.425	0.076	0.200	0.036	0.111	0.187	0.122	0.183	23.417

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	55	59	53	53	49	109	153	90
N.S.	1	1.00	0.93	1.00	0.90	0.90	0.83	1.85	2.59	1.53
time (sec)	N/A	0.388	0.044	0.151	0.031	0.110	0.144	0.114	0.193	23.231

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	36	35	29	72	149	51
N.S.	1	1.00	1.00	0.98	0.90	0.88	0.72	1.80	3.72	1.28
time (sec)	N/A	0.348	0.030	0.144	0.027	0.069	0.109	0.136	0.160	23.120

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	32	22	19	22	15	37	143	21
N.S.	1	1.00	1.39	0.96	0.83	0.96	0.65	1.61	6.22	0.91
time (sec)	N/A	0.301	0.013	0.089	0.032	0.069	0.078	0.136	0.169	0.064

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	34	42	47	27	99	68	157	34
N.S.	1	1.00	0.83	1.02	1.15	0.66	2.41	1.66	3.83	0.83
time (sec)	N/A	0.361	0.030	0.165	0.028	0.077	0.422	0.138	0.174	23.359

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	42	69	110	40	158	95	176	100
N.S.	1	1.00	0.68	1.11	1.77	0.65	2.55	1.53	2.84	1.61
time (sec)	N/A	0.381	0.035	0.191	0.037	0.076	0.296	0.131	0.162	23.357

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	81	66	109	160	69	226	142	191	156
N.S.	1	0.98	0.80	1.31	1.93	0.83	2.72	1.71	2.30	1.88
time (sec)	N/A	0.417	0.049	0.216	0.037	0.115	0.427	0.137	0.171	23.350

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	102	91	150	218	94	286	183	197	199
N.S.	1	0.98	0.88	1.44	2.10	0.90	2.75	1.76	1.89	1.91
time (sec)	N/A	0.445	0.062	0.243	0.038	0.087	0.573	0.140	0.173	23.434

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	342	299	445	1368	264	0	334	361	0
N.S.	1	0.95	0.83	1.24	3.80	0.73	0.00	0.93	1.00	0.00
time (sec)	N/A	0.843	0.882	1.209	0.148	0.095	0.000	0.177	7.634	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	264	244	351	979	216	0	285	361	0
N.S.	1	0.94	0.87	1.24	3.47	0.77	0.00	1.01	1.28	0.00
time (sec)	N/A	0.715	0.566	0.849	0.138	0.144	0.000	0.139	4.904	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	232	198	268	624	174	0	241	361	0
N.S.	1	1.01	0.86	1.17	2.72	0.76	0.00	1.05	1.58	0.00
time (sec)	N/A	0.604	0.441	0.902	0.119	0.095	0.000	0.146	0.936	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	172	157	195	293	136	0	210	357	0
N.S.	1	1.06	0.96	1.20	1.80	0.83	0.00	1.29	2.19	0.00
time (sec)	N/A	0.509	0.395	0.810	0.110	0.099	0.000	0.156	0.501	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	109	45	129	103	99	0	180	353	0
N.S.	1	1.16	0.48	1.37	1.10	1.05	0.00	1.91	3.76	0.00
time (sec)	N/A	0.408	0.048	0.802	0.107	0.087	0.000	0.133	0.481	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	134	163	189	1067	0	356	0	252	29	0
N.S.	1	1.22	1.41	7.96	0.00	2.66	0.00	1.88	0.22	0.00
time (sec)	N/A	0.577	0.762	0.793	0.000	0.148	0.000	0.253	200.022	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	187	145	187	0	389	0	0	29	0
N.S.	1	1.05	0.81	1.05	0.00	2.19	0.00	0.00	0.16	0.00
time (sec)	N/A	0.498	0.198	1.487	0.000	0.144	0.000	0.000	200.026	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	255	194	282	0	574	0	0	29	0
N.S.	1	0.97	0.73	1.07	0.00	2.17	0.00	0.00	0.11	0.00
time (sec)	N/A	0.587	0.282	1.451	0.000	0.164	0.000	0.000	200.022	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	363	275	392	0	839	0	0	29	0
N.S.	1	1.07	0.81	1.16	0.00	2.47	0.00	0.00	0.09	0.00
time (sec)	N/A	0.801	0.486	2.716	0.000	0.161	0.000	0.000	200.026	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	391	414	121	0	0	554	0	0	30	0
N.S.	1	1.06	0.31	0.00	0.00	1.42	0.00	0.00	0.08	0.00
time (sec)	N/A	0.888	0.107	0.000	0.000	0.155	0.000	0.000	200.027	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	314	355	81	0	0	415	0	0	28	0
N.S.	1	1.13	0.26	0.00	0.00	1.32	0.00	0.00	0.09	0.00
time (sec)	N/A	0.766	0.056	0.000	0.000	0.142	0.000	0.000	200.024	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	245	299	45	0	0	255	0	0	26	0
N.S.	1	1.22	0.18	0.00	0.00	1.04	0.00	0.00	0.11	0.00
time (sec)	N/A	0.659	0.020	0.000	0.000	0.143	0.000	0.000	200.025	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	319	437	124	0	0	414	0	0	30	0
N.S.	1	1.37	0.39	0.00	0.00	1.30	0.00	0.00	0.09	0.00
time (sec)	N/A	0.894	0.116	0.000	0.000	0.143	0.000	0.000	200.022	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	205	261	110	0	0	598	0	0	30	0
N.S.	1	1.27	0.54	0.00	0.00	2.92	0.00	0.00	0.15	0.00
time (sec)	N/A	0.518	0.035	0.000	0.000	0.157	0.000	0.000	200.029	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	391	414	121	0	0	561	0	0	30	0
N.S.	1	1.06	0.31	0.00	0.00	1.43	0.00	0.00	0.08	0.00
time (sec)	N/A	0.889	0.117	0.000	0.000	0.141	0.000	0.000	200.026	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	314	355	79	0	0	431	0	0	28	0
N.S.	1	1.13	0.25	0.00	0.00	1.37	0.00	0.00	0.09	0.00
time (sec)	N/A	0.773	0.051	0.000	0.000	0.131	0.000	0.000	200.025	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	245	299	45	0	0	268	0	0	26	0
N.S.	1	1.22	0.18	0.00	0.00	1.09	0.00	0.00	0.11	0.00
time (sec)	N/A	0.680	0.019	0.000	0.000	0.132	0.000	0.000	200.022	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	344	427	122	0	0	690	0	0	30	0
N.S.	1	1.24	0.35	0.00	0.00	2.01	0.00	0.00	0.09	0.00
time (sec)	N/A	0.963	0.094	0.000	0.000	0.098	0.000	0.000	200.027	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	211	224	106	0	0	694	0	0	30	0
N.S.	1	1.06	0.50	0.00	0.00	3.29	0.00	0.00	0.14	0.00
time (sec)	N/A	0.563	0.031	0.000	0.000	0.093	0.000	0.000	200.033	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	391	414	99	0	0	561	0	0	30	0
N.S.	1	1.06	0.25	0.00	0.00	1.43	0.00	0.00	0.08	0.00
time (sec)	N/A	0.947	0.114	0.000	0.000	0.096	0.000	0.000	200.031	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	314	355	84	0	0	421	0	0	28	0
N.S.	1	1.13	0.27	0.00	0.00	1.34	0.00	0.00	0.09	0.00
time (sec)	N/A	0.803	0.050	0.000	0.000	0.093	0.000	0.000	200.024	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	245	299	45	0	0	266	0	0	26	0
N.S.	1	1.22	0.18	0.00	0.00	1.09	0.00	0.00	0.11	0.00
time (sec)	N/A	0.677	0.021	0.000	0.000	0.113	0.000	0.000	200.021	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	319	437	126	0	0	470	0	0	30	0
N.S.	1	1.37	0.39	0.00	0.00	1.47	0.00	0.00	0.09	0.00
time (sec)	N/A	0.886	0.040	0.000	0.000	0.098	0.000	0.000	200.027	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	210	261	107	0	0	707	0	0	30	0
N.S.	1	1.24	0.51	0.00	0.00	3.37	0.00	0.00	0.14	0.00
time (sec)	N/A	0.524	0.037	0.000	0.000	0.088	0.000	0.000	200.028	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	391	414	98	0	0	555	0	0	30	0
N.S.	1	1.06	0.25	0.00	0.00	1.42	0.00	0.00	0.08	0.00
time (sec)	N/A	0.982	0.114	0.000	0.000	0.131	0.000	0.000	200.026	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	314	355	84	0	0	433	0	0	28	0
N.S.	1	1.13	0.27	0.00	0.00	1.38	0.00	0.00	0.09	0.00
time (sec)	N/A	0.797	0.048	0.000	0.000	0.144	0.000	0.000	200.026	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	245	299	43	0	0	255	0	0	26	0
N.S.	1	1.22	0.18	0.00	0.00	1.04	0.00	0.00	0.11	0.00
time (sec)	N/A	0.671	0.022	0.000	0.000	0.129	0.000	0.000	200.040	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	344	427	128	0	0	629	0	0	30	0
N.S.	1	1.24	0.37	0.00	0.00	1.83	0.00	0.00	0.09	0.00
time (sec)	N/A	0.961	0.041	0.000	0.000	0.165	0.000	0.000	200.022	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	211	226	107	0	0	613	0	0	30	0
N.S.	1	1.07	0.51	0.00	0.00	2.91	0.00	0.00	0.14	0.00
time (sec)	N/A	0.569	0.034	0.000	0.000	0.141	0.000	0.000	200.020	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	140	0	0	0	0	0	0	16	0
N.S.	1	1.04	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.470	0.000	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	320	266	272	0	0	0	0	0	16	0
N.S.	1	0.83	0.85	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.683	0.393	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	220	226	160	0	0	0	0	0	16	0
N.S.	1	1.03	0.73	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.592	0.199	0.000	0.000	0.000	0.000	0.000	0.151	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	128	0	0	0	0	0	14	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.454	0.163	0.000	0.000	0.000	0.000	0.000	0.151	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	60	0	0	0	0	0	12	0
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.338	0.047	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	191	203	170	0	0	0	0	0	16	0
N.S.	1	1.06	0.89	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.495	0.049	0.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	125	0	0	0	0	0	57	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.434	0.029	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	207	205	173	0	0	0	0	0	362	0
N.S.	1	0.99	0.84	0.00	0.00	0.00	0.00	0.00	1.75	0.00
time (sec)	N/A	0.593	0.095	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	102	0	0	0	0	0	21	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.536	0.039	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0	46	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.363	0.020	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	27	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.344	0.018	0.000	0.000	0.000	0.000	0.000	0.152	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	45	0	0	0	0	0	8	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.302	0.032	0.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	35	13	12	12	12	12	13	12
N.S.	1	1.00	2.69	1.00	0.92	0.92	0.92	0.92	1.00	0.92
time (sec)	N/A	0.319	0.011	0.360	0.118	0.078	0.440	0.114	0.168	23.324

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	60	39	0	39	95	109	39	44
N.S.	1	1.00	1.20	0.78	0.00	0.78	1.90	2.18	0.78	0.88
time (sec)	N/A	0.476	0.026	2.372	0.000	0.077	1.130	0.139	0.162	23.352

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	85	89	55	0	66	223	203	63	74
N.S.	1	1.02	1.07	0.66	0.00	0.80	2.69	2.45	0.76	0.89
time (sec)	N/A	0.650	0.174	11.075	0.000	0.120	3.095	0.142	0.166	23.454

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	120	123	71	0	93	398	297	87	104
N.S.	1	1.03	1.06	0.61	0.00	0.80	3.43	2.56	0.75	0.90
time (sec)	N/A	0.866	0.315	36.436	0.000	0.098	8.308	0.143	0.154	23.338

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	155	153	87	0	120	620	391	111	134
N.S.	1	1.04	1.03	0.58	0.00	0.81	4.16	2.62	0.74	0.90
time (sec)	N/A	1.096	0.361	100.004	0.000	0.086	23.449	0.213	0.166	23.496

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	0	0	0	0	0	50	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.529	0.031	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	0	0	0	0	0	22	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.526	0.028	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	0	0	0	0	0	26	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.551	0.030	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	37	0	42	0	64	42	33
N.S.	1	1.00	1.00	1.06	0.00	1.20	0.00	1.83	1.20	0.94
time (sec)	N/A	0.351	0.020	0.064	0.000	0.134	0.000	0.188	0.165	0.210

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	60	54	0	70	0	107	67	78
N.S.	1	1.00	0.83	0.75	0.00	0.97	0.00	1.49	0.93	1.08
time (sec)	N/A	0.543	0.045	0.063	0.000	0.119	0.000	0.169	0.162	0.255

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	116	79	70	0	97	0	250	91	120
N.S.	1	1.07	0.73	0.65	0.00	0.90	0.00	2.31	0.84	1.11
time (sec)	N/A	0.753	0.047	0.069	0.000	0.146	0.000	0.191	0.191	23.555

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	0	0	0	0	0	23	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.540	0.030	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0	52	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.98	0.00
time (sec)	N/A	0.369	0.019	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	31	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.343	0.018	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	0	0	0	0	0	10	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.306	0.037	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	34	16	15	15	15	15	16	15
N.S.	1	1.00	1.89	0.89	0.83	0.83	0.83	0.83	0.89	0.83
time (sec)	N/A	0.329	0.012	0.367	0.120	0.112	0.449	0.117	0.153	23.269

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	55	40	0	40	99	119	40	46
N.S.	1	1.00	1.04	0.75	0.00	0.75	1.87	2.25	0.75	0.87
time (sec)	N/A	0.514	0.027	2.622	0.000	0.131	1.185	0.127	0.172	23.265

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	90	86	57	0	68	231	221	65	79
N.S.	1	1.02	0.98	0.65	0.00	0.77	2.62	2.51	0.74	0.90
time (sec)	N/A	0.665	0.157	11.234	0.000	0.131	3.240	0.154	0.158	23.414

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	127	122	73	0	95	410	323	89	111
N.S.	1	1.03	0.99	0.59	0.00	0.77	3.33	2.63	0.72	0.90
time (sec)	N/A	0.897	0.320	36.803	0.000	0.154	8.580	0.161	0.152	23.626

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	0	0	0	0	0	54	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.531	0.030	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0	24	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.527	0.025	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0	28	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.522	0.026	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	39	0	44	0	70	44	35
N.S.	1	1.00	1.00	1.05	0.00	1.19	0.00	1.89	1.19	0.95
time (sec)	N/A	0.360	0.021	0.059	0.000	0.087	0.000	0.194	0.163	23.107

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	62	56	0	72	0	170	69	80
N.S.	1	1.00	0.82	0.74	0.00	0.95	0.00	2.24	0.91	1.05
time (sec)	N/A	0.542	0.045	0.062	0.000	0.082	0.000	0.141	0.158	0.244

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	122	81	72	0	99	0	272	93	122
N.S.	1	1.07	0.71	0.63	0.00	0.87	0.00	2.39	0.82	1.07
time (sec)	N/A	0.763	0.047	0.063	0.000	0.092	0.000	0.165	0.194	22.954

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	102	0	0	0	0	0	23	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.530	0.035	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0	52	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	0.356	0.022	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	31	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.340	0.018	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	45	0	0	0	0	0	10	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.304	0.034	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	36	16	23	15	15	15	16	15
N.S.	1	1.00	2.25	1.00	1.44	0.94	0.94	0.94	1.00	0.94
time (sec)	N/A	0.328	0.013	0.549	0.043	0.111	3.165	0.113	0.151	22.878

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	60	41	0	41	116	98	41	47
N.S.	1	1.00	1.11	0.76	0.00	0.76	2.15	1.81	0.76	0.87
time (sec)	N/A	0.506	0.028	3.535	0.000	0.114	19.748	0.140	0.160	23.067

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	91	91	57	0	68	291	182	237	80
N.S.	1	1.02	1.02	0.64	0.00	0.76	3.27	2.04	2.66	0.90
time (sec)	N/A	0.692	0.191	14.631	0.000	0.116	90.233	0.135	0.155	23.079

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	128	127	73	0	95	0	266	369	112
N.S.	1	1.03	1.02	0.59	0.00	0.77	0.00	2.15	2.98	0.90
time (sec)	N/A	0.885	0.433	46.457	0.000	0.108	0.000	0.151	0.163	23.260

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	0	0	0	0	0	54	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.531	0.031	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	0	0	0	0	0	24	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.517	0.029	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	0	0	0	0	0	28	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.522	0.026	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	37	39	0	44	0	94	44	35
N.S.	1	1.00	0.97	1.03	0.00	1.16	0.00	2.47	1.16	0.92
time (sec)	N/A	0.362	0.026	0.059	0.000	0.141	0.000	0.184	0.162	23.150

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	62	56	0	72	0	105	77	81
N.S.	1	1.00	0.81	0.73	0.00	0.94	0.00	1.36	1.00	1.05
time (sec)	N/A	0.558	0.053	0.062	0.000	0.130	0.000	0.170	0.174	23.295

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	123	81	72	0	99	0	224	101	123
N.S.	1	1.07	0.70	0.63	0.00	0.86	0.00	1.95	0.88	1.07
time (sec)	N/A	0.847	0.058	0.065	0.000	0.103	0.000	0.178	0.168	22.969

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	0	0	0	0	0	25	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.530	0.031	0.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0	58	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.09	0.00
time (sec)	N/A	0.362	0.021	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	35	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.339	0.020	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	0	0	0	0	0	12	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.304	0.038	0.000	0.000	0.000	0.000	0.000	0.150	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	34	18	23	15	19	15	18	15
N.S.	1	1.00	1.89	1.00	1.28	0.83	1.06	0.83	1.00	0.83
time (sec)	N/A	0.321	0.012	0.553	0.043	0.078	6.659	0.133	0.150	23.205

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	55	42	0	40	124	104	43	47
N.S.	1	1.00	1.02	0.78	0.00	0.74	2.30	1.93	0.80	0.87
time (sec)	N/A	0.489	0.026	3.657	0.000	0.074	38.443	0.123	0.167	24.144

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	91	85	59	0	68	316	192	270	79
N.S.	1	1.02	0.96	0.66	0.00	0.76	3.55	2.16	3.03	0.89
time (sec)	N/A	0.669	0.207	14.869	0.000	0.124	173.990	0.125	0.156	23.898

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	128	121	75	0	95	0	280	420	111
N.S.	1	1.03	0.98	0.60	0.00	0.77	0.00	2.26	3.39	0.90
time (sec)	N/A	0.898	0.481	47.540	0.000	0.121	0.000	0.149	0.153	23.406

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	0	0	0	0	0	58	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	0.528	0.033	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0	26	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.522	0.029	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0	30	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.541	0.031	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	37	41	0	44	0	94	46	35
N.S.	1	1.00	0.97	1.08	0.00	1.16	0.00	2.47	1.21	0.92
time (sec)	N/A	0.364	0.028	0.059	0.000	0.119	0.000	0.154	0.159	0.193

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	62	58	0	72	0	148	83	81
N.S.	1	1.00	0.81	0.75	0.00	0.94	0.00	1.92	1.08	1.05
time (sec)	N/A	0.548	0.052	0.063	0.000	0.142	0.000	0.163	0.164	23.401

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	123	81	74	0	99	0	236	109	123
N.S.	1	1.07	0.70	0.64	0.00	0.86	0.00	2.05	0.95	1.07
time (sec)	N/A	0.790	0.060	0.062	0.000	0.145	0.000	0.184	0.177	23.710

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	42	34	63	53	36	30	124	49
N.S.	1	1.00	0.84	0.68	1.26	1.06	0.72	0.60	2.48	0.98
time (sec)	N/A	0.406	0.030	0.152	0.102	0.084	0.139	0.109	0.156	0.149

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	A	F	A	B	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	73	48	178	112	86	0	24	155	92
N.S.	1	1.06	0.70	2.58	1.62	1.25	0.00	0.35	2.25	1.33
time (sec)	N/A	0.404	0.018	0.189	0.032	0.080	0.000	0.154	0.151	23.666

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	28	43	31	19	24	80	28
N.S.	1	1.00	1.00	0.93	1.43	1.03	0.63	0.80	2.67	0.93
time (sec)	N/A	0.384	0.024	0.121	0.102	0.073	0.083	0.132	0.157	23.387

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	41	52	87	40	54	0	0	92	55
N.S.	1	1.08	1.37	2.29	1.05	1.42	0.00	0.00	2.42	1.45
time (sec)	N/A	0.364	0.040	0.132	0.026	0.079	0.000	0.000	0.159	0.074

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	24	15	8	11	24	15
N.S.	1	1.00	1.00	0.93	1.60	1.00	0.53	0.73	1.60	1.00
time (sec)	N/A	0.341	0.009	0.075	0.103	0.102	0.022	0.142	0.160	22.931

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	14	12	15	10	12	13	15
N.S.	1	1.00	1.00	0.88	0.75	0.94	0.62	0.75	0.81	0.94
time (sec)	N/A	0.335	0.009	0.053	0.029	0.093	0.024	0.112	0.155	23.789

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	41	56	149	33	54	0	0	31	56
N.S.	1	1.08	1.47	3.92	0.87	1.42	0.00	0.00	0.82	1.47
time (sec)	N/A	0.388	0.276	0.132	0.104	0.113	0.000	0.000	0.158	23.401

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	30	41	31	19	24	97	29
N.S.	1	1.00	1.00	0.94	1.28	0.97	0.59	0.75	3.03	0.91
time (sec)	N/A	0.395	0.024	0.105	0.031	0.104	0.085	0.140	0.168	23.971

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	73	82	305	107	86	0	24	226	93
N.S.	1	1.06	1.19	4.42	1.55	1.25	0.00	0.35	3.28	1.35
time (sec)	N/A	0.397	0.104	0.185	0.108	0.082	0.000	0.132	0.540	0.112

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	78	69	82	0	364	0	0	129	0
N.S.	1	0.60	0.53	0.63	0.00	2.78	0.00	0.00	0.98	0.00
time (sec)	N/A	0.620	0.070	0.141	0.000	0.098	0.000	0.000	0.165	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	112	71	526	0	186	0	132	160	0
N.S.	1	1.17	0.74	5.48	0.00	1.94	0.00	1.38	1.67	0.00
time (sec)	N/A	0.509	0.069	0.349	0.000	0.148	0.000	0.220	0.162	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	58	55	61	0	357	0	0	85	0
N.S.	1	0.69	0.65	0.73	0.00	4.25	0.00	0.00	1.01	0.00
time (sec)	N/A	0.578	0.044	0.128	0.000	0.177	0.000	0.000	0.152	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	68	91	204	0	152	0	70	97	0
N.S.	1	1.08	1.44	3.24	0.00	2.41	0.00	1.11	1.54	0.00
time (sec)	N/A	0.429	0.052	0.164	0.000	0.120	0.000	0.150	0.156	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	38	0	253	0	0	29	0
N.S.	1	1.00	1.00	0.90	0.00	6.02	0.00	0.00	0.69	0.00
time (sec)	N/A	0.502	0.027	0.094	0.000	0.164	0.000	0.000	0.167	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	39	15	253	0	0	18	0
N.S.	1	1.00	1.00	0.91	0.35	5.88	0.00	0.00	0.42	0.00
time (sec)	N/A	0.498	0.027	0.095	0.034	0.112	0.000	0.000	0.161	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	68	117	87	40	152	0	70	115	0
N.S.	1	1.08	1.86	1.38	0.63	2.41	0.00	1.11	1.83	0.00
time (sec)	N/A	0.423	0.132	0.190	0.109	0.086	0.000	0.182	0.163	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	60	60	66	35	357	0	0	102	0
N.S.	1	0.70	0.70	0.77	0.41	4.15	0.00	0.00	1.19	0.00
time (sec)	N/A	0.569	0.050	0.136	0.035	0.100	0.000	0.000	0.151	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	112	132	188	76	186	0	132	231	0
N.S.	1	1.17	1.38	1.96	0.79	1.94	0.00	1.38	2.41	0.00
time (sec)	N/A	0.508	0.116	0.237	0.113	0.093	0.000	0.187	0.544	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	24	20	59	35	39	18	53	21
N.S.	1	1.00	0.69	0.57	1.69	1.00	1.11	0.51	1.51	0.60
time (sec)	N/A	0.419	0.057	0.149	0.104	0.070	0.154	0.104	0.154	0.107

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	67	47	32	95	75	0	111	140	41
N.S.	1	1.03	0.72	0.49	1.46	1.15	0.00	1.71	2.15	0.63
time (sec)	N/A	0.389	0.040	0.194	0.027	0.115	0.000	0.140	0.185	22.850

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	15	35	21	20	12	41	24
N.S.	1	1.00	0.95	0.79	1.84	1.11	1.05	0.63	2.16	1.26
time (sec)	N/A	0.346	0.021	0.101	0.105	0.091	0.089	0.114	0.159	22.831

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	67	48	31	45	51	0	67	86	33
N.S.	1	1.37	0.98	0.63	0.92	1.04	0.00	1.37	1.76	0.67
time (sec)	N/A	0.390	0.021	0.137	0.026	0.113	0.000	0.125	0.152	23.001

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	21	24	28	49	32	35	43	23
N.S.	1	1.00	0.75	0.86	1.00	1.75	1.14	1.25	1.54	0.82
time (sec)	N/A	0.401	0.025	0.110	0.105	0.106	0.103	0.108	0.143	22.960

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	21	24	0	49	34	35	25	25
N.S.	1	1.00	0.72	0.83	0.00	1.69	1.17	1.21	0.86	0.86
time (sec)	N/A	0.391	0.022	0.129	0.000	0.118	0.102	0.130	0.161	23.137

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	67	48	33	58	51	0	67	51	31
N.S.	1	1.37	0.98	0.67	1.18	1.04	0.00	1.37	1.04	0.63
time (sec)	N/A	0.381	0.020	0.137	0.102	0.144	0.000	0.147	0.184	0.066

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	15	13	21	22	13	29	24
N.S.	1	1.00	0.95	0.79	0.68	1.11	1.16	0.68	1.53	1.26
time (sec)	N/A	0.351	0.021	0.079	0.027	0.062	0.084	0.112	0.147	0.057

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	67	47	33	99	75	0	111	49	40
N.S.	1	1.03	0.72	0.51	1.52	1.15	0.00	1.71	0.75	0.62
time (sec)	N/A	0.384	0.023	0.194	0.031	0.087	0.000	0.153	0.334	22.960

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	66	56	44	0	101	0	0	58	48
N.S.	1	0.69	0.59	0.46	0.00	1.06	0.00	0.00	0.61	0.51
time (sec)	N/A	0.590	0.046	0.144	0.000	0.077	0.000	0.000	0.157	23.995

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	77	77	44	0	66	0	134	145	46
N.S.	1	1.12	1.12	0.64	0.00	0.96	0.00	1.94	2.10	0.67
time (sec)	N/A	0.444	0.050	0.256	0.000	0.103	0.000	0.143	0.160	23.284

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	48	42	0	71	0	0	46	41
N.S.	1	1.00	0.98	0.86	0.00	1.45	0.00	0.00	0.94	0.84
time (sec)	N/A	0.497	0.038	0.110	0.000	0.083	0.000	0.000	0.160	23.583

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	59	78	33	0	47	0	76	91	32
N.S.	1	1.09	1.44	0.61	0.00	0.87	0.00	1.41	1.69	0.59
time (sec)	N/A	0.398	0.041	0.164	0.000	0.090	0.000	0.146	0.169	0.238

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	59	51	58	0	317	0	0	48	0
N.S.	1	0.67	0.58	0.66	0.00	3.60	0.00	0.00	0.55	0.00
time (sec)	N/A	0.572	0.052	0.125	0.000	0.150	0.000	0.000	0.160	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	60	60	86	52	317	0	0	31	0
N.S.	1	0.67	0.67	0.97	0.58	3.56	0.00	0.00	0.35	0.00
time (sec)	N/A	0.579	0.047	0.138	0.037	0.154	0.000	0.000	0.155	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	59	78	45	59	47	0	76	58	32
N.S.	1	1.09	1.44	0.83	1.09	0.87	0.00	1.41	1.07	0.59
time (sec)	N/A	0.417	0.045	0.163	0.033	0.129	0.000	0.179	0.160	23.441

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	48	42	29	71	0	0	34	49
N.S.	1	1.00	0.98	0.86	0.59	1.45	0.00	0.00	0.69	1.00
time (sec)	N/A	0.503	0.074	0.106	0.042	0.107	0.000	0.000	0.164	24.170

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	77	77	45	119	66	0	134	54	45
N.S.	1	1.12	1.12	0.65	1.72	0.96	0.00	1.94	0.78	0.65
time (sec)	N/A	0.442	0.061	0.231	0.035	0.176	0.000	0.140	0.377	24.258

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	90	0	0	0	0	0	52	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	0.411	0.032	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	88	0	0	0	0	0	31	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.406	0.026	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	56	0	0	0	0	0	10	0
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.346	0.024	0.000	0.000	0.000	0.000	0.000	0.158	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	208	141	0	0	0	0	0	29	0
N.S.	1	1.59	1.08	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.667	0.175	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	121	0	0	0	0	0	29	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.579	0.153	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	120	109	0	0	0	0	0	27	0
N.S.	1	0.98	0.89	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.457	0.070	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	42	18	17	17	26	17	18	17
N.S.	1	1.00	2.33	1.00	0.94	0.94	1.44	0.94	1.00	0.94
time (sec)	N/A	0.339	0.011	0.344	0.113	0.109	0.479	0.111	0.157	22.881

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	120	120	0	0	0	0	0	26	0
N.S.	1	1.85	1.85	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.495	0.048	0.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	162	142	0	0	0	0	0	62	0
N.S.	1	1.80	1.58	0.00	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.604	0.065	0.000	0.000	0.000	0.000	0.000	0.149	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	223	174	0	0	0	0	0	110	0
N.S.	1	1.77	1.38	0.00	0.00	0.00	0.00	0.00	0.87	0.00
time (sec)	N/A	0.706	0.098	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	158	165	166	0	298	0	1346	277	281
N.S.	1	0.87	0.91	0.92	0.00	1.65	0.00	7.44	1.53	1.55
time (sec)	N/A	1.080	0.551	42.053	0.000	0.154	0.000	0.189	0.173	23.242

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	118	0	0	0	0	0	54	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.626	0.126	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	117	0	0	0	0	0	24	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.582	0.066	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	117	0	0	0	0	0	28	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.613	0.051	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	283	236	217	0	0	0	0	0	57	0
N.S.	1	0.83	0.77	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.992	0.311	0.000	0.000	0.000	0.000	0.000	0.388	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	280	235	214	0	0	0	0	0	27	0
N.S.	1	0.84	0.76	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.921	0.244	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	390	275	248	0	0	0	0	0	31	0
N.S.	1	0.71	0.64	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.097	0.373	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	291	252	206	0	0	0	0	0	31	0
N.S.	1	0.87	0.71	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.996	0.225	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	202	184	176	0	0	0	0	0	29	0
N.S.	1	0.91	0.87	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.759	0.170	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	117	0	0	0	0	0	28	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.594	0.033	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	120	0	0	0	0	0	31	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.809	0.064	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	196	170	142	0	0	0	0	0	31	0
N.S.	1	0.87	0.72	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.823	0.081	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	281	230	159	0	0	0	0	0	31	0
N.S.	1	0.82	0.57	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.899	0.102	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	120	0	0	0	0	0	26	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.600	0.084	0.000	0.000	0.000	0.000	0.000	0.211	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	120	0	0	0	0	0	26	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.612	0.057	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	120	0	0	0	0	0	26	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.613	0.057	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	123	0	0	0	0	0	47	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.612	0.072	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	0	0	0	0	0	0	38	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.415	0.000	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	96	0	0	0	0	0	43	0
N.S.	1	1.00	1.88	0.00	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.446	0.190	0.000	0.000	0.000	0.000	0.000	0.158	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	0	0	0	0	0	0	276	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	5.41	0.00
time (sec)	N/A	0.445	0.000	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	0	0	0	0	0	0	483	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	9.47	0.00
time (sec)	N/A	0.445	0.000	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	0	0	0	0	0	0	31	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.717	0.000	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	0	0	0	0	0	0	52	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.745	0.000	0.000	0.000	0.000	0.000	0.000	4.231	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	0	0	0	0	0	0	70	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	0.758	0.000	0.000	0.000	0.000	0.000	0.000	0.273	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	115	0	0	0	0	0	23	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.564	0.044	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	39	41	76	42	0	53	40	54
N.S.	1	1.00	0.74	0.77	1.43	0.79	0.00	1.00	0.75	1.02
time (sec)	N/A	0.510	0.039	0.642	0.054	0.077	0.000	0.256	0.151	0.208

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	39	41	0	44	0	53	40	54
N.S.	1	1.00	0.74	0.77	0.00	0.83	0.00	1.00	0.75	1.02
time (sec)	N/A	0.489	0.034	1.100	0.000	0.079	0.000	0.252	0.148	22.997

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	55	59	0	78	0	333	61	0
N.S.	1	1.00	0.92	0.98	0.00	1.30	0.00	5.55	1.02	0.00
time (sec)	N/A	0.477	0.033	5.021	0.000	0.089	0.000	0.429	0.176	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	36	34	292	379	439	299	234	0
N.S.	1	1.00	0.95	0.89	7.68	9.97	11.55	7.87	6.16	0.00
time (sec)	N/A	0.425	1.290	0.859	0.187	0.539	2.757	0.126	0.161	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	36	34	155	169	194	139	129	160
N.S.	1	1.00	0.95	0.89	4.08	4.45	5.11	3.66	3.39	4.21
time (sec)	N/A	0.414	0.242	0.450	0.120	0.134	0.847	0.117	0.160	27.196

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	36	34	62	49	54	45	56	47
N.S.	1	1.00	0.95	0.89	1.63	1.29	1.42	1.18	1.47	1.24
time (sec)	N/A	0.412	0.041	0.187	0.111	0.108	0.212	0.128	0.159	23.041

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	36	34	0	49	53	80	60	65
N.S.	1	1.00	0.95	0.89	0.00	1.29	1.39	2.11	1.58	1.71
time (sec)	N/A	0.418	0.041	0.217	0.000	0.107	0.207	0.136	0.156	22.991

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	36	34	0	169	192	139	151	159
N.S.	1	1.00	0.95	0.89	0.00	4.45	5.05	3.66	3.97	4.18
time (sec)	N/A	0.421	0.249	0.581	0.000	0.129	0.768	0.118	0.161	26.967

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	63	57	0	496	0	0	180	46
N.S.	1	1.00	0.97	0.88	0.00	7.63	0.00	0.00	2.77	0.71
time (sec)	N/A	0.716	0.594	0.438	0.000	0.191	0.000	0.000	0.151	2.454

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	57	0	192	0	0	91	48
N.S.	1	1.00	1.00	0.88	0.00	2.95	0.00	0.00	1.40	0.74
time (sec)	N/A	0.728	0.123	0.243	0.000	0.131	0.000	0.000	0.156	24.251

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	83	74	87	0	0	0	0	80	0
N.S.	1	0.58	0.52	0.61	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.794	0.066	0.165	0.000	0.000	0.000	0.000	0.174	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	84	75	86	54	0	0	0	58	0
N.S.	1	0.59	0.52	0.60	0.38	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.793	0.071	0.171	0.039	0.000	0.000	0.000	0.155	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	50	93	192	0	0	105	57
N.S.	1	1.00	1.00	0.77	1.43	2.95	0.00	0.00	1.62	0.88
time (sec)	N/A	0.722	0.129	0.237	0.043	0.088	0.000	0.000	0.158	25.225

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	63	50	274	496	0	0	239	47
N.S.	1	1.00	0.97	0.77	4.22	7.63	0.00	0.00	3.68	0.72
time (sec)	N/A	0.741	0.612	0.359	0.121	0.190	0.000	0.000	0.169	27.197

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [146] had the largest ratio of [1.56250000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	F	0	0	N/A	0.000	N/A
2	F	0	0	N/A	0.000	N/A
3	F	0	0	N/A	0.000	N/A
4	F	0	0	N/A	0.000	N/A
5	F	0	0	N/A	0.000	N/A
6	F	0	0	N/A	0.000	N/A
7	F	0	0	N/A	0.000	N/A
8	F	0	0	N/A	0.000	N/A
9	F	0	0	N/A	0.000	N/A
10	F	0	0	N/A	0.000	N/A
11	F	0	0	N/A	0.000	N/A
12	F	0	0	N/A	0.000	N/A
13	N/A	3	0	1.00	14	0.000
14	F	0	0	N/A	0.000	N/A
15	F	0	0	N/A	0.000	N/A
16	A	13	13	1.22	14	0.929
17	A	10	10	1.19	14	0.714
18	A	8	8	1.15	14	0.571
19	A	5	5	1.13	12	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
20	A	3	3	1.00	10	0.300
21	A	7	6	1.00	14	0.429
22	A	6	5	1.00	14	0.357
23	A	9	8	1.00	14	0.571
24	A	11	10	1.02	14	0.714
25	A	14	13	1.04	14	0.929
26	A	3	3	1.00	14	0.214
27	A	3	3	1.00	14	0.214
28	A	3	3	1.00	12	0.250
29	A	3	3	1.00	10	0.300
30	A	3	3	1.00	14	0.214
31	A	3	3	1.00	14	0.214
32	A	3	3	1.00	14	0.214
33	A	3	3	1.00	14	0.214
34	A	15	15	1.22	14	1.071
35	A	12	12	1.19	14	0.857
36	A	12	12	1.07	12	1.000
37	A	6	6	1.00	10	0.600
38	A	10	9	1.00	14	0.643
39	A	3	3	0.95	14	0.214
40	A	3	3	1.01	14	0.214
41	A	3	3	0.98	14	0.214
42	A	3	3	1.00	14	0.214
43	A	3	3	1.00	14	0.214
44	A	3	3	1.00	12	0.250
45	A	3	3	1.00	10	0.300
46	A	3	3	1.00	14	0.214
47	A	3	3	1.00	14	0.214
48	A	3	3	1.00	14	0.214
49	A	3	3	1.00	14	0.214
50	A	11	11	1.19	14	0.786
51	A	8	8	1.15	14	0.571

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
52	A	6	6	1.13	12	0.500
53	A	3	3	1.00	10	0.300
54	A	7	6	1.00	14	0.429
55	A	6	5	1.00	14	0.357
56	A	8	7	1.02	14	0.500
57	A	11	10	1.02	14	0.714
58	A	13	12	1.04	14	0.857
59	A	3	3	1.00	14	0.214
60	A	3	3	1.00	14	0.214
61	A	3	3	1.00	12	0.250
62	A	3	3	1.00	10	0.300
63	A	3	3	1.00	14	0.214
64	A	3	3	1.00	14	0.214
65	A	3	3	1.00	14	0.214
66	A	3	3	1.00	14	0.214
67	A	15	15	1.24	14	1.071
68	A	12	12	1.21	14	0.857
69	A	12	12	1.07	12	1.000
70	A	6	6	1.03	10	0.600
71	A	10	9	1.04	14	0.643
72	A	3	3	1.00	14	0.214
73	A	3	3	1.04	14	0.214
74	A	3	3	1.00	14	0.214
75	A	3	3	1.03	14	0.214
76	A	16	15	1.18	16	0.938
77	A	14	13	1.24	14	0.929
78	A	13	12	1.23	12	1.000
79	A	17	16	1.31	16	1.000
80	A	7	6	1.04	16	0.375
81	A	8	7	1.02	16	0.438
82	A	12	11	1.01	16	0.688
83	A	14	13	1.03	16	0.812

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
84	A	16	15	1.03	16	0.938
85	A	16	15	1.13	16	0.938
86	A	16	15	1.18	16	0.938
87	A	14	13	1.24	14	0.929
88	A	13	12	1.23	12	1.000
89	A	18	17	1.30	16	1.062
90	A	8	7	1.04	16	0.438
91	A	9	8	1.02	16	0.500
92	A	13	12	1.01	16	0.750
93	A	15	14	1.02	16	0.875
94	A	18	17	1.14	16	1.062
95	A	17	16	1.18	16	1.000
96	A	15	14	1.23	14	1.000
97	A	14	13	1.21	12	1.083
98	A	20	19	1.27	16	1.188
99	A	8	7	1.05	16	0.438
100	A	9	8	1.02	16	0.500
101	A	14	13	1.01	16	0.812
102	A	16	15	1.03	16	0.938
103	A	16	15	1.13	16	0.938
104	A	16	15	1.18	16	0.938
105	A	14	13	1.24	14	0.929
106	A	13	12	1.23	12	1.000
107	A	18	17	1.31	16	1.062
108	A	8	7	1.04	16	0.438
109	A	9	8	1.02	16	0.500
110	A	13	12	1.01	16	0.750
111	A	15	14	1.03	16	0.875
112	A	16	15	1.13	16	0.938
113	A	16	15	1.18	16	0.938
114	A	14	13	1.24	14	0.929
115	A	13	12	1.23	12	1.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
116	A	17	16	1.30	16	1.000
117	A	7	6	1.04	16	0.375
118	A	8	7	1.02	16	0.438
119	A	12	11	1.01	16	0.688
120	A	14	13	1.02	16	0.812
121	A	18	17	1.14	16	1.062
122	A	17	16	1.18	16	1.000
123	A	15	14	1.23	14	1.000
124	A	14	13	1.21	12	1.083
125	A	21	20	1.27	16	1.250
126	A	9	8	1.05	16	0.500
127	A	10	9	1.02	16	0.562
128	A	15	14	1.01	16	0.875
129	A	17	16	1.03	16	1.000
130	A	16	15	1.18	14	1.071
131	A	14	13	1.25	12	1.083
132	A	13	12	1.21	10	1.200
133	A	17	16	1.41	14	1.143
134	A	12	11	1.30	14	0.786
135	A	13	12	1.29	14	0.857
136	A	17	16	1.24	14	1.143
137	A	6	6	1.05	14	0.429
138	A	4	4	1.11	12	0.333
139	A	3	3	1.06	10	0.300
140	A	4	4	1.07	14	0.286
141	A	3	3	1.02	14	0.214
142	A	4	4	1.05	14	0.286
143	A	16	15	1.29	16	0.938
144	A	14	13	1.33	14	0.929
145	A	13	12	1.30	12	1.000
146	A	26	25	1.46	16	1.562
147	A	16	15	1.28	16	0.938

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
148	A	17	16	1.25	16	1.000
149	A	5	5	0.92	14	0.357
150	A	6	6	1.00	14	0.429
151	A	3	3	1.00	14	0.214
152	A	3	3	1.00	14	0.214
153	A	6	6	1.00	14	0.429
154	A	5	5	0.93	14	0.357
155	A	7	7	1.36	14	0.500
156	A	3	3	1.00	14	0.214
157	A	3	3	1.00	14	0.214
158	A	7	7	1.36	14	0.500
159	A	2	2	1.00	16	0.125
160	A	2	2	1.00	16	0.125
161	A	2	2	1.00	16	0.125
162	A	2	2	1.00	16	0.125
163	A	2	2	1.00	16	0.125
164	A	2	2	1.00	12	0.167
165	A	2	2	1.00	12	0.167
166	A	2	2	1.00	16	0.125
167	A	2	2	1.00	15	0.133
168	A	5	5	0.86	15	0.333
169	A	5	5	1.04	15	0.333
170	A	3	3	1.00	13	0.231
171	A	2	2	1.00	11	0.182
172	A	4	4	1.00	15	0.267
173	A	2	2	1.00	15	0.133
174	A	3	3	1.00	15	0.200
175	A	7	7	1.02	15	0.467
176	A	11	10	0.90	16	0.625
177	A	9	8	0.87	16	0.500
178	A	9	8	1.05	16	0.500
179	A	7	6	1.10	14	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
180	A	6	5	1.25	12	0.417
181	A	9	8	1.25	16	0.500
182	A	5	4	1.00	16	0.250
183	A	6	5	0.99	16	0.312
184	A	10	9	1.04	16	0.562
185	A	3	3	1.00	16	0.188
186	A	3	3	1.00	16	0.188
187	A	3	3	1.00	16	0.188
188	A	3	3	1.00	14	0.214
189	A	3	3	1.00	12	0.250
190	A	3	3	1.00	16	0.188
191	A	3	3	1.00	16	0.188
192	A	3	3	1.00	16	0.188
193	A	3	3	1.00	16	0.188
194	A	14	13	0.95	16	0.812
195	A	12	11	0.94	16	0.688
196	A	10	9	1.01	16	0.562
197	A	8	7	1.05	14	0.500
198	A	7	6	1.16	12	0.500
199	A	10	9	1.21	16	0.562
200	A	6	5	1.05	16	0.312
201	A	7	6	0.96	16	0.375
202	A	14	13	1.05	16	0.812
203	A	12	11	0.90	16	0.688
204	A	9	8	0.89	16	0.500
205	A	9	8	1.05	16	0.500
206	A	7	6	1.10	14	0.429
207	A	6	5	1.25	12	0.417
208	A	9	8	1.25	16	0.500
209	A	5	4	1.00	16	0.250
210	A	6	5	0.99	16	0.312
211	A	12	11	1.04	16	0.688

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
212	A	3	3	1.00	16	0.188
213	A	3	3	1.00	16	0.188
214	A	3	3	1.00	16	0.188
215	A	3	3	1.00	14	0.214
216	A	3	3	1.00	12	0.250
217	A	3	3	1.00	16	0.188
218	A	3	3	1.00	16	0.188
219	A	3	3	0.98	16	0.188
220	A	3	3	0.98	16	0.188
221	A	13	12	0.95	16	0.750
222	A	11	10	0.94	16	0.625
223	A	11	10	1.01	16	0.625
224	A	8	7	1.06	14	0.500
225	A	7	6	1.16	12	0.500
226	A	10	9	1.22	16	0.562
227	A	6	5	1.05	16	0.312
228	A	7	6	0.97	16	0.375
229	A	13	12	1.07	16	0.750
230	A	16	15	1.06	18	0.833
231	A	14	13	1.13	16	0.812
232	A	13	12	1.22	14	0.857
233	A	15	14	1.37	18	0.778
234	A	7	6	1.27	18	0.333
235	A	16	15	1.06	18	0.833
236	A	14	13	1.13	16	0.812
237	A	13	12	1.22	14	0.857
238	A	19	18	1.24	18	1.000
239	A	8	7	1.06	18	0.389
240	A	16	15	1.06	18	0.833
241	A	14	13	1.13	16	0.812
242	A	13	12	1.22	14	0.857
243	A	14	13	1.37	18	0.722

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
244	A	6	5	1.24	18	0.278
245	A	16	15	1.06	18	0.833
246	A	14	13	1.13	16	0.812
247	A	13	12	1.22	14	0.857
248	A	18	17	1.24	18	0.944
249	A	7	6	1.07	18	0.333
250	A	4	4	1.04	14	0.286
251	A	5	5	0.83	14	0.357
252	A	5	5	1.03	14	0.357
253	A	3	3	1.00	12	0.250
254	A	2	2	1.00	10	0.200
255	A	5	5	1.06	14	0.357
256	A	2	2	1.00	14	0.143
257	A	3	3	0.99	14	0.214
258	A	3	3	1.00	19	0.158
259	A	2	2	1.00	19	0.105
260	A	2	2	1.00	17	0.118
261	A	2	2	1.00	6	0.333
262	A	1	1	1.00	19	0.053
263	A	3	3	1.00	19	0.158
264	A	4	4	1.02	19	0.211
265	A	5	5	1.03	19	0.263
266	A	6	6	1.04	19	0.316
267	A	3	3	1.00	21	0.143
268	A	3	3	1.00	21	0.143
269	A	3	3	1.00	21	0.143
270	A	1	1	1.00	21	0.048
271	A	2	2	1.00	21	0.095
272	A	3	3	1.07	21	0.143
273	A	3	3	1.00	21	0.143
274	A	2	2	1.00	21	0.095
275	A	2	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
276	A	2	2	1.00	8	0.250
277	A	1	1	1.00	21	0.048
278	A	3	3	1.00	21	0.143
279	A	4	4	1.02	21	0.190
280	A	5	5	1.03	21	0.238
281	A	3	3	1.00	23	0.130
282	A	3	3	1.00	23	0.130
283	A	3	3	1.00	23	0.130
284	A	1	1	1.00	23	0.043
285	A	2	2	1.00	23	0.087
286	A	3	3	1.07	23	0.130
287	A	3	3	1.00	21	0.143
288	A	2	2	1.00	21	0.095
289	A	2	2	1.00	19	0.105
290	A	2	2	1.00	8	0.250
291	A	1	1	1.00	21	0.048
292	A	3	3	1.00	21	0.143
293	A	4	4	1.02	21	0.190
294	A	5	5	1.03	21	0.238
295	A	3	3	1.00	23	0.130
296	A	3	3	1.00	23	0.130
297	A	3	3	1.00	23	0.130
298	A	1	1	1.00	23	0.043
299	A	2	2	1.00	23	0.087
300	A	3	3	1.07	23	0.130
301	A	3	3	1.00	21	0.143
302	A	2	2	1.00	21	0.095
303	A	2	2	1.00	19	0.105
304	A	2	2	1.00	8	0.250
305	A	1	1	1.00	21	0.048
306	A	3	3	1.00	21	0.143
307	A	4	4	1.02	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
308	A	5	5	1.03	21	0.238
309	A	3	3	1.00	23	0.130
310	A	3	3	1.00	23	0.130
311	A	3	3	1.00	23	0.130
312	A	1	1	1.00	23	0.043
313	A	2	2	1.00	23	0.087
314	A	3	3	1.07	23	0.130
315	A	3	3	1.00	24	0.125
316	A	5	5	1.06	24	0.208
317	A	3	3	1.00	24	0.125
318	A	4	4	1.08	24	0.167
319	A	2	2	1.00	24	0.083
320	A	2	2	1.00	24	0.083
321	A	4	4	1.08	24	0.167
322	A	3	3	1.00	24	0.125
323	A	5	5	1.06	24	0.208
324	A	4	4	0.60	25	0.160
325	A	6	5	1.17	25	0.200
326	A	4	4	0.69	25	0.160
327	A	5	4	1.08	25	0.160
328	A	3	3	1.00	25	0.120
329	A	3	3	1.00	25	0.120
330	A	5	4	1.08	25	0.160
331	A	4	4	0.70	25	0.160
332	A	6	5	1.17	25	0.200
333	A	3	3	1.00	24	0.125
334	A	3	3	1.03	24	0.125
335	A	2	2	1.00	24	0.083
336	A	3	3	1.37	24	0.125
337	A	3	3	1.00	24	0.125
338	A	3	3	1.00	24	0.125
339	A	3	3	1.37	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	A	2	2	1.00	24	0.083
341	A	3	3	1.03	24	0.125
342	A	4	4	0.69	25	0.160
343	A	3	3	1.12	25	0.120
344	A	3	3	1.00	25	0.120
345	A	3	3	1.09	25	0.120
346	A	4	4	0.67	25	0.160
347	A	4	4	0.67	25	0.160
348	A	3	3	1.09	25	0.120
349	A	3	3	1.00	25	0.120
350	A	3	3	1.12	25	0.120
351	A	2	2	1.00	21	0.095
352	A	2	2	1.00	19	0.105
353	A	2	2	1.00	8	0.250
354	A	5	5	1.59	24	0.208
355	A	5	5	1.00	24	0.208
356	A	3	3	0.98	22	0.136
357	A	1	1	1.00	21	0.048
358	A	3	3	1.85	24	0.125
359	A	8	8	1.80	24	0.333
360	A	10	10	1.77	24	0.417
361	A	5	5	0.87	21	0.238
362	A	3	3	1.00	23	0.130
363	A	3	3	1.00	23	0.130
364	A	3	3	1.00	23	0.130
365	A	6	6	0.83	26	0.231
366	A	6	6	0.84	26	0.231
367	A	6	6	0.71	26	0.231
368	A	6	6	0.87	26	0.231
369	A	4	4	0.91	24	0.167
370	A	3	3	1.00	23	0.130
371	A	3	3	1.00	26	0.115

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
372	A	4	4	0.87	26	0.154
373	A	8	8	0.82	26	0.308
374	A	3	3	1.00	23	0.130
375	A	3	3	1.00	23	0.130
376	A	3	3	1.00	23	0.130
377	A	3	3	1.00	23	0.130
378	A	2	2	1.00	22	0.091
379	A	2	2	1.00	24	0.083
380	A	2	2	1.00	24	0.083
381	A	2	2	1.00	24	0.083
382	A	3	3	1.00	26	0.115
383	A	3	3	1.00	26	0.115
384	A	3	3	1.00	26	0.115
385	A	3	3	1.00	21	0.143
386	A	3	3	1.00	24	0.125
387	A	3	3	1.00	24	0.125
388	A	1	1	1.00	35	0.029
389	A	2	2	1.00	26	0.077
390	A	2	2	1.00	26	0.077
391	A	2	2	1.00	26	0.077
392	A	2	2	1.00	26	0.077
393	A	2	2	1.00	26	0.077
394	A	3	3	1.00	28	0.107
395	A	3	3	1.00	28	0.107
396	A	4	4	0.58	28	0.143
397	A	4	4	0.59	28	0.143
398	A	3	3	1.00	28	0.107
399	A	3	3	1.00	28	0.107

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int e^{c+6i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 69

$$\int e^{c+6i\arctan(a+bx)}\,dx=-e^cx-\frac{4ie^c}{b(1-ia-ibx)^2}-\frac{12e^c}{b(i+a+bx)}+\frac{6ie^c\log(i+a+bx)}{b}$$

output
```
-exp(c)*x-4*I*exp(c)/b/(1-I*a-I*b*x)^2-12*exp(c)/b/(I+a+b*x)+6*I*exp(c)*ln
(I+a+b*x)/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int e^{c+6i\arctan(a+bx)}\,dx =\frac{e^c\left(-a-bx+\frac{4i}{(i+a+bx)^2}-\frac{12}{i+a+bx}+6\arctan(a+bx)+3i\log\left(1+(a+bx)^2\right)\right)}{b}$$

input
```
Integrate[E^(c + (6*I)*ArcTan[a + b*x]),x]
```

output
```
(E^c*(-a - b*x + (4*I)/(I + a + b*x)^2 - 12/(I + a + b*x) + 6*ArcTan[a + b
*x] + (3*I)*Log[1 + (a + b*x)^2]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+6i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{ 7281}$$

$$\frac{\int e^{c+6i\arctan(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 7299}$$

$$\frac{\int e^{c+6i\arctan(a+bx)}d(a+bx)}{b}$$

input
```
Int[E^(c + (6*I)*ArcTan[a + b*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \mathrm{e}^{c+6i\arctan(bx+a)}dx$$

input
```
int(exp(c+6*I*arctan(b*x+a)),x)
```

output
```
int(exp(c+6*I*arctan(b*x+a)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(53) = 106$.

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.61

$$\int e^{c+6i\arctan(a+bx)}\,dx = -\frac{6\left(-i\,b^2x^2+2\left(-i\,a+1\right)bx-i\,a^2+2\,a+i\right)e^c\log\left(\frac{bx+a+i}{b}\right)+\left(b^3x^3+2\left(a+i\right)b^2x^2+\left(a^2+2i\,a+11\right)b}{b^3x^2+2\left(a+i\right)b^2x+\left(a^2+2i\,a-1\right)b}$$

input
```
integrate(exp(c+6*I*arctan(b*x+a)),x, algorithm="fricas")
```

output
```
-(6*(-I*b^2*x^2 + 2*(-I*a + 1)*b*x - I*a^2 + 2*a + I)*e^c*log((b*x + a + I
)/b) + (b^3*x^3 + 2*(a + I)*b^2*x^2 + (a^2 + 2*I*a + 11)*b*x + 12*a + 8*I)
*e^c)/(b^3*x^2 + 2*(a + I)*b^2*x + (a^2 + 2*I*a - 1)*b)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int e^{c+6i\arctan(a+bx)}\,dx = -xe^c - \frac{12ae^c+12bxe^c+8ie^c}{a^2b+2iab+b^3x^2-b+x\left(2ab^2+2ib^2\right)} + \frac{6ie^c\log\left(a+bx+i\right)}{b}$$

input
```
integrate(exp(c+6*I*atan(b*x+a)),x)
```

output
```
-x*exp(c) - (12*a*exp(c) + 12*b*x*exp(c) + 8*I*exp(c))/(a**2*b + 2*I*a*b +
 b**3*x**2 - b + x*(2*a*b**2 + 2*I*b**2)) + 6*I*exp(c)*log(a + b*x + I)/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(53) = 106.

Time = 0.06 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.46

$$\int e^{c+6i\arctan(a+bx)}\,dx = -\frac{b^3x^3e^c + 2\,(a+i)b^2x^2e^c + (a^2+2i\,a+11)bxe^c + 6\,(b^2x^2e^c + 2\,(a+i)bxe^c + (a^2+2i\,a-1)e^c)\arctan}{b^3x^2 + 2\,(a+}$$

input
```
integrate(exp(c+6*I*arctan(b*x+a)),x, algorithm="maxima")
```

output
```
-(b^3*x^3*e^c + 2*(a + I)*b^2*x^2*e^c + (a^2 + 2*I*a + 11)*b*x*e^c + 6*(b^
2*x^2*e^c + 2*(a + I)*b*x*e^c + (a^2 + 2*I*a - 1)*e^c)*arctan2(b*x + a, -1
) + 4*(3*a + 2*I)*e^c + 3*(-I*b^2*x^2*e^c + 2*(-I*a + 1)*b*x*e^c + (-I*a^2
 + 2*a + I)*e^c)*log(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^3*x^2 + 2*(a + I)*b^
2*x + (a^2 + 2*I*a - 1)*b)
```

Giac [F(-1)]

Timed out.

$$\int e^{c+6i\arctan(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(exp(c+6*I*arctan(b*x+a)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+6i\arctan(a+bx)}\,dx = \int e^{c+\operatorname{atan}(a+b\,x)\,6\mathrm{i}}\,dx$$

input
```
int(exp(c + atan(a + b*x)*6i),x)
```

output
```
int(exp(c + atan(a + b*x)*6i), x)
```

Reduce [F]

$$\int e^{c+6i\arctan(a+bx)}\,dx = e^{c}\left(\int e^{6atan(bx+a)i}dx\right)$$

input
```
int(exp(c+6*I*atan(b*x+a)),x)
```

output
```
e**c*int(e**(6*atan(a + b*x)*i),x)
```

3.2 $\int e^{c+4i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 43

$$\int e^{c+4i\arctan(a+bx)}\,dx = e^c x + \frac{4e^c}{b(i+a+bx)} - \frac{4ie^c\log(i+a+bx)}{b}$$

output
```
exp(c)*x+4*exp(c)/b/(I+a+b*x)-4*I*exp(c)*ln(I+a+b*x)/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int e^{c+4i\arctan(a+bx)}\,dx = \frac{e^c\left(a+bx+\frac{4}{i+a+bx}-4\arctan(a+bx)-2i\log\left(1+(a+bx)^2\right)\right)}{b}$$

input
```
Integrate[E^(c + (4*I)*ArcTan[a + b*x]),x]
```

output
```
(E^c*(a + b*x + 4/(I + a + b*x) - 4*ArcTan[a + b*x] - (2*I)*Log[1 + (a + b
*x)^2]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+4i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{ 7281}$$

$$\frac{\int e^{c+4i\arctan(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 7299}$$

$$\frac{\int e^{c+4i\arctan(a+bx)}d(a+bx)}{b}$$

input
```
Int[E^(c + (4*I)*ArcTan[a + b*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \mathrm{e}^{c+4i\arctan(bx+a)}dx$$

input
```
int(exp(c+4*I*arctan(b*x+a)),x)
```

output
```
int(exp(c+4*I*arctan(b*x+a)),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.37

$$\int e^{c+4i\arctan(a+bx)}\,dx = -\frac{4\,(i\,bx+i\,a-1)e^c\log\left(\frac{bx+a+i}{b}\right)-(b^2x^2+(a+i)bx+4)e^c}{b^2x+(a+i)b}$$

input
```
integrate(exp(c+4*I*arctan(b*x+a)),x, algorithm="fricas")
```

output
```
-(4*(I*b*x + I*a - 1)*e^c*log((b*x + a + I)/b) - (b^2*x^2 + (a + I)*b*x +
4)*e^c)/(b^2*x + (a + I)*b)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int e^{c+4i\arctan(a+bx)}\,dx = xe^c + \frac{4e^c}{ab+b^2x+ib} - \frac{4ie^c\log(a+bx+i)}{b}$$

input
```
integrate(exp(c+4*I*atan(b*x+a)),x)
```

output
```
x*exp(c) + 4*exp(c)/(a*b + b**2*x + I*b) - 4*I*exp(c)*log(a + b*x + I)/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(34) = 68.

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.33

$$\int e^{c+4i\arctan(a+bx)}\,dx$$
$$= \frac{b^2x^2e^c + (a+4\arctan(1,bx+a)+i)bxe^c + 4\,(a\arctan(1,bx+a)+i\arctan(1,bx+a)+1)e^c + 2\,(-}{b^2x+(a+i)b}$$

input
```
integrate(exp(c+4*I*arctan(b*x+a)),x, algorithm="maxima")
```

output
```
(b^2*x^2*e^c + (a + 4*arctan2(1, b*x + a) + I)*b*x*e^c + 4*(a*arctan2(1, b
*x + a) + I*arctan2(1, b*x + a) + 1)*e^c + 2*(-I*b*x*e^c + (-I*a + 1)*e^c)
*log(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^2*x + (a + I)*b)
```

Giac [F(-1)]

Timed out.

$$\int e^{c+4i\arctan(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(exp(c+4*I*arctan(b*x+a)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+4i\arctan(a+bx)}\,dx = \int \mathrm{e}^{c+\mathrm{atan}(a+b\,x)\,4\mathrm{i}}\,dx$$

input
```
int(exp(c + atan(a + b*x)*4i),x)
```

output
```
int(exp(c + atan(a + b*x)*4i), x)
```

Reduce [F]

$$\int e^{c+4i\arctan(a+bx)}\,dx = e^c\left(\int e^{4atan(bx+a)i}dx\right)$$

input
```
int(exp(c+4*I*atan(b*x+a)),x)
```

output
```
e**c*int(e**(4*atan(a + b*x)*i),x)
```

3.3 $\int e^{c+2i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 26

$$\int e^{c+2i\arctan(a+bx)}\,dx = -e^cx + \frac{2ie^c\log(i+a+bx)}{b}$$

output
```
-exp(c)*x+2*I*exp(c)*ln(I+a+b*x)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.35

$$\int e^{c+2i\arctan(a+bx)}\,dx = -\frac{e^c(a+bx-2\arctan(a+bx)-i\log(1+(a+bx)^2))}{b}$$

input
```
Integrate[E^(c + (2*I)*ArcTan[a + b*x]),x]
```

output
```
-((E^c*(a + b*x - 2*ArcTan[a + b*x] - I*Log[1 + (a + b*x)^2]))/b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+2i\arctan(a+bx)}dx$$

$$\Big\downarrow \text{ 7281}$$

$$\frac{\int e^{c+2i\arctan(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 7299}$$

$$\frac{\int e^{c+2i\arctan(a+bx)}d(a+bx)}{b}$$

input
```
Int[E^(c + (2*I)*ArcTan[a + b*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int e^{c+2i\arctan(bx+a)}dx$$

input
```
int(exp(c+2*I*arctan(b*x+a)),x)
```

output
```
int(exp(c+2*I*arctan(b*x+a)),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^{c+2i\arctan(a+bx)}\,dx = -\frac{bxe^c - 2i\,e^c\log\left(\frac{bx+a+i}{b}\right)}{b}$$

input
```
integrate(exp(c+2*I*arctan(b*x+a)),x, algorithm="fricas")
```

output
```
-(b*x*e^c - 2*I*e^c*log((b*x + a + I)/b))/b
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int e^{c+2i\arctan(a+bx)}\,dx = -xe^c + \frac{2ie^c\log(a+bx+i)}{b}$$

input
```
integrate(exp(c+2*I*atan(b*x+a)),x)
```

output
```
-x*exp(c) + 2*I*exp(c)*log(a + b*x + I)/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(20) = 40$.

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.69

$$\int e^{c+2i\arctan(a+bx)}\,dx = -\frac{bxe^c + 2\arctan(bx+a,-1)\,e^c - i\,e^c\log(b^2x^2 + 2\,abx + a^2 + 1)}{b}$$

input
```
integrate(exp(c+2*I*arctan(b*x+a)),x, algorithm="maxima")
```

output
```
-(b*x*e^c + 2*arctan2(b*x + a, -1)*e^c - I*e^c*log(b^2*x^2 + 2*a*b*x + a^2
 + 1))/b
```

Giac [F(-1)]

Timed out.

$$\int e^{c+2i\arctan(a+bx)}\,dx=\text{Timed out}$$

input
```
integrate(exp(c+2*I*arctan(b*x+a)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+2i\arctan(a+bx)}\,dx=\int \mathrm{e}^{c+\mathrm{atan}(a+b\,x)\,2\mathrm{i}}\,dx$$

input
```
int(exp(c + atan(a + b*x)*2i),x)
```

output
```
int(exp(c + atan(a + b*x)*2i), x)
```

Reduce [F]

$$\int e^{c+2i\arctan(a+bx)}\,dx=e^{c}\left(\int e^{2atan(bx+a)i}dx\right)$$

input
```
int(exp(c+2*I*atan(b*x+a)),x)
```

output
```
e**c*int(e**(2*atan(a + b*x)*i),x)
```

3.4 $\int e^{c-2i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 29

$$\int e^{c-2i\arctan(a+bx)}\,dx = -e^c x - \frac{2ie^c \log(i-a-bx)}{b}$$

output
```
-exp(c)*x-2*I*exp(c)*ln(I-a-b*x)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int e^{c-2i\arctan(a+bx)}\,dx = -\frac{e^c(a+bx-2\arctan(a+bx)+i\log(1+(a+bx)^2))}{b}$$

input
```
Integrate[E^(c - (2*I)*ArcTan[a + b*x]),x]
```

output
```
-((E^c*(a + b*x - 2*ArcTan[a + b*x] + I*Log[1 + (a + b*x)^2]))/b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c-2i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{ 7281}$$

$$\frac{\int e^{c-2i\arctan(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 7299}$$

$$\frac{\int e^{c-2i\arctan(a+bx)}d(a+bx)}{b}$$

input
```
Int[E^(c - (2*I)*ArcTan[a + b*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \mathrm{e}^{c-2i\arctan(bx+a)}dx$$

input
```
int(exp(c-2*I*arctan(b*x+a)),x)
```

output
```
int(exp(c-2*I*arctan(b*x+a)),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int e^{c-2i\arctan(a+bx)}\,dx = -\frac{bxe^c + 2i\,e^c\log\left(\frac{bx+a-i}{b}\right)}{b}$$

input
```
integrate(exp(c-2*I*arctan(b*x+a)),x, algorithm="fricas")
```

output
```
-(b*x*e^c + 2*I*e^c*log((b*x + a - I)/b))/b
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int e^{c-2i\arctan(a+bx)}\,dx = -xe^c - \frac{2ie^c\log\left(a+bx-i\right)}{b}$$

input
```
integrate(exp(c-2*I*atan(b*x+a)),x)
```

output
```
-x*exp(c) - 2*I*exp(c)*log(a + b*x - I)/b
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int e^{c-2i\arctan(a+bx)}\,dx = -\frac{bxe^c - 2\,\arctan\left(bx+a\right)e^c + i\,e^c\log\left(b^2x^2 + 2\,abx + a^2 + 1\right)}{b}$$

input
```
integrate(exp(c-2*I*arctan(b*x+a)),x, algorithm="maxima")
```

output
```
-(b*x*e^c - 2*arctan(b*x + a)*e^c + I*e^c*log(b^2*x^2 + 2*a*b*x + a^2 + 1)
)/b
```

Giac [F(-1)]

Timed out.

$$\int e^{c-2i\arctan(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(exp(c-2*I*arctan(b*x+a)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int e^{c-2i\arctan(a+bx)}\,dx = \int \mathrm{e}^{c-\mathrm{atan}(a+b\,x)\,2\mathrm{i}}\,dx$$

input
```
int(exp(c - atan(a + b*x)*2i),x)
```

output
```
int(exp(c - atan(a + b*x)*2i), x)
```

Reduce [F]

$$\int e^{c-2i\arctan(a+bx)}\,dx = e^c\left(\int \frac{1}{e^{2atan(bx+a)i}}dx\right)$$

input
```
int(exp(c-2*I*atan(b*x+a)),x)
```

output
```
e**c*int(1/e**(2*atan(a + b*x)*i),x)
```

3.5 $\int e^{c-4i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 49

$$\int e^{c-4i\arctan(a+bx)}\,dx=e^cx-\frac{4e^c}{b(i-a-bx)}+\frac{4ie^c\log(i-a-bx)}{b}$$

output
```
exp(c)*x-4*exp(c)/b/(I-a-b*x)+4*I*exp(c)*ln(I-a-b*x)/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int e^{c-4i\arctan(a+bx)}\,dx=\frac{e^c\left(a+bx+\frac{4}{-i+a+bx}-4\arctan(a+bx)+2i\log\left(1+(a+bx)^2\right)\right)}{b}$$

input
```
Integrate[E^(c - (4*I)*ArcTan[a + b*x]),x]
```

output
```
(E^c*(a + b*x + 4/(-I + a + b*x) - 4*ArcTan[a + b*x] + (2*I)*Log[1 + (a +
b*x)^2]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c-4i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int e^{c-4i\arctan(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{7299}$$

$$\frac{\int e^{c-4i\arctan(a+bx)}d(a+bx)}{b}$$

input
```
Int[E^(c - (4*I)*ArcTan[a + b*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \mathrm{e}^{c-4i\arctan(bx+a)}dx$$

input
```
int(exp(c-4*I*arctan(b*x+a)),x)
```

output
```
int(exp(c-4*I*arctan(b*x+a)),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.20

$$\int e^{c-4i\arctan(a+bx)}\,dx = -\frac{4\,(-i\,bx-i\,a-1)e^c\log\left(\frac{bx+a-i}{b}\right)-(b^2x^2+(a-i)bx+4)e^c}{b^2x+(a-i)b}$$

input
```
integrate(exp(c-4*I*arctan(b*x+a)),x, algorithm="fricas")
```

output
```
-(4*(-I*b*x - I*a - 1)*e^c*log((b*x + a - I)/b) - (b^2*x^2 + (a - I)*b*x +
 4)*e^c)/(b^2*x + (a - I)*b)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int e^{c-4i\arctan(a+bx)}\,dx = xe^c + \frac{4e^c}{ab+b^2x-ib} + \frac{4ie^c\log{(a+bx-i)}}{b}$$

input
```
integrate(exp(c-4*I*atan(b*x+a)),x)
```

output
```
x*exp(c) + 4*exp(c)/(a*b + b**2*x - I*b) + 4*I*exp(c)*log(a + b*x - I)/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(37) = 74.

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.04

$$\int e^{c-4i\arctan(a+bx)}\,dx$$
$$= \frac{b^2x^2e^c+(a+4\arctan(1,bx+a)-i)bxe^c+4\,(a\arctan(1,bx+a)-i\arctan(1,bx+a)+1)e^c-2\,(-}{b^2x+(a-i)b}$$

input
```
integrate(exp(c-4*I*arctan(b*x+a)),x, algorithm="maxima")
```

output
```
(b^2*x^2*e^c + (a + 4*arctan2(1, b*x + a) - I)*b*x*e^c + 4*(a*arctan2(1, b
*x + a) - I*arctan2(1, b*x + a) + 1)*e^c - 2*(-I*b*x*e^c + (-I*a - 1)*e^c)
*log(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^2*x + (a - I)*b)
```

Giac [F(-1)]

Timed out.

$$\int e^{c-4i\arctan(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(exp(c-4*I*arctan(b*x+a)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int e^{c-4i\arctan(a+bx)}\,dx = \int \mathrm{e}^{c-\mathrm{atan}(a+b\,x)\,4\mathrm{i}}\,dx$$

input
```
int(exp(c - atan(a + b*x)*4i),x)
```

output
```
int(exp(c - atan(a + b*x)*4i), x)
```

Reduce [F]

$$\int e^{c-4i\arctan(a+bx)}\,dx = e^c\left(\int \frac{1}{e^{4atan(bx+a)i}}dx\right)$$

input
```
int(exp(c-4*I*atan(b*x+a)),x)
```

output
```
e**c*int(1/e**(4*atan(a + b*x)*i),x)
```

3.6 $\int e^{c-6 i \arctan(a+b x)} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 75

$$\int e^{c-6 i \arctan(a+b x)} \, dx = -e^c x + \frac{12 e^c}{b(i-a-b x)} + \frac{4 i e^c}{b(1+i a+i b x)^2} - \frac{6 i e^c \log(i-a-b x)}{b}$$

output
```
-exp(c)*x+12*exp(c)/b/(I-a-b*x)+4*I*exp(c)/b/(1+I*a+I*b*x)^2-6*I*exp(c)*ln
(I-a-b*x)/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

$$\int e^{c-6 i \arctan(a+b x)} \, dx = \frac{e^c \left(-a-b x-\frac{4 i}{(-i+a+b x)^2}-\frac{12}{-i+a+b x}+6 \arctan(a+b x)-3 i \log \left(1+(a+b x)^2\right)\right)}{b}$$

input
```
Integrate[E^(c - (6*I)*ArcTan[a + b*x]),x]
```

output
```
(E^c*(-a - b*x - (4*I)/(-I + a + b*x)^2 - 12/(-I + a + b*x) + 6*ArcTan[a +
 b*x] - (3*I)*Log[1 + (a + b*x)^2]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c-6i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int e^{c-6i\arctan(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{7299}$$

$$\frac{\int e^{c-6i\arctan(a+bx)}d(a+bx)}{b}$$

input
```
Int[E^(c - (6*I)*ArcTan[a + b*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \mathrm{e}^{c-6i\arctan(bx+a)}dx$$

input
```
int(exp(c-6*I*arctan(b*x+a)),x)
```

output
```
int(exp(c-6*I*arctan(b*x+a)),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.48

$$\int e^{c-6i\arctan(a+bx)}\,dx = -\frac{6\,(i\,b^2x^2+2\,(i\,a+1)bx+i\,a^2+2\,a-i)e^c\log\left(\frac{bx+a-i}{b}\right)+(b^3x^3+2\,(a-i)b^2x^2+(a^2-2i\,a+11)bx+}{b^3x^2+2\,(a-i)b^2x+(a^2-2i\,a-1)b}$$

input
```
integrate(exp(c-6*I*arctan(b*x+a)),x, algorithm="fricas")
```

output
```
-(6*(I*b^2*x^2 + 2*(I*a + 1)*b*x + I*a^2 + 2*a - I)*e^c*log((b*x + a - I)/
b) + (b^3*x^3 + 2*(a - I)*b^2*x^2 + (a^2 - 2*I*a + 11)*b*x + 12*a - 8*I)*e
^c)/(b^3*x^2 + 2*(a - I)*b^2*x + (a^2 - 2*I*a - 1)*b)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07

$$\int e^{c-6i\arctan(a+bx)}\,dx = -xe^c - \frac{12ae^c+12bxe^c-8ie^c}{a^2b-2iab+b^3x^2-b+x\,(2ab^2-2ib^2)} - \frac{6ie^c\log(a+bx-i)}{b}$$

input
```
integrate(exp(c-6*I*atan(b*x+a)),x)
```

output
```
-x*exp(c) - (12*a*exp(c) + 12*b*x*exp(c) - 8*I*exp(c))/(a**2*b - 2*I*a*b +
 b**3*x**2 - b + x*(2*a*b**2 - 2*I*b**2)) - 6*I*exp(c)*log(a + b*x - I)/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(56) = 112.

Time = 0.05 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.25

$$\int e^{c-6i\arctan(a+bx)}\,dx = -\frac{b^3x^3e^c + 2\,(a-i)b^2x^2e^c + (a^2-2i\,a+11)bxe^c - 6\,(b^2x^2e^c + 2\,(a-i)bxe^c + (a^2-2i\,a-1)e^c)\arctan}{b^3x^2 + 2\,(a-i)}$$

input
```
integrate(exp(c-6*I*arctan(b*x+a)),x, algorithm="maxima")
```

output
```
-(b^3*x^3*e^c + 2*(a - I)*b^2*x^2*e^c + (a^2 - 2*I*a + 11)*b*x*e^c - 6*(b^
2*x^2*e^c + 2*(a - I)*b*x*e^c + (a^2 - 2*I*a - 1)*e^c)*arctan(b*x + a) + 4
*(3*a - 2*I)*e^c - 3*(-I*b^2*x^2*e^c + 2*(-I*a - 1)*b*x*e^c + (-I*a^2 - 2*
a + I)*e^c)*log(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^3*x^2 + 2*(a - I)*b^2*x +
 (a^2 - 2*I*a - 1)*b)
```

Giac [F(-1)]

Timed out.

$$\int e^{c-6i\arctan(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(exp(c-6*I*arctan(b*x+a)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int e^{c-6i\arctan(a+bx)}\,dx = \int \mathrm{e}^{c-\mathrm{atan}(a+b\,x)\,6\mathrm{i}}\,dx$$

input
```
int(exp(c - atan(a + b*x)*6i),x)
```

output
```
int(exp(c - atan(a + b*x)*6i), x)
```

Reduce [F]

$$\int e^{c-6i\arctan(a+bx)}\,dx = e^c\left(\int \frac{1}{e^{6atan(bx+a)i}}dx\right)$$

input
```
int(exp(c-6*I*atan(b*x+a)),x)
```

output
```
e**c*int(1/e**(6*atan(a + b*x)*i),x)
```

3.7 $\int e^{c+5i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 171

$$\int e^{c+5i\arctan(a+bx)}\,dx=\frac{5ie^c\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b}+\frac{10ie^c(1+ia+ibx)^{3/2}}{3b\sqrt{1-ia-ibx}}-\frac{2ie^c(1+ia+ibx)^{5/2}}{3b(1-ia-ibx)^{3/2}}-\frac{10ie^c\arcsin\left(\frac{\sqrt{1+ia+ibx}}{\sqrt{2}}\right)}{b}$$

output
```
5*I*exp(c)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b+10/3*I*exp(c)*(1+I*a+
I*b*x)^(3/2)/b/(1-I*a-I*b*x)^(1/2)-2/3*I*exp(c)*(1+I*a+I*b*x)^(5/2)/b/(1-I
*a-I*b*x)^(3/2)-10*I*exp(c)*arcsin(1/2*(1+I*a+I*b*x)^(1/2)*2^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.37

$$\int e^{c+5i\arctan(a+bx)}\,dx=\frac{e^c\left(\sqrt{1+(a+bx)^2}\left(3i+\frac{8i}{(i+a+bx)^2}-\frac{28}{i+a+bx}\right)+15\text{arcsinh}(a+bx)\right)}{3b}$$

input
```
Integrate[E^(c + (5*I)*ArcTan[a + b*x]),x]
```

output
```
(E^c*(Sqrt[1 + (a + b*x)^2]*(3*I + (8*I)/(I + a + b*x)^2 - 28/(I + a + b*x
)) + 15*ArcSinh[a + b*x]))/(3*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+5i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int e^{c+5i\arctan(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{7299}$$

$$\frac{\int e^{c+5i\arctan(a+bx)}d(a+bx)}{b}$$

input
```
Int[E^(c + (5*I)*ArcTan[a + b*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int e^{c+5i\arctan(bx+a)}dx$$

input
```
int(exp(c+5*I*arctan(b*x+a)),x)
```

output
```
int(exp(c+5*I*arctan(b*x+a)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.13

$$\int e^{c+5i\arctan(a+bx)}\,dx =$$

$$-\frac{15\,(b^2x^2+2\,(a+i)bx+a^2+2i\,a-1)e^c\log\left(\sqrt{\frac{bx+a+i}{bx+a-i}}+1\right)-15\,(b^2x^2+2\,(a+i)bx+a^2+2i\,a-1)\ldots}{3\,(b^3x^2+2\ldots}$$

input
```
integrate(exp(c+5*I*arctan(b*x+a)),x, algorithm="fricas")
```

output
```
-1/3*(15*(b^2*x^2 + 2*(a + I)*b*x + a^2 + 2*I*a - 1)*e^c*log(sqrt((b*x + a
 + I)/(b*x + a - I)) + 1) - 15*(b^2*x^2 + 2*(a + I)*b*x + a^2 + 2*I*a - 1)
*e^c*log(sqrt((b*x + a + I)/(b*x + a - I)) - 1) - (-3*I*b^3*x^3 + (-9*I*a
+ 31)*b^2*x^2 - 3*I*a^3 + (-9*I*a^2 + 62*a - 11*I)*b*x + 31*a^2 - 11*I*a +
 23)*sqrt((b*x + a + I)/(b*x + a - I))*e^c)/(b^3*x^2 + 2*(a + I)*b^2*x + (
a^2 + 2*I*a - 1)*b)
```

Sympy [F]

$$\int e^{c+5i\arctan(a+bx)}\,dx = e^c\int e^{5i\,\mathrm{atan}\,(a+bx)}\,dx$$

input
```
integrate(exp(c+5*I*atan(b*x+a)),x)
```

output
```
exp(c)*Integral(exp(5*I*atan(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.87

$$\int e^{c+5i\arctan(a+bx)}\,dx =$$
$$-\frac{15\,(b^2x^2e^c+2\,(a+i)bxe^c+(a^2+2i\,a-1)e^c)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-(3i\,b^2x^2e^c-}{3\,(b^3x^2+2\,(a+i)b^2x+(a^2+2i\,a-}$$

input
```
integrate(exp(c+5*I*arctan(b*x+a)),x, algorithm="maxima")
```

output
```
-1/3*(15*(b^2*x^2*e^c + 2*(a + I)*b*x*e^c + (a^2 + 2*I*a - 1)*e^c)*log(-b*
x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - (3*I*b^2*x^2*e^c - 2*(-3*I*a
+ 17)*b*x*e^c + (3*I*a^2 - 34*a - 23*I)*e^c)*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1))/(b^3*x^2 + 2*(a + I)*b^2*x + (a^2 + 2*I*a - 1)*b)
```

Giac [F(-1)]

Timed out.

$$\int e^{c+5i\arctan(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(exp(c+5*I*arctan(b*x+a)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+5i\arctan(a+bx)}\,dx=\int e^{c+\operatorname{atan}(a+b\,x)\,5\mathrm{i}}\,dx$$

input
```
int(exp(c + atan(a + b*x)*5i),x)
```

output
```
int(exp(c + atan(a + b*x)*5i), x)
```

Reduce [F]

$$\int e^{c+5i\arctan(a+bx)}\,dx=e^c\left(\int e^{5atan(bx+a)i}dx\right)$$

input
```
int(exp(c+5*I*atan(b*x+a)),x)
```

output
```
e**c*int(e**(5*atan(a + b*x)*i),x)
```

3.8 $\int e^{c+3i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 123

$$\int e^{c+3i\arctan(a+bx)}\,dx=-\frac{3ie^c\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b}-\frac{2ie^c(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}}+\frac{6ie^c\arcsin\left(\frac{\sqrt{1+ia+ibx}}{\sqrt{2}}\right)}{b}$$

output
```
-3*I*exp(c)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b-2*I*exp(c)*(1+I*a+I*
b*x)^(3/2)/b/(1-I*a-I*b*x)^(1/2)+6*I*exp(c)*arcsin(1/2*(1+I*a+I*b*x)^(1/2)
*2^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.37

$$\int e^{c+3i\arctan(a+bx)}\,dx=\frac{e^c\left(\sqrt{1+(a+bx)^2}\left(-i+\frac{4}{i+a+bx}\right)-3\text{arcsinh}(a+bx)\right)}{b}$$

input
```
Integrate[E^(c + (3*I)*ArcTan[a + b*x]),x]
```

output
```
(E^c*(Sqrt[1 + (a + b*x)^2]*(-I + 4/(I + a + b*x)) - 3*ArcSinh[a + b*x]))/
b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+3i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{ 7281}$$

$$\frac{\int e^{c+3i\arctan(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 7299}$$

$$\frac{\int e^{c+3i\arctan(a+bx)}d(a+bx)}{b}$$

input
```
Int[E^(c + (3*I)*ArcTan[a + b*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \mathrm{e}^{c+3i\arctan(bx+a)}dx$$

input
```
int(exp(c+3*I*arctan(b*x+a)),x)
```

output
```
int(exp(c+3*I*arctan(b*x+a)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.99

$$\int e^{c+3i\arctan(a+bx)}\,dx = \frac{3\,(bx+a+i)e^c\log\left(\sqrt{\frac{bx+a+i}{bx+a-i}}+1\right)-3\,(bx+a+i)e^c\log\left(\sqrt{\frac{bx+a+i}{bx+a-i}}-1\right)+(i\,b^2x^2-2\,(-i\,a+2)bx+i}{b^2x+(a+i)b}$$

input
```
integrate(exp(c+3*I*arctan(b*x+a)),x, algorithm="fricas")
```

output
```
(3*(b*x + a + I)*e^c*log(sqrt((b*x + a + I)/(b*x + a - I)) + 1) - 3*(b*x +
 a + I)*e^c*log(sqrt((b*x + a + I)/(b*x + a - I)) - 1) + (I*b^2*x^2 - 2*(-
I*a + 2)*b*x + I*a^2 - 4*a + 5*I)*sqrt((b*x + a + I)/(b*x + a - I))*e^c)/(
b^2*x + (a + I)*b)
```

Sympy [F]

$$\int e^{c+3i\arctan(a+bx)}\,dx = e^c\int e^{3i\,\mathrm{atan}\,(a+bx)}\,dx$$

input
```
integrate(exp(c+3*I*atan(b*x+a)),x)
```

output
```
exp(c)*Integral(exp(3*I*atan(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.76

$$\int e^{c+3i\arctan(a+bx)}\,dx = \frac{3\left(bxe^c+(a+i)e^c\right)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\sqrt{b^2x^2+2\,abx+a^2+1}(i\,bxe^c+(i\,a-5}{b^2x+(a+i)b}$$

input
```
integrate(exp(c+3*I*arctan(b*x+a)),x, algorithm="maxima")
```

output
```
(3*(b*x*e^c + (a + I)*e^c)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(I*b*x*e^c + (I*a - 5)*e^c))/(b^2*x
 + (a + I)*b)
```

Giac [F(-1)]

Timed out.

$$\int e^{c+3i\arctan(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(exp(c+3*I*arctan(b*x+a)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+3i\arctan(a+bx)}\,dx = \int \mathrm{e}^{c+\mathrm{atan}(a+b\,x)\,3{}\mathrm{i}}\,dx$$

input
```
int(exp(c + atan(a + b*x)*3i),x)
```

output
```
int(exp(c + atan(a + b*x)*3i), x)
```

Reduce [F]

$$\int e^{c+3i\arctan(a+bx)}\,dx = e^c\left(\int e^{3atan(bx+a)i}dx\right)$$

input
```
int(exp(c+3*I*atan(b*x+a)),x)
```

output
```
e**c*int(e**(3*atan(a + b*x)*i),x)
```

3.9 $\int e^{c+i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 79

$$\int e^{c+i\arctan(a+bx)}\,dx=\frac{ie^c\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b}-\frac{2ie^c\arcsin\left(\frac{\sqrt{1+ia+ibx}}{\sqrt{2}}\right)}{b}$$

output
```
I*exp(c)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b-2*I*exp(c)*arcsin(1/2*(
1+I*a+I*b*x)^(1/2)*2^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

$$\int e^{c+i\arctan(a+bx)}\,dx=\frac{e^c\left(i\sqrt{1+(a+bx)^2}+\operatorname{arcsinh}(a+bx)\right)}{b}$$

input
```
Integrate[E^(c + I*ArcTan[a + b*x]),x]
```

output
```
(E^c*(I*Sqrt[1 + (a + b*x)^2] + ArcSinh[a + b*x]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+i\arctan(a+bx)}\,dx$$
$$\Big\downarrow \text{ 7281}$$
$$\frac{\int e^{c+i\arctan(a+bx)}d(a+bx)}{b}$$
$$\Big\downarrow \text{ 7299}$$
$$\frac{\int e^{c+i\arctan(a+bx)}d(a+bx)}{b}$$

input
```
Int[E^(c + I*ArcTan[a + b*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \mathrm{e}^{c+i\arctan(bx+a)}dx$$

input
```
int(exp(c+I*arctan(b*x+a)),x)
```

output
```
int(exp(c+I*arctan(b*x+a)),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03

$$\int e^{c+i\arctan(a+bx)}\,dx = \frac{(-i\,bx-i\,a-1)\sqrt{\frac{bx+a+i}{bx+a-i}}e^{c}-e^{c}\log\left(\sqrt{\frac{bx+a+i}{bx+a-i}}+1\right)+e^{c}\log\left(\sqrt{\frac{bx+a+i}{bx+a-i}}-1\right)}{b}$$

input
```
integrate(exp(c+I*arctan(b*x+a)),x, algorithm="fricas")
```

output
```
((-I*b*x - I*a - 1)*sqrt((b*x + a + I)/(b*x + a - I))*e^c - e^c*log(sqrt((
b*x + a + I)/(b*x + a - I)) + 1) + e^c*log(sqrt((b*x + a + I)/(b*x + a - I
)) - 1))/b
```

Sympy [F]

$$\int e^{c+i\arctan(a+bx)}\,dx = e^{c}\int e^{i\,\mathrm{atan}\,(a+bx)}\,dx$$

input
```
integrate(exp(c+I*atan(b*x+a)),x)
```

output
```
exp(c)*Integral(exp(I*atan(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.76

$$\int e^{c+i\arctan(a+bx)}\,dx = -\frac{e^{c}\log\left(-bx-a+\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}\right)-i\,\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}e^{c}}{b}$$

input
```
integrate(exp(c+I*arctan(b*x+a)),x, algorithm="maxima")
```

output
```
-(e^c*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - I*sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)*e^c)/b
```

Giac [F(-1)]

Timed out.

$$\int e^{c+i\arctan(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(exp(c+I*arctan(b*x+a)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+i\arctan(a+bx)}\,dx = \int \mathrm{e}^{c+\mathrm{atan}(a+b\,x)\,1\mathrm{i}}\,dx$$

input
```
int(exp(c + atan(a + b*x)*1i),x)
```

output
```
int(exp(c + atan(a + b*x)*1i), x)
```

Reduce [F]

$$\int e^{c+i\arctan(a+bx)}\,dx = e^c\left(\int e^{atan(bx+a)i}dx\right)$$

input
```
int(exp(c+I*atan(b*x+a)),x)
```

output
```
e**c*int(e**(atan(a + b*x)*i),x)
```

3.10 $\int e^{c-i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 79

$$\int e^{c-i\arctan(a+bx)}\,dx=-\frac{ie^c\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b}-\frac{2ie^c\arcsin\left(\frac{\sqrt{1+ia+ibx}}{\sqrt{2}}\right)}{b}$$

output
```
-I*exp(c)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b-2*I*exp(c)*arcsin(1/2*
(1+I*a+I*b*x)^(1/2)*2^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

$$\int e^{c-i\arctan(a+bx)}\,dx=\frac{e^c\left(-i\sqrt{1+(a+bx)^2}+\operatorname{arcsinh}(a+bx)\right)}{b}$$

input
```
Integrate[E^(c - I*ArcTan[a + b*x]),x]
```

output
```
(E^c*((-I)*Sqrt[1 + (a + b*x)^2] + ArcSinh[a + b*x]))/b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c-i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int e^{c-i\arctan(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{7299}$$

$$\frac{\int e^{c-i\arctan(a+bx)}d(a+bx)}{b}$$

input
```
Int[E^(c - I*ArcTan[a + b*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \mathrm{e}^{c-i\arctan(bx+a)}dx$$

input
```
int(exp(c-I*arctan(b*x+a)),x)
```

output
```
int(exp(c-I*arctan(b*x+a)),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03

$$\int e^{c-i\arctan(a+bx)}\,dx = \frac{(i\,bx+i\,a+1)\sqrt{\frac{bx+a+i}{bx+a-i}}e^{c}-e^{c}\log\left(\sqrt{\frac{bx+a+i}{bx+a-i}}+1\right)+e^{c}\log\left(\sqrt{\frac{bx+a+i}{bx+a-i}}-1\right)}{b}$$

input
```
integrate(exp(c-I*arctan(b*x+a)),x, algorithm="fricas")
```

output
```
((I*b*x + I*a + 1)*sqrt((b*x + a + I)/(b*x + a - I))*e^c - e^c*log(sqrt((b
*x + a + I)/(b*x + a - I)) + 1) + e^c*log(sqrt((b*x + a + I)/(b*x + a - I)
) - 1))/b
```

Sympy [F]

$$\int e^{c-i\arctan(a+bx)}\,dx = e^{c}\int e^{-i\operatorname{atan}(a+bx)}\,dx$$

input
```
integrate(exp(c-I*atan(b*x+a)),x)
```

output
```
exp(c)*Integral(exp(-I*atan(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.71

$$\int e^{c-i\arctan(a+bx)}\,dx = \frac{e^{c}\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-i\,\sqrt{b^2x^2+2\,abx+a^2+1}e^{c}}{b}$$

input
```
integrate(exp(c-I*arctan(b*x+a)),x, algorithm="maxima")
```

output
```
(e^c*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - I*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1)*e^c)/b
```

Giac [F(-1)]

Timed out.

$$\int e^{c-i\arctan(a+bx)}\,dx=\text{Timed out}$$

input
```
integrate(exp(c-I*arctan(b*x+a)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int e^{c-i\arctan(a+bx)}\,dx=\int \mathrm{e}^{c-\mathrm{atan}(a+b\,x)\,1\mathrm{i}}\,dx$$

input
```
int(exp(c - atan(a + b*x)*1i),x)
```

output
```
int(exp(c - atan(a + b*x)*1i), x)
```

Reduce [F]

$$\int e^{c-i\arctan(a+bx)}\,dx=e^{c}\left(\int \frac{1}{e^{atan(bx+a)i}}dx\right)$$

input
```
int(exp(c-I*atan(b*x+a)),x)
```

output
```
e**c*int(1/e**(atan(a + b*x)*i),x)
```

3.11 $\int e^{c-3i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 123

$$\int e^{c-3i\arctan(a+bx)}\,dx=\frac{2ie^c(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}}+\frac{3ie^c\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b}+\frac{6ie^c\arcsin\left(\frac{\sqrt{1+ia+ibx}}{\sqrt{2}}\right)}{b}$$

output
```
2*I*exp(c)*(1-I*a-I*b*x)^(3/2)/b/(1+I*a+I*b*x)^(1/2)+3*I*exp(c)*(1-I*a-I*b
*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b+6*I*exp(c)*arcsin(1/2*(1+I*a+I*b*x)^(1/2)*
2^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.37

$$\int e^{c-3i\arctan(a+bx)}\,dx=\frac{e^c\left(\sqrt{1+(a+bx)^2}\left(i+\frac{4}{-i+a+bx}\right)-3\text{arcsinh}(a+bx)\right)}{b}$$

input
```
Integrate[E^(c - (3*I)*ArcTan[a + b*x]),x]
```

output
```
(E^c*(Sqrt[1 + (a + b*x)^2]*(I + 4/(-I + a + b*x)) - 3*ArcSinh[a + b*x]))/
b
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c-3i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \; 7281$$

$$\frac{\int e^{c-3i\arctan(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \; 7299$$

$$\frac{\int e^{c-3i\arctan(a+bx)}d(a+bx)}{b}$$

input
```
Int[E^(c - (3*I)*ArcTan[a + b*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int e^{c-3i\arctan(bx+a)}dx$$

input
```
int(exp(c-3*I*arctan(b*x+a)),x)
```

output
```
int(exp(c-3*I*arctan(b*x+a)),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.67

$$\int e^{c-3i\arctan(a+bx)}\,dx = \frac{(-i\,bx - i\,a - 5)\sqrt{\frac{bx+a+i}{bx+a-i}}e^{c} + 3\,e^{c}\log\left(\sqrt{\frac{bx+a+i}{bx+a-i}} + 1\right) - 3\,e^{c}\log\left(\sqrt{\frac{bx+a+i}{bx+a-i}} - 1\right)}{b}$$

input
```
integrate(exp(c-3*I*arctan(b*x+a)),x, algorithm="fricas")
```

output
```
((-I*b*x - I*a - 5)*sqrt((b*x + a + I)/(b*x + a - I))*e^c + 3*e^c*log(sqrt
((b*x + a + I)/(b*x + a - I)) + 1) - 3*e^c*log(sqrt((b*x + a + I)/(b*x + a
 - I)) - 1))/b
```

Sympy [F]

$$\int e^{c-3i\arctan(a+bx)}\,dx = e^{c}\int e^{-3i\operatorname{atan}(a+bx)}\,dx$$

input
```
integrate(exp(c-3*I*atan(b*x+a)),x)
```

output
```
exp(c)*Integral(exp(-3*I*atan(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.74

$$\int e^{c-3i\arctan(a+bx)}\,dx =$$

$$-\frac{3\left(bxe^{c}+(a-i)e^{c}\right)\log\left(bx+a+\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}\right)-\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}(i\,bxe^{c}+(i\,a+5}{b^{2}x+(a-i)b}$$

input
```
integrate(exp(c-3*I*arctan(b*x+a)),x, algorithm="maxima")
```

output
```
-(3*(b*x*e^c + (a - I)*e^c)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(I*b*x*e^c + (I*a + 5)*e^c))/(b^2*x
 + (a - I)*b)
```

Giac [F(-1)]

Timed out.

$$\int e^{c-3i\arctan(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(exp(c-3*I*arctan(b*x+a)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int e^{c-3i\arctan(a+bx)}\,dx = \int \mathrm{e}^{c-\mathrm{atan}(a+b\,x)\,3i}\,dx$$

input
```
int(exp(c - atan(a + b*x)*3i),x)
```

output
```
int(exp(c - atan(a + b*x)*3i), x)
```

Reduce [F]

$$\int e^{c-3i\arctan(a+bx)}\,dx = e^c\left(\int \frac{1}{e^{3atan(bx+a)i}}dx\right)$$

input
```
int(exp(c-3*I*atan(b*x+a)),x)
```

output
```
e**c*int(1/e**(3*atan(a + b*x)*i),x)
```

3.12 $\int e^{c-5i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 171

$$\int e^{c-5i\arctan(a+bx)}\,dx=\frac{2ie^c(1-ia-ibx)^{5/2}}{3b(1+ia+ibx)^{3/2}}-\frac{10ie^c(1-ia-ibx)^{3/2}}{3b\sqrt{1+ia+ibx}}-\frac{5ie^c\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b}-\frac{10ie^c\arcsin\left(\frac{\sqrt{1+ia+ibx}}{\sqrt{2}}\right)}{b}$$

output
```
2/3*I*exp(c)*(1-I*a-I*b*x)^(5/2)/b/(1+I*a+I*b*x)^(3/2)-10/3*I*exp(c)*(1-I*
a-I*b*x)^(3/2)/b/(1+I*a+I*b*x)^(1/2)-5*I*exp(c)*(1-I*a-I*b*x)^(1/2)*(1+I*a
+I*b*x)^(1/2)/b-10*I*exp(c)*arcsin(1/2*(1+I*a+I*b*x)^(1/2)*2^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.37

$$\int e^{c-5i\arctan(a+bx)}\,dx=\frac{e^c\left(\sqrt{1+(a+bx)^2}\left(-3i-\frac{8i}{(-i+a+bx)^2}-\frac{28}{-i+a+bx}\right)+15\text{arcsinh}(a+bx)\right)}{3b}$$

input
```
Integrate[E^(c - (5*I)*ArcTan[a + b*x]),x]
```

output
```
(E^c*(Sqrt[1 + (a + b*x)^2]*(-3*I - (8*I)/(-I + a + b*x)^2 - 28/(-I + a +
b*x)) + 15*ArcSinh[a + b*x]))/(3*b)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c-5i\arctan(a+bx)}\,dx$$
$$\Big\downarrow \text{ 7281}$$
$$\frac{\int e^{c-5i\arctan(a+bx)}d(a+bx)}{b}$$
$$\Big\downarrow \text{ 7299}$$
$$\frac{\int e^{c-5i\arctan(a+bx)}d(a+bx)}{b}$$

input
```
Int[E^(c - (5*I)*ArcTan[a + b*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int e^{c-5i\arctan(bx+a)}dx$$

input
```
int(exp(c-5*I*arctan(b*x+a)),x)
```

output
```
int(exp(c-5*I*arctan(b*x+a)),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.73

$$\int e^{c-5i\arctan(a+bx)}\,dx = -\frac{15\,(bx+a-i)e^c\log\left(\sqrt{\frac{bx+a+i}{bx+a-i}}+1\right)-15\,(bx+a-i)e^c\log\left(\sqrt{\frac{bx+a+i}{bx+a-i}}-1\right)-(3i\,b^2x^2-2\,(-3i\,a-1}{3\,(b^2x+(a-i)b)}$$

input
```
integrate(exp(c-5*I*arctan(b*x+a)),x, algorithm="fricas")
```

output
```
-1/3*(15*(b*x + a - I)*e^c*log(sqrt((b*x + a + I)/(b*x + a - I)) + 1) - 15
*(b*x + a - I)*e^c*log(sqrt((b*x + a + I)/(b*x + a - I)) - 1) - (3*I*b^2*x
^2 - 2*(-3*I*a - 17)*b*x + 3*I*a^2 + 34*a - 23*I)*sqrt((b*x + a + I)/(b*x
+ a - I))*e^c)/(b^2*x + (a - I)*b)
```

Sympy [F]

$$\int e^{c-5i\arctan(a+bx)}\,dx = e^c\int e^{-5i\,\mathrm{atan}\,(a+bx)}\,dx$$

input
```
integrate(exp(c-5*I*atan(b*x+a)),x)
```

output
```
exp(c)*Integral(exp(-5*I*atan(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.85

$$\int e^{c-5i\arctan(a+bx)}\,dx$$
$$= \frac{15\left(b^2x^2e^c + 2\,(a-i)bxe^c + (a^2-2i\,a-1)e^c\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right) - \left(3i\,b^2x^2e^c - 2\,(\right.}{3\,(b^3x^2+2\,(a-i)b^2x+(a^2-2i\,a-1)b}$$

input
```
integrate(exp(c-5*I*arctan(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*(15*(b^2*x^2*e^c + 2*(a - I)*b*x*e^c + (a^2 - 2*I*a - 1)*e^c)*log(b*x
+ a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - (3*I*b^2*x^2*e^c - 2*(-3*I*a -
17)*b*x*e^c + (3*I*a^2 + 34*a - 23*I)*e^c)*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1))/(b^3*x^2 + 2*(a - I)*b^2*x + (a^2 - 2*I*a - 1)*b)
```

Giac [F(-1)]

Timed out.

$$\int e^{c-5i\arctan(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(exp(c-5*I*arctan(b*x+a)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int e^{c-5i\arctan(a+bx)}\,dx = \int e^{c-\mathrm{atan}(a+b\,x)\,5\mathrm{i}}\,dx$$

input
```
int(exp(c - atan(a + b*x)*5i),x)
```

output
```
int(exp(c - atan(a + b*x)*5i), x)
```

Reduce [F]

$$\int e^{c-5i\arctan(a+bx)}\,dx = e^c\left(\int \frac{1}{e^{5atan(bx+a)i}}dx\right)$$

input
```
int(exp(c-5*I*atan(b*x+a)),x)
```

output
```
e**c*int(1/e**(5*atan(a + b*x)*i),x)
```

3.13 $\int F^{c+d\arctan(a+bx)^n}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int F^{c+d\arctan(a+bx)^n}\,dx=\text{Int}\left(F^{c+d\arctan(a+bx)^n},x\right)$$

output
```
Defer(Int)(F^(c+d*arctan(b*x+a)^n),x)
```

Mathematica [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int F^{c+d\arctan(a+bx)^n}\,dx=\int F^{c+d\arctan(a+bx)^n}\,dx$$

input
```
Integrate[F^(c + d*ArcTan[a + b*x]^n),x]
```

output
```
Integrate[F^(c + d*ArcTan[a + b*x]^n), x]
```

Rubi [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7281, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{d\arctan(a+bx)^n+c}dx$$
$$\Big\downarrow \text{ 7281}$$
$$\frac{\int F^{d\arctan(a+bx)^n+c}d(a+bx)}{b}$$
$$\Big\downarrow \text{ 7299}$$
$$\frac{\int F^{d\arctan(a+bx)^n+c}d(a+bx)}{b}$$

input
```
Int[F^(c + d*ArcTan[a + b*x]^n),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int F^{c+d\arctan(bx+a)^n}dx$$

input
```
int(F^(c+d*arctan(b*x+a)^n),x)
```

output
```
int(F^(c+d*arctan(b*x+a)^n),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int F^{c+d\arctan(a+bx)^n}\,dx = \int F^{d\arctan(bx+a)^n+c}\,dx$$

input
```
integrate(F^(c+d*arctan(b*x+a)^n),x, algorithm="fricas")
```

output
```
integral(F^(d*arctan(b*x + a)^n + c), x)
```

Sympy [N/A]

Not integrable

Time = 3.55 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int F^{c+d\arctan(a+bx)^n}\,dx = \int F^{c+d\operatorname{atan}^n(a+bx)}\,dx$$

input
```
integrate(F**(c+d*atan(b*x+a)**n),x)
```

output
```
Integral(F**(c + d*atan(a + b*x)**n), x)
```

Maxima [F(-2)]

Exception generated.

$$\int F^{c+d\arctan(a+bx)^n}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(F^(c+d*arctan(b*x+a)^n),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int F^{c+d\arctan(a+bx)^n}\,dx = \int F^{d\arctan(bx+a)^n+c}\,dx$$

input
```
integrate(F^(c+d*arctan(b*x+a)^n),x, algorithm="giac")
```

output
```
integrate(F^(d*arctan(b*x + a)^n + c), x)
```

Mupad [N/A]

Not integrable

Time = 22.72 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int F^{c+d\arctan(a+bx)^n}\,dx = \int F^{c+d\,\mathrm{atan}(a+b\,x)^n}\,dx$$

input
```
int(F^(c + d*atan(a + b*x)^n),x)
```

output
```
int(F^(c + d*atan(a + b*x)^n), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int F^{c+d\arctan(a+bx)^n}\,dx = f^c\left(\int f^{atan(bx+a)^n d}dx\right)$$

input
```
int(F^(c+d*atan(b*x+a)^n),x)
```

output
```
f**c*int(f**(atan(a + b*x)**n*d),x)
```

3.14 $\int F^{c+d\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 104

$$\int F^{c+d\arctan(a+bx)}\,dx = \frac{2^{1+\frac{1}{2}id\log(F)}F^c(1+ia+ibx)^{1-\frac{1}{2}id\log(F)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}id\log(F),1-\frac{1}{2}id\log(F),2-\frac{1}{2}id\log(F)\right.}{b(2i+d\log(F))}$$

output
```
2^(1+1/2*I*d*ln(F))*F^c*(1+I*a+I*b*x)^(1-1/2*I*d*ln(F))*hypergeom([-1/2*I*
d*ln(F), 1-1/2*I*d*ln(F)],[2-1/2*I*d*ln(F)],1/2+1/2*I*a+1/2*I*b*x)/b/(2*I+
d*ln(F))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.74

$$\int F^{c+d\arctan(a+bx)}\,dx = \frac{4e^{2i\arctan(a+bx)}F^{c+d\arctan(a+bx)}\,\text{Hypergeometric2F1}\left(2,1-\frac{1}{2}id\log(F),2-\frac{1}{2}id\log(F),-e^{2i\arctan(a+bx)}\right)}{b(2i+d\log(F))}$$

input
```
Integrate[F^(c + d*ArcTan[a + b*x]),x]
```

output
```
(4*E^((2*I)*ArcTan[a + b*x])*F^(c + d*ArcTan[a + b*x])*Hypergeometric2F1[2
, 1 - (I/2)*d*Log[F], 2 - (I/2)*d*Log[F], -E^((2*I)*ArcTan[a + b*x])])/(b*
(2*I + d*Log[F]))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{d\arctan(a+bx)+c}\,dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int F^{c+d\arctan(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{7299}$$

$$\frac{\int F^{c+d\arctan(a+bx)}d(a+bx)}{b}$$

input
```
Int[F^(c + d*ArcTan[a + b*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int F^{c+d\arctan(bx+a)}dx$$

input
```
int(F^(c+d*arctan(b*x+a)),x)
```

output
```
int(F^(c+d*arctan(b*x+a)),x)
```

Fricas [F]

$$\int F^{c+d\arctan(a+bx)}\,dx = \int F^{d\arctan(bx+a)+c}\,dx$$

input
```
integrate(F^(c+d*arctan(b*x+a)),x, algorithm="fricas")
```

output
```
integral(F^(d*arctan(b*x + a) + c), x)
```

Sympy [F]

$$\int F^{c+d\arctan(a+bx)}\,dx = \int F^{c+d\operatorname{atan}(a+bx)}\,dx$$

input
```
integrate(F**(c+d*atan(b*x+a)),x)
```

output
```
Integral(F**(c + d*atan(a + b*x)), x)
```

Maxima [F]

$$\int F^{c+d\arctan(a+bx)}\,dx = \int F^{d\arctan(bx+a)+c}\,dx$$

input
```
integrate(F^(c+d*arctan(b*x+a)),x, algorithm="maxima")
```

output
```
integrate(F^(d*arctan(b*x + a) + c), x)
```

Giac [F(-1)]

Timed out.

$$\int F^{c+d\arctan(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(F^(c+d*arctan(b*x+a)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int F^{c+d\arctan(a+bx)}\,dx = \int F^{c+d\,\mathrm{atan}(a+b\,x)}\,dx$$

input
```
int(F^(c + d*atan(a + b*x)),x)
```

output
```
int(F^(c + d*atan(a + b*x)), x)
```

Reduce [F]

$$\int F^{c+d\arctan(a+bx)}\,dx = f^c\left(\int f^{atan(bx+a)d}dx\right)$$

input
```
int(F^(c+d*atan(b*x+a)),x)
```

output
```
f**c*int(f**(atan(a + b*x)*d),x)
```

3.15 $\int e^{c+id\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 82

$$\int e^{c+id\arctan(a+bx)}\,dx = -\frac{i2^{1-\frac{d}{2}}e^c(1+ia+ibx)^{\frac{2+d}{2}}\operatorname{Hypergeometric2F1}\left(\frac{d}{2},\frac{2+d}{2},\frac{4+d}{2},\frac{1}{2}(1+ia+ibx)\right)}{b(2+d)}$$

output
```
-I*2^(1-1/2*d)*exp(c)*(1+I*a+I*b*x)^(1+1/2*d)*hypergeom([1/2*d, 1+1/2*d],[
2+1/2*d],1/2+1/2*I*a+1/2*I*b*x)/b/(2+d)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.72

$$\int e^{c+id\arctan(a+bx)}\,dx = -\frac{4ie^{c+i(2+d)\arctan(a+bx)}\operatorname{Hypergeometric2F1}\left(2,1+\frac{d}{2},2+\frac{d}{2},-e^{2i\arctan(a+bx)}\right)}{b(2+d)}$$

input
```
Integrate[E^(c + I*d*ArcTan[a + b*x]),x]
```

output
```
((-4*I)*E^(c + I*(2 + d)*ArcTan[a + b*x])*Hypergeometric2F1[2, 1 + d/2, 2
+ d/2, -E^((2*I)*ArcTan[a + b*x])])/(b*(2 + d))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+id\arctan(a+bx)}\,dx$$

$$\Big\downarrow \; 7281$$

$$\frac{\int e^{c+id\arctan(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \; 7299$$

$$\frac{\int e^{c+id\arctan(a+bx)}d(a+bx)}{b}$$

input
```
Int[E^(c + I*d*ArcTan[a + b*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int e^{c+id\arctan(bx+a)}dx$$

input
```
int(exp(c+I*d*arctan(b*x+a)),x)
```

output
```
int(exp(c+I*d*arctan(b*x+a)),x)
```

Fricas [F]

$$\int e^{c+id\arctan(a+bx)}\,dx = \int e^{(i\,d\arctan(bx+a)+c)}\,dx$$

input
```
integrate(exp(c+I*d*arctan(b*x+a)),x, algorithm="fricas")
```

output
```
integral(e^(-1/2*d*log(-(b*x + a + I)/(b*x + a - I)) + c), x)
```

Sympy [F]

$$\int e^{c+id\arctan(a+bx)}\,dx = e^c\int e^{id\,\mathrm{atan}\,(a+bx)}\,dx$$

input
```
integrate(exp(c+I*d*atan(b*x+a)),x)
```

output
```
exp(c)*Integral(exp(I*d*atan(a + b*x)), x)
```

Maxima [F]

$$\int e^{c+id\arctan(a+bx)}\,dx = \int e^{(i\,d\arctan(bx+a)+c)}\,dx$$

input
```
integrate(exp(c+I*d*arctan(b*x+a)),x, algorithm="maxima")
```

output
```
integrate(e^(I*d*arctan(b*x + a) + c), x)
```

Giac [F(-1)]

Timed out.

$$\int e^{c+id\arctan(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(exp(c+I*d*arctan(b*x+a)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int e^{c+id\arctan(a+bx)}\,dx = \int \mathrm{e}^{c+d\,\mathrm{atan}(a+b\,x)\,1\mathrm{i}}\,dx$$

input
```
int(exp(c + d*atan(a + b*x)*1i),x)
```

output
```
int(exp(c + d*atan(a + b*x)*1i), x)
```

Reduce [F]

$$\int e^{c+id\arctan(a+bx)}\,dx = e^c\left(\int e^{atan(bx+a)di}dx\right)$$

input
```
int(exp(c+I*d*atan(b*x+a)),x)
```

output
```
e**c*int(e**(atan(a + b*x)*d*i),x)
```

3.16 $\int e^{i \arctan(ax)} x^4 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 128

$$\int e^{i \arctan(ax)} x^4 \, dx = \frac{8i\sqrt{1+a^2x^2}}{15a^5} - \frac{3x\sqrt{1+a^2x^2}}{8a^4} - \frac{4ix^2\sqrt{1+a^2x^2}}{15a^3} + \frac{x^3\sqrt{1+a^2x^2}}{4a^2} + \frac{ix^4\sqrt{1+a^2x^2}}{5a} + \frac{3\text{arcsinh}(ax)}{8a^5}$$

output
```
8/15*I*(a^2*x^2+1)^(1/2)/a^5-3/8*x*(a^2*x^2+1)^(1/2)/a^4-4/15*I*x^2*(a^2*x
^2+1)^(1/2)/a^3+1/4*x^3*(a^2*x^2+1)^(1/2)/a^2+1/5*I*x^4*(a^2*x^2+1)^(1/2)/
a+3/8*arcsinh(a*x)/a^5
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.50

$$\int e^{i \arctan(ax)} x^4 \, dx = \frac{\sqrt{1+a^2x^2}(64i - 45ax - 32ia^2x^2 + 30a^3x^3 + 24ia^4x^4) + 45\text{arcsinh}(ax)}{120a^5}$$

input
```
Integrate[E^(I*ArcTan[a*x])*x^4,x]
```

output
```
(Sqrt[1 + a^2*x^2]*(64*I - 45*a*x - (32*I)*a^2*x^2 + 30*a^3*x^3 + (24*I)*a
^4*x^4) + 45*ArcSinh[a*x])/(120*a^5)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.22, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {5583, 533, 27, 533, 25, 27, 533, 27, 533, 25, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{x^4(1+iax)}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{533}$$

$$\frac{ix^4\sqrt{a^2x^2+1}}{5a}-\frac{\int \frac{ax^3(4i-5ax)}{\sqrt{a^2x^2+1}}dx}{5a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{ix^4\sqrt{a^2x^2+1}}{5a}-\frac{\int \frac{x^3(4i-5ax)}{\sqrt{a^2x^2+1}}dx}{5a}$$

$$\Big\downarrow \text{533}$$

$$\frac{ix^4\sqrt{a^2x^2+1}}{5a}-\frac{-\frac{5x^3\sqrt{a^2x^2+1}}{4a}-\frac{\int -\frac{ax^2(16iax+15)}{\sqrt{a^2x^2+1}}dx}{4a^2}}{5a}$$

$$\Big\downarrow \text{25}$$

$$\frac{ix^4\sqrt{a^2x^2+1}}{5a}-\frac{-\frac{5x^3\sqrt{a^2x^2+1}}{4a}+\frac{\int \frac{ax^2(16iax+15)}{\sqrt{a^2x^2+1}}dx}{4a^2}}{5a}$$

$$\Big\downarrow \text{27}$$

$$\frac{ix^4\sqrt{a^2x^2+1}}{5a}-\frac{-\frac{5x^3\sqrt{a^2x^2+1}}{4a}+\frac{\int \frac{x^2(16iax+15)}{\sqrt{a^2x^2+1}}dx}{4a}}{5a}$$

$\Big\downarrow$ 533

$$\frac{ix^4\sqrt{a^2x^2+1}}{5a}-\frac{-\frac{5x^3\sqrt{a^2x^2+1}}{4a}+\frac{\frac{16ix^2\sqrt{a^2x^2+1}}{3a}-\frac{\int\frac{ax(32i-45ax)}{\sqrt{a^2x^2+1}}dx}{3a^2}}{4a}}{5a}$$

$\Big\downarrow$ 27

$$\frac{ix^4\sqrt{a^2x^2+1}}{5a}-\frac{-\frac{5x^3\sqrt{a^2x^2+1}}{4a}+\frac{\frac{16ix^2\sqrt{a^2x^2+1}}{3a}-\frac{\int\frac{x(32i-45ax)}{\sqrt{a^2x^2+1}}dx}{3a}}{4a}}{5a}$$

$\Big\downarrow$ 533

$$\frac{ix^4\sqrt{a^2x^2+1}}{5a}-\frac{-\frac{5x^3\sqrt{a^2x^2+1}}{4a}+\frac{\frac{16ix^2\sqrt{a^2x^2+1}}{3a}-\frac{-\frac{45x\sqrt{a^2x^2+1}}{2a}-\frac{\int-\frac{a(64iax+45)}{\sqrt{a^2x^2+1}}dx}{2a^2}}{3a}}{4a}}{5a}$$

$\Big\downarrow$ 25

$$\frac{ix^4\sqrt{a^2x^2+1}}{5a}-\frac{-\frac{5x^3\sqrt{a^2x^2+1}}{4a}+\frac{\frac{16ix^2\sqrt{a^2x^2+1}}{3a}-\frac{-\frac{45x\sqrt{a^2x^2+1}}{2a}+\frac{\int\frac{a(64iax+45)}{\sqrt{a^2x^2+1}}dx}{2a^2}}{3a}}{4a}}{5a}$$

$\Big\downarrow$ 27

$$\frac{ix^4\sqrt{a^2x^2+1}}{5a}-\frac{-\frac{5x^3\sqrt{a^2x^2+1}}{4a}+\frac{\frac{16ix^2\sqrt{a^2x^2+1}}{3a}-\frac{-\frac{45x\sqrt{a^2x^2+1}}{2a}+\frac{\int\frac{64iax+45}{\sqrt{a^2x^2+1}}dx}{2a}}{3a}}{4a}}{5a}$$

$\Big\downarrow$ 455

$$\frac{ix^4\sqrt{a^2x^2+1}}{5a}-\frac{-\frac{5x^3\sqrt{a^2x^2+1}}{4a}+\frac{\frac{16ix^2\sqrt{a^2x^2+1}}{3a}-\frac{-\frac{45x\sqrt{a^2x^2+1}}{2a}+\frac{45\int\frac{1}{\sqrt{a^2x^2+1}}dx+\frac{64i\sqrt{a^2x^2+1}}{a}}{2a}}{3a}}{4a}}{5a}$$

$\Big\downarrow$ 222

$$\frac{ix^4\sqrt{a^2x^2+1}}{5a}-\frac{-\frac{5x^3\sqrt{a^2x^2+1}}{4a}+\frac{\frac{16ix^2\sqrt{a^2x^2+1}}{3a}-\frac{-\frac{45x\sqrt{a^2x^2+1}}{2a}+\frac{\frac{45\operatorname{arcsinh}(ax)}{a}+\frac{64i\sqrt{a^2x^2+1}}{a}}{2a}}{3a}}{4a}}{5a}$$

input
```
Int[E^(I*ArcTan[a*x])*x^4,x]
```

output
```
((I/5)*x^4*Sqrt[1 + a^2*x^2])/a - ((-5*x^3*Sqrt[1 + a^2*x^2])/(4*a) + ((((
16*I)/3)*x^2*Sqrt[1 + a^2*x^2])/a - ((-45*x*Sqrt[1 + a^2*x^2])/(2*a) + (((
64*I)*Sqrt[1 + a^2*x^2])/a + (45*ArcSinh[a*x])/a)/(2*a))/(3*a))/(4*a))/(5*
a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{i(24a^4x^4-30ia^3x^3-32a^2x^2+45iax+64)\sqrt{a^2x^2+1}}{120a^5}+\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{8a^4\sqrt{a^2}}$	84
meijerg	$\frac{-\frac{\sqrt{\pi}\,x(a^2)^{\frac{5}{2}}(-10a^2x^2+15)\sqrt{a^2x^2+1}}{20a^4}+\frac{3\sqrt{\pi}\,(a^2)^{\frac{5}{2}}\operatorname{arcsinh}(ax)}{4a^5}}{2a^4\sqrt{\pi}\,\sqrt{a^2}}+\frac{i\left(-\frac{16\sqrt{\pi}}{15}+\frac{\sqrt{\pi}\,(6a^4x^4-8a^2x^2+16)\sqrt{a^2x^2+1}}{15}\right)}{2a^5\sqrt{\pi}}$	117
default	$\frac{x^3\sqrt{a^2x^2+1}}{4a^2}-\frac{3\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2a^2\sqrt{a^2}}\right)}{4a^2}+ia\left(\frac{x^4\sqrt{a^2x^2+1}}{5a^2}-\frac{4\left(\frac{x^2\sqrt{a^2x^2+1}}{3a^2}-\frac{2\sqrt{a^2x^2+1}}{3a^4}\right)}{5a^2}\right)$	142

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/120*I*(24*a^4*x^4-30*I*a^3*x^3-32*a^2*x^2+45*I*a*x+64)*(a^2*x^2+1)^(1/2)
/a^5+3/8/a^4*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.52

$$\int e^{i\arctan(ax)}x^4\,dx = \frac{(24i\,a^4x^4+30\,a^3x^3-32i\,a^2x^2-45\,ax+64i)\sqrt{a^2x^2+1}-45\,\log\left(-ax+\sqrt{a^2x^2+1}\right)}{120\,a^5}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^4,x, algorithm="fricas")
```

output
```
1/120*((24*I*a^4*x^4 + 30*a^3*x^3 - 32*I*a^2*x^2 - 45*a*x + 64*I)*sqrt(a^2
*x^2 + 1) - 45*log(-a*x + sqrt(a^2*x^2 + 1)))/a^5
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.89

$$\int e^{i\arctan(ax)}x^4\,dx = \begin{cases} \sqrt{a^2x^2+1}\left(\frac{ix^4}{5a}+\frac{x^3}{4a^2}-\frac{4ix^2}{15a^3}-\frac{3x}{8a^4}+\frac{8i}{15a^5}\right)+\frac{3\log\left(2a^2x+2\sqrt{a^2x^2+1}\sqrt{a^2}\right)}{8a^4\sqrt{a^2}} & \text{for } a^2\neq 0 \\ \frac{iax^6}{6}+\frac{x^5}{5} & \text{otherwise} \end{cases}$$

input
```
integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)*x**4,x)
```

output
```
Piecewise((sqrt(a**2*x**2 + 1)*(I*x**4/(5*a) + x**3/(4*a**2) - 4*I*x**2/(1
5*a**3) - 3*x/(8*a**4) + 8*I/(15*a**5)) + 3*log(2*a**2*x + 2*sqrt(a**2*x**
2 + 1)*sqrt(a**2))/(8*a**4*sqrt(a**2)), Ne(a**2, 0)), (I*a*x**6/6 + x**5/5
, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.78

$$\int e^{i\arctan(ax)}x^4\,dx = \frac{i\sqrt{a^2x^2+1}x^4}{5a}+\frac{\sqrt{a^2x^2+1}x^3}{4a^2}-\frac{4i\sqrt{a^2x^2+1}x^2}{15a^3}-\frac{3\sqrt{a^2x^2+1}x}{8a^4}+\frac{3\operatorname{arsinh}(ax)}{8a^5}+\frac{8i\sqrt{a^2x^2+1}}{15a^5}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^4,x, algorithm="maxima")
```

output
```
1/5*I*sqrt(a^2*x^2 + 1)*x^4/a + 1/4*sqrt(a^2*x^2 + 1)*x^3/a^2 - 4/15*I*sqr
t(a^2*x^2 + 1)*x^2/a^3 - 3/8*sqrt(a^2*x^2 + 1)*x/a^4 + 3/8*arcsinh(a*x)/a^
5 + 8/15*I*sqrt(a^2*x^2 + 1)/a^5
```

Giac [F(-2)]

Exception generated.

$$\int e^{i\arctan(ax)}x^4\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.77

$$\int e^{i\arctan(ax)}x^4\,dx = \frac{\sqrt{a^2\,x^2+1}\,\left(\frac{x^3\,{\left(a^2\right)}^{3/2}}{4\,a^4}-\frac{3\,x\,\sqrt{a^2}}{8\,a^4}+\frac{a\,8{}\mathrm{i}}{15\,{\left(a^2\right)}^{5/2}}-\frac{a^3\,x^2\,4{}\mathrm{i}}{15\,{\left(a^2\right)}^{5/2}}+\frac{a^5\,x^4\,1{}\mathrm{i}}{5\,{\left(a^2\right)}^{5/2}}\right)}{\sqrt{a^2}} + \frac{3\,\mathrm{asinh}\left(x\,\sqrt{a^2}\right)}{8\,a^4\,\sqrt{a^2}}$$

input
```
int((x^4*(a*x*1i + 1))/(a^2*x^2 + 1)^(1/2),x)
```

output
```
((a^2*x^2 + 1)^(1/2)*((a*8i)/(15*(a^2)^(5/2)) - (a^3*x^2*4i)/(15*(a^2)^(5/
2)) + (x^3*(a^2)^(3/2))/(4*a^4) + (a^5*x^4*1i)/(5*(a^2)^(5/2)) - (3*x*(a^2
)^(1/2))/(8*a^4)))/(a^2)^(1/2) + (3*asinh(x*(a^2)^(1/2)))/(8*a^4*(a^2)^(1/
2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.83

$$\int e^{i\arctan(ax)}x^4\,dx$$
$$= \frac{24\sqrt{a^2x^2+1}\,a^4i\,x^4+30\sqrt{a^2x^2+1}\,a^3x^3-32\sqrt{a^2x^2+1}\,a^2i\,x^2-45\sqrt{a^2x^2+1}\,ax+64\sqrt{a^2x^2+1}\,i+45}{120a^5}$$

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^4,x)
```

output
```
(24*sqrt(a**2*x**2 + 1)*a**4*i*x**4 + 30*sqrt(a**2*x**2 + 1)*a**3*x**3 - 3
2*sqrt(a**2*x**2 + 1)*a**2*i*x**2 - 45*sqrt(a**2*x**2 + 1)*a*x + 64*sqrt(a
**2*x**2 + 1)*i + 45*log(sqrt(a**2*x**2 + 1) + a*x))/(120*a**5)
```

3.17 $\int e^{i\arctan(ax)}x^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 105

$$\int e^{i\arctan(ax)}x^3\,dx = -\frac{2\sqrt{1+a^2x^2}}{3a^4}-\frac{3ix\sqrt{1+a^2x^2}}{8a^3}+\frac{x^2\sqrt{1+a^2x^2}}{3a^2}+\frac{ix^3\sqrt{1+a^2x^2}}{4a}+\frac{3i\text{arcsinh}(ax)}{8a^4}$$

output
```
-2/3*(a^2*x^2+1)^(1/2)/a^4-3/8*I*x*(a^2*x^2+1)^(1/2)/a^3+1/3*x^2*(a^2*x^2+
1)^(1/2)/a^2+1/4*I*x^3*(a^2*x^2+1)^(1/2)/a+3/8*I*arcsinh(a*x)/a^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.53

$$\int e^{i\arctan(ax)}x^3\,dx = \frac{\sqrt{1+a^2x^2}(-16-9iax+8a^2x^2+6ia^3x^3)+9i\text{arcsinh}(ax)}{24a^4}$$

input
```
Integrate[E^(I*ArcTan[a*x])*x^3,x]
```

output
```
(Sqrt[1 + a^2*x^2]*(-16 - (9*I)*a*x + 8*a^2*x^2 + (6*I)*a^3*x^3) + (9*I)*A
rcSinh[a*x])/(24*a^4)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5583, 533, 27, 533, 25, 27, 533, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{x^3(1+iax)}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{533}$$

$$\frac{ix^3\sqrt{a^2x^2+1}}{4a}-\frac{\int \frac{ax^2(3i-4ax)}{\sqrt{a^2x^2+1}}dx}{4a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{ix^3\sqrt{a^2x^2+1}}{4a}-\frac{\int \frac{x^2(3i-4ax)}{\sqrt{a^2x^2+1}}dx}{4a}$$

$$\Big\downarrow \text{533}$$

$$\frac{ix^3\sqrt{a^2x^2+1}}{4a}-\frac{-\frac{4x^2\sqrt{a^2x^2+1}}{3a}-\frac{\int -\frac{ax(9iax+8)}{\sqrt{a^2x^2+1}}dx}{3a^2}}{4a}$$

$$\Big\downarrow \text{25}$$

$$\frac{ix^3\sqrt{a^2x^2+1}}{4a}-\frac{-\frac{4x^2\sqrt{a^2x^2+1}}{3a}+\frac{\int \frac{ax(9iax+8)}{\sqrt{a^2x^2+1}}dx}{3a^2}}{4a}$$

$$\Big\downarrow \text{27}$$

$$\frac{ix^3\sqrt{a^2x^2+1}}{4a}-\frac{-\frac{4x^2\sqrt{a^2x^2+1}}{3a}+\frac{\int \frac{x(9iax+8)}{\sqrt{a^2x^2+1}}dx}{3a}}{4a}$$

$$\Big\downarrow \text{533}$$

$$\frac{ix^3\sqrt{a^2x^2+1}}{4a}-\frac{-\frac{4x^2\sqrt{a^2x^2+1}}{3a}+\frac{\frac{9ix\sqrt{a^2x^2+1}}{2a}-\frac{\int \frac{a(9i-16ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}}{3a}}{4a}$$

$$\Big\downarrow \text{27}$$

$$\frac{ix^3\sqrt{a^2x^2+1}}{4a}-\frac{-\frac{4x^2\sqrt{a^2x^2+1}}{3a}+\frac{\frac{9ix\sqrt{a^2x^2+1}}{2a}-\frac{\int\frac{9i-16ax}{\sqrt{a^2x^2+1}}dx}{2a}}{3a}}{4a}$$

$$\Big\downarrow \text{455}$$

$$\frac{ix^3\sqrt{a^2x^2+1}}{4a}-\frac{-\frac{4x^2\sqrt{a^2x^2+1}}{3a}+\frac{\frac{9ix\sqrt{a^2x^2+1}}{2a}-\frac{-\frac{16\sqrt{a^2x^2+1}}{a}+9i\int\frac{1}{\sqrt{a^2x^2+1}}dx}{2a}}{3a}}{4a}$$

$$\Big\downarrow \text{222}$$

$$\frac{ix^3\sqrt{a^2x^2+1}}{4a}-\frac{-\frac{4x^2\sqrt{a^2x^2+1}}{3a}+\frac{\frac{9ix\sqrt{a^2x^2+1}}{2a}-\frac{-\frac{16\sqrt{a^2x^2+1}}{a}+\frac{9i\text{arcsinh}(ax)}{a}}{2a}}{3a}}{4a}$$

input
```
Int[E^(I*ArcTan[a*x])*x^3,x]
```

output
```
((I/4)*x^3*Sqrt[1 + a^2*x^2])/a - ((-4*x^2*Sqrt[1 + a^2*x^2])/(3*a) + ((((
9*I)/2)*x*Sqrt[1 + a^2*x^2])/a - ((-16*Sqrt[1 + a^2*x^2])/a + ((9*I)*ArcSi
nh[a*x])/a)/(2*a))/(3*a))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.73

method	result	size
risch	$\frac{i\left(6a^{3}x^{3}-8ia^{2}x^{2}-9ax+16i\right)\sqrt{a^{2}x^{2}+1}}{24a^{4}}+\frac{3i\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}+1}\right)}{8a^{3}\sqrt{a^{2}}}$	77
meijerg	$\frac{\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(-4a^{2}x^{2}+8\right)\sqrt{a^{2}x^{2}+1}}{6}}{2a^{4}\sqrt{\pi}}+\frac{i\left(-\frac{\sqrt{\pi}\,x\left(a^{2}\right)^{\frac{5}{2}}\left(-10a^{2}x^{2}+15\right)\sqrt{a^{2}x^{2}+1}}{20a^{4}}+\frac{3\sqrt{\pi}\left(a^{2}\right)^{\frac{5}{2}}\operatorname{arcsinh}\left(ax\right)}{4a^{5}}\right)}{2a^{3}\sqrt{\pi}\sqrt{a^{2}}}$	109
default	$\frac{x^{2}\sqrt{a^{2}x^{2}+1}}{3a^{2}}-\frac{2\sqrt{a^{2}x^{2}+1}}{3a^{4}}+ia\left(\frac{x^{3}\sqrt{a^{2}x^{2}+1}}{4a^{2}}-\frac{3\left(\frac{x\sqrt{a^{2}x^{2}+1}}{2a^{2}}-\frac{\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}+1}\right)}{2a^{2}\sqrt{a^{2}}}\right)}{4a^{2}}\right)$	117

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/24*I*(6*a^3*x^3-8*I*a^2*x^2-9*a*x+16*I)*(a^2*x^2+1)^(1/2)/a^4+3/8*I/a^3*
ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.56

$$\int e^{i\arctan(ax)}x^3\,dx = \frac{(6i\,a^3x^3+8\,a^2x^2-9i\,ax-16)\sqrt{a^2x^2+1}-9i\,\log\left(-ax+\sqrt{a^2x^2+1}\right)}{24\,a^4}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^3,x, algorithm="fricas")
```

output
```
1/24*((6*I*a^3*x^3 + 8*a^2*x^2 - 9*I*a*x - 16)*sqrt(a^2*x^2 + 1) - 9*I*log
(-a*x + sqrt(a^2*x^2 + 1)))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99

$$\int e^{i\arctan(ax)}x^3\,dx = \begin{cases} \sqrt{a^2x^2+1}\left(\frac{ix^3}{4a}+\frac{x^2}{3a^2}-\frac{3ix}{8a^3}-\frac{2}{3a^4}\right)+\frac{3i\log\left(2a^2x+2\sqrt{a^2x^2+1}\sqrt{a^2}\right)}{8a^3\sqrt{a^2}} & \text{for } a^2\neq 0 \\ \frac{iax^5}{5}+\frac{x^4}{4} & \text{otherwise} \end{cases}$$

input
```
integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)*x**3,x)
```

output
```
Piecewise((sqrt(a**2*x**2 + 1)*(I*x**3/(4*a) + x**2/(3*a**2) - 3*I*x/(8*a*
*3) - 2/(3*a**4)) + 3*I*log(2*a**2*x + 2*sqrt(a**2*x**2 + 1)*sqrt(a**2))/(
8*a**3*sqrt(a**2)), Ne(a**2, 0)), (I*a*x**5/5 + x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.77

$$\int e^{i\arctan(ax)}x^3\,dx = \frac{i\sqrt{a^2x^2+1}x^3}{4\,a} + \frac{\sqrt{a^2x^2+1}x^2}{3\,a^2} - \frac{3i\sqrt{a^2x^2+1}x}{8\,a^3} + \frac{3i\,\operatorname{arsinh}(ax)}{8\,a^4} - \frac{2\sqrt{a^2x^2+1}}{3\,a^4}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^3,x, algorithm="maxima")
```

output
```
1/4*I*sqrt(a^2*x^2 + 1)*x^3/a + 1/3*sqrt(a^2*x^2 + 1)*x^2/a^2 - 3/8*I*sqrt
(a^2*x^2 + 1)*x/a^3 + 3/8*I*arcsinh(a*x)/a^4 - 2/3*sqrt(a^2*x^2 + 1)/a^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.67

$$\int e^{i\arctan(ax)}x^3\,dx = -\frac{1}{24}\sqrt{a^2x^2+1}\left(\left(2\,x\left(-\frac{3i\,x}{a}-\frac{4}{a^2}\right)+\frac{9i}{a^3}\right)x+\frac{16}{a^4}\right) - \frac{3i\,\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{8\,a^3|a|}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^3,x, algorithm="giac")
```

output
```
-1/24*sqrt(a^2*x^2 + 1)*((2*x*(-3*I*x/a - 4/a^2) + 9*I/a^3)*x + 16/a^4) -
3/8*I*log(-x*abs(a) + sqrt(a^2*x^2 + 1))/(a^3*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.81

$$\int e^{i\arctan(ax)}x^3\,dx = \frac{\operatorname{asinh}\left(x\sqrt{a^2}\right)\,3\mathrm{i}}{8\,a^3\sqrt{a^2}} - \frac{\sqrt{a^2\,x^2+1}\left(\frac{2}{3\,(a^2)^{3/2}} - \frac{a^2\,x^2}{3\,(a^2)^{3/2}} - \frac{x^3\,(a^2)^{3/2}\,1\mathrm{i}}{4\,a^3} + \frac{x\sqrt{a^2}\,3\mathrm{i}}{8\,a^3}\right)}{\sqrt{a^2}}$$

input
```
int((x^3*(a*x*1i + 1))/(a^2*x^2 + 1)^(1/2),x)
```

output
```
(asinh(x*(a^2)^(1/2))*3i)/(8*a^3*(a^2)^(1/2)) - ((a^2*x^2 + 1)^(1/2)*(2/(3
*(a^2)^(3/2)) - (a^2*x^2)/(3*(a^2)^(3/2)) - (x^3*(a^2)^(3/2)*1i)/(4*a^3) +
 (x*(a^2)^(1/2)*3i)/(8*a^3)))/(a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.84

$$\int e^{i\arctan(ax)}x^3\,dx = \frac{6\sqrt{a^2x^2+1}\,a^3i\,x^3 + 8\sqrt{a^2x^2+1}\,a^2x^2 - 9\sqrt{a^2x^2+1}\,aix - 16\sqrt{a^2x^2+1} + 9\log\left(\sqrt{a^2x^2+1}+ax\right)i}{24a^4}$$

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^3,x)
```

output
```
(6*sqrt(a**2*x**2 + 1)*a**3*i*x**3 + 8*sqrt(a**2*x**2 + 1)*a**2*x**2 - 9*s
qrt(a**2*x**2 + 1)*a*i*x - 16*sqrt(a**2*x**2 + 1) + 9*log(sqrt(a**2*x**2 +
 1) + a*x)*i)/(24*a**4)
```

3.18 $\int e^{i\arctan(ax)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 80

$$\int e^{i\arctan(ax)}x^2\,dx=-\frac{2i\sqrt{1+a^2x^2}}{3a^3}+\frac{x\sqrt{1+a^2x^2}}{2a^2}+\frac{ix^2\sqrt{1+a^2x^2}}{3a}-\frac{\operatorname{arcsinh}(ax)}{2a^3}$$

output
```
-2/3*I*(a^2*x^2+1)^(1/2)/a^3+1/2*x*(a^2*x^2+1)^(1/2)/a^2+1/3*I*x^2*(a^2*x^
2+1)^(1/2)/a-1/2*arcsinh(a*x)/a^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int e^{i\arctan(ax)}x^2\,dx=\frac{(-4i+3ax+2ia^2x^2)\sqrt{1+a^2x^2}-3\operatorname{arcsinh}(ax)}{6a^3}$$

input
```
Integrate[E^(I*ArcTan[a*x])*x^2,x]
```

output
```
((-4*I + 3*a*x + (2*I)*a^2*x^2)*Sqrt[1 + a^2*x^2] - 3*ArcSinh[a*x])/(6*a^3
)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5583, 533, 27, 533, 25, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{x^2(1+iax)}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{533}$$

$$\frac{ix^2\sqrt{a^2x^2+1}}{3a}-\frac{\int \frac{ax(2i-3ax)}{\sqrt{a^2x^2+1}}dx}{3a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{ix^2\sqrt{a^2x^2+1}}{3a}-\frac{\int \frac{x(2i-3ax)}{\sqrt{a^2x^2+1}}dx}{3a}$$

$$\Big\downarrow \text{533}$$

$$\frac{ix^2\sqrt{a^2x^2+1}}{3a}-\frac{-\frac{3x\sqrt{a^2x^2+1}}{2a}-\frac{\int -\frac{a(4iax+3)}{\sqrt{a^2x^2+1}}dx}{2a^2}}{3a}$$

$$\Big\downarrow \text{25}$$

$$\frac{ix^2\sqrt{a^2x^2+1}}{3a}-\frac{-\frac{3x\sqrt{a^2x^2+1}}{2a}+\frac{\int \frac{a(4iax+3)}{\sqrt{a^2x^2+1}}dx}{2a^2}}{3a}$$

$$\Big\downarrow \text{27}$$

$$\frac{ix^2\sqrt{a^2x^2+1}}{3a}-\frac{-\frac{3x\sqrt{a^2x^2+1}}{2a}+\frac{\int \frac{4iax+3}{\sqrt{a^2x^2+1}}dx}{2a}}{3a}$$

$$\Big\downarrow \text{455}$$

$$\frac{ix^2\sqrt{a^2x^2+1}}{3a}-\frac{-\frac{3x\sqrt{a^2x^2+1}}{2a}+\frac{3\int \frac{1}{\sqrt{a^2x^2+1}}dx+\frac{4i\sqrt{a^2x^2+1}}{a}}{2a}}{3a}$$

$$\Big\downarrow \text{222}$$

$$\frac{ix^2\sqrt{a^2x^2+1}}{3a}-\frac{-\frac{3x\sqrt{a^2x^2+1}}{2a}+\frac{\frac{3\text{arcsinh}(ax)}{a}+\frac{4i\sqrt{a^2x^2+1}}{a}}{2a}}{3a}$$

input
```
Int[E^(I*ArcTan[a*x])*x^2,x]
```

output
```
((I/3)*x^2*Sqrt[1 + a^2*x^2])/a - ((-3*x*Sqrt[1 + a^2*x^2])/(2*a) + (((4*I
)*Sqrt[1 + a^2*x^2])/a + (3*ArcSinh[a*x])/a)/(2*a))/(3*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{i\left(2a^{2}x^{2}-3iax-4\right)\sqrt{a^{2}x^{2}+1}}{6a^{3}}-\frac{\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}+1}\right)}{2a^{2}\sqrt{a^{2}}}$	67
default	$\frac{x\sqrt{a^{2}x^{2}+1}}{2a^{2}}-\frac{\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}+1}\right)}{2a^{2}\sqrt{a^{2}}}+ia\left(\frac{x^{2}\sqrt{a^{2}x^{2}+1}}{3a^{2}}-\frac{2\sqrt{a^{2}x^{2}+1}}{3a^{4}}\right)$	92
meijerg	$\frac{\frac{\sqrt{\pi}\,x\left(a^{2}\right)^{\frac{3}{2}}\sqrt{a^{2}x^{2}+1}}{a^{2}}-\frac{\sqrt{\pi}\left(a^{2}\right)^{\frac{3}{2}}\operatorname{arcsinh}(ax)}{a^{3}}}{2a^{2}\sqrt{\pi}\,\sqrt{a^{2}}}+\frac{i\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(-4a^{2}x^{2}+8\right)\sqrt{a^{2}x^{2}+1}}{6}\right)}{2a^{3}\sqrt{\pi}}$	98

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*I*(2*a^2*x^2-3*I*a*x-4)*(a^2*x^2+1)^(1/2)/a^3-1/2/a^2*ln(a^2*x/(a^2)^(
1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.64

$$\int e^{i\arctan(ax)}x^{2}\,dx=\frac{\sqrt{a^{2}x^{2}+1}(2i\,a^{2}x^{2}+3\,ax-4i)+3\,\log\left(-ax+\sqrt{a^{2}x^{2}+1}\right)}{6\,a^{3}}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2,x, algorithm="fricas")
```

output
```
1/6*(sqrt(a^2*x^2 + 1)*(2*I*a^2*x^2 + 3*a*x - 4*I) + 3*log(-a*x + sqrt(a^2
*x^2 + 1)))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.12

$$\int e^{i\arctan(ax)}x^2\,dx = \begin{cases} \sqrt{a^2x^2+1}\left(\frac{ix^2}{3a}+\frac{x}{2a^2}-\frac{2i}{3a^3}\right)-\frac{\log\left(2a^2x+2\sqrt{a^2x^2+1}\sqrt{a^2}\right)}{2a^2\sqrt{a^2}} & \text{for } a^2\neq 0 \\ \frac{iax^4}{4}+\frac{x^3}{3} & \text{otherwise} \end{cases}$$

input
```
integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)*x**2,x)
```

output
```
Piecewise((sqrt(a**2*x**2 + 1)*(I*x**2/(3*a) + x/(2*a**2) - 2*I/(3*a**3))
- log(2*a**2*x + 2*sqrt(a**2*x**2 + 1)*sqrt(a**2))/(2*a**2*sqrt(a**2)), Ne
(a**2, 0)), (I*a*x**4/4 + x**3/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int e^{i\arctan(ax)}x^2\,dx = \frac{i\sqrt{a^2x^2+1}x^2}{3a}+\frac{\sqrt{a^2x^2+1}x}{2a^2}-\frac{\operatorname{arsinh}(ax)}{2a^3}-\frac{2i\sqrt{a^2x^2+1}}{3a^3}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2,x, algorithm="maxima")
```

output
```
1/3*I*sqrt(a^2*x^2 + 1)*x^2/a + 1/2*sqrt(a^2*x^2 + 1)*x/a^2 - 1/2*arcsinh(
a*x)/a^3 - 2/3*I*sqrt(a^2*x^2 + 1)/a^3
```

Giac [F(-2)]

Exception generated.

$$\int e^{i\arctan(ax)}x^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.61 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int e^{i\arctan(ax)}x^2\,dx=\frac{\sqrt{a^2\,x^2+1}\,\left(\frac{x\,\sqrt{a^2}}{2\,a^2}-\frac{a\,2\mathrm{i}}{3\,{\left(a^2\right)}^{3/2}}+\frac{a^3\,x^2\,1\mathrm{i}}{3\,{\left(a^2\right)}^{3/2}}\right)}{\sqrt{a^2}}-\frac{\mathrm{asinh}\left(x\,\sqrt{a^2}\right)}{2\,a^2\,\sqrt{a^2}}$$

input
```
int((x^2*(a*x*1i + 1))/(a^2*x^2 + 1)^(1/2),x)
```

output
```
((a^2*x^2 + 1)^(1/2)*((a^3*x^2*1i)/(3*(a^2)^(3/2)) - (a*2i)/(3*(a^2)^(3/2)
) + (x*(a^2)^(1/2))/(2*a^2)))/(a^2)^(1/2) - asinh(x*(a^2)^(1/2))/(2*a^2*(a
^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$\int e^{i\arctan(ax)}x^2\,dx$$
$$=\frac{2\sqrt{a^2x^2+1}\,a^2i\,x^2+3\sqrt{a^2x^2+1}\,ax-4\sqrt{a^2x^2+1}\,i-3\log\left(\sqrt{a^2x^2+1}+ax\right)}{6a^3}$$

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2,x)
```

output
```
(2*sqrt(a**2*x**2 + 1)*a**2*i*x**2 + 3*sqrt(a**2*x**2 + 1)*a*x - 4*sqrt(a*
*2*x**2 + 1)*i - 3*log(sqrt(a**2*x**2 + 1) + a*x))/(6*a**3)
```

3.19 $\int e^{i \arctan(ax)} x \, dx$

Optimal result

Integrand size = 12, antiderivative size = 54

$$\int e^{i \arctan(ax)} x \, dx = \frac{\sqrt{1+a^2x^2}}{a^2} + \frac{ix\sqrt{1+a^2x^2}}{2a} - \frac{i\text{arcsinh}(ax)}{2a^2}$$

output
```
(a^2*x^2+1)^(1/2)/a^2+1/2*I*x*(a^2*x^2+1)^(1/2)/a-1/2*I*arcsinh(a*x)/a^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.70

$$\int e^{i \arctan(ax)} x \, dx = \frac{(2+iax)\sqrt{1+a^2x^2} - i\text{arcsinh}(ax)}{2a^2}$$

input
```
Integrate[E^(I*ArcTan[a*x])*x,x]
```

output
```
((2 + I*a*x)*Sqrt[1 + a^2*x^2] - I*ArcSinh[a*x])/(2*a^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5583, 533, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{i\arctan(ax)}dx$$

$$\Big\downarrow \ 5583$$

$$\int \frac{x(1+iax)}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \ 533$$

$$\frac{ix\sqrt{a^2x^2+1}}{2a}-\frac{\int \frac{a(i-2ax)}{\sqrt{a^2x^2+1}}dx}{2a^2}$$

$$\Big\downarrow \ 27$$

$$\frac{ix\sqrt{a^2x^2+1}}{2a}-\frac{\int \frac{i-2ax}{\sqrt{a^2x^2+1}}dx}{2a}$$

$$\Big\downarrow \ 455$$

$$\frac{ix\sqrt{a^2x^2+1}}{2a}-\frac{-\frac{2\sqrt{a^2x^2+1}}{a}+i\int \frac{1}{\sqrt{a^2x^2+1}}dx}{2a}$$

$$\Big\downarrow \ 222$$

$$\frac{ix\sqrt{a^2x^2+1}}{2a}-\frac{-\frac{2\sqrt{a^2x^2+1}}{a}+\frac{i\text{arcsinh}(ax)}{a}}{2a}$$

input
```
Int[E^(I*ArcTan[a*x])*x,x]
```

output
```
((I/2)*x*Sqrt[1 + a^2*x^2])/a - ((-2*Sqrt[1 + a^2*x^2])/a + (I*ArcSinh[a*x
])/a)/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

method	result	size
risch	$\frac{i(ax-2i)\sqrt{a^2x^2+1}}{2a^2}-\frac{i\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2a\sqrt{a^2}}$	59
default	$\frac{\sqrt{a^2x^2+1}}{a^2}+ia\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2a^2\sqrt{a^2}}\right)$	72
meijerg	$\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{a^2x^2+1}}{2a^2\sqrt{\pi}}+\frac{i\left(\frac{\sqrt{\pi}\,x\left(a^2\right)^{\frac{3}{2}}\sqrt{a^2x^2+1}}{a^2}-\frac{\sqrt{\pi}\left(a^2\right)^{\frac{3}{2}}\operatorname{arcsinh}(ax)}{a^3}\right)}{2a\sqrt{\pi}\,\sqrt{a^2}}$	88

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*I*(a*x-2*I)*(a^2*x^2+1)^(1/2)/a^2-1/2*I/a*ln(a^2*x/(a^2)^(1/2)+(a^2*x^
2+1)^(1/2))/(a^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int e^{i\arctan(ax)}x\,dx = \frac{\sqrt{a^2x^2+1}(i\,ax+2)+i\,\log\left(-ax+\sqrt{a^2x^2+1}\right)}{2\,a^2}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x,x, algorithm="fricas")
```

output
```
1/2*(sqrt(a^2*x^2 + 1)*(I*a*x + 2) + I*log(-a*x + sqrt(a^2*x^2 + 1)))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.44

$$\int e^{i\arctan(ax)}x\,dx = \begin{cases} \sqrt{a^2x^2+1}\left(\frac{ix}{2a}+\frac{1}{a^2}\right)-\frac{i\log\left(2a^2x+2\sqrt{a^2x^2+1}\sqrt{a^2}\right)}{2a\sqrt{a^2}} & \text{for } a^2\neq 0 \\ \frac{iax^3}{3}+\frac{x^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)*x,x)
```

output
```
Piecewise((sqrt(a**2*x**2 + 1)*(I*x/(2*a) + a**(-2)) - I*log(2*a**2*x + 2*
sqrt(a**2*x**2 + 1)*sqrt(a**2))/(2*a*sqrt(a**2)), Ne(a**2, 0)), (I*a*x**3/
3 + x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int e^{i\arctan(ax)}x\,dx = \frac{i\sqrt{a^2x^2+1}x}{2\,a} - \frac{i\,\mathrm{arsinh}\,(ax)}{2\,a^2} + \frac{\sqrt{a^2x^2+1}}{a^2}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x,x, algorithm="maxima")
```

output
```
1/2*I*sqrt(a^2*x^2 + 1)*x/a - 1/2*I*arcsinh(a*x)/a^2 + sqrt(a^2*x^2 + 1)/a
^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int e^{i\arctan(ax)}x\,dx = -\frac{1}{2}\sqrt{a^2x^2+1}\left(-\frac{i\,x}{a} - \frac{2}{a^2}\right) + \frac{i\,\log\left(-x|a| + \sqrt{a^2x^2+1}\right)}{2\,a|a|}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x,x, algorithm="giac")
```

output
```
-1/2*sqrt(a^2*x^2 + 1)*(-I*x/a - 2/a^2) + 1/2*I*log(-x*abs(a) + sqrt(a^2*x
^2 + 1))/(a*abs(a))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int e^{i\arctan(ax)}x\,dx = \frac{\left(\frac{1}{\sqrt{a^2}} + \frac{x\,\sqrt{a^2}\,1\mathrm{i}}{2\,a}\right)\,\sqrt{a^2\,x^2+1} - \frac{\mathrm{asinh}\left(x\,\sqrt{a^2}\right)\,1\mathrm{i}}{2\,a}}{\sqrt{a^2}}$$

input
```
int((x*(a*x*1i + 1))/(a^2*x^2 + 1)^(1/2),x)
```

output
```
((1/(a^2)^(1/2) + (x*(a^2)^(1/2)*1i)/(2*a))*(a^2*x^2 + 1)^(1/2) - (asinh(x
*(a^2)^(1/2))*1i)/(2*a))/(a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int e^{i\arctan(ax)}x\,dx = \frac{\sqrt{a^2x^2+1}\,aix+2\sqrt{a^2x^2+1}-\log\left(\sqrt{a^2x^2+1}+ax\right)i}{2a^2}$$

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x,x)
```

output
```
(sqrt(a**2*x**2 + 1)*a*i*x + 2*sqrt(a**2*x**2 + 1) - log(sqrt(a**2*x**2 +
1) + a*x)*i)/(2*a**2)
```

3.20 $\int e^{i \arctan(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 29

$$\int e^{i \arctan(ax)}\,dx=\frac{i\sqrt{1+a^2x^2}}{a}+\frac{\operatorname{arcsinh}(ax)}{a}$$

output
```
I*(a^2*x^2+1)^(1/2)/a+arcsinh(a*x)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int e^{i \arctan(ax)}\,dx=\frac{i\sqrt{1+a^2x^2}+\operatorname{arcsinh}(ax)}{a}$$

input
```
Integrate[E^(I*ArcTan[a*x]),x]
```

output
```
(I*Sqrt[1 + a^2*x^2] + ArcSinh[a*x])/a
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5582, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{i \arctan (ax)} dx$$

$$\Big\downarrow \text{5582}$$

$$\int \frac{1+iax}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{455}$$

$$\int \frac{1}{\sqrt{a^2x^2+1}}dx+\frac{i\sqrt{a^2x^2+1}}{a}$$

$$\Big\downarrow \text{222}$$

$$\frac{\text{arcsinh}(ax)}{a}+\frac{i\sqrt{a^2x^2+1}}{a}$$

input
```
Int[E^(I*ArcTan[a*x]),x]
```

output
```
(I*Sqrt[1 + a^2*x^2])/a + ArcSinh[a*x]/a
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

rule 5582
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 - I*a*x)^((I*n + 1)/2)
/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2]), x] /; FreeQ[a, x] && Intege
rQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

method	result	size
meijerg	$\frac{\operatorname{arcsinh}(ax)}{a}+\frac{i\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{a^{2}x^{2}+1}\right)}{2a\sqrt{\pi}}$	41
default	$\frac{\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}+1}\right)}{\sqrt{a^{2}}}+\frac{i\sqrt{a^{2}x^{2}+1}}{a}$	48
risch	$\frac{\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}+1}\right)}{\sqrt{a^{2}}}+\frac{i\sqrt{a^{2}x^{2}+1}}{a}$	48

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(a*x)/a+1/2*I/a/Pi^(1/2)*(-2*Pi^(1/2)+2*Pi^(1/2)*(a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int e^{i\arctan(ax)}\,dx=\frac{i\sqrt{a^{2}x^{2}+1}-\log\left(-ax+\sqrt{a^{2}x^{2}+1}\right)}{a}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
(I*sqrt(a^2*x^2 + 1) - log(-a*x + sqrt(a^2*x^2 + 1)))/a
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(20) = 40.

Time = 0.43 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.10

$$\int e^{i\arctan(ax)}\,dx = \begin{cases} \frac{\log\left(2a^2x+2\sqrt{a^2x^2+1}\sqrt{a^2}\right)}{\sqrt{a^2}} + \frac{i\sqrt{a^2x^2+1}}{a} & \text{for } a^2 \neq 0 \\ \frac{iax^2}{2} + x & \text{otherwise} \end{cases}$$

input
```
integrate((1+I*a*x)/(a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((log(2*a**2*x + 2*sqrt(a**2*x**2 + 1)*sqrt(a**2))/sqrt(a**2) + I
*sqrt(a**2*x**2 + 1)/a, Ne(a**2, 0)), (I*a*x**2/2 + x, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int e^{i\arctan(ax)}\,dx = \frac{\operatorname{arsinh}(ax)}{a} + \frac{i\sqrt{a^2x^2+1}}{a}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(a*x)/a + I*sqrt(a^2*x^2 + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int e^{i\arctan(ax)}\,dx = -\frac{\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{|a|} + \frac{i\sqrt{a^2x^2+1}}{a}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a) + I*sqrt(a^2*x^2 + 1)/a
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int e^{i\arctan(ax)}\,dx = \frac{\operatorname{asinh}\left(x\sqrt{a^2}\right)}{\sqrt{a^2}} + \frac{\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{a}$$

input
```
int((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2),x)
```

output
```
((a^2*x^2 + 1)^(1/2)*1i)/a + asinh(x*(a^2)^(1/2))/(a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int e^{i\arctan(ax)}\,dx = \frac{\sqrt{a^2x^2+1}\,i + \log\left(\sqrt{a^2x^2+1}+ax\right)}{a}$$

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2),x)
```

output
```
(sqrt(a**2*x**2 + 1)*i + log(sqrt(a**2*x**2 + 1) + a*x))/a
```

3.21 $\int \frac{e^{i \arctan(ax)}}{x} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \frac{e^{i \arctan(ax)}}{x} \, dx = i\text{arcsinh}(ax) - \text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

output
```
I*arcsinh(a*x)-arctanh((a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{e^{i \arctan(ax)}}{x} \, dx = i\text{arcsinh}(ax) + \log(x) - \log\left(1+\sqrt{1+a^2x^2}\right)$$

input
```
Integrate[E^(I*ArcTan[a*x])/x,x]
```

output
```
I*ArcSinh[a*x] + Log[x] - Log[1 + Sqrt[1 + a^2*x^2]]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5583, 538, 222, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{i\arctan(ax)}}{x}dx$$

$$\Big\downarrow \; 5583$$

$$\int \frac{1+iax}{x\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \; 538$$

$$\int \frac{1}{x\sqrt{a^2x^2+1}}dx+ia\int \frac{1}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \; 222$$

$$\int \frac{1}{x\sqrt{a^2x^2+1}}dx+i\text{arcsinh}(ax)$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{a^2x^2+1}}dx^2+i\text{arcsinh}(ax)$$

$$\Big\downarrow \; 73$$

$$\frac{\int \frac{1}{\frac{x^4}{a^2}-\frac{1}{a^2}}d\sqrt{a^2x^2+1}}{a^2}+i\text{arcsinh}(ax)$$

$$\Big\downarrow \; 221$$

$$-\text{arctanh}\left(\sqrt{a^2x^2+1}\right)+i\text{arcsinh}(ax)$$

input
```
Int[E^(I*ArcTan[a*x])/x,x]
```

output
```
I*ArcSinh[a*x] - ArcTanh[Sqrt[1 + a^2*x^2]]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(22) = 44$.

Time $= 0.13$ (sec) , antiderivative size $= 48$, normalized size of antiderivative $= 1.92$

method	result	size
default	$-\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)+\frac{ia\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{\sqrt{a^2}}$	48
meijerg	$\frac{(-2\ln(2)+2\ln(x)+\ln(a^2))\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{a^2x^2+1}}{2}\right)}{2\sqrt{\pi}}+i\operatorname{arcsinh}(ax)$	53

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-arctanh(1/(a^2*x^2+1)^(1/2))+I*a*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/
(a^2)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(21) = 42.

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

$$\int \frac{e^{i\arctan(ax)}}{x}\,dx = -\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-i\log\left(-ax+\sqrt{a^2x^2+1}\right) + \log\left(-ax+\sqrt{a^2x^2+1}-1\right)$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x,x, algorithm="fricas")
```

output
```
-log(-a*x + sqrt(a^2*x^2 + 1) + 1) - I*log(-a*x + sqrt(a^2*x^2 + 1)) + log
(-a*x + sqrt(a^2*x^2 + 1) - 1)
```

Sympy [A] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.92

$$\int \frac{e^{i\arctan(ax)}}{x}\,dx = ia\left(\begin{cases}\frac{\log\left(2a^2x+2\sqrt{a^2x^2+1}\sqrt{a^2}\right)}{\sqrt{a^2}} & \text{for } a^2\neq 0\\ x & \text{otherwise}\end{cases}\right) - \operatorname{asinh}\left(\frac{1}{ax}\right)$$

input
```
integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/x,x)
```

output
```
I*a*Piecewise((log(2*a**2*x + 2*sqrt(a**2*x**2 + 1)*sqrt(a**2))/sqrt(a**2)
, Ne(a**2, 0)), (x, True)) - asinh(1/(a*x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \frac{e^{i\arctan(ax)}}{x}\,dx = i\operatorname{arsinh}(ax) - \operatorname{arsinh}\left(\frac{1}{a|x|}\right)$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x,x, algorithm="maxima")
```

output
```
I*arcsinh(a*x) - arcsinh(1/(a*abs(x)))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(21) = 42.

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.72

$$\int \frac{e^{i\arctan(ax)}}{x}\,dx = -\frac{i\,a\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{|a|} - \log\left(\left|-x|a|+\sqrt{a^2x^2+1}+1\right|\right) + \log\left(\left|-x|a|+\sqrt{a^2x^2+1}-1\right|\right)$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x,x, algorithm="giac")
```

output
```
-I*a*log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a) - log(abs(-x*abs(a) + sqrt(
a^2*x^2 + 1) + 1)) + log(abs(-x*abs(a) + sqrt(a^2*x^2 + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{e^{i\arctan(ax)}}{x}\,dx = -\mathrm{atanh}\left(\sqrt{a^2\,x^2+1}\right) + \frac{a\,\mathrm{asinh}\left(x\,\sqrt{a^2}\right)\,1\mathrm{i}}{\sqrt{a^2}}$$

input
```
int((a*x*1i + 1)/(x*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
(a*asinh(x*(a^2)^(1/2))*1i)/(a^2)^(1/2) - atanh((a^2*x^2 + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.08

$$\int \frac{e^{i\arctan(ax)}}{x}\,dx = \log\left(\sqrt{a^2x^2+1}+ax-1\right) - \log\left(\sqrt{a^2x^2+1}+ax+1\right) + \log\left(\sqrt{a^2x^2+1}+ax\right)i$$

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2)/x,x)
```

output
```
log(sqrt(a**2*x**2 + 1) + a*x - 1) - log(sqrt(a**2*x**2 + 1) + a*x + 1) +
log(sqrt(a**2*x**2 + 1) + a*x)*i
```

3.22 $\int \frac{e^{i \arctan(ax)}}{x^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 38

$$\int \frac{e^{i \arctan(ax)}}{x^2} \, dx = -\frac{\sqrt{1+a^2x^2}}{x} - ia\text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

output
```
-(a^2*x^2+1)^(1/2)/x-I*a*arctanh((a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{e^{i \arctan(ax)}}{x^2} \, dx = -\frac{\sqrt{1+a^2x^2}}{x} + ia\log(x) - ia\log\left(1+\sqrt{1+a^2x^2}\right)$$

input
```
Integrate[E^(I*ArcTan[a*x])/x^2,x]
```

output
```
-(Sqrt[1 + a^2*x^2]/x) + I*a*Log[x] - I*a*Log[1 + Sqrt[1 + a^2*x^2]]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5583, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{i\arctan(ax)}}{x^2}dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{1+iax}{x^2\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{534}$$

$$-\frac{\sqrt{a^2x^2+1}}{x}+ia\int \frac{1}{x\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{243}$$

$$-\frac{\sqrt{a^2x^2+1}}{x}+\frac{1}{2}ia\int \frac{1}{x^2\sqrt{a^2x^2+1}}dx^2$$

$$\Big\downarrow \text{73}$$

$$-\frac{\sqrt{a^2x^2+1}}{x}+\frac{i\int \frac{1}{\frac{x^4}{a^2}-\frac{1}{a^2}}d\sqrt{a^2x^2+1}}{a}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\sqrt{a^2x^2+1}}{x}-ia\text{arctanh}\left(\sqrt{a^2x^2+1}\right)$$

input
```
Int[E^(I*ArcTan[a*x])/x^2,x]
```

output
```
-(Sqrt[1 + a^2*x^2]/x) - I*a*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{\sqrt{a^{2} x^{2}+1}}{x}-i a \,\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2} x^{2}+1}}\right)$	34
risch	$-\frac{\sqrt{a^{2} x^{2}+1}}{x}-i a \,\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2} x^{2}+1}}\right)$	34
meijerg	$-\frac{\sqrt{a^{2} x^{2}+1}}{x}+\frac{i a \left(\left(-2 \ln \left(2\right)+2 \ln \left(x \right)+\ln \left(a^{2}\right)\right) \sqrt{\pi}-2 \sqrt{\pi}\, \ln \left(\frac{1}{2}+\frac{\sqrt{a^{2} x^{2}+1}}{2}\right)\right)}{2 \sqrt{\pi}}$	64

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(a^2*x^2+1)^(1/2)/x-I*a*arctanh(1/(a^2*x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(32) = 64$.

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \frac{e^{i\arctan(ax)}}{x^2}\,dx = \frac{-i\,ax\log\left(-ax+\sqrt{a^2x^2+1}+1\right)+i\,ax\log\left(-ax+\sqrt{a^2x^2+1}-1\right)-ax-\sqrt{a^2x^2+1}}{x}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
(-I*a*x*log(-a*x + sqrt(a^2*x^2 + 1) + 1) + I*a*x*log(-a*x + sqrt(a^2*x^2
+ 1) - 1) - a*x - sqrt(a^2*x^2 + 1))/x
```

Sympy [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \frac{e^{i\arctan(ax)}}{x^2}\,dx = -a\sqrt{1+\frac{1}{a^2x^2}} - ia\operatorname{asinh}\left(\frac{1}{ax}\right)$$

input
```
integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/x**2,x)
```

output
```
-a*sqrt(1 + 1/(a**2*x**2)) - I*a*asinh(1/(a*x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{e^{i\arctan(ax)}}{x^2}\,dx = -i\,a\,\mathrm{arsinh}\left(\frac{1}{a|x|}\right) - \frac{\sqrt{a^2x^2+1}}{x}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-I*a*arcsinh(1/(a*abs(x))) - sqrt(a^2*x^2 + 1)/x
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(32) = 64.

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.97

$$\int \frac{e^{i\arctan(ax)}}{x^2}\,dx = -i\,a\log\left(\left|-x|a| + \sqrt{a^2x^2+1} + 1\right|\right) + i\,a\log\left(\left|-x|a| + \sqrt{a^2x^2+1} - 1\right|\right) + \frac{2\,|a|}{\left(x|a| - \sqrt{a^2x^2+1}\right)^2 - 1}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac")
```

output
```
-I*a*log(abs(-x*abs(a) + sqrt(a^2*x^2 + 1) + 1)) + I*a*log(abs(-x*abs(a) +
 sqrt(a^2*x^2 + 1) - 1)) + 2*abs(a)/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{e^{i \arctan(ax)}}{x^2} \, dx = -\frac{\sqrt{a^2 \, x^2 + 1}}{x} - a \, \mathrm{atanh}\left(\sqrt{a^2 \, x^2 + 1}\right) \, 1\mathrm{i}$$

input
```
int((a*x*1i + 1)/(x^2*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
- a*atanh((a^2*x^2 + 1)^(1/2))*1i - (a^2*x^2 + 1)^(1/2)/x
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.63

$$\int \frac{e^{i \arctan(ax)}}{x^2} \, dx = \frac{-\sqrt{a^2x^2+1} + \log\left(\sqrt{a^2x^2+1} + ax - 1\right) aix - \log\left(\sqrt{a^2x^2+1} + ax + 1\right) aix - ax}{x}$$

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^2,x)
```

output
```
( - sqrt(a**2*x**2 + 1) + log(sqrt(a**2*x**2 + 1) + a*x - 1)*a*i*x - log(s
qrt(a**2*x**2 + 1) + a*x + 1)*a*i*x - a*x)/x
```

3.23 $\int \frac{e^{i \arctan(ax)}}{x^3} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 63

$$\int \frac{e^{i \arctan(ax)}}{x^3} \, dx = -\frac{\sqrt{1+a^2x^2}}{2x^2} - \frac{ia\sqrt{1+a^2x^2}}{x} + \frac{1}{2}a^2 \operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

output
```
-1/2*(a^2*x^2+1)^(1/2)/x^2-I*a*(a^2*x^2+1)^(1/2)/x+1/2*a^2*arctanh((a^2*x^
2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{e^{i \arctan(ax)}}{x^3} \, dx = \frac{1}{2}\left(\frac{(-1-2iax)\sqrt{1+a^2x^2}}{x^2} - a^2 \log(x) + a^2 \log\left(1+\sqrt{1+a^2x^2}\right)\right)$$

input
```
Integrate[E^(I*ArcTan[a*x])/x^3,x]
```

output
```
(((-1 - (2*I)*a*x)*Sqrt[1 + a^2*x^2])/x^2 - a^2*Log[x] + a^2*Log[1 + Sqrt[
1 + a^2*x^2]])/2
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5583, 539, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{i\arctan(ax)}}{x^3}dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{1+iax}{x^3\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{539}$$

$$-\frac{\sqrt{a^2x^2+1}}{2x^2}-\frac{1}{2}\int -\frac{a(2i-ax)}{x^2\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{1}{2}\int \frac{a(2i-ax)}{x^2\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{1}{2}a\int \frac{2i-ax}{x^2\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{534}$$

$$-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{1}{2}a\left(-a\int \frac{1}{x\sqrt{a^2x^2+1}}dx-\frac{2i\sqrt{a^2x^2+1}}{x}\right)$$

$$\Big\downarrow \text{243}$$

$$-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{1}{2}a\left(-\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2x^2+1}}dx^2-\frac{2i\sqrt{a^2x^2+1}}{x}\right)$$

$$\Big\downarrow \text{73}$$

$$-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{1}{2}a\left(-\frac{\int \frac{1}{\frac{x^4}{a^2}-\frac{1}{a^2}}d\sqrt{a^2x^2+1}}{a}-\frac{2i\sqrt{a^2x^2+1}}{x}\right)$$

$$\downarrow \text{221}$$

$$-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{1}{2}a\left(a\text{arctanh}\left(\sqrt{a^2x^2+1}\right)-\frac{2i\sqrt{a^2x^2+1}}{x}\right)$$

input
```
Int[E^(I*ArcTan[a*x])/x^3,x]
```

output
```
-1/2*Sqrt[1 + a^2*x^2]/x^2 + (a*(((-2*I)*Sqrt[1 + a^2*x^2])/x + a*ArcTanh[
Sqrt[1 + a^2*x^2]]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{\sqrt{a^{2} x^{2}+1}}{2 x^{2}}+\frac{a^{2} \operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2} x^{2}+1}}\right)}{2}-\frac{i a \sqrt{a^{2} x^{2}+1}}{x}$	53
risch	$-\frac{i \left(2 a^{3} x^{3}-i a^{2} x^{2}+2 a x -i\right)}{2 x^{2} \sqrt{a^{2} x^{2}+1}}+\frac{a^{2} \operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2} x^{2}+1}}\right)}{2}$	60
meijerg	$\frac{a^{2} \left(-\frac{\sqrt{\pi}}{x^{2} a^{2}}-\frac{\left(1-2 \ln \left(2\right)+2 \ln \left(x \right)+\ln \left(a^{2}\right)\right) \sqrt{\pi}}{2}+\frac{\sqrt{\pi}\, \left(4 a^{2} x^{2}+8\right)}{8 a^{2} x^{2}}-\frac{\sqrt{\pi}\, \sqrt{a^{2} x^{2}+1}}{a^{2} x^{2}}+\sqrt{\pi}\, \ln \left(\frac{1}{2}+\frac{\sqrt{a^{2} x^{2}+1}}{2}\right)\right)}{2 \sqrt{\pi}}-\frac{i a \sqrt{a^{2} x^{2}+1}}{x}$	122

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a^2*x^2+1)^(1/2)/x^2+1/2*a^2*arctanh(1/(a^2*x^2+1)^(1/2))-I*a*(a^2*x
^2+1)^(1/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.32

$$\int \frac{e^{i\arctan(ax)}}{x^3}\,dx = \frac{a^2x^2\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-a^2x^2\log\left(-ax+\sqrt{a^2x^2+1}-1\right)-2i\,a^2x^2+\sqrt{a^2x^2+1}(-2i\,ax-1)}{2\,x^2}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
1/2*(a^2*x^2*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - a^2*x^2*log(-a*x + sqrt(a
^2*x^2 + 1) - 1) - 2*I*a^2*x^2 + sqrt(a^2*x^2 + 1)*(-2*I*a*x - 1))/x^2
```

Sympy [A] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int \frac{e^{i\arctan(ax)}}{x^3}\,dx = -ia^2\sqrt{1+\frac{1}{a^2x^2}}+\frac{a^2\operatorname{asinh}\left(\frac{1}{ax}\right)}{2}-\frac{a\sqrt{1+\frac{1}{a^2x^2}}}{2x}$$

input
```
integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/x**3,x)
```

output
```
-I*a**2*sqrt(1 + 1/(a**2*x**2)) + a**2*asinh(1/(a*x))/2 - a*sqrt(1 + 1/(a*
*2*x**2))/(2*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int \frac{e^{i\arctan(ax)}}{x^3}\,dx = \frac{1}{2}\,a^2\operatorname{arsinh}\left(\frac{1}{a|x|}\right)-\frac{i\sqrt{a^2x^2+1}a}{x}-\frac{\sqrt{a^2x^2+1}}{2\,x^2}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
1/2*a^2*arcsinh(1/(a*abs(x))) - I*sqrt(a^2*x^2 + 1)*a/x - 1/2*sqrt(a^2*x^2
 + 1)/x^2
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(51) = 102$.

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.43

$$\int \frac{e^{i\arctan(ax)}}{x^3}\,dx$$
$$= \frac{1}{2}a^2\log\left(\left|-x|a|+\sqrt{a^2x^2+1}+1\right|\right)-\frac{1}{2}a^2\log\left(\left|-x|a|+\sqrt{a^2x^2+1}-1\right|\right)$$
$$+\frac{\left(x|a|-\sqrt{a^2x^2+1}\right)^3a^2+2i\left(x|a|-\sqrt{a^2x^2+1}\right)^2a|a|+\left(x|a|-\sqrt{a^2x^2+1}\right)a^2-2i\,a|a|}{\left(\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1\right)^2}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^3,x, algorithm="giac")
```

output
```
1/2*a^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 + 1) + 1)) - 1/2*a^2*log(abs(-x*a
bs(a) + sqrt(a^2*x^2 + 1) - 1)) + ((x*abs(a) - sqrt(a^2*x^2 + 1))^3*a^2 +
2*I*(x*abs(a) - sqrt(a^2*x^2 + 1))^2*a*abs(a) + (x*abs(a) - sqrt(a^2*x^2 +
 1))*a^2 - 2*I*a*abs(a))/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{e^{i\arctan(ax)}}{x^3}\,dx = \frac{a^2\,\mathrm{atanh}\left(\sqrt{a^2\,x^2+1}\right)}{2}-\frac{\sqrt{a^2\,x^2+1}}{2\,x^2}-\frac{a\,\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{x}$$

input
```
int((a*x*1i + 1)/(x^3*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
(a^2*atanh((a^2*x^2 + 1)^(1/2)))/2 - (a^2*x^2 + 1)^(1/2)/(2*x^2) - (a*(a^2
*x^2 + 1)^(1/2)*1i)/x
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int \frac{e^{i\arctan(ax)}}{x^3}\,dx$$
$$= \frac{-2\sqrt{a^2x^2+1}\,aix - \sqrt{a^2x^2+1} - \log\left(\sqrt{a^2x^2+1}+ax-1\right)a^2x^2 + \log\left(\sqrt{a^2x^2+1}+ax+1\right)a^2x^2}{2x^2}$$

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^3,x)
```

output
```
( - 2*sqrt(a**2*x**2 + 1)*a*i*x - sqrt(a**2*x**2 + 1) - log(sqrt(a**2*x**2
 + 1) + a*x - 1)*a**2*x**2 + log(sqrt(a**2*x**2 + 1) + a*x + 1)*a**2*x**2)
/(2*x**2)
```

3.24 $\int \frac{e^{i\arctan(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 90

$$\int \frac{e^{i\arctan(ax)}}{x^4}\,dx = -\frac{\sqrt{1+a^2x^2}}{3x^3} - \frac{ia\sqrt{1+a^2x^2}}{2x^2} + \frac{2a^2\sqrt{1+a^2x^2}}{3x} + \frac{1}{2}ia^3\operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

output
```
-1/3*(a^2*x^2+1)^(1/2)/x^3-1/2*I*a*(a^2*x^2+1)^(1/2)/x^2+2/3*a^2*(a^2*x^2+
1)^(1/2)/x+1/2*I*a^3*arctanh((a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.78

$$\int \frac{e^{i\arctan(ax)}}{x^4}\,dx = \frac{1}{6}\left(\frac{\sqrt{1+a^2x^2}(-2-3iax+4a^2x^2)}{x^3} - 3ia^3\log(x) + 3ia^3\log\left(1+\sqrt{1+a^2x^2}\right)\right)$$

input
```
Integrate[E^(I*ArcTan[a*x])/x^4,x]
```

output
```
((Sqrt[1 + a^2*x^2]*(-2 - (3*I)*a*x + 4*a^2*x^2))/x^3 - (3*I)*a^3*Log[x] +
 (3*I)*a^3*Log[1 + Sqrt[1 + a^2*x^2]])/6
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5583, 539, 25, 27, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{i\arctan(ax)}}{x^4}dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{1+iax}{x^4\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{539}$$

$$-\frac{\sqrt{a^2x^2+1}}{3x^3}-\frac{1}{3}\int -\frac{a(3i-2ax)}{x^3\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{a^2x^2+1}}{3x^3}+\frac{1}{3}\int \frac{a(3i-2ax)}{x^3\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{a^2x^2+1}}{3x^3}+\frac{1}{3}a\int \frac{3i-2ax}{x^3\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{539}$$

$$-\frac{\sqrt{a^2x^2+1}}{3x^3}+\frac{1}{3}a\left(-\frac{1}{2}\int \frac{a(3iax+4)}{x^2\sqrt{a^2x^2+1}}dx-\frac{3i\sqrt{a^2x^2+1}}{2x^2}\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{a^2x^2+1}}{3x^3}+\frac{1}{3}a\left(-\frac{1}{2}a\int \frac{3iax+4}{x^2\sqrt{a^2x^2+1}}dx-\frac{3i\sqrt{a^2x^2+1}}{2x^2}\right)$$

$$\Big\downarrow \ 534$$

$$-\frac{\sqrt{a^2x^2+1}}{3x^3}+\frac{1}{3}a\left(-\frac{1}{2}a\left(-\frac{4\sqrt{a^2x^2+1}}{x}+3ia\int\frac{1}{x\sqrt{a^2x^2+1}}dx\right)-\frac{3i\sqrt{a^2x^2+1}}{2x^2}\right)$$

$$\Big\downarrow \ 243$$

$$-\frac{\sqrt{a^2x^2+1}}{3x^3}+\frac{1}{3}a\left(-\frac{1}{2}a\left(-\frac{4\sqrt{a^2x^2+1}}{x}+\frac{3}{2}ia\int\frac{1}{x^2\sqrt{a^2x^2+1}}dx^2\right)-\frac{3i\sqrt{a^2x^2+1}}{2x^2}\right)$$

$$\Big\downarrow \ 73$$

$$-\frac{\sqrt{a^2x^2+1}}{3x^3}+\frac{1}{3}a\left(-\frac{1}{2}a\left(-\frac{4\sqrt{a^2x^2+1}}{x}+\frac{3i\int\frac{1}{\frac{x^4}{a^2}-\frac{1}{a^2}}d\sqrt{a^2x^2+1}}{a}\right)-\frac{3i\sqrt{a^2x^2+1}}{2x^2}\right)$$

$$\Big\downarrow \ 221$$

$$-\frac{\sqrt{a^2x^2+1}}{3x^3}+\frac{1}{3}a\left(-\frac{1}{2}a\left(-\frac{4\sqrt{a^2x^2+1}}{x}-3ia\text{arctanh}\left(\sqrt{a^2x^2+1}\right)\right)-\frac{3i\sqrt{a^2x^2+1}}{2x^2}\right)$$

input
```
Int[E^(I*ArcTan[a*x])/x^4,x]
```

output
```
-1/3*Sqrt[1 + a^2*x^2]/x^3 + (a*((((-3*I)/2)*Sqrt[1 + a^2*x^2])/x^2 - (a*(
(-4*Sqrt[1 + a^2*x^2])/x - (3*I)*a*ArcTanh[Sqrt[1 + a^2*x^2]]))/2))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

method	result
risch	$\frac{4a^4x^4-3ia^3x^3+2a^2x^2-3iax-2}{6x^3\sqrt{a^2x^2+1}}+\frac{ia^3\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)}{2}$
default	$-\frac{\sqrt{a^2x^2+1}}{3x^3}+\frac{2a^2\sqrt{a^2x^2+1}}{3x}+ia\left(-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{a^2\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)}{2}\right)$
meijerg	$-\frac{(-2a^2x^2+1)\sqrt{a^2x^2+1}}{3x^3}+\frac{ia^3\left(-\frac{\sqrt{\pi}}{x^2a^2}-\frac{\left(1-2\ln(2)+2\ln(x)+\ln\left(a^2\right)\right)\sqrt{\pi}}{2}+\frac{\sqrt{\pi}\left(4a^2x^2+8\right)}{8a^2x^2}-\frac{\sqrt{\pi}\sqrt{a^2x^2+1}}{a^2x^2}+\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{a^2x^2+1}}{2}\right)\right)}{2\sqrt{\pi}}$

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*(4*a^4*x^4-3*I*a^3*x^3+2*a^2*x^2-3*I*a*x-2)/x^3/(a^2*x^2+1)^(1/2)+1/2*
I*a^3*arctanh(1/(a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{e^{i\arctan(ax)}}{x^4}\,dx$$
$$= \frac{3i\,a^3x^3\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-3i\,a^3x^3\log\left(-ax+\sqrt{a^2x^2+1}-1\right)+4\,a^3x^3+\left(4\,a^2x^2-3i\,ax-2\right.}{6\,x^3}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
1/6*(3*I*a^3*x^3*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - 3*I*a^3*x^3*log(-a*x
+ sqrt(a^2*x^2 + 1) - 1) + 4*a^3*x^3 + (4*a^2*x^2 - 3*I*a*x - 2)*sqrt(a^2*
x^2 + 1))/x^3
```

Sympy [A] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.83

$$\int \frac{e^{i\arctan(ax)}}{x^4}\,dx = \frac{2a^3\sqrt{1+\frac{1}{a^2x^2}}}{3} + \frac{ia^3\operatorname{asinh}\left(\frac{1}{ax}\right)}{2} - \frac{ia^2\sqrt{1+\frac{1}{a^2x^2}}}{2x} - \frac{a\sqrt{1+\frac{1}{a^2x^2}}}{3x^2}$$

input
```
integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/x**4,x)
```

output
```
2*a**3*sqrt(1 + 1/(a**2*x**2))/3 + I*a**3*asinh(1/(a*x))/2 - I*a**2*sqrt(1
 + 1/(a**2*x**2))/(2*x) - a*sqrt(1 + 1/(a**2*x**2))/(3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int \frac{e^{i\arctan(ax)}}{x^4}\,dx = \frac{1}{2}i\,a^3\operatorname{arsinh}\left(\frac{1}{a|x|}\right) + \frac{2\sqrt{a^2x^2+1}a^2}{3\,x} - \frac{i\sqrt{a^2x^2+1}a}{2\,x^2} - \frac{\sqrt{a^2x^2+1}}{3\,x^3}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
1/2*I*a^3*arcsinh(1/(a*abs(x))) + 2/3*sqrt(a^2*x^2 + 1)*a^2/x - 1/2*I*sqrt
(a^2*x^2 + 1)*a/x^2 - 1/3*sqrt(a^2*x^2 + 1)/x^3
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(70) = 140.

Time = 0.14 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.79

$$\int \frac{e^{i\arctan(ax)}}{x^4}\,dx$$
$$= \frac{1}{2}i\,a^3\log\left(\left|-x|a|+\sqrt{a^2x^2+1}+1\right|\right) - \frac{1}{2}i\,a^3\log\left(\left|-x|a|+\sqrt{a^2x^2+1}-1\right|\right)$$
$$-\frac{-3i\left(x|a|-\sqrt{a^2x^2+1}\right)^5a^3 - 12\left(x|a|-\sqrt{a^2x^2+1}\right)^2a^2|a| + 3\left(i\,x|a|-i\sqrt{a^2x^2+1}\right)a^3 + 4\,a^2|a|}{3\left(\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1\right)^3}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^4,x, algorithm="giac")
```

output
```
1/2*I*a^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 + 1) + 1)) - 1/2*I*a^3*log(abs(
-x*abs(a) + sqrt(a^2*x^2 + 1) - 1)) - 1/3*(-3*I*(x*abs(a) - sqrt(a^2*x^2 +
 1))^5*a^3 - 12*(x*abs(a) - sqrt(a^2*x^2 + 1))^2*a^2*abs(a) + 3*(I*x*abs(a
) - I*sqrt(a^2*x^2 + 1))*a^3 + 4*a^2*abs(a))/((x*abs(a) - sqrt(a^2*x^2 + 1
))^2 - 1)^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.82

$$\int \frac{e^{i\arctan(ax)}}{x^4}\,dx = \frac{a^3\,\mathrm{atan}\left(\sqrt{a^2\,x^2+1}\,1\mathrm{i}\right)}{2} - \frac{\sqrt{a^2\,x^2+1}}{3\,x^3} + \frac{2\,a^2\,\sqrt{a^2\,x^2+1}}{3\,x} - \frac{a\,\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{2\,x^2}$$

input
```
int((a*x*1i + 1)/(x^4*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
(a^3*atan((a^2*x^2 + 1)^(1/2)*1i))/2 - (a^2*x^2 + 1)^(1/2)/(3*x^3) - (a*(a
^2*x^2 + 1)^(1/2)*1i)/(2*x^2) + (2*a^2*(a^2*x^2 + 1)^(1/2))/(3*x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.21

$$\int \frac{e^{i\arctan(ax)}}{x^4}\,dx = \frac{4\sqrt{a^2x^2+1}\,a^2x^2 - 3\sqrt{a^2x^2+1}\,aix - 2\sqrt{a^2x^2+1} - 3\log\left(\sqrt{a^2x^2+1}+ax-1\right)a^3i\,x^3 + 3\log\left(\sqrt{a^2x^2+\right.}{6x^3}$$

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^4,x)
```

output

```
(4*sqrt(a**2*x**2 + 1)*a**2*x**2 - 3*sqrt(a**2*x**2 + 1)*a*i*x - 2*sqrt(a*
*2*x**2 + 1) - 3*log(sqrt(a**2*x**2 + 1) + a*x - 1)*a**3*i*x**3 + 3*log(sq
rt(a**2*x**2 + 1) + a*x + 1)*a**3*i*x**3 - 4*a**3*x**3)/(6*x**3)
```

3.25 $\int \frac{e^{i\arctan(ax)}}{x^5}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 113

$$\int \frac{e^{i\arctan(ax)}}{x^5}\,dx = -\frac{\sqrt{1+a^2x^2}}{4x^4}-\frac{ia\sqrt{1+a^2x^2}}{3x^3}+\frac{3a^2\sqrt{1+a^2x^2}}{8x^2}+\frac{2ia^3\sqrt{1+a^2x^2}}{3x}-\frac{3}{8}a^4\text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

output
```
-1/4*(a^2*x^2+1)^(1/2)/x^4-1/3*I*a*(a^2*x^2+1)^(1/2)/x^3+3/8*a^2*(a^2*x^2+
1)^(1/2)/x^2+2/3*I*a^3*(a^2*x^2+1)^(1/2)/x-3/8*a^4*arctanh((a^2*x^2+1)^(1/
2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.67

$$\int \frac{e^{i\arctan(ax)}}{x^5}\,dx = \frac{1}{24}\left(\frac{\sqrt{1+a^2x^2}(-6-8iax+9a^2x^2+16ia^3x^3)}{x^4}+9a^4\log(x)-9a^4\log\left(1+\sqrt{1+a^2x^2}\right)\right)$$

input
```
Integrate[E^(I*ArcTan[a*x])/x^5,x]
```

output
```
((Sqrt[1 + a^2*x^2]*(-6 - (8*I)*a*x + 9*a^2*x^2 + (16*I)*a^3*x^3))/x^4 + 9
*a^4*Log[x] - 9*a^4*Log[1 + Sqrt[1 + a^2*x^2]])/24
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {5583, 539, 25, 27, 539, 27, 539, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{i\arctan(ax)}}{x^5}dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{1+iax}{x^5\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{539}$$

$$-\frac{\sqrt{a^2x^2+1}}{4x^4}-\frac{1}{4}\int -\frac{a(4i-3ax)}{x^4\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{a^2x^2+1}}{4x^4}+\frac{1}{4}\int \frac{a(4i-3ax)}{x^4\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{a^2x^2+1}}{4x^4}+\frac{1}{4}a\int \frac{4i-3ax}{x^4\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{539}$$

$$-\frac{\sqrt{a^2x^2+1}}{4x^4}+\frac{1}{4}a\left(-\frac{1}{3}\int \frac{a(8iax+9)}{x^3\sqrt{a^2x^2+1}}dx-\frac{4i\sqrt{a^2x^2+1}}{3x^3}\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{a^2x^2+1}}{4x^4}+\frac{1}{4}a\left(-\frac{1}{3}a\int \frac{8iax+9}{x^3\sqrt{a^2x^2+1}}dx-\frac{4i\sqrt{a^2x^2+1}}{3x^3}\right)$$

$$\Big\downarrow \text{539}$$

$$-\frac{\sqrt{a^2x^2+1}}{4x^4}+\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{9\sqrt{a^2x^2+1}}{2x^2}-\frac{1}{2}\int -\frac{a(16i-9ax)}{x^2\sqrt{a^2x^2+1}}dx\right)-\frac{4i\sqrt{a^2x^2+1}}{3x^3}\right)$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{a^2x^2+1}}{4x^4}+\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{9\sqrt{a^2x^2+1}}{2x^2}+\frac{1}{2}\int \frac{a(16i-9ax)}{x^2\sqrt{a^2x^2+1}}dx\right)-\frac{4i\sqrt{a^2x^2+1}}{3x^3}\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{a^2x^2+1}}{4x^4}+\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{9\sqrt{a^2x^2+1}}{2x^2}+\frac{1}{2}a\int \frac{16i-9ax}{x^2\sqrt{a^2x^2+1}}dx\right)-\frac{4i\sqrt{a^2x^2+1}}{3x^3}\right)$$

$$\Big\downarrow \text{534}$$

$$-\frac{\sqrt{a^2x^2+1}}{4x^4}+\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{9\sqrt{a^2x^2+1}}{2x^2}+\frac{1}{2}a\left(-9a\int \frac{1}{x\sqrt{a^2x^2+1}}dx-\frac{16i\sqrt{a^2x^2+1}}{x}\right)\right)-\frac{4i\sqrt{a^2x^2+1}}{3x^3}\right)$$

$$\Big\downarrow \text{243}$$

$$-\frac{\sqrt{a^2x^2+1}}{4x^4}+\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{9\sqrt{a^2x^2+1}}{2x^2}+\frac{1}{2}a\left(-\frac{9}{2}a\int \frac{1}{x^2\sqrt{a^2x^2+1}}dx^2-\frac{16i\sqrt{a^2x^2+1}}{x}\right)\right)-\frac{4i\sqrt{a^2x^2+1}}{3x^3}\right)$$

$$\Big\downarrow \text{73}$$

$$-\frac{\sqrt{a^2x^2+1}}{4x^4}+\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{9\sqrt{a^2x^2+1}}{2x^2}+\frac{1}{2}a\left(-\frac{9\int \frac{1}{\frac{x^4}{a^2}-\frac{1}{a^2}}d\sqrt{a^2x^2+1}}{a}-\frac{16i\sqrt{a^2x^2+1}}{x}\right)\right)-\frac{4i\sqrt{a^2x^2+1}}{3x^3}\right)$$

$$\Big\downarrow \text{221}$$

$$-\frac{\sqrt{a^2x^2+1}}{4x^4}+\frac{1}{4}a\left(-\frac{1}{3}a\left(-\frac{9\sqrt{a^2x^2+1}}{2x^2}+\frac{1}{2}a\left(9a\text{arctanh}\left(\sqrt{a^2x^2+1}\right)-\frac{16i\sqrt{a^2x^2+1}}{x}\right)\right)-\frac{4i\sqrt{a^2x^2+1}}{3x^3}\right)$$

input

```
Int[E^(I*ArcTan[a*x])/x^5,x]
```

output
```
-1/4*Sqrt[1 + a^2*x^2]/x^4 + (a*((((-4*I)/3)*Sqrt[1 + a^2*x^2])/x^3 - (a*(
(-9*Sqrt[1 + a^2*x^2])/(2*x^2) + (a*(((-16*I)*Sqrt[1 + a^2*x^2])/x + 9*a*A
rcTanh[Sqrt[1 + a^2*x^2]]))/2))/3))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539

```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 5583

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.68

method	result
risch	$\frac{i\left(16 a^{5} x^{5}-9 i a^{4} x^{4}+8 a^{3} x^{3}-3 i a^{2} x^{2}-8 a x +6 i\right)}{24 x^{4} \sqrt{a^{2} x^{2}+1}}-\frac{3 a^{4} \operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2} x^{2}+1}}\right)}{8}$
default	$-\frac{\sqrt{a^{2} x^{2}+1}}{4 x^{4}}-\frac{3 a^{2} \left(-\frac{\sqrt{a^{2} x^{2}+1}}{2 x^{2}}+\frac{a^{2} \operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2} x^{2}+1}}\right)}{2}\right)}{4}+i a \left(-\frac{\sqrt{a^{2} x^{2}+1}}{3 x^{3}}+\frac{2 a^{2} \sqrt{a^{2} x^{2}+1}}{3 x}\right)$
meijerg	$\frac{a^{4} \left(-\frac{\sqrt{\pi}}{2 x^{4} a^{4}}+\frac{\sqrt{\pi}}{2 x^{2} a^{2}}+\frac{3 \left(\frac{7}{6}-2 \ln \left(2\right)+2 \ln \left(x \right)+\ln \left(a^{2}\right)\right) \sqrt{\pi}}{8}+\frac{\sqrt{\pi}\, \left(-7 a^{4} x^{4}-8 a^{2} x^{2}+8\right)}{16 a^{4} x^{4}}-\frac{\sqrt{\pi}\, \left(-12 a^{2} x^{2}+8\right) \sqrt{a^{2} x^{2}+1}}{16 a^{4} x^{4}}-\frac{3 \sqrt{\pi}\, \ln \left(\frac{1}{2}+\frac{\sqrt{a^{2} x^{2}+1}}{2}\right)}{4}\right.}{2 \sqrt{\pi}}$

input

```
int((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output

```
1/24*I*(16*a^5*x^5-9*I*a^4*x^4+8*a^3*x^3-3*I*a^2*x^2-8*a*x+6*I)/x^4/(a^2*x
^2+1)^(1/2)-3/8*a^4*arctanh(1/(a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int \frac{e^{i\arctan(ax)}}{x^5}\,dx = -\frac{9\,a^4x^4\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-9\,a^4x^4\log\left(-ax+\sqrt{a^2x^2+1}-1\right)-16i\,a^4x^4-(16i\,a^3x^3+9\,a^2}{24\,x^4}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
-1/24*(9*a^4*x^4*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - 9*a^4*x^4*log(-a*x +
sqrt(a^2*x^2 + 1) - 1) - 16*I*a^4*x^4 - (16*I*a^3*x^3 + 9*a^2*x^2 - 8*I*a*
x - 6)*sqrt(a^2*x^2 + 1))/x^4
```

Sympy [A] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.08

$$\int \frac{e^{i\arctan(ax)}}{x^5}\,dx = \frac{2ia^4\sqrt{1+\frac{1}{a^2x^2}}}{3}-\frac{3a^4\operatorname{asinh}\left(\frac{1}{ax}\right)}{8}+\frac{3a^3}{8x\sqrt{1+\frac{1}{a^2x^2}}} - \frac{ia^2\sqrt{1+\frac{1}{a^2x^2}}}{3x^2}+\frac{a}{8x^3\sqrt{1+\frac{1}{a^2x^2}}}-\frac{1}{4ax^5\sqrt{1+\frac{1}{a^2x^2}}}$$

input
```
integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/x**5,x)
```

output
```
2*I*a**4*sqrt(1 + 1/(a**2*x**2))/3 - 3*a**4*asinh(1/(a*x))/8 + 3*a**3/(8*x
*sqrt(1 + 1/(a**2*x**2))) - I*a**2*sqrt(1 + 1/(a**2*x**2))/(3*x**2) + a/(8
*x**3*sqrt(1 + 1/(a**2*x**2))) - 1/(4*a*x**5*sqrt(1 + 1/(a**2*x**2)))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.76

$$\int \frac{e^{i\arctan(ax)}}{x^5}\,dx = -\frac{3}{8}\,a^4\operatorname{arsinh}\left(\frac{1}{a|x|}\right) + \frac{2i\,\sqrt{a^2x^2+1}a^3}{3\,x} + \frac{3\,\sqrt{a^2x^2+1}a^2}{8\,x^2} - \frac{i\,\sqrt{a^2x^2+1}a}{3\,x^3} - \frac{\sqrt{a^2x^2+1}}{4\,x^4}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-3/8*a^4*arcsinh(1/(a*abs(x))) + 2/3*I*sqrt(a^2*x^2 + 1)*a^3/x + 3/8*sqrt(
a^2*x^2 + 1)*a^2/x^2 - 1/3*I*sqrt(a^2*x^2 + 1)*a/x^3 - 1/4*sqrt(a^2*x^2 +
1)/x^4
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(89) = 178.

Time = 0.13 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.10

$$\int \frac{e^{i\arctan(ax)}}{x^5}\,dx$$
$$= -\frac{3}{8}\,a^4\log\left(\left|-x|a| + \sqrt{a^2x^2+1} + 1\right|\right) + \frac{3}{8}\,a^4\log\left(\left|-x|a| + \sqrt{a^2x^2+1} - 1\right|\right) - \frac{9\,\left(x|a| - \sqrt{a^2x^2+1}\right)^7a^4 - 33\,\left(x|a| - \sqrt{a^2x^2+1}\right)^5a^4 - 48i\,\left(x|a| - \sqrt{a^2x^2+1}\right)^4a^3|a| - 33\,\left(x|a| - \sqrt{a\right.}{12\left(\left(x|a| - \sqrt{a^2x^2+1}\right)^2\right.}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^5,x, algorithm="giac")
```

output
```
-3/8*a^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 + 1) + 1)) + 3/8*a^4*log(abs(-x*
abs(a) + sqrt(a^2*x^2 + 1) - 1)) - 1/12*(9*(x*abs(a) - sqrt(a^2*x^2 + 1))^
7*a^4 - 33*(x*abs(a) - sqrt(a^2*x^2 + 1))^5*a^4 - 48*I*(x*abs(a) - sqrt(a^
2*x^2 + 1))^4*a^3*abs(a) - 33*(x*abs(a) - sqrt(a^2*x^2 + 1))^3*a^4 + 64*I*
(x*abs(a) - sqrt(a^2*x^2 + 1))^2*a^3*abs(a) + 9*(x*abs(a) - sqrt(a^2*x^2 +
 1))*a^4 - 16*I*a^3*abs(a))/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.84

$$\int \frac{e^{i\arctan(ax)}}{x^5}\,dx = \frac{a^4\,\mathrm{atan}\left(\sqrt{a^2\,x^2+1}\,1\mathrm{i}\right)\,3\mathrm{i}}{8} - \frac{\sqrt{a^2\,x^2+1}}{4\,x^4} - \frac{a\,\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{3\,x^3} + \frac{3\,a^2\,\sqrt{a^2\,x^2+1}}{8\,x^2} + \frac{a^3\,\sqrt{a^2\,x^2+1}\,2\mathrm{i}}{3\,x}$$

input
```
int((a*x*1i + 1)/(x^5*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
(a^4*atan((a^2*x^2 + 1)^(1/2)*1i)*3i)/8 - (a^2*x^2 + 1)^(1/2)/(4*x^4) - (a
*(a^2*x^2 + 1)^(1/2)*1i)/(3*x^3) + (3*a^2*(a^2*x^2 + 1)^(1/2))/(8*x^2) + (
a^3*(a^2*x^2 + 1)^(1/2)*2i)/(3*x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.12

$$\int \frac{e^{i\arctan(ax)}}{x^5}\,dx = \frac{16\sqrt{a^2x^2+1}\,a^3i\,x^3 + 9\sqrt{a^2x^2+1}\,a^2x^2 - 8\sqrt{a^2x^2+1}\,aix - 6\sqrt{a^2x^2+1} + 9\log\left(\sqrt{a^2x^2+1}+ax-1\right)a}{24x^4}$$

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2)/x^5,x)
```

output
```
(16*sqrt(a**2*x**2 + 1)*a**3*i*x**3 + 9*sqrt(a**2*x**2 + 1)*a**2*x**2 - 8*
sqrt(a**2*x**2 + 1)*a*i*x - 6*sqrt(a**2*x**2 + 1) + 9*log(sqrt(a**2*x**2 +
 1) + a*x - 1)*a**4*x**4 - 9*log(sqrt(a**2*x**2 + 1) + a*x + 1)*a**4*x**4
- 16*a**4*i*x**4)/(24*x**4)
```

3.26 $\int e^{2i\arctan(ax)}x^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 48

$$\int e^{2i\arctan(ax)}x^3\,dx=-\frac{2ix}{a^3}+\frac{x^2}{a^2}+\frac{2ix^3}{3a}-\frac{x^4}{4}-\frac{2\log(i+ax)}{a^4}$$

output
```
-2*I*x/a^3+x^2/a^2+2/3*I*x^3/a-1/4*x^4-2*ln(I+a*x)/a^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int e^{2i\arctan(ax)}x^3\,dx=-\frac{2ix}{a^3}+\frac{x^2}{a^2}+\frac{2ix^3}{3a}-\frac{x^4}{4}-\frac{2\log(i+ax)}{a^4}$$

input
```
Integrate[E^((2*I)*ArcTan[a*x])*x^3,x]
```

output
```
((-2*I)*x)/a^3 + x^2/a^2 + (((2*I)/3)*x^3)/a - x^4/4 - (2*Log[I + a*x])/a^
4
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5585, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{2i\arctan(ax)} dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{x^3(1+iax)}{1-iax}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{2}{a^3(ax+i)}-\frac{2i}{a^3}+\frac{2x}{a^2}+\frac{2ix^2}{a}-x^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\log(ax+i)}{a^4}-\frac{2ix}{a^3}+\frac{x^2}{a^2}+\frac{2ix^3}{3a}-\frac{x^4}{4}$$

input
```
Int[E^((2*I)*ArcTan[a*x])*x^3,x]
```

output
```
((-2*I)*x)/a^3 + x^2/a^2 + (((2*I)/3)*x^3)/a - x^4/4 - (2*Log[I + a*x])/a^
4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

method	result	size
parallelrisch	$\frac{-3a^4x^4+8ia^3x^3+12a^2x^2-24iax-24\ln(ax+i)}{12a^4}$	46
risch	$-\frac{x^4}{4}+\frac{2ix^3}{3a}+\frac{x^2}{a^2}-\frac{2ix}{a^3}-\frac{\ln(a^2x^2+1)}{a^4}+\frac{2i\arctan(ax)}{a^4}$	55
default	$\frac{-\frac{1}{4}a^3x^4+\frac{2}{3}ia^2x^3+a\,x^2-2ix}{a^3}+\frac{-\frac{\ln\left(a^2x^2+1\right)}{a}+\frac{2i\arctan(ax)}{a}}{a^3}$	63
meijerg	$\frac{a^2x^2-\ln(a^2x^2+1)}{2a^4}+\frac{i\left(-\frac{2x\left(a^2\right)^{\frac{5}{2}}\left(-5a^2x^2+15\right)}{15a^4}+\frac{2\left(a^2\right)^{\frac{5}{2}}\arctan(ax)}{a^5}\right)}{a^3\sqrt{a^2}}-\frac{-\frac{x^2a^2\left(-3a^2x^2+6\right)}{6}+\ln(a^2x^2+1)}{2a^4}$	108

input
```
int((1+I*a*x)^2/(a^2*x^2+1)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*(-3*a^4*x^4+8*I*x^3*a^3+12*a^2*x^2-24*I*a*x-24*ln(I+a*x))/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int e^{2i\arctan(ax)}x^3\,dx = -\frac{3\,a^4x^4-8i\,a^3x^3-12\,a^2x^2+24i\,ax+24\,\log\left(\frac{ax+i}{a}\right)}{12\,a^4}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)*x^3,x, algorithm="fricas")
```

output
```
-1/12*(3*a^4*x^4 - 8*I*a^3*x^3 - 12*a^2*x^2 + 24*I*a*x + 24*log((a*x + I)/
a))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int e^{2i\arctan(ax)}x^3\,dx = -\frac{x^4}{4} + \frac{2ix^3}{3a} + \frac{x^2}{a^2} - \frac{2ix}{a^3} - \frac{2\log(ax+i)}{a^4}$$

input
```
integrate((1+I*a*x)**2/(a**2*x**2+1)*x**3,x)
```

output
```
-x**4/4 + 2*I*x**3/(3*a) + x**2/a**2 - 2*I*x/a**3 - 2*log(a*x + I)/a**4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.17

$$\int e^{2i\arctan(ax)}x^3\,dx = -\frac{3\,a^3x^4 - 8i\,a^2x^3 - 12\,ax^2 + 24i\,x}{12\,a^3} + \frac{2i\,\arctan(ax)}{a^4} - \frac{\log(a^2x^2+1)}{a^4}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)*x^3,x, algorithm="maxima")
```

output
```
-1/12*(3*a^3*x^4 - 8*I*a^2*x^3 - 12*a*x^2 + 24*I*x)/a^3 + 2*I*arctan(a*x)/
a^4 - log(a^2*x^2 + 1)/a^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int e^{2i\arctan(ax)}x^3\,dx = -\frac{3\,a^4x^4 - 8i\,a^3x^3 - 12\,a^2x^2 + 24i\,ax}{12\,a^4} - \frac{2\,\log\left(ax + i\right)}{a^4}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)*x^3,x, algorithm="giac")
```

output
```
-1/12*(3*a^4*x^4 - 8*I*a^3*x^3 - 12*a^2*x^2 + 24*I*a*x)/a^4 - 2*log(a*x +
I)/a^4
```

Mupad [B] (verification not implemented)

Time = 22.51 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int e^{2i\arctan(ax)}x^3\,dx = \frac{x^2}{a^2} - \frac{x^4}{4} - \frac{2\,\ln\left(x + \frac{1\mathrm{i}}{a}\right)}{a^4} - \frac{x\,2\mathrm{i}}{a^3} + \frac{x^3\,2\mathrm{i}}{3\,a}$$

input
```
int((x^3*(a*x*1i + 1)^2)/(a^2*x^2 + 1),x)
```

output
```
(x^3*2i)/(3*a) - (x*2i)/a^3 - x^4/4 - (2*log(x + 1i/a))/a^4 + x^2/a^2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int e^{2i\arctan(ax)}x^3\,dx = \frac{24atan(ax)\,i - 12\log(a^2x^2+1) - 3a^4x^4 + 8a^3i\,x^3 + 12a^2x^2 - 24aix}{12a^4}$$

input
```
int((1+I*a*x)^2/(a^2*x^2+1)*x^3,x)
```

output
```
(24*atan(a*x)*i - 12*log(a**2*x**2 + 1) - 3*a**4*x**4 + 8*a**3*i*x**3 + 12
*a**2*x**2 - 24*a*i*x)/(12*a**4)
```

3.27 $\int e^{2i\arctan(ax)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int e^{2i\arctan(ax)}x^2\,dx=\frac{2x}{a^2}+\frac{ix^2}{a}-\frac{x^3}{3}-\frac{2i\log(i+ax)}{a^3}$$

output
```
2*x/a^2+I*x^2/a-1/3*x^3-2*I*ln(I+a*x)/a^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int e^{2i\arctan(ax)}x^2\,dx=\frac{2x}{a^2}+\frac{ix^2}{a}-\frac{x^3}{3}-\frac{2i\log(i+ax)}{a^3}$$

input
```
Integrate[E^((2*I)*ArcTan[a*x])*x^2,x]
```

output
```
(2*x)/a^2 + (I*x^2)/a - x^3/3 - ((2*I)*Log[I + a*x])/a^3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5585, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{2i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{x^2(1+iax)}{1-iax}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{2i}{a^2(ax+i)}+\frac{2}{a^2}+\frac{2ix}{a}-x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2i\log(ax+i)}{a^3}+\frac{2x}{a^2}+\frac{ix^2}{a}-\frac{x^3}{3}$$

input
```
Int[E^((2*I)*ArcTan[a*x])*x^2,x]
```

output
```
(2*x)/a^2 + (I*x^2)/a - x^3/3 - ((2*I)*Log[I + a*x])/a^3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

method	result	size
parallelrisch	$-\frac{a^{3} x^{3}-3 i a^{2} x^{2}+6 i \ln \left(a x +i\right)-6 a x}{3 a^{3}}$	37
risch	$\frac{2 x}{a^{2}}-\frac{x^{3}}{3}+\frac{i x^{2}}{a}-\frac{i \ln \left(a^{2} x^{2}+1\right)}{a^{3}}-\frac{2 \arctan \left(a x \right)}{a^{3}}$	47
default	$\frac{2 x -\frac{1}{3} a^{2} x^{3}+i a \,x^{2}}{a^{2}}+\frac{-\frac{i \ln \left(a^{2} x^{2}+1\right)}{a}-\frac{2 \arctan \left(a x \right)}{a}}{a^{2}}$	55
meijerg	$\frac{\frac{2 x \left(a^{2}\right)^{\frac{3}{2}}}{a^{2}}-\frac{2 \left(a^{2}\right)^{\frac{3}{2}} \arctan \left(a x \right)}{a^{3}}}{2 a^{2} \sqrt{a^{2}}}+\frac{i \left(a^{2} x^{2}-\ln \left(a^{2} x^{2}+1\right)\right)}{a^{3}}-\frac{-\frac{2 x \left(a^{2}\right)^{\frac{5}{2}} \left(-5 a^{2} x^{2}+15\right)}{15 a^{4}}+\frac{2 \left(a^{2}\right)^{\frac{5}{2}} \arctan \left(a x \right)}{a^{5}}}{2 a^{2} \sqrt{a^{2}}}$	110

input
```
int((1+I*a*x)^2/(a^2*x^2+1)*x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a^3*x^3-3*I*a^2*x^2+6*I*ln(I+a*x)-6*a*x)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int e^{2 i \arctan (a x)} x^2 \, dx=-\frac{a^{3} x^{3} - 3 i \, a^{2} x^{2} - 6 \, a x + 6 i \, \log \left(\frac{a x + i}{a}\right)}{3 \, a^{3}}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)*x^2,x, algorithm="fricas")
```

output
```
-1/3*(a^3*x^3 - 3*I*a^2*x^2 - 6*a*x + 6*I*log((a*x + I)/a))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int e^{2i\arctan(ax)}x^2\,dx = -\frac{x^3}{3} + \frac{ix^2}{a} + \frac{2x}{a^2} - \frac{2i\log(ax+i)}{a^3}$$

input
```
integrate((1+I*a*x)**2/(a**2*x**2+1)*x**2,x)
```

output
```
-x**3/3 + I*x**2/a + 2*x/a**2 - 2*I*log(a*x + I)/a**3
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int e^{2i\arctan(ax)}x^2\,dx = -\frac{a^2x^3 - 3i\,ax^2 - 6\,x}{3\,a^2} - \frac{2\,\arctan(ax)}{a^3} - \frac{i\,\log(a^2x^2+1)}{a^3}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)*x^2,x, algorithm="maxima")
```

output
```
-1/3*(a^2*x^3 - 3*I*a*x^2 - 6*x)/a^2 - 2*arctan(a*x)/a^3 - I*log(a^2*x^2 +
 1)/a^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int e^{2i\arctan(ax)}x^2\,dx = -\frac{a^3x^3 - 3i\,a^2x^2 - 6\,ax}{3\,a^3} - \frac{2i\,\log(ax+i)}{a^3}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)*x^2,x, algorithm="giac")
```

output
```
-1/3*(a^3*x^3 - 3*I*a^2*x^2 - 6*a*x)/a^3 - 2*I*log(a*x + I)/a^3
```

Mupad [B] (verification not implemented)

Time = 22.63 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int e^{2i\arctan(ax)}x^2\,dx = \frac{2\,x}{a^2} - \frac{\ln\left(x+\frac{1\mathrm{i}}{a}\right)\,2\mathrm{i}}{a^3} - \frac{x^3}{3} + \frac{x^2\,1\mathrm{i}}{a}$$

input
```
int((x^2*(a*x*1i + 1)^2)/(a^2*x^2 + 1),x)
```

output
```
(2*x)/a^2 - (log(x + 1i/a)*2i)/a^3 - x^3/3 + (x^2*1i)/a
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int e^{2i\arctan(ax)}x^2\,dx = \frac{-6atan(ax) - 3\log(a^2x^2+1)\,i - a^3x^3 + 3a^2i\,x^2 + 6ax}{3a^3}$$

input
```
int((1+I*a*x)^2/(a^2*x^2+1)*x^2,x)
```

output
```
( - 6*atan(a*x) - 3*log(a**2*x**2 + 1)*i - a**3*x**3 + 3*a**2*i*x**2 + 6*a
*x)/(3*a**3)
```

3.28 $\int e^{2i\arctan(ax)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 29

$$\int e^{2i\arctan(ax)}x\,dx=\frac{2ix}{a}-\frac{x^2}{2}+\frac{2\log(i+ax)}{a^2}$$

output
```
2*I*x/a-1/2*x^2+2*ln(I+a*x)/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int e^{2i\arctan(ax)}x\,dx=\frac{2ix}{a}-\frac{x^2}{2}+\frac{2\log(i+ax)}{a^2}$$

input
```
Integrate[E^((2*I)*ArcTan[a*x])*x,x]
```

output
```
((2*I)*x)/a - x^2/2 + (2*Log[I + a*x])/a^2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5585, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{2i\arctan(ax)}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{x(1+iax)}{1-iax}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{2}{a(ax+i)}+\frac{2i}{a}-x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\log(ax+i)}{a^2}+\frac{2ix}{a}-\frac{x^2}{2}$$

input
```
Int[E^((2*I)*ArcTan[a*x])*x,x]
```

output
```
((2*I)*x)/a - x^2/2 + (2*Log[I + a*x])/a^2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{-a^2x^2+4iax+4\ln(ax+i)}{2a^2}$	29
risch	$-\frac{x^2}{2}+\frac{2ix}{a}+\frac{\ln(a^2x^2+1)}{a^2}-\frac{2i\arctan(ax)}{a^2}$	38
default	$\frac{-\frac{1}{2}a\,x^2+2ix}{a}+\frac{\frac{\ln\left(a^2x^2+1\right)}{a}-\frac{2i\arctan(ax)}{a}}{a}$	46
meijerg	$\frac{\ln(a^2x^2+1)}{2a^2}+\frac{i\left(\frac{2x\left(a^2\right)^{\frac{3}{2}}}{a^2}-\frac{2\left(a^2\right)^{\frac{3}{2}}\arctan(ax)}{a^3}\right)}{a\sqrt{a^2}}-\frac{a^2x^2-\ln(a^2x^2+1)}{2a^2}$	79

input
```
int((1+I*a*x)^2/(a^2*x^2+1)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-a^2*x^2+4*I*a*x+4*ln(I+a*x))/a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int e^{2i\arctan(ax)}x\,dx=-\frac{a^2x^2-4i\,ax-4\,\log\left(\frac{ax+i}{a}\right)}{2\,a^2}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)*x,x, algorithm="fricas")
```

output
```
-1/2*(a^2*x^2 - 4*I*a*x - 4*log((a*x + I)/a))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int e^{2i\arctan(ax)}x\,dx = -\frac{x^2}{2} + \frac{2ix}{a} + \frac{2\log(ax+i)}{a^2}$$

input
```
integrate((1+I*a*x)**2/(a**2*x**2+1)*x,x)
```

output
```
-x**2/2 + 2*I*x/a + 2*log(a*x + I)/a**2
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.31

$$\int e^{2i\arctan(ax)}x\,dx = -\frac{ax^2 - 4i\,x}{2\,a} - \frac{2i\,\arctan(ax)}{a^2} + \frac{\log(a^2x^2+1)}{a^2}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)*x,x, algorithm="maxima")
```

output
```
-1/2*(a*x^2 - 4*I*x)/a - 2*I*arctan(a*x)/a^2 + log(a^2*x^2 + 1)/a^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int e^{2i\arctan(ax)}x\,dx = -\frac{a^2x^2 - 4i\,ax}{2\,a^2} + \frac{2\,\log(ax+i)}{a^2}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)*x,x, algorithm="giac")
```

output
```
-1/2*(a^2*x^2 - 4*I*a*x)/a^2 + 2*log(a*x + I)/a^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int e^{2i\arctan(ax)}x\,dx = \frac{2\,\ln\left(x+\frac{1\mathrm{i}}{a}\right)}{a^2}-\frac{x^2}{2}+\frac{x\,2\mathrm{i}}{a}$$

input
```
int((x*(a*x*1i + 1)^2)/(a^2*x^2 + 1),x)
```

output
```
(2*log(x + 1i/a))/a^2 + (x*2i)/a - x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.31

$$\int e^{2i\arctan(ax)}x\,dx = \frac{-4atan(ax)\,i+2\log(a^2x^2+1)-a^2x^2+4aix}{2a^2}$$

input
```
int((1+I*a*x)^2/(a^2*x^2+1)*x,x)
```

output
```
( - 4*atan(a*x)*i + 2*log(a**2*x**2 + 1) - a**2*x**2 + 4*a*i*x)/(2*a**2)
```

3.29 $\int e^{2i\arctan(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 19

$$\int e^{2i\arctan(ax)}\,dx = -x + \frac{2i\log(i+ax)}{a}$$

output
```
-x+2*I*ln(I+a*x)/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int e^{2i\arctan(ax)}\,dx = -x + \frac{2\arctan(ax)}{a} + \frac{i\log(1+a^2x^2)}{a}$$

input
```
Integrate[E^((2*I)*ArcTan[a*x]),x]
```

output
```
-x + (2*ArcTan[a*x])/a + (I*Log[1 + a^2*x^2])/a
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5584, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2i\arctan(ax)}dx$$

$$\Big\downarrow \text{5584}$$

$$\int \frac{1+iax}{1-iax}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-1+\frac{2i}{ax+i}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-x+\frac{2i\log(ax+i)}{a}$$

input
```
Int[E^((2*I)*ArcTan[a*x]),x]
```

output
```
-x + ((2*I)*Log[I + a*x])/a
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5584
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
parallelrisch	$\frac{2 i \ln \left(a x +i\right)-a x}{a}$	20
default	$-x +\frac{i \ln \left(a^{2} x^{2}+1\right)}{a}+\frac{2 \arctan \left(a x \right)}{a}$	30
risch	$-x +\frac{i \ln \left(a^{2} x^{2}+1\right)}{a}+\frac{2 \arctan \left(a x \right)}{a}$	30
meijerg	$\frac{\arctan \left(a x \right)}{a}+\frac{i \ln \left(a^{2} x^{2}+1\right)}{a}-\frac{\frac{2 x \left(a^{2}\right)^{\frac{3}{2}}}{a^{2}}-\frac{2 \left(a^{2}\right)^{\frac{3}{2}} \arctan \left(a x \right)}{a^{3}}}{2 \sqrt{a^{2}}}$	59

input
```
int((1+I*a*x)^2/(a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
(2*I*ln(I+a*x)-a*x)/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int e^{2i \arctan (ax)}\,dx=-\frac{ax-2i\,\log\left(\frac{ax+i}{a}\right)}{a}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1),x, algorithm="fricas")
```

output
```
-(a*x - 2*I*log((a*x + I)/a))/a
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int e^{2i\arctan(ax)}\,dx = -x + \frac{2i\log\left(ax+i\right)}{a}$$

input
```
integrate((1+I*a*x)**2/(a**2*x**2+1),x)
```

output
```
-x + 2*I*log(a*x + I)/a
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int e^{2i\arctan(ax)}\,dx = -x + \frac{2\arctan\left(ax\right)}{a} + \frac{i\log\left(a^2x^2+1\right)}{a}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1),x, algorithm="maxima")
```

output
```
-x + 2*arctan(a*x)/a + I*log(a^2*x^2 + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int e^{2i\arctan(ax)}\,dx = -x + \frac{2i\log\left(ax+i\right)}{a}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1),x, algorithm="giac")
```

output
```
-x + 2*I*log(a*x + I)/a
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int e^{2i\arctan(ax)}\,dx = -x + \frac{\ln\left(x + \frac{1\mathrm{i}}{a}\right)\,2\mathrm{i}}{a}$$

input
```
int((a*x*1i + 1)^2/(a^2*x^2 + 1),x)
```

output
```
(log(x + 1i/a)*2i)/a - x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int e^{2i\arctan(ax)}\,dx = \frac{2atan(ax) + \log(a^2x^2+1)\,i - ax}{a}$$

input
```
int((1+I*a*x)^2/(a^2*x^2+1),x)
```

output
```
(2*atan(a*x) + log(a**2*x**2 + 1)*i - a*x)/a
```

3.30 $\int \frac{e^{2i\arctan(ax)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 13

$$\int \frac{e^{2i\arctan(ax)}}{x}\,dx = \log(x) - 2\log(i + ax)$$

output
```
ln(x)-2*ln(I+a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^{2i\arctan(ax)}}{x}\,dx = \log(x) - 2\log(i + ax)$$

input
```
Integrate[E^((2*I)*ArcTan[a*x])/x,x]
```

output
```
Log[x] - 2*Log[I + a*x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5585, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2i\arctan(ax)}}{x}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{1+iax}{x(1-iax)}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{1}{x}-\frac{2a}{ax+i}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\log(x)-2\log(ax+i)$$

input
```
Int[E^((2*I)*ArcTan[a*x])/x,x]
```

output
```
Log[x] - 2*Log[I + a*x]
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\ln(x) - 2\ln(ax+i)$	13
risch	$\ln(-x) - \ln(a^2x^2+1) + 2i\arctan(ax)$	25
meijerg	$\ln(x) + \frac{\ln(a^2)}{2} - \ln(a^2x^2+1) + 2i\arctan(ax)$	29
default	$2a\left(-\frac{\ln(a^2x^2+1)}{2a} + \frac{i\arctan(ax)}{a}\right) + \ln(x)$	33

input
```
int((1+I*a*x)^2/(a^2*x^2+1)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)-2*ln(I+a*x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{e^{2i\arctan(ax)}}{x}\,dx = \log(x) - 2\,\log\left(\frac{ax+i}{a}\right)$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)/x,x, algorithm="fricas")
```

output
```
log(x) - 2*log((a*x + I)/a)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{e^{2i\arctan(ax)}}{x}\,dx = \log(3ax) - 2\log(3ax + 3i)$$

input
```
integrate((1+I*a*x)**2/(a**2*x**2+1)/x,x)
```

output
```
log(3*a*x) - 2*log(3*a*x + 3*I)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{e^{2i\arctan(ax)}}{x}\,dx = 2i\arctan(ax) - \log(a^2x^2 + 1) + \log(x)$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)/x,x, algorithm="maxima")
```

output
```
2*I*arctan(a*x) - log(a^2*x^2 + 1) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{e^{2i\arctan(ax)}}{x}\,dx = -2\log(ax + i) + \log(|x|)$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)/x,x, algorithm="giac")
```

output
```
-2*log(a*x + I) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 22.68 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{e^{2i\arctan(ax)}}{x}\,dx = \ln(x) - 2\ln\left(x + \frac{1\mathrm{i}}{a}\right)$$

input
```
int((a*x*1i + 1)^2/(x*(a^2*x^2 + 1)),x)
```

output
```
log(x) - 2*log(x + 1i/a)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{e^{2i\arctan(ax)}}{x}\,dx = 2atan(ax)\,i - \log\left(a^2x^2 + 1\right) + \log(x)$$

input
```
int((1+I*a*x)^2/(a^2*x^2+1)/x,x)
```

output
```
2*atan(a*x)*i - log(a**2*x**2 + 1) + log(x)
```

3.31 $\int \frac{e^{2i\arctan(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 26

$$\int \frac{e^{2i\arctan(ax)}}{x^2}\,dx = -\frac{1}{x} + 2ia\log(x) - 2ia\log(i + ax)$$

output
```
-1/x+2*I*a*ln(x)-2*I*a*ln(I+a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{e^{2i\arctan(ax)}}{x^2}\,dx = -\frac{1}{x} + 2ia\log(x) - 2ia\log(i + ax)$$

input
```
Integrate[E^((2*I)*ArcTan[a*x])/x^2,x]
```

output
```
-x^(-1) + (2*I)*a*Log[x] - (2*I)*a*Log[I + a*x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5585, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2i\arctan(ax)}}{x^2}dx$$
$$\Big\downarrow \text{5585}$$
$$\int \frac{1+iax}{x^2(1-iax)}dx$$
$$\Big\downarrow \text{86}$$
$$\int \left(-\frac{2ia^2}{ax+i}+\frac{2ia}{x}+\frac{1}{x^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$2ia\log(x)-2ia\log(ax+i)-\frac{1}{x}$$

input
```
Int[E^((2*I)*ArcTan[a*x])/x^2,x]
```

output
```
-x^(-1) + (2*I)*a*Log[x] - (2*I)*a*Log[I + a*x]
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{2ia\ln(x)x-2ia\ln(ax+i)x-1}{x}$	26
risch	$-\frac{1}{x}-2a\arctan\left(ax\right)-ia\ln\left(a^{2}x^{2}+1\right)+2ia\ln\left(x\right)$	34
default	$-2a^{2}\left(\frac{i\ln\left(a^{2}x^{2}+1\right)}{2a}+\frac{\arctan(ax)}{a}\right)-\frac{1}{x}+2ia\ln\left(x\right)$	43
meijerg	$\frac{a^{2}\left(-\frac{2}{x\sqrt{a^{2}}}-\frac{2a\arctan(ax)}{\sqrt{a^{2}}}\right)}{2\sqrt{a^{2}}}+ia(2\ln\left(x\right)+\ln\left(a^{2}\right)-\ln\left(a^{2}x^{2}+1\right))-a\arctan\left(ax\right)$	67

input
```
int((1+I*a*x)^2/(a^2*x^2+1)/x^2,x,method=_RETURNVERBOSE)
```

output
```
(2*I*a*ln(x)*x-2*I*a*ln(I+a*x)*x-1)/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int\frac{e^{2i\arctan(ax)}}{x^{2}}\,dx=\frac{2i\,ax\log\left(x\right)-2i\,ax\log\left(\frac{ax+i}{a}\right)-1}{x}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)/x^2,x, algorithm="fricas")
```

output
```
(2*I*a*x*log(x) - 2*I*a*x*log((a*x + I)/a) - 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \frac{e^{2i\arctan(ax)}}{x^2}\,dx = -2a\left(-i\log\left(4a^2x\right) + i\log\left(4a^2x + 4ia\right)\right) - \frac{1}{x}$$

input
```
integrate((1+I*a*x)**2/(a**2*x**2+1)/x**2,x)
```

output
```
-2*a*(-I*log(4*a**2*x) + I*log(4*a**2*x + 4*I*a)) - 1/x
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \frac{e^{2i\arctan(ax)}}{x^2}\,dx = -2\,a\arctan\left(ax\right) - i\,a\log\left(a^2x^2 + 1\right) + 2i\,a\log\left(x\right) - \frac{1}{x}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)/x^2,x, algorithm="maxima")
```

output
```
-2*a*arctan(a*x) - I*a*log(a^2*x^2 + 1) + 2*I*a*log(x) - 1/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{e^{2i\arctan(ax)}}{x^2}\,dx = -2i\,a\log\left(ax + i\right) + 2i\,a\log\left(|x|\right) - \frac{1}{x}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)/x^2,x, algorithm="giac")
```

output
```
-2*I*a*log(a*x + I) + 2*I*a*log(abs(x)) - 1/x
```

Mupad [B] (verification not implemented)

Time = 23.67 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{e^{2i\arctan(ax)}}{x^2}\,dx = -4\,a\,\mathrm{atan}(2\,a\,x + 1\mathrm{i}) - \frac{1}{x}$$

input
```
int((a*x*1i + 1)^2/(x^2*(a^2*x^2 + 1)),x)
```

output
```
- 4*a*atan(2*a*x + 1i) - 1/x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \frac{e^{2i\arctan(ax)}}{x^2}\,dx = \frac{-2atan(ax)\,ax - \log(a^2x^2 + 1)\,aix + 2\log(x)\,aix - 1}{x}$$

input
```
int((1+I*a*x)^2/(a^2*x^2+1)/x^2,x)
```

output
```
( - 2*atan(a*x)*a*x - log(a**2*x**2 + 1)*a*i*x + 2*log(x)*a*i*x - 1)/x
```

3.32 $\int \frac{e^{2i\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 36

$$\int \frac{e^{2i\arctan(ax)}}{x^3}\,dx = -\frac{1}{2x^2} - \frac{2ia}{x} - 2a^2\log(x) + 2a^2\log(i+ax)$$

output
```
-1/2/x^2-2*I*a/x-2*a^2*ln(x)+2*a^2*ln(I+a*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{e^{2i\arctan(ax)}}{x^3}\,dx = -\frac{1}{2x^2} - \frac{2ia}{x} - 2a^2\log(x) + 2a^2\log(i+ax)$$

input
```
Integrate[E^((2*I)*ArcTan[a*x])/x^3,x]
```

output
```
-1/2*1/x^2 - ((2*I)*a)/x - 2*a^2*Log[x] + 2*a^2*Log[I + a*x]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5585, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2i\arctan(ax)}}{x^3}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{1+iax}{x^3(1-iax)}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{2a^3}{ax+i}-\frac{2a^2}{x}+\frac{2ia}{x^2}+\frac{1}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-2a^2\log(x)+2a^2\log(ax+i)-\frac{2ia}{x}-\frac{1}{2x^2}$$

input
```
Int[E^((2*I)*ArcTan[a*x])/x^3,x]
```

output
```
-1/2*1/x^2 - ((2*I)*a)/x - 2*a^2*Log[x] + 2*a^2*Log[I + a*x]
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

method	result	size
parallelrisch	$-\frac{4a^2\ln(x)x^2-4a^2\ln(ax+i)x^2+4iax+1}{2x^2}$	38
risch	$\frac{-2iax-\frac{1}{2}}{x^2}-2a^2\ln\left(x\right)-2ia^2\arctan\left(ax\right)+a^2\ln\left(a^2x^2+1\right)$	44
default	$-2a^3\left(-\frac{\ln(a^2x^2+1)}{2a}+\frac{i\arctan(ax)}{a}\right)-\frac{1}{2x^2}-\frac{2ia}{x}-2a^2\ln\left(x\right)$	52
meijerg	$\frac{a^2\left(-\frac{1}{a^2x^2}-2\ln(x)-\ln(a^2)+\ln(a^2x^2+1)\right)}{2}+\frac{ia^3\left(-\frac{2}{x\sqrt{a^2}}-\frac{2a\arctan(ax)}{\sqrt{a^2}}\right)}{\sqrt{a^2}}-\frac{a^2\left(2\ln(x)+\ln(a^2)-\ln(a^2x^2+1)\right)}{2}$	96

input
```
int((1+I*a*x)^2/(a^2*x^2+1)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(4*a^2*ln(x)*x^2-4*a^2*ln(I+a*x)*x^2+4*I*a*x+1)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int\frac{e^{2i\arctan(ax)}}{x^3}\,dx=-\frac{4\,a^2x^2\log\left(x\right)-4\,a^2x^2\log\left(\frac{ax+i}{a}\right)+4i\,ax+1}{2\,x^2}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(4*a^2*x^2*log(x) - 4*a^2*x^2*log((a*x + I)/a) + 4*I*a*x + 1)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

$$\int \frac{e^{2i\arctan(ax)}}{x^3} \, dx = -2a^2\left(\log\left(4a^3x\right) - \log\left(4a^3x + 4ia^2\right)\right) - \frac{4iax + 1}{2x^2}$$

input
```
integrate((1+I*a*x)**2/(a**2*x**2+1)/x**3,x)
```

output
```
-2*a**2*(log(4*a**3*x) - log(4*a**3*x + 4*I*a**2)) - (4*I*a*x + 1)/(2*x**2
)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

$$\int \frac{e^{2i\arctan(ax)}}{x^3} \, dx = -2i\,a^2\arctan(ax) + a^2\log\left(a^2x^2 + 1\right) - 2\,a^2\log(x) - \frac{4i\,ax + 1}{2\,x^2}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)/x^3,x, algorithm="maxima")
```

output
```
-2*I*a^2*arctan(a*x) + a^2*log(a^2*x^2 + 1) - 2*a^2*log(x) - 1/2*(4*I*a*x
+ 1)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{e^{2i\arctan(ax)}}{x^3} \, dx = 2\,a^2\log(ax + i) - 2\,a^2\log(|x|) - \frac{4i\,ax + 1}{2\,x^2}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)/x^3,x, algorithm="giac")
```

output
```
2*a^2*log(a*x + I) - 2*a^2*log(abs(x)) - 1/2*(4*I*a*x + 1)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \frac{e^{2i\arctan(ax)}}{x^3}\,dx = -a^2\,\mathrm{atan}(2\,a\,x+1\mathrm{i})\;4\mathrm{i}-\frac{\frac{1}{2}+a\,x\,2\mathrm{i}}{x^2}$$

input
```
int((a*x*1i + 1)^2/(x^3*(a^2*x^2 + 1)),x)
```

output
```
- a^2*atan(2*a*x + 1i)*4i - (a*x*2i + 1/2)/x^2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.47

$$\int \frac{e^{2i\arctan(ax)}}{x^3}\,dx = \frac{-4atan(ax)\,a^2i\,x^2+2\log(a^2x^2+1)\,a^2x^2-4\log(x)\,a^2x^2-4aix-1}{2x^2}$$

input
```
int((1+I*a*x)^2/(a^2*x^2+1)/x^3,x)
```

output
```
( - 4*atan(a*x)*a**2*i*x**2 + 2*log(a**2*x**2 + 1)*a**2*x**2 - 4*log(x)*a*
*2*x**2 - 4*a*i*x - 1)/(2*x**2)
```

3.33 $\int \frac{e^{2i\arctan(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 48

$$\int \frac{e^{2i\arctan(ax)}}{x^4}\,dx = -\frac{1}{3x^3} - \frac{ia}{x^2} + \frac{2a^2}{x} - 2ia^3\log(x) + 2ia^3\log(i+ax)$$

output
```
-1/3/x^3-I/x^2*a+2*a^2/x-2*I*a^3*ln(x)+2*I*a^3*ln(I+a*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{e^{2i\arctan(ax)}}{x^4}\,dx = -\frac{1}{3x^3} - \frac{ia}{x^2} + \frac{2a^2}{x} - 2ia^3\log(x) + 2ia^3\log(i+ax)$$

input
```
Integrate[E^((2*I)*ArcTan[a*x])/x^4,x]
```

output
```
-1/3*1/x^3 - (I*a)/x^2 + (2*a^2)/x - (2*I)*a^3*Log[x] + (2*I)*a^3*Log[I +
a*x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5585, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2i\arctan(ax)}}{x^4}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{1+iax}{x^4(1-iax)}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{2ia^4}{ax+i}-\frac{2ia^3}{x}-\frac{2a^2}{x^2}+\frac{2ia}{x^3}+\frac{1}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-2ia^3\log(x)+2ia^3\log(ax+i)+\frac{2a^2}{x}-\frac{ia}{x^2}-\frac{1}{3x^3}$$

input
```
Int[E^((2*I)*ArcTan[a*x])/x^4,x]
```

output
```
-1/3*1/x^3 - (I*a)/x^2 + (2*a^2)/x - (2*I)*a^3*Log[x] + (2*I)*a^3*Log[I +
a*x]
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

method	result
parallelrisch	$-\frac{6ia^3\ln(x)x^3-6ia^3\ln(ax+i)x^3+1-6a^2x^2+3iax}{3x^3}$
risch	$\frac{2a^2x^2-iax-\frac{1}{3}}{x^3}+2a^3\arctan\left(ax\right)+ia^3\ln\left(a^2x^2+1\right)-2ia^3\ln\left(-x\right)$
default	$2a^4\left(\frac{i\ln(a^2x^2+1)}{2a}+\frac{\arctan(ax)}{a}\right)-\frac{1}{3x^3}-2ia^3\ln\left(x\right)-\frac{ia}{x^2}+\frac{2a^2}{x}$
meijerg	$\frac{a^4\left(\frac{2a^2}{x(a^2)^{\frac{3}{2}}}-\frac{2}{3x^3(a^2)^{\frac{3}{2}}}+\frac{2a^3\arctan(ax)}{(a^2)^{\frac{3}{2}}}\right)}{2\sqrt{a^2}}+ia^3\left(-\frac{1}{a^2x^2}-2\ln\left(x\right)-\ln\left(a^2\right)+\ln\left(a^2x^2+1\right)\right)-$

input
```
int((1+I*a*x)^2/(a^2*x^2+1)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(6*I*a^3*ln(x)*x^3-6*I*a^3*ln(I+a*x)*x^3+1-6*a^2*x^2+3*I*a*x)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int \frac{e^{2i\arctan(ax)}}{x^4}\,dx=\frac{-6i\,a^3x^3\log\left(x\right)+6i\,a^3x^3\log\left(\frac{ax+i}{a}\right)+6\,a^2x^2-3i\,ax-1}{3\,x^3}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)/x^4,x, algorithm="fricas")
```

output
```
1/3*(-6*I*a^3*x^3*log(x) + 6*I*a^3*x^3*log((a*x + I)/a) + 6*a^2*x^2 - 3*I*
a*x - 1)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{e^{2i\arctan(ax)}}{x^4}\,dx = -2a^3\left(i\log\left(4a^4x\right) - i\log\left(4a^4x + 4ia^3\right)\right) - \frac{-6a^2x^2 + 3iax + 1}{3x^3}$$

input
```
integrate((1+I*a*x)**2/(a**2*x**2+1)/x**4,x)
```

output
```
-2*a**3*(I*log(4*a**4*x) - I*log(4*a**4*x + 4*I*a**3)) - (-6*a**2*x**2 + 3
*I*a*x + 1)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{e^{2i\arctan(ax)}}{x^4}\,dx = 2\,a^3\arctan(ax) + i\,a^3\log\left(a^2x^2 + 1\right) - 2i\,a^3\log(x) + \frac{6\,a^2x^2 - 3i\,ax - 1}{3\,x^3}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)/x^4,x, algorithm="maxima")
```

output
```
2*a^3*arctan(a*x) + I*a^3*log(a^2*x^2 + 1) - 2*I*a^3*log(x) + 1/3*(6*a^2*x
^2 - 3*I*a*x - 1)/x^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{e^{2i\arctan(ax)}}{x^4}\,dx = 2i\,a^3\log(ax+i) - 2i\,a^3\log(|x|) + \frac{6\,a^2x^2 - 3i\,ax - 1}{3\,x^3}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)/x^4,x, algorithm="giac")
```

output
```
2*I*a^3*log(a*x + I) - 2*I*a^3*log(abs(x)) + 1/3*(6*a^2*x^2 - 3*I*a*x - 1)
/x^3
```

Mupad [B] (verification not implemented)

Time = 22.95 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.71

$$\int \frac{e^{2i\arctan(ax)}}{x^4}\,dx = 4\,a^3\,\mathrm{atan}(2\,a\,x + 1\mathrm{i}) - \frac{-2\,a^2\,x^2 + a\,x\,1\mathrm{i} + \frac{1}{3}}{x^3}$$

input
```
int((a*x*1i + 1)^2/(x^4*(a^2*x^2 + 1)),x)
```

output
```
4*a^3*atan(2*a*x + 1i) - (a*x*1i - 2*a^2*x^2 + 1/3)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.29

$$\int \frac{e^{2i\arctan(ax)}}{x^4}\,dx = \frac{6atan(ax)\,a^3x^3 + 3\log(a^2x^2+1)\,a^3i\,x^3 - 6\log(x)\,a^3i\,x^3 + 6a^2x^2 - 3aix - 1}{3x^3}$$

input
```
int((1+I*a*x)^2/(a^2*x^2+1)/x^4,x)
```

output
```
(6*atan(a*x)*a**3*x**3 + 3*log(a**2*x**2 + 1)*a**3*i*x**3 - 6*log(x)*a**3*
i*x**3 + 6*a**2*x**2 - 3*a*i*x - 1)/(3*x**3)
```

3.34 $\int e^{3i\arctan(ax)}x^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 126

$$\int e^{3i\arctan(ax)}x^3\,dx=\frac{7\sqrt{1+a^2x^2}}{a^4}+\frac{19ix\sqrt{1+a^2x^2}}{8a^3}-\frac{ix^3\sqrt{1+a^2x^2}}{4a}+\frac{4\sqrt{1+a^2x^2}}{a^4(1-iax)}-\frac{(1+a^2x^2)^{3/2}}{a^4}-\frac{51i\operatorname{arcsinh}(ax)}{8a^4}$$

output
```
7*(a^2*x^2+1)^(1/2)/a^4+19/8*I*x*(a^2*x^2+1)^(1/2)/a^3-1/4*I*x^3*(a^2*x^2+
1)^(1/2)/a+4*(a^2*x^2+1)^(1/2)/a^4/(1-I*a*x)-(a^2*x^2+1)^(3/2)/a^4-51/8*I*
arcsinh(a*x)/a^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.63

$$\int e^{3i\arctan(ax)}x^3\,dx=\sqrt{1+a^2x^2}\left(\frac{6}{a^4}+\frac{19ix}{8a^3}-\frac{x^2}{a^2}-\frac{ix^3}{4a}+\frac{4i}{a^4(i+ax)}\right)-\frac{51i\operatorname{arcsinh}(ax)}{8a^4}$$

input
```
Integrate[E^((3*I)*ArcTan[a*x])*x^3,x]
```

output
```
Sqrt[1 + a^2*x^2]*(6/a^4 + (((19*I)/8)*x)/a^3 - x^2/a^2 - ((I/4)*x^3)/a +
(4*I)/(a^4*(I + a*x))) - (((51*I)/8)*ArcSinh[a*x])/a^4
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.22, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.071$, Rules used = {5583, 2164, 2027, 2164, 25, 27, 563, 25, 2346, 2346, 27, 2346, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{3i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{x^3(1+iax)^2}{(1-iax)\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{2164}$$

$$-ia\int \frac{\sqrt{a^2x^2+1}\left(\frac{ix^3}{a}-x^4\right)}{(1-iax)^2}dx$$

$$\Big\downarrow \text{2027}$$

$$-ia\int \frac{\left(\frac{i}{a}-x\right)x^3\sqrt{a^2x^2+1}}{(1-iax)^2}dx$$

$$\Big\downarrow \text{2164}$$

$$-a^2\int -\frac{x^3\left(a^2x^2+1\right)^{3/2}}{a^2(1-iax)^3}dx$$

$$\Big\downarrow \text{25}$$

$$a^2\int \frac{x^3\left(a^2x^2+1\right)^{3/2}}{a^2(1-iax)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^3\left(a^2x^2+1\right)^{3/2}}{(1-iax)^3}dx$$

$$\Big\downarrow \ 563$$

$$\frac{i\int -\frac{a^4x^4-3ia^3x^3-4a^2x^2+4iax+4}{\sqrt{a^2x^2+1}}dx}{a^3}+\frac{4\sqrt{a^2x^2+1}}{a^4(1-iax)}$$

$$\Big\downarrow \ 25$$

$$\frac{4\sqrt{a^2x^2+1}}{a^4(1-iax)}-\frac{i\int \frac{a^4x^4-3ia^3x^3-4a^2x^2+4iax+4}{\sqrt{a^2x^2+1}}dx}{a^3}$$

$$\Big\downarrow \ 2346$$

$$\frac{4\sqrt{a^2x^2+1}}{a^4(1-iax)}-\frac{i\left(\frac{1}{4}a^2x^3\sqrt{a^2x^2+1}+\frac{\int \frac{-12ix^3a^5-19x^2a^4+16ixa^3+16a^2}{\sqrt{a^2x^2+1}}dx}{4a^2}\right)}{a^3}$$

$$\Big\downarrow \ 2346$$

$$\frac{4\sqrt{a^2x^2+1}}{a^4(1-iax)}-\frac{i\left(\frac{1}{4}a^2x^3\sqrt{a^2x^2+1}+\frac{\frac{\int \frac{3\left(-19x^2a^6+24ixa^5+16a^4\right)}{\sqrt{a^2x^2+1}}dx}{3a^2}-4ia^3x^2\sqrt{a^2x^2+1}}{4a^2}\right)}{a^3}$$

$$\Big\downarrow \ 27$$

$$\frac{4\sqrt{a^2x^2+1}}{a^4(1-iax)}-\frac{i\left(\frac{1}{4}a^2x^3\sqrt{a^2x^2+1}+\frac{\frac{\int \frac{-19x^2a^6+24ixa^5+16a^4}{\sqrt{a^2x^2+1}}dx}{a^2}-4ia^3x^2\sqrt{a^2x^2+1}}{4a^2}\right)}{a^3}$$

$$\Big\downarrow \ 2346$$

$$\frac{4\sqrt{a^2x^2+1}}{a^4(1-iax)}-\frac{i\left(\frac{1}{4}a^2x^3\sqrt{a^2x^2+1}+\frac{\frac{-\frac{19}{2}a^4x\sqrt{a^2x^2+1}+\frac{\int \frac{3a^6(16iax+17)}{\sqrt{a^2x^2+1}}dx}{2a^2}}{a^2}-4ia^3x^2\sqrt{a^2x^2+1}}{4a^2}\right)}{a^3}$$

$$\Big\downarrow \ 27$$

$$\frac{4\sqrt{a^2x^2+1}}{a^4(1-iax)}-\frac{i\left(\frac{1}{4}a^2x^3\sqrt{a^2x^2+1}+\frac{\frac{-\frac{19}{2}a^4x\sqrt{a^2x^2+1}+\frac{3}{2}a^4\int \frac{16iax+17}{\sqrt{a^2x^2+1}}dx}{a^2}-4ia^3x^2\sqrt{a^2x^2+1}}{4a^2}\right)}{a^3}$$

$$\Big\downarrow \ 455$$

$$\frac{4\sqrt{a^2x^2+1}}{a^4(1-iax)}-\frac{i\left(\frac{1}{4}a^2x^3\sqrt{a^2x^2+1}+\frac{\frac{-\frac{19}{2}a^4x\sqrt{a^2x^2+1}+\frac{3}{2}a^4\left(17\int\frac{1}{\sqrt{a^2x^2+1}}dx+\frac{16i\sqrt{a^2x^2+1}}{a}\right)}{a^2}-4ia^3x^2\sqrt{a^2x^2+1}}{4a^2}\right)}{a^3}$$

$$\Big\downarrow \text{222}$$

$$\frac{4\sqrt{a^2x^2+1}}{a^4(1-iax)}-\frac{i\left(\frac{1}{4}a^2x^3\sqrt{a^2x^2+1}+\frac{\frac{-\frac{19}{2}a^4x\sqrt{a^2x^2+1}+\frac{3}{2}a^4\left(\frac{17\text{arcsinh}(ax)}{a}+\frac{16i\sqrt{a^2x^2+1}}{a}\right)}{a^2}-4ia^3x^2\sqrt{a^2x^2+1}}{4a^2}\right)}{a^3}$$

input
```
Int[E^((3*I)*ArcTan[a*x])*x^3,x]
```

output
```
(4*Sqrt[1 + a^2*x^2])/(a^4*(1 - I*a*x)) - (I*((a^2*x^3*Sqrt[1 + a^2*x^2])/
4 + ((-4*I)*a^3*x^2*Sqrt[1 + a^2*x^2] + ((-19*a^4*x*Sqrt[1 + a^2*x^2])/2 +
 (3*a^4*(((16*I)*Sqrt[1 + a^2*x^2])/a + (17*ArcSinh[a*x])/a))/2)/a^2)/(4*a
^2)))/a^3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2164
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[d*e   Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + b*d*x, x]*
(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x]
 && EqQ[b*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + b*d*x, x], 0
]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{i(2a^3x^3-8ia^2x^2-19ax+48i)\sqrt{a^2x^2+1}}{8a^4}-\frac{i\left(-\frac{32\sqrt{\left(x+\frac{i}{a}\right)^2a^2-2ia\left(x+\frac{i}{a}\right)}}{a^2\left(x+\frac{i}{a}\right)}+\frac{51\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{\sqrt{a^2}}\right)}{8a^3}$
meijerg	$\frac{-2\sqrt{\pi}+\frac{\sqrt{\pi}\left(4a^2x^2+8\right)}{4\sqrt{a^2x^2+1}}}{a^4\sqrt{\pi}}+\frac{3i\left(\frac{\sqrt{\pi}\,x\left(a^2\right)^{\frac{5}{2}}\left(5a^2x^2+15\right)}{10a^4\sqrt{a^2x^2+1}}-\frac{3\sqrt{\pi}\left(a^2\right)^{\frac{5}{2}}\operatorname{arcsinh}(ax)}{2a^5}\right)}{a^3\sqrt{\pi}\sqrt{a^2}}-\frac{3\left(\frac{8\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(-2a^4x^4+8a^2x^2+16\right)}{6\sqrt{a^2x^2+1}}\right)}{a^4\sqrt{\pi}}-\frac{i\left(-\right.}{}$
default	$\frac{x^2}{a^2\sqrt{a^2x^2+1}}+\frac{2}{a^4\sqrt{a^2x^2+1}}-ia^3\left(\frac{x^5}{4a^2\sqrt{a^2x^2+1}}-\frac{5\left(\frac{x^3}{2a^2\sqrt{a^2x^2+1}}-\frac{3\left(-\frac{x}{a^2\sqrt{a^2x^2+1}}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{a^2\sqrt{a^2}}\right)}{2a^2}\right)}{4a^2}\right)+3i$

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*I*(2*a^3*x^3-8*I*a^2*x^2-19*a*x+48*I)*(a^2*x^2+1)^(1/2)/a^4-1/8*I/a^3
*(-32/a^2/(x+I/a)*((x+I/a)^2*a^2-2*I*a*(x+I/a))^(1/2)+51*ln(a^2*x/(a^2)^(1
/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.70

$$\int e^{3i\arctan(ax)}x^3\,dx$$
$$=\frac{32i\,ax-51\left(-i\,ax+1\right)\log\left(-ax+\sqrt{a^2x^2+1}\right)+\left(-2i\,a^4x^4-6\,a^3x^3+11i\,a^2x^2+29\,ax+80i\right)\sqrt{a^2x^2+}}{8\left(a^5x+i\,a^4\right)}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^3,x, algorithm="fricas")
```

output
```
1/8*(32*I*a*x - 51*(-I*a*x + 1)*log(-a*x + sqrt(a^2*x^2 + 1)) + (-2*I*a^4*
x^4 - 6*a^3*x^3 + 11*I*a^2*x^2 + 29*a*x + 80*I)*sqrt(a^2*x^2 + 1) - 32)/(a
^5*x + I*a^4)
```

Sympy [F]

$$\int e^{3i\arctan(ax)}x^3\,dx = -i\left(\int \frac{ix^3}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx + \int \left(-\frac{3ax^4}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx + \int \frac{a^3x^6}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx + \int \left(-\frac{3ia^2x^5}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx\right)$$

input
```
integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)*x**3,x)
```

output
```
-I*(Integral(I*x**3/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)),
 x) + Integral(-3*a*x**4/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 +
 1)), x) + Integral(a**3*x**6/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x
**2 + 1)), x) + Integral(-3*I*a**2*x**5/(a**2*x**2*sqrt(a**2*x**2 + 1) + s
qrt(a**2*x**2 + 1)), x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int e^{3i\arctan(ax)}x^3\,dx = -\frac{i\,ax^5}{4\sqrt{a^2x^2+1}} - \frac{x^4}{\sqrt{a^2x^2+1}} + \frac{17i\,x^3}{8\sqrt{a^2x^2+1}a} + \frac{5\,x^2}{\sqrt{a^2x^2+1}a^2} + \frac{51i\,x}{8\sqrt{a^2x^2+1}a^3} - \frac{51i\,\operatorname{arsinh}(ax)}{8\,a^4} + \frac{10}{\sqrt{a^2x^2+1}a^4}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^3,x, algorithm="maxima")
```

output
```
-1/4*I*a*x^5/sqrt(a^2*x^2 + 1) - x^4/sqrt(a^2*x^2 + 1) + 17/8*I*x^3/(sqrt(
a^2*x^2 + 1)*a) + 5*x^2/(sqrt(a^2*x^2 + 1)*a^2) + 51/8*I*x/(sqrt(a^2*x^2 +
 1)*a^3) - 51/8*I*arcsinh(a*x)/a^4 + 10/(sqrt(a^2*x^2 + 1)*a^4)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3i\arctan(ax)}x^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.09

$$\int e^{3i\arctan(ax)}x^3\,dx = \frac{\sqrt{a^2\,x^2+1}\,\left(\frac{4}{{\left(a^2\right)}^{3/2}}+\frac{2\,\sqrt{a^2}}{a^4}-\frac{x^2\,\sqrt{a^2}}{a^2}-\frac{x^3\,{\left(a^2\right)}^{3/2}\,1{}\mathrm{i}}{4\,a^3}+\frac{x\,\sqrt{a^2}\,19{}\mathrm{i}}{8\,a^3}\right)}{\sqrt{a^2}} - \frac{\mathrm{asinh}\left(x\,\sqrt{a^2}\right)\,51{}\mathrm{i}}{8\,a^3\,\sqrt{a^2}}+\frac{\sqrt{a^2\,x^2+1}\,4{}\mathrm{i}}{a^3\,\left(x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1{}\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

input
```
int((x^3*(a*x*1i + 1)^3)/(a^2*x^2 + 1)^(3/2),x)
```

output
```
((a^2*x^2 + 1)^(1/2)*(4/(a^2)^(3/2) + (2*(a^2)^(1/2))/a^4 - (x^2*(a^2)^(1/
2))/a^2 - (x^3*(a^2)^(3/2)*1i)/(4*a^3) + (x*(a^2)^(1/2)*19i)/(8*a^3)))/(a^
2)^(1/2) - (asinh(x*(a^2)^(1/2))*51i)/(8*a^3*(a^2)^(1/2)) + ((a^2*x^2 + 1)
^(1/2)*4i)/(a^3*(((a^2)^(1/2)*1i)/a + x*(a^2)^(1/2))*(a^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.37

$$\int e^{3i\arctan(ax)}x^3\,dx$$
$$= \frac{-2\sqrt{a^2x^2+1}\,a^5i\,x^5 - 8\sqrt{a^2x^2+1}\,a^4x^4 + 17\sqrt{a^2x^2+1}\,a^3i\,x^3 + 40\sqrt{a^2x^2+1}\,a^2x^2 + 51\sqrt{a^2x^2+1}\,aix +}{8a^4\,(a^2x^2+1}$$

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^3,x)
```

output
```
( - 2*sqrt(a**2*x**2 + 1)*a**5*i*x**5 - 8*sqrt(a**2*x**2 + 1)*a**4*x**4 +
17*sqrt(a**2*x**2 + 1)*a**3*i*x**3 + 40*sqrt(a**2*x**2 + 1)*a**2*x**2 + 51
*sqrt(a**2*x**2 + 1)*a*i*x + 80*sqrt(a**2*x**2 + 1) - 51*log(sqrt(a**2*x**
2 + 1) + a*x)*a**2*i*x**2 - 51*log(sqrt(a**2*x**2 + 1) + a*x)*i + 37*a**2*
i*x**2 + 37*i)/(8*a**4*(a**2*x**2 + 1))
```

3.35 $\int e^{3i\arctan(ax)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 105

$$\int e^{3i\arctan(ax)}x^2\,dx=\frac{5i\sqrt{1+a^2x^2}}{a^3}-\frac{3x\sqrt{1+a^2x^2}}{2a^2}+\frac{4i\sqrt{1+a^2x^2}}{a^3(1-iax)}-\frac{i(1+a^2x^2)^{3/2}}{3a^3}+\frac{11\text{arcsinh}(ax)}{2a^3}$$

output
```
5*I*(a^2*x^2+1)^(1/2)/a^3-3/2*x*(a^2*x^2+1)^(1/2)/a^2+4*I*(a^2*x^2+1)^(1/2
)/a^3/(1-I*a*x)-1/3*I*(a^2*x^2+1)^(3/2)/a^3+11/2*arcsinh(a*x)/a^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.60

$$\int e^{3i\arctan(ax)}x^2\,dx=\frac{\frac{\sqrt{1+a^2x^2}(-52+19iax-7a^2x^2-2ia^3x^3)}{i+ax}+33\text{arcsinh}(ax)}{6a^3}$$

input
```
Integrate[E^((3*I)*ArcTan[a*x])*x^2,x]
```

output
```
((Sqrt[1 + a^2*x^2]*(-52 + (19*I)*a*x - 7*a^2*x^2 - (2*I)*a^3*x^3))/(I + a
*x) + 33*ArcSinh[a*x])/(6*a^3)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.19, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {5583, 2164, 2027, 2164, 25, 27, 563, 2346, 2346, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{3i\arctan(ax)}dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{x^2(1+iax)^2}{(1-iax)\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{2164}$$

$$-ia\int \frac{\sqrt{a^2x^2+1}\left(\frac{ix^2}{a}-x^3\right)}{(1-iax)^2}dx$$

$$\Big\downarrow \text{2027}$$

$$-ia\int \frac{\left(\frac{i}{a}-x\right)x^2\sqrt{a^2x^2+1}}{(1-iax)^2}dx$$

$$\Big\downarrow \text{2164}$$

$$-a^2\int -\frac{x^2\left(a^2x^2+1\right)^{3/2}}{a^2(1-iax)^3}dx$$

$$\Big\downarrow \text{25}$$

$$a^2\int \frac{x^2\left(a^2x^2+1\right)^{3/2}}{a^2(1-iax)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^2\left(a^2x^2+1\right)^{3/2}}{(1-iax)^3}dx$$

$$\Big\downarrow \text{563}$$

$$\frac{\int \frac{-ia^3x^3-3a^2x^2+4iax+4}{\sqrt{a^2x^2+1}}dx}{a^2}+\frac{4i\sqrt{a^2x^2+1}}{a^3(1-iax)}$$

$$\Big\downarrow \; 2346$$

$$\frac{\frac{\int \frac{-9x^2a^4+14iax a^3+12a^2}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{1}{3}iax^2\sqrt{a^2x^2+1}}{a^2}+\frac{4i\sqrt{a^2x^2+1}}{a^3(1-iax)}$$

$$\Big\downarrow \; 2346$$

$$\frac{\frac{-\frac{9}{2}a^2x\sqrt{a^2x^2+1}+\frac{\int \frac{a^4(28iax+33)}{\sqrt{a^2x^2+1}}dx}{2a^2}}{3a^2}-\frac{1}{3}iax^2\sqrt{a^2x^2+1}}{a^2}+\frac{4i\sqrt{a^2x^2+1}}{a^3(1-iax)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{-\frac{9}{2}a^2x\sqrt{a^2x^2+1}+\frac{1}{2}a^2\int \frac{28iax+33}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{1}{3}iax^2\sqrt{a^2x^2+1}}{a^2}+\frac{4i\sqrt{a^2x^2+1}}{a^3(1-iax)}$$

$$\Big\downarrow \; 455$$

$$\frac{\frac{-\frac{9}{2}a^2x\sqrt{a^2x^2+1}+\frac{1}{2}a^2\left(33\int \frac{1}{\sqrt{a^2x^2+1}}dx+\frac{28i\sqrt{a^2x^2+1}}{a}\right)}{3a^2}-\frac{1}{3}iax^2\sqrt{a^2x^2+1}}{a^2}+\frac{4i\sqrt{a^2x^2+1}}{a^3(1-iax)}$$

$$\Big\downarrow \; 222$$

$$\frac{\frac{-\frac{9}{2}a^2x\sqrt{a^2x^2+1}+\frac{1}{2}a^2\left(\frac{33\text{arcsinh}(ax)}{a}+\frac{28i\sqrt{a^2x^2+1}}{a}\right)}{3a^2}-\frac{1}{3}iax^2\sqrt{a^2x^2+1}}{a^2}+\frac{4i\sqrt{a^2x^2+1}}{a^3(1-iax)}$$

input
```
Int[E^((3*I)*ArcTan[a*x])*x^2,x]
```

output
```
((4*I)*Sqrt[1 + a^2*x^2])/(a^3*(1 - I*a*x)) + ((-1/3*I)*a*x^2*Sqrt[1 + a^2
*x^2] + ((-9*a^2*x*Sqrt[1 + a^2*x^2])/2 + (a^2*(((28*I)*Sqrt[1 + a^2*x^2])
/a + (33*ArcSinh[a*x])/a))/2)/(3*a^2))/a^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2164
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[d*e   Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + b*d*x, x]*
(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x]
 && EqQ[b*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + b*d*x, x], 0
]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.06

method	result
risch	$-\frac{i\left(2a^{2}x^{2}-9iax-28\right)\sqrt{a^{2}x^{2}+1}}{6a^{3}}-\frac{4\sqrt{\left(x+\frac{i}{a}\right)^{2}a^{2}-2ia\left(x+\frac{i}{a}\right)}}{a^{4}\left(x+\frac{i}{a}\right)}+\frac{11\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}+1}\right)}{2a^{2}\sqrt{a^{2}}}$
meijerg	$\frac{-\frac{\sqrt{\pi}\,x\left(a^{2}\right)^{\frac{3}{2}}}{a^{2}\sqrt{a^{2}x^{2}+1}}+\frac{\sqrt{\pi}\left(a^{2}\right)^{\frac{3}{2}}\operatorname{arcsinh}(ax)}{a^{3}}}{a^{2}\sqrt{\pi}\sqrt{a^{2}}}+\frac{3i\left(-2\sqrt{\pi}+\frac{\sqrt{\pi}\left(4a^{2}x^{2}+8\right)}{4\sqrt{a^{2}x^{2}+1}}\right)}{a^{3}\sqrt{\pi}}-\frac{3\left(\frac{\sqrt{\pi}\,x\left(a^{2}\right)^{\frac{5}{2}}\left(5a^{2}x^{2}+15\right)}{10a^{4}\sqrt{a^{2}x^{2}+1}}-\frac{3\sqrt{\pi}\left(a^{2}\right)^{\frac{5}{2}}\operatorname{arcsinh}(ax)}{2a^{5}}\right)}{a^{2}\sqrt{\pi}\sqrt{a^{2}}}-$
default	$-\frac{x}{a^{2}\sqrt{a^{2}x^{2}+1}}+\frac{\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}+1}\right)}{a^{2}\sqrt{a^{2}}}-ia^{3}\left(\frac{x^{4}}{3a^{2}\sqrt{a^{2}x^{2}+1}}-\frac{4\left(\frac{x^{2}}{a^{2}\sqrt{a^{2}x^{2}+1}}+\frac{2}{a^{4}\sqrt{a^{2}x^{2}+1}}\right)}{3a^{2}}\right)+3ia\left(\frac{x^{2}}{a^{2}\sqrt{a^{2}x^{2}+1}}+\right.$

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*I*(2*a^2*x^2-9*I*a*x-28)*(a^2*x^2+1)^(1/2)/a^3-4/a^4/(x+I/a)*((x+I/a)
^2*a^2-2*I*a*(x+I/a))^(1/2)+11/2/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2
))/(a^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.76

$$\int e^{3i\arctan(ax)}x^2\,dx = -\frac{24\,ax+33\,(ax+i)\log\left(-ax+\sqrt{a^2x^2+1}\right)-\left(-2i\,a^3x^3-7\,a^2x^2+19i\,ax-52\right)\sqrt{a^2x^2+1}+24i}{6\,(a^4x+i\,a^3)}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2,x, algorithm="fricas")
```

output
```
-1/6*(24*a*x + 33*(a*x + I)*log(-a*x + sqrt(a^2*x^2 + 1)) - (-2*I*a^3*x^3
- 7*a^2*x^2 + 19*I*a*x - 52)*sqrt(a^2*x^2 + 1) + 24*I)/(a^4*x + I*a^3)
```

Sympy [F]

$$\int e^{3i\arctan(ax)}x^2\,dx = -i\left(\int\frac{ix^2}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx + \int\left(-\frac{3ax^3}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx + \int\frac{a^3x^5}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx + \int\left(-\frac{3ia^2x^4}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx\right)$$

input
```
integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)*x**2,x)
```

output
```
-I*(Integral(I*x**2/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)),
 x) + Integral(-3*a*x**3/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 +
 1)), x) + Integral(a**3*x**5/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x
**2 + 1)), x) + Integral(-3*I*a**2*x**4/(a**2*x**2*sqrt(a**2*x**2 + 1) + s
qrt(a**2*x**2 + 1)), x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$\int e^{3i\arctan(ax)}x^2\,dx = -\frac{i\,ax^4}{3\,\sqrt{a^2x^2+1}} - \frac{3\,x^3}{2\,\sqrt{a^2x^2+1}} + \frac{13i\,x^2}{3\,\sqrt{a^2x^2+1}a} - \frac{11\,x}{2\,\sqrt{a^2x^2+1}a^2} + \frac{11\,\operatorname{arsinh}(ax)}{2\,a^3} + \frac{26i}{3\,\sqrt{a^2x^2+1}a^3}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2,x, algorithm="maxima")
```

output
```
-1/3*I*a*x^4/sqrt(a^2*x^2 + 1) - 3/2*x^3/sqrt(a^2*x^2 + 1) + 13/3*I*x^2/(s
qrt(a^2*x^2 + 1)*a) - 11/2*x/(sqrt(a^2*x^2 + 1)*a^2) + 11/2*arcsinh(a*x)/a
^3 + 26/3*I/(sqrt(a^2*x^2 + 1)*a^3)
```

Giac [F]

$$\int e^{3i\arctan(ax)}x^2\,dx = \int \frac{(i\,ax+1)^3x^2}{(a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.09

$$\int e^{3i\arctan(ax)}x^2\,dx = \frac{11\,\operatorname{asinh}\left(x\,\sqrt{a^2}\right)}{2\,a^2\,\sqrt{a^2}} - \frac{\sqrt{a^2\,x^2+1}\,\left(\frac{3\,x\,\sqrt{a^2}}{2\,a^2} - \frac{a\,14\mathrm{i}}{3\,{\left(a^2\right)}^{3/2}} + \frac{a^3\,x^2\,1\mathrm{i}}{3\,{\left(a^2\right)}^{3/2}}\right)}{\sqrt{a^2}} - \frac{4\,\sqrt{a^2\,x^2+1}}{a^2\,\left(x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

input
```
int((x^2*(a*x*1i + 1)^3)/(a^2*x^2 + 1)^(3/2),x)
```

output
```
(11*asinh(x*(a^2)^(1/2)))/(2*a^2*(a^2)^(1/2)) - ((a^2*x^2 + 1)^(1/2)*((a^3
*x^2*1i)/(3*(a^2)^(3/2)) - (a*14i)/(3*(a^2)^(3/2)) + (3*x*(a^2)^(1/2))/(2*
a^2)))/(a^2)^(1/2) - (4*(a^2*x^2 + 1)^(1/2))/(a^2*(((a^2)^(1/2)*1i)/a + x*
(a^2)^(1/2))*(a^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.42

$$\int e^{3i\arctan(ax)}x^2\,dx$$
$$= \frac{-2\sqrt{a^2x^2+1}\,a^4 i\,x^4 - 9\sqrt{a^2x^2+1}\,a^3x^3 + 26\sqrt{a^2x^2+1}\,a^2 i\,x^2 - 33\sqrt{a^2x^2+1}\,ax + 52\sqrt{a^2x^2+1}\,i + 33\mathrm{l}}{6a^3\left(a^2x^2+1\right)}$$

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2,x)
```

output
```
( - 2*sqrt(a**2*x**2 + 1)*a**4*i*x**4 - 9*sqrt(a**2*x**2 + 1)*a**3*x**3 +
26*sqrt(a**2*x**2 + 1)*a**2*i*x**2 - 33*sqrt(a**2*x**2 + 1)*a*x + 52*sqrt(
a**2*x**2 + 1)*i + 33*log(sqrt(a**2*x**2 + 1) + a*x)*a**2*x**2 + 33*log(sq
rt(a**2*x**2 + 1) + a*x) - 24*a**2*x**2 - 24)/(6*a**3*(a**2*x**2 + 1))
```

3.36 $\int e^{3i\arctan(ax)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 83

$$\int e^{3i\arctan(ax)}x\,dx=-\frac{3\sqrt{1+a^2x^2}}{a^2}-\frac{ix\sqrt{1+a^2x^2}}{2a}-\frac{4\sqrt{1+a^2x^2}}{a^2(1-iax)}+\frac{9i\text{arcsinh}(ax)}{2a^2}$$

output
```
-3*(a^2*x^2+1)^(1/2)/a^2-1/2*I*x*(a^2*x^2+1)^(1/2)/a-4*(a^2*x^2+1)^(1/2)/a
^2/(1-I*a*x)+9/2*I*arcsinh(a*x)/a^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.65

$$\int e^{3i\arctan(ax)}x\,dx=-\frac{i\left(\frac{\sqrt{1+a^2x^2}(14-5iax+a^2x^2)}{i+ax}-9\text{arcsinh}(ax)\right)}{2a^2}$$

input
```
Integrate[E^((3*I)*ArcTan[a*x])*x,x]
```

output
```
((-1/2*I)*((Sqrt[1 + a^2*x^2]*(14 - (5*I)*a*x + a^2*x^2))/(I + a*x) - 9*Ar
cSinh[a*x]))/a^2
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5583, 2164, 2027, 2164, 25, 27, 563, 25, 2346, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{3i\arctan(ax)}dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{x(1+iax)^2}{(1-iax)\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{2164}$$

$$-ia\int \frac{\left(\frac{ix}{a}-x^2\right)\sqrt{a^2x^2+1}}{(1-iax)^2}dx$$

$$\Big\downarrow \text{2027}$$

$$-ia\int \frac{\left(\frac{i}{a}-x\right)x\sqrt{a^2x^2+1}}{(1-iax)^2}dx$$

$$\Big\downarrow \text{2164}$$

$$-a^2\int -\frac{x\left(a^2x^2+1\right)^{3/2}}{a^2(1-iax)^3}dx$$

$$\Big\downarrow \text{25}$$

$$a^2\int \frac{x\left(a^2x^2+1\right)^{3/2}}{a^2(1-iax)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x\left(a^2x^2+1\right)^{3/2}}{(1-iax)^3}dx$$

$$\Big\downarrow \text{563}$$

$$-\frac{i\int -\frac{-a^2x^2+3iax+4}{\sqrt{a^2x^2+1}}dx}{a}-\frac{4\sqrt{a^2x^2+1}}{a^2(1-iax)}$$

$$\Big\downarrow \text{25}$$

$$\frac{i\int \frac{-a^2x^2+3iax+4}{\sqrt{a^2x^2+1}}dx}{a}-\frac{4\sqrt{a^2x^2+1}}{a^2(1-iax)}$$

$\big\downarrow$ 2346

$$\frac{i\left(-\frac{1}{2}x\sqrt{a^2x^2+1}+\frac{\int \frac{3a^2(2iax+3)}{\sqrt{a^2x^2+1}}dx}{2a^2}\right)}{a}-\frac{4\sqrt{a^2x^2+1}}{a^2(1-iax)}$$

$\big\downarrow$ 27

$$\frac{i\left(-\frac{1}{2}x\sqrt{a^2x^2+1}+\frac{3}{2}\int \frac{2iax+3}{\sqrt{a^2x^2+1}}dx\right)}{a}-\frac{4\sqrt{a^2x^2+1}}{a^2(1-iax)}$$

$\big\downarrow$ 455

$$\frac{i\left(-\frac{1}{2}x\sqrt{a^2x^2+1}+\frac{3}{2}\left(3\int \frac{1}{\sqrt{a^2x^2+1}}dx+\frac{2i\sqrt{a^2x^2+1}}{a}\right)\right)}{a}-\frac{4\sqrt{a^2x^2+1}}{a^2(1-iax)}$$

$\big\downarrow$ 222

$$\frac{i\left(-\frac{1}{2}x\sqrt{a^2x^2+1}+\frac{3}{2}\left(\frac{3\text{arcsinh}(ax)}{a}+\frac{2i\sqrt{a^2x^2+1}}{a}\right)\right)}{a}-\frac{4\sqrt{a^2x^2+1}}{a^2(1-iax)}$$

input
```
Int[E^((3*I)*ArcTan[a*x])*x,x]
```

output
```
(-4*Sqrt[1 + a^2*x^2])/(a^2*(1 - I*a*x)) + (I*(-1/2*(x*Sqrt[1 + a^2*x^2])
+ (3*(((2*I)*Sqrt[1 + a^2*x^2])/a + (3*ArcSinh[a*x])/a))/2))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2164
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[d*e   Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + b*d*x, x]*
(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x]
 && EqQ[b*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + b*d*x, x], 0
]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.28

method	result
risch	$-\frac{i(ax-6i)\sqrt{a^2x^2+1}}{2a^2}+\frac{i\left(-\frac{8\sqrt{\left(x+\frac{i}{a}\right)^2a^2-2ia\left(x+\frac{i}{a}\right)}}{a^2\left(x+\frac{i}{a}\right)}+\frac{9\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{\sqrt{a^2}}\right)}{2a}$
meijerg	$\frac{\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{a^2x^2+1}}}{a^2\sqrt{\pi}}+\frac{3i\left(-\frac{\sqrt{\pi}\,x\left(a^2\right)^{\frac{3}{2}}}{a^2\sqrt{a^2x^2+1}}+\frac{\sqrt{\pi}\left(a^2\right)^{\frac{3}{2}}\operatorname{arcsinh}(ax)}{a^3}\right)}{a\sqrt{\pi}\sqrt{a^2}}-\frac{3\left(-2\sqrt{\pi}+\frac{\sqrt{\pi}\left(4a^2x^2+8\right)}{4\sqrt{a^2x^2+1}}\right)}{a^2\sqrt{\pi}}-\frac{i\left(\frac{\sqrt{\pi}\,x\left(a^2\right)^{\frac{5}{2}}\left(5a^2x^2+15\right)}{10a^4\sqrt{a^2x^2+1}}-\frac{3\sqrt{\cdots}}{\cdots}\right.}{a\sqrt{\pi}\sqrt{a^2}\cdots}$
default	$-\frac{1}{a^2\sqrt{a^2x^2+1}}-ia^3\left(\frac{x^3}{2a^2\sqrt{a^2x^2+1}}-\frac{3\left(-\frac{x}{a^2\sqrt{a^2x^2+1}}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{a^2\sqrt{a^2}}\right)}{2a^2}\right)+3ia\left(-\frac{x}{a^2\sqrt{a^2x^2+1}}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\cdots\right.}{a^2\cdots}\right.$

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*(a*x-6*I)*(a^2*x^2+1)^(1/2)/a^2+1/2*I/a*(-8/a^2/(x+I/a)*((x+I/a)^2*
a^2-2*I*a*(x+I/a))^(1/2)+9*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(
1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int e^{3i\arctan(ax)}x\,dx$$
$$=\frac{-8i\,ax-9\,(i\,ax-1)\log\left(-ax+\sqrt{a^2x^2+1}\right)+\sqrt{a^2x^2+1}(-i\,a^2x^2-5\,ax-14i)+8}{2\,(a^3x+i\,a^2)}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x,x, algorithm="fricas")
```

output
```
1/2*(-8*I*a*x - 9*(I*a*x - 1)*log(-a*x + sqrt(a^2*x^2 + 1)) + sqrt(a^2*x^2
 + 1)*(-I*a^2*x^2 - 5*a*x - 14*I) + 8)/(a^3*x + I*a^2)
```

Sympy [F]

$$\int e^{3i\arctan(ax)}x\,dx = -i\left(\int \frac{ix}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx + \int \left(-\frac{3ax^2}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx + \int \frac{a^3x^4}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx + \int \left(-\frac{3ia^2x^3}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx\right)$$

input
```
integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)*x,x)
```

output
```
-I*(Integral(I*x/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)), x)
 + Integral(-3*a*x**2/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)
), x) + Integral(a**3*x**4/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2
 + 1)), x) + Integral(-3*I*a**2*x**3/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt
(a**2*x**2 + 1)), x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.92

$$\int e^{3i\arctan(ax)}x\,dx = -\frac{i\,ax^3}{2\sqrt{a^2x^2+1}} - \frac{3\,x^2}{\sqrt{a^2x^2+1}} - \frac{9i\,x}{2\sqrt{a^2x^2+1}a} + \frac{9i\,\operatorname{arsinh}(ax)}{2\,a^2} - \frac{7}{\sqrt{a^2x^2+1}a^2}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x,x, algorithm="maxima")
```

output
```
-1/2*I*a*x^3/sqrt(a^2*x^2 + 1) - 3*x^2/sqrt(a^2*x^2 + 1) - 9/2*I*x/(sqrt(a
^2*x^2 + 1)*a) + 9/2*I*arcsinh(a*x)/a^2 - 7/(sqrt(a^2*x^2 + 1)*a^2)
```

Giac [F]

$$\int e^{3i\arctan(ax)}x\,dx=\int\frac{(i\,ax+1)^3x}{(a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 23.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.25

$$\int e^{3i\arctan(ax)}x\,dx=-\frac{\sqrt{a^2\,x^2+1}\,\left(\frac{3\,\sqrt{a^2}}{a^2}+\frac{x\,\sqrt{a^2}\,1\mathrm{i}}{2\,a}\right)}{\sqrt{a^2}}+\frac{\mathrm{asinh}\left(x\,\sqrt{a^2}\right)\,9\mathrm{i}}{2\,a\,\sqrt{a^2}}-\frac{\sqrt{a^2\,x^2+1}\,4\mathrm{i}}{a\,\left(x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

input
```
int((x*(a*x*1i + 1)^3)/(a^2*x^2 + 1)^(3/2),x)
```

output
```
(asinh(x*(a^2)^(1/2))*9i)/(2*a*(a^2)^(1/2)) - ((a^2*x^2 + 1)^(1/2)*((3*(a^
2)^(1/2))/a^2 + (x*(a^2)^(1/2)*1i)/(2*a)))/(a^2)^(1/2) - ((a^2*x^2 + 1)^(1
/2)*4i)/(a*(((a^2)^(1/2)*1i)/a + x*(a^2)^(1/2))*(a^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.63

$$\int e^{3i\arctan(ax)}x\,dx=\frac{-\sqrt{a^2x^2+1}\,a^3i\,x^3-6\sqrt{a^2x^2+1}\,a^2x^2-9\sqrt{a^2x^2+1}\,aix-14\sqrt{a^2x^2+1}+9\log\left(\sqrt{a^2x^2+1}+ax\right)a^2i\,x}{2a^2\left(a^2x^2+1\right)}$$

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x,x)
```

output
```
( - sqrt(a**2*x**2 + 1)*a**3*i*x**3 - 6*sqrt(a**2*x**2 + 1)*a**2*x**2 - 9*
sqrt(a**2*x**2 + 1)*a*i*x - 14*sqrt(a**2*x**2 + 1) + 9*log(sqrt(a**2*x**2
+ 1) + a*x)*a**2*i*x**2 + 9*log(sqrt(a**2*x**2 + 1) + a*x)*i - 8*a**2*i*x*
*2 - 8*i)/(2*a**2*(a**2*x**2 + 1))
```

3.37 $\int e^{3i\arctan(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 60

$$\int e^{3i\arctan(ax)}\,dx=-\frac{i\sqrt{1+a^2x^2}}{a}-\frac{4i\sqrt{1+a^2x^2}}{a(1-iax)}-\frac{3\text{arcsinh}(ax)}{a}$$

output
```
-I*(a^2*x^2+1)^(1/2)/a-4*I*(a^2*x^2+1)^(1/2)/a/(1-I*a*x)-3*arcsinh(a*x)/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int e^{3i\arctan(ax)}\,dx=\frac{\sqrt{1+a^2x^2}\left(-i+\frac{4}{i+ax}\right)}{a}-\frac{3\text{arcsinh}(ax)}{a}$$

input
```
Integrate[E^((3*I)*ArcTan[a*x]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(-I + 4/(I + a*x)))/a - (3*ArcSinh[a*x])/a
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5582, 711, 25, 27, 671, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{3i\arctan(ax)}dx$$

$$\Big\downarrow \text{5582}$$

$$\int \frac{(1+iax)^2}{(1-iax)\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{711}$$

$$-\frac{\int -\frac{a^4(3iax+1)}{(1-iax)\sqrt{a^2x^2+1}}dx}{a^4}-\frac{i\sqrt{a^2x^2+1}}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a^4(3iax+1)}{(1-iax)\sqrt{a^2x^2+1}}dx}{a^4}-\frac{i\sqrt{a^2x^2+1}}{a}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{3iax+1}{(1-iax)\sqrt{a^2x^2+1}}dx-\frac{i\sqrt{a^2x^2+1}}{a}$$

$$\Big\downarrow \text{671}$$

$$-3\int \frac{1}{\sqrt{a^2x^2+1}}dx-\frac{i\sqrt{a^2x^2+1}}{a}-\frac{4i\sqrt{a^2x^2+1}}{a(1-iax)}$$

$$\Big\downarrow \text{222}$$

$$-\frac{i\sqrt{a^2x^2+1}}{a}-\frac{4i\sqrt{a^2x^2+1}}{a(1-iax)}-\frac{3\text{arcsinh}(ax)}{a}$$

input
```
Int[E^((3*I)*ArcTan[a*x]),x]
```

output
```
((-I)*Sqrt[1 + a^2*x^2])/a - ((4*I)*Sqrt[1 + a^2*x^2])/(a*(1 - I*a*x)) - (
3*ArcSinh[a*x])/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 711
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + c*x^2)^(p + 1)
/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m + n + 2*p + 1))
Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c*e^n*(m + n + 2*p + 1)*(f + g*x)
^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - 2*e*g^n*(m + p + n)*(d + e*x)^(n
 - 2)*(a*e - c*d*x), x], x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && Eq
Q[c*d^2 + a*e^2, 0] && IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

rule 5582
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 - I*a*x)^((I*n + 1)/2)
/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2]), x] /; FreeQ[a, x] && Intege
rQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.55

method	result	si
risch	$-\frac{i\sqrt{a^{2}x^{2}+1}}{a}+\frac{4\sqrt{\left(x+\frac{i}{a}\right)^{2}a^{2}-2ia\left(x+\frac{i}{a}\right)}}{a^{2}\left(x+\frac{i}{a}\right)}-\frac{3\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}+1}\right)}{\sqrt{a^{2}}}$	93
default	$\frac{x}{\sqrt{a^{2}x^{2}+1}}-ia^{3}\left(\frac{x^{2}}{a^{2}\sqrt{a^{2}x^{2}+1}}+\frac{2}{a^{4}\sqrt{a^{2}x^{2}+1}}\right)-\frac{3i}{a\sqrt{a^{2}x^{2}+1}}-3a^{2}\left(-\frac{x}{a^{2}\sqrt{a^{2}x^{2}+1}}+\frac{\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}+1}\right)}{a^{2}\sqrt{a^{2}}}\right)$	12
meijerg	$\frac{x}{\sqrt{a^{2}x^{2}+1}}+\frac{3i\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{a^{2}x^{2}+1}}\right)}{a\sqrt{\pi}}-\frac{3\left(-\frac{\sqrt{\pi}\,x\left(a^{2}\right)^{\frac{3}{2}}}{a^{2}\sqrt{a^{2}x^{2}+1}}+\frac{\sqrt{\pi}\left(a^{2}\right)^{\frac{3}{2}}\operatorname{arcsinh}(ax)}{a^{3}}\right)}{\sqrt{\pi}\sqrt{a^{2}}}-\frac{i\left(-2\sqrt{\pi}+\frac{\sqrt{\pi}\left(4a^{2}x^{2}+8\right)}{4\sqrt{a^{2}x^{2}+1}}\right)}{a\sqrt{\pi}}$	13

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-I*(a^2*x^2+1)^(1/2)/a+4/a^2/(x+I/a)*((x+I/a)^2*a^2-2*I*a*(x+I/a))^(1/2)-3
*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int e^{3i\arctan(ax)}\,dx = \frac{4\,ax+3\,(ax+i)\log\left(-ax+\sqrt{a^{2}x^{2}+1}\right)+\sqrt{a^{2}x^{2}+1}(-i\,ax+5)+4i}{a^{2}x+i\,a}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
(4*a*x + 3*(a*x + I)*log(-a*x + sqrt(a^2*x^2 + 1)) + sqrt(a^2*x^2 + 1)*(-I
*a*x + 5) + 4*I)/(a^2*x + I*a)
```

Sympy [F]

$$\int e^{3i\arctan(ax)}\,dx = -i\left(\int \frac{i}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx + \int\left(-\frac{3ax}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx + \int \frac{a^3x^3}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx + \int\left(-\frac{3ia^2x^2}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx\right)$$

input
```
integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2),x)
```

output
```
-I*(Integral(I/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)), x) +
 Integral(-3*a*x/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)), x)
 + Integral(a**3*x**3/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)
), x) + Integral(-3*I*a**2*x**2/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2
*x**2 + 1)), x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int e^{3i\arctan(ax)}\,dx = -\frac{i\,ax^2}{\sqrt{a^2x^2+1}} + \frac{4\,x}{\sqrt{a^2x^2+1}} - \frac{3\,\mathrm{arsinh}\,(ax)}{a} - \frac{5i}{\sqrt{a^2x^2+1}a}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
-I*a*x^2/sqrt(a^2*x^2 + 1) + 4*x/sqrt(a^2*x^2 + 1) - 3*arcsinh(a*x)/a - 5*
I/(sqrt(a^2*x^2 + 1)*a)
```

Giac [F]

$$\int e^{3i\arctan(ax)}\,dx = \int \frac{(i\,ax+1)^3}{(a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 22.74 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.20

$$\int e^{3i\arctan(ax)}\,dx = -\frac{\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{a} - \frac{3\,\mathrm{asinh}\left(x\,\sqrt{a^2}\right)}{\sqrt{a^2}} + \frac{4\,\sqrt{a^2\,x^2+1}}{\left(x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

input
```
int((a*x*1i + 1)^3/(a^2*x^2 + 1)^(3/2),x)
```

output
```
(4*(a^2*x^2 + 1)^(1/2))/((((a^2)^(1/2)*1i)/a + x*(a^2)^(1/2))*(a^2)^(1/2))
 - (3*asinh(x*(a^2)^(1/2)))/(a^2)^(1/2) - ((a^2*x^2 + 1)^(1/2)*1i)/a
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.85

$$\int e^{3i\arctan(ax)}\,dx = \frac{-\sqrt{a^2x^2+1}\,a^2i\,x^2+4\sqrt{a^2x^2+1}\,ax-5\sqrt{a^2x^2+1}\,i-3\log\left(\sqrt{a^2x^2+1}+ax\right)a^2x^2-3\log\left(\sqrt{a^2x^2+1}-\right.}{a\,(a^2x^2+1)}$$

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^(3/2),x)
```

output
```
( - sqrt(a**2*x**2 + 1)*a**2*i*x**2 + 4*sqrt(a**2*x**2 + 1)*a*x - 5*sqrt(a
**2*x**2 + 1)*i - 3*log(sqrt(a**2*x**2 + 1) + a*x)*a**2*x**2 - 3*log(sqrt(
a**2*x**2 + 1) + a*x) + 4*a**2*x**2 + 4)/(a*(a**2*x**2 + 1))
```

3.38 $\int \frac{e^{3i\arctan(ax)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \frac{e^{3i\arctan(ax)}}{x}\,dx = \frac{4\sqrt{1+a^2x^2}}{1-iax} - i\operatorname{arcsinh}(ax) - \operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

output
```
4*(a^2*x^2+1)^(1/2)/(1-I*a*x)-I*arcsinh(a*x)-arctanh((a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10

$$\int \frac{e^{3i\arctan(ax)}}{x}\,dx = \frac{4i\sqrt{1+a^2x^2}}{i+ax} - i\operatorname{arcsinh}(ax) + \log(x) - \log\left(1+\sqrt{1+a^2x^2}\right)$$

input
```
Integrate[E^((3*I)*ArcTan[a*x])/x,x]
```

output
```
((4*I)*Sqrt[1 + a^2*x^2])/(I + a*x) - I*ArcSinh[a*x] + Log[x] - Log[1 + Sq
rt[1 + a^2*x^2]]
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {5583, 2351, 564, 25, 243, 73, 221, 671, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3i\arctan(ax)}}{x}dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{(1+iax)^2}{x(1-iax)\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{2351}$$

$$\int \frac{1}{x(1-iax)\sqrt{a^2x^2+1}}dx+\int \frac{2ia-a^2x}{(1-iax)\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{564}$$

$$-\int -\frac{1}{x\sqrt{a^2x^2+1}}dx+\int \frac{2ia-a^2x}{(1-iax)\sqrt{a^2x^2+1}}dx+\frac{\sqrt{a^2x^2+1}}{1-iax}$$

$$\Big\downarrow \text{25}$$

$$\int \frac{1}{x\sqrt{a^2x^2+1}}dx+\int \frac{2ia-a^2x}{(1-iax)\sqrt{a^2x^2+1}}dx+\frac{\sqrt{a^2x^2+1}}{1-iax}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{a^2x^2+1}}dx^2+\int \frac{2ia-a^2x}{(1-iax)\sqrt{a^2x^2+1}}dx+\frac{\sqrt{a^2x^2+1}}{1-iax}$$

$$\Big\downarrow \text{73}$$

$$\int \frac{2ia-a^2x}{(1-iax)\sqrt{a^2x^2+1}}dx+\frac{\int \frac{1}{\frac{x^4}{a^2}-\frac{1}{a^2}}d\sqrt{a^2x^2+1}}{a^2}+\frac{\sqrt{a^2x^2+1}}{1-iax}$$

$$\Big\downarrow \text{221}$$

$$\int \frac{2ia-a^2x}{(1-iax)\sqrt{a^2x^2+1}}dx-\operatorname{arctanh}\left(\sqrt{a^2x^2+1}\right)+\frac{\sqrt{a^2x^2+1}}{1-iax}$$

$$\Big\downarrow \text{671}$$

$$-ia\int \frac{1}{\sqrt{a^2x^2+1}}dx-\operatorname{arctanh}\left(\sqrt{a^2x^2+1}\right)+\frac{4\sqrt{a^2x^2+1}}{1-iax}$$

$$\Big\downarrow \text{222}$$

$$-\operatorname{arctanh}\left(\sqrt{a^2x^2+1}\right)+\frac{4\sqrt{a^2x^2+1}}{1-iax}-i\operatorname{arcsinh}(ax)$$

input
```
Int[E^((3*I)*ArcTan[a*x])/x,x]
```

output
```
(4*Sqrt[1 + a^2*x^2])/(1 - I*a*x) - I*ArcSinh[a*x] - ArcTanh[Sqrt[1 + a^2*
x^2]]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 564
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(44) = 88$.

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.02

method	result
default	$\frac{4}{\sqrt{a^2x^2+1}}-\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)+\frac{3iax}{\sqrt{a^2x^2+1}}-ia^3\left(-\frac{x}{a^2\sqrt{a^2x^2+1}}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{a^2\sqrt{a^2}}\right)$
meijerg	$\frac{\frac{\left(2-2\ln(2)+2\ln(x)+\ln\left(a^2\right)\right)\sqrt{\pi}}{2}-\sqrt{\pi}+\frac{\sqrt{\pi}}{\sqrt{a^2x^2+1}}-\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{a^2x^2+1}}{2}\right)}{\sqrt{\pi}}+\frac{3iax}{\sqrt{a^2x^2+1}}-\frac{3\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{a^2x^2+1}}\right)}{\sqrt{\pi}}-\frac{ia\left(-\frac{\sqrt{\pi}\,x\left(a^2\right)}{a^2\sqrt{a^2x^2}}\right.}{}$

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
4/(a^2*x^2+1)^(1/2)-arctanh(1/(a^2*x^2+1)^(1/2))+3*I*a*x/(a^2*x^2+1)^(1/2)
-I*a^3*(-x/a^2/(a^2*x^2+1)^(1/2)+1/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1
/2))/(a^2)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(42) = 84$.

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.00

$$\int \frac{e^{3i\arctan(ax)}}{x}\,dx$$
$$= \frac{4i\,ax-(ax+i)\log\left(-ax+\sqrt{a^2x^2+1}+1\right)+(i\,ax-1)\log\left(-ax+\sqrt{a^2x^2+1}\right)+(ax+i)\log\left(-ax+\right.}{ax+i}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x,x, algorithm="fricas")
```

output
```
(4*I*a*x - (a*x + I)*log(-a*x + sqrt(a^2*x^2 + 1) + 1) + (I*a*x - 1)*log(-
a*x + sqrt(a^2*x^2 + 1)) + (a*x + I)*log(-a*x + sqrt(a^2*x^2 + 1) - 1) + 4
*I*sqrt(a^2*x^2 + 1) - 4)/(a*x + I)
```

Sympy [F]

$$\int \frac{e^{3i\arctan(ax)}}{x}\,dx = -i\left(\int \frac{i}{a^2x^3\sqrt{a^2x^2+1}+x\sqrt{a^2x^2+1}}\,dx + \int \left(-\frac{3ax}{a^2x^3\sqrt{a^2x^2+1}+x\sqrt{a^2x^2+1}}\right)dx + \int \frac{a^3x^3}{a^2x^3\sqrt{a^2x^2+1}+x\sqrt{a^2x^2+1}}\,dx + \int \left(-\frac{3ia^2x^2}{a^2x^3\sqrt{a^2x^2+1}+x\sqrt{a^2x^2+1}}\right)dx\right)$$

input
```
integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)/x,x)
```

output
```
-I*(Integral(I/(a**2*x**3*sqrt(a**2*x**2 + 1) + x*sqrt(a**2*x**2 + 1)), x)
 + Integral(-3*a*x/(a**2*x**3*sqrt(a**2*x**2 + 1) + x*sqrt(a**2*x**2 + 1))
, x) + Integral(a**3*x**3/(a**2*x**3*sqrt(a**2*x**2 + 1) + x*sqrt(a**2*x**
2 + 1)), x) + Integral(-3*I*a**2*x**2/(a**2*x**3*sqrt(a**2*x**2 + 1) + x*s
qrt(a**2*x**2 + 1)), x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{e^{3i\arctan(ax)}}{x}\,dx = \frac{4i\,ax}{\sqrt{a^2x^2+1}} + \frac{4}{\sqrt{a^2x^2+1}} - i\,\operatorname{arsinh}(ax) - \operatorname{arsinh}\left(\frac{1}{a|x|}\right)$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x,x, algorithm="maxima")
```

output
```
4*I*a*x/sqrt(a^2*x^2 + 1) + 4/sqrt(a^2*x^2 + 1) - I*arcsinh(a*x) - arcsinh
(1/(a*abs(x)))
```

Giac [F]

$$\int \frac{e^{3i\arctan(ax)}}{x}\,dx = \int \frac{(i\,ax+1)^3}{(a^2x^2+1)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 23.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.46

$$\int \frac{e^{3i\arctan(ax)}}{x}\,dx = -\mathrm{atanh}\left(\sqrt{a^2\,x^2+1}\right) - \frac{a\,\mathrm{asinh}\left(x\,\sqrt{a^2}\right)\,1\mathrm{i}}{\sqrt{a^2}} + \frac{a\,\sqrt{a^2\,x^2+1}\,4\mathrm{i}}{\left(x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

input
```
int((a*x*1i + 1)^3/(x*(a^2*x^2 + 1)^(3/2)),x)
```

output
```
(a*(a^2*x^2 + 1)^(1/2)*4i)/((((a^2)^(1/2)*1i)/a + x*(a^2)^(1/2))*(a^2)^(1/
2)) - (a*asinh(x*(a^2)^(1/2))*1i)/(a^2)^(1/2) - atanh((a^2*x^2 + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 175, normalized size of antiderivative = 3.50

$$\int \frac{e^{3i\arctan(ax)}}{x}\,dx$$
$$= \frac{4\sqrt{a^2x^2+1}\,aix + 4\sqrt{a^2x^2+1} + \log\left(\sqrt{a^2x^2+1}+ax-1\right)a^2x^2 + \log\left(\sqrt{a^2x^2+1}+ax-1\right) - \log\left(\sqrt{a^2x^2}\right.}{}$$

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x,x)
```

output

```
(4*sqrt(a**2*x**2 + 1)*a*i*x + 4*sqrt(a**2*x**2 + 1) + log(sqrt(a**2*x**2
+ 1) + a*x - 1)*a**2*x**2 + log(sqrt(a**2*x**2 + 1) + a*x - 1) - log(sqrt(
a**2*x**2 + 1) + a*x + 1)*a**2*x**2 - log(sqrt(a**2*x**2 + 1) + a*x + 1) -
 log(sqrt(a**2*x**2 + 1) + a*x)*a**2*i*x**2 - log(sqrt(a**2*x**2 + 1) + a*
x)*i + 4*a**2*i*x**2 + 4*i)/(a**2*x**2 + 1)
```

3.39 $\int \frac{e^{3i\arctan(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 66

$$\int \frac{e^{3i\arctan(ax)}}{x^2}\,dx = -\frac{\sqrt{1+a^2x^2}}{x} + \frac{4ia\sqrt{1+a^2x^2}}{1-iax} - 3ia\text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

output
```
-(a^2*x^2+1)^(1/2)/x+4*I*a*(a^2*x^2+1)^(1/2)/(1-I*a*x)-3*I*a*arctanh((a^2*
x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{e^{3i\arctan(ax)}}{x^2}\,dx = \sqrt{1+a^2x^2}\left(-\frac{1}{x}-\frac{4a}{i+ax}\right) + 3ia\log(x) - 3ia\log\left(1+\sqrt{1+a^2x^2}\right)$$

input
```
Integrate[E^((3*I)*ArcTan[a*x])/x^2,x]
```

output
```
Sqrt[1 + a^2*x^2]*(-x^(-1) - (4*a)/(I + a*x)) + (3*I)*a*Log[x] - (3*I)*a*L
og[1 + Sqrt[1 + a^2*x^2]]
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5583, 2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3i\arctan(ax)}}{x^2}dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{(1+iax)^2}{x^2(1-iax)\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{2353}$$

$$\int \left(-\frac{4ia^2}{(ax+i)\sqrt{a^2x^2+1}}+\frac{3ia}{x\sqrt{a^2x^2+1}}+\frac{1}{x^2\sqrt{a^2x^2+1}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-3ia\text{arctanh}\left(\sqrt{a^2x^2+1}\right)-\frac{4a\sqrt{a^2x^2+1}}{ax+i}-\frac{\sqrt{a^2x^2+1}}{x}$$

input
```
Int[E^((3*I)*ArcTan[a*x])/x^2,x]
```

output
```
-(Sqrt[1 + a^2*x^2]/x) - (4*a*Sqrt[1 + a^2*x^2])/(I + a*x) - (3*I)*a*ArcTa
nh[Sqrt[1 + a^2*x^2]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.21

method	result
default	$-\frac{1}{x\sqrt{a^2x^2+1}}-\frac{5a^2x}{\sqrt{a^2x^2+1}}+\frac{ia}{\sqrt{a^2x^2+1}}+3ia\left(\frac{1}{\sqrt{a^2x^2+1}}-\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)\right)$
risch	$-\frac{\sqrt{a^2x^2+1}}{x}+ia\left(-3\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)+\frac{4i\sqrt{\left(x+\frac{i}{a}\right)^2a^2-2ia\left(x+\frac{i}{a}\right)}}{a\left(x+\frac{i}{a}\right)}\right)$
meijerg	$-\frac{2a^2x^2+1}{x\sqrt{a^2x^2+1}}+\frac{3ia\left(\frac{\left(2-2\ln(2)+2\ln(x)+\ln\left(a^2\right)\right)\sqrt{\pi}}{2}-\sqrt{\pi}+\frac{\sqrt{\pi}}{\sqrt{a^2x^2+1}}-\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{a^2x^2+1}}{2}\right)\right)}{\sqrt{\pi}}-\frac{3a^2x}{\sqrt{a^2x^2+1}}-\frac{ia\left(\sqrt{\pi}-\frac{\sqrt{}}{\sqrt{a^2x}}\right.}{\sqrt{\pi}}$

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x/(a^2*x^2+1)^(1/2)-5/(a^2*x^2+1)^(1/2)*a^2*x+I*a/(a^2*x^2+1)^(1/2)+3*I
*a*(1/(a^2*x^2+1)^(1/2)-arctanh(1/(a^2*x^2+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(54) = 108.

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.65

$$\int \frac{e^{3i\arctan(ax)}}{x^2}\,dx = -\frac{5a^2x^2 + 5i\,ax + 3\left(i\,a^2x^2 - ax\right)\log\left(-ax + \sqrt{a^2x^2+1} + 1\right) + 3\left(-i\,a^2x^2 + ax\right)\log\left(-ax + \sqrt{a^2x^2+1}\right.}{ax^2 + i\,x}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
-(5*a^2*x^2 + 5*I*a*x + 3*(I*a^2*x^2 - a*x)*log(-a*x + sqrt(a^2*x^2 + 1) +
 1) + 3*(-I*a^2*x^2 + a*x)*log(-a*x + sqrt(a^2*x^2 + 1) - 1) + sqrt(a^2*x^
2 + 1)*(5*a*x + I))/(a*x^2 + I*x)
```

Sympy [F]

$$\int \frac{e^{3i\arctan(ax)}}{x^2}\,dx = -i\left(\int \frac{i}{a^2x^4\sqrt{a^2x^2+1} + x^2\sqrt{a^2x^2+1}}\,dx + \int \left(-\frac{3ax}{a^2x^4\sqrt{a^2x^2+1} + x^2\sqrt{a^2x^2+1}}\right)dx + \int \frac{a^3x^3}{a^2x^4\sqrt{a^2x^2+1} + x^2\sqrt{a^2x^2+1}}\,dx + \int \left(-\frac{3ia^2x^2}{a^2x^4\sqrt{a^2x^2+1} + x^2\sqrt{a^2x^2+1}}\right)dx\right)$$

input
```
integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)/x**2,x)
```

output
```
-I*(Integral(I/(a**2*x**4*sqrt(a**2*x**2 + 1) + x**2*sqrt(a**2*x**2 + 1)),
 x) + Integral(-3*a*x/(a**2*x**4*sqrt(a**2*x**2 + 1) + x**2*sqrt(a**2*x**2
 + 1)), x) + Integral(a**3*x**3/(a**2*x**4*sqrt(a**2*x**2 + 1) + x**2*sqrt
(a**2*x**2 + 1)), x) + Integral(-3*I*a**2*x**2/(a**2*x**4*sqrt(a**2*x**2 +
 1) + x**2*sqrt(a**2*x**2 + 1)), x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.91

$$\int \frac{e^{3i\arctan(ax)}}{x^2}\,dx = -\frac{5\,a^2x}{\sqrt{a^2x^2+1}} - 3i\,a\,\text{arsinh}\left(\frac{1}{a|x|}\right) + \frac{4i\,a}{\sqrt{a^2x^2+1}} - \frac{1}{\sqrt{a^2x^2+1}x}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
-5*a^2*x/sqrt(a^2*x^2 + 1) - 3*I*a*arcsinh(1/(a*abs(x))) + 4*I*a/sqrt(a^2*
x^2 + 1) - 1/(sqrt(a^2*x^2 + 1)*x)
```

Giac [F]

$$\int \frac{e^{3i\arctan(ax)}}{x^2}\,dx = \int \frac{(i\,ax+1)^3}{(a^2x^2+1)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^2,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

$$\int \frac{e^{3i\arctan(ax)}}{x^2}\,dx = -a\,\text{atanh}\left(\sqrt{a^2\,x^2+1}\right)\,3\text{i} - \frac{\sqrt{a^2\,x^2+1}}{x} - \frac{4\,a^2\,\sqrt{a^2\,x^2+1}}{\left(x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\text{i}}{a}\right)\,\sqrt{a^2}}$$

input
```
int((a*x*1i + 1)^3/(x^2*(a^2*x^2 + 1)^(3/2)),x)
```

output
```
- a*atanh((a^2*x^2 + 1)^(1/2))*3i - (a^2*x^2 + 1)^(1/2)/x - (4*a^2*(a^2*x^
2 + 1)^(1/2))/((((a^2)^(1/2)*1i)/a + x*(a^2)^(1/2))*(a^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.50

$$\int \frac{e^{3i\arctan(ax)}}{x^2}\,dx = \frac{-5\sqrt{a^2x^2+1}\,a^2x^2+4\sqrt{a^2x^2+1}\,aix-\sqrt{a^2x^2+1}+3\log\left(\sqrt{a^2x^2+1}+ax-1\right)a^3i\,x^3+3\log\left(\sqrt{a^2x^2+\cdots}\right.}{x\,(a^2x^2\cdots}$$

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^2,x)
```

output
```
( - 5*sqrt(a**2*x**2 + 1)*a**2*x**2 + 4*sqrt(a**2*x**2 + 1)*a*i*x - sqrt(a
**2*x**2 + 1) + 3*log(sqrt(a**2*x**2 + 1) + a*x - 1)*a**3*i*x**3 + 3*log(s
qrt(a**2*x**2 + 1) + a*x - 1)*a*i*x - 3*log(sqrt(a**2*x**2 + 1) + a*x + 1)
*a**3*i*x**3 - 3*log(sqrt(a**2*x**2 + 1) + a*x + 1)*a*i*x - 5*a**3*x**3 -
5*a*x)/(x*(a**2*x**2 + 1))
```

3.40 $\int \frac{e^{3i\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 91

$$\int \frac{e^{3i\arctan(ax)}}{x^3}\,dx = -\frac{\sqrt{1+a^2x^2}}{2x^2}-\frac{3ia\sqrt{1+a^2x^2}}{x}-\frac{4a^2\sqrt{1+a^2x^2}}{1-iax}+\frac{9}{2}a^2\operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

output
```
-1/2*(a^2*x^2+1)^(1/2)/x^2-3*I*a*(a^2*x^2+1)^(1/2)/x-4*a^2*(a^2*x^2+1)^(1/
2)/(1-I*a*x)+9/2*a^2*arctanh((a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.87

$$\int \frac{e^{3i\arctan(ax)}}{x^3}\,dx = \sqrt{1+a^2x^2}\left(-\frac{1}{2x^2}-\frac{3ia}{x}-\frac{4ia^2}{i+ax}\right)-\frac{9}{2}a^2\log(x)+\frac{9}{2}a^2\log\left(1+\sqrt{1+a^2x^2}\right)$$

input
```
Integrate[E^((3*I)*ArcTan[a*x])/x^3,x]
```

output
```
Sqrt[1 + a^2*x^2]*(-1/2*1/x^2 - ((3*I)*a)/x - ((4*I)*a^2)/(I + a*x)) - (9*
a^2*Log[x])/2 + (9*a^2*Log[1 + Sqrt[1 + a^2*x^2]])/2
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5583, 2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3i\arctan(ax)}}{x^3}dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{(1+iax)^2}{x^3(1-iax)\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{2353}$$

$$\int \left(-\frac{4a^2}{x\sqrt{a^2x^2+1}}+\frac{3ia}{x^2\sqrt{a^2x^2+1}}+\frac{1}{x^3\sqrt{a^2x^2+1}}+\frac{4a^3}{(ax+i)\sqrt{a^2x^2+1}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{9}{2}a^2\text{arctanh}\left(\sqrt{a^2x^2+1}\right)-\frac{4ia^2\sqrt{a^2x^2+1}}{ax+i}-\frac{3ia\sqrt{a^2x^2+1}}{x}-\frac{\sqrt{a^2x^2+1}}{2x^2}$$

input
```
Int[E^((3*I)*ArcTan[a*x])/x^3,x]
```

output
```
-1/2*Sqrt[1 + a^2*x^2]/x^2 - ((3*I)*a*Sqrt[1 + a^2*x^2])/x - ((4*I)*a^2*Sq
rt[1 + a^2*x^2])/(I + a*x) + (9*a^2*ArcTanh[Sqrt[1 + a^2*x^2]])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.15

method	result
default	$-\frac{1}{2x^2\sqrt{a^2x^2+1}}-\frac{9a^2\left(\frac{1}{\sqrt{a^2x^2+1}}-\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)\right)}{2}-\frac{ia^3x}{\sqrt{a^2x^2+1}}+3ia\left(-\frac{1}{x\sqrt{a^2x^2+1}}-\frac{2a^2x}{\sqrt{a^2x^2+1}}\right)$
risch	$-\frac{i\left(6a^3x^3-ia^2x^2+6ax-i\right)}{2x^2\sqrt{a^2x^2+1}}-\frac{a^2\left(-9\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)+\frac{8i\sqrt{\left(x+\frac{i}{a}\right)^2a^2-2ia\left(x+\frac{i}{a}\right)}}{a\left(x+\frac{i}{a}\right)}\right)}{2}$
meijerg	$\frac{a^2\left(-\frac{\sqrt{\pi}}{2x^2a^2}-\frac{3\left(\frac{5}{3}-2\ln(2)+2\ln(x)+\ln\left(a^2\right)\right)\sqrt{\pi}}{4}+\frac{\sqrt{\pi}\left(20a^2x^2+8\right)}{16a^2x^2}-\frac{\sqrt{\pi}\left(24a^2x^2+8\right)}{16a^2x^2\sqrt{a^2x^2+1}}+\frac{3\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{a^2x^2+1}}{2}\right)}{2}\right)}{\sqrt{\pi}}-\frac{3ia\left(2a^2x^2+1\right)}{x\sqrt{a^2x^2+1}}-$

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2/(a^2*x^2+1)^(1/2)-9/2*a^2*(1/(a^2*x^2+1)^(1/2)-arctanh(1/(a^2*x^2
+1)^(1/2)))-I*a^3*x/(a^2*x^2+1)^(1/2)+3*I*a*(-1/x/(a^2*x^2+1)^(1/2)-2/(a^2
*x^2+1)^(1/2)*a^2*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.43

$$\int \frac{e^{3i\arctan(ax)}}{x^3}\,dx$$
$$= \frac{-14i\,a^3x^3 + 14\,a^2x^2 + 9\,(a^3x^3 + i\,a^2x^2)\log\left(-ax + \sqrt{a^2x^2+1}+1\right) - 9\,(a^3x^3 + i\,a^2x^2)\log\left(-ax + \sqrt{a^2}\right.}{2\,(ax^3 + i\,x^2)}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
1/2*(-14*I*a^3*x^3 + 14*a^2*x^2 + 9*(a^3*x^3 + I*a^2*x^2)*log(-a*x + sqrt(
a^2*x^2 + 1) + 1) - 9*(a^3*x^3 + I*a^2*x^2)*log(-a*x + sqrt(a^2*x^2 + 1) -
 1) + sqrt(a^2*x^2 + 1)*(-14*I*a^2*x^2 + 5*a*x - I))/(a*x^3 + I*x^2)
```

Sympy [F]

$$\int \frac{e^{3i\arctan(ax)}}{x^3}\,dx = -i\left(\int \frac{i}{a^2x^5\sqrt{a^2x^2+1} + x^3\sqrt{a^2x^2+1}}\,dx + \int \left(-\frac{3ax}{a^2x^5\sqrt{a^2x^2+1} + x^3\sqrt{a^2x^2+1}}\right)dx + \int \frac{a^3x^3}{a^2x^5\sqrt{a^2x^2+1} + x^3\sqrt{a^2x^2+1}}\,dx + \int \left(-\frac{3ia^2x^2}{a^2x^5\sqrt{a^2x^2+1} + x^3\sqrt{a^2x^2+1}}\right)dx\right)$$

input
```
integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)/x**3,x)
```

output
```
-I*(Integral(I/(a**2*x**5*sqrt(a**2*x**2 + 1) + x**3*sqrt(a**2*x**2 + 1)),
 x) + Integral(-3*a*x/(a**2*x**5*sqrt(a**2*x**2 + 1) + x**3*sqrt(a**2*x**2
 + 1)), x) + Integral(a**3*x**3/(a**2*x**5*sqrt(a**2*x**2 + 1) + x**3*sqrt
(a**2*x**2 + 1)), x) + Integral(-3*I*a**2*x**2/(a**2*x**5*sqrt(a**2*x**2 +
 1) + x**3*sqrt(a**2*x**2 + 1)), x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.89

$$\int \frac{e^{3i\arctan(ax)}}{x^3}\,dx = -\frac{7i\,a^3x}{\sqrt{a^2x^2+1}} + \frac{9}{2}\,a^2\,\mathrm{arsinh}\left(\frac{1}{a|x|}\right) - \frac{9\,a^2}{2\,\sqrt{a^2x^2+1}} - \frac{3i\,a}{\sqrt{a^2x^2+1}x} - \frac{1}{2\,\sqrt{a^2x^2+1}x^2}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
-7*I*a^3*x/sqrt(a^2*x^2 + 1) + 9/2*a^2*arcsinh(1/(a*abs(x))) - 9/2*a^2/sqr
t(a^2*x^2 + 1) - 3*I*a/(sqrt(a^2*x^2 + 1)*x) - 1/2/(sqrt(a^2*x^2 + 1)*x^2)
```

Giac [F]

$$\int \frac{e^{3i\arctan(ax)}}{x^3}\,dx = \int \frac{(i\,ax+1)^3}{(a^2x^2+1)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^3,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 22.79 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.09

$$\int \frac{e^{3i\arctan(ax)}}{x^3}\,dx = -\frac{a^2\,\mathrm{atan}\left(\sqrt{a^2\,x^2+1}\,1\mathrm{i}\right)\,9\mathrm{i}}{2} - \frac{\sqrt{a^2\,x^2+1}}{2\,x^2} - \frac{a\,\sqrt{a^2\,x^2+1}\,3\mathrm{i}}{x} - \frac{a^3\,\sqrt{a^2\,x^2+1}\,4\mathrm{i}}{\left(x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

input
```
int((a*x*1i + 1)^3/(x^3*(a^2*x^2 + 1)^(3/2)),x)
```

output
```
- (a^2*atan((a^2*x^2 + 1)^(1/2)*1i)*9i)/2 - (a^2*x^2 + 1)^(1/2)/(2*x^2) -
(a*(a^2*x^2 + 1)^(1/2)*3i)/x - (a^3*(a^2*x^2 + 1)^(1/2)*4i)/((((a^2)^(1/2)
*1i)/a + x*(a^2)^(1/2))*(a^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.14

$$\int \frac{e^{3i\arctan(ax)}}{x^3}\,dx = \frac{-14\sqrt{a^2x^2+1}\,a^3i\,x^3-9\sqrt{a^2x^2+1}\,a^2x^2-6\sqrt{a^2x^2+1}\,aix-\sqrt{a^2x^2+1}-9\log\left(\sqrt{a^2x^2+1}+ax-1\right)a$$

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^3,x)
```

output
```
( - 14*sqrt(a**2*x**2 + 1)*a**3*i*x**3 - 9*sqrt(a**2*x**2 + 1)*a**2*x**2 -
 6*sqrt(a**2*x**2 + 1)*a*i*x - sqrt(a**2*x**2 + 1) - 9*log(sqrt(a**2*x**2
+ 1) + a*x - 1)*a**4*x**4 - 9*log(sqrt(a**2*x**2 + 1) + a*x - 1)*a**2*x**2
 + 9*log(sqrt(a**2*x**2 + 1) + a*x + 1)*a**4*x**4 + 9*log(sqrt(a**2*x**2 +
 1) + a*x + 1)*a**2*x**2 + 18*a**4*i*x**4 + 18*a**2*i*x**2)/(2*x**2*(a**2*
x**2 + 1))
```

3.41 $\int \frac{e^{3i\arctan(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 120

$$\int \frac{e^{3i\arctan(ax)}}{x^4}\,dx = -\frac{\sqrt{1+a^2x^2}}{3x^3}-\frac{3ia\sqrt{1+a^2x^2}}{2x^2}+\frac{14a^2\sqrt{1+a^2x^2}}{3x}-\frac{4ia^3\sqrt{1+a^2x^2}}{1-iax}+\frac{11}{2}ia^3\operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

output
```
-1/3*(a^2*x^2+1)^(1/2)/x^3-3/2*I*a*(a^2*x^2+1)^(1/2)/x^2+14/3*a^2*(a^2*x^2
+1)^(1/2)/x-4*I*a^3*(a^2*x^2+1)^(1/2)/(1-I*a*x)+11/2*I*a^3*arctanh((a^2*x^
2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int \frac{e^{3i\arctan(ax)}}{x^4}\,dx = \frac{1}{6}\left(\frac{\sqrt{1+a^2x^2}(-2i+7ax+19ia^2x^2+52a^3x^3)}{x^3(i+ax)}-33ia^3\log(x)+33ia^3\log\left(1+\sqrt{1+a^2x^2}\right)\right)$$

input
```
Integrate[E^((3*I)*ArcTan[a*x])/x^4,x]
```

output
```
((Sqrt[1 + a^2*x^2]*(-2*I + 7*a*x + (19*I)*a^2*x^2 + 52*a^3*x^3))/(x^3*(I
+ a*x)) - (33*I)*a^3*Log[x] + (33*I)*a^3*Log[1 + Sqrt[1 + a^2*x^2]])/6
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5583, 2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3i\arctan(ax)}}{x^4}dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{(1+iax)^2}{x^4(1-iax)\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{2353}$$

$$\int \left(-\frac{4a^2}{x^2\sqrt{a^2x^2+1}}+\frac{1}{x^4\sqrt{a^2x^2+1}}+\frac{3ia}{x^3\sqrt{a^2x^2+1}}+\frac{4ia^4}{(ax+i)\sqrt{a^2x^2+1}}-\frac{4ia^3}{x\sqrt{a^2x^2+1}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{14a^2\sqrt{a^2x^2+1}}{3x}-\frac{3ia\sqrt{a^2x^2+1}}{2x^2}-\frac{\sqrt{a^2x^2+1}}{3x^3}+\frac{11}{2}ia^3\text{arctanh}\left(\sqrt{a^2x^2+1}\right)+\frac{4a^3\sqrt{a^2x^2+1}}{ax+i}$$

input
```
Int[E^((3*I)*ArcTan[a*x])/x^4,x]
```

output
```
-1/3*Sqrt[1 + a^2*x^2]/x^3 - (((3*I)/2)*a*Sqrt[1 + a^2*x^2])/x^2 + (14*a^2
*Sqrt[1 + a^2*x^2])/(3*x) + (4*a^3*Sqrt[1 + a^2*x^2])/(I + a*x) + ((11*I)/
2)*a^3*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.97

method	result
risch	$\frac{28a^4x^4-9ia^3x^3+26a^2x^2-9iax-2}{6x^3\sqrt{a^2x^2+1}}-\frac{ia^3\left(-11\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)+\frac{8i\sqrt{\left(x+\frac{i}{a}\right)^2a^2-2ia\left(x+\frac{i}{a}\right)}}{a\left(x+\frac{i}{a}\right)}\right)}{2}$
default	$-\frac{1}{3x^3\sqrt{a^2x^2+1}}-\frac{13a^2\left(-\frac{1}{x\sqrt{a^2x^2+1}}-\frac{2a^2x}{\sqrt{a^2x^2+1}}\right)}{3}-ia^3\left(\frac{1}{\sqrt{a^2x^2+1}}-\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)\right)+3ia\left(-\frac{1}{2x^2\sqrt{a^2x^2}}\right.$
meijerg	$-\frac{-8a^4x^4-4a^2x^2+1}{3x^3\sqrt{a^2x^2+1}}+\frac{3ia^3\left(-\frac{\sqrt{\pi}}{2x^2a^2}-\frac{3\left(\frac{5}{3}-2\ln(2)+2\ln(x)+\ln\left(a^2\right)\right)\sqrt{\pi}}{4}+\frac{\sqrt{\pi}\left(20a^2x^2+8\right)}{16a^2x^2}-\frac{\sqrt{\pi}\left(24a^2x^2+8\right)}{16a^2x^2\sqrt{a^2x^2+1}}+\frac{3\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{a^2x^2+}}{2}\right.}{2}\right.}{\sqrt{\pi}}$

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*(28*a^4*x^4-9*I*a^3*x^3+26*a^2*x^2-9*I*a*x-2)/x^3/(a^2*x^2+1)^(1/2)-1/
2*I*a^3*(-11*arctanh(1/(a^2*x^2+1)^(1/2))+8*I/a/(x+I/a)*((x+I/a)^2*a^2-2*I
*a*(x+I/a))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.16

$$\int \frac{e^{3i\arctan(ax)}}{x^4}\,dx = \frac{52\,a^4x^4 + 52i\,a^3x^3 - 33\left(-i\,a^4x^4 + a^3x^3\right)\log\left(-ax + \sqrt{a^2x^2+1} + 1\right) - 33\left(i\,a^4x^4 - a^3x^3\right)\log\left(-ax + \sqrt{}\right.}{6\left(ax^4 + i\,x^3\right)}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
1/6*(52*a^4*x^4 + 52*I*a^3*x^3 - 33*(-I*a^4*x^4 + a^3*x^3)*log(-a*x + sqrt
(a^2*x^2 + 1) + 1) - 33*(I*a^4*x^4 - a^3*x^3)*log(-a*x + sqrt(a^2*x^2 + 1)
 - 1) + (52*a^3*x^3 + 19*I*a^2*x^2 + 7*a*x - 2*I)*sqrt(a^2*x^2 + 1))/(a*x^
4 + I*x^3)
```

Sympy [F]

$$\int \frac{e^{3i\arctan(ax)}}{x^4}\,dx = -i\left(\int \frac{i}{a^2x^6\sqrt{a^2x^2+1} + x^4\sqrt{a^2x^2+1}}\,dx + \int \left(-\frac{3ax}{a^2x^6\sqrt{a^2x^2+1} + x^4\sqrt{a^2x^2+1}}\right)dx + \int \frac{a^3x^3}{a^2x^6\sqrt{a^2x^2+1} + x^4\sqrt{a^2x^2+1}}\,dx + \int \left(-\frac{3ia^2x^2}{a^2x^6\sqrt{a^2x^2+1} + x^4\sqrt{a^2x^2+1}}\right)dx\right)$$

input
```
integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)/x**4,x)
```

output
```
-I*(Integral(I/(a**2*x**6*sqrt(a**2*x**2 + 1) + x**4*sqrt(a**2*x**2 + 1)),
 x) + Integral(-3*a*x/(a**2*x**6*sqrt(a**2*x**2 + 1) + x**4*sqrt(a**2*x**2
 + 1)), x) + Integral(a**3*x**3/(a**2*x**6*sqrt(a**2*x**2 + 1) + x**4*sqrt
(a**2*x**2 + 1)), x) + Integral(-3*I*a**2*x**2/(a**2*x**6*sqrt(a**2*x**2 +
 1) + x**4*sqrt(a**2*x**2 + 1)), x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\int \frac{e^{3i\arctan(ax)}}{x^4}\,dx = \frac{26\,a^4x}{3\,\sqrt{a^2x^2+1}} + \frac{11}{2}i\,a^3\,\mathrm{arsinh}\left(\frac{1}{a|x|}\right) - \frac{11i\,a^3}{2\,\sqrt{a^2x^2+1}} + \frac{13\,a^2}{3\,\sqrt{a^2x^2+1}x} - \frac{3i\,a}{2\,\sqrt{a^2x^2+1}x^2} - \frac{1}{3\,\sqrt{a^2x^2+1}x^3}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
26/3*a^4*x/sqrt(a^2*x^2 + 1) + 11/2*I*a^3*arcsinh(1/(a*abs(x))) - 11/2*I*a
^3/sqrt(a^2*x^2 + 1) + 13/3*a^2/(sqrt(a^2*x^2 + 1)*x) - 3/2*I*a/(sqrt(a^2*
x^2 + 1)*x^2) - 1/3/(sqrt(a^2*x^2 + 1)*x^3)
```

Giac [F]

$$\int \frac{e^{3i\arctan(ax)}}{x^4}\,dx = \int \frac{(i\,ax+1)^3}{(a^2x^2+1)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^4,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 22.60 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.97

$$\int \frac{e^{3i\arctan(ax)}}{x^4}\,dx = \frac{11\,a^3\,\mathrm{atan}\left(\sqrt{a^2\,x^2+1}\,1\mathrm{i}\right)}{2} - \frac{\sqrt{a^2\,x^2+1}}{3\,x^3} - \frac{a\,\sqrt{a^2\,x^2+1}\,3\mathrm{i}}{2\,x^2} + \frac{14\,a^2\,\sqrt{a^2\,x^2+1}}{3\,x} + \frac{4\,a^4\,\sqrt{a^2\,x^2+1}}{\left(x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

input
```
int((a*x*1i + 1)^3/(x^4*(a^2*x^2 + 1)^(3/2)),x)
```

output
```
(11*a^3*atan((a^2*x^2 + 1)^(1/2)*1i))/2 - (a^2*x^2 + 1)^(1/2)/(3*x^3) - (a
*(a^2*x^2 + 1)^(1/2)*3i)/(2*x^2) + (14*a^2*(a^2*x^2 + 1)^(1/2))/(3*x) + (4
*a^4*(a^2*x^2 + 1)^(1/2))/((((a^2)^(1/2)*1i)/a + x*(a^2)^(1/2))*(a^2)^(1/2
))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.79

$$\int \frac{e^{3i\arctan(ax)}}{x^4}\,dx = \frac{52\sqrt{a^2x^2+1}\,a^4x^4 - 33\sqrt{a^2x^2+1}\,a^3i\,x^3 + 26\sqrt{a^2x^2+1}\,a^2x^2 - 9\sqrt{a^2x^2+1}\,aix - 2\sqrt{a^2x^2+1} - 33\log\left(\right.}{}$$

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/x^4,x)
```

output
```
(52*sqrt(a**2*x**2 + 1)*a**4*x**4 - 33*sqrt(a**2*x**2 + 1)*a**3*i*x**3 + 2
6*sqrt(a**2*x**2 + 1)*a**2*x**2 - 9*sqrt(a**2*x**2 + 1)*a*i*x - 2*sqrt(a**
2*x**2 + 1) - 33*log(sqrt(a**2*x**2 + 1) + a*x - 1)*a**5*i*x**5 - 33*log(s
qrt(a**2*x**2 + 1) + a*x - 1)*a**3*i*x**3 + 33*log(sqrt(a**2*x**2 + 1) + a
*x + 1)*a**5*i*x**5 + 33*log(sqrt(a**2*x**2 + 1) + a*x + 1)*a**3*i*x**3 -
52*a**5*x**5 - 52*a**3*x**3)/(6*x**3*(a**2*x**2 + 1))
```

3.42 $\int e^{4i\arctan(ax)}x^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 65

$$\int e^{4i\arctan(ax)}x^3\,dx=\frac{12ix}{a^3}-\frac{4x^2}{a^2}-\frac{4ix^3}{3a}+\frac{x^4}{4}+\frac{4i}{a^4(i+ax)}+\frac{16\log(i+ax)}{a^4}$$

output
```
12*I*x/a^3-4*x^2/a^2-4/3*I*x^3/a+1/4*x^4+4*I/a^4/(I+a*x)+16*ln(I+a*x)/a^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int e^{4i\arctan(ax)}x^3\,dx=\frac{12ix}{a^3}-\frac{4x^2}{a^2}-\frac{4ix^3}{3a}+\frac{x^4}{4}+\frac{4i}{a^4(i+ax)}+\frac{16\log(i+ax)}{a^4}$$

input
```
Integrate[E^((4*I)*ArcTan[a*x])*x^3,x]
```

output
```
((12*I)*x)/a^3 - (4*x^2)/a^2 - (((4*I)/3)*x^3)/a + x^4/4 + (4*I)/(a^4*(I + 
 a*x)) + (16*Log[I + a*x])/a^4
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5585, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{4 i \arctan (a x)} \, dx$$

$$\Big\downarrow \text{ 5585}$$

$$\int \frac{x^3 (1+i a x)^2}{(1-i a x)^2}dx$$

$$\Big\downarrow \text{ 99}$$

$$\int \left(\frac{16}{a^3 (a x+i)}-\frac{4 i}{a^3 (a x+i)^2}+\frac{12 i}{a^3}-\frac{8 x}{a^2}-\frac{4 i x^2}{a}+x^3\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4 i}{a^4 (a x+i)}+\frac{16 \log (a x+i)}{a^4}+\frac{12 i x}{a^3}-\frac{4 x^2}{a^2}-\frac{4 i x^3}{3 a}+\frac{x^4}{4}$$

input
```
Int[E^((4*I)*ArcTan[a*x])*x^3,x]
```

output
```
((12*I)*x)/a^3 - (4*x^2)/a^2 - (((4*I)/3)*x^3)/a + x^4/4 + (4*I)/(a^4*(I +
 a*x)) + (16*Log[I + a*x])/a^4
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

method	result
default	$-\frac{-\frac{1}{4}a^3x^4+\frac{4}{3}ia^2x^3+4a\,x^2-12ix}{a^3}-\frac{4\left(-\frac{i}{a(ax+i)}-\frac{4\ln(ax+i)}{a}\right)}{a^3}$
risch	$\frac{x^4}{4}-\frac{4ix^3}{3a}-\frac{4x^2}{a^2}+\frac{12ix}{a^3}+\frac{4i}{a^4(ax+i)}+\frac{8\ln(a^2x^2+1)}{a^4}-\frac{16i\arctan(ax)}{a^4}$
parallelrisch	$-\frac{-3x^6a^6+16ix^5a^5+45a^4x^4-128ia^3x^3-192a^2\ln(ax+i)x^2+96a^2x^2-192iax-192\ln(ax+i)}{12a^4(a^2x^2+1)}$
meijerg	$\frac{-\frac{a^2x^2}{a^2x^2+1}+\ln(a^2x^2+1)}{2a^4}+\frac{2i\left(\frac{x(a^2)^{\frac{5}{2}}(10a^2x^2+15)}{5a^4(a^2x^2+1)}-\frac{3(a^2)^{\frac{5}{2}}\arctan(ax)}{a^5}\right)}{a^3\sqrt{a^2}}-\frac{3\left(\frac{x^2a^2(3a^2x^2+6)}{3a^2x^2+3}-2\ln(a^2x^2+1)\right)}{a^4}-\frac{2i\left(\cdots\right.}{\cdots}$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^2*x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/a^3*(-1/4*a^3*x^4+4/3*I*a^2*x^3+4*a*x^2-12*I*x)-4/a^3*(-I/a/(I+a*x)-4/a
*ln(I+a*x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\int e^{4i\arctan(ax)}x^3\,dx = \frac{3\,a^5x^5-13i\,a^4x^4-32\,a^3x^3+96i\,a^2x^2-144\,ax+192\,(ax+i)\log\left(\frac{ax+i}{a}\right)+48i}{12\,(a^5x+i\,a^4)}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^3,x, algorithm="fricas")
```

output
```
1/12*(3*a^5*x^5 - 13*I*a^4*x^4 - 32*a^3*x^3 + 96*I*a^2*x^2 - 144*a*x + 192
*(a*x + I)*log((a*x + I)/a) + 48*I)/(a^5*x + I*a^4)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int e^{4i\arctan(ax)}x^3\,dx = \frac{x^4}{4} + \frac{4i}{a^5x+ia^4} - \frac{4ix^3}{3a} - \frac{4x^2}{a^2} + \frac{12ix}{a^3} + \frac{16\log{(ax+i)}}{a^4}$$

input
```
integrate((1+I*a*x)**4/(a**2*x**2+1)**2*x**3,x)
```

output
```
x**4/4 + 4*I/(a**5*x + I*a**4) - 4*I*x**3/(3*a) - 4*x**2/a**2 + 12*I*x/a**
3 + 16*log(a*x + I)/a**4
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18

$$\int e^{4i\arctan(ax)}x^3\,dx = -\frac{4\,(-i\,ax-1)}{a^6x^2+a^4} + \frac{3\,a^3x^4 - 16i\,a^2x^3 - 48\,ax^2 + 144i\,x}{12\,a^3} - \frac{16i\,\arctan{(ax)}}{a^4} + \frac{8\,\log{(a^2x^2+1)}}{a^4}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^3,x, algorithm="maxima")
```

output
```
-4*(-I*a*x - 1)/(a^6*x^2 + a^4) + 1/12*(3*a^3*x^4 - 16*I*a^2*x^3 - 48*a*x^
2 + 144*I*x)/a^3 - 16*I*arctan(a*x)/a^4 + 8*log(a^2*x^2 + 1)/a^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int e^{4i\arctan(ax)}x^3\,dx$$
$$= \frac{16\,\log(ax+i)}{a^4} + \frac{4i}{(ax+i)a^4} + \frac{3\,a^8x^4 - 16i\,a^7x^3 - 48\,a^6x^2 + 144i\,a^5x}{12\,a^8}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^3,x, algorithm="giac")
```

output
```
16*log(a*x + I)/a^4 + 4*I/((a*x + I)*a^4) + 1/12*(3*a^8*x^4 - 16*I*a^7*x^3
 - 48*a^6*x^2 + 144*I*a^5*x)/a^8
```

Mupad [B] (verification not implemented)

Time = 22.70 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int e^{4i\arctan(ax)}x^3\,dx = \frac{16\,\ln\left(x+\frac{1\mathrm{i}}{a}\right)}{a^4} + \frac{x^4}{4} - \frac{4\,x^2}{a^2} + \frac{4\mathrm{i}}{a^5\,\left(x+\frac{1\mathrm{i}}{a}\right)} + \frac{x\,12\mathrm{i}}{a^3} - \frac{x^3\,4\mathrm{i}}{3\,a}$$

input
```
int((x^3*(a*x*1i + 1)^4)/(a^2*x^2 + 1)^2,x)
```

output
```
4i/(a^5*(x + 1i/a)) + (16*log(x + 1i/a))/a^4 + (x*12i)/a^3 + x^4/4 - (x^3*
4i)/(3*a) - (4*x^2)/a^2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.75

$$\int e^{4i\arctan(ax)}x^3\,dx$$
$$= \frac{-192atan(ax)\,a^2i\,x^2 - 192atan(ax)\,i + 96\log(a^2x^2+1)\,a^2x^2 + 96\log(a^2x^2+1) + 3a^6x^6 - 16a^5i\,x^5 - 4}{12a^4\,(a^2x^2+1)}$$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^2*x^3,x)
```

output
```
( - 192*atan(a*x)*a**2*i*x**2 - 192*atan(a*x)*i + 96*log(a**2*x**2 + 1)*a*
*2*x**2 + 96*log(a**2*x**2 + 1) + 3*a**6*x**6 - 16*a**5*i*x**5 - 45*a**4*x
**4 + 128*a**3*i*x**3 - 96*a**2*x**2 + 192*a*i*x)/(12*a**4*(a**2*x**2 + 1)
)
```

3.43 $\int e^{4i\arctan(ax)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 53

$$\int e^{4i\arctan(ax)}x^2\,dx=-\frac{8x}{a^2}-\frac{2ix^2}{a}+\frac{x^3}{3}-\frac{4}{a^3(i+ax)}+\frac{12i\log(i+ax)}{a^3}$$

output
```
-8*x/a^2-2*I*x^2/a+1/3*x^3-4/a^3/(I+a*x)+12*I*ln(I+a*x)/a^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int e^{4i\arctan(ax)}x^2\,dx=-\frac{8x}{a^2}-\frac{2ix^2}{a}+\frac{x^3}{3}-\frac{4}{a^3(i+ax)}+\frac{12i\log(i+ax)}{a^3}$$

input
```
Integrate[E^((4*I)*ArcTan[a*x])*x^2,x]
```

output
```
(-8*x)/a^2 - ((2*I)*x^2)/a + x^3/3 - 4/(a^3*(I + a*x)) + ((12*I)*Log[I + a
*x])/a^3
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5585, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{4i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{ 5585}$$

$$\int \frac{x^2(1+iax)^2}{(1-iax)^2}dx$$

$$\Big\downarrow \text{ 99}$$

$$\int \left(\frac{12i}{a^2(ax+i)}+\frac{4}{a^2(ax+i)^2}-\frac{8}{a^2}-\frac{4ix}{a}+x^2\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{4}{a^3(ax+i)}+\frac{12i\log(ax+i)}{a^3}-\frac{8x}{a^2}-\frac{2ix^2}{a}+\frac{x^3}{3}$$

input
```
Int[E^((4*I)*ArcTan[a*x])*x^2,x]
```

output
```
(-8*x)/a^2 - ((2*I)*x^2)/a + x^3/3 - 4/(a^3*(I + a*x)) + ((12*I)*Log[I + a
*x])/a^3
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

method	result
default	$-\frac{8x-\frac{1}{3}a^2x^3+2ia\,x^2}{a^2}+\frac{-\frac{4}{a(ax+i)}+\frac{12i\ln(ax+i)}{a}}{a^2}$
risch	$-\frac{8x}{a^2}+\frac{x^3}{3}-\frac{2ix^2}{a}-\frac{4}{a^3(ax+i)}+\frac{6i\ln(a^2x^2+1)}{a^3}+\frac{12\arctan(ax)}{a^3}$
parallelrisch	$\frac{a^5x^5-6ia^4x^4+36i\ln(ax+i)x^2a^2-23a^3x^3-18ia^2x^2+36i\ln(ax+i)-36ax}{3a^3(a^2x^2+1)}$
meijerg	$\frac{-\frac{x(a^2)^{\frac{3}{2}}}{a^2(a^2x^2+1)}+\frac{(a^2)^{\frac{3}{2}}\arctan(ax)}{a^3}}{2a^2\sqrt{a^2}}+\frac{2i\left(-\frac{a^2x^2}{a^2x^2+1}+\ln(a^2x^2+1)\right)}{a^3}-\frac{3\left(\frac{x(a^2)^{\frac{5}{2}}(10a^2x^2+15)}{5a^4(a^2x^2+1)}-\frac{3(a^2)^{\frac{5}{2}}\arctan(ax)}{a^5}\right)}{a^2\sqrt{a^2}}-\frac{2i}{}$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^2*x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*(8*x-1/3*a^2*x^3+2*I*a*x^2)+4/a^2*(-1/a/(I+a*x)+3*I*ln(I+a*x)/a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

$$\int e^{4i\arctan(ax)}x^2\,dx=\frac{a^4x^4-5i\,a^3x^3-18\,a^2x^2-24i\,ax-36\,(-i\,ax+1)\log\left(\frac{ax+i}{a}\right)-12}{3\,(a^4x+i\,a^3)}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^2,x, algorithm="fricas")
```

output
```
1/3*(a^4*x^4 - 5*I*a^3*x^3 - 18*a^2*x^2 - 24*I*a*x - 36*(-I*a*x + 1)*log((
a*x + I)/a) - 12)/(a^4*x + I*a^3)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int e^{4i\arctan(ax)}x^2\,dx = \frac{x^3}{3} - \frac{4}{a^4x + ia^3} - \frac{2ix^2}{a} - \frac{8x}{a^2} + \frac{12i\log{(ax + i)}}{a^3}$$

input
```
integrate((1+I*a*x)**4/(a**2*x**2+1)**2*x**2,x)
```

output
```
x**3/3 - 4/(a**4*x + I*a**3) - 2*I*x**2/a - 8*x/a**2 + 12*I*log(a*x + I)/a
**3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.26

$$\int e^{4i\arctan(ax)}x^2\,dx = -\frac{4\,(ax - i)}{a^5x^2 + a^3} + \frac{a^2x^3 - 6i\,ax^2 - 24\,x}{3\,a^2} + \frac{12\,\arctan{(ax)}}{a^3} + \frac{6i\,\log{(a^2x^2 + 1)}}{a^3}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^2,x, algorithm="maxima")
```

output
```
-4*(a*x - I)/(a^5*x^2 + a^3) + 1/3*(a^2*x^3 - 6*I*a*x^2 - 24*x)/a^2 + 12*a
rctan(a*x)/a^3 + 6*I*log(a^2*x^2 + 1)/a^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int e^{4i\arctan(ax)}x^2\,dx = \frac{12i\,\log{(ax + i)}}{a^3} - \frac{4}{(ax + i)a^3} + \frac{a^6x^3 - 6i\,a^5x^2 - 24\,a^4x}{3\,a^6}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^2,x, algorithm="giac")
```

output
```
12*I*log(a*x + I)/a^3 - 4/((a*x + I)*a^3) + 1/3*(a^6*x^3 - 6*I*a^5*x^2 - 2
4*a^4*x)/a^6
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int e^{4i\arctan(ax)}x^2\,dx = \frac{x^3}{3} + \frac{\ln\left(x+\frac{1\mathrm{i}}{a}\right)\,12\mathrm{i}}{a^3} - \frac{8\,x}{a^2} - \frac{4}{a^4\,\left(x+\frac{1\mathrm{i}}{a}\right)} - \frac{x^2\,2\mathrm{i}}{a}$$

input
```
int((x^2*(a*x*1i + 1)^4)/(a^2*x^2 + 1)^2,x)
```

output
```
(log(x + 1i/a)*12i)/a^3 - 4/(a^4*(x + 1i/a)) - (8*x)/a^2 + x^3/3 - (x^2*2i
)/a
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.96

$$\int e^{4i\arctan(ax)}x^2\,dx$$
$$= \frac{36atan(ax)\,a^2x^2 + 36atan(ax) + 18\log(a^2x^2+1)\,a^2i\,x^2 + 18\log(a^2x^2+1)\,i + a^5x^5 - 6a^4i\,x^4 - 23a^3x^3}{3a^3\,(a^2x^2+1)}$$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^2*x^2,x)
```

output
```
(36*atan(a*x)*a**2*x**2 + 36*atan(a*x) + 18*log(a**2*x**2 + 1)*a**2*i*x**2
 + 18*log(a**2*x**2 + 1)*i + a**5*x**5 - 6*a**4*i*x**4 - 23*a**3*x**3 - 18
*a**2*i*x**2 - 36*a*x)/(3*a**3*(a**2*x**2 + 1))
```

3.44 $\int e^{4 i \arctan (a x)} x \, dx$

Optimal result

Integrand size = 12, antiderivative size = 45

$$\int e^{4 i \arctan (a x)} x \, dx = -\frac{4 i x}{a}+\frac{x^2}{2}-\frac{4 i}{a^2 (i+a x)}-\frac{8 \log (i+a x)}{a^2}$$

output
```
-4*I*x/a+1/2*x^2-4*I/a^2/(I+a*x)-8*ln(I+a*x)/a^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int e^{4 i \arctan (a x)} x \, dx = -\frac{4 i x}{a}+\frac{x^2}{2}-\frac{4 i}{a^2 (i+a x)}-\frac{8 \log (i+a x)}{a^2}$$

input
```
Integrate[E^((4*I)*ArcTan[a*x])*x,x]
```

output
```
((-4*I)*x)/a + x^2/2 - (4*I)/(a^2*(I + a*x)) - (8*Log[I + a*x])/a^2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5585, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{4i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{ 5585}$$

$$\int \frac{x(1+iax)^2}{(1-iax)^2}dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(-\frac{8}{a(ax+i)}+\frac{4i}{a(ax+i)^2}-\frac{4i}{a}+x\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{4i}{a^2(ax+i)}-\frac{8\log(ax+i)}{a^2}-\frac{4ix}{a}+\frac{x^2}{2}$$

input
```
Int[E^((4*I)*ArcTan[a*x])*x,x]
```

output
```
((-4*I)*x)/a + x^2/2 - (4*I)/(a^2*(I + a*x)) - (8*Log[I + a*x])/a^2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

method	result
default	$-\frac{-\frac{1}{2}a\,x^2+4ix}{a}+\frac{-\frac{4i}{a(ax+i)}-\frac{8\ln(ax+i)}{a}}{a}$
risch	$\frac{x^2}{2}-\frac{4ix}{a}-\frac{4i}{a^2(ax+i)}-\frac{4\ln(a^2x^2+1)}{a^2}+\frac{8i\arctan(ax)}{a^2}$
parallelrisch	$-\frac{-a^4x^4+8ia^3x^3+16a^2\ln(ax+i)x^2-9a^2x^2+16iax+16\ln(ax+i)}{2a^2(a^2x^2+1)}$
meijerg	$\frac{x^2}{2a^2x^2+2}+\frac{2i\left(-\frac{x\left(a^2\right)^{\frac{3}{2}}}{a^2\left(a^2x^2+1\right)}+\frac{\left(a^2\right)^{\frac{3}{2}}\arctan(ax)}{a^3}\right)}{a\sqrt{a^2}}-\frac{3\left(-\frac{a^2x^2}{a^2x^2+1}+\ln(a^2x^2+1)\right)}{a^2}-\frac{2i\left(\frac{x\left(a^2\right)^{\frac{5}{2}}\left(10a^2x^2+15\right)}{5a^4\left(a^2x^2+1\right)}-\frac{3\left(a^2\right)^{\frac{5}{2}}}{}\right.}{a\sqrt{a^2}}$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^2*x,x,method=_RETURNVERBOSE)
```

output
```
-1/a*(-1/2*a*x^2+4*I*x)+4/a*(-I/a/(I+a*x)-2/a*ln(I+a*x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int e^{4i\arctan(ax)}x\,dx=\frac{a^3x^3-7i\,a^2x^2+8\,ax-16\,(ax+i)\log\left(\frac{ax+i}{a}\right)-8i}{2\,(a^3x+i\,a^2)}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x,x, algorithm="fricas")
```

output
```
1/2*(a^3*x^3 - 7*I*a^2*x^2 + 8*a*x - 16*(a*x + I)*log((a*x + I)/a) - 8*I)/
(a^3*x + I*a^2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int e^{4i\arctan(ax)}x\,dx = \frac{x^2}{2} - \frac{4i}{a^3x + ia^2} - \frac{4ix}{a} - \frac{8\log(ax+i)}{a^2}$$

input
```
integrate((1+I*a*x)**4/(a**2*x**2+1)**2*x,x)
```

output
```
x**2/2 - 4*I/(a**3*x + I*a**2) - 4*I*x/a - 8*log(a*x + I)/a**2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int e^{4i\arctan(ax)}x\,dx = -\frac{4(i\,ax+1)}{a^4x^2+a^2} + \frac{ax^2 - 8i\,x}{2\,a} + \frac{8i\,\arctan(ax)}{a^2} - \frac{4\,\log(a^2x^2+1)}{a^2}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x,x, algorithm="maxima")
```

output
```
-4*(I*a*x + 1)/(a^4*x^2 + a^2) + 1/2*(a*x^2 - 8*I*x)/a + 8*I*arctan(a*x)/a
^2 - 4*log(a^2*x^2 + 1)/a^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int e^{4i\arctan(ax)}x\,dx = -\frac{8\,\log(ax+i)}{a^2} + \frac{a^4x^2 - 8i\,a^3x}{2\,a^4} - \frac{4i}{(ax+i)a^2}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x,x, algorithm="giac")
```

output
```
-8*log(a*x + I)/a^2 + 1/2*(a^4*x^2 - 8*I*a^3*x)/a^4 - 4*I/((a*x + I)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int e^{4i\arctan(ax)}x\,dx = \frac{x^2}{2} - \frac{8\,\ln\left(x + \frac{1\mathrm{i}}{a}\right)}{a^2} - \frac{4\mathrm{i}}{a^3\,\left(x + \frac{1\mathrm{i}}{a}\right)} - \frac{x\,4\mathrm{i}}{a}$$

input
```
int((x*(a*x*1i + 1)^4)/(a^2*x^2 + 1)^2,x)
```

output
```
x^2/2 - (8*log(x + 1i/a))/a^2 - (x*4i)/a - 4i/(a^3*(x + 1i/a))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.13

$$\int e^{4i\arctan(ax)}x\,dx$$
$$= \frac{16atan(ax)\,a^2i\,x^2 + 16atan(ax)\,i - 8\log(a^2x^2+1)\,a^2x^2 - 8\log(a^2x^2+1) + a^4x^4 - 8a^3i\,x^3 + 9a^2x^2 - 1}{2a^2\,(a^2x^2+1)}$$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^2*x,x)
```

output
```
(16*atan(a*x)*a**2*i*x**2 + 16*atan(a*x)*i - 8*log(a**2*x**2 + 1)*a**2*x**
2 - 8*log(a**2*x**2 + 1) + a**4*x**4 - 8*a**3*i*x**3 + 9*a**2*x**2 - 16*a*
i*x)/(2*a**2*(a**2*x**2 + 1))
```

3.45 $\int e^{4i\arctan(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 31

$$\int e^{4i\arctan(ax)}\,dx = x + \frac{4}{a(i+ax)} - \frac{4i\log(i+ax)}{a}$$

output
```
x+4/a/(I+a*x)-4*I*ln(I+a*x)/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int e^{4i\arctan(ax)}\,dx = x + \frac{4}{a(i+ax)} - \frac{4\arctan(ax)}{a} - \frac{2i\log(1+a^2x^2)}{a}$$

input
```
Integrate[E^((4*I)*ArcTan[a*x]),x]
```

output
```
x + 4/(a*(I + a*x)) - (4*ArcTan[a*x])/a - ((2*I)*Log[1 + a^2*x^2])/a
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5584, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{4i\arctan(ax)}dx$$

$$\Big\downarrow \text{5584}$$

$$\int \frac{(1+iax)^2}{(1-iax)^2}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{4i}{ax+i}-\frac{4}{(ax+i)^2}+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{a(ax+i)}-\frac{4i\log(ax+i)}{a}+x$$

input
```
Int[E^((4*I)*ArcTan[a*x]),x]
```

output
```
x + 4/(a*(I + a*x)) - ((4*I)*Log[I + a*x])/a
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5584
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] && !IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

method	result
default	$x-4a\left(-\frac{1}{a^{2}(ax+i)}+\frac{i\ln(ax+i)}{a^{2}}\right)$
risch	$x+\frac{4}{a(ax+i)}-\frac{2i\ln(a^{2}x^{2}+1)}{a}-\frac{4\arctan(ax)}{a}$
parallelrisch	$-\frac{4i\ln(ax+i)x^{2}a^{2}-a^{3}x^{3}-4ia^{2}x^{2}+4i\ln(ax+i)-5ax}{(a^{2}x^{2}+1)a}$
meijerg	$\frac{\frac{2x\sqrt{a^{2}}}{2a^{2}x^{2}+2}+\frac{\sqrt{a^{2}}\arctan(ax)}{a}}{2\sqrt{a^{2}}}+\frac{2ia\,x^{2}}{a^{2}x^{2}+1}-\frac{3\left(-\frac{x\left(a^{2}\right)^{\frac{3}{2}}}{a^{2}\left(a^{2}x^{2}+1\right)}+\frac{\left(a^{2}\right)^{\frac{3}{2}}\arctan(ax)}{a^{3}}\right)}{\sqrt{a^{2}}}-\frac{2i\left(-\frac{a^{2}x^{2}}{a^{2}x^{2}+1}+\ln\left(a^{2}x^{2}+1\right)\right)}{a}+$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
x-4*a*(-1/a^2/(I+a*x)+I/a^2*ln(I+a*x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.39

$$\int e^{4i\arctan(ax)}\,dx=\frac{a^{2}x^{2}+i\,ax-4\,(i\,ax-1)\log\left(\frac{ax+i}{a}\right)+4}{a^{2}x+i\,a}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2,x, algorithm="fricas")
```

output
```
(a^2*x^2 + I*a*x - 4*(I*a*x - 1)*log((a*x + I)/a) + 4)/(a^2*x + I*a)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int e^{4i\arctan(ax)}\,dx = x + \frac{4}{a^2x + ia} - \frac{4i\log(ax + i)}{a}$$

input
```
integrate((1+I*a*x)**4/(a**2*x**2+1)**2,x)
```

output
```
x + 4/(a**2*x + I*a) - 4*I*log(a*x + I)/a
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int e^{4i\arctan(ax)}\,dx = x + \frac{4(ax - i)}{a^3x^2 + a} - \frac{4\arctan(ax)}{a} - \frac{2i\log(a^2x^2 + 1)}{a}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2,x, algorithm="maxima")
```

output
```
x + 4*(a*x - I)/(a^3*x^2 + a) - 4*arctan(a*x)/a - 2*I*log(a^2*x^2 + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int e^{4i\arctan(ax)}\,dx = x - \frac{4i\log(ax + i)}{a} + \frac{4}{(ax + i)a}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2,x, algorithm="giac")
```

output
```
x - 4*I*log(a*x + I)/a + 4/((a*x + I)*a)
```

Mupad [B] (verification not implemented)

Time = 22.72 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int e^{4i\arctan(ax)}\,dx = x + \frac{4}{a^2\,\left(x+\frac{1\mathrm{i}}{a}\right)} - \frac{\ln\left(x+\frac{1\mathrm{i}}{a}\right)\,4\mathrm{i}}{a}$$

input
```
int((a*x*1i + 1)^4/(a^2*x^2 + 1)^2,x)
```

output
```
x + 4/(a^2*(x + 1i/a)) - (log(x + 1i/a)*4i)/a
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.77

$$\int e^{4i\arctan(ax)}\,dx = \frac{-4atan(ax)\,a^2x^2 - 4atan(ax) - 2\log(a^2x^2+1)\,a^2i\,x^2 - 2\log(a^2x^2+1)\,i + a^3x^3 + 4a^2i\,x^2 + 5ax}{a\,(a^2x^2+1)}$$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^2,x)
```

output
```
( - 4*atan(a*x)*a**2*x**2 - 4*atan(a*x) - 2*log(a**2*x**2 + 1)*a**2*i*x**2
 - 2*log(a**2*x**2 + 1)*i + a**3*x**3 + 4*a**2*i*x**2 + 5*a*x)/(a*(a**2*x*
*2 + 1))
```

3.46 $\int \frac{e^{4i\arctan(ax)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 16

$$\int \frac{e^{4i\arctan(ax)}}{x}\,dx = \frac{4i}{i+ax} + \log(x)$$

output
```
4*I/(I+a*x)+ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{e^{4i\arctan(ax)}}{x}\,dx = \frac{4i}{i+ax} + \log(x)$$

input
```
Integrate[E^((4*I)*ArcTan[a*x])/x,x]
```

output
```
(4*I)/(I + a*x) + Log[x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5585, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4 i \arctan (a x)}}{x} \, dx$$
$$\Big\downarrow \text{ 5585}$$
$$\int \frac{(1+i a x)^2}{x (1-i a x)^2}dx$$
$$\Big\downarrow \text{ 99}$$
$$\int \left(\frac{1}{x}-\frac{4 i a}{(a x+i)^2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\log (x)+\frac{4 i}{a x+i}$$

input
```
Int[E^((4*I)*ArcTan[a*x])/x,x]
```

output
```
(4*I)/(I + a*x) + Log[x]
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
default	$\frac{4i}{ax+i}+\ln\left(x\right)$
risch	$\frac{4i}{ax+i}+\ln\left(-x\right)$
norman	$\frac{-4a^{2}x^{2}+4iax}{a^{2}x^{2}+1}+\ln\left(x\right)$
parallelrisch	$\frac{a^{2}\ln(x)x^{2}-4a^{2}x^{2}+4iax+\ln(x)}{a^{2}x^{2}+1}$
meijerg	$\frac{1}{2}+\ln\left(x\right)+\frac{\ln(a^{2})}{2}-\frac{a^{2}x^{2}}{2a^{2}x^{2}+2}+\frac{2ia\left(\frac{2x\sqrt{a^{2}}}{2a^{2}x^{2}+2}+\frac{\sqrt{a^{2}}\,\arctan(ax)}{a}\right)}{\sqrt{a^{2}}}-\frac{7a^{2}x^{2}}{2(a^{2}x^{2}+1)}-\frac{2ia\left(-\frac{x(a^{2})^{\frac{3}{2}}}{a^{2}(a^{2}x^{2}+1)}+\frac{(a^{2})^{\frac{3}{2}}}{}\right.}{\sqrt{a^{2}}}$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^2/x,x,method=_RETURNVERBOSE)
```

output
```
4*I/(I+a*x)+ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{e^{4i\arctan(ax)}}{x}\,dx=\frac{(ax+i)\log\left(x\right)+4i}{ax+i}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x,x, algorithm="fricas")
```

output
```
((a*x + I)*log(x) + 4*I)/(a*x + I)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{e^{4i\arctan(ax)}}{x}\,dx = \log(x) + \frac{4i}{ax+i}$$

input
```
integrate((1+I*a*x)**4/(a**2*x**2+1)**2/x,x)
```

output
```
log(x) + 4*I/(a*x + I)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{e^{4i\arctan(ax)}}{x}\,dx = -\frac{4(-iax-1)}{a^2x^2+1} + \log(x)$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x,x, algorithm="maxima")
```

output
```
-4*(-I*a*x - 1)/(a^2*x^2 + 1) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{e^{4i\arctan(ax)}}{x}\,dx = \frac{4i}{ax+i} + \log(|x|)$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x,x, algorithm="giac")
```

output
```
4*I/(a*x + I) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 22.78 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{e^{4i\arctan(ax)}}{x}\,dx = \ln(x) + \frac{4\mathrm{i}}{a\,x + 1\mathrm{i}}$$

input
```
int((a*x*1i + 1)^4/(x*(a^2*x^2 + 1)^2),x)
```

output
```
log(x) + 4i/(a*x + 1i)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.31

$$\int \frac{e^{4i\arctan(ax)}}{x}\,dx = \frac{\log(x)\,a^2x^2 + \log(x) - 4a^2x^2 + 4aix}{a^2x^2 + 1}$$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^2/x,x)
```

output
```
(log(x)*a**2*x**2 + log(x) - 4*a**2*x**2 + 4*a*i*x)/(a**2*x**2 + 1)
```

3.47 $\int \frac{e^{4i\arctan(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 38

$$\int \frac{e^{4i\arctan(ax)}}{x^2}\,dx = -\frac{1}{x}-\frac{4a}{i+ax}+4ia\log(x)-4ia\log(i+ax)$$

output
```
-1/x-4*a/(I+a*x)+4*I*a*ln(x)-4*I*a*ln(I+a*x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{e^{4i\arctan(ax)}}{x^2}\,dx = -\frac{1}{x}-\frac{4a}{i+ax}+4ia\log(x)-4ia\log(i+ax)$$

input
```
Integrate[E^((4*I)*ArcTan[a*x])/x^2,x]
```

output
```
-x^(-1) - (4*a)/(I + a*x) + (4*I)*a*Log[x] - (4*I)*a*Log[I + a*x]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5585, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4i\arctan(ax)}}{x^2}dx$$
$$\Big\downarrow \text{ 5585}$$
$$\int \frac{(1+iax)^2}{x^2(1-iax)^2}dx$$
$$\Big\downarrow \text{ 99}$$
$$\int \left(-\frac{4ia^2}{ax+i}+\frac{4a^2}{(ax+i)^2}+\frac{4ia}{x}+\frac{1}{x^2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{4a}{ax+i}+4ia\log(x)-4ia\log(ax+i)-\frac{1}{x}$$

input
```
Int[E^((4*I)*ArcTan[a*x])/x^2,x]
```

output
```
-x^(-1) - (4*a)/(I + a*x) + (4*I)*a*Log[x] - (4*I)*a*Log[I + a*x]
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

method	result
default	$-4a^2\left(\frac{1}{a(ax+i)}+\frac{i\ln(ax+i)}{a}\right)-\frac{1}{x}+4ia\ln(x)$
risch	$\frac{-5ax-i}{(ax+i)x}-4a\arctan(ax)-2ia\ln(a^2x^2+1)+4ia\ln(x)$
parallelrisch	$\frac{4ia^3\ln(x)x^3-4ia^3\ln(ax+i)x^3-4ia^3x^3-1+4ia\ln(x)x-4ia\ln(ax+i)x-5a^2x^2}{(a^2x^2+1)x}$
meijerg	$\frac{a^2\left(-\frac{2(3a^2x^2+2)}{x\sqrt{a^2}(2a^2x^2+2)}-\frac{3a\arctan(ax)}{\sqrt{a^2}}\right)}{2\sqrt{a^2}}+2ia\left(1+2\ln(x)+\ln(a^2)-\frac{2a^2x^2}{2a^2x^2+2}-\ln(a^2x^2+1)\right)-\frac{3a^2}{}$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-4*a^2*(1/a/(I+a*x)+I*ln(I+a*x)/a)-1/x+4*I*a*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.58

$$\int\frac{e^{4i\arctan(ax)}}{x^2}\,dx=-\frac{5\,ax+4\,(-i\,a^2x^2+ax)\log(x)+4\,(i\,a^2x^2-ax)\log\left(\frac{ax+i}{a}\right)+i}{ax^2+i\,x}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^2,x, algorithm="fricas")
```

output
```
-(5*a*x + 4*(-I*a^2*x^2 + a*x)*log(x) + 4*(I*a^2*x^2 - a*x)*log((a*x + I)/
a) + I)/(a*x^2 + I*x)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{e^{4i\arctan(ax)}}{x^2}\,dx = 4a\left(i\log\left(8a^2x\right) - i\log\left(8a^2x + 8ia\right)\right) + \frac{-5ax - i}{ax^2 + ix}$$

input
```
integrate((1+I*a*x)**4/(a**2*x**2+1)**2/x**2,x)
```

output
```
4*a*(I*log(8*a**2*x) - I*log(8*a**2*x + 8*I*a)) + (-5*a*x - I)/(a*x**2 + I
*x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.39

$$\int \frac{e^{4i\arctan(ax)}}{x^2}\,dx = -4\,a\arctan\left(ax\right) - 2i\,a\log\left(a^2x^2 + 1\right) + 4i\,a\log\left(x\right) - \frac{5\,a^2x^2 - 4i\,ax + 1}{a^2x^3 + x}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^2,x, algorithm="maxima")
```

output
```
-4*a*arctan(a*x) - 2*I*a*log(a^2*x^2 + 1) + 4*I*a*log(x) - (5*a^2*x^2 - 4*
I*a*x + 1)/(a^2*x^3 + x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{e^{4i\arctan(ax)}}{x^2}\,dx = -4i\,a\log\left(ax + i\right) + 4i\,a\log\left(|x|\right) - \frac{5\,ax + i}{ax^2 + i\,x}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^2,x, algorithm="giac")
```

output
```
-4*I*a*log(a*x + I) + 4*I*a*log(abs(x)) - (5*a*x + I)/(a*x^2 + I*x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{e^{4i\arctan(ax)}}{x^2}\,dx = -8\,a\,\mathrm{atan}(2\,a\,x+1\mathrm{i}) - \frac{5\,x+\frac{1\mathrm{i}}{a}}{x^2+\frac{x\,1\mathrm{i}}{a}}$$

input
```
int((a*x*1i + 1)^4/(x^2*(a^2*x^2 + 1)^2),x)
```

output
```
- 8*a*atan(2*a*x + 1i) - (5*x + 1i/a)/((x*1i)/a + x^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.79

$$\int \frac{e^{4i\arctan(ax)}}{x^2}\,dx = \frac{-4atan(ax)\,a^3x^3 - 4atan(ax)\,ax - 2\log(a^2x^2+1)\,a^3i\,x^3 - 2\log(a^2x^2+1)\,aix + 4\log(x)\,a^3i\,x^3 + 4\log}{x\,(a^2x^2+1)}$$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^2/x^2,x)
```

output
```
( - 4*atan(a*x)*a**3*x**3 - 4*atan(a*x)*a*x - 2*log(a**2*x**2 + 1)*a**3*i*
x**3 - 2*log(a**2*x**2 + 1)*a*i*x + 4*log(x)*a**3*i*x**3 + 4*log(x)*a*i*x
- 4*a**3*i*x**3 - 5*a**2*x**2 - 1)/(x*(a**2*x**2 + 1))
```

3.48 $\int \frac{e^{4i\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 52

$$\int \frac{e^{4i\arctan(ax)}}{x^3}\,dx=-\frac{1}{2x^2}-\frac{4ia}{x}-\frac{4ia^2}{i+ax}-8a^2\log(x)+8a^2\log(i+ax)$$

output
```
-1/2/x^2-4*I*a/x-4*I*a^2/(I+a*x)-8*a^2*ln(x)+8*a^2*ln(I+a*x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{e^{4i\arctan(ax)}}{x^3}\,dx=-\frac{1}{2x^2}-\frac{4ia}{x}-\frac{4ia^2}{i+ax}-8a^2\log(x)+8a^2\log(i+ax)$$

input
```
Integrate[E^((4*I)*ArcTan[a*x])/x^3,x]
```

output
```
-1/2*1/x^2 - ((4*I)*a)/x - ((4*I)*a^2)/(I + a*x) - 8*a^2*Log[x] + 8*a^2*Lo
g[I + a*x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5585, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4i\arctan(ax)}}{x^3}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{(1+iax)^2}{x^3(1-iax)^2}dx$$

$$\Big\downarrow \text{99}$$

$$\int \left(\frac{8a^3}{ax+i}+\frac{4ia^3}{(ax+i)^2}-\frac{8a^2}{x}+\frac{4ia}{x^2}+\frac{1}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4ia^2}{ax+i}-8a^2\log(x)+8a^2\log(ax+i)-\frac{4ia}{x}-\frac{1}{2x^2}$$

input
```
Int[E^((4*I)*ArcTan[a*x])/x^3,x]
```

output
```
-1/2*1/x^2 - ((4*I)*a)/x - ((4*I)*a^2)/(I + a*x) - 8*a^2*Log[x] + 8*a^2*Lo
g[I + a*x]
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

method	result
default	$-4a^3\left(\frac{i}{a(ax+i)}-\frac{2\ln(ax+i)}{a}\right)-\frac{1}{2x^2}-\frac{4ia}{x}-8a^2\ln(x)$
risch	$\frac{-8ia^2x^2+\frac{7}{2}ax-\frac{1}{2}i}{(ax+i)x^2}-8a^2\ln(x)-8ia^2\arctan(ax)+4a^2\ln(a^2x^2+1)$
parallelrisch	$-\frac{16\ln(x)x^4a^4-16\ln(ax+i)x^4a^4-9a^4x^4+16ia^3x^3+1+16a^2\ln(x)x^2-16a^2\ln(ax+i)x^2+8iax}{2(a^2x^2+1)x^2}$
meijerg	$\frac{a^2\left(-\frac{1}{a^2x^2}-1-4\ln(x)-2\ln(a^2)+\frac{3a^2x^2}{3a^2x^2+3}+2\ln(a^2x^2+1)\right)}{2}+\frac{2ia^3\left(-\frac{2(3a^2x^2+2)}{x\sqrt{a^2}(2a^2x^2+2)}-\frac{3a\arctan(ax)}{\sqrt{a^2}}\right)}{\sqrt{a^2}}-3a^2\Big(1+2$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-4*a^3*(I/a/(I+a*x)-2/a*ln(I+a*x))-1/2/x^2-4*I*a/x-8*a^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.48

$$\int \frac{e^{4i\arctan(ax)}}{x^3}\,dx = \frac{-16i\,a^2x^2+7\,ax-16\,(a^3x^3+i\,a^2x^2)\log(x)+16\,(a^3x^3+i\,a^2x^2)\log\left(\frac{ax+i}{a}\right)-i}{2\,(ax^3+i\,x^2)}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^3,x, algorithm="fricas")
```

output
```
1/2*(-16*I*a^2*x^2 + 7*a*x - 16*(a^3*x^3 + I*a^2*x^2)*log(x) + 16*(a^3*x^3
 + I*a^2*x^2)*log((a*x + I)/a) - I)/(a*x^3 + I*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

$$\int \frac{e^{4i\arctan(ax)}}{x^3}\,dx = 8a^2\left(-\log\left(16a^3x\right)+\log\left(16a^3x+16ia^2\right)\right)+\frac{-16ia^2x^2+7ax-i}{2ax^3+2ix^2}$$

input
```
integrate((1+I*a*x)**4/(a**2*x**2+1)**2/x**3,x)
```

output
```
8*a**2*(-log(16*a**3*x) + log(16*a**3*x + 16*I*a**2)) + (-16*I*a**2*x**2 +
 7*a*x - I)/(2*a*x**3 + 2*I*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33

$$\int \frac{e^{4i\arctan(ax)}}{x^3}\,dx = -8i\,a^2\arctan(ax)+4\,a^2\log\left(a^2x^2+1\right) - 8\,a^2\log(x)+\frac{-16i\,a^3x^3-9\,a^2x^2-8i\,ax-1}{2\,(a^2x^4+x^2)}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^3,x, algorithm="maxima")
```

output
```
-8*I*a^2*arctan(a*x) + 4*a^2*log(a^2*x^2 + 1) - 8*a^2*log(x) + 1/2*(-16*I*
a^3*x^3 - 9*a^2*x^2 - 8*I*a*x - 1)/(a^2*x^4 + x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{e^{4i\arctan(ax)}}{x^3}\,dx = 8\,a^2\log(ax+i)-8\,a^2\log(|x|)-\frac{16i\,a^2x^2-7\,ax+i}{2\,(ax+i)x^2}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^3,x, algorithm="giac")
```

output
```
8*a^2*log(a*x + I) - 8*a^2*log(abs(x)) - 1/2*(16*I*a^2*x^2 - 7*a*x + I)/((
a*x + I)*x^2)
```

Mupad [B] (verification not implemented)

Time = 22.50 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \frac{e^{4i\arctan(ax)}}{x^3}\,dx = -a^2\,\mathrm{atan}(2\,a\,x + 1\mathrm{i})\;16\mathrm{i} + \frac{8\,a^2\,x^2 + \frac{a\,x\,7\mathrm{i}}{2} + \frac{1}{2}}{x^2\,\left(-1 + a\,x\,1\mathrm{i}\right)}$$

input
```
int((a*x*1i + 1)^4/(x^3*(a^2*x^2 + 1)^2),x)
```

output
```
((a*x*7i)/2 + 8*a^2*x^2 + 1/2)/(x^2*(a*x*1i - 1)) - a^2*atan(2*a*x + 1i)*1
6i
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.35

$$\int \frac{e^{4i\arctan(ax)}}{x^3}\,dx$$
$$= \frac{-16atan(ax)\,a^4 i\,x^4 - 16atan(ax)\,a^2 i\,x^2 + 8\log(a^2x^2+1)\,a^4x^4 + 8\log(a^2x^2+1)\,a^2x^2 - 16\log(x)\,a^4x^4 -}{2x^2\,(a^2x^2+1)}$$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^2/x^3,x)
```

output
```
( - 16*atan(a*x)*a**4*i*x**4 - 16*atan(a*x)*a**2*i*x**2 + 8*log(a**2*x**2
+ 1)*a**4*x**4 + 8*log(a**2*x**2 + 1)*a**2*x**2 - 16*log(x)*a**4*x**4 - 16
*log(x)*a**2*x**2 + 9*a**4*x**4 - 16*a**3*i*x**3 - 8*a*i*x - 1)/(2*x**2*(a
**2*x**2 + 1))
```

3.49 $\int \frac{e^{4i\arctan(ax)}}{x^4} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 62

$$\int \frac{e^{4i\arctan(ax)}}{x^4} \, dx = -\frac{1}{3x^3} - \frac{2ia}{x^2} + \frac{8a^2}{x} + \frac{4a^3}{i+ax} - 12ia^3\log(x) + 12ia^3\log(i+ax)$$

output
```
-1/3/x^3-2*I*a/x^2+8*a^2/x+4*a^3/(I+a*x)-12*I*a^3*ln(x)+12*I*a^3*ln(I+a*x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{e^{4i\arctan(ax)}}{x^4} \, dx = -\frac{1}{3x^3} - \frac{2ia}{x^2} + \frac{8a^2}{x} + \frac{4a^3}{i+ax} - 12ia^3\log(x) + 12ia^3\log(i+ax)$$

input
```
Integrate[E^((4*I)*ArcTan[a*x])/x^4,x]
```

output
```
-1/3*1/x^3 - ((2*I)*a)/x^2 + (8*a^2)/x + (4*a^3)/(I + a*x) - (12*I)*a^3*Lo
g[x] + (12*I)*a^3*Log[I + a*x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5585, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4i\arctan(ax)}}{x^4}dx$$

$$\Big\downarrow \text{ 5585}$$

$$\int \frac{(1+iax)^2}{x^4(1-iax)^2}dx$$

$$\Big\downarrow \text{ 99}$$

$$\int \left(\frac{12ia^4}{ax+i}-\frac{4a^4}{(ax+i)^2}-\frac{12ia^3}{x}-\frac{8a^2}{x^2}+\frac{4ia}{x^3}+\frac{1}{x^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4a^3}{ax+i}-12ia^3\log(x)+12ia^3\log(ax+i)+\frac{8a^2}{x}-\frac{2ia}{x^2}-\frac{1}{3x^3}$$

input
```
Int[E^((4*I)*ArcTan[a*x])/x^4,x]
```

output
```
-1/3*1/x^3 - ((2*I)*a)/x^2 + (8*a^2)/x + (4*a^3)/(I + a*x) - (12*I)*a^3*Lo
g[x] + (12*I)*a^3*Log[I + a*x]
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

method	result
default	$4a^{4}\left(\frac{1}{a(ax+i)}+\frac{3i\ln(ax+i)}{a}\right)-\frac{1}{3x^{3}}-12ia^{3}\ln\left(x\right)-\frac{2ia}{x^{2}}+\frac{8a^{2}}{x}$
risch	$\frac{12a^{3}x^{3}+6ia^{2}x^{2}+\frac{5}{3}ax-\frac{1}{3}i}{(ax+i)x^{3}}+12a^{3}\arctan\left(ax\right)+6ia^{3}\ln\left(a^{2}x^{2}+1\right)-12ia^{3}\ln\left(-x\right)$
parallelrisch	$-\frac{36i\ln(x)x^{5}a^{5}-36i\ln(ax+i)x^{5}a^{5}-18ix^{5}a^{5}+36ia^{3}\ln(x)x^{3}-36ia^{3}\ln(ax+i)x^{3}+1-36a^{4}x^{4}-23a^{2}x^{2}+6iax}{3(a^{2}x^{2}+1)x^{3}}$
meijerg	$\frac{a^{4}\left(-\frac{2\left(-15a^{4}x^{4}-10a^{2}x^{2}+2\right)}{3x^{3}\left(a^{2}\right)^{\frac{3}{2}}\left(2a^{2}x^{2}+2\right)}+\frac{5a^{3}\arctan(ax)}{\left(a^{2}\right)^{\frac{3}{2}}}\right)}{2\sqrt{a^{2}}}+2ia^{3}\left(-\frac{1}{a^{2}x^{2}}-1-4\ln\left(x\right)-2\ln\left(a^{2}\right)+\frac{3a^{2}x^{2}}{3a^{2}x^{2}+3}+2\ln\right.$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
4*a^4*(1/a/(I+a*x)+3*I*ln(I+a*x)/a)-1/3/x^3-12*I*a^3*ln(x)-2*I*a/x^2+8*a^2
/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.39

$$\int \frac{e^{4i\arctan(ax)}}{x^4}\,dx = \frac{36\,a^{3}x^{3}+18i\,a^{2}x^{2}+5\,ax-36\left(i\,a^{4}x^{4}-a^{3}x^{3}\right)\log\left(x\right)-36\left(-i\,a^{4}x^{4}+a^{3}x^{3}\right)\log\left(\frac{ax+i}{a}\right)-i}{3\left(ax^{4}+i\,x^{3}\right)}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^4,x, algorithm="fricas")
```

output
```
1/3*(36*a^3*x^3 + 18*I*a^2*x^2 + 5*a*x - 36*(I*a^4*x^4 - a^3*x^3)*log(x) -
 36*(-I*a^4*x^4 + a^3*x^3)*log((a*x + I)/a) - I)/(a*x^4 + I*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \frac{e^{4i\arctan(ax)}}{x^4}\,dx = 12a^3\left(-i\log\left(24a^4x\right)+i\log\left(24a^4x+24ia^3\right)\right) + \frac{36a^3x^3+18ia^2x^2+5ax-i}{3ax^4+3ix^3}$$

input
```
integrate((1+I*a*x)**4/(a**2*x**2+1)**2/x**4,x)
```

output
```
12*a**3*(-I*log(24*a**4*x) + I*log(24*a**4*x + 24*I*a**3)) + (36*a**3*x**3
 + 18*I*a**2*x**2 + 5*a*x - I)/(3*a*x**4 + 3*I*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.24

$$\int \frac{e^{4i\arctan(ax)}}{x^4}\,dx = 12\,a^3\arctan\left(ax\right)+6i\,a^3\log\left(a^2x^2+1\right)-12i\,a^3\log\left(x\right) + \frac{36\,a^4x^4-18i\,a^3x^3+23\,a^2x^2-6i\,ax-1}{3\left(a^2x^5+x^3\right)}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^4,x, algorithm="maxima")
```

output
```
12*a^3*arctan(a*x) + 6*I*a^3*log(a^2*x^2 + 1) - 12*I*a^3*log(x) + 1/3*(36*
a^4*x^4 - 18*I*a^3*x^3 + 23*a^2*x^2 - 6*I*a*x - 1)/(a^2*x^5 + x^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{e^{4i\arctan(ax)}}{x^4}\,dx = 12i\,a^3\log(ax+i) - 12i\,a^3\log(|x|) + \frac{36\,a^3x^3 + 18i\,a^2x^2 + 5\,ax - i}{3\,(ax+i)x^3}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2/x^4,x, algorithm="giac")
```

output
```
12*I*a^3*log(a*x + I) - 12*I*a^3*log(abs(x)) + 1/3*(36*a^3*x^3 + 18*I*a^2*
x^2 + 5*a*x - I)/((a*x + I)*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \frac{e^{4i\arctan(ax)}}{x^4}\,dx = 24\,a^3\,\mathrm{atan}(2\,a\,x+1\mathrm{i}) + \frac{\frac{5\,x}{3} + 12\,a^2\,x^3 + a\,x^2\,6\mathrm{i} - \frac{1\mathrm{i}}{3\,a}}{x^4 + \frac{x^3\,1\mathrm{i}}{a}}$$

input
```
int((a*x*1i + 1)^4/(x^4*(a^2*x^2 + 1)^2),x)
```

output
```
24*a^3*atan(2*a*x + 1i) + ((5*x)/3 + a*x^2*6i - 1i/(3*a) + 12*a^2*x^3)/(x^
4 + (x^3*1i)/a)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.13

$$\int \frac{e^{4i\arctan(ax)}}{x^4}\,dx$$
$$= \frac{36atan(ax)\,a^5x^5 + 36atan(ax)\,a^3x^3 + 18\log(a^2x^2+1)\,a^5i\,x^5 + 18\log(a^2x^2+1)\,a^3i\,x^3 - 36\log(x)\,a^5i\,x^5}{3x^3\,(a^2x^2+1)}$$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^2/x^4,x)
```

output
```
(36*atan(a*x)*a**5*x**5 + 36*atan(a*x)*a**3*x**3 + 18*log(a**2*x**2 + 1)*a
**5*i*x**5 + 18*log(a**2*x**2 + 1)*a**3*i*x**3 - 36*log(x)*a**5*i*x**5 - 3
6*log(x)*a**3*i*x**3 + 18*a**5*i*x**5 + 36*a**4*x**4 + 23*a**2*x**2 - 6*a*
i*x - 1)/(3*x**3*(a**2*x**2 + 1))
```

3.50 $\int e^{-i\arctan(ax)}x^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 105

$$\int e^{-i\arctan(ax)}x^3\,dx = -\frac{2\sqrt{1+a^2x^2}}{3a^4}+\frac{3ix\sqrt{1+a^2x^2}}{8a^3}+\frac{x^2\sqrt{1+a^2x^2}}{3a^2} - \frac{ix^3\sqrt{1+a^2x^2}}{4a}-\frac{3i\text{arcsinh}(ax)}{8a^4}$$

output
```
-2/3*(a^2*x^2+1)^(1/2)/a^4+3/8*I*x*(a^2*x^2+1)^(1/2)/a^3+1/3*x^2*(a^2*x^2+
1)^(1/2)/a^2-1/4*I*x^3*(a^2*x^2+1)^(1/2)/a-3/8*I*arcsinh(a*x)/a^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.53

$$\int e^{-i\arctan(ax)}x^3\,dx = \frac{\sqrt{1+a^2x^2}(-16+9iax+8a^2x^2-6ia^3x^3)-9i\text{arcsinh}(ax)}{24a^4}$$

input
```
Integrate[x^3/E^(I*ArcTan[a*x]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(-16 + (9*I)*a*x + 8*a^2*x^2 - (6*I)*a^3*x^3) - (9*I)*A
rcSinh[a*x])/(24*a^4)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5583, 533, 25, 27, 533, 27, 533, 25, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{-i\arctan(ax)}dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{x^3(1-iax)}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{533}$$

$$-\frac{\int -\frac{ax^2(4ax+3i)}{\sqrt{a^2x^2+1}}dx}{4a^2}-\frac{ix^3\sqrt{a^2x^2+1}}{4a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{ax^2(4ax+3i)}{\sqrt{a^2x^2+1}}dx}{4a^2}-\frac{ix^3\sqrt{a^2x^2+1}}{4a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x^2(4ax+3i)}{\sqrt{a^2x^2+1}}dx}{4a}-\frac{ix^3\sqrt{a^2x^2+1}}{4a}$$

$$\Big\downarrow \text{533}$$

$$\frac{\frac{4x^2\sqrt{a^2x^2+1}}{3a}-\frac{\int \frac{ax(8-9iax)}{\sqrt{a^2x^2+1}}dx}{3a^2}}{4a}-\frac{ix^3\sqrt{a^2x^2+1}}{4a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{4x^2\sqrt{a^2x^2+1}}{3a}-\frac{\int \frac{x(8-9iax)}{\sqrt{a^2x^2+1}}dx}{3a}}{4a}-\frac{ix^3\sqrt{a^2x^2+1}}{4a}$$

$$\Big\downarrow \text{533}$$

$$\frac{\frac{4x^2\sqrt{a^2x^2+1}}{3a}-\frac{-\frac{\int -\frac{a(16ax+9i)}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{9ix\sqrt{a^2x^2+1}}{2a}}{3a}}{4a}-\frac{ix^3\sqrt{a^2x^2+1}}{4a}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{4x^2\sqrt{a^2x^2+1}}{3a}-\frac{\frac{\int \frac{a(16ax+9i)}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{9ix\sqrt{a^2x^2+1}}{2a}}{3a}}{4a}-\frac{ix^3\sqrt{a^2x^2+1}}{4a}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{4x^2\sqrt{a^2x^2+1}}{3a}-\frac{\frac{\int \frac{16ax+9i}{\sqrt{a^2x^2+1}}dx}{2a}-\frac{9ix\sqrt{a^2x^2+1}}{2a}}{3a}}{4a}-\frac{ix^3\sqrt{a^2x^2+1}}{4a}$$

$$\Big\downarrow \; 455$$

$$\frac{\frac{4x^2\sqrt{a^2x^2+1}}{3a}-\frac{\frac{\frac{16\sqrt{a^2x^2+1}}{a}+9i\int \frac{1}{\sqrt{a^2x^2+1}}dx}{2a}-\frac{9ix\sqrt{a^2x^2+1}}{2a}}{3a}}{4a}-\frac{ix^3\sqrt{a^2x^2+1}}{4a}$$

$$\Big\downarrow \; 222$$

$$\frac{\frac{4x^2\sqrt{a^2x^2+1}}{3a}-\frac{\frac{\frac{16\sqrt{a^2x^2+1}}{a}+\frac{9i\text{arcsinh}(ax)}{a}}{2a}-\frac{9ix\sqrt{a^2x^2+1}}{2a}}{3a}}{4a}-\frac{ix^3\sqrt{a^2x^2+1}}{4a}$$

input
```
Int[x^3/E^(I*ArcTan[a*x]),x]
```

output
```
((-1/4*I)*x^3*Sqrt[1 + a^2*x^2])/a + ((4*x^2*Sqrt[1 + a^2*x^2])/(3*a) - ((
((-9*I)/2)*x*Sqrt[1 + a^2*x^2])/a + ((16*Sqrt[1 + a^2*x^2])/a + ((9*I)*Arc
Sinh[a*x])/a)/(2*a))/(3*a))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{i\left(6a^3x^3+8ia^2x^2-9ax-16i\right)\sqrt{a^2x^2+1}}{24a^4}-\frac{3i\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{8a^3\sqrt{a^2}}$
default	$\frac{i\left(\frac{\sqrt{a^2x^2+1}\,x}{2}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2\sqrt{a^2}}\right)}{a^3}+\frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{3a^4}-\frac{i\left(\frac{x\left(a^2x^2+1\right)^{\frac{3}{2}}}{4a^2}-\frac{\frac{\sqrt{a^2x^2+1}\,x}{2}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2\sqrt{a^2}}}{4a^2}\right)}{a}-\frac{\sqrt{\left(x-\frac{i}{a}\right)^2 a}}{}$

input
```
int(x^3/(1+I*a*x)*(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*I*(6*a^3*x^3+8*I*a^2*x^2-9*a*x-16*I)*(a^2*x^2+1)^(1/2)/a^4-3/8*I/a^3
*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.56

$$\int e^{-i\arctan(ax)}x^3\,dx = \frac{\left(-6i\,a^3x^3+8\,a^2x^2+9i\,ax-16\right)\sqrt{a^2x^2+1}+9i\,\log\left(-ax+\sqrt{a^2x^2+1}\right)}{24\,a^4}$$

input
```
integrate(x^3/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/24*((-6*I*a^3*x^3 + 8*a^2*x^2 + 9*I*a*x - 16)*sqrt(a^2*x^2 + 1) + 9*I*lo
g(-a*x + sqrt(a^2*x^2 + 1)))/a^4
```

Sympy [F]

$$\int e^{-i\arctan(ax)}x^3\,dx = -i\int\frac{x^3\sqrt{a^2x^2+1}}{ax-i}\,dx$$

input
```
integrate(x**3/(1+I*a*x)*(a**2*x**2+1)**(1/2),x)
```

output
```
-I*Integral(x**3*sqrt(a**2*x**2 + 1)/(a*x - I), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.72

$$\int e^{-i\arctan(ax)}x^3\,dx = -\frac{i\,(a^2x^2+1)^{\frac{3}{2}}x}{4\,a^3}+\frac{5i\,\sqrt{a^2x^2+1}x}{8\,a^3}+\frac{(a^2x^2+1)^{\frac{3}{2}}}{3\,a^4}-\frac{3i\,\operatorname{arsinh}(ax)}{8\,a^4}-\frac{\sqrt{a^2x^2+1}}{a^4}$$

input
```
integrate(x^3/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*I*(a^2*x^2 + 1)^(3/2)*x/a^3 + 5/8*I*sqrt(a^2*x^2 + 1)*x/a^3 + 1/3*(a^
2*x^2 + 1)^(3/2)/a^4 - 3/8*I*arcsinh(a*x)/a^4 - sqrt(a^2*x^2 + 1)/a^4
```

Giac [F(-2)]

Exception generated.

$$\int e^{-i\arctan(ax)}x^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.81

$$\int e^{-i\arctan(ax)}x^3\,dx = -\frac{\operatorname{asinh}\left(x\sqrt{a^2}\right)\,3\mathrm{i}}{8\,a^3\sqrt{a^2}} - \frac{\sqrt{a^2\,x^2+1}\left(\frac{2}{3\,{\left(a^2\right)}^{3/2}}-\frac{a^2\,x^2}{3\,{\left(a^2\right)}^{3/2}}+\frac{x^3\,{\left(a^2\right)}^{3/2}\,1\mathrm{i}}{4\,a^3}-\frac{x\,\sqrt{a^2}\,3\mathrm{i}}{8\,a^3}\right)}{\sqrt{a^2}}$$

input
```
int((x^3*(a^2*x^2 + 1)^(1/2))/(a*x*1i + 1),x)
```

output
```
- (asinh(x*(a^2)^(1/2))*3i)/(8*a^3*(a^2)^(1/2)) - ((a^2*x^2 + 1)^(1/2)*(2/
(3*(a^2)^(3/2)) - (a^2*x^2)/(3*(a^2)^(3/2)) + (x^3*(a^2)^(3/2)*1i)/(4*a^3)
 - (x*(a^2)^(1/2)*3i)/(8*a^3)))/(a^2)^(1/2)
```

Reduce [F]

$$\int e^{-i\arctan(ax)}x^3\,dx = \int \frac{\sqrt{a^2x^2+1}\,x^3}{aix+1}dx$$

input
```
int(x^3/(1+I*a*x)*(a^2*x^2+1)^(1/2),x)
```

output
```
int((sqrt(a**2*x**2 + 1)*x**3)/(a*i*x + 1),x)
```

3.51 $\int e^{-i\arctan(ax)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 80

$$\int e^{-i\arctan(ax)}x^2\,dx=\frac{2i\sqrt{1+a^2x^2}}{3a^3}+\frac{x\sqrt{1+a^2x^2}}{2a^2}-\frac{ix^2\sqrt{1+a^2x^2}}{3a}-\frac{\operatorname{arcsinh}(ax)}{2a^3}$$

output
```
2/3*I*(a^2*x^2+1)^(1/2)/a^3+1/2*x*(a^2*x^2+1)^(1/2)/a^2-1/3*I*x^2*(a^2*x^2
+1)^(1/2)/a-1/2*arcsinh(a*x)/a^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int e^{-i\arctan(ax)}x^2\,dx=\frac{(4i+3ax-2ia^2x^2)\sqrt{1+a^2x^2}-3\operatorname{arcsinh}(ax)}{6a^3}$$

input
```
Integrate[x^2/E^(I*ArcTan[a*x]),x]
```

output
```
((4*I + 3*a*x - (2*I)*a^2*x^2)*Sqrt[1 + a^2*x^2] - 3*ArcSinh[a*x])/(6*a^3)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5583, 533, 25, 27, 533, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{-i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{ 5583}$$

$$\int \frac{x^2(1-iax)}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{ 533}$$

$$-\frac{\int -\frac{ax(3ax+2i)}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{ix^2\sqrt{a^2x^2+1}}{3a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{ax(3ax+2i)}{\sqrt{a^2x^2+1}}dx}{3a^2}-\frac{ix^2\sqrt{a^2x^2+1}}{3a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{x(3ax+2i)}{\sqrt{a^2x^2+1}}dx}{3a}-\frac{ix^2\sqrt{a^2x^2+1}}{3a}$$

$$\Big\downarrow \text{ 533}$$

$$\frac{\frac{3x\sqrt{a^2x^2+1}}{2a}-\frac{\int \frac{a(3-4iax)}{\sqrt{a^2x^2+1}}dx}{2a^2}}{3a}-\frac{ix^2\sqrt{a^2x^2+1}}{3a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{3x\sqrt{a^2x^2+1}}{2a}-\frac{\int \frac{3-4iax}{\sqrt{a^2x^2+1}}dx}{2a}}{3a}-\frac{ix^2\sqrt{a^2x^2+1}}{3a}$$

$$\Big\downarrow \text{ 455}$$

$$\frac{\frac{3x\sqrt{a^2x^2+1}}{2a}-\frac{3\int \frac{1}{\sqrt{a^2x^2+1}}dx-\frac{4i\sqrt{a^2x^2+1}}{a}}{2a}}{3a}-\frac{ix^2\sqrt{a^2x^2+1}}{3a}$$

$$\Big\downarrow \text{222}$$

$$\frac{\frac{3x\sqrt{a^2x^2+1}}{2a}-\frac{\frac{3\text{arcsinh}(ax)}{a}-\frac{4i\sqrt{a^2x^2+1}}{a}}{2a}}{3a}-\frac{ix^2\sqrt{a^2x^2+1}}{3a}$$

input
```
Int[x^2/E^(I*ArcTan[a*x]),x]
```

output
```
((-1/3*I)*x^2*Sqrt[1 + a^2*x^2])/a + ((3*x*Sqrt[1 + a^2*x^2])/(2*a) - (((-
4*I)*Sqrt[1 + a^2*x^2])/a + (3*ArcSinh[a*x])/a)/(2*a))/(3*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{i(2a^2x^2+3iax-4)\sqrt{a^2x^2+1}}{6a^3}-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2a^2\sqrt{a^2}}$
default	$\frac{\frac{\sqrt{a^2x^2+1}\,x}{2}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2\sqrt{a^2}}}{a^2}-\frac{i(a^2x^2+1)^{\frac{3}{2}}}{3a^3}+\frac{i\left(\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}+\frac{ia\ln\left(\frac{ia+\left(x-\frac{i}{a}\right)a^2}{\sqrt{a^2}}+\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}\right.}{\sqrt{a^2}}\right.}{a^3}$

input
```
int(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*I*(2*a^2*x^2+3*I*a*x-4)*(a^2*x^2+1)^(1/2)/a^3-1/2/a^2*ln(a^2*x/(a^2)^
(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.64

$$\int e^{-i\arctan(ax)}x^2\,dx=\frac{\sqrt{a^2x^2+1}(-2i\,a^2x^2+3\,ax+4i)+3\,\log\left(-ax+\sqrt{a^2x^2+1}\right)}{6\,a^3}$$

input
```
integrate(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(a^2*x^2 + 1)*(-2*I*a^2*x^2 + 3*a*x + 4*I) + 3*log(-a*x + sqrt(a^
2*x^2 + 1)))/a^3
```

Sympy [F]

$$\int e^{-i\arctan(ax)}x^2\,dx = -i\int \frac{x^2\sqrt{a^2x^2+1}}{ax-i}\,dx$$

input
```
integrate(x**2/(1+I*a*x)*(a**2*x**2+1)**(1/2),x)
```

output
```
-I*Integral(x**2*sqrt(a**2*x**2 + 1)/(a*x - I), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.74

$$\int e^{-i\arctan(ax)}x^2\,dx = \frac{\sqrt{a^2x^2+1}x}{2a^2} - \frac{i(a^2x^2+1)^{\frac{3}{2}}}{3a^3} - \frac{\operatorname{arsinh}(ax)}{2a^3} + \frac{i\sqrt{a^2x^2+1}}{a^3}$$

input
```
integrate(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(a^2*x^2 + 1)*x/a^2 - 1/3*I*(a^2*x^2 + 1)^(3/2)/a^3 - 1/2*arcsinh(
a*x)/a^3 + I*sqrt(a^2*x^2 + 1)/a^3
```

Giac [F(-2)]

Exception generated.

$$\int e^{-i\arctan(ax)}x^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.71 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int e^{-i\arctan(ax)}x^2\,dx = \frac{\sqrt{a^2\,x^2+1}\,\left(\frac{x\,\sqrt{a^2}}{2\,a^2}+\frac{a\,2\mathrm{i}}{3\,{\left(a^2\right)}^{3/2}}-\frac{a^3\,x^2\,1\mathrm{i}}{3\,{\left(a^2\right)}^{3/2}}\right)}{\sqrt{a^2}}-\frac{\mathrm{asinh}\left(x\,\sqrt{a^2}\right)}{2\,a^2\,\sqrt{a^2}}$$

input
```
int((x^2*(a^2*x^2 + 1)^(1/2))/(a*x*1i + 1),x)
```

output
```
((a^2*x^2 + 1)^(1/2)*((a*2i)/(3*(a^2)^(3/2)) - (a^3*x^2*1i)/(3*(a^2)^(3/2)
) + (x*(a^2)^(1/2))/(2*a^2)))/(a^2)^(1/2) - asinh(x*(a^2)^(1/2))/(2*a^2*(a
^2)^(1/2))
```

Reduce [F]

$$\int e^{-i\arctan(ax)}x^2\,dx = \int \frac{\sqrt{a^2x^2+1}\,x^2}{aix+1}dx$$

input
```
int(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2),x)
```

output
```
int((sqrt(a**2*x**2 + 1)*x**2)/(a*i*x + 1),x)
```

3.52 $\int e^{-i\arctan(ax)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 54

$$\int e^{-i\arctan(ax)}x\,dx=\frac{\sqrt{1+a^2x^2}}{a^2}-\frac{ix\sqrt{1+a^2x^2}}{2a}+\frac{i\text{arcsinh}(ax)}{2a^2}$$

output
```
(a^2*x^2+1)^(1/2)/a^2-1/2*I*x*(a^2*x^2+1)^(1/2)/a+1/2*I*arcsinh(a*x)/a^2
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.70

$$\int e^{-i\arctan(ax)}x\,dx=\frac{(2-iax)\sqrt{1+a^2x^2}+i\text{arcsinh}(ax)}{2a^2}$$

input
```
Integrate[x/E^(I*ArcTan[a*x]),x]
```

output
```
((2 - I*a*x)*Sqrt[1 + a^2*x^2] + I*ArcSinh[a*x])/(2*a^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5583, 533, 25, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{ 5583}$$

$$\int \frac{x(1-iax)}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{ 533}$$

$$-\frac{\int -\frac{a(2ax+i)}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{ix\sqrt{a^2x^2+1}}{2a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{a(2ax+i)}{\sqrt{a^2x^2+1}}dx}{2a^2}-\frac{ix\sqrt{a^2x^2+1}}{2a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{2ax+i}{\sqrt{a^2x^2+1}}dx}{2a}-\frac{ix\sqrt{a^2x^2+1}}{2a}$$

$$\Big\downarrow \text{ 455}$$

$$\frac{\frac{2\sqrt{a^2x^2+1}}{a}+i\int \frac{1}{\sqrt{a^2x^2+1}}dx}{2a}-\frac{ix\sqrt{a^2x^2+1}}{2a}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{\frac{2\sqrt{a^2x^2+1}}{a}+\frac{i\text{arcsinh}(ax)}{a}}{2a}-\frac{ix\sqrt{a^2x^2+1}}{2a}$$

input
```
Int[x/E^(I*ArcTan[a*x]),x]
```

output
```
((-1/2*I)*x*Sqrt[1 + a^2*x^2])/a + ((2*Sqrt[1 + a^2*x^2])/a + (I*ArcSinh[a
*x])/a)/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

method	result	size
risch	$-\frac{i(ax+2i)\sqrt{a^2x^2+1}}{2a^2}+\frac{i\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2a\sqrt{a^2}}$	59
default	$-\frac{i\left(\frac{\sqrt{a^2x^2+1}\,x}{2}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2\sqrt{a^2}}\right)}{a}+\frac{\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}+\frac{ia\ln\left(\frac{ia+\left(x-\frac{i}{a}\right)a^2}{\sqrt{a^2}}+\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}\right)}{\sqrt{a^2}}}{a^2}$	150

input
```
int(x/(1+I*a*x)*(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*(a*x+2*I)*(a^2*x^2+1)^(1/2)/a^2+1/2*I/a*ln(a^2*x/(a^2)^(1/2)+(a^2*x
^2+1)^(1/2))/(a^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int e^{-i\arctan(ax)}x\,dx=\frac{\sqrt{a^2x^2+1}(-i\,ax+2)-i\,\log\left(-ax+\sqrt{a^2x^2+1}\right)}{2\,a^2}$$

input
```
integrate(x/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(a^2*x^2 + 1)*(-I*a*x + 2) - I*log(-a*x + sqrt(a^2*x^2 + 1)))/a^2
```

Sympy [F]

$$\int e^{-i\arctan(ax)}x\,dx=-i\int\frac{x\sqrt{a^2x^2+1}}{ax-i}\,dx$$

input
```
integrate(x/(1+I*a*x)*(a**2*x**2+1)**(1/2),x)
```

output
```
-I*Integral(x*sqrt(a**2*x**2 + 1)/(a*x - I), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int e^{-i\arctan(ax)}x\,dx = -\frac{i\sqrt{a^2x^2+1}x}{2a} + \frac{i\,\operatorname{arsinh}(ax)}{2a^2} + \frac{\sqrt{a^2x^2+1}}{a^2}$$

input
```
integrate(x/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*I*sqrt(a^2*x^2 + 1)*x/a + 1/2*I*arcsinh(a*x)/a^2 + sqrt(a^2*x^2 + 1)/
a^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int e^{-i\arctan(ax)}x\,dx = -\frac{1}{2}\sqrt{a^2x^2+1}\left(\frac{ix}{a} - \frac{2}{a^2}\right) - \frac{i\log\left(-x|a| + \sqrt{a^2x^2+1}\right)}{2a|a|}$$

input
```
integrate(x/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(a^2*x^2 + 1)*(I*x/a - 2/a^2) - 1/2*I*log(-x*abs(a) + sqrt(a^2*x^
2 + 1))/(a*abs(a))
```

Mupad [B] (verification not implemented)

Time = 22.89 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int e^{-i\arctan(ax)}x\,dx = \frac{\left(\frac{1}{\sqrt{a^2}} - \frac{x\sqrt{a^2}\,1\mathrm{i}}{2\,a}\right)\sqrt{a^2\,x^2+1} + \frac{\mathrm{asinh}\left(x\,\sqrt{a^2}\right)\,1\mathrm{i}}{2\,a}}{\sqrt{a^2}}$$

input
```
int((x*(a^2*x^2 + 1)^(1/2))/(a*x*1i + 1),x)
```

output
```
((1/(a^2)^(1/2) - (x*(a^2)^(1/2)*1i)/(2*a))*(a^2*x^2 + 1)^(1/2) + (asinh(x
*(a^2)^(1/2))*1i)/(2*a))/(a^2)^(1/2)
```

Reduce [F]

$$\int e^{-i\arctan(ax)}x\,dx = \int \frac{\sqrt{a^2x^2+1}\,x}{aix+1}dx$$

input
```
int(x/(1+I*a*x)*(a^2*x^2+1)^(1/2),x)
```

output
```
int((sqrt(a**2*x**2 + 1)*x)/(a*i*x + 1),x)
```

3.53 $\int e^{-i\arctan(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 29

$$\int e^{-i\arctan(ax)}\,dx = -\frac{i\sqrt{1+a^2x^2}}{a}+\frac{\operatorname{arcsinh}(ax)}{a}$$

output
```
-I*(a^2*x^2+1)^(1/2)/a+arcsinh(a*x)/a
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int e^{-i\arctan(ax)}\,dx = \frac{-i\sqrt{1+a^2x^2}+\operatorname{arcsinh}(ax)}{a}$$

input
```
Integrate[E^((-I)*ArcTan[a*x]),x]
```

output
```
((-I)*Sqrt[1 + a^2*x^2] + ArcSinh[a*x])/a
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5582, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{ 5582}$$

$$\int \frac{1-iax}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{ 455}$$

$$\int \frac{1}{\sqrt{a^2x^2+1}}dx - \frac{i\sqrt{a^2x^2+1}}{a}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{\text{arcsinh}(ax)}{a} - \frac{i\sqrt{a^2x^2+1}}{a}$$

input
```
Int[E^((-I)*ArcTan[a*x]),x]
```

output
```
((-I)*Sqrt[1 + a^2*x^2])/a + ArcSinh[a*x]/a
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

rule 5582
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 - I*a*x)^((I*n + 1)/2)
/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2]), x] /; FreeQ[a, x] && Intege
rQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.66

method	result	size
risch	$-\frac{i\sqrt{a^2x^2+1}}{a}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{\sqrt{a^2}}$	48
default	$-\frac{i\left(\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}+\frac{ia\ln\left(\frac{ia+\left(x-\frac{i}{a}\right)a^2}{\sqrt{a^2}}+\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}\right)}{\sqrt{a^2}}\right)}{a}$	100

input
```
int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*(a^2*x^2+1)^(1/2)/a+ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int e^{-i\arctan(ax)}\,dx=\frac{-i\sqrt{a^2x^2+1}-\log\left(-ax+\sqrt{a^2x^2+1}\right)}{a}$$

input
```
integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(a^2*x^2 + 1) - log(-a*x + sqrt(a^2*x^2 + 1)))/a
```

Sympy [F]

$$\int e^{-i\arctan(ax)}\,dx = -i\int \frac{\sqrt{a^2x^2+1}}{ax-i}\,dx$$

input
```
integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2),x)
```

output
```
-I*Integral(sqrt(a**2*x**2 + 1)/(a*x - I), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int e^{-i\arctan(ax)}\,dx = \frac{\operatorname{arsinh}(ax)}{a} - \frac{i\sqrt{a^2x^2+1}}{a}$$

input
```
integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(a*x)/a - I*sqrt(a^2*x^2 + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int e^{-i\arctan(ax)}\,dx = -\frac{\log\left(-x|a| + \sqrt{a^2x^2+1}\right)}{|a|} - \frac{i\sqrt{a^2x^2+1}}{a}$$

input
```
integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a) - I*sqrt(a^2*x^2 + 1)/a
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int e^{-i\arctan(ax)}\,dx=\frac{\operatorname{asinh}\left(x\sqrt{a^2}\right)}{\sqrt{a^2}}-\frac{\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{a}$$

input
```
int((a^2*x^2 + 1)^(1/2)/(a*x*1i + 1),x)
```

output
```
asinh(x*(a^2)^(1/2))/(a^2)^(1/2) - ((a^2*x^2 + 1)^(1/2)*1i)/a
```

Reduce [F]

$$\int e^{-i\arctan(ax)}\,dx=\int\frac{\sqrt{a^2x^2+1}}{aix+1}dx$$

input
```
int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2),x)
```

output
```
int(sqrt(a**2*x**2 + 1)/(a*i*x + 1),x)
```

3.54 $\int \frac{e^{-i\arctan(ax)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int \frac{e^{-i\arctan(ax)}}{x}\,dx = -i\text{arcsinh}(ax) - \text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

output
```
-I*arcsinh(a*x)-arctanh((a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{e^{-i\arctan(ax)}}{x}\,dx = -i\text{arcsinh}(ax) + \log(x) - \log\left(1+\sqrt{1+a^2x^2}\right)$$

input
```
Integrate[1/(E^(I*ArcTan[a*x])*x),x]
```

output
```
(-I)*ArcSinh[a*x] + Log[x] - Log[1 + Sqrt[1 + a^2*x^2]]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5583, 538, 222, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-i\arctan(ax)}}{x}dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{1-iax}{x\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{538}$$

$$\int \frac{1}{x\sqrt{a^2x^2+1}}dx - ia\int \frac{1}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{222}$$

$$\int \frac{1}{x\sqrt{a^2x^2+1}}dx - i\text{arcsinh}(ax)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{a^2x^2+1}}dx^2 - i\text{arcsinh}(ax)$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{x^4}{a^2}-\frac{1}{a^2}}d\sqrt{a^2x^2+1}}{a^2} - i\text{arcsinh}(ax)$$

$$\Big\downarrow \text{221}$$

$$-\text{arctanh}\left(\sqrt{a^2x^2+1}\right) - i\text{arcsinh}(ax)$$

input
```
Int[1/(E^(I*ArcTan[a*x])*x),x]
```

output
```
(-I)*ArcSinh[a*x] - ArcTanh[Sqrt[1 + a^2*x^2]]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(22) = 44$.

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 4.84

method	result
default	$\sqrt{a^2x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)-\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}-\frac{ia\ln\left(\frac{ia+\left(x-\frac{i}{a}\right)a^2}{\sqrt{a^2}}+\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2i\right.}{\sqrt{a^2}}$

input
```
int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(a^2*x^2+1)^(1/2)-arctanh(1/(a^2*x^2+1)^(1/2))-((x-I/a)^2*a^2+2*I*a*(x-I/a
))^(1/2)-I*a*ln((I*a+(x-I/a)*a^2)/(a^2)^(1/2)+((x-I/a)^2*a^2+2*I*a*(x-I/a)
)^(1/2))/(a^2)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(21) = 42$.

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

$$\int \frac{e^{-i\arctan(ax)}}{x}\,dx = -\log\left(-ax+\sqrt{a^2x^2+1}+1\right)+i\log\left(-ax+\sqrt{a^2x^2+1}\right)+\log\left(-ax+\sqrt{a^2x^2+1}-1\right)$$

input
```
integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x,x, algorithm="fricas")
```

output
```
-log(-a*x + sqrt(a^2*x^2 + 1) + 1) + I*log(-a*x + sqrt(a^2*x^2 + 1)) + log
(-a*x + sqrt(a^2*x^2 + 1) - 1)
```

Sympy [F]

$$\int \frac{e^{-i\arctan(ax)}}{x}\,dx = -i\int \frac{\sqrt{a^2x^2+1}}{ax^2-ix}\,dx$$

input
```
integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/x,x)
```

output
```
-I*Integral(sqrt(a**2*x**2 + 1)/(a*x**2 - I*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{e^{-i\arctan(ax)}}{x}\,dx = -i\,a\left(\frac{\operatorname{arsinh}(ax)}{a} - \frac{i\,\operatorname{arsinh}\left(\frac{1}{a|x|}\right)}{a}\right)$$

input
```
integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x,x, algorithm="maxima")
```

output
```
-I*a*(arcsinh(a*x)/a - I*arcsinh(1/(a*abs(x)))/a)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(21) = 42.

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.72

$$\int \frac{e^{-i\arctan(ax)}}{x}\,dx = \frac{i\,a\log\left(-x|a| + \sqrt{a^2x^2+1}\right)}{|a|} - \log\left(\left|-x|a| + \sqrt{a^2x^2+1}+1\right|\right) + \log\left(\left|-x|a| + \sqrt{a^2x^2+1}-1\right|\right)$$

input
```
integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x,x, algorithm="giac")
```

output
```
I*a*log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a) - log(abs(-x*abs(a) + sqrt(a
^2*x^2 + 1) + 1)) + log(abs(-x*abs(a) + sqrt(a^2*x^2 + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{e^{-i\arctan(ax)}}{x}\,dx = -\mathrm{atanh}\left(\sqrt{a^2\,x^2+1}\right) - \frac{a\,\mathrm{asinh}\left(x\,\sqrt{a^2}\right)\,1\mathrm{i}}{\sqrt{a^2}}$$

input
```
int((a^2*x^2 + 1)^(1/2)/(x*(a*x*1i + 1)),x)
```

output
```
- atanh((a^2*x^2 + 1)^(1/2)) - (a*asinh(x*(a^2)^(1/2))*1i)/(a^2)^(1/2)
```

Reduce [F]

$$\int \frac{e^{-i\arctan(ax)}}{x}\,dx = \int \frac{\sqrt{a^2x^2+1}}{ai\,x^2+x}dx$$

input
```
int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x,x)
```

output
```
int(sqrt(a**2*x**2 + 1)/(a*i*x**2 + x),x)
```

3.55 $\int \frac{e^{-i\arctan(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 38

$$\int \frac{e^{-i\arctan(ax)}}{x^2}\,dx = -\frac{\sqrt{1+a^2x^2}}{x} + ia\text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

output
```
-(a^2*x^2+1)^(1/2)/x+I*a*arctanh((a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{e^{-i\arctan(ax)}}{x^2}\,dx = -\frac{\sqrt{1+a^2x^2}}{x} - ia\log(x) + ia\log\left(1+\sqrt{1+a^2x^2}\right)$$

input
```
Integrate[1/(E^(I*ArcTan[a*x])*x^2),x]
```

output
```
-(Sqrt[1 + a^2*x^2]/x) - I*a*Log[x] + I*a*Log[1 + Sqrt[1 + a^2*x^2]]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5583, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-i\arctan(ax)}}{x^2}dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{1-iax}{x^2\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{534}$$

$$-\frac{\sqrt{a^2x^2+1}}{x}-ia\int \frac{1}{x\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{243}$$

$$-\frac{\sqrt{a^2x^2+1}}{x}-\frac{1}{2}ia\int \frac{1}{x^2\sqrt{a^2x^2+1}}dx^2$$

$$\Big\downarrow \text{73}$$

$$-\frac{\sqrt{a^2x^2+1}}{x}-\frac{i\int \frac{1}{\frac{x^4}{a^2}-\frac{1}{a^2}}d\sqrt{a^2x^2+1}}{a}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\sqrt{a^2x^2+1}}{x}+ia\text{arctanh}\left(\sqrt{a^2x^2+1}\right)$$

input
```
Int[1/(E^(I*ArcTan[a*x])*x^2),x]
```

output
```
-(Sqrt[1 + a^2*x^2]/x) + I*a*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result
risch	$-\frac{\sqrt{a^{2}x^{2}+1}}{x}+ia\,\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2}x^{2}+1}}\right)$
default	$-\frac{\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}}{x}+2a^{2}\left(\frac{\sqrt{a^{2}x^{2}+1}\,x}{2}+\frac{\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}+1}\right)}{2\sqrt{a^{2}}}\right)-ia\left(\sqrt{a^{2}x^{2}+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2}x^{2}+1}}\right)\right)+ia$

input
```
int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(a^2*x^2+1)^(1/2)/x+I*a*arctanh(1/(a^2*x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(32) = 64$.

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \frac{e^{-i\arctan(ax)}}{x^2}\,dx = \frac{i\,ax\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-i\,ax\log\left(-ax+\sqrt{a^2x^2+1}-1\right)-ax-\sqrt{a^2x^2+1}}{x}$$

input
```
integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
(I*a*x*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - I*a*x*log(-a*x + sqrt(a^2*x^2 +
 1) - 1) - a*x - sqrt(a^2*x^2 + 1))/x
```

Sympy [F]

$$\int \frac{e^{-i\arctan(ax)}}{x^2}\,dx = -i\int \frac{\sqrt{a^2x^2+1}}{ax^3-ix^2}\,dx$$

input
```
integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/x**2,x)
```

output
```
-I*Integral(sqrt(a**2*x**2 + 1)/(a*x**3 - I*x**2), x)
```

Maxima [F]

$$\int \frac{e^{-i\arctan(ax)}}{x^2}\,dx = \int \frac{\sqrt{a^2x^2+1}}{(i\,ax+1)x^2}\,dx$$

input
```
integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*x^2 + 1)/((I*a*x + 1)*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-i\arctan(ax)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.80 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{e^{-i\arctan(ax)}}{x^2}\,dx = -\frac{\sqrt{a^2\,x^2+1}}{x} + a\,\mathrm{atanh}\left(\sqrt{a^2\,x^2+1}\right)\,1\mathrm{i}$$

input
```
int((a^2*x^2 + 1)^(1/2)/(x^2*(a*x*1i + 1)),x)
```

output
```
a*atanh((a^2*x^2 + 1)^(1/2))*1i - (a^2*x^2 + 1)^(1/2)/x
```

Reduce [F]

$$\int \frac{e^{-i\arctan(ax)}}{x^2}\,dx = \int \frac{\sqrt{a^2x^2+1}}{ai\,x^3+x^2}dx$$

input
```
int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^2,x)
```

output
```
int(sqrt(a**2*x**2 + 1)/(a*i*x**3 + x**2),x)
```

3.56 $\int \frac{e^{-i\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 63

$$\int \frac{e^{-i\arctan(ax)}}{x^3}\,dx = -\frac{\sqrt{1+a^2x^2}}{2x^2}+\frac{ia\sqrt{1+a^2x^2}}{x}+\frac{1}{2}a^2\text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

output
```
-1/2*(a^2*x^2+1)^(1/2)/x^2+I*a*(a^2*x^2+1)^(1/2)/x+1/2*a^2*arctanh((a^2*x^
2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{e^{-i\arctan(ax)}}{x^3}\,dx = \frac{1}{2}\left(\frac{(-1+2iax)\sqrt{1+a^2x^2}}{x^2}-a^2\log(x)+a^2\log\left(1+\sqrt{1+a^2x^2}\right)\right)$$

input
```
Integrate[1/(E^(I*ArcTan[a*x])*x^3),x]
```

output
```
(((-1 + (2*I)*a*x)*Sqrt[1 + a^2*x^2])/x^2 - a^2*Log[x] + a^2*Log[1 + Sqrt[
1 + a^2*x^2]])/2
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5583, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-i\arctan(ax)}}{x^3}dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{1-iax}{x^3\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{539}$$

$$-\frac{\sqrt{a^2x^2+1}}{2x^2}-\frac{1}{2}\int \frac{a(ax+2i)}{x^2\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{a^2x^2+1}}{2x^2}-\frac{1}{2}a\int \frac{ax+2i}{x^2\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{534}$$

$$-\frac{\sqrt{a^2x^2+1}}{2x^2}-\frac{1}{2}a\left(a\int \frac{1}{x\sqrt{a^2x^2+1}}dx-\frac{2i\sqrt{a^2x^2+1}}{x}\right)$$

$$\Big\downarrow \text{243}$$

$$-\frac{\sqrt{a^2x^2+1}}{2x^2}-\frac{1}{2}a\left(\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2x^2+1}}dx^2-\frac{2i\sqrt{a^2x^2+1}}{x}\right)$$

$$\Big\downarrow \text{73}$$

$$-\frac{\sqrt{a^2x^2+1}}{2x^2}-\frac{1}{2}a\left(\frac{\int \frac{1}{\frac{x^4}{a^2}-\frac{1}{a^2}}d\sqrt{a^2x^2+1}}{a}-\frac{2i\sqrt{a^2x^2+1}}{x}\right)$$

$$\Big\downarrow \text{221}$$

$$-\frac{\sqrt{a^2x^2+1}}{2x^2}-\frac{1}{2}a\left(-a\text{arctanh}\left(\sqrt{a^2x^2+1}\right)-\frac{2i\sqrt{a^2x^2+1}}{x}\right)$$

input
```
Int[1/(E^(I*ArcTan[a*x])*x^3),x]
```

output
```
-1/2*Sqrt[1 + a^2*x^2]/x^2 - (a*(((-2*I)*Sqrt[1 + a^2*x^2])/x - a*ArcTanh[
Sqrt[1 + a^2*x^2]]))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

method	result
risch	$\frac{i(2a^3x^3+ia^2x^2+2ax+i)}{2x^2\sqrt{a^2x^2+1}}+\frac{a^2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)}{2}$
default	$-\frac{(a^2x^2+1)^{\frac{3}{2}}}{2x^2}-\frac{a^2\left(\sqrt{a^2x^2+1}-\mathrm{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)\right)}{2}+a^2\left(\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}+\frac{ia\ln\left(\frac{ia+\left(x-\frac{i}{a}\right)a^2}{\sqrt{a^2}}\right.}{}\right.$

input
```
int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*I*(2*a^3*x^3+I*a^2*x^2+2*a*x+I)/x^2/(a^2*x^2+1)^(1/2)+1/2*a^2*arctanh(
1/(a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.32

$$\int\frac{e^{-i\arctan(ax)}}{x^3}\,dx$$
$$=\frac{a^2x^2\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-a^2x^2\log\left(-ax+\sqrt{a^2x^2+1}-1\right)+2i\,a^2x^2+\sqrt{a^2x^2+1}(2i\,ax-1)}{2\,x^2}$$

input
```
integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
1/2*(a^2*x^2*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - a^2*x^2*log(-a*x + sqrt(a
^2*x^2 + 1) - 1) + 2*I*a^2*x^2 + sqrt(a^2*x^2 + 1)*(2*I*a*x - 1))/x^2
```

Sympy [F]

$$\int \frac{e^{-i\arctan(ax)}}{x^3}\,dx = -i\int \frac{\sqrt{a^2x^2+1}}{ax^4-ix^3}\,dx$$

input
```
integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/x**3,x)
```

output
```
-I*Integral(sqrt(a**2*x**2 + 1)/(a*x**4 - I*x**3), x)
```

Maxima [F]

$$\int \frac{e^{-i\arctan(ax)}}{x^3}\,dx = \int \frac{\sqrt{a^2x^2+1}}{(i\,ax+1)x^3}\,dx$$

input
```
integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*x^2 + 1)/((I*a*x + 1)*x^3), x)
```

Giac [F]

$$\int \frac{e^{-i\arctan(ax)}}{x^3}\,dx = \int \frac{\sqrt{a^2x^2+1}}{(i\,ax+1)x^3}\,dx$$

input
```
integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^3,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 23.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{e^{-i\arctan(ax)}}{x^3}\,dx = \frac{a^2\,\mathrm{atanh}\left(\sqrt{a^2\,x^2+1}\right)}{2} - \frac{\sqrt{a^2\,x^2+1}}{2\,x^2} + \frac{a\,\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{x}$$

input
```
int((a^2*x^2 + 1)^(1/2)/(x^3*(a*x*1i + 1)),x)
```

output
```
(a^2*atanh((a^2*x^2 + 1)^(1/2)))/2 - (a^2*x^2 + 1)^(1/2)/(2*x^2) + (a*(a^2
*x^2 + 1)^(1/2)*1i)/x
```

Reduce [F]

$$\int \frac{e^{-i\arctan(ax)}}{x^3}\,dx = \int \frac{\sqrt{a^2x^2+1}}{ai\,x^4+x^3}dx$$

input
```
int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^3,x)
```

output
```
int(sqrt(a**2*x**2 + 1)/(a*i*x**4 + x**3),x)
```

3.57 $\int \frac{e^{-i \arctan(ax)}}{x^4} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 90

$$\int \frac{e^{-i \arctan(ax)}}{x^4} \, dx = -\frac{\sqrt{1+a^2x^2}}{3x^3} + \frac{ia\sqrt{1+a^2x^2}}{2x^2} + \frac{2a^2\sqrt{1+a^2x^2}}{3x} - \frac{1}{2}ia^3 \text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

output
```
-1/3*(a^2*x^2+1)^(1/2)/x^3+1/2*I*a*(a^2*x^2+1)^(1/2)/x^2+2/3*a^2*(a^2*x^2+
1)^(1/2)/x-1/2*I*a^3*arctanh((a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.78

$$\int \frac{e^{-i \arctan(ax)}}{x^4} \, dx = \frac{1}{6}\left(\frac{\sqrt{1+a^2x^2}(-2+3iax+4a^2x^2)}{x^3} + 3ia^3 \log(x) - 3ia^3 \log\left(1+\sqrt{1+a^2x^2}\right)\right)$$

input
```
Integrate[1/(E^(I*ArcTan[a*x])*x^4),x]
```

output
```
((Sqrt[1 + a^2*x^2]*(-2 + (3*I)*a*x + 4*a^2*x^2))/x^3 + (3*I)*a^3*Log[x] -
 (3*I)*a^3*Log[1 + Sqrt[1 + a^2*x^2]])/6
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5583, 539, 27, 539, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-i\arctan(ax)}}{x^4}dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{1-iax}{x^4\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{539}$$

$$-\frac{\sqrt{a^2x^2+1}}{3x^3}-\frac{1}{3}\int \frac{a(2ax+3i)}{x^3\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{a^2x^2+1}}{3x^3}-\frac{1}{3}a\int \frac{2ax+3i}{x^3\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{539}$$

$$-\frac{\sqrt{a^2x^2+1}}{3x^3}-\frac{1}{3}a\left(-\frac{1}{2}\int -\frac{a(4-3iax)}{x^2\sqrt{a^2x^2+1}}dx-\frac{3i\sqrt{a^2x^2+1}}{2x^2}\right)$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{a^2x^2+1}}{3x^3}-\frac{1}{3}a\left(\frac{1}{2}\int \frac{a(4-3iax)}{x^2\sqrt{a^2x^2+1}}dx-\frac{3i\sqrt{a^2x^2+1}}{2x^2}\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{a^2x^2+1}}{3x^3}-\frac{1}{3}a\left(\frac{1}{2}a\int \frac{4-3iax}{x^2\sqrt{a^2x^2+1}}dx-\frac{3i\sqrt{a^2x^2+1}}{2x^2}\right)$$

$$\Big\downarrow \; 534$$

$$-\frac{\sqrt{a^2x^2+1}}{3x^3}-\frac{1}{3}a\left(\frac{1}{2}a\left(-\frac{4\sqrt{a^2x^2+1}}{x}-3ia\int \frac{1}{x\sqrt{a^2x^2+1}}dx\right)-\frac{3i\sqrt{a^2x^2+1}}{2x^2}\right)$$

$$\Big\downarrow \; 243$$

$$-\frac{\sqrt{a^2x^2+1}}{3x^3}-\frac{1}{3}a\left(\frac{1}{2}a\left(-\frac{4\sqrt{a^2x^2+1}}{x}-\frac{3}{2}ia\int \frac{1}{x^2\sqrt{a^2x^2+1}}dx^2\right)-\frac{3i\sqrt{a^2x^2+1}}{2x^2}\right)$$

$$\Big\downarrow \; 73$$

$$-\frac{\sqrt{a^2x^2+1}}{3x^3}-\frac{1}{3}a\left(\frac{1}{2}a\left(-\frac{4\sqrt{a^2x^2+1}}{x}-\frac{3i\int \frac{1}{\frac{x^4}{a^2}-\frac{1}{a^2}}d\sqrt{a^2x^2+1}}{a}\right)-\frac{3i\sqrt{a^2x^2+1}}{2x^2}\right)$$

$$\Big\downarrow \; 221$$

$$-\frac{\sqrt{a^2x^2+1}}{3x^3}-\frac{1}{3}a\left(\frac{1}{2}a\left(-\frac{4\sqrt{a^2x^2+1}}{x}+3ia\text{arctanh}\left(\sqrt{a^2x^2+1}\right)\right)-\frac{3i\sqrt{a^2x^2+1}}{2x^2}\right)$$

input
```
Int[1/(E^(I*ArcTan[a*x])*x^4),x]
```

output
```
-1/3*Sqrt[1 + a^2*x^2]/x^3 - (a*((((-3*I)/2)*Sqrt[1 + a^2*x^2])/x^2 + (a*(
(-4*Sqrt[1 + a^2*x^2])/x + (3*I)*a*ArcTanh[Sqrt[1 + a^2*x^2]]))/2))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

method	result
risch	$\frac{4a^4x^4+3ia^3x^3+2a^2x^2+3iax-2}{6x^3\sqrt{a^2x^2+1}}-\frac{ia^3\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)}{2}$
default	$-\frac{(a^2x^2+1)^{\frac{3}{2}}}{3x^3}-ia\left(-\frac{(a^2x^2+1)^{\frac{3}{2}}}{2x^2}+\frac{a^2\left(\sqrt{a^2x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)\right)}{2}\right)-ia^3\left(\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\right.}\right.$

input
```
int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*(4*a^4*x^4+3*I*a^3*x^3+2*a^2*x^2+3*I*a*x-2)/x^3/(a^2*x^2+1)^(1/2)-1/2*
I*a^3*arctanh(1/(a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{e^{-i\arctan(ax)}}{x^4}\,dx = \frac{-3i\,a^3x^3\log\left(-ax+\sqrt{a^2x^2+1}+1\right)+3i\,a^3x^3\log\left(-ax+\sqrt{a^2x^2+1}-1\right)+4\,a^3x^3+\left(4\,a^2x^2+3i\,ax-\right.}{6\,x^3}$$

input
```
integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
1/6*(-3*I*a^3*x^3*log(-a*x + sqrt(a^2*x^2 + 1) + 1) + 3*I*a^3*x^3*log(-a*x
 + sqrt(a^2*x^2 + 1) - 1) + 4*a^3*x^3 + (4*a^2*x^2 + 3*I*a*x - 2)*sqrt(a^2
*x^2 + 1))/x^3
```

Sympy [F]

$$\int \frac{e^{-i\arctan(ax)}}{x^4}\,dx = -i\int \frac{\sqrt{a^2x^2+1}}{ax^5-ix^4}\,dx$$

input
```
integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/x**4,x)
```

output
```
-I*Integral(sqrt(a**2*x**2 + 1)/(a*x**5 - I*x**4), x)
```

Maxima [F]

$$\int \frac{e^{-i\arctan(ax)}}{x^4}\,dx = \int \frac{\sqrt{a^2x^2+1}}{(i\,ax+1)x^4}\,dx$$

input
```
integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*x^2 + 1)/((I*a*x + 1)*x^4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-i\arctan(ax)}}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.82

$$\int \frac{e^{-i\arctan(ax)}}{x^4}\,dx = \frac{2\,a^2\,\sqrt{a^2\,x^2+1}}{3\,x} - \frac{\sqrt{a^2\,x^2+1}}{3\,x^3} - \frac{a^3\,\mathrm{atan}\left(\sqrt{a^2\,x^2+1}\,1\mathrm{i}\right)}{2} + \frac{a\,\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{2\,x^2}$$

input
```
int((a^2*x^2 + 1)^(1/2)/(x^4*(a*x*1i + 1)),x)
```

output
```
(a*(a^2*x^2 + 1)^(1/2)*1i)/(2*x^2) - (a^2*x^2 + 1)^(1/2)/(3*x^3) - (a^3*at
an((a^2*x^2 + 1)^(1/2)*1i))/2 + (2*a^2*(a^2*x^2 + 1)^(1/2))/(3*x)
```

Reduce [F]

$$\int \frac{e^{-i\arctan(ax)}}{x^4}\,dx = \int \frac{\sqrt{a^2x^2+1}}{ai\,x^5+x^4}dx$$

input
```
int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^4,x)
```

output
```
int(sqrt(a**2*x**2 + 1)/(a*i*x**5 + x**4),x)
```

3.58 $\int \frac{e^{-i\arctan(ax)}}{x^5}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 113

$$\int \frac{e^{-i\arctan(ax)}}{x^5}\,dx = -\frac{\sqrt{1+a^2x^2}}{4x^4}+\frac{ia\sqrt{1+a^2x^2}}{3x^3}+\frac{3a^2\sqrt{1+a^2x^2}}{8x^2}-\frac{2ia^3\sqrt{1+a^2x^2}}{3x}-\frac{3}{8}a^4\text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

output
```
-1/4*(a^2*x^2+1)^(1/2)/x^4+1/3*I*a*(a^2*x^2+1)^(1/2)/x^3+3/8*a^2*(a^2*x^2+
1)^(1/2)/x^2-2/3*I*a^3*(a^2*x^2+1)^(1/2)/x-3/8*a^4*arctanh((a^2*x^2+1)^(1/
2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.67

$$\int \frac{e^{-i\arctan(ax)}}{x^5}\,dx = \frac{1}{24}\left(\frac{\sqrt{1+a^2x^2}(-6+8iax+9a^2x^2-16ia^3x^3)}{x^4}+9a^4\log(x)-9a^4\log\left(1+\sqrt{1+a^2x^2}\right)\right)$$

input
```
Integrate[1/(E^(I*ArcTan[a*x])*x^5),x]
```

output
```
((Sqrt[1 + a^2*x^2]*(-6 + (8*I)*a*x + 9*a^2*x^2 - (16*I)*a^3*x^3))/x^4 + 9
*a^4*Log[x] - 9*a^4*Log[1 + Sqrt[1 + a^2*x^2]])/24
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {5583, 539, 27, 539, 25, 27, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-i\arctan(ax)}}{x^5}dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{1-iax}{x^5\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{539}$$

$$-\frac{\sqrt{a^2x^2+1}}{4x^4}-\frac{1}{4}\int \frac{a(3ax+4i)}{x^4\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{a^2x^2+1}}{4x^4}-\frac{1}{4}a\int \frac{3ax+4i}{x^4\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{539}$$

$$-\frac{\sqrt{a^2x^2+1}}{4x^4}-\frac{1}{4}a\left(-\frac{1}{3}\int -\frac{a(9-8iax)}{x^3\sqrt{a^2x^2+1}}dx-\frac{4i\sqrt{a^2x^2+1}}{3x^3}\right)$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{a^2x^2+1}}{4x^4}-\frac{1}{4}a\left(\frac{1}{3}\int \frac{a(9-8iax)}{x^3\sqrt{a^2x^2+1}}dx-\frac{4i\sqrt{a^2x^2+1}}{3x^3}\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{a^2x^2+1}}{4x^4}-\frac{1}{4}a\left(\frac{1}{3}a\int \frac{9-8iax}{x^3\sqrt{a^2x^2+1}}dx-\frac{4i\sqrt{a^2x^2+1}}{3x^3}\right)$$

$$\Big\downarrow \; 539$$

$$-\frac{\sqrt{a^2x^2+1}}{4x^4}-\frac{1}{4}a\left(\frac{1}{3}a\left(-\frac{9\sqrt{a^2x^2+1}}{2x^2}-\frac{1}{2}\int\frac{a(9ax+16i)}{x^2\sqrt{a^2x^2+1}}dx\right)-\frac{4i\sqrt{a^2x^2+1}}{3x^3}\right)$$

$$\Big\downarrow \; 27$$

$$-\frac{\sqrt{a^2x^2+1}}{4x^4}-\frac{1}{4}a\left(\frac{1}{3}a\left(-\frac{9\sqrt{a^2x^2+1}}{2x^2}-\frac{1}{2}a\int\frac{9ax+16i}{x^2\sqrt{a^2x^2+1}}dx\right)-\frac{4i\sqrt{a^2x^2+1}}{3x^3}\right)$$

$$\Big\downarrow \; 534$$

$$-\frac{\sqrt{a^2x^2+1}}{4x^4}-\frac{1}{4}a\left(\frac{1}{3}a\left(-\frac{9\sqrt{a^2x^2+1}}{2x^2}-\frac{1}{2}a\left(9a\int\frac{1}{x\sqrt{a^2x^2+1}}dx-\frac{16i\sqrt{a^2x^2+1}}{x}\right)\right)-\frac{4i\sqrt{a^2x^2+1}}{3x^3}\right)$$

$$\Big\downarrow \; 243$$

$$-\frac{\sqrt{a^2x^2+1}}{4x^4}-\frac{1}{4}a\left(\frac{1}{3}a\left(-\frac{9\sqrt{a^2x^2+1}}{2x^2}-\frac{1}{2}a\left(\frac{9}{2}a\int\frac{1}{x^2\sqrt{a^2x^2+1}}dx^2-\frac{16i\sqrt{a^2x^2+1}}{x}\right)\right)-\frac{4i\sqrt{a^2x^2+1}}{3x^3}\right)$$

$$\Big\downarrow \; 73$$

$$-\frac{\sqrt{a^2x^2+1}}{4x^4}-\frac{1}{4}a\left(\frac{1}{3}a\left(-\frac{9\sqrt{a^2x^2+1}}{2x^2}-\frac{1}{2}a\left(\frac{9\int\frac{1}{\frac{x^4}{a^2}-\frac{1}{a^2}}d\sqrt{a^2x^2+1}}{a}-\frac{16i\sqrt{a^2x^2+1}}{x}\right)\right)-\frac{4i\sqrt{a^2x^2+1}}{3x^3}\right)$$

$$\Big\downarrow \; 221$$

$$-\frac{\sqrt{a^2x^2+1}}{4x^4}-\frac{1}{4}a\left(\frac{1}{3}a\left(-\frac{9\sqrt{a^2x^2+1}}{2x^2}-\frac{1}{2}a\left(-9a\text{arctanh}\left(\sqrt{a^2x^2+1}\right)-\frac{16i\sqrt{a^2x^2+1}}{x}\right)\right)-\frac{4i\sqrt{a^2x^2+1}}{3x^3}\right)$$

input
```
Int[1/(E^(I*ArcTan[a*x])*x^5),x]
```

output
```
-1/4*Sqrt[1 + a^2*x^2]/x^4 - (a*((((-4*I)/3)*Sqrt[1 + a^2*x^2])/x^3 + (a*(
(-9*Sqrt[1 + a^2*x^2])/(2*x^2) - (a*(((-16*I)*Sqrt[1 + a^2*x^2])/x - 9*a*A
rcTanh[Sqrt[1 + a^2*x^2]]))/2))/3))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{i\left(16 a^{5} x^{5}+9 i a^{4} x^{4}+8 a^{3} x^{3}+3 i a^{2} x^{2}-8 a x -6 i \right)}{24 x^{4} \sqrt{a^{2} x^{2}+1}}-\frac{3 a^{4} \operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2} x^{2}+1}}\right)}{8}$
default	$-\frac{\left(a^{2} x^{2}+1\right)^{\frac{3}{2}}}{4 x^{4}}-\frac{5 a^{2} \left(-\frac{\left(a^{2} x^{2}+1\right)^{\frac{3}{2}}}{2 x^{2}}+\frac{a^{2} \left(\sqrt{a^{2} x^{2}+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2} x^{2}+1}}\right)\right)}{2}\right)}{4}+a^{4} \left(\sqrt{a^{2} x^{2}+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2} x^{2}+1}}\right.\right.$

input
```
int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/24*I*(16*a^5*x^5+9*I*a^4*x^4+8*a^3*x^3+3*I*a^2*x^2-8*a*x-6*I)/x^4/(a^2*
x^2+1)^(1/2)-3/8*a^4*arctanh(1/(a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int \frac{e^{-i \arctan(ax)}}{x^5} dx = -\frac{9 a^4 x^4 \log\left(-ax+\sqrt{a^2x^2+1}+1\right)-9 a^4 x^4 \log\left(-ax+\sqrt{a^2x^2+1}-1\right)+16 i a^4 x^4-\left(-16 i a^3 x^3+9 a\right.}{24 x^4}$$

input
```
integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
-1/24*(9*a^4*x^4*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - 9*a^4*x^4*log(-a*x +
sqrt(a^2*x^2 + 1) - 1) + 16*I*a^4*x^4 - (-16*I*a^3*x^3 + 9*a^2*x^2 + 8*I*a
*x - 6)*sqrt(a^2*x^2 + 1))/x^4
```

Sympy [F]

$$\int \frac{e^{-i\arctan(ax)}}{x^5}\,dx = -i\int \frac{\sqrt{a^2x^2+1}}{ax^6-ix^5}\,dx$$

input
```
integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/x**5,x)
```

output
```
-I*Integral(sqrt(a**2*x**2 + 1)/(a*x**6 - I*x**5), x)
```

Maxima [F]

$$\int \frac{e^{-i\arctan(ax)}}{x^5}\,dx = \int \frac{\sqrt{a^2x^2+1}}{(i\,ax+1)x^5}\,dx$$

input
```
integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*x^2 + 1)/((I*a*x + 1)*x^5), x)
```

Giac [F]

$$\int \frac{e^{-i\arctan(ax)}}{x^5}\,dx = \int \frac{\sqrt{a^2x^2+1}}{(i\,ax+1)x^5}\,dx$$

input
```
integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^5,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.84

$$\int \frac{e^{-i\arctan(ax)}}{x^5}\,dx = \frac{a^4\,\mathrm{atan}\left(\sqrt{a^2\,x^2+1}\,1\mathrm{i}\right)\,3\mathrm{i}}{8} - \frac{\sqrt{a^2\,x^2+1}}{4\,x^4} + \frac{a\,\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{3\,x^3} + \frac{3\,a^2\,\sqrt{a^2\,x^2+1}}{8\,x^2} - \frac{a^3\,\sqrt{a^2\,x^2+1}\,2\mathrm{i}}{3\,x}$$

input
```
int((a^2*x^2 + 1)^(1/2)/(x^5*(a*x*1i + 1)),x)
```

output
```
(a^4*atan((a^2*x^2 + 1)^(1/2)*1i)*3i)/8 - (a^2*x^2 + 1)^(1/2)/(4*x^4) + (a
*(a^2*x^2 + 1)^(1/2)*1i)/(3*x^3) + (3*a^2*(a^2*x^2 + 1)^(1/2))/(8*x^2) - (
a^3*(a^2*x^2 + 1)^(1/2)*2i)/(3*x)
```

Reduce [F]

$$\int \frac{e^{-i\arctan(ax)}}{x^5}\,dx = \int \frac{\sqrt{a^2x^2+1}}{ai\,x^6+x^5}dx$$

input
```
int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/x^5,x)
```

output
```
int(sqrt(a**2*x***2 + 1)/(a*i*x***6 + x**5),x)
```

3.59 $\int e^{-2i\arctan(ax)}x^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 49

$$\int e^{-2i\arctan(ax)}x^3\,dx=\frac{2ix}{a^3}+\frac{x^2}{a^2}-\frac{2ix^3}{3a}-\frac{x^4}{4}-\frac{2\log(i-ax)}{a^4}$$

output
```
2*I*x/a^3+x^2/a^2-2/3*I*x^3/a-1/4*x^4-2*ln(I-a*x)/a^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int e^{-2i\arctan(ax)}x^3\,dx=\frac{2ix}{a^3}+\frac{x^2}{a^2}-\frac{2ix^3}{3a}-\frac{x^4}{4}-\frac{2\log(i-ax)}{a^4}$$

input
```
Integrate[x^3/E^((2*I)*ArcTan[a*x]),x]
```

output
```
((2*I)*x)/a^3 + x^2/a^2 - (((2*I)/3)*x^3)/a - x^4/4 - (2*Log[I - a*x])/a^4
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5585, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{-2i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{ 5585}$$

$$\int \frac{x^3(1-iax)}{1+iax}dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(-\frac{2}{a^3(ax-i)}+\frac{2i}{a^3}+\frac{2x}{a^2}-\frac{2ix^2}{a}-x^3\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2\log(-ax+i)}{a^4}+\frac{2ix}{a^3}+\frac{x^2}{a^2}-\frac{2ix^3}{3a}-\frac{x^4}{4}$$

input
```
Int[x^3/E^((2*I)*ArcTan[a*x]),x]
```

output
```
((2*I)*x)/a^3 + x^2/a^2 - (((2*I)/3)*x^3)/a - x^4/4 - (2*Log[I - a*x])/a^4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{\frac{1}{4}a^3x^4+\frac{2}{3}ia^2x^3-a\,x^2-2ix}{a^3}-\frac{2\ln(-ax+i)}{a^4}$	48
risch	$-\frac{x^4}{4}-\frac{2ix^3}{3a}+\frac{x^2}{a^2}+\frac{2ix}{a^3}-\frac{\ln(a^2x^2+1)}{a^4}-\frac{2i\arctan(ax)}{a^4}$	55
parallelrisch	$-\frac{-3a^5x^5-5ia^4x^4+4a^3x^3+24i+12ia^2x^2-24\ln(ax-i)xa+24i\ln(ax-i)}{12a^4(-ax+i)}$	73
meijerg	$-\frac{-\frac{ixa\left(-3a^4x^4-5ia^3x^3+10a^2x^2+30iax+60\right)}{12(iax+1)}+5\ln(iax+1)}{a^4}+\frac{-\frac{iax\left(2a^2x^2+6iax+12\right)}{4(iax+1)}+3\ln(iax+1)}{a^4}$	108

input
```
int(x^3/(1+I*a*x)^2*(a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/a^3*(1/4*a^3*x^4+2/3*I*a^2*x^3-a*x^2-2*I*x)-2*ln(I-a*x)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int e^{-2i\arctan(ax)}x^3\,dx=-\frac{3\,a^4x^4+8i\,a^3x^3-12\,a^2x^2-24i\,ax+24\,\log\left(\frac{ax-i}{a}\right)}{12\,a^4}$$

input
```
integrate(x^3/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="fricas")
```

output
```
-1/12*(3*a^4*x^4 + 8*I*a^3*x^3 - 12*a^2*x^2 - 24*I*a*x + 24*log((a*x - I)/
a))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int e^{-2i\arctan(ax)}x^3\,dx = -\frac{x^4}{4} - \frac{2ix^3}{3a} + \frac{x^2}{a^2} + \frac{2ix}{a^3} - \frac{2\log{(ax-i)}}{a^4}$$

input
```
integrate(x**3/(1+I*a*x)**2*(a**2*x**2+1),x)
```

output
```
-x**4/4 - 2*I*x**3/(3*a) + x**2/a**2 + 2*I*x/a**3 - 2*log(a*x - I)/a**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int e^{-2i\arctan(ax)}x^3\,dx = -\frac{i\,(-3i\,a^3x^4 + 8\,a^2x^3 + 12i\,ax^2 - 24\,x)}{12\,a^3} - \frac{2\,\log{(i\,ax+1)}}{a^4}$$

input
```
integrate(x^3/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="maxima")
```

output
```
-1/12*I*(-3*I*a^3*x^4 + 8*a^2*x^3 + 12*I*a*x^2 - 24*x)/a^3 - 2*log(I*a*x +
 1)/a^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.39

$$\int e^{-2i\arctan(ax)}x^3\,dx = \frac{(i\,ax+1)^4\left(\frac{20}{i\,ax+1} - \frac{54}{(i\,ax+1)^2} + \frac{84}{(i\,ax+1)^3} - 3\right)}{12\,a^4} + \frac{2\,\log\left(\frac{1}{\sqrt{a^2x^2+1}|a|}\right)}{a^4}$$

input
```
integrate(x^3/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="giac")
```

output
```
1/12*(I*a*x + 1)^4*(20/(I*a*x + 1) - 54/(I*a*x + 1)^2 + 84/(I*a*x + 1)^3 -
 3)/a^4 + 2*log(1/(sqrt(a^2*x^2 + 1)*abs(a)))/a^4
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int e^{-2i\arctan(ax)}x^3\,dx=\frac{x^2}{a^2}-\frac{x^4}{4}-\frac{2\,\ln\left(x-\frac{1\mathrm{i}}{a}\right)}{a^4}+\frac{x\,2\mathrm{i}}{a^3}-\frac{x^3\,2\mathrm{i}}{3\,a}$$

input
```
int((x^3*(a^2*x^2 + 1))/(a*x*1i + 1)^2,x)
```

output
```
(x*2i)/a^3 - (2*log(x - 1i/a))/a^4 - x^4/4 - (x^3*2i)/(3*a) + x^2/a^2
```

Reduce [F]

$$\int e^{-2i\arctan(ax)}x^3\,dx=-\left(\int\frac{x^5}{a^2x^2-2aix-1}dx\right)a^2-\left(\int\frac{x^3}{a^2x^2-2aix-1}dx\right)$$

input
```
int(x^3/(1+I*a*x)^2*(a^2*x^2+1),x)
```

output
```
- (int(x**5/(a**2*x**2 - 2*a*i*x - 1),x)*a**2 + int(x**3/(a**2*x**2 - 2*a
*i*x - 1),x))
```

3.60 $\int e^{-2i\arctan(ax)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 40

$$\int e^{-2i\arctan(ax)}x^2\,dx=\frac{2x}{a^2}-\frac{ix^2}{a}-\frac{x^3}{3}+\frac{2i\log(i-ax)}{a^3}$$

output
```
2*x/a^2-I*x^2/a-1/3*x^3+2*I*ln(I-a*x)/a^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int e^{-2i\arctan(ax)}x^2\,dx=\frac{2x}{a^2}-\frac{ix^2}{a}-\frac{x^3}{3}+\frac{2i\log(i-ax)}{a^3}$$

input
```
Integrate[x^2/E^((2*I)*ArcTan[a*x]),x]
```

output
```
(2*x)/a^2 - (I*x^2)/a - x^3/3 + ((2*I)*Log[I - a*x])/a^3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5585, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{-2i\arctan(ax)}\,dx$$

$$\Big\downarrow \ 5585$$

$$\int \frac{x^2(1-iax)}{1+iax}dx$$

$$\Big\downarrow \ 86$$

$$\int \left(\frac{2i}{a^2(ax-i)}+\frac{2}{a^2}-\frac{2ix}{a}-x^2\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2i\log(-ax+i)}{a^3}+\frac{2x}{a^2}-\frac{ix^2}{a}-\frac{x^3}{3}$$

input
```
Int[x^2/E^((2*I)*ArcTan[a*x]),x]
```

output
```
(2*x)/a^2 - (I*x^2)/a - x^3/3 + ((2*I)*Log[I - a*x])/a^3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{\frac{1}{3}a^2x^3+ia\,x^2-2x}{a^2}+\frac{2i\ln(-ax+i)}{a^3}$	40
risch	$-\frac{x^3}{3}-\frac{ix^2}{a}+\frac{2x}{a^2}+\frac{i\ln(a^2x^2+1)}{a^3}-\frac{2\arctan(ax)}{a^3}$	47
parallelrisch	$-\frac{-a^4x^4-2ia^3x^3+6+6i\ln(ax-i)xa+3a^2x^2+6\ln(ax-i)}{3a^3(-ax+i)}$	63
meijerg	$-\frac{i\left(\frac{ixa\left(-5ia^3x^3+10a^2x^2+30iax+60\right)}{15iax+15}-4\ln(iax+1)\right)}{a^3}+\frac{i\left(\frac{iax(3iax+6)}{3iax+3}-2\ln(iax+1)\right)}{a^3}$	95

input
```
int(x^2/(1+I*a*x)^2*(a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*(1/3*a^2*x^3+I*a*x^2-2*x)+2*I*ln(I-a*x)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int e^{-2i\arctan(ax)}x^2\,dx=-\frac{a^3x^3+3i\,a^2x^2-6\,ax-6i\,\log\left(\frac{ax-i}{a}\right)}{3\,a^3}$$

input
```
integrate(x^2/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="fricas")
```

output
```
-1/3*(a^3*x^3 + 3*I*a^2*x^2 - 6*a*x - 6*I*log((a*x - I)/a))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int e^{-2i\arctan(ax)}x^2\,dx = -\frac{x^3}{3} - \frac{ix^2}{a} + \frac{2x}{a^2} + \frac{2i\log(ax-i)}{a^3}$$

input
```
integrate(x**2/(1+I*a*x)**2*(a**2*x**2+1),x)
```

output
```
-x**3/3 - I*x**2/a + 2*x/a**2 + 2*I*log(a*x - I)/a**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int e^{-2i\arctan(ax)}x^2\,dx = -\frac{a^2x^3 + 3i\,ax^2 - 6\,x}{3\,a^2} + \frac{2i\,\log(i\,ax+1)}{a^3}$$

input
```
integrate(x^2/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="maxima")
```

output
```
-1/3*(a^2*x^3 + 3*I*a*x^2 - 6*x)/a^2 + 2*I*log(I*a*x + 1)/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int e^{-2i\arctan(ax)}x^2\,dx = \frac{i\,(i\,ax+1)^3\left(\frac{6}{i\,ax+1} - \frac{15}{(i\,ax+1)^2} - 1\right)}{3\,a^3} - \frac{2i\,\log\left(\frac{1}{\sqrt{a^2x^2+1}|a|}\right)}{a^3}$$

input
```
integrate(x^2/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="giac")
```

output
```
1/3*I*(I*a*x + 1)^3*(6/(I*a*x + 1) - 15/(I*a*x + 1)^2 - 1)/a^3 - 2*I*log(1
/(sqrt(a^2*x^2 + 1)*abs(a)))/a^3
```

Mupad [B] (verification not implemented)

Time = 23.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int e^{-2i\arctan(ax)}x^2\,dx = \frac{\ln\left(x-\frac{1\mathrm{i}}{a}\right)\,2\mathrm{i}}{a^3}+\frac{2\,x}{a^2}-\frac{x^3}{3}-\frac{x^2\,1\mathrm{i}}{a}$$

input
```
int((x^2*(a^2*x^2 + 1))/(a*x*1i + 1)^2,x)
```

output
```
(log(x - 1i/a)*2i)/a^3 + (2*x)/a^2 - x^3/3 - (x^2*1i)/a
```

Reduce [F]

$$\int e^{-2i\arctan(ax)}x^2\,dx = -\left(\int\frac{x^4}{a^2x^2-2aix-1}dx\right)a^2-\left(\int\frac{x^2}{a^2x^2-2aix-1}dx\right)$$

input
```
int(x^2/(1+I*a*x)^2*(a^2*x^2+1),x)
```

output
```
- (int(x**4/(a**2*x**2 - 2*a*i*x - 1),x)*a**2 + int(x**2/(a**2*x**2 - 2*a
*i*x - 1),x))
```

3.61 $\int e^{-2i\arctan(ax)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 30

$$\int e^{-2i\arctan(ax)}x\,dx=-\frac{2ix}{a}-\frac{x^2}{2}+\frac{2\log(i-ax)}{a^2}$$

output
```
-2*I*x/a-1/2*x^2+2*ln(I-a*x)/a^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int e^{-2i\arctan(ax)}x\,dx=-\frac{2ix}{a}-\frac{x^2}{2}+\frac{2\log(i-ax)}{a^2}$$

input
```
Integrate[x/E^((2*I)*ArcTan[a*x]),x]
```

output
```
((-2*I)*x)/a - x^2/2 + (2*Log[I - a*x])/a^2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5585, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-2i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{ 5585}$$

$$\int \frac{x(1-iax)}{1+iax}dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(\frac{2}{a(ax-i)}-\frac{2i}{a}-x\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2\log(-ax+i)}{a^2}-\frac{2ix}{a}-\frac{x^2}{2}$$

input
```
Int[x/E^((2*I)*ArcTan[a*x]),x]
```

output
```
((-2*I)*x)/a - x^2/2 + (2*Log[I - a*x])/a^2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

method	result	size
default	$-\frac{\frac{1}{2}a\,x^{2}+2ix}{a}+\frac{2\ln \left(-ax+i\right)}{a^{2}}$	31
risch	$-\frac{x^{2}}{2}-\frac{2ix}{a}+\frac{\ln \left(a^{2}x^{2}+1\right)}{a^{2}}+\frac{2i\arctan \left(ax\right)}{a^{2}}$	38
parallelrisch	$\frac{a^{3}x^{3}+3ia^{2}x^{2}-4\ln \left(ax-i\right)xa+4i\ln \left(ax-i\right)+4ax}{2a^{2}\left(-ax+i\right)}$	57
meijerg	$\frac{-\frac{iax\left(2a^{2}x^{2}+6iax+12\right)}{4\left(iax+1\right)}+3\ln \left(iax+1\right)}{a^{2}}-\frac{-\frac{iax}{iax+1}+\ln \left(iax+1\right)}{a^{2}}$	74

input
```
int(x/(1+I*a*x)^2*(a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/a*(1/2*a*x^2+2*I*x)+2*ln(I-a*x)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int e^{-2i\arctan(ax)}x\,dx=-\frac{a^{2}x^{2}+4i\,ax-4\,\log\left(\frac{ax-i}{a}\right)}{2\,a^{2}}$$

input
```
integrate(x/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="fricas")
```

output
```
-1/2*(a^2*x^2 + 4*I*a*x - 4*log((a*x - I)/a))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int e^{-2i\arctan(ax)}x\,dx = -\frac{x^2}{2} - \frac{2ix}{a} + \frac{2\log(ax-i)}{a^2}$$

input
```
integrate(x/(1+I*a*x)**2*(a**2*x**2+1),x)
```

output
```
-x**2/2 - 2*I*x/a + 2*log(a*x - I)/a**2
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int e^{-2i\arctan(ax)}x\,dx = \frac{i\,(i\,ax^2 - 4\,x)}{2\,a} + \frac{2\,\log(i\,ax+1)}{a^2}$$

input
```
integrate(x/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="maxima")
```

output
```
1/2*I*(I*a*x^2 - 4*x)/a + 2*log(I*a*x + 1)/a^2
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(24) = 48.

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.73

$$\int e^{-2i\arctan(ax)}x\,dx = -\frac{i\left(\frac{(i\,ax+1)^2\left(-\frac{6i}{i\,ax+1}+i\right)}{a} - \frac{4i\,\log\left(\frac{1}{\sqrt{a^2x^2+1}|a|}\right)}{a}\right)}{2\,a}$$

input
```
integrate(x/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="giac")
```

output
```
-1/2*I*((I*a*x + 1)^2*(-6*I/(I*a*x + 1) + I)/a - 4*I*log(1/(sqrt(a^2*x^2 +
 1)*abs(a)))/a)/a
```

Mupad [B] (verification not implemented)

Time = 22.89 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int e^{-2i\arctan(ax)}x\,dx = \frac{2\ln\left(x-\frac{1i}{a}\right)}{a^2} - \frac{x^2}{2} - \frac{x\,2i}{a}$$

input
```
int((x*(a^2*x^2 + 1))/(a*x*1i + 1)^2,x)
```

output
```
(2*log(x - 1i/a))/a^2 - (x*2i)/a - x^2/2
```

Reduce [F]

$$\int e^{-2i\arctan(ax)}x\,dx = -\left(\int\frac{x^3}{a^2x^2-2aix-1}dx\right)a^2 - \left(\int\frac{x}{a^2x^2-2aix-1}dx\right)$$

input
```
int(x/(1+I*a*x)^2*(a^2*x^2+1),x)
```

output
```
 - (int(x**3/(a**2*x**2 - 2*a*i*x - 1),x)*a**2 + int(x/(a**2*x**2 - 2*a*i*
x - 1),x))
```

3.62 $\int e^{-2i\arctan(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 20

$$\int e^{-2i\arctan(ax)}\,dx=-x-\frac{2i\log(i-ax)}{a}$$

output
```
-x-2*I*ln(I-a*x)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

$$\int e^{-2i\arctan(ax)}\,dx=-x+\frac{2\arctan(ax)}{a}-\frac{i\log(1+a^2x^2)}{a}$$

input
```
Integrate[E^((-2*I)*ArcTan[a*x]),x]
```

output
```
-x + (2*ArcTan[a*x])/a - (I*Log[1 + a^2*x^2])/a
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5584, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2i\arctan(ax)}dx$$

$$\Big\downarrow \text{ 5584}$$

$$\int \frac{1-iax}{1+iax}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(-1-\frac{2i}{ax-i}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-x-\frac{2i\log(-ax+i)}{a}$$

input
```
Int[E^((-2*I)*ArcTan[a*x]),x]
```

output
```
-x - ((2*I)*Log[I - a*x])/a
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5584
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$-x-\frac{2 i \ln \left(-a x +i\right)}{a}$	19
risch	$-x-\frac{i \ln \left(a^{2} x^{2}+1\right)}{a}+\frac{2 \arctan \left(a x \right)}{a}$	30
parallelrisch	$\frac{2 i \ln \left(a x -i\right) x a +a^{2} x^{2}+1+2 \ln \left(a x -i\right)}{a \left(-a x +i\right)}$	44
meijerg	$\frac{i \left(\frac{i a x \left(3 i a x +6\right)}{3 i a x +3}-2 \ln \left(i a x +1\right)\right)}{a}+\frac{x}{i a x +1}$	51

input
```
int(1/(1+I*a*x)^2*(a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-x-2*I*ln(I-a*x)/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int e^{-2 i \arctan (a x)} \, dx=-\frac{a x + 2 i \, \log \left(\frac{a x - i}{a}\right)}{a}$$

input
```
integrate(1/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="fricas")
```

output
```
-(a*x + 2*I*log((a*x - I)/a))/a
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int e^{-2i\arctan(ax)}\,dx = -x - \frac{2i\log(ax-i)}{a}$$

input
```
integrate(1/(1+I*a*x)**2*(a**2*x**2+1),x)
```

output
```
-x - 2*I*log(a*x - I)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int e^{-2i\arctan(ax)}\,dx = -x - \frac{2i\log(i\,ax+1)}{a}$$

input
```
integrate(1/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="maxima")
```

output
```
-x - 2*I*log(I*a*x + 1)/a
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(16) = 32$.

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.25

$$\int e^{-2i\arctan(ax)}\,dx = a^2\left(\frac{i\,(i\,ax+1)}{a^3} + \frac{2i\,\log\left(\frac{1}{\sqrt{a^2x^2+1}|a|}\right)}{a^3} - \frac{i}{(i\,ax+1)a^3}\right) + \frac{i}{(i\,ax+1)a}$$

input
```
integrate(1/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="giac")
```

output
```
a^2*(I*(I*a*x + 1)/a^3 + 2*I*log(1/(sqrt(a^2*x^2 + 1)*abs(a)))/a^3 - I/((I
*a*x + 1)*a^3)) + I/((I*a*x + 1)*a)
```

Mupad [B] (verification not implemented)

Time = 22.88 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int e^{-2i\arctan(ax)}\,dx = -x - \frac{\ln\left(x - \frac{1\mathrm{i}}{a}\right)\,2\mathrm{i}}{a}$$

input
```
int((a^2*x^2 + 1)/(a*x*1i + 1)^2,x)
```

output
```
- x - (log(x - 1i/a)*2i)/a
```

Reduce [F]

$$\int e^{-2i\arctan(ax)}\,dx = -\left(\int \frac{x^2}{a^2x^2 - 2aix - 1}dx\right)a^2 - \left(\int \frac{1}{a^2x^2 - 2aix - 1}dx\right)$$

input
```
int(1/(1+I*a*x)^2*(a^2*x^2+1),x)
```

output
```
- (int(x**2/(a**2*x**2 - 2*a*i*x - 1),x)*a**2 + int(1/(a**2*x**2 - 2*a*i*
x - 1),x))
```

3.63 $\int \frac{e^{-2i\arctan(ax)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{e^{-2i\arctan(ax)}}{x}\,dx = \log(x) - 2\log(i - ax)$$

output
```
ln(x)-2*ln(I-a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2i\arctan(ax)}}{x}\,dx = \log(x) - 2\log(i - ax)$$

input
```
Integrate[1/(E^((2*I)*ArcTan[a*x])*x),x]
```

output
```
Log[x] - 2*Log[I - a*x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5585, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2i\arctan(ax)}}{x}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{1-iax}{x(1+iax)}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{1}{x}-\frac{2a}{ax-i}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\log(x)-2\log(-ax+i)$$

input
```
Int[1/(E^((2*I)*ArcTan[a*x])*x),x]
```

output
```
Log[x] - 2*Log[I - a*x]
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

method	result	size
default	$\ln(x) - 2\ln(-ax + i)$	14
parallelrisch	$\frac{\ln(x)a - 2\ln(ax - i)a}{a}$	20
risch	$\ln(x) - \ln(a^2x^2 + 1) - 2i\arctan(ax)$	23
meijerg	$\frac{iax}{iax+1} - 2\ln(iax + 1) + 1 + \ln(x) + \ln(ia) - \frac{2iax}{2iax+2}$	48

input
```
int(1/(1+I*a*x)^2*(a^2*x^2+1)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)-2*ln(I-a*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{e^{-2i\arctan(ax)}}{x}\,dx = \log(x) - 2\,\log\left(\frac{ax - i}{a}\right)$$

input
```
integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x,x, algorithm="fricas")
```

output
```
log(x) - 2*log((a*x - I)/a)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \frac{e^{-2i\arctan(ax)}}{x}\,dx = \log(3ax) - 2\log(3ax - 3i)$$

input
```
integrate(1/(1+I*a*x)**2*(a**2*x**2+1)/x,x)
```

output
```
log(3*a*x) - 2*log(3*a*x - 3*I)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{e^{-2i\arctan(ax)}}{x}\,dx = -2\,\log(i\,ax + 1) + \log(x)$$

input
```
integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x,x, algorithm="maxima")
```

output
```
-2*log(I*a*x + 1) + log(x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(12) = 24.

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 3.14

$$\int \frac{e^{-2i\arctan(ax)}}{x}\,dx = i\,a\left(-\frac{i\,\log\left(\frac{i}{i\,ax+1} - i\right)}{a} - \frac{i\,\log\left(\frac{1}{\sqrt{a^2x^2+1}|a|}\right)}{a}\right)$$

input
```
integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x,x, algorithm="giac")
```

output
```
I*a*(-I*log(I/(I*a*x + 1) - I)/a - I*log(1/(sqrt(a^2*x^2 + 1)*abs(a)))/a)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2i\arctan(ax)}}{x}\,dx = \ln(x) - 2\ln\left(x - \frac{1\mathrm{i}}{a}\right)$$

input
```
int((a^2*x^2 + 1)/(x*(a*x*1i + 1)^2),x)
```

output
```
log(x) - 2*log(x - 1i/a)
```

Reduce [F]

$$\int \frac{e^{-2i\arctan(ax)}}{x}\,dx = -\left(\int \frac{x}{a^2x^2 - 2aix - 1}dx\right)a^2 - \left(\int \frac{1}{a^2x^3 - 2ai\,x^2 - x}dx\right)$$

input
```
int(1/(1+I*a*x)^2*(a^2*x^2+1)/x,x)
```

output
```
 - (int(x/(a**2*x**2 - 2*a*i*x - 1),x)*a**2 + int(1/(a**2*x**3 - 2*a*i*x**
2 - x),x))
```

3.64 $\int \frac{e^{-2i\arctan(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \frac{e^{-2i\arctan(ax)}}{x^2}\,dx=-\frac{1}{x}-2ia\log(x)+2ia\log(i-ax)$$

output
```
-1/x-2*I*a*ln(x)+2*I*a*ln(I-a*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2i\arctan(ax)}}{x^2}\,dx=-\frac{1}{x}-2ia\log(x)+2ia\log(i-ax)$$

input
```
Integrate[1/(E^((2*I)*ArcTan[a*x])*x^2),x]
```

output
```
-x^(-1) - (2*I)*a*Log[x] + (2*I)*a*Log[I - a*x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5585, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2i\arctan(ax)}}{x^2}dx$$
$$\Big\downarrow \text{5585}$$
$$\int \frac{1-iax}{x^2(1+iax)}dx$$
$$\Big\downarrow \text{86}$$
$$\int \left(\frac{2ia^2}{ax-i}-\frac{2ia}{x}+\frac{1}{x^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-2ia\log(x)+2ia\log(-ax+i)-\frac{1}{x}$$

input
```
Int[1/(E^((2*I)*ArcTan[a*x])*x^2),x]
```

output
```
-x^(-1) - (2*I)*a*Log[x] + (2*I)*a*Log[I - a*x]
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{1}{x}-2ia\ln\left(x\right)+2ia\ln\left(-ax+i\right)$	25
risch	$-\frac{1}{x}-2ia\ln\left(x\right)-2a\arctan\left(ax\right)+ia\ln\left(a^{2}x^{2}+1\right)$	34
parallelrisch	$-\frac{2ia^{2}\ln(x)x-2ia^{2}\ln(ax-i)x+a}{ax}$	34
meijerg	$\frac{a^{2}x}{iax+1}+ia\left(\frac{i}{xa}-1-2\ln\left(x\right)-2\ln\left(ia\right)+\frac{3iax}{3iax+3}+2\ln\left(iax+1\right)\right)$	66

input
```
int(1/(1+I*a*x)^2*(a^2*x^2+1)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x-2*I*a*ln(x)+2*I*a*ln(I-a*x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int\frac{e^{-2i\arctan(ax)}}{x^{2}}\,dx=\frac{-2i\,ax\log\left(x\right)+2i\,ax\log\left(\frac{ax-i}{a}\right)-1}{x}$$

input
```
integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^2,x, algorithm="fricas")
```

output
```
(-2*I*a*x*log(x) + 2*I*a*x*log((a*x - I)/a) - 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{e^{-2i\arctan(ax)}}{x^2}\,dx = -2a\big(i\log\left(4a^2x\right) - i\log\left(4a^2x - 4ia\right)\big) - \frac{1}{x}$$

input
```
integrate(1/(1+I*a*x)**2*(a**2*x**2+1)/x**2,x)
```

output
```
-2*a*(I*log(4*a**2*x) - I*log(4*a**2*x - 4*I*a)) - 1/x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{e^{-2i\arctan(ax)}}{x^2}\,dx = 2i\,a\log\left(i\,ax+1\right) - 2i\,a\log\left(x\right) - \frac{ax-i}{ax^2-i\,x}$$

input
```
integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^2,x, algorithm="maxima")
```

output
```
2*I*a*log(I*a*x + 1) - 2*I*a*log(x) - (a*x - I)/(a*x^2 - I*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{e^{-2i\arctan(ax)}}{x^2}\,dx = -2i\,a\log\left(\frac{i}{i\,ax+1} - i\right) - \frac{a}{\frac{i}{i\,ax+1} - i}$$

input
```
integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^2,x, algorithm="giac")
```

output
```
-2*I*a*log(I/(I*a*x + 1) - I) - a/(I/(I*a*x + 1) - I)
```

Mupad [B] (verification not implemented)

Time = 23.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{e^{-2i\arctan(ax)}}{x^2}\,dx = -4\,a\,\mathrm{atan}(2\,a\,x-\mathrm{i})-\frac{1}{x}$$

input
```
int((a^2*x^2 + 1)/(x^2*(a*x*1i + 1)^2),x)
```

output
```
- 4*a*atan(2*a*x - 1i) - 1/x
```

Reduce [F]

$$\int \frac{e^{-2i\arctan(ax)}}{x^2}\,dx = -\left(\int \frac{1}{a^2x^4-2ai\,x^3-x^2}dx\right)-\left(\int \frac{1}{a^2x^2-2aix-1}dx\right)a^2$$

input
```
int(1/(1+I*a*x)^2*(a^2*x^2+1)/x^2,x)
```

output
```
- (int(1/(a**2*x**4 - 2*a*i*x**3 - x**2),x) + int(1/(a**2*x**2 - 2*a*i*x
- 1),x)*a**2)
```

3.65 $\int \frac{e^{-2i\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 37

$$\int \frac{e^{-2i\arctan(ax)}}{x^3}\,dx = -\frac{1}{2x^2} + \frac{2ia}{x} - 2a^2\log(x) + 2a^2\log(i - ax)$$

output
```
-1/2/x^2+2*I*a/x-2*a^2*ln(x)+2*a^2*ln(I-a*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2i\arctan(ax)}}{x^3}\,dx = -\frac{1}{2x^2} + \frac{2ia}{x} - 2a^2\log(x) + 2a^2\log(i - ax)$$

input
```
Integrate[1/(E^((2*I)*ArcTan[a*x])*x^3),x]
```

output
```
-1/2*1/x^2 + ((2*I)*a)/x - 2*a^2*Log[x] + 2*a^2*Log[I - a*x]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5585, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2i\arctan(ax)}}{x^3}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{1-iax}{x^3(1+iax)}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{2a^3}{ax-i}-\frac{2a^2}{x}-\frac{2ia}{x^2}+\frac{1}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-2a^2\log(x)+2a^2\log(-ax+i)+\frac{2ia}{x}-\frac{1}{2x^2}$$

input
```
Int[1/(E^((2*I)*ArcTan[a*x])*x^3),x]
```

output
```
-1/2*1/x^2 + ((2*I)*a)/x - 2*a^2*Log[x] + 2*a^2*Log[I - a*x]
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result
default	$-\frac{1}{2x^2}+\frac{2ia}{x}-2a^2\ln(x)+2a^2\ln(-ax+i)$
risch	$\frac{2iax-\frac{1}{2}}{x^2}-2a^2\ln(-x)+2ia^2\arctan(ax)+a^2\ln(a^2x^2+1)$
parallelrisch	$-\frac{-4\ln(x)x^3a^3+4\ln(ax-i)x^3a^3+4i\ln(x)x^2a^2-4i\ln(ax-i)x^2a^2+i+4a^3x^3+3ax}{2(-ax+i)x^2}$
meijerg	$a^2\left(1+\ln(x)+\ln(ia)-\frac{2iax}{2iax+2}-\ln(iax+1)\right)-a^2\left(\frac{1}{2a^2x^2}-\frac{2i}{xa}+1+3\ln(x)+3\ln(ia)-\right.$

input
```
int(1/(1+I*a*x)^2*(a^2*x^2+1)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2+2*I*a/x-2*a^2*ln(x)+2*a^2*ln(I-a*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \frac{e^{-2i\arctan(ax)}}{x^3}\,dx = -\frac{4\,a^2x^2\log(x)-4\,a^2x^2\log\left(\frac{ax-i}{a}\right)-4i\,ax+1}{2\,x^2}$$

input
```
integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(4*a^2*x^2*log(x) - 4*a^2*x^2*log((a*x - I)/a) - 4*I*a*x + 1)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{e^{-2i\arctan(ax)}}{x^3}\,dx = -2a^2\left(\log\left(4a^3x\right) - \log\left(4a^3x - 4ia^2\right)\right) - \frac{-4iax+1}{2x^2}$$

input
```
integrate(1/(1+I*a*x)**2*(a**2*x**2+1)/x**3,x)
```

output
```
-2*a**2*(log(4*a**3*x) - log(4*a**3*x - 4*I*a**2)) - (-4*I*a*x + 1)/(2*x**
2)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int \frac{e^{-2i\arctan(ax)}}{x^3}\,dx = 2\,a^2\log\left(i\,ax+1\right) - 2\,a^2\log\left(x\right) - \frac{4\,a^2x^2 - 3i\,ax + 1}{2i\,ax^3 + 2\,x^2}$$

input
```
integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^3,x, algorithm="maxima")
```

output
```
2*a^2*log(I*a*x + 1) - 2*a^2*log(x) - (4*a^2*x^2 - 3*I*a*x + 1)/(2*I*a*x^3
 + 2*x^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46

$$\int \frac{e^{-2i\arctan(ax)}}{x^3}\,dx = -2\,a^2\log\left(\frac{i}{i\,ax+1} - i\right) + \frac{5\,a^2 - \frac{6\,a^2}{i\,ax+1}}{2\left(\frac{i}{i\,ax+1} - i\right)^2}$$

input
```
integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^3,x, algorithm="giac")
```

output
```
-2*a^2*log(I/(I*a*x + 1) - I) + 1/2*(5*a^2 - 6*a^2/(I*a*x + 1))/(I/(I*a*x
+ 1) - I)^2
```

Mupad [B] (verification not implemented)

Time = 23.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{e^{-2i\arctan(ax)}}{x^3}\,dx = a^2\,\mathrm{atan}(2\,a\,x - \mathrm{i})\;4\mathrm{i} + \frac{-\frac{1}{2} + a\,x\,2\mathrm{i}}{x^2}$$

input
```
int((a^2*x^2 + 1)/(x^3*(a*x*1i + 1)^2),x)
```

output
```
a^2*atan(2*a*x - 1i)*4i + (a*x*2i - 1/2)/x^2
```

Reduce [F]

$$\int \frac{e^{-2i\arctan(ax)}}{x^3}\,dx = -\left(\int \frac{1}{a^2x^5 - 2ai\,x^4 - x^3}dx\right) - \left(\int \frac{1}{a^2x^3 - 2ai\,x^2 - x}dx\right)a^2$$

input
```
int(1/(1+I*a*x)^2*(a^2*x^2+1)/x^3,x)
```

output
```
- (int(1/(a**2*x**5 - 2*a*i*x**4 - x**3),x) + int(1/(a**2*x**3 - 2*a*i*x*
*2 - x),x)*a**2)
```

3.66 $\int \frac{e^{-2i\arctan(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 49

$$\int \frac{e^{-2i\arctan(ax)}}{x^4}\,dx = -\frac{1}{3x^3} + \frac{ia}{x^2} + \frac{2a^2}{x} + 2ia^3\log(x) - 2ia^3\log(i-ax)$$

output
```
-1/3/x^3+I*a/x^2+2*a^2/x+2*I*a^3*ln(x)-2*I*a^3*ln(I-a*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2i\arctan(ax)}}{x^4}\,dx = -\frac{1}{3x^3} + \frac{ia}{x^2} + \frac{2a^2}{x} + 2ia^3\log(x) - 2ia^3\log(i-ax)$$

input
```
Integrate[1/(E^((2*I)*ArcTan[a*x])*x^4),x]
```

output
```
-1/3*1/x^3 + (I*a)/x^2 + (2*a^2)/x + (2*I)*a^3*Log[x] - (2*I)*a^3*Log[I -
a*x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5585, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2i\arctan(ax)}}{x^4}\,dx$$

$$\Big\downarrow \text{ 5585}$$

$$\int \frac{1-iax}{x^4(1+iax)}dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(-\frac{2ia^4}{ax-i}+\frac{2ia^3}{x}-\frac{2a^2}{x^2}-\frac{2ia}{x^3}+\frac{1}{x^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$2ia^3\log(x)-2ia^3\log(-ax+i)+\frac{2a^2}{x}+\frac{ia}{x^2}-\frac{1}{3x^3}$$

input
```
Int[1/(E^((2*I)*ArcTan[a*x])*x^4),x]
```

output
```
-1/3*1/x^3 + (I*a)/x^2 + (2*a^2)/x + (2*I)*a^3*Log[x] - (2*I)*a^3*Log[I -
a*x]
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

method	result
default	$-\frac{1}{3x^3}+\frac{ia}{x^2}+\frac{2a^2}{x}+2ia^3\ln(x)-2ia^3\ln(-ax+i)$
risch	$\frac{2a^2x^2+iax-\frac{1}{3}}{x^3}+2a^3\arctan(ax)-ia^3\ln(a^2x^2+1)+2ia^3\ln(-x)$
parallelrisch	$-\frac{6i\ln(x)x^4a^5-6i\ln(ax-i)x^4a^5+6\ln(x)x^3a^4-6\ln(ax-i)x^3a^4+6a^4x^3-3ia^3x^2+2a^2x+ia}{3a(-ax+i)x^3}$
meijerg	$ia^3\left(\frac{i}{xa}-1-2\ln(x)-2\ln(ia)+\frac{3iax}{3iax+3}+2\ln(iax+1)\right)-ia^3\left(-\frac{i}{3x^3a^3}-\frac{1}{a^2x^2}+\frac{3i}{xa}-1-\right.$

input
```
int(1/(1+I*a*x)^2*(a^2*x^2+1)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^3+I*a/x^2+2*a^2/x+2*I*a^3*ln(x)-2*I*a^3*ln(I-a*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{e^{-2i\arctan(ax)}}{x^4}\,dx=\frac{6i\,a^3x^3\log(x)-6i\,a^3x^3\log\left(\frac{ax-i}{a}\right)+6\,a^2x^2+3i\,ax-1}{3\,x^3}$$

input
```
integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^4,x, algorithm="fricas")
```

output
```
1/3*(6*I*a^3*x^3*log(x) - 6*I*a^3*x^3*log((a*x - I)/a) + 6*a^2*x^2 + 3*I*a
*x - 1)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10

$$\int \frac{e^{-2i\arctan(ax)}}{x^4}\,dx = -2a^3\left(-i\log\left(4a^4x\right)+i\log\left(4a^4x-4ia^3\right)\right)-\frac{-6a^2x^2-3iax+1}{3x^3}$$

input
```
integrate(1/(1+I*a*x)**2*(a**2*x**2+1)/x**4,x)
```

output
```
-2*a**3*(-I*log(4*a**4*x) + I*log(4*a**4*x - 4*I*a**3)) - (-6*a**2*x**2 -
3*I*a*x + 1)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int \frac{e^{-2i\arctan(ax)}}{x^4}\,dx = -2i\,a^3\log\left(i\,ax+1\right)+2i\,a^3\log\left(x\right)+\frac{6i\,a^3x^3+3\,a^2x^2+2i\,ax-1}{3i\,ax^4+3\,x^3}$$

input
```
integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^4,x, algorithm="maxima")
```

output
```
-2*I*a^3*log(I*a*x + 1) + 2*I*a^3*log(x) + (6*I*a^3*x^3 + 3*a^2*x^2 + 2*I*
a*x - 1)/(3*I*a*x^4 + 3*x^3)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.37

$$\int \frac{e^{-2i\arctan(ax)}}{x^4}\,dx = 2i\,a^3\log\left(\frac{i}{i\,ax+1}-i\right)-\frac{10\,a^3-\frac{24\,a^3}{i\,ax+1}+\frac{15\,a^3}{(i\,ax+1)^2}}{3\left(\frac{i}{i\,ax+1}-i\right)^3}$$

input
```
integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/x^4,x, algorithm="giac")
```

output
```
2*I*a^3*log(I/(I*a*x + 1) - I) - 1/3*(10*a^3 - 24*a^3/(I*a*x + 1) + 15*a^3
/(I*a*x + 1)^2)/(I/(I*a*x + 1) - I)^3
```

Mupad [B] (verification not implemented)

Time = 23.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

$$\int \frac{e^{-2i\arctan(ax)}}{x^4}\,dx = 4\,a^3\,\mathrm{atan}(2\,a\,x - \mathrm{i}) + \frac{2\,a^2\,x^2 + a\,x\,1\mathrm{i} - \frac{1}{3}}{x^3}$$

input
```
int((a^2*x^2 + 1)/(x^4*(a*x*1i + 1)^2),x)
```

output
```
4*a^3*atan(2*a*x - 1i) + (a*x*1i + 2*a^2*x^2 - 1/3)/x^3
```

Reduce [F]

$$\int \frac{e^{-2i\arctan(ax)}}{x^4}\,dx = -\left(\int \frac{1}{a^2x^6 - 2ai\,x^5 - x^4}dx\right) - \left(\int \frac{1}{a^2x^4 - 2ai\,x^3 - x^2}dx\right)a^2$$

input
```
int(1/(1+I*a*x)^2*(a^2*x^2+1)/x^4,x)
```

output
```
- (int(1/(a**2*x**6 - 2*a*i*x**5 - x**4),x) + int(1/(a**2*x**4 - 2*a*i*x*
*3 - x**2),x)*a**2)
```

3.67 $\int e^{-3i\arctan(ax)}x^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 124

$$\int e^{-3i\arctan(ax)}x^3\,dx=\frac{4(1-iax)}{a^4\sqrt{1+a^2x^2}}+\frac{7\sqrt{1+a^2x^2}}{a^4}-\frac{19ix\sqrt{1+a^2x^2}}{8a^3}+\frac{ix^3\sqrt{1+a^2x^2}}{4a}-\frac{(1+a^2x^2)^{3/2}}{a^4}+\frac{51i\operatorname{arcsinh}(ax)}{8a^4}$$

output
```
4*(1-I*a*x)/a^4/(a^2*x^2+1)^(1/2)+7*(a^2*x^2+1)^(1/2)/a^4-19/8*I*x*(a^2*x^
2+1)^(1/2)/a^3+1/4*I*x^3*(a^2*x^2+1)^(1/2)/a-(a^2*x^2+1)^(3/2)/a^4+51/8*I*
arcsinh(a*x)/a^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.65

$$\int e^{-3i\arctan(ax)}x^3\,dx=\sqrt{1+a^2x^2}\left(\frac{6}{a^4}-\frac{19ix}{8a^3}-\frac{x^2}{a^2}+\frac{ix^3}{4a}-\frac{4i}{a^4(-i+ax)}\right)+\frac{51i\operatorname{arcsinh}(ax)}{8a^4}$$

input
```
Integrate[x^3/E^((3*I)*ArcTan[a*x]),x]
```

output
```
Sqrt[1 + a^2*x^2]*(6/a^4 - (((19*I)/8)*x)/a^3 - x^2/a^2 + ((I/4)*x^3)/a -
(4*I)/(a^4*(-I + a*x))) + (((51*I)/8)*ArcSinh[a*x])/a^4
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.24, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.071$, Rules used = {5583, 2164, 25, 2027, 2164, 27, 563, 25, 2346, 2346, 27, 2346, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{-3i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{x^3(1-iax)^2}{(1+iax)\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{2164}$$

$$ia\int -\frac{\sqrt{a^2x^2+1}\left(x^4+\frac{ix^3}{a}\right)}{(iax+1)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-ia\int \frac{\sqrt{a^2x^2+1}\left(x^4+\frac{ix^3}{a}\right)}{(iax+1)^2}dx$$

$$\Big\downarrow \text{2027}$$

$$-ia\int \frac{x^3\left(x+\frac{i}{a}\right)\sqrt{a^2x^2+1}}{(iax+1)^2}dx$$

$$\Big\downarrow \text{2164}$$

$$a^2\int \frac{x^3\left(a^2x^2+1\right)^{3/2}}{a^2(iax+1)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^3\left(a^2x^2+1\right)^{3/2}}{(1+iax)^3}dx$$

$$\Big\downarrow \ 563$$

$$\frac{4\sqrt{a^2x^2+1}}{a^4(1+iax)}-\frac{i\int -\frac{a^4x^4+3ia^3x^3-4a^2x^2-4iax+4}{\sqrt{a^2x^2+1}}dx}{a^3}$$

$$\Big\downarrow \ 25$$

$$\frac{i\int \frac{a^4x^4+3ia^3x^3-4a^2x^2-4iax+4}{\sqrt{a^2x^2+1}}dx}{a^3}+\frac{4\sqrt{a^2x^2+1}}{a^4(1+iax)}$$

$$\Big\downarrow \ 2346$$

$$\frac{i\left(\frac{1}{4}a^2x^3\sqrt{a^2x^2+1}+\frac{\int \frac{12ix^3a^5-19x^2a^4-16ixa^3+16a^2}{\sqrt{a^2x^2+1}}dx}{4a^2}\right)}{a^3}+\frac{4\sqrt{a^2x^2+1}}{a^4(1+iax)}$$

$$\Big\downarrow \ 2346$$

$$\frac{i\left(\frac{1}{4}a^2x^3\sqrt{a^2x^2+1}+\frac{\frac{\int \frac{3\left(-19x^2a^6-24ixa^5+16a^4\right)}{\sqrt{a^2x^2+1}}dx}{3a^2}+4ia^3x^2\sqrt{a^2x^2+1}}{4a^2}\right)}{a^3}+\frac{4\sqrt{a^2x^2+1}}{a^4(1+iax)}$$

$$\Big\downarrow \ 27$$

$$\frac{i\left(\frac{1}{4}a^2x^3\sqrt{a^2x^2+1}+\frac{\frac{\int \frac{-19x^2a^6-24ixa^5+16a^4}{\sqrt{a^2x^2+1}}dx}{a^2}+4ia^3x^2\sqrt{a^2x^2+1}}{4a^2}\right)}{a^3}+\frac{4\sqrt{a^2x^2+1}}{a^4(1+iax)}$$

$$\Big\downarrow \ 2346$$

$$\frac{i\left(\frac{1}{4}a^2x^3\sqrt{a^2x^2+1}+\frac{\frac{-\frac{19}{2}a^4x\sqrt{a^2x^2+1}+\frac{\int \frac{3a^6(17-16iax)}{\sqrt{a^2x^2+1}}dx}{2a^2}}{a^2}+4ia^3x^2\sqrt{a^2x^2+1}}{4a^2}\right)}{a^3}+\frac{4\sqrt{a^2x^2+1}}{a^4(1+iax)}$$

$$\Big\downarrow \ 27$$

$$\frac{i\left(\frac{1}{4}a^2x^3\sqrt{a^2x^2+1}+\frac{\frac{-\frac{19}{2}a^4x\sqrt{a^2x^2+1}+\frac{3}{2}a^4\int \frac{17-16iax}{\sqrt{a^2x^2+1}}dx}{a^2}+4ia^3x^2\sqrt{a^2x^2+1}}{4a^2}\right)}{a^3}+\frac{4\sqrt{a^2x^2+1}}{a^4(1+iax)}$$

$$\Big\downarrow \ 455$$

$$\frac{i\left(\frac{1}{4}a^2x^3\sqrt{a^2x^2+1}+\frac{\frac{-\frac{19}{2}a^4x\sqrt{a^2x^2+1}+\frac{3}{2}a^4\left(17\int\frac{1}{\sqrt{a^2x^2+1}}dx-\frac{16i\sqrt{a^2x^2+1}}{a}\right)}{a^2}+4ia^3x^2\sqrt{a^2x^2+1}}{4a^2}\right)}{a^3}+\frac{4\sqrt{a^2x^2+1}}{a^4(1+iax)}$$

$$\Big\downarrow \text{222}$$

$$\frac{4\sqrt{a^2x^2+1}}{a^4(1+iax)}+\frac{i\left(\frac{1}{4}a^2x^3\sqrt{a^2x^2+1}+\frac{\frac{-\frac{19}{2}a^4x\sqrt{a^2x^2+1}+\frac{3}{2}a^4\left(\frac{17\text{arcsinh}(ax)}{a}-\frac{16i\sqrt{a^2x^2+1}}{a}\right)}{a^2}+4ia^3x^2\sqrt{a^2x^2+1}}{4a^2}\right)}{a^3}$$

input
```
Int[x^3/E^((3*I)*ArcTan[a*x]),x]
```

output
```
(4*Sqrt[1 + a^2*x^2])/(a^4*(1 + I*a*x)) + (I*((a^2*x^3*Sqrt[1 + a^2*x^2])/
4 + ((4*I)*a^3*x^2*Sqrt[1 + a^2*x^2] + ((-19*a^4*x*Sqrt[1 + a^2*x^2])/2 +
(3*a^4*(((-16*I)*Sqrt[1 + a^2*x^2])/a + (17*ArcSinh[a*x])/a))/2)/a^2)/(4*a
^2)))/a^3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2164
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[d*e   Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + b*d*x, x]*
(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x]
 && EqQ[b*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + b*d*x, x], 0
]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00

method	result
risch	$\frac{i(2a^3x^3+8ia^2x^2-19ax-48i)\sqrt{a^2x^2+1}}{8a^4}+\frac{i\left(-\frac{32\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{a^2\left(x-\frac{i}{a}\right)}+\frac{51\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{\sqrt{a^2}}\right)}{8a^3}$
default	$\frac{i\left(\frac{\left(a^2x^2+1\right)^{\frac{3}{2}}x}{4}+\frac{3\sqrt{a^2x^2+1}\,x}{8}+\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{8\sqrt{a^2}}\right)}{a^3}+\frac{\frac{i\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a\left(x-\frac{i}{a}\right)^3}-2ia\left(-\frac{i\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a\left(x-\frac{i}{a}\right)^2}+3ia\right.}{}$

input
```
int(x^3/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*I*(2*a^3*x^3+8*I*a^2*x^2-19*a*x-48*I)*(a^2*x^2+1)^(1/2)/a^4+1/8*I/a^3*
(-32/a^2/(x-I/a)*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2)+51*ln(a^2*x/(a^2)^(1/
2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.71

$$\int e^{-3i\arctan(ax)}x^3\,dx$$
$$=\frac{-32i\,ax-51\,(i\,ax+1)\log\left(-ax+\sqrt{a^2x^2+1}\right)+(2i\,a^4x^4-6\,a^3x^3-11i\,a^2x^2+29\,ax-80i)\sqrt{a^2x^2+1}}{8\,(a^5x-i\,a^4)}$$

input
```
integrate(x^3/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
1/8*(-32*I*a*x - 51*(I*a*x + 1)*log(-a*x + sqrt(a^2*x^2 + 1)) + (2*I*a^4*x
^4 - 6*a^3*x^3 - 11*I*a^2*x^2 + 29*a*x - 80*I)*sqrt(a^2*x^2 + 1) - 32)/(a^
5*x - I*a^4)
```

Sympy [F]

$$\int e^{-3i\arctan(ax)}x^3\,dx = i\left(\int \frac{x^3\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx+\int \frac{a^2x^5\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx\right)$$

input
```
integrate(x**3/(1+I*a*x)**3*(a**2*x**2+1)**(3/2),x)
```

output
```
I*(Integral(x**3*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2 - 3*a*x +
I), x) + Integral(a**2*x**5*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2
 - 3*a*x + I), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(100) = 200$.

Time = 0.11 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.74

$$\int e^{-3i\arctan(ax)}x^3\,dx = \frac{(a^2x^2+1)^{\frac{3}{2}}}{a^6x^2-2i\,a^5x-a^4}+\frac{3\,(a^2x^2+1)^{\frac{3}{2}}}{2i\,a^5x+2\,a^4}+\frac{6\sqrt{a^2x^2+1}}{i\,a^5x+a^4}+\frac{i\,(a^2x^2+1)^{\frac{3}{2}}x}{4\,a^3}+\frac{3i\sqrt{a^2x^2+1}x}{8\,a^3}-\frac{3i\sqrt{-a^2x^2+4i\,ax+3}x}{2\,a^3}-\frac{(a^2x^2+1)^{\frac{3}{2}}}{a^4}+\frac{3i\,\arcsin(i\,ax+2)}{2\,a^4}+\frac{63i\,\text{arsinh}(ax)}{8\,a^4}+\frac{9\sqrt{a^2x^2+1}}{2\,a^4}-\frac{3\sqrt{-a^2x^2+4i\,ax+3}}{a^4}$$

input
```
integrate(x^3/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
(a^2*x^2 + 1)^(3/2)/(a^6*x^2 - 2*I*a^5*x - a^4) + 3*(a^2*x^2 + 1)^(3/2)/(2
*I*a^5*x + 2*a^4) + 6*sqrt(a^2*x^2 + 1)/(I*a^5*x + a^4) + 1/4*I*(a^2*x^2 +
 1)^(3/2)*x/a^3 + 3/8*I*sqrt(a^2*x^2 + 1)*x/a^3 - 3/2*I*sqrt(-a^2*x^2 + 4*
I*a*x + 3)*x/a^3 - (a^2*x^2 + 1)^(3/2)/a^4 + 3/2*I*arcsin(I*a*x + 2)/a^4 +
 63/8*I*arcsinh(a*x)/a^4 + 9/2*sqrt(a^2*x^2 + 1)/a^4 - 3*sqrt(-a^2*x^2 + 4
*I*a*x + 3)/a^4
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3i\arctan(ax)}x^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.87 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.11

$$\int e^{-3i\arctan(ax)}x^3\,dx = \frac{\sqrt{a^2\,x^2+1}\,\left(\frac{4}{{\left(a^2\right)}^{3/2}}+\frac{2\,\sqrt{a^2}}{a^4}-\frac{x^2\,\sqrt{a^2}}{a^2}+\frac{x^3\,{\left(a^2\right)}^{3/2}\,1{}\mathrm{i}}{4\,a^3}-\frac{x\,\sqrt{a^2}\,19{}\mathrm{i}}{8\,a^3}\right)}{\sqrt{a^2}} + \frac{\mathrm{asinh}\left(x\,\sqrt{a^2}\right)\,51{}\mathrm{i}}{8\,a^3\,\sqrt{a^2}}+\frac{\sqrt{a^2\,x^2+1}\,4{}\mathrm{i}}{a^3\,\left(-x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1{}\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

input
```
int((x^3*(a^2*x^2 + 1)^(3/2))/(a*x*1i + 1)^3,x)
```

output
```
((a^2*x^2 + 1)^(1/2)*(4/(a^2)^(3/2) + (2*(a^2)^(1/2))/a^4 - (x^2*(a^2)^(1/
2))/a^2 + (x^3*(a^2)^(3/2)*1i)/(4*a^3) - (x*(a^2)^(1/2)*19i)/(8*a^3)))/(a^
2)^(1/2) + (asinh(x*(a^2)^(1/2))*51i)/(8*a^3*(a^2)^(1/2)) + ((a^2*x^2 + 1)
^(1/2)*4i)/(a^3*(((a^2)^(1/2)*1i)/a - x*(a^2)^(1/2))*(a^2)^(1/2))
```

Reduce [F]

$$\int e^{-3i\arctan(ax)}x^3\,dx = -\left(\int \frac{\sqrt{a^2x^2+1}\,x^5}{a^3i\,x^3+3a^2x^2-3aix-1}dx\right)a^2 - \left(\int \frac{\sqrt{a^2x^2+1}\,x^3}{a^3i\,x^3+3a^2x^2-3aix-1}dx\right)$$

input
```
int(x^3/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x)
```

output
```
 - (int((sqrt(a**2*x**2 + 1)*x**5)/(a**3*i*x**3 + 3*a**2*x**2 - 3*a*i*x -
1),x)*a**2 + int((sqrt(a**2*x**2 + 1)*x**3)/(a**3*i*x**3 + 3*a**2*x**2 - 3
*a*i*x - 1),x))
```

3.68 $\int e^{-3i\arctan(ax)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 103

$$\int e^{-3i\arctan(ax)}x^2\,dx = -\frac{4i(1-iax)}{a^3\sqrt{1+a^2x^2}}-\frac{5i\sqrt{1+a^2x^2}}{a^3}-\frac{3x\sqrt{1+a^2x^2}}{2a^2}+\frac{i(1+a^2x^2)^{3/2}}{3a^3}+\frac{11\text{arcsinh}(ax)}{2a^3}$$

output
```
-4*I*(1-I*a*x)/a^3/(a^2*x^2+1)^(1/2)-5*I*(a^2*x^2+1)^(1/2)/a^3-3/2*x*(a^2*
x^2+1)^(1/2)/a^2+1/3*I*(a^2*x^2+1)^(3/2)/a^3+11/2*arcsinh(a*x)/a^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.61

$$\int e^{-3i\arctan(ax)}x^2\,dx = \frac{\frac{\sqrt{1+a^2x^2}(-52-19iax-7a^2x^2+2ia^3x^3)}{-i+ax}+33\text{arcsinh}(ax)}{6a^3}$$

input
```
Integrate[x^2/E^((3*I)*ArcTan[a*x]),x]
```

output
```
((Sqrt[1 + a^2*x^2]*(-52 - (19*I)*a*x - 7*a^2*x^2 + (2*I)*a^3*x^3))/(-I +
a*x) + 33*ArcSinh[a*x])/(6*a^3)
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {5583, 2164, 25, 2027, 2164, 27, 563, 2346, 2346, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{-3i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{ 5583}$$

$$\int \frac{x^2(1-iax)^2}{(1+iax)\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{ 2164}$$

$$ia\int -\frac{\sqrt{a^2x^2+1}\left(x^3+\frac{ix^2}{a}\right)}{(iax+1)^2}dx$$

$$\Big\downarrow \text{ 25}$$

$$-ia\int \frac{\sqrt{a^2x^2+1}\left(x^3+\frac{ix^2}{a}\right)}{(iax+1)^2}dx$$

$$\Big\downarrow \text{ 2027}$$

$$-ia\int \frac{x^2\left(x+\frac{i}{a}\right)\sqrt{a^2x^2+1}}{(iax+1)^2}dx$$

$$\Big\downarrow \text{ 2164}$$

$$a^2\int \frac{x^2\left(a^2x^2+1\right)^{3/2}}{a^2(iax+1)^3}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x^2\left(a^2x^2+1\right)^{3/2}}{(1+iax)^3}dx$$

$$\Big\downarrow \text{ 563}$$

$$\frac{\int \frac{ia^3x^3-3a^2x^2-4iax+4}{\sqrt{a^2x^2+1}}dx}{a^2}-\frac{4i\sqrt{a^2x^2+1}}{a^3(1+iax)}$$

$$\Big\downarrow \; 2346$$

$$\frac{\frac{\int \frac{-9x^2a^4-14ixa^3+12a^2}{\sqrt{a^2x^2+1}}dx}{3a^2}+\frac{1}{3}iax^2\sqrt{a^2x^2+1}}{a^2}-\frac{4i\sqrt{a^2x^2+1}}{a^3(1+iax)}$$

$$\Big\downarrow \; 2346$$

$$\frac{\frac{-\frac{9}{2}a^2x\sqrt{a^2x^2+1}+\frac{\int \frac{a^4(33-28iax)}{\sqrt{a^2x^2+1}}dx}{2a^2}}{3a^2}+\frac{1}{3}iax^2\sqrt{a^2x^2+1}}{a^2}-\frac{4i\sqrt{a^2x^2+1}}{a^3(1+iax)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{-\frac{9}{2}a^2x\sqrt{a^2x^2+1}+\frac{1}{2}a^2\int \frac{33-28iax}{\sqrt{a^2x^2+1}}dx}{3a^2}+\frac{1}{3}iax^2\sqrt{a^2x^2+1}}{a^2}-\frac{4i\sqrt{a^2x^2+1}}{a^3(1+iax)}$$

$$\Big\downarrow \; 455$$

$$\frac{\frac{-\frac{9}{2}a^2x\sqrt{a^2x^2+1}+\frac{1}{2}a^2\left(33\int \frac{1}{\sqrt{a^2x^2+1}}dx-\frac{28i\sqrt{a^2x^2+1}}{a}\right)}{3a^2}+\frac{1}{3}iax^2\sqrt{a^2x^2+1}}{a^2}-\frac{4i\sqrt{a^2x^2+1}}{a^3(1+iax)}$$

$$\Big\downarrow \; 222$$

$$\frac{\frac{-\frac{9}{2}a^2x\sqrt{a^2x^2+1}+\frac{1}{2}a^2\left(\frac{33\text{arcsinh}(ax)}{a}-\frac{28i\sqrt{a^2x^2+1}}{a}\right)}{3a^2}+\frac{1}{3}iax^2\sqrt{a^2x^2+1}}{a^2}-\frac{4i\sqrt{a^2x^2+1}}{a^3(1+iax)}$$

input
```
Int[x^2/E^((3*I)*ArcTan[a*x]),x]
```

output
```
((-4*I)*Sqrt[1 + a^2*x^2])/(a^3*(1 + I*a*x)) + ((I/3)*a*x^2*Sqrt[1 + a^2*x
^2] + ((-9*a^2*x*Sqrt[1 + a^2*x^2])/2 + (a^2*(((-28*I)*Sqrt[1 + a^2*x^2])/
a + (33*ArcSinh[a*x])/a))/2)/(3*a^2))/a^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2164
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[d*e   Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + b*d*x, x]*
(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x]
 && EqQ[b*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + b*d*x, x], 0
]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.08

method	result
risch	$\frac{i(2a^2x^2+9iax-28)\sqrt{a^2x^2+1}}{6a^3}-\frac{4\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{a^4\left(x-\frac{i}{a}\right)}+\frac{11\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2a^2\sqrt{a^2}}$
default	$-\frac{i\left(\frac{i\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a\left(x-\frac{i}{a}\right)^3}-2ia\left(-\frac{i\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a\left(x-\frac{i}{a}\right)^2}+3ia\left(\frac{\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}}}{3}+ia\left(\frac{\left(2\left(x-\frac{i}{a}\right)a^2+2ia\right)\sqrt{(\cdots}}{4a\cdots}\right.\right.\right.\right.}{a^5}$

input
```
int(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*I*(2*a^2*x^2+9*I*a*x-28)*(a^2*x^2+1)^(1/2)/a^3-4/a^4/(x-I/a)*((x-I/a)^
2*a^2+2*I*a*(x-I/a))^(1/2)+11/2/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2)
)/(a^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

$$\int e^{-3i\arctan(ax)}x^2\,dx = -\frac{24\,ax+33\,(ax-i)\log\left(-ax+\sqrt{a^2x^2+1}\right)-\left(2i\,a^3x^3-7\,a^2x^2-19i\,ax-52\right)\sqrt{a^2x^2+1}-24i}{6\,(a^4x-i\,a^3)}$$

input
```
integrate(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(24*a*x + 33*(a*x - I)*log(-a*x + sqrt(a^2*x^2 + 1)) - (2*I*a^3*x^3 -
 7*a^2*x^2 - 19*I*a*x - 52)*sqrt(a^2*x^2 + 1) - 24*I)/(a^4*x - I*a^3)
```

Sympy [F]

$$\int e^{-3i\arctan(ax)}x^2\,dx = i\left(\int\frac{x^2\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx+\int\frac{a^2x^4\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx\right)$$

input
```
integrate(x**2/(1+I*a*x)**3*(a**2*x**2+1)**(3/2),x)
```

output
```
I*(Integral(x**2*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2 - 3*a*x +
I), x) + Integral(a**2*x**4*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2
 - 3*a*x + I), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(80) = 160$.

Time = 0.11 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.76

$$\int e^{-3i\arctan(ax)}x^2\,dx = -\frac{i\,(a^2x^2+1)^{\frac{3}{2}}}{a^5x^2-2i\,a^4x-a^3} - \frac{i\,(a^2x^2+1)^{\frac{3}{2}}}{i\,a^4x+a^3} - \frac{6i\,\sqrt{a^2x^2+1}}{i\,a^4x+a^3} - \frac{\sqrt{-a^2x^2+4i\,ax+3}\,x}{2\,a^2} + \frac{i\,(a^2x^2+1)^{\frac{3}{2}}}{3\,a^3} + \frac{\arcsin(i\,ax+2)}{2\,a^3} + \frac{6\,\mathrm{arsinh}\,(ax)}{a^3} - \frac{3i\,\sqrt{a^2x^2+1}}{a^3} + \frac{i\,\sqrt{-a^2x^2+4i\,ax+3}}{a^3}$$

input
```
integrate(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
-I*(a^2*x^2 + 1)^(3/2)/(a^5*x^2 - 2*I*a^4*x - a^3) - I*(a^2*x^2 + 1)^(3/2)
/(I*a^4*x + a^3) - 6*I*sqrt(a^2*x^2 + 1)/(I*a^4*x + a^3) - 1/2*sqrt(-a^2*x
^2 + 4*I*a*x + 3)*x/a^2 + 1/3*I*(a^2*x^2 + 1)^(3/2)/a^3 + 1/2*arcsin(I*a*x
 + 2)/a^3 + 6*arcsinh(a*x)/a^3 - 3*I*sqrt(a^2*x^2 + 1)/a^3 + I*sqrt(-a^2*x
^2 + 4*I*a*x + 3)/a^3
```

Giac [F]

$$\int e^{-3i\arctan(ax)}x^2\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}x^2}{(i\,ax+1)^3}\,dx$$

input
```
integrate(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 22.78 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.12

$$\int e^{-3i\arctan(ax)}x^2\,dx = \frac{11\,\mathrm{asinh}\left(x\sqrt{a^2}\right)}{2\,a^2\sqrt{a^2}} - \frac{\sqrt{a^2\,x^2+1}\left(\frac{3\,x\sqrt{a^2}}{2\,a^2}+\frac{a\,14\mathrm{i}}{3\,(a^2)^{3/2}}-\frac{a^3\,x^2\,1\mathrm{i}}{3\,(a^2)^{3/2}}\right)}{\sqrt{a^2}} + \frac{4\sqrt{a^2\,x^2+1}}{a^2\left(-x\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\sqrt{a^2}}$$

input
```
int((x^2*(a^2*x^2 + 1)^(3/2))/(a*x*1i + 1)^3,x)
```

output
```
(11*asinh(x*(a^2)^(1/2)))/(2*a^2*(a^2)^(1/2)) - ((a^2*x^2 + 1)^(1/2)*((a*1
4i)/(3*(a^2)^(3/2)) - (a^3*x^2*1i)/(3*(a^2)^(3/2)) + (3*x*(a^2)^(1/2))/(2*
a^2)))/(a^2)^(1/2) + (4*(a^2*x^2 + 1)^(1/2))/(a^2*(((a^2)^(1/2)*1i)/a - x*
(a^2)^(1/2))*(a^2)^(1/2))
```

Reduce [F]

$$\int e^{-3i\arctan(ax)}x^2\,dx = -\left(\int \frac{\sqrt{a^2x^2+1}\,x^4}{a^3i\,x^3+3a^2x^2-3aix-1}dx\right)a^2 - \left(\int \frac{\sqrt{a^2x^2+1}\,x^2}{a^3i\,x^3+3a^2x^2-3aix-1}dx\right)$$

input
```
int(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x)
```

output
```
- (int((sqrt(a**2*x**2 + 1)*x**4)/(a**3*i*x**3 + 3*a**2*x**2 - 3*a*i*x -
1),x)*a**2 + int((sqrt(a**2*x**2 + 1)*x**2)/(a**3*i*x**3 + 3*a**2*x**2 - 3
*a*i*x - 1),x))
```

3.69 $\int e^{-3i\arctan(ax)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 83

$$\int e^{-3i\arctan(ax)}x\,dx=-\frac{(1-iax)^3}{a^2\sqrt{1+a^2x^2}}-\frac{6\sqrt{1+a^2x^2}}{a^2}+\frac{3ix\sqrt{1+a^2x^2}}{2a}-\frac{9i\text{arcsinh}(ax)}{2a^2}$$

output
```
-(1-I*a*x)^3/a^2/(a^2*x^2+1)^(1/2)-6*(a^2*x^2+1)^(1/2)/a^2+3/2*I*x*(a^2*x^
2+1)^(1/2)/a-9/2*I*arcsinh(a*x)/a^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int e^{-3i\arctan(ax)}x\,dx=\sqrt{1+a^2x^2}\left(-\frac{3}{a^2}+\frac{ix}{2a}+\frac{4i}{a^2(-i+ax)}\right)-\frac{9i\text{arcsinh}(ax)}{2a^2}$$

input
```
Integrate[x/E^((3*I)*ArcTan[a*x]),x]
```

output
```
Sqrt[1 + a^2*x^2]*(-3/a^2 + ((I/2)*x)/a + (4*I)/(a^2*(-I + a*x))) - (((9*I
)/2)*ArcSinh[a*x])/a^2
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5583, 2164, 25, 2027, 2164, 27, 563, 25, 2346, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-3i\arctan(ax)}dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{x(1-iax)^2}{(1+iax)\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{2164}$$

$$ia\int -\frac{\left(x^2+\frac{ix}{a}\right)\sqrt{a^2x^2+1}}{(iax+1)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-ia\int \frac{\left(x^2+\frac{ix}{a}\right)\sqrt{a^2x^2+1}}{(iax+1)^2}dx$$

$$\Big\downarrow \text{2027}$$

$$-ia\int \frac{x\left(x+\frac{i}{a}\right)\sqrt{a^2x^2+1}}{(iax+1)^2}dx$$

$$\Big\downarrow \text{2164}$$

$$a^2\int \frac{x\left(a^2x^2+1\right)^{3/2}}{a^2(iax+1)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x\left(a^2x^2+1\right)^{3/2}}{(1+iax)^3}dx$$

$$\Big\downarrow \text{563}$$

$$\frac{i\int -\frac{-a^2x^2-3iax+4}{\sqrt{a^2x^2+1}}dx}{a}-\frac{4\sqrt{a^2x^2+1}}{a^2(1+iax)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{i\int \frac{-a^2x^2-3iax+4}{\sqrt{a^2x^2+1}}dx}{a}-\frac{4\sqrt{a^2x^2+1}}{a^2(1+iax)}$$

$$\Big\downarrow \; 2346$$

$$-\frac{i\left(-\frac{1}{2}x\sqrt{a^2x^2+1}+\frac{\int \frac{3a^2(3-2iax)}{\sqrt{a^2x^2+1}}dx}{2a^2}\right)}{a}-\frac{4\sqrt{a^2x^2+1}}{a^2(1+iax)}$$

$$\Big\downarrow \; 27$$

$$-\frac{i\left(-\frac{1}{2}x\sqrt{a^2x^2+1}+\frac{3}{2}\int \frac{3-2iax}{\sqrt{a^2x^2+1}}dx\right)}{a}-\frac{4\sqrt{a^2x^2+1}}{a^2(1+iax)}$$

$$\Big\downarrow \; 455$$

$$-\frac{i\left(-\frac{1}{2}x\sqrt{a^2x^2+1}+\frac{3}{2}\left(3\int \frac{1}{\sqrt{a^2x^2+1}}dx-\frac{2i\sqrt{a^2x^2+1}}{a}\right)\right)}{a}-\frac{4\sqrt{a^2x^2+1}}{a^2(1+iax)}$$

$$\Big\downarrow \; 222$$

$$-\frac{i\left(-\frac{1}{2}x\sqrt{a^2x^2+1}+\frac{3}{2}\left(\frac{3\text{arcsinh}(ax)}{a}-\frac{2i\sqrt{a^2x^2+1}}{a}\right)\right)}{a}-\frac{4\sqrt{a^2x^2+1}}{a^2(1+iax)}$$

input
```
Int[x/E^((3*I)*ArcTan[a*x]),x]
```

output
```
(-4*Sqrt[1 + a^2*x^2])/(a^2*(1 + I*a*x)) - (I*(-1/2*(x*Sqrt[1 + a^2*x^2])
+ (3*(((-2*I)*Sqrt[1 + a^2*x^2])/a + (3*ArcSinh[a*x])/a))/2))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 563
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*
b^(n + 2)*(c + d*x))), x] - Simp[d^(2*n - m + 2)/b^(n + 1)   Int[(1/Sqrt[a
+ b*x^2])*ExpandToSum[(2^(-n - 1)*(-c)^(m - n - 1) - d^m*x^m*(-c + d*x)^(-n
 - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2
, 0] && IGtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2164
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Simp[d*e   Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + b*d*x, x]*
(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x]
 && EqQ[b*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + b*d*x, x], 0
]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.28

method	result
risch	$\frac{i(ax+6i)\sqrt{a^2x^2+1}}{2a^2}-\frac{i\left(-\frac{8\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{a^2\left(x-\frac{i}{a}\right)}+\frac{9\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{\sqrt{a^2}}\right)}{2a}$
default	$\frac{i\left(-\frac{i\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a\left(x-\frac{i}{a}\right)^2}+3ia\left(\frac{\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}}}{3}+ia\left(\frac{\left(2\left(x-\frac{i}{a}\right)a^2+2ia\right)\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{4a^2}+\frac{\ln\left(\frac{ia+\left(x-\frac{i}{a}\right)a}{\sqrt{a^2}}\right.}{}\right.\right.\right.}{a^3}$

input
```
int(x/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*(a*x+6*I)*(a^2*x^2+1)^(1/2)/a^2-1/2*I/a*(-8/a^2/(x-I/a)*((x-I/a)^2*a
^2+2*I*a*(x-I/a))^(1/2)+9*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1
/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int e^{-3i\arctan(ax)}x\,dx = \frac{8i\,ax-9\,(-i\,ax-1)\log\left(-ax+\sqrt{a^2x^2+1}\right)+\sqrt{a^2x^2+1}(i\,a^2x^2-5\,ax+14i)+8}{2\,(a^3x-i\,a^2)}$$

input
```
integrate(x/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(8*I*a*x - 9*(-I*a*x - 1)*log(-a*x + sqrt(a^2*x^2 + 1)) + sqrt(a^2*x^2
 + 1)*(I*a^2*x^2 - 5*a*x + 14*I) + 8)/(a^3*x - I*a^2)
```

Sympy [F]

$$\int e^{-3i\arctan(ax)}x\,dx = i\left(\int \frac{x\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx + \int \frac{a^2x^3\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx\right)$$

input
```
integrate(x/(1+I*a*x)**3*(a**2*x**2+1)**(3/2),x)
```

output
```
I*(Integral(x*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2 - 3*a*x + I),
 x) + Integral(a**2*x**3*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2 -
3*a*x + I), x))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

$$\int e^{-3i\arctan(ax)}x\,dx = -\frac{(a^2x^2+1)^{\frac{3}{2}}}{a^4x^2-2i\,a^3x-a^2} - \frac{(a^2x^2+1)^{\frac{3}{2}}}{2i\,a^3x+2\,a^2} - \frac{6\sqrt{a^2x^2+1}}{i\,a^3x+a^2} - \frac{9i\,\mathrm{arsinh}\,(ax)}{2\,a^2} - \frac{3\sqrt{a^2x^2+1}}{2\,a^2}$$

input
```
integrate(x/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
-(a^2*x^2 + 1)^(3/2)/(a^4*x^2 - 2*I*a^3*x - a^2) - (a^2*x^2 + 1)^(3/2)/(2*
I*a^3*x + 2*a^2) - 6*sqrt(a^2*x^2 + 1)/(I*a^3*x + a^2) - 9/2*I*arcsinh(a*x
)/a^2 - 3/2*sqrt(a^2*x^2 + 1)/a^2
```

Giac [F]

$$\int e^{-3i\arctan(ax)}x\,dx=\int\frac{(a^2x^2+1)^{\frac{3}{2}}x}{(i\,ax+1)^3}\,dx$$

input
```
integrate(x/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.27

$$\int e^{-3i\arctan(ax)}x\,dx=-\frac{\sqrt{a^2\,x^2+1}\,\left(\frac{3\,\sqrt{a^2}}{a^2}-\frac{x\,\sqrt{a^2}\,1\mathrm{i}}{2\,a}\right)}{\sqrt{a^2}}-\frac{\mathrm{asinh}\left(x\,\sqrt{a^2}\right)\,9\mathrm{i}}{2\,a\,\sqrt{a^2}}-\frac{\sqrt{a^2\,x^2+1}\,4\mathrm{i}}{a\,\left(-x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

input
```
int((x*(a^2*x^2 + 1)^(3/2))/(a*x*1i + 1)^3,x)
```

output
```
- ((a^2*x^2 + 1)^(1/2)*((3*(a^2)^(1/2))/a^2 - (x*(a^2)^(1/2)*1i)/(2*a)))/(
a^2)^(1/2) - (asinh(x*(a^2)^(1/2))*9i)/(2*a*(a^2)^(1/2)) - ((a^2*x^2 + 1)^
(1/2)*4i)/(a*(((a^2)^(1/2)*1i)/a - x*(a^2)^(1/2))*(a^2)^(1/2))
```

Reduce [F]

$$\int e^{-3i\arctan(ax)}x\,dx=-\left(\int\frac{\sqrt{a^2x^2+1}\,x^3}{a^3i\,x^3+3a^2x^2-3aix-1}dx\right)a^2-\left(\int\frac{\sqrt{a^2x^2+1}\,x}{a^3i\,x^3+3a^2x^2-3aix-1}dx\right)$$

input
```
int(x/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x)
```

output
```
 - (int((sqrt(a**2*x**2 + 1)*x**3)/(a**3*i*x**3 + 3*a**2*x**2 - 3*a*i*x -
1),x)*a**2 + int((sqrt(a**2*x**2 + 1)*x)/(a**3*i*x**3 + 3*a**2*x**2 - 3*a*
i*x - 1),x))
```

3.70 $\int e^{-3i\arctan(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 58

$$\int e^{-3i\arctan(ax)}\,dx=\frac{4i(1-iax)}{a\sqrt{1+a^2x^2}}+\frac{i\sqrt{1+a^2x^2}}{a}-\frac{3\mathrm{arcsinh}(ax)}{a}$$

output
```
4*I*(1-I*a*x)/a/(a^2*x^2+1)^(1/2)+I*(a^2*x^2+1)^(1/2)/a-3*arcsinh(a*x)/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.72

$$\int e^{-3i\arctan(ax)}\,dx=\frac{\sqrt{1+a^2x^2}\left(i+\frac{4}{-i+ax}\right)}{a}-\frac{3\mathrm{arcsinh}(ax)}{a}$$

input
```
Integrate[E^((-3*I)*ArcTan[a*x]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(I + 4/(-I + a*x)))/a - (3*ArcSinh[a*x])/a
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5582, 711, 25, 27, 671, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-3i\arctan(ax)}dx$$

$$\Big\downarrow \text{5582}$$

$$\int \frac{(1-iax)^2}{(1+iax)\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{711}$$

$$\frac{i\sqrt{a^2x^2+1}}{a}-\frac{\int -\frac{a^4(1-3iax)}{(iax+1)\sqrt{a^2x^2+1}}dx}{a^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a^4(1-3iax)}{(iax+1)\sqrt{a^2x^2+1}}dx}{a^4}+\frac{i\sqrt{a^2x^2+1}}{a}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1-3iax}{(iax+1)\sqrt{a^2x^2+1}}dx+\frac{i\sqrt{a^2x^2+1}}{a}$$

$$\Big\downarrow \text{671}$$

$$-3\int \frac{1}{\sqrt{a^2x^2+1}}dx+\frac{i\sqrt{a^2x^2+1}}{a}+\frac{4i\sqrt{a^2x^2+1}}{a(1+iax)}$$

$$\Big\downarrow \text{222}$$

$$\frac{i\sqrt{a^2x^2+1}}{a}+\frac{4i\sqrt{a^2x^2+1}}{a(1+iax)}-\frac{3\text{arcsinh}(ax)}{a}$$

input

```
Int[E^((-3*I)*ArcTan[a*x]),x]
```

output
```
(I*Sqrt[1 + a^2*x^2])/a + ((4*I)*Sqrt[1 + a^2*x^2])/(a*(1 + I*a*x)) - (3*A
rcSinh[a*x])/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 711
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + c*x^2)^(p + 1)
/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m + n + 2*p + 1))
Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c*e^n*(m + n + 2*p + 1)*(f + g*x)
^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - 2*e*g^n*(m + p + n)*(d + e*x)^(n
 - 2)*(a*e - c*d*x), x], x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && Eq
Q[c*d^2 + a*e^2, 0] && IGtQ[n, 0] && NeQ[m + n + 2*p + 1, 0]
```

rule 5582
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 - I*a*x)^((I*n + 1)/2)
/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2]), x] /; FreeQ[a, x] && Intege
rQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.60

method	result
risch	$\frac{i\sqrt{a^2x^2+1}}{a}+\frac{4\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{a^2\left(x-\frac{i}{a}\right)}-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{\sqrt{a^2}}$
default	$\frac{i\left(\frac{i\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a\left(x-\frac{i}{a}\right)^3}-2ia\left(-\frac{i\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a\left(x-\frac{i}{a}\right)^2}+3ia\left(\frac{\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}}}{3}+ia\left(\frac{\left(2\left(x-\frac{i}{a}\right)a^2+2ia\right)\sqrt{\left(x-\right.}}{4a^2}\right.\right.\right.\right.}{a^3}$

input
```
int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
I*(a^2*x^2+1)^(1/2)/a+4/a^2/(x-I/a)*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2)-3*
ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int e^{-3i\arctan(ax)}\,dx = \frac{4\,ax+3\,(ax-i)\log\left(-ax+\sqrt{a^2x^2+1}\right)+\sqrt{a^2x^2+1}(i\,ax+5)-4i}{a^2x-i\,a}$$

input
```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
(4*a*x + 3*(a*x - I)*log(-a*x + sqrt(a^2*x^2 + 1)) + sqrt(a^2*x^2 + 1)*(I*
a*x + 5) - 4*I)/(a^2*x - I*a)
```

Sympy [F]

$$\int e^{-3i\arctan(ax)}\,dx = i\left(\int \frac{\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx + \int \frac{a^2x^2\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx\right)$$

input
```
integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2),x)
```

output
```
I*(Integral(sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2 - 3*a*x + I), x
) + Integral(a**2*x**2*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2 - 3*
a*x + I), x))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12

$$\int e^{-3i\arctan(ax)}\,dx = \frac{i\,(a^2x^2+1)^{\frac{3}{2}}}{a^3x^2-2i\,a^2x-a} - \frac{3\,\operatorname{arsinh}(ax)}{a} + \frac{6i\,\sqrt{a^2x^2+1}}{i\,a^2x+a}$$

input
```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
I*(a^2*x^2 + 1)^(3/2)/(a^3*x^2 - 2*I*a^2*x - a) - 3*arcsinh(a*x)/a + 6*I*s
qrt(a^2*x^2 + 1)/(I*a^2*x + a)
```

Giac [F]

$$\int e^{-3i\arctan(ax)}\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}}{(i\,ax+1)^3}\,dx$$

input
```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 22.71 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.26

$$\int e^{-3i\arctan(ax)}\,dx = \frac{\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{a} - \frac{3\,\mathrm{asinh}\left(x\,\sqrt{a^2}\right)}{\sqrt{a^2}} - \frac{4\,\sqrt{a^2\,x^2+1}}{\left(-x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

input
```
int((a^2*x^2 + 1)^(3/2)/(a*x*1i + 1)^3,x)
```

output
```
((a^2*x^2 + 1)^(1/2)*1i)/a - (3*asinh(x*(a^2)^(1/2)))/(a^2)^(1/2) - (4*(a^
2*x^2 + 1)^(1/2))/((((a^2)^(1/2)*1i)/a - x*(a^2)^(1/2))*(a^2)^(1/2))
```

Reduce [F]

$$\int e^{-3i\arctan(ax)}\,dx = -\left(\int \frac{\sqrt{a^2x^2+1}}{a^3i\,x^3+3a^2x^2-3aix-1}dx\right) - \left(\int \frac{\sqrt{a^2x^2+1}\,x^2}{a^3i\,x^3+3a^2x^2-3aix-1}dx\right)a^2$$

input
```
int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x)
```

output
```
- (int(sqrt(a**2*x**2 + 1)/(a**3*i*x**3 + 3*a**2*x**2 - 3*a*i*x - 1),x) +
 int((sqrt(a**2*x**2 + 1)*x**2)/(a**3*i*x**3 + 3*a**2*x**2 - 3*a*i*x - 1),
x)*a**2)
```

3.71 $\int \frac{e^{-3i\arctan(ax)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 48

$$\int \frac{e^{-3i\arctan(ax)}}{x}\,dx = \frac{4(1-iax)}{\sqrt{1+a^2x^2}} + i\text{arcsinh}(ax) - \text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

output
```
4*(1-I*a*x)/(a^2*x^2+1)^(1/2)+I*arcsinh(a*x)-arctanh((a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int \frac{e^{-3i\arctan(ax)}}{x}\,dx = -\frac{4i\sqrt{1+a^2x^2}}{-i+ax} + i\text{arcsinh}(ax) + \log(x) - \log\left(1+\sqrt{1+a^2x^2}\right)$$

input
```
Integrate[1/(E^((3*I)*ArcTan[a*x])*x),x]
```

output
```
((-4*I)*Sqrt[1 + a^2*x^2])/(-I + a*x) + I*ArcSinh[a*x] + Log[x] - Log[1 +
Sqrt[1 + a^2*x^2]]
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5583, 2351, 564, 25, 243, 73, 221, 671, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3i\arctan(ax)}}{x} \, dx$$
$$\Big\downarrow \text{5583}$$
$$\int \frac{(1-iax)^2}{x(1+iax)\sqrt{a^2x^2+1}}dx$$
$$\Big\downarrow \text{2351}$$
$$\int \frac{1}{x(iax+1)\sqrt{a^2x^2+1}}dx+\int \frac{-xa^2-2ia}{(iax+1)\sqrt{a^2x^2+1}}dx$$
$$\Big\downarrow \text{564}$$
$$-\int -\frac{1}{x\sqrt{a^2x^2+1}}dx+\int \frac{-xa^2-2ia}{(iax+1)\sqrt{a^2x^2+1}}dx+\frac{\sqrt{a^2x^2+1}}{1+iax}$$
$$\Big\downarrow \text{25}$$
$$\int \frac{1}{x\sqrt{a^2x^2+1}}dx+\int \frac{-xa^2-2ia}{(iax+1)\sqrt{a^2x^2+1}}dx+\frac{\sqrt{a^2x^2+1}}{1+iax}$$
$$\Big\downarrow \text{243}$$
$$\frac{1}{2}\int \frac{1}{x^2\sqrt{a^2x^2+1}}dx^2+\int \frac{-xa^2-2ia}{(iax+1)\sqrt{a^2x^2+1}}dx+\frac{\sqrt{a^2x^2+1}}{1+iax}$$
$$\Big\downarrow \text{73}$$
$$\int \frac{-xa^2-2ia}{(iax+1)\sqrt{a^2x^2+1}}dx+\frac{\int \frac{1}{\frac{x^4}{a^2}-\frac{1}{a^2}}d\sqrt{a^2x^2+1}}{a^2}+\frac{\sqrt{a^2x^2+1}}{1+iax}$$
$$\Big\downarrow \text{221}$$
$$\int \frac{-xa^2-2ia}{(iax+1)\sqrt{a^2x^2+1}}dx-\operatorname{arctanh}\left(\sqrt{a^2x^2+1}\right)+\frac{\sqrt{a^2x^2+1}}{1+iax}$$

$$\Big\downarrow \text{671}$$

$$ia\int \frac{1}{\sqrt{a^2x^2+1}}dx - \text{arctanh}\left(\sqrt{a^2x^2+1}\right) + \frac{4\sqrt{a^2x^2+1}}{1+iax}$$

$$\Big\downarrow \text{222}$$

$$-\text{arctanh}\left(\sqrt{a^2x^2+1}\right) + \frac{4\sqrt{a^2x^2+1}}{1+iax} + i\text{arcsinh}(ax)$$

input
```
Int[1/(E^((3*I)*ArcTan[a*x])*x),x]
```

output
```
(4*Sqrt[1 + a^2*x^2])/(1 + I*a*x) + I*ArcSinh[a*x] - ArcTanh[Sqrt[1 + a^2*
x^2]]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 564
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(-(-c)^(m - n - 2))*d^(2*n - m + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b
^(n + 2)*(c + d*x))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(x^m/Sqrt[a + b
*x^2])*ExpandToSum[((2^(-n - 1)*(-c)^(m - n - 1))/(d^m*x^m) - (-c + d*x)^(-
n - 1))/(c + d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^
2, 0] && ILtQ[m, 0] && ILtQ[n, 0] && EqQ[n + p, -3/2]
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(42) = 84$.

Time = 0.19 (sec) , antiderivative size = 649, normalized size of antiderivative = 13.52

method	result
default	$\frac{(a^2x^2+1)^{\frac{3}{2}}}{3}+\sqrt{a^2x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)+\frac{\frac{i\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a\left(x-\frac{i}{a}\right)^3}-2ia\left(-\frac{i\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a\left(x-\frac{i}{a}\right)^2}\right.}{}$

input
```
int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*(a^2*x^2+1)^(3/2)+(a^2*x^2+1)^(1/2)-arctanh(1/(a^2*x^2+1)^(1/2))+1/a^2
*(I/a/(x-I/a)^3*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(5/2)-2*I*a*(-I/a/(x-I/a)^2*
((x-I/a)^2*a^2+2*I*a*(x-I/a))^(5/2)+3*I*a*(1/3*((x-I/a)^2*a^2+2*I*a*(x-I/a
))^(3/2)+I*a*(1/4*(2*(x-I/a)*a^2+2*I*a)/a^2*((x-I/a)^2*a^2+2*I*a*(x-I/a))^
(1/2)+1/2*ln((I*a+(x-I/a)*a^2)/(a^2)^(1/2)+((x-I/a)^2*a^2+2*I*a*(x-I/a))^(
1/2))/(a^2)^(1/2)))))+I/a*(-I/a/(x-I/a)^2*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(5
/2)+3*I*a*(1/3*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(3/2)+I*a*(1/4*(2*(x-I/a)*a^2
+2*I*a)/a^2*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2)+1/2*ln((I*a+(x-I/a)*a^2)/(
a^2)^(1/2)+((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(1/2))))-1/3*((x-I/a
)^2*a^2+2*I*a*(x-I/a))^(3/2)-I*a*(1/4*(2*(x-I/a)*a^2+2*I*a)/a^2*((x-I/a)^2
*a^2+2*I*a*(x-I/a))^(1/2)+1/2*ln((I*a+(x-I/a)*a^2)/(a^2)^(1/2)+((x-I/a)^2*
a^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(40) = 80$.

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.08

$$\int \frac{e^{-3i\arctan(ax)}}{x}\,dx$$
$$= \frac{-4i\,ax - (ax-i)\log\left(-ax+\sqrt{a^2x^2+1}+1\right) + (-i\,ax-1)\log\left(-ax+\sqrt{a^2x^2+1}\right) + (ax-i)\log\left(-a\right.}{ax-i}$$

input
```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x,x, algorithm="fricas")
```

output
```
(-4*I*a*x - (a*x - I)*log(-a*x + sqrt(a^2*x^2 + 1) + 1) + (-I*a*x - 1)*log
(-a*x + sqrt(a^2*x^2 + 1)) + (a*x - I)*log(-a*x + sqrt(a^2*x^2 + 1) - 1) -
 4*I*sqrt(a^2*x^2 + 1) - 4)/(a*x - I)
```

Sympy [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x}\,dx = i\left(\int \frac{\sqrt{a^2x^2+1}}{a^3x^4-3ia^2x^3-3ax^2+ix}\,dx + \int \frac{a^2x^2\sqrt{a^2x^2+1}}{a^3x^4-3ia^2x^3-3ax^2+ix}\,dx\right)$$

input
```
integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/x,x)
```

output
```
I*(Integral(sqrt(a**2*x**2 + 1)/(a**3*x**4 - 3*I*a**2*x**3 - 3*a*x**2 + I*
x), x) + Integral(a**2*x**2*sqrt(a**2*x**2 + 1)/(a**3*x**4 - 3*I*a**2*x**3
 - 3*a*x**2 + I*x), x))
```

Maxima [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x}\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}}{(i\,ax+1)^3x}\,dx$$

input
```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate((a^2*x^2 + 1)^(3/2)/((I*a*x + 1)^3*x), x)
```

Giac [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x}\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}}{(i\,ax+1)^3x}\,dx$$

input
```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 22.88 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.54

$$\int \frac{e^{-3i\arctan(ax)}}{x}\,dx$$
$$= -\operatorname{atanh}\left(\sqrt{a^2x^2+1}\right) + \frac{a\operatorname{asinh}\left(x\sqrt{a^2}\right)\,1\mathrm{i}}{\sqrt{a^2}} + \frac{a\sqrt{a^2x^2+1}\,4\mathrm{i}}{\left(-x\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\sqrt{a^2}}$$

input
```
int((a^2*x^2 + 1)^(3/2)/(x*(a*x*1i + 1)^3),x)
```

output
```
(a*asinh(x*(a^2)^(1/2))*1i)/(a^2)^(1/2) - atanh((a^2*x^2 + 1)^(1/2)) + (a*
(a^2*x^2 + 1)^(1/2)*4i)/((((a^2)^(1/2)*1i)/a - x*(a^2)^(1/2))*(a^2)^(1/2))
```

Reduce [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x}\,dx = \frac{\sqrt{a^2x^2+1}}{3}$$
$$-3\left(\int -\frac{\sqrt{a^2x^2+1}}{a^5x^5-3a^4i\,x^4-2a^3x^3-2a^2i\,x^2-3ax+i}dx\right)a$$
$$+\frac{\left(\int -\frac{\sqrt{a^2x^2+1}\,x^4}{a^5x^5-3a^4i\,x^4-2a^3x^3-2a^2i\,x^2-3ax+i}dx\right)a^5}{3}$$
$$-\left(\int -\frac{\sqrt{a^2x^2+1}\,x^3}{a^5x^5-3a^4i\,x^4-2a^3x^3-2a^2i\,x^2-3ax+i}dx\right)a^4i$$
$$-\frac{11\left(\int -\frac{\sqrt{a^2x^2+1}\,x}{a^5x^5-3a^4i\,x^4-2a^3x^3-2a^2i\,x^2-3ax+i}dx\right)a^2i}{3}$$
$$-\left(\int \frac{\sqrt{a^2x^2+1}\,x}{a^3i\,x^3+3a^2x^2-3aix-1}dx\right)a^2$$
$$+\frac{\log\left(\sqrt{a^2x^2+1}-1\right)}{2}-\frac{\log\left(\sqrt{a^2x^2+1}+1\right)}{2}$$

input
```
int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x,x)
```

output
```
(2*sqrt(a**2*x**2 + 1) - 18*int(( - sqrt(a**2*x**2 + 1))/(a**5*x**5 - 3*a*
*4*i*x**4 - 2*a**3*x**3 - 2*a**2*i*x**2 - 3*a*x + i),x)*a + 2*int(( - sqrt
(a**2*x**2 + 1)*x**4)/(a**5*x**5 - 3*a**4*i*x**4 - 2*a**3*x**3 - 2*a**2*i*
x**2 - 3*a*x + i),x)*a**5 - 6*int(( - sqrt(a**2*x**2 + 1)*x**3)/(a**5*x**5
 - 3*a**4*i*x**4 - 2*a**3*x**3 - 2*a**2*i*x**2 - 3*a*x + i),x)*a**4*i - 22
*int(( - sqrt(a**2*x**2 + 1)*x)/(a**5*x**5 - 3*a**4*i*x**4 - 2*a**3*x**3 -
 2*a**2*i*x**2 - 3*a*x + i),x)*a**2*i - 6*int((sqrt(a**2*x**2 + 1)*x)/(a**
3*i*x**3 + 3*a**2*x**2 - 3*a*i*x - 1),x)*a**2 + 3*log(sqrt(a**2*x**2 + 1)
- 1) - 3*log(sqrt(a**2*x**2 + 1) + 1))/6
```

3.72 $\int \frac{e^{-3i\arctan(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 64

$$\int \frac{e^{-3i\arctan(ax)}}{x^2}\,dx = -\frac{4ia(1-iax)}{\sqrt{1+a^2x^2}} - \frac{\sqrt{1+a^2x^2}}{x} + 3ia\text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

output
```
-4*I*a*(1-I*a*x)/(a^2*x^2+1)^(1/2)-(a^2*x^2+1)^(1/2)/x+3*I*a*arctanh((a^2*
x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{e^{-3i\arctan(ax)}}{x^2}\,dx = \sqrt{1+a^2x^2}\left(-\frac{1}{x} - \frac{4a}{-i+ax}\right) - 3ia\log(x) + 3ia\log\left(1+\sqrt{1+a^2x^2}\right)$$

input
```
Integrate[1/(E^((3*I)*ArcTan[a*x])*x^2),x]
```

output
```
Sqrt[1 + a^2*x^2]*(-x^(-1) - (4*a)/(-I + a*x)) - (3*I)*a*Log[x] + (3*I)*a*
Log[1 + Sqrt[1 + a^2*x^2]]
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5583, 2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3i\arctan(ax)}}{x^2}dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{(1-iax)^2}{x^2(1+iax)\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{2353}$$

$$\int \left(\frac{4ia^2}{(ax-i)\sqrt{a^2x^2+1}}-\frac{3ia}{x\sqrt{a^2x^2+1}}+\frac{1}{x^2\sqrt{a^2x^2+1}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$3ia\text{arctanh}\left(\sqrt{a^2x^2+1}\right)+\frac{4a\sqrt{a^2x^2+1}}{-ax+i}-\frac{\sqrt{a^2x^2+1}}{x}$$

input
```
Int[1/(E^((3*I)*ArcTan[a*x])*x^2),x]
```

output
```
-(Sqrt[1 + a^2*x^2]/x) + (4*a*Sqrt[1 + a^2*x^2])/(I - a*x) + (3*I)*a*ArcTa
nh[Sqrt[1 + a^2*x^2]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.28

method	result
risch	$-\frac{\sqrt{a^2x^2+1}}{x}+ia\left(3\ \operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)+\frac{4i\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{a\left(x-\frac{i}{a}\right)}\right)$
default	$-\frac{\left(a^2x^2+1\right)^{\frac{5}{2}}}{x}+4a^2\left(\frac{\left(a^2x^2+1\right)^{\frac{3}{2}}x}{4}+\frac{3\sqrt{a^2x^2+1}\,x}{8}+\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{8\sqrt{a^2}}\right)-3ia\left(\frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{3}+\sqrt{a^2x^2+1}\right.$

input
```
int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(a^2*x^2+1)^(1/2)/x+I*a*(3*arctanh(1/(a^2*x^2+1)^(1/2))+4*I/a/(x-I/a)*((x
-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(52) = 104.

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.70

$$\int \frac{e^{-3i\arctan(ax)}}{x^2}\,dx =$$
$$-\frac{5\,a^2x^2 - 5i\,ax + 3\left(-i\,a^2x^2 - ax\right)\log\left(-ax + \sqrt{a^2x^2+1} + 1\right) + 3\left(i\,a^2x^2 + ax\right)\log\left(-ax + \sqrt{a^2x^2+ \right.}{ax^2 - i\,x}$$

input
```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
-(5*a^2*x^2 - 5*I*a*x + 3*(-I*a^2*x^2 - a*x)*log(-a*x + sqrt(a^2*x^2 + 1)
+ 1) + 3*(I*a^2*x^2 + a*x)*log(-a*x + sqrt(a^2*x^2 + 1) - 1) + sqrt(a^2*x^
2 + 1)*(5*a*x - I))/(a*x^2 - I*x)
```

Sympy [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^2}\,dx = i\left(\int \frac{\sqrt{a^2x^2+1}}{a^3x^5 - 3ia^2x^4 - 3ax^3 + ix^2}\,dx + \int \frac{a^2x^2\sqrt{a^2x^2+1}}{a^3x^5 - 3ia^2x^4 - 3ax^3 + ix^2}\,dx\right)$$

input
```
integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/x**2,x)
```

output
```
I*(Integral(sqrt(a**2*x**2 + 1)/(a**3*x**5 - 3*I*a**2*x**4 - 3*a*x**3 + I*
x**2), x) + Integral(a**2*x**2*sqrt(a**2*x**2 + 1)/(a**3*x**5 - 3*I*a**2*x
**4 - 3*a*x**3 + I*x**2), x))
```

Maxima [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^2}\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}}{(i\,ax+1)^3x^2}\,dx$$

input
```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((a^2*x^2 + 1)^(3/2)/((I*a*x + 1)^3*x^2), x)
```

Giac [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^2}\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}}{(i\,ax+1)^3x^2}\,dx$$

input
```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^2,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 22.97 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.19

$$\int \frac{e^{-3i\arctan(ax)}}{x^2}\,dx = a\,\mathrm{atanh}\left(\sqrt{a^2\,x^2+1}\right)\,3\mathrm{i} - \frac{\sqrt{a^2\,x^2+1}}{x} + \frac{4\,a^2\,\sqrt{a^2\,x^2+1}}{\left(-x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

input
```
int((a^2*x^2 + 1)^(3/2)/(x^2*(a*x*1i + 1)^3),x)
```

output
```
a*atanh((a^2*x^2 + 1)^(1/2))*3i - (a^2*x^2 + 1)^(1/2)/x + (4*a^2*(a^2*x^2
+ 1)^(1/2))/((((a^2)^(1/2)*1i)/a - x*(a^2)^(1/2))*(a^2)^(1/2))
```

Reduce [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^2}\,dx = -\left(\int \frac{\sqrt{a^2x^2+1}}{a^3i\,x^5+3a^2x^4-3ai\,x^3-x^2}dx\right) - \left(\int \frac{\sqrt{a^2x^2+1}}{a^3i\,x^3+3a^2x^2-3aix-1}dx\right)a^2$$

input
```
int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^2,x)
```

output
```
- (int(sqrt(a**2*x**2 + 1)/(a**3*i*x**5 + 3*a**2*x**4 - 3*a*i*x**3 - x**2
),x) + int(sqrt(a**2*x**2 + 1)/(a**3*i*x**3 + 3*a**2*x**2 - 3*a*i*x - 1),x
)*a**2)
```

3.73 $\int \frac{e^{-3i\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 89

$$\int \frac{e^{-3i\arctan(ax)}}{x^3}\,dx = -\frac{4a^2(1-iax)}{\sqrt{1+a^2x^2}} - \frac{\sqrt{1+a^2x^2}}{2x^2} + \frac{3ia\sqrt{1+a^2x^2}}{x} + \frac{9}{2}a^2\operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

output
```
-4*a^2*(1-I*a*x)/(a^2*x^2+1)^(1/2)-1/2*(a^2*x^2+1)^(1/2)/x^2+3*I*a*(a^2*x^
2+1)^(1/2)/x+9/2*a^2*arctanh((a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

$$\int \frac{e^{-3i\arctan(ax)}}{x^3}\,dx = \sqrt{1+a^2x^2}\left(-\frac{1}{2x^2} + \frac{3ia}{x} + \frac{4ia^2}{-i+ax}\right) - \frac{9}{2}a^2\log(x) + \frac{9}{2}a^2\log\left(1+\sqrt{1+a^2x^2}\right)$$

input
```
Integrate[1/(E^((3*I)*ArcTan[a*x])*x^3),x]
```

output
```
Sqrt[1 + a^2*x^2]*(-1/2*1/x^2 + ((3*I)*a)/x + ((4*I)*a^2)/(-I + a*x)) - (9
*a^2*Log[x])/2 + (9*a^2*Log[1 + Sqrt[1 + a^2*x^2]])/2
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5583, 2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3i\arctan(ax)}}{x^3}dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{(1-iax)^2}{x^3(1+iax)\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{2353}$$

$$\int \left(-\frac{4a^2}{x\sqrt{a^2x^2+1}}-\frac{3ia}{x^2\sqrt{a^2x^2+1}}+\frac{1}{x^3\sqrt{a^2x^2+1}}+\frac{4a^3}{(ax-i)\sqrt{a^2x^2+1}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{9}{2}a^2\text{arctanh}\left(\sqrt{a^2x^2+1}\right)-\frac{4ia^2\sqrt{a^2x^2+1}}{-ax+i}+\frac{3ia\sqrt{a^2x^2+1}}{x}-\frac{\sqrt{a^2x^2+1}}{2x^2}$$

input
```
Int[1/(E^((3*I)*ArcTan[a*x])*x^3),x]
```

output
```
-1/2*Sqrt[1 + a^2*x^2]/x^2 + ((3*I)*a*Sqrt[1 + a^2*x^2])/x - ((4*I)*a^2*Sq
rt[1 + a^2*x^2])/(I - a*x) + (9*a^2*ArcTanh[Sqrt[1 + a^2*x^2]])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.21

method	result
risch	$\frac{i\left(6a^3x^3+i a^2x^2+6ax+i\right)}{2x^2\sqrt{a^2x^2+1}}-\frac{a^2\left(-9\,\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)-\frac{8i\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{a\left(x-\frac{i}{a}\right)}\right)}{2}$
default	$-\frac{\left(a^2x^2+1\right)^{\frac{5}{2}}}{2x^2}-\frac{9a^2\left(\frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{3}+\sqrt{a^2x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)\right)}{2}-3ia\left(-\frac{\left(a^2x^2+1\right)^{\frac{5}{2}}}{x}+4a^2\left(\frac{\left(a^2x^2+1\right)^{\frac{3}{2}}x}{4}+\right.\right.$

input
```
int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*I*(6*a^3*x^3+I*a^2*x^2+6*a*x+I)/x^2/(a^2*x^2+1)^(1/2)-1/2*a^2*(-9*arct
anh(1/(a^2*x^2+1)^(1/2))-8*I/a/(x-I/a)*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2)
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.46

$$\int \frac{e^{-3i\arctan(ax)}}{x^3}\,dx$$
$$= \frac{14i\,a^3x^3 + 14\,a^2x^2 + 9\,(a^3x^3 - i\,a^2x^2)\log\left(-ax + \sqrt{a^2x^2+1} + 1\right) - 9\,(a^3x^3 - i\,a^2x^2)\log\left(-ax + \sqrt{a^2x^2}\right.}{2\,(ax^3 - i\,x^2)}$$

input
```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
1/2*(14*I*a^3*x^3 + 14*a^2*x^2 + 9*(a^3*x^3 - I*a^2*x^2)*log(-a*x + sqrt(a
^2*x^2 + 1) + 1) - 9*(a^3*x^3 - I*a^2*x^2)*log(-a*x + sqrt(a^2*x^2 + 1) -
1) + sqrt(a^2*x^2 + 1)*(14*I*a^2*x^2 + 5*a*x + I))/(a*x^3 - I*x^2)
```

Sympy [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^3}\,dx = i\left(\int \frac{\sqrt{a^2x^2+1}}{a^3x^6 - 3ia^2x^5 - 3ax^4 + ix^3}\,dx + \int \frac{a^2x^2\sqrt{a^2x^2+1}}{a^3x^6 - 3ia^2x^5 - 3ax^4 + ix^3}\,dx\right)$$

input
```
integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/x**3,x)
```

output
```
I*(Integral(sqrt(a**2*x**2 + 1)/(a**3*x**6 - 3*I*a**2*x**5 - 3*a*x**4 + I*
x**3), x) + Integral(a**2*x**2*sqrt(a**2*x**2 + 1)/(a**3*x**6 - 3*I*a**2*x
**5 - 3*a*x**4 + I*x**3), x))
```

Maxima [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^3}\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}}{(i\,ax+1)^3x^3}\,dx$$

input
```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((a^2*x^2 + 1)^(3/2)/((I*a*x + 1)^3*x^3), x)
```

Giac [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^3}\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}}{(i\,ax+1)^3x^3}\,dx$$

input
```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^3,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 22.64 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.12

$$\int \frac{e^{-3i\arctan(ax)}}{x^3}\,dx = -\frac{a^2\,\mathrm{atan}\left(\sqrt{a^2\,x^2+1}\,1\mathrm{i}\right)\,9\mathrm{i}}{2} - \frac{\sqrt{a^2\,x^2+1}}{2\,x^2} + \frac{a\,\sqrt{a^2\,x^2+1}\,3\mathrm{i}}{x} - \frac{a^3\,\sqrt{a^2\,x^2+1}\,4\mathrm{i}}{\left(-x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

input
```
int((a^2*x^2 + 1)^(3/2)/(x^3*(a*x*1i + 1)^3),x)
```

output
```
(a*(a^2*x^2 + 1)^(1/2)*3i)/x - (a^2*x^2 + 1)^(1/2)/(2*x^2) - (a^2*atan((a^
2*x^2 + 1)^(1/2)*1i)*9i)/2 - (a^3*(a^2*x^2 + 1)^(1/2)*4i)/((((a^2)^(1/2)*1
i)/a - x*(a^2)^(1/2))*(a^2)^(1/2))
```

Reduce [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^3}\,dx$$
$$= \frac{-16\sqrt{a^2x^2+1}\,a^2x^2+164\sqrt{a^2x^2+1}\,aix-6\sqrt{a^2x^2+1}+288\left(\int \frac{\sqrt{a^2x^2+1}}{a^5x^7-3a^4i\,x^6-2a^3x^5-2a^2i\,x^4-3a\,x^3+i\,x^2}dx\right)a}{}$$

input
```
int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^3,x)
```

output
```
( - 16*sqrt(a**2*x**2 + 1)*a**2*x**2 + 164*sqrt(a**2*x**2 + 1)*a*i*x - 6*s
qrt(a**2*x**2 + 1) + 288*int(sqrt(a**2*x**2 + 1)/(a**5*x**7 - 3*a**4*i*x**
6 - 2*a**3*x**5 - 2*a**2*i*x**4 - 3*a*x**3 + i*x**2),x)*a*x**2 + 240*int(s
qrt(a**2*x**2 + 1)/(a**5*x**6 - 3*a**4*i*x**5 - 2*a**3*x**4 - 2*a**2*i*x**
3 - 3*a*x**2 + i*x),x)*a**2*i*x**2 - 416*int(sqrt(a**2*x**2 + 1)/(a**5*i*x
**7 + 3*a**4*x**6 - 2*a**3*i*x**5 + 2*a**2*x**4 - 3*a*i*x**3 - x**2),x)*a*
i*x**2 + 624*int(sqrt(a**2*x**2 + 1)/(a**5*i*x**6 + 3*a**4*x**5 - 2*a**3*i
*x**4 + 2*a**2*x**3 - 3*a*i*x**2 - x),x)*a**2*x**2 + 336*int(sqrt(a**2*x**
2 + 1)/(a**5*i*x**5 + 3*a**4*x**4 - 2*a**3*i*x**3 + 2*a**2*x**2 - 3*a*i*x
- 1),x)*a**3*i*x**2 + 16*int((sqrt(a**2*x**2 + 1)*x**4)/(a**5*i*x**5 + 3*a
**4*x**4 - 2*a**3*i*x**3 + 2*a**2*x**2 - 3*a*i*x - 1),x)*a**7*i*x**2 + 48*
int((sqrt(a**2*x**2 + 1)*x**3)/(a**5*i*x**5 + 3*a**4*x**4 - 2*a**3*i*x**3
+ 2*a**2*x**2 - 3*a*i*x - 1),x)*a**6*x**2 + 48*int((sqrt(a**2*x**2 + 1)*x)
/(a**5*x**5 - 3*a**4*i*x**4 - 2*a**3*x**3 - 2*a**2*i*x**2 - 3*a*x + i),x)*
a**4*i*x**2 - 27*log(sqrt(a**2*x**2 + 1) - 1)*a**2*x**2 + 27*log(sqrt(a**2
*x**2 + 1) + 1)*a**2*x**2)/(12*x**2)
```

3.74 $\int \frac{e^{-3i\arctan(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 118

$$\int \frac{e^{-3i\arctan(ax)}}{x^4}\,dx = \frac{4ia^3(1-iax)}{\sqrt{1+a^2x^2}} - \frac{\sqrt{1+a^2x^2}}{3x^3} + \frac{3ia\sqrt{1+a^2x^2}}{2x^2} + \frac{14a^2\sqrt{1+a^2x^2}}{3x} - \frac{11}{2}ia^3\text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

output
```
4*I*a^3*(1-I*a*x)/(a^2*x^2+1)^(1/2)-1/3*(a^2*x^2+1)^(1/2)/x^3+3/2*I*a*(a^2
*x^2+1)^(1/2)/x^2+14/3*a^2*(a^2*x^2+1)^(1/2)/x-11/2*I*a^3*arctanh((a^2*x^2
+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int \frac{e^{-3i\arctan(ax)}}{x^4}\,dx = \frac{1}{6}\left(\frac{\sqrt{1+a^2x^2}(2i+7ax-19ia^2x^2+52a^3x^3)}{x^3(-i+ax)} + 33ia^3\log(x) - 33ia^3\log\left(1+\sqrt{1+a^2x^2}\right)\right)$$

input
```
Integrate[1/(E^((3*I)*ArcTan[a*x])*x^4),x]
```

output
```
((Sqrt[1 + a^2*x^2]*(2*I + 7*a*x - (19*I)*a^2*x^2 + 52*a^3*x^3))/(x^3*(-I
+ a*x)) + (33*I)*a^3*Log[x] - (33*I)*a^3*Log[1 + Sqrt[1 + a^2*x^2]])/6
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5583, 2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3i\arctan(ax)}}{x^4}dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{(1-iax)^2}{x^4(1+iax)\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{2353}$$

$$\int \left(-\frac{4a^2}{x^2\sqrt{a^2x^2+1}}+\frac{1}{x^4\sqrt{a^2x^2+1}}-\frac{3ia}{x^3\sqrt{a^2x^2+1}}-\frac{4ia^4}{(ax-i)\sqrt{a^2x^2+1}}+\frac{4ia^3}{x\sqrt{a^2x^2+1}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{14a^2\sqrt{a^2x^2+1}}{3x}+\frac{3ia\sqrt{a^2x^2+1}}{2x^2}-\frac{\sqrt{a^2x^2+1}}{3x^3}-\frac{11}{2}ia^3\text{arctanh}\left(\sqrt{a^2x^2+1}\right)-\frac{4a^3\sqrt{a^2x^2+1}}{-ax+i}$$

input
```
Int[1/(E^((3*I)*ArcTan[a*x])*x^4),x]
```

output
```
-1/3*Sqrt[1 + a^2*x^2]/x^3 + (((3*I)/2)*a*Sqrt[1 + a^2*x^2])/x^2 + (14*a^2
*Sqrt[1 + a^2*x^2])/(3*x) - (4*a^3*Sqrt[1 + a^2*x^2])/(I - a*x) - ((11*I)/
2)*a^3*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.98

method	result
risch	$\frac{28a^4x^4+9ia^3x^3+26a^2x^2+9iax-2}{6x^3\sqrt{a^2x^2+1}}+\frac{ia^3\left(-11\,\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)-\frac{8i\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{a\left(x-\frac{i}{a}\right)}\right)}{2}$
default	$-\frac{\left(a^2x^2+1\right)^{\frac{5}{2}}}{3x^3}-\frac{16a^2\left(-\frac{\left(a^2x^2+1\right)^{\frac{5}{2}}}{x}+4a^2\left(\frac{\left(a^2x^2+1\right)^{\frac{3}{2}}x}{4}+\frac{3\sqrt{a^2x^2+1}\,x}{8}+\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{8\sqrt{a^2}}\right)\right)}{3}-10ia^3\left(\frac{\left(\left(x-\frac{i}{a}\right)^2a^2\right.}{}\right.$

input
```
int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*(28*a^4*x^4+9*I*a^3*x^3+26*a^2*x^2+9*I*a*x-2)/x^3/(a^2*x^2+1)^(1/2)+1/
2*I*a^3*(-11*arctanh(1/(a^2*x^2+1)^(1/2))-8*I/a/(x-I/a)*((x-I/a)^2*a^2+2*I
*a*(x-I/a))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.18

$$\int \frac{e^{-3i\arctan(ax)}}{x^4}\,dx$$
$$= \frac{52\,a^4x^4 - 52i\,a^3x^3 - 33\left(i\,a^4x^4 + a^3x^3\right)\log\left(-ax + \sqrt{a^2x^2+1} + 1\right) - 33\left(-i\,a^4x^4 - a^3x^3\right)\log\left(-ax + \sqrt{}\right.}{6\left(ax^4 - i\,x^3\right)}$$

input
```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
1/6*(52*a^4*x^4 - 52*I*a^3*x^3 - 33*(I*a^4*x^4 + a^3*x^3)*log(-a*x + sqrt(
a^2*x^2 + 1) + 1) - 33*(-I*a^4*x^4 - a^3*x^3)*log(-a*x + sqrt(a^2*x^2 + 1)
 - 1) + (52*a^3*x^3 - 19*I*a^2*x^2 + 7*a*x + 2*I)*sqrt(a^2*x^2 + 1))/(a*x^
4 - I*x^3)
```

Sympy [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^4}\,dx = i\left(\int \frac{\sqrt{a^2x^2+1}}{a^3x^7 - 3ia^2x^6 - 3ax^5 + ix^4}\,dx + \int \frac{a^2x^2\sqrt{a^2x^2+1}}{a^3x^7 - 3ia^2x^6 - 3ax^5 + ix^4}\,dx\right)$$

input
```
integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/x**4,x)
```

output
```
I*(Integral(sqrt(a**2*x**2 + 1)/(a**3*x**7 - 3*I*a**2*x**6 - 3*a*x**5 + I*
x**4), x) + Integral(a**2*x**2*sqrt(a**2*x**2 + 1)/(a**3*x**7 - 3*I*a**2*x
**6 - 3*a*x**5 + I*x**4), x))
```

Maxima [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^4}\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}}{(i\,ax+1)^3x^4}\,dx$$

input
```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((a^2*x^2 + 1)^(3/2)/((I*a*x + 1)^3*x^4), x)
```

Giac [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^4}\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}}{(i\,ax+1)^3x^4}\,dx$$

input
```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^4,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 22.82 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.99

$$\int \frac{e^{-3i\arctan(ax)}}{x^4}\,dx = \frac{14\,a^2\,\sqrt{a^2\,x^2+1}}{3\,x} - \frac{\sqrt{a^2\,x^2+1}}{3\,x^3} + \frac{a\,\sqrt{a^2\,x^2+1}\,3\mathrm{i}}{2\,x^2} - \frac{11\,a^3\,\mathrm{atan}\left(\sqrt{a^2\,x^2+1}\,1\mathrm{i}\right)}{2} - \frac{4\,a^4\,\sqrt{a^2\,x^2+1}}{\left(-x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

input
```
int((a^2*x^2 + 1)^(3/2)/(x^4*(a*x*1i + 1)^3),x)
```

output
```
(a*(a^2*x^2 + 1)^(1/2)*3i)/(2*x^2) - (a^2*x^2 + 1)^(1/2)/(3*x^3) - (11*a^3
*atan((a^2*x^2 + 1)^(1/2)*1i))/2 + (14*a^2*(a^2*x^2 + 1)^(1/2))/(3*x) - (4
*a^4*(a^2*x^2 + 1)^(1/2))/((((a^2)^(1/2)*1i)/a - x*(a^2)^(1/2))*(a^2)^(1/2
))
```

Reduce [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^4}\,dx = -\left(\int \frac{\sqrt{a^2x^2+1}}{a^3i\,x^7+3a^2x^6-3ai\,x^5-x^4}dx\right) - \left(\int \frac{\sqrt{a^2x^2+1}}{a^3i\,x^5+3a^2x^4-3ai\,x^3-x^2}dx\right)a^2$$

input
```
int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^4,x)
```

output
```
- (int(sqrt(a**2*x**2 + 1)/(a**3*i*x**7 + 3*a**2*x**6 - 3*a*i*x**5 - x**4
),x) + int(sqrt(a**2*x**2 + 1)/(a**3*i*x**5 + 3*a**2*x**4 - 3*a*i*x**3 - x
**2),x)*a**2)
```

3.75 $\int \frac{e^{-3i\arctan(ax)}}{x^5}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 135

$$\int \frac{e^{-3i\arctan(ax)}}{x^5}\,dx = \frac{4a^4(1-iax)}{\sqrt{1+a^2x^2}} - \frac{\sqrt{1+a^2x^2}}{4x^4} + \frac{ia\sqrt{1+a^2x^2}}{x^3} + \frac{19a^2\sqrt{1+a^2x^2}}{8x^2} - \frac{6ia^3\sqrt{1+a^2x^2}}{x} - \frac{51}{8}a^4\operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

output
```
4*a^4*(1-I*a*x)/(a^2*x^2+1)^(1/2)-1/4*(a^2*x^2+1)^(1/2)/x^4+I*a*(a^2*x^2+1
)^(1/2)/x^3+19/8*a^2*(a^2*x^2+1)^(1/2)/x^2-6*I*a^3*(a^2*x^2+1)^(1/2)/x-51/
8*a^4*arctanh((a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.70

$$\int \frac{e^{-3i\arctan(ax)}}{x^5}\,dx = \frac{1}{8}\left(\frac{\sqrt{1+a^2x^2}(2i+6ax-11ia^2x^2-29a^3x^3-80ia^4x^4)}{x^4(-i+ax)} + 51a^4\log(x) - 51a^4\log\left(1+\sqrt{1+a^2x^2}\right)\right)$$

input
```
Integrate[1/(E^((3*I)*ArcTan[a*x])*x^5),x]
```

output
```
((Sqrt[1 + a^2*x^2]*(2*I + 6*a*x - (11*I)*a^2*x^2 - 29*a^3*x^3 - (80*I)*a^
4*x^4))/(x^4*(-I + a*x)) + 51*a^4*Log[x] - 51*a^4*Log[1 + Sqrt[1 + a^2*x^2
]])/8
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5583, 2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3i\arctan(ax)}}{x^5}dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{(1-iax)^2}{x^5(1+iax)\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{2353}$$

$$\int \left(\frac{1}{x^5\sqrt{a^2x^2+1}}-\frac{3ia}{x^4\sqrt{a^2x^2+1}}-\frac{4a^2}{x^3\sqrt{a^2x^2+1}}-\frac{4a^5}{(ax-i)\sqrt{a^2x^2+1}}+\frac{4a^4}{x\sqrt{a^2x^2+1}}+\frac{4ia^3}{x^2\sqrt{a^2x^2+1}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{19a^2\sqrt{a^2x^2+1}}{8x^2}-\frac{\sqrt{a^2x^2+1}}{4x^4}+\frac{ia\sqrt{a^2x^2+1}}{x^3}-\frac{51}{8}a^4\text{arctanh}\left(\sqrt{a^2x^2+1}\right)+\frac{4ia^4\sqrt{a^2x^2+1}}{-ax+i}-\frac{6ia^3\sqrt{a^2x^2+1}}{x}$$

input
```
Int[1/(E^((3*I)*ArcTan[a*x])*x^5),x]
```

output
```
-1/4*Sqrt[1 + a^2*x^2]/x^4 + (I*a*Sqrt[1 + a^2*x^2])/x^3 + (19*a^2*Sqrt[1
+ a^2*x^2])/(8*x^2) - ((6*I)*a^3*Sqrt[1 + a^2*x^2])/x + ((4*I)*a^4*Sqrt[1
+ a^2*x^2])/(I - a*x) - (51*a^4*ArcTanh[Sqrt[1 + a^2*x^2]])/8
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.93

method	result	size
risch	$-\frac{i\left(48a^5x^5+19ia^4x^4+40a^3x^3+17ia^2x^2-8ax-2i\right)}{8x^4\sqrt{a^2x^2+1}}+\frac{a^4\left(-51\,\mathrm{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)-\frac{32i\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{a\left(x-\frac{i}{a}\right)}\right)}{8}$	125
default	Expression too large to display	937

input
```
int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/8*I*(48*a^5*x^5+19*I*a^4*x^4+40*a^3*x^3+17*I*a^2*x^2-8*a*x-2*I)/x^4/(a^
2*x^2+1)^(1/2)+1/8*a^4*(-51*arctanh(1/(a^2*x^2+1)^(1/2))-32*I/a/(x-I/a)*((
x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.08

$$\int \frac{e^{-3i\arctan(ax)}}{x^5}\,dx = \frac{-80i\,a^5x^5 - 80\,a^4x^4 - 51\left(a^5x^5 - i\,a^4x^4\right)\log\left(-ax + \sqrt{a^2x^2+1} + 1\right) + 51\left(a^5x^5 - i\,a^4x^4\right)\log\left(-ax + \sqrt{\right.}{8\left(ax^5 - i\,x^4\right)}$$

input
```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
1/8*(-80*I*a^5*x^5 - 80*a^4*x^4 - 51*(a^5*x^5 - I*a^4*x^4)*log(-a*x + sqrt
(a^2*x^2 + 1) + 1) + 51*(a^5*x^5 - I*a^4*x^4)*log(-a*x + sqrt(a^2*x^2 + 1)
 - 1) + (-80*I*a^4*x^4 - 29*a^3*x^3 - 11*I*a^2*x^2 + 6*a*x + 2*I)*sqrt(a^2
*x^2 + 1))/(a*x^5 - I*x^4)
```

Sympy [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^5}\,dx = i\left(\int \frac{\sqrt{a^2x^2+1}}{a^3x^8 - 3ia^2x^7 - 3ax^6 + ix^5}\,dx + \int \frac{a^2x^2\sqrt{a^2x^2+1}}{a^3x^8 - 3ia^2x^7 - 3ax^6 + ix^5}\,dx\right)$$

input
```
integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/x**5,x)
```

output
```
I*(Integral(sqrt(a**2*x**2 + 1)/(a**3*x**8 - 3*I*a**2*x**7 - 3*a*x**6 + I*
x**5), x) + Integral(a**2*x**2*sqrt(a**2*x**2 + 1)/(a**3*x**8 - 3*I*a**2*x
**7 - 3*a*x**6 + I*x**5), x))
```

Maxima [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^5}\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}}{(i\,ax+1)^3x^5}\,dx$$

input
```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
integrate((a^2*x^2 + 1)^(3/2)/((I*a*x + 1)^3*x^5), x)
```

Giac [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^5}\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}}{(i\,ax+1)^3x^5}\,dx$$

input
```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^5,x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 23.15 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.03

$$\int \frac{e^{-3i\arctan(ax)}}{x^5}\,dx = \frac{a^4\,\mathrm{atan}\left(\sqrt{a^2\,x^2+1}\,1\mathrm{i}\right)\,51\mathrm{i}}{8} - \frac{\sqrt{a^2\,x^2+1}}{4\,x^4} + \frac{a\,\sqrt{a^2\,x^2+1}\,1\mathrm{i}}{x^3} + \frac{19\,a^2\,\sqrt{a^2\,x^2+1}}{8\,x^2} - \frac{a^3\,\sqrt{a^2\,x^2+1}\,6\mathrm{i}}{x} + \frac{a^5\,\sqrt{a^2\,x^2+1}\,4\mathrm{i}}{\left(-x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}}$$

input
```
int((a^2*x^2 + 1)^(3/2)/(x^5*(a*x*1i + 1)^3),x)
```

output
```
(a^4*atan((a^2*x^2 + 1)^(1/2)*1i)*51i)/8 - (a^2*x^2 + 1)^(1/2)/(4*x^4) + (
a*(a^2*x^2 + 1)^(1/2)*1i)/x^3 + (19*a^2*(a^2*x^2 + 1)^(1/2))/(8*x^2) - (a^
3*(a^2*x^2 + 1)^(1/2)*6i)/x + (a^5*(a^2*x^2 + 1)^(1/2)*4i)/((((a^2)^(1/2)*
1i)/a - x*(a^2)^(1/2))*(a^2)^(1/2))
```

Reduce [F]

$$\int \frac{e^{-3i\arctan(ax)}}{x^5}\,dx = \frac{448\sqrt{a^2x^2+1}\,a^4x^4 - 4512\sqrt{a^2x^2+1}\,a^3i\,x^3 - 1038\sqrt{a^2x^2+1}\,a^2x^2 + 368\sqrt{a^2x^2+1}\,aix - 12\sqrt{a^2x^2+1}}{}$$

input
```
int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/x^5,x)
```

output
```
(448*sqrt(a**2*x**2 + 1)*a**4*x**4 - 4512*sqrt(a**2*x**2 + 1)*a**3*i*x**3
- 1038*sqrt(a**2*x**2 + 1)*a**2*x**2 + 368*sqrt(a**2*x**2 + 1)*a*i*x - 12*
sqrt(a**2*x**2 + 1) + 2496*int(sqrt(a**2*x**2 + 1)/(a**5*x**9 - 3*a**4*i*x
**8 - 2*a**3*x**7 - 2*a**2*i*x**6 - 3*a*x**5 + i*x**4),x)*a*x**4 - 960*int
(sqrt(a**2*x**2 + 1)/(a**5*x**7 - 3*a**4*i*x**6 - 2*a**3*x**5 - 2*a**2*i*x
**4 - 3*a*x**3 + i*x**2),x)*a**3*x**4 - 3456*int(sqrt(a**2*x**2 + 1)/(a**5
*i*x**9 + 3*a**4*x**8 - 2*a**3*i*x**7 + 2*a**2*x**6 - 3*a*i*x**5 - x**4),x
)*a*i*x**4 + 5184*int(sqrt(a**2*x**2 + 1)/(a**5*i*x**8 + 3*a**4*x**7 - 2*a
**3*i*x**6 + 2*a**2*x**5 - 3*a*i*x**4 - x**3),x)*a**2*x**4 + 14336*int(sqr
t(a**2*x**2 + 1)/(a**5*i*x**7 + 3*a**4*x**6 - 2*a**3*i*x**5 + 2*a**2*x**4
- 3*a*i*x**3 - x**2),x)*a**3*i*x**4 - 17472*int(sqrt(a**2*x**2 + 1)/(a**5*
i*x**6 + 3*a**4*x**5 - 2*a**3*i*x**4 + 2*a**2*x**3 - 3*a*i*x**2 - x),x)*a*
*4*x**4 - 9408*int(sqrt(a**2*x**2 + 1)/(a**5*i*x**5 + 3*a**4*x**4 - 2*a**3
*i*x**3 + 2*a**2*x**2 - 3*a*i*x - 1),x)*a**5*i*x**4 - 448*int((sqrt(a**2*x
**2 + 1)*x**4)/(a**5*i*x**5 + 3*a**4*x**4 - 2*a**3*i*x**3 + 2*a**2*x**2 -
3*a*i*x - 1),x)*a**9*i*x**4 - 1344*int((sqrt(a**2*x**2 + 1)*x**3)/(a**5*i*
x**5 + 3*a**4*x**4 - 2*a**3*i*x**3 + 2*a**2*x**2 - 3*a*i*x - 1),x)*a**8*x*
*4 + 192*int((sqrt(a**2*x**2 + 1)*x)/(a**5*i*x**5 + 3*a**4*x**4 - 2*a**3*i
*x**3 + 2*a**2*x**2 - 3*a*i*x - 1),x)*a**6*x**4 + 153*log(sqrt(a**2*x**2 +
 1) - 1)*a**4*x**4 - 153*log(sqrt(a**2*x**2 + 1) + 1)*a**4*x**4)/(48*x*...
```

3.76 $\int e^{\frac{1}{2}i\arctan(ax)}x^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 268

$$\begin{aligned}\int e^{\frac{1}{2}i\arctan(ax)}x^2\,dx=&-\frac{3i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3}-\frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3}\\&+\frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2}+\frac{3i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}\\&-\frac{3i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}\\&+\frac{3i\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)}{8\sqrt{2}a^3}\end{aligned}$$

output
```
-3/8*I*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a^3-1/12*I*(1-I*a*x)^(3/4)*(1+I*a*x
)^(5/4)/a^3+1/3*x*(1-I*a*x)^(3/4)*(1+I*a*x)^(5/4)/a^2+3/16*I*arctan(1-2^(1
/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))*2^(1/2)/a^3-3/16*I*arctan(1+2^(1/2)*(
1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))*2^(1/2)/a^3+3/16*I*arctanh(2^(1/2)*(1-I*a*
x)^(1/4)/(1+I*a*x)^(1/4)/(1+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2)))*2^(1/2)/a^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.31

$$\int e^{\frac{1}{2}i\arctan(ax)}x^2\,dx = \frac{(1-iax)^{3/4}\left(\sqrt[4]{1+iax}(-i+5ax+4ia^2x^2)-6i\sqrt[4]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-iax)\right)\right)}{12a^3}$$

input
```
Integrate[E^((I/2)*ArcTan[a*x])*x^2,x]
```

output
```
((1 - I*a*x)^(3/4)*((1 + I*a*x)^(1/4)*(-I + 5*a*x + (4*I)*a^2*x^2) - (6*I)
*2^(1/4)*Hypergeometric2F1[-1/4, 3/4, 7/4, (1 - I*a*x)/2]))/(12*a^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.75 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.18, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.938$, Rules used = {5585, 101, 27, 90, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{\frac{1}{2}i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{x^2\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}dx$$

$$\Big\downarrow \text{101}$$

$$\frac{\int -\frac{\sqrt[4]{iax+1}(iax+2)}{2\sqrt[4]{1-iax}}dx}{3a^2}+\frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2}-\frac{\int \frac{\sqrt[4]{iax+1}(iax+2)}{\sqrt[4]{1-iax}}dx}{6a^2}$$

$$\Big\downarrow \text{90}$$

$$\frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2}-\frac{\frac{9}{4}\int \frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}dx+\frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{2a}}{6a^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2}-\frac{\frac{9}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt[4]{1-iax}(iax+1)^{3/4}}dx+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)+\frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{2a}}{6a^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2}-\frac{\frac{9}{4}\left(\frac{2i\int \frac{\sqrt{1-iax}}{(iax+1)^{3/4}}d\sqrt[4]{1-iax}}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)+\frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{2a}}{6a^2}$$

$$\Big\downarrow \text{854}$$

$$\frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2}-\frac{\frac{9}{4}\left(\frac{2i\int \frac{\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)+\frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{2a}}{6a^2}$$

$$\Big\downarrow \text{826}$$

$$\frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2}-\frac{\frac{9}{4}\left(\frac{2i\left(\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)+\frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{2a}}{6a^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2}-\frac{\frac{9}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a} -\right.}{6a^2}$$

$\downarrow$ 1082

$$\frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2}-\frac{\frac{9}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{\cdots}}{a}\right.}{6a^2}$$

$\downarrow$ 217

$$\frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2}-\frac{\frac{9}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)+\frac{i(\cdots}{6a^2}$$

$\downarrow$ 1479

$$\frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2}-\frac{\frac{9}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\cdots\right.\right.}{a}\cdots\right)}{6a^2}$$

$$\Big\downarrow \; 25$$

$$\frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2}-\frac{\frac{9}{4}\left(\frac{2i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\cdots\right.\right.}{a}\cdots\right)}{6a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2}-\frac{\frac{9}{4}\left(\frac{2i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\cdots\right.\right)}{a}\cdots\right)}{6a^2}$$

$\downarrow$ 1103

$$\frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2}-\frac{\frac{9}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\cdots\right)}{2\sqrt{2}}\cdots\right.\right)}{a}\cdots\right)}{6a^2}$$

input
```
Int[E^((I/2)*ArcTan[a*x])*x^2,x]
```

output
```
(x*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(5/4))/(3*a^2) - (((I/2)*(1 - I*a*x)^(3/4
)*(1 + I*a*x)^(5/4))/a + (9*((I*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a + (
(2*I)*((-(ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2
]) + ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2])/2
+ (Log[1 + Sqrt[1 - I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)
]/(2*Sqrt[2]) - Log[1 + Sqrt[1 - I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 +
 I*a*x)^(1/4)]/(2*Sqrt[2]))/2))/a))/4)/(6*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\, x^2 dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^2,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.91

$$\int e^{\frac{1}{2}i\arctan(ax)}x^2\,dx = -\frac{12\,a^3\sqrt{\frac{9i}{64\,a^6}}\log\left(\frac{8}{3}i\,a^3\sqrt{\frac{9i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3\sqrt{\frac{9i}{64\,a^6}}\log\left(-\frac{8}{3}i\,a^3\sqrt{\frac{9i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+12\,a^3}{}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^2,x, algorithm="fricas")
```

output
```
-1/24*(12*a^3*sqrt(9/64*I/a^6)*log(8/3*I*a^3*sqrt(9/64*I/a^6) + sqrt(I*sqr
t(a^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(9/64*I/a^6)*log(-8/3*I*a^3*sqrt(9
/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 12*a^3*sqrt(-9/64*I/a^
6)*log(8/3*I*a^3*sqrt(-9/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
- 12*a^3*sqrt(-9/64*I/a^6)*log(-8/3*I*a^3*sqrt(-9/64*I/a^6) + sqrt(I*sqrt(
a^2*x^2 + 1)/(a*x + I))) - (8*a^3*x^3 - 2*I*a^2*x^2 - a*x - 11*I)*sqrt(I*s
qrt(a^2*x^2 + 1)/(a*x + I)))/a^3
```

Sympy [F]

$$\int e^{\frac{1}{2}i\arctan(ax)}x^2\,dx = \int x^2\sqrt{\frac{i\,(ax-i)}{\sqrt{a^2x^2+1}}}\,dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)*x**2,x)
```

output
```
Integral(x**2*sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int e^{\frac{1}{2}i\arctan(ax)}x^2\,dx = \int x^2\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^2,x, algorithm="maxima")
```

output
```
integrate(x^2*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{1}{2}i\arctan(ax)}x^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{2}i\arctan(ax)}x^2\,dx = \int x^2\,\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}\,dx$$

input
```
int(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)
```

output
```
int(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int e^{\frac{1}{2}i\arctan(ax)}x^2\,dx = \int \sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,x^2dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^2,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^2,x)
```

3.77 $\int e^{\frac{1}{2}i\arctan(ax)}x\,dx$

Optimal result

Integrand size = 14, antiderivative size = 226

$$\int e^{\frac{1}{2}i\arctan(ax)}x\,dx = \frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}+\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)}{4\sqrt{2}a^2}$$

output

```
1/4*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a^2+1/2*(1-I*a*x)^(3/4)*(1+I*a*x)^(5/4
)/a^2-1/8*arctan(1-2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))*2^(1/2)/a^2+1/
8*arctan(1+2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))*2^(1/2)/a^2-1/8*arctan
h(2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)/(1+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/
2)))*2^(1/2)/a^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.28

$$\int e^{\frac{1}{2}i\arctan(ax)}x\,dx = \frac{(1-iax)^{3/4}\left(3(1+iax)^{5/4}+2\sqrt[4]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-iax)\right)\right)}{6a^2}$$

input
```
Integrate[E^((I/2)*ArcTan[a*x])*x,x]
```

output
```
((1 - I*a*x)^(3/4)*(3*(1 + I*a*x)^(5/4) + 2*2^(1/4)*Hypergeometric2F1[-1/4
, 3/4, 7/4, (1 - I*a*x)/2]))/(6*a^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.70 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.24, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {5585, 90, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\frac{1}{2}i\arctan(ax)}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{x\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}dx$$

$$\Big\downarrow \text{90}$$

$$\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}-\frac{i\int \frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}dx}{4a}$$

$$\Big\downarrow \text{60}$$

$$\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}-\frac{i\left(\frac{1}{2}\int\frac{1}{\sqrt[4]{1-iax}(iax+1)^{3/4}}dx+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)}{4a}$$

$\Big\downarrow$ 73

$$\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}-\frac{i\left(\frac{2i\int\frac{\sqrt{1-iax}}{(iax+1)^{3/4}}d\sqrt[4]{1-iax}}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)}{4a}$$

$\Big\downarrow$ 854

$$\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}-\frac{i\left(\frac{2i\int\frac{\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)}{4a}$$

$\Big\downarrow$ 826

$$\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}-\frac{i\left(\frac{2i\left(\frac{1}{2}\int\frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}-\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)}{4a}$$

$\Big\downarrow$ 1476

$$\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}-\frac{i\left(\frac{2i\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int\frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)-\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+\cdots}{4a}$$

$\Big\downarrow$ 1082

$$\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}-\frac{i\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1\cdots}}{a}\cdots\right)}{4a}$$

$$\Big\downarrow \; 217$$

$$\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}-\frac{i\left(\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)}{4a}$$

$$\Big\downarrow \; 1479$$

$$\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}-\frac{i\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\cdots\right)\cdots\right)}{a}\cdots\right)}{4a}$$

$$\Big\downarrow \; 25$$

$$\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}-\frac{i\left(\frac{2i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\ldots\right)\right)}{a}\ldots\right)}{4a}$$

$\downarrow$ 27

$$\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}-\frac{i\left(\frac{2i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\ldots\right)\right)}{a}\ldots\right)}{4a}$$

$\downarrow$ 1103

$$\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2} - \frac{i\left(\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\right.}{2\sqrt{2}}\right.\right.}{a}\right.}{4a}$$

input
```
Int[E^((I/2)*ArcTan[a*x])*x,x]
```

output
```
((1 - I*a*x)^(3/4)*(1 + I*a*x)^(5/4))/(2*a^2) - ((I/4)*((I*(1 - I*a*x)^(3/
4)*(1 + I*a*x)^(1/4))/a + ((2*I)*((-(ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4)
)/(1 + I*a*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1
+ I*a*x)^(1/4)]/Sqrt[2])/2 + (Log[1 + Sqrt[1 - I*a*x] - (Sqrt[2]*(1 - I*a*
x)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]) - Log[1 + Sqrt[1 - I*a*x] + (Sqrt
[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]))/2))/a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\, x dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.04

$$\int e^{\frac{1}{2}i\arctan(ax)}x\,dx = -\frac{2\,a^2\sqrt{\frac{i}{16\,a^4}}\log\left(4\,a^2\sqrt{\frac{i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2\,a^2\sqrt{\frac{i}{16\,a^4}}\log\left(-4\,a^2\sqrt{\frac{i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+2\,a^2\sqrt{-\frac{i}{16}}}{}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x,x, algorithm="fricas")
```

output
```
-1/4*(2*a^2*sqrt(1/16*I/a^4)*log(4*a^2*sqrt(1/16*I/a^4) + sqrt(I*sqrt(a^2*
x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(1/16*I/a^4)*log(-4*a^2*sqrt(1/16*I/a^4)
+ sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 2*a^2*sqrt(-1/16*I/a^4)*log(4*a^2
*sqrt(-1/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(-1/
16*I/a^4)*log(-4*a^2*sqrt(-1/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I
))) - (2*a^2*x^2 - I*a*x + 3)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a^2
```

Sympy [F]

$$\int e^{\frac{1}{2}i\arctan(ax)}x\,dx = \int x\sqrt{\frac{i\,(ax-i)}{\sqrt{a^2x^2+1}}}\,dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)*x,x)
```

output
```
Integral(x*sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int e^{\frac{1}{2}i\arctan(ax)}x\,dx = \int x\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}\,dx$$

input
```
integrate((((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x,x, algorithm="maxima")
```

output
```
integrate(x*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{1}{2}i\arctan(ax)}x\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{2}i\arctan(ax)}x\,dx = \int x\,\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}\,dx$$

input
```
int(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)
```

output
```
int(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int e^{\frac{1}{2}i\arctan(ax)}x\,dx = \int \sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,xdx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x,x)
```

3.78 $\int e^{\frac{1}{2}i\arctan(ax)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 195

$$\int e^{\frac{1}{2}i\arctan(ax)}\,dx=\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}-\frac{i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}+\frac{i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}-\frac{i\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)}{\sqrt{2}a}$$

output
```
I*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a-1/2*I*arctan(1-2^(1/2)*(1-I*a*x)^(1/4)
/(1+I*a*x)^(1/4))*2^(1/2)/a+1/2*I*arctan(1+2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*
x)^(1/4))*2^(1/2)/a-1/2*I*arctanh(2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)/
(1+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2)))*2^(1/2)/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.21

$$\int e^{\frac{1}{2}i\arctan(ax)}\,dx=-\frac{8ie^{\frac{5}{2}i\arctan(ax)}\,\text{Hypergeometric2F1}\left(\frac{5}{4},2,\frac{9}{4},-e^{2i\arctan(ax)}\right)}{5a}$$

input
```
Integrate[E^((I/2)*ArcTan[a*x]),x]
```

output
```
(((-8*I)/5)*E^(((5*I)/2)*ArcTan[a*x])*Hypergeometric2F1[5/4, 2, 9/4, -E^((
2*I)*ArcTan[a*x])])/a
```

Rubi [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.23, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5584, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{1}{2}i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5584}$$

$$\int \frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\int \frac{1}{\sqrt[4]{1-iax}(iax+1)^{3/4}}dx+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}$$

$$\Big\downarrow \text{73}$$

$$\frac{2i\int \frac{\sqrt{1-iax}}{(iax+1)^{3/4}}d\sqrt[4]{1-iax}}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}$$

$$\Big\downarrow \; 854$$

$$\frac{2i\int \frac{\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}$$

$$\Big\downarrow \; 826$$

$$\frac{2i\left(\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}$$

$$\Big\downarrow \; 1476$$

$$\frac{2i\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-\cdots}}{\sqrt[4]{iax\cdots}}\right.}{a}$$

$$\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}$$

$$\Big\downarrow \; 1082$$

$$\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+$$

$$\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}$$

$$\Big\downarrow \; 217$$

$$\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+$$

$$\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}$$

$$\Big\downarrow \; 1479$$

$$\frac{2i\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right.\right.}{\frac{a}{\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}}}$$

$\downarrow$ 25

$$\frac{2i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right.\right.}{\frac{a}{\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}}}$$

$\downarrow$ 27

$$\frac{2i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{1+i}}\right.}{\sqrt{2}}\right.\right.}{\frac{a}{\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}}}$$

$\downarrow$ 1103

$$\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{2\sqrt{2}}\right.\right.}{a}$$

$$\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}$$

input
```
Int[E^((I/2)*ArcTan[a*x]),x]
```

output
```
(I*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a + ((2*I)*((-(ArcTan[1 - (Sqrt[2]
*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 -
 I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2])/2 + (Log[1 + Sqrt[1 - I*a*x] -
(Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]) - Log[1 + Sqrt[
1 - I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]))/2
))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5584
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Maple [F]

$$\int \sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.07

$$\int e^{\frac{1}{2}i\arctan(ax)}\,dx = \frac{a\sqrt{\frac{i}{a^2}}\log\left(i\,a\sqrt{\frac{i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-a\sqrt{\frac{i}{a^2}}\log\left(-i\,a\sqrt{\frac{i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+a\sqrt{-\frac{i}{a^2}}\log\left(i\,a\sqrt{-\frac{i}{a^2}}+\sqrt{\ldots}\right.}{2\,a}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(a*sqrt(I/a^2)*log(I*a*sqrt(I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I
))) - a*sqrt(I/a^2)*log(-I*a*sqrt(I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
 I))) + a*sqrt(-I/a^2)*log(I*a*sqrt(-I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*
x + I))) - a*sqrt(-I/a^2)*log(-I*a*sqrt(-I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)
/(a*x + I))) + 2*(a*x + I)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a
```

Sympy [F]

$$\int e^{\frac{1}{2}i\arctan(ax)}\,dx = \int\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2),x)
```

output
```
Integral(sqrt((I*a*x + 1)/sqrt(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int e^{\frac{1}{2}i\arctan(ax)}\,dx = \int\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{1}{2}i\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{2}i\arctan(ax)}\,dx = \int \sqrt{\frac{1 + a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}\,dx$$

input
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)
```

output
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int e^{\frac{1}{2}i\arctan(ax)}\,dx = \int \sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

3.79 $\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 203

$$\begin{aligned}\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x}\,dx &= -2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)\\ &\quad-\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-2\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\\ &\quad+\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)\end{aligned}$$

output
```
-2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+2^(1/2)*arctan(1-2^(1/2)*(1-I*a
*x)^(1/4)/(1+I*a*x)^(1/4))-2^(1/2)*arctan(1+2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a
*x)^(1/4))-2*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+2^(1/2)*arctanh(2^(1
/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)/(1+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.48

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x}\,dx = -\frac{2(1-iax)^{3/4}\left(\sqrt[4]{2}(1+iax)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-iax)\right)+2\,\text{Hypergeometric2F1}\left(\frac{3}{4},1\right.}{3(1+iax)^{3/4}}$$

input
```
Integrate[E^((I/2)*ArcTan[a*x])/x,x]
```

output
```
(-2*(1 - I*a*x)^(3/4)*(2^(1/4)*(1 + I*a*x)^(3/4)*Hypergeometric2F1[3/4, 3/
4, 7/4, (1 - I*a*x)/2] + 2*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a
*x)]))/(3*(1 + I*a*x)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.71 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.31, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5585, 140, 73, 104, 756, 216, 219, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x}\,dx$$

$$\Big\downarrow \text{ 5585}$$

$$\int \frac{\sqrt[4]{1+iax}}{x\sqrt[4]{1-iax}}dx$$

$$\Big\downarrow \text{ 140}$$

$$ia\int \frac{1}{\sqrt[4]{1-iax}(iax+1)^{3/4}}dx+\int \frac{1}{x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx$$

$$\Big\downarrow \text{ 73}$$

$$\int \frac{1}{x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-4\int \frac{\sqrt{1-iax}}{(iax+1)^{3/4}}d\sqrt[4]{1-iax}$$

$$\Big\downarrow \text{104}$$

$$4\int \frac{1}{\frac{iax+1}{1-iax}-1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-4\int \frac{\sqrt{1-iax}}{(iax+1)^{3/4}}d\sqrt[4]{1-iax}$$

$$\Big\downarrow \text{756}$$

$$4\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-4\int \frac{\sqrt{1-iax}}{(iax+1)^{3/4}}d\sqrt[4]{1-iax}$$

$$\Big\downarrow \text{216}$$

$$4\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-4\int \frac{\sqrt{1-iax}}{(iax+1)^{3/4}}d\sqrt[4]{1-iax}$$

$$\Big\downarrow \text{219}$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-4\int \frac{\sqrt{1-iax}}{(iax+1)^{3/4}}d\sqrt[4]{1-iax}$$

$$\Big\downarrow \text{854}$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-4\int \frac{\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}$$

$$\Big\downarrow \text{826}$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-4\left(\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)$$

$$\Big\downarrow \text{1476}$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-4\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)-\frac{1}{2}\int 1\right.$$

$\Big\downarrow$ 1082

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-$$
$$4\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-ia}}{\sqrt[4]{iax+}}\right.$$

$\Big\downarrow$ 217

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-$$
$$4\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)$$

$\Big\downarrow$ 1479

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-$$
$$4\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{\sqrt{2}}\right.\right.$$

$\Big\downarrow$ 25

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-$$
$$4\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{1}}\right.}{\sqrt{2}}\right.\right.$$

$\Big\downarrow$ 27

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-$$
$$4\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\right.}{}\right.\right.$$

$$\Big\downarrow \text{1103}$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-$$
$$4\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}}-\right.\right.$$

input
```
Int[E^((I/2)*ArcTan[a*x])/x,x]
```

output
```
4*(-1/2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - ArcTanh[(1 + I*a*x)^
(1/4)/(1 - I*a*x)^(1/4)]/2) - 4*((-(ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))
/(1 + I*a*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 +
 I*a*x)^(1/4)]/Sqrt[2])/2 + (Log[1 + Sqrt[1 - I*a*x] - (Sqrt[2]*(1 - I*a*x
)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]) - Log[1 + Sqrt[1 - I*a*x] + (Sqrt[
2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}{x}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.20

$$\begin{aligned}\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x}\,dx =& \frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)\\&-\frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)\\&+\frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)\\&-\frac{1}{2}\sqrt{-4i}\log\left(-\frac{1}{2}\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)\\&-\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-i\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)\\&+i\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)+\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-1\right)\end{aligned}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

output
```
1/2*sqrt(4*I)*log(1/2*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 1
/2*sqrt(4*I)*log(-1/2*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 1
/2*sqrt(-4*I)*log(1/2*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) -
1/2*sqrt(-4*I)*log(-1/2*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
- log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - I*log(sqrt(I*sqrt(a^2*x^2
 + 1)/(a*x + I)) + I) + I*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) + l
og(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1)
```

Sympy [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x}\,dx = \int \frac{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x}\,dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x,x)
```

output
```
Integral(sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))/x, x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x}\,dx = \int \frac{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}{x}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x}\,dx = \int \frac{\sqrt{\frac{1+a\,x\,1i}{\sqrt{a^2\,x^2+1}}}}{x}\,dx$$

input
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x,x)
```

output
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x, x)
```

Reduce [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x}\,dx = \int \frac{\sqrt{aix+1}\,(a^2x^2+1)^{\frac{3}{4}}}{a^2x^3+x}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x)
```

output
```
int((sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4))/(a**2*x**3 + x),x)
```

3.80 $\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 92

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x} - ia\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - ia\,\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

output
```
-(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x-I*a*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1
/4))-I*a*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.77

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = -\frac{i(1-iax)^{3/4}\left(-3i+3ax+2ax\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{i+ax}{i-ax}\right)\right)}{3x(1+iax)^{3/4}}$$

input
```
Integrate[E^((I/2)*ArcTan[a*x])/x^2,x]
```

output
```
((-1/3*I)*(1 - I*a*x)^(3/4)*(-3*I + 3*a*x + 2*a*x*Hypergeometric2F1[3/4, 1
, 7/4, (I + a*x)/(I - a*x)]))/(x*(1 + I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5585, 105, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^2}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{\sqrt[4]{1+iax}}{x^2\sqrt[4]{1-iax}}dx$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{2}ia\int \frac{1}{x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}$$

$$\Big\downarrow \text{104}$$

$$2ia\int \frac{1}{\frac{iax+1}{1-iax}-1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}$$

$$\Big\downarrow \text{756}$$

$$2ia\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}$$

$$\Big\downarrow \text{216}$$

$$2ia\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}$$

$$\Big\downarrow 219$$

$$2ia\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}$$

input
```
Int[E^((I/2)*ArcTan[a*x])/x^2,x]
```

output
```
-(((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x) + (2*I)*a*(-1/2*ArcTan[(1 + I*a
*x)^(1/4)/(1 - I*a*x)^(1/4)] - ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)
]/2)
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}{x^2}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(64) = 128$.

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.64

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^2}\,dx$$
$$= \frac{-i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)+i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2}}{ax+}}\right.}{2\,x}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
1/2*(-I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + a*x*log(sqrt(I*
sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x
+ I)) - I) + I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1) - 2*(-I*a*
x + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x
```

Sympy [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x^2}\,dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))/x**2, x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}{x^2}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}{x^2}\,dx$$

input
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^2,x)
```

output
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^2, x)
```

Reduce [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}{x^2}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x)
```

3.81 $\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 132

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = -\frac{ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x}-\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2}+\frac{1}{4}a^2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{1}{4}a^2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

output
```
-1/4*I*a*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x-1/2*(1-I*a*x)^(3/4)*(1+I*a*x)^(
5/4)/x^2+1/4*a^2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+1/4*a^2*arctanh((
1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.61

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = \frac{(1-iax)^{3/4}\left(-6-15iax+9a^2x^2+2a^2x^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{i+ax}{i-ax}\right)\right)}{12x^2(1+iax)^{3/4}}$$

input
```
Integrate[E^((I/2)*ArcTan[a*x])/x^3,x]
```

output
```
((1 - I*a*x)^(3/4)*(-6 - (15*I)*a*x + 9*a^2*x^2 + 2*a^2*x^2*Hypergeometric
2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)]))/(12*x^2*(1 + I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5585, 107, 105, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^3}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{\sqrt[4]{1+iax}}{x^3\sqrt[4]{1-iax}}dx$$

$$\Big\downarrow \text{107}$$

$$\frac{1}{4}ia\int \frac{\sqrt[4]{iax+1}}{x^2\sqrt[4]{1-iax}}dx-\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2}$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{4}ia\left(\frac{1}{2}ia\int \frac{1}{x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2}$$

$$\Big\downarrow \text{104}$$

$$\frac{1}{4}ia\left(2ia\int \frac{1}{\frac{iax+1}{1-iax}-1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2}$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{4}ia\left(2ia\left(-\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int\frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2}$$

$\downarrow$ 216

$$\frac{1}{4}ia\left(2ia\left(-\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2}$$

$\downarrow$ 219

$$\frac{1}{4}ia\left(2ia\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{(1-iax)^{3/4}(1+iax)^{5/4}}{2x^2}$$

input
```
Int[E^((I/2)*ArcTan[a*x])/x^3,x]
```

output
```
-1/2*((1 - I*a*x)^(3/4)*(1 + I*a*x)^(5/4))/x^2 + (I/4)*a*(-(((1 - I*a*x)^(
3/4)*(1 + I*a*x)^(1/4))/x) + (2*I)*a*(-1/2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I
*a*x)^(1/4)] - ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/2))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}{x^3}\,dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.33

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = \frac{a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-a^2x^2\log\left(\sqrt{\right.}\right.}{8\,x^2}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
1/8*(a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + I*a^2*x^2*log(
sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - I*a^2*x^2*log(sqrt(I*sqrt(a^2*x
^2 + 1)/(a*x + I)) - I) - a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))
- 1) - 2*(3*a^2*x^2 + I*a*x + 2)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^2
```

Sympy [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x^3}\,dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**3,x)
```

output
```
Integral(sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))/x**3, x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}{x^3}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}{x^3}\,dx$$

input
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^3,x)
```

output
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^3, x)
```

Reduce [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}{x^3}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
```

3.82 $\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 170

$$\begin{aligned}\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^4}\,dx =& -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}-\frac{5ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2}\\&+\frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x}\\&+\frac{3}{8}ia^3\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{3}{8}ia^3\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\end{aligned}$$

output

```
-1/3*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^3-5/12*I*a*(1-I*a*x)^(3/4)*(1+I*a*x
)^(1/4)/x^2+11/24*a^2*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x+3/8*I*a^3*arctan((
1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+3/8*I*a^3*arctanh((1+I*a*x)^(1/4)/(1-I*a*x
)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.55

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = \frac{(1-iax)^{3/4}\left(-8-18iax+21a^2x^2+11ia^3x^3+6ia^3x^3\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{i+ax}{i-ax}\right)\right)}{24x^3(1+iax)^{3/4}}$$

input
```
Integrate[E^((I/2)*ArcTan[a*x])/x^4,x]
```

output
```
((1 - I*a*x)^(3/4)*(-8 - (18*I)*a*x + 21*a^2*x^2 + (11*I)*a^3*x^3 + (6*I)*
a^3*x^3*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)]))/(24*x^3*(1 +
 I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {5585, 110, 27, 168, 27, 168, 27, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^4}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{\sqrt[4]{1+iax}}{x^4\sqrt[4]{1-iax}}dx$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{3}\int \frac{a(5i-4ax)}{2x^3\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}a\int \frac{5i-4ax}{x^3\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{6}a\left(-\frac{1}{2}\int \frac{a(10iax+11)}{2x^2\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{2x^2}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}a\left(-\frac{1}{4}a\int \frac{10iax+11}{x^2\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{2x^2}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{6}a\left(-\frac{1}{4}a\left(-\int -\frac{9ia}{2x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{11(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{2x^2}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}a\left(-\frac{1}{4}a\left(\frac{9}{2}ia\int \frac{1}{x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{11(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{2x^2}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}$$

$$\Big\downarrow \text{104}$$

$$\frac{1}{6}a\left(-\frac{1}{4}a\left(18ia\int \frac{1}{\frac{iax+1}{1-iax}-1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{11(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{2x^2}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{6}a\left(-\frac{1}{4}a\left(18ia\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{11(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{6}a\left(-\frac{1}{4}a\left(18ia\left(-\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-\frac{11(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{5i(1\right.\right.$$
$$\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{6}a\left(-\frac{1}{4}a\left(18ia\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-\frac{11(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{5i(1-ia\right.$$
$$\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}$$

input
```
Int[E^((I/2)*ArcTan[a*x])/x^4,x]
```

output
```
-1/3*((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x^3 + (a*((((-5*I)/2)*(1 - I*a*
x)^(3/4)*(1 + I*a*x)^(1/4))/x^2 - (a*((-11*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(
1/4))/x + (18*I)*a*(-1/2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - Arc
Tanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/2)))/4))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}{x^4}\,dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.08

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = \frac{9i\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-9\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+9\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-9i\,a^3x^3\log}{48\,x^3}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="fricas")
```

output
```
1/48*(9*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 9*a^3*x^3
*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 9*a^3*x^3*log(sqrt(I*sqrt(
a^2*x^2 + 1)/(a*x + I)) - I) - 9*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a
*x + I)) - 1) - 2*(11*I*a^3*x^3 - a^2*x^2 + 2*I*a*x + 8)*sqrt(I*sqrt(a^2*x
^2 + 1)/(a*x + I)))/x^3
```

Sympy [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x^4}\,dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**4,x)
```

output
```
Integral(sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))/x**4, x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}{x^4}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}{x^4}\,dx$$

input
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^4,x)
```

output
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^4, x)
```

Reduce [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}{x^4}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
```

3.83 $\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 202

$$\begin{aligned}\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = & -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}-\frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x^3} \\ & +\frac{29a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{96x^2}+\frac{83ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{192x} \\ & -\frac{11}{64}a^4\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{11}{64}a^4\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\end{aligned}$$

output
```
-1/4*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^4-7/24*I*a*(1-I*a*x)^(3/4)*(1+I*a*x
)^(1/4)/x^3+29/96*a^2*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^2+83/192*I*a^3*(1-
I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x-11/64*a^4*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^
(1/4))-11/64*a^4*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.49

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = -\frac{(1-iax)^{3/4}\left(48+104iax-114a^2x^2-141ia^3x^3+83a^4x^4+22a^4x^4\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{i+ax}{i-ax}\right)\right)}{192x^4(1+iax)^{3/4}}$$

input
```
Integrate[E^((I/2)*ArcTan[a*x])/x^5,x]
```

output
```
-1/192*((1 - I*a*x)^(3/4)*(48 + (104*I)*a*x - 114*a^2*x^2 - (141*I)*a^3*x^
3 + 83*a^4*x^4 + 22*a^4*x^4*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I -
a*x)]))/(x^4*(1 + I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {5585, 110, 27, 168, 27, 168, 27, 168, 27, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^5}\,dx$$

$$\Big\downarrow \text{ 5585}$$

$$\int \frac{\sqrt[4]{1+iax}}{x^5\sqrt[4]{1-iax}}dx$$

$$\Big\downarrow \text{ 110}$$

$$\frac{1}{4}\int \frac{a(7i-6ax)}{2x^4\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{8}a\int \frac{7i-6ax}{x^4\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}$$

$\big\downarrow$ 168

$$\frac{1}{8}a\left(-\frac{1}{3}\int \frac{a(28iax+29)}{2x^3\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{7i(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}$$

$\big\downarrow$ 27

$$\frac{1}{8}a\left(-\frac{1}{6}a\int \frac{28iax+29}{x^3\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{7i(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}$$

$\big\downarrow$ 168

$$\frac{1}{8}a\left(-\frac{1}{6}a\left(-\frac{1}{2}\int -\frac{a(83i-58ax)}{2x^2\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{29(1-iax)^{3/4}\sqrt[4]{1+iax}}{2x^2}\right)-\frac{7i(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}$$

$\big\downarrow$ 27

$$\frac{1}{8}a\left(-\frac{1}{6}a\left(\frac{1}{4}a\int \frac{83i-58ax}{x^2\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{29(1-iax)^{3/4}\sqrt[4]{1+iax}}{2x^2}\right)-\frac{7i(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}$$

$\big\downarrow$ 168

$$\frac{1}{8}a\Bigg(-\frac{1}{6}a\Bigg(\frac{1}{4}a\left(-\int \frac{33a}{2x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{83i(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{29(1-iax)^{3/4}\sqrt[4]{1+iax}}{2x^2}\Bigg)-7 \ldots$$
$$\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}$$

$\big\downarrow$ 27

$$\frac{1}{8}a\Bigg(-\frac{1}{6}a\Bigg(\frac{1}{4}a\left(-\frac{33}{2}a\int \frac{1}{x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{83i(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{29(1-iax)^{3/4}\sqrt[4]{1+iax}}{2x^2}\Bigg) \ldots$$
$$\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}$$

$\big\downarrow$ 104

$$\frac{1}{8}a\left(-\frac{1}{6}a\left(\frac{1}{4}a\left(-66a\int \frac{1}{\frac{iax+1}{1-iax}-1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{83i(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{29(1-iax)^{3/4}\sqrt[4]{1+iax}}{2x^2}\right)-\frac{7i(1\right.$$
$$\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}$$

$\big\downarrow$ 756

$$\frac{1}{8}a\left(-\frac{1}{6}a\left(\frac{1}{4}a\left(-66a\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{83i(1-iax)^{3/4}\sqrt[4]{1+ia}}{x}\right.\right.\right.$$
$$\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}$$

$\big\downarrow$ 216

$$\frac{1}{8}a\left(-\frac{1}{6}a\left(\frac{1}{4}a\left(-66a\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-\frac{83i(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)\right.\right.$$
$$\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}$$

$\big\downarrow$ 219

$$\frac{1}{8}a\left(-\frac{1}{6}a\left(\frac{1}{4}a\left(-66a\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-\frac{83i(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{29\right.\right.$$
$$\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}$$

input
```
Int[E^((I/2)*ArcTan[a*x])/x^5,x]
```

output
```
-1/4*((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x^4 + (a*((((-7*I)/3)*(1 - I*a*
x)^(3/4)*(1 + I*a*x)^(1/4))/x^3 - (a*((-29*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(
1/4))/(2*x^2) + (a*(((-83*I)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x - 66*a
*(-1/2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - ArcTanh[(1 + I*a*x)^(
1/4)/(1 - I*a*x)^(1/4)]/2)))/4))/6))/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}{x^5}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.95

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = -\frac{33\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+33i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-33i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-33}{384\,x^4}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="fricas")
```

output
```
-1/384*(33*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 33*I*a^4
*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 33*I*a^4*x^4*log(sqrt(
I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 33*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 +
 1)/(a*x + I)) - 1) - 2*(83*a^4*x^4 + 25*I*a^3*x^3 + 2*a^2*x^2 - 8*I*a*x -
 48)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^4
```

Sympy [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x^5}\,dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**5,x)
```

output
```
Integral(sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))/x**5, x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}{x^5}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{\sqrt{\frac{1+a\,x\,1i}{\sqrt{a^2\,x^2+1}}}}{x^5}\,dx$$

input
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^5,x)
```

output
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^5, x)
```

Reduce [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^5}\,dx$$
$$= \frac{66\sqrt{aix+1}\,(a^2x^2+1)^{\frac{3}{4}}\,a^3i\,x^3+100\sqrt{aix+1}\,(a^2x^2+1)^{\frac{3}{4}}\,a^2x^2-16\sqrt{aix+1}\,(a^2x^2+1)^{\frac{3}{4}}\,aix-96\sqrt{ai}}{384x^4}$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x)
```

output
```
(66*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4)*a**3*i*x**3 + 100*sqrt(a*i*x +
1)*(a**2*x**2 + 1)**(3/4)*a**2*x**2 - 16*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**
(3/4)*a*i*x - 96*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4) + 33*int((sqrt(a*i
*x + 1)*(a**2*x**2 + 1)**(3/4))/(a**2*x**3 + x),x)*a**4*x**4 - 66*int((sqr
t(a*i*x + 1)*(a**2*x**2 + 1)**(3/4))/(a**2*x**2 + 1),x)*a**5*i*x**4)/(384*
x**4)
```

3.84 $\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 240

$$\begin{aligned}\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^6}\,dx = &-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5}-\frac{9ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{40x^4}\\&+\frac{11a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{48x^3}+\frac{269ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{960x^2}\\&-\frac{611a^4(1-iax)^{3/4}\sqrt[4]{1+iax}}{1920x}\\&-\frac{31}{128}ia^5\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{31}{128}ia^5\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\end{aligned}$$

output
```
-1/5*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^5-9/40*I*a*(1-I*a*x)^(3/4)*(1+I*a*x
)^(1/4)/x^4+11/48*a^2*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^3+269/960*I*a^3*(1
-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^2-611/1920*a^4*(1-I*a*x)^(3/4)*(1+I*a*x)^(
1/4)/x-31/128*I*a^5*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-31/128*I*a^5*a
rctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.46

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^6}\,dx = \frac{(1-iax)^{3/4}\left(-384-816iax+872a^2x^2+978ia^3x^3-1149a^4x^4-611ia^5x^5-310ia^5x^5\,\text{Hypergeometric}\right.}{1920x^5(1+iax)^{3/4}}$$

input
```
Integrate[E^((I/2)*ArcTan[a*x])/x^6,x]
```

output
```
((1 - I*a*x)^(3/4)*(-384 - (816*I)*a*x + 872*a^2*x^2 + (978*I)*a^3*x^3 - 1
149*a^4*x^4 - (611*I)*a^5*x^5 - (310*I)*a^5*x^5*Hypergeometric2F1[3/4, 1,
7/4, (I + a*x)/(I - a*x)]))/(1920*x^5*(1 + I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.03, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.938$, Rules used = {5585, 110, 27, 168, 27, 168, 27, 168, 27, 168, 27, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^6}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{\sqrt[4]{1+iax}}{x^6\sqrt[4]{1-iax}}dx$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{5}\int \frac{a(9i-8ax)}{2x^5\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}a\int \frac{9i-8ax}{x^5\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5}$$

$$\Big\downarrow \text{ 168}$$

$$\frac{1}{10}a\left(-\frac{1}{4}\int \frac{a(54iax+55)}{2x^4\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{9i(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{10}a\left(-\frac{1}{8}a\int \frac{54iax+55}{x^4\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{9i(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5}$$

$$\Big\downarrow \text{ 168}$$

$$\frac{1}{10}a\left(-\frac{1}{8}a\left(-\frac{1}{3}\int -\frac{a(269i-220ax)}{2x^3\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{55(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}\right)-\frac{9i(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}\right)-$$
$$\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{10}a\left(-\frac{1}{8}a\left(\frac{1}{6}a\int \frac{269i-220ax}{x^3\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{55(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}\right)-\frac{9i(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}\right)-$$
$$\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5}$$

$$\Big\downarrow \text{ 168}$$

$$\frac{1}{10}a\left(-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{2}\int \frac{a(538iax+611)}{2x^2\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{269i(1-iax)^{3/4}\sqrt[4]{1+iax}}{2x^2}\right)-\frac{55(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}\right)\right.\right.$$
$$\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{10}a\left(-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{4}a\int \frac{538iax+611}{x^2\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{269i(1-iax)^{3/4}\sqrt[4]{1+iax}}{2x^2}\right)-\frac{55(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}\right)\right.\right.$$
$$\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5}$$

$$\Big\downarrow \text{ 168}$$

$$\frac{1}{10}a\left(-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{4}a\left(-\int -\frac{465ia}{2x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{611(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{269i(1-iax)^{3/4}\sqrt[4]{1}}{2x^2}\right.\right.\right.\right.$$
$$\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5}$$

$\downarrow$ 27

$$\frac{1}{10}a\left(-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{4}a\left(\frac{465}{2}ia\int \frac{1}{x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{611(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{269i(1-iax)^{3/4}\sqrt[4]{1}}{2x^2}\right.\right.\right.\right.$$
$$\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5}$$

$\downarrow$ 104

$$\frac{1}{10}a\left(-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{4}a\left(930ia\int \frac{1}{\frac{iax+1}{1-iax}-1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{611(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{269i(1-iax)^{3/4}\sqrt[4]{1+ia}}{2x^2}\right.\right.\right.\right.$$
$$\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5}$$

$\downarrow$ 756

$$\frac{1}{10}a\left(-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{4}a\left(930ia\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{611(1-iax)^{3/}}{x}\right.\right.\right.\right.$$
$$\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5}$$

$\downarrow$ 216

$$\frac{1}{10}a\left(-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{4}a\left(930ia\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-\frac{611(1-iax)^{3/4}\sqrt[4]{1}}{x}\right.\right.\right.\right.$$
$$\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5}$$

$\downarrow$ 219

$$\frac{1}{10}a\left(-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{4}a\left(930ia\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-\frac{611(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right.\right.\right.\right.$$
$$\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{5x^5}$$

input
```
Int[E^((I/2)*ArcTan[a*x])/x^6,x]
```

output
```
-1/5*((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x^5 + (a*((((-9*I)/4)*(1 - I*a*
x)^(3/4)*(1 + I*a*x)^(1/4))/x^4 - (a*((-55*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(
1/4))/(3*x^3) + (a*((((-269*I)/2)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x^2
 - (a*((-611*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x + (930*I)*a*(-1/2*ArcT
an[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - ArcTanh[(1 + I*a*x)^(1/4)/(1 - I
*a*x)^(1/4)]/2)))/4))/6))/8))/10
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}{x^6}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^6,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^6,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.83

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^6}\,dx$$
$$= \frac{-465i\,a^5x^5\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+465\,a^5x^5\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-465\,a^5x^5\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)+}{384}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^6,x, algorithm="fricas")
```

output
```
1/3840*(-465*I*a^5*x^5*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 465*
a^5*x^5*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 465*a^5*x^5*log(sqr
t(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) + 465*I*a^5*x^5*log(sqrt(I*sqrt(a^2*
x^2 + 1)/(a*x + I)) - 1) - 2*(-611*I*a^5*x^5 + 73*a^4*x^4 - 98*I*a^3*x^3 -
 8*a^2*x^2 + 48*I*a*x + 384)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^5
```

Sympy [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^6}\,dx = \int \frac{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x^6}\,dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**6,x)
```

output
```
Integral(sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))/x**6, x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^6}\,dx = \int \frac{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}{x^6}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))/x^6, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^6}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^6,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^6}\,dx = \int \frac{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}{x^6}\,dx$$

input
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^6,x)
```

output
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)/x^6, x)
```

Reduce [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(ax)}}{x^6}\,dx$$
$$= \frac{-1860\sqrt{aix+1}\,(a^2x^2+1)^{\frac{3}{4}}\,a^5 i\,x^5 - 930\sqrt{aix+1}\,(a^2x^2+1)^{\frac{3}{4}}\,a^4x^4 + 292\sqrt{aix+1}\,(a^2x^2+1)^{\frac{3}{4}}\,a^3 i\,x^3 +}{}$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^6,x)
```

output
```
( - 1860*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4)*a**5*i*x**5 - 930*sqrt(a*i
*x + 1)*(a**2*x**2 + 1)**(3/4)*a**4*x**4 + 292*sqrt(a*i*x + 1)*(a**2*x**2
+ 1)**(3/4)*a**3*i*x**3 + 784*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4)*a**2*
x**2 - 96*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4)*a*i*x - 768*sqrt(a*i*x +
1)*(a**2*x**2 + 1)**(3/4) + 3720*int((sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/
4)*x)/(a**2*x**2 + 1),x)*a**7*i*x**5 + 465*int((sqrt(a*i*x + 1)*(a**2*x**2
 + 1)**(3/4))/(a**2*x**3 + x),x)*a**5*i*x**5)/(3840*x**5)
```

3.85 $\int e^{\frac{3}{2}i\arctan(ax)}x^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 291

$$\begin{aligned}\int e^{\frac{3}{2}i\arctan(ax)}x^3\,dx = & -\frac{41\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4}-\frac{15\sqrt[4]{1-iax}(1+iax)^{7/4}}{32a^4} \\ & +\frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2}+\frac{(1-iax)^{5/4}(1+iax)^{7/4}}{8a^4} \\ & +\frac{123\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4} \\ & -\frac{123\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4} \\ & -\frac{123\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)}{64\sqrt{2}a^4}\end{aligned}$$

output

```
-41/64*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/a^4-15/32*(1-I*a*x)^(1/4)*(1+I*a*x)
^(7/4)/a^4+1/4*x^2*(1-I*a*x)^(1/4)*(1+I*a*x)^(7/4)/a^2+1/8*(1-I*a*x)^(5/4)
*(1+I*a*x)^(7/4)/a^4+123/128*arctan(1-2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1
/4))*2^(1/2)/a^4-123/128*arctan(1+2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))
*2^(1/2)/a^4-123/128*arctanh(2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)/(1+(1
-I*a*x)^(1/2)/(1+I*a*x)^(1/2)))*2^(1/2)/a^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.15 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.51

$$\int e^{\frac{3}{2}i\arctan(ax)}x^3\,dx$$
$$= \frac{\sqrt[4]{1-iax}\left(a^2x^2(1+iax)^{3/4}+ia^3x^3(1+iax)^{3/4}-24\,2^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{11}{4},\frac{1}{4},\frac{5}{4},\frac{1}{2}(1-iax)\right)+\right.}{4a}$$

input
```
Integrate[E^(((3*I)/2)*ArcTan[a*x])*x^3,x]
```

output
```
((1 - I*a*x)^(1/4)*(a^2*x^2*(1 + I*a*x)^(3/4) + I*a^3*x^3*(1 + I*a*x)^(3/4
) - 24*2^(3/4)*Hypergeometric2F1[-11/4, 1/4, 5/4, (1 - I*a*x)/2] + 8*2^(3/
4)*Hypergeometric2F1[-7/4, 1/4, 5/4, (1 - I*a*x)/2] + 2*2^(3/4)*Hypergeome
tric2F1[-3/4, 1/4, 5/4, (1 - I*a*x)/2]))/(4*a^4)
```

Rubi [A] (warning: unable to verify)

Time = 0.79 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.13, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.938$, Rules used = {5585, 111, 27, 164, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{\frac{3}{2}i\arctan(ax)}\,dx$$
$$\Big\downarrow \text{5585}$$
$$\int \frac{x^3(1+iax)^{3/4}}{(1-iax)^{3/4}}dx$$
$$\Big\downarrow \text{111}$$
$$\frac{\int -\frac{x(iax+1)^{3/4}(3iax+4)}{2(1-iax)^{3/4}}dx}{4a^2}+\frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2}$$
$$\Big\downarrow \text{27}$$

$$\frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2}-\frac{\int \frac{x(iax+1)^{3/4}(3iax+4)}{(1-iax)^{3/4}}dx}{8a^2}$$

$\big\downarrow$ 164

$$\frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2}-\frac{\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{4a^2}-\frac{41i\int \frac{(iax+1)^{3/4}}{(1-iax)^{3/4}}dx}{8a}}{8a^2}$$

$\big\downarrow$ 60

$$\frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2}-\frac{\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{4a^2}-\frac{41i\left(\frac{3}{2}\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{iax+1}}dx+\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)}{8a}}{8a^2}$$

$\big\downarrow$ 73

$$\frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2}-\frac{\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{4a^2}-\frac{41i\left(\frac{6i\int \frac{1}{\sqrt[4]{iax+1}}d\sqrt[4]{1-iax}}{a}+\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)}{8a}}{8a^2}$$

$\big\downarrow$ 770

$$\frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2}-\frac{\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{4a^2}-\frac{41i\left(\frac{6i\int \frac{1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{a}+\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)}{8a}}{8a^2}$$

$\big\downarrow$ 755

$$\frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2}-\frac{\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{4a^2}-\frac{41i\left(\frac{6i\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)}{8a}}{8a^2}$$

$\big\downarrow$ 1476

$$\frac{\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{4a^2}-\frac{41i\left(\frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2}-\frac{6i\left(\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int\frac{}{\sqrt{1-iax}+\sqrt{}}\right.\right.}{a}\right.}{8a}}{8a^2}$$

$\downarrow$ 1082

$$\frac{\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{4a^2}-\frac{41i\left(\frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2}-\frac{6i\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}\right.}{a}\right.}{8a}}{8a^2}$$

$\downarrow$ 217

$$\frac{\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{4a^2}-\frac{41i\left(\frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2}-\frac{6i\left(\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)\right)}{a}+\right.}{8a}}{8a^2}$$

$\downarrow$ 1479

$$\frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2}-$$

$$\frac{\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{4a^2}-\frac{41i\left(\frac{6i\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-i}}{\sqrt[4]{iax+}}}{2\sqrt{2}}\right.\right.}{a}\right.}{8a}}{8a^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2}-$$

$$\frac{\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{4a^2}-\frac{41i\left(\frac{6i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)\right.}{a}\right.}{8a}}{8a^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{4a^2}-\frac{41i\left(\frac{6i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right.\right.}{a}\right.}{8a}}{8a^2}\ \frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2}-$$

$\downarrow$ 1103

$$\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}(11+4iax)}{4a^2}-\frac{41i\left(\frac{6i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right.}{2\sqrt{2}}\right.\right.}{a}\right.}{8a}}{8a^2}\ \frac{x^2\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^2}-$$

input

```
Int[E^(((3*I)/2)*ArcTan[a*x])*x^3,x]
```

output

```
(x^2*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(7/4))/(4*a^2) - (((1 - I*a*x)^(1/4)*(1
 + I*a*x)^(7/4)*(11 + (4*I)*a*x))/(4*a^2) - (((41*I)/8)*((I*(1 - I*a*x)^(1
/4)*(1 + I*a*x)^(3/4))/a + ((6*I)*((-(ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4
))/(1 + I*a*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1
 + I*a*x)^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 + Sqrt[1 - I*a*x] - (Sqrt[2]*(1
- I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - I*a*x] + (Sq
rt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]))/2))/a))/a)/(8*a^2
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} x^3 dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^3,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.87

$$\int e^{\frac{3}{2}i\arctan(ax)}x^3\,dx = \frac{32\,a^4\sqrt{\frac{15129i}{4096\,a^8}}\log\left(\frac{64}{123}i\,a^4\sqrt{\frac{15129i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-32\,a^4\sqrt{\frac{15129i}{4096\,a^8}}\log\left(-\frac{64}{123}i\,a^4\sqrt{\frac{15129i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)}{}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^3,x, algorithm="fricas")
```

output
```
1/64*(32*a^4*sqrt(15129/4096*I/a^8)*log(64/123*I*a^4*sqrt(15129/4096*I/a^8
) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 32*a^4*sqrt(15129/4096*I/a^8)*l
og(-64/123*I*a^4*sqrt(15129/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
I))) - 32*a^4*sqrt(-15129/4096*I/a^8)*log(64/123*I*a^4*sqrt(-15129/4096*I/
a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 32*a^4*sqrt(-15129/4096*I/a^
8)*log(-64/123*I*a^4*sqrt(-15129/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a
*x + I))) + (16*I*a^3*x^3 + 24*a^2*x^2 - 30*I*a*x - 63)*sqrt(a^2*x^2 + 1)*
sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a^4
```

Sympy [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}x^3\,dx = \int x^3\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)*x**3,x)
```

output
```
Integral(x**3*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}x^3\,dx = \int x^3\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^3,x, algorithm="maxima")
```

output
```
integrate(x^3*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{3}{2}i\arctan(ax)}x^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}i\arctan(ax)}x^3\,dx = \int x^3\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}\,dx$$

input
```
int(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
```

output
```
int(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

Reduce [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}x^3\,dx = \int\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}x^3dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^3,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^3,x)
```

3.86 $\int e^{\frac{3}{2}i\arctan(ax)}x^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 268

$$\int e^{\frac{3}{2}i\arctan(ax)}x^2\,dx=-\frac{17i\sqrt[4]{1-iax}(1+iax)^{3/4}}{24a^3}-\frac{i\sqrt[4]{1-iax}(1+iax)^{7/4}}{4a^3}+\frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2}+\frac{17i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}-\frac{17i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}-\frac{17i\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)}{8\sqrt{2}a^3}$$

output
```
-17/24*I*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/a^3-1/4*I*(1-I*a*x)^(1/4)*(1+I*a*
x)^(7/4)/a^3+1/3*x*(1-I*a*x)^(1/4)*(1+I*a*x)^(7/4)/a^2+17/16*I*arctan(1-2^
(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))*2^(1/2)/a^3-17/16*I*arctan(1+2^(1/2
)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))*2^(1/2)/a^3-17/16*I*arctanh(2^(1/2)*(1-
I*a*x)^(1/4)/(1+I*a*x)^(1/4)/(1+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2)))*2^(1/2)/
a^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.31

$$\int e^{\frac{3}{2}i\arctan(ax)}x^2\,dx = \frac{\sqrt[4]{1-iax}\left((1+iax)^{3/4}\left(-3i+7ax+4ia^2x^2\right)-34i2^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{4},\frac{5}{4},\frac{1}{2}(1-iax)\right)\right)}{12a^3}$$

input
```
Integrate[E^(((3*I)/2)*ArcTan[a*x])*x^2,x]
```

output
```
((1 - I*a*x)^(1/4)*((1 + I*a*x)^(3/4)*(-3*I + 7*a*x + (4*I)*a^2*x^2) - (34
*I)*2^(3/4)*Hypergeometric2F1[-3/4, 1/4, 5/4, (1 - I*a*x)/2]))/(12*a^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.75 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.18, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.938, Rules used = {5585, 101, 27, 90, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2e^{\frac{3}{2}i\arctan(ax)}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{x^2(1+iax)^{3/4}}{(1-iax)^{3/4}}dx$$

$$\Big\downarrow \text{101}$$

$$\frac{\int -\frac{(iax+1)^{3/4}(3iax+2)}{2(1-iax)^{3/4}}dx}{3a^2}+\frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2}-\frac{\int \frac{(iax+1)^{3/4}(3iax+2)}{(1-iax)^{3/4}}dx}{6a^2}$$

$$\Big\downarrow \; 90$$

$$\frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2}-\frac{\frac{17}{4}\int \frac{(iax+1)^{3/4}}{(1-iax)^{3/4}}dx+\frac{3i\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a}}{6a^2}$$

$$\Big\downarrow \; 60$$

$$\frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2}-\frac{\frac{17}{4}\left(\frac{3}{2}\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{iax+1}}dx+\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)+\frac{3i\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a}}{6a^2}$$

$$\Big\downarrow \; 73$$

$$\frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2}-\frac{\frac{17}{4}\left(\frac{6i\int \frac{1}{\sqrt[4]{iax+1}}d\sqrt[4]{1-iax}}{a}+\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)+\frac{3i\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a}}{6a^2}$$

$$\Big\downarrow \; 770$$

$$\frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2}-\frac{\frac{17}{4}\left(\frac{6i\int \frac{1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{a}+\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)+\frac{3i\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a}}{6a^2}$$

$$\Big\downarrow \; 755$$

$$\frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2}-\frac{\frac{17}{4}\left(\frac{6i\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)+\frac{3i\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a}}{6a^2}$$

$$\Big\downarrow \; 1476$$

$$\frac{\frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2}-\frac{17}{4}\left(\frac{6i\left(\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int\frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)\right)}{a}\cdots\right.}{6a^2}$$

$\downarrow$ 1082

$$\frac{\frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2}-\frac{17}{4}\left(\frac{6i\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+\frac{i\sqrt[4]{1-iax}(\cdots}{a}\cdots\right.}{6a^2}$$

$\downarrow$ 217

$$\frac{\frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2}-\frac{17}{4}\left(\frac{6i\left(\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)\right)}{a}+\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)+\frac{3\cdots}{}}{6a^2}$$

$\downarrow$ 1479

$$\frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2}-\frac{\frac{17}{4}\Bigg(\frac{6i\Bigg(\frac{1}{2}\Bigg(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\Bigg)+\frac{1}{2}\Bigg(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\cdots}{a}\cdots}{6a^2}$$

$\downarrow$ 25

$$\frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2}-\frac{\frac{17}{4}\Bigg(\frac{6i\Bigg(\frac{1}{2}\Bigg(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\Bigg)+\frac{1}{2}\Bigg(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\cdots}{a}\cdots}{6a^2}$$

$\downarrow$ 27

$$\frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2}-\frac{\frac{17}{4}\left(\frac{6i\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\right.\right.}{a}\right.}{6a^2}$$

$$\Big\downarrow \text{1103}$$

$$\frac{x\sqrt[4]{1-iax}(1+iax)^{7/4}}{3a^2}-\frac{\frac{17}{4}\left(\frac{6i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right.}{2\sqrt{2}}\right.\right.}{a}\right.}{6a^2}$$

input
```
Int[E^(((3*I)/2)*ArcTan[a*x])*x^2,x]
```

output
```
(x*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(7/4))/(3*a^2) - ((((3*I)/2)*(1 - I*a*x)^
(1/4)*(1 + I*a*x)^(7/4))/a + (17*((I*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/
a + ((6*I)*((-(ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/S
qrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2
])/2 + (-1/2*Log[1 + Sqrt[1 - I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*
a*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4)
)/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]))/2))/a))/4)/(6*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} x^2dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^2,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.92

$$\int e^{\frac{3}{2}i\arctan(ax)}x^2\,dx = -\frac{12\,a^3\sqrt{\frac{289i}{64\,a^6}}\log\left(\frac{8}{17}\,a^3\sqrt{\frac{289i}{64\,a^6}}+\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3\sqrt{\frac{289i}{64\,a^6}}\log\left(-\frac{8}{17}\,a^3\sqrt{\frac{289i}{64\,a^6}}+\sqrt{\frac{i\,\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3}{}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^2,x, algorithm="fricas")
```

output
```
-1/24*(12*a^3*sqrt(289/64*I/a^6)*log(8/17*a^3*sqrt(289/64*I/a^6) + sqrt(I*
sqrt(a^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(289/64*I/a^6)*log(-8/17*a^3*sq
rt(289/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(-289
/64*I/a^6)*log(8/17*a^3*sqrt(-289/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*
x + I))) + 12*a^3*sqrt(-289/64*I/a^6)*log(-8/17*a^3*sqrt(-289/64*I/a^6) +
sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - sqrt(a^2*x^2 + 1)*(8*I*a^2*x^2 + 14
*a*x - 23*I)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a^3
```

Sympy [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}x^2\,dx=\int x^2\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)*x**2,x)
```

output
```
Integral(x**2*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}x^2\,dx=\int x^2\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^2,x, algorithm="maxima")
```

output
```
integrate(x^2*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{3}{2}i\arctan(ax)}x^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}i\arctan(ax)}x^2\,dx=\int x^2\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}\,dx$$

input
```
int(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
```

output
```
int(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

Reduce [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}x^2\,dx=\int\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}x^2dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^2,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^2,x)
```

3.87 $\int e^{\frac{3}{2}i\arctan(ax)}x\,dx$

Optimal result

Integrand size = 14, antiderivative size = 226

$$\int e^{\frac{3}{2}i\arctan(ax)}x\,dx=\frac{3\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2}+\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2}-\frac{9\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}+\frac{9\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}+\frac{9\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)}{4\sqrt{2}a^2}$$

output
```
3/4*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/a^2+1/2*(1-I*a*x)^(1/4)*(1+I*a*x)^(7/4
)/a^2-9/8*arctan(1-2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))*2^(1/2)/a^2+9/
8*arctan(1+2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))*2^(1/2)/a^2+9/8*arctan
h(2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)/(1+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/
2)))*2^(1/2)/a^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.27

$$\int e^{\frac{3}{2}i\arctan(ax)}x\,dx = \frac{\sqrt[4]{1-iax}\left((1+iax)^{7/4}+6\ 2^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{4},\frac{5}{4},\frac{1}{2}(1-iax)\right)\right)}{2a^2}$$

input
```
Integrate[E^(((3*I)/2)*ArcTan[a*x])*x,x]
```

output
```
((1 - I*a*x)^(1/4)*((1 + I*a*x)^(7/4) + 6*2^(3/4)*Hypergeometric2F1[-3/4,
1/4, 5/4, (1 - I*a*x)/2]))/(2*a^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.70 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.24, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {5585, 90, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\frac{3}{2}i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{x(1+iax)^{3/4}}{(1-iax)^{3/4}}dx$$

$$\Big\downarrow \text{90}$$

$$\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2}-\frac{3i\int \frac{(iax+1)^{3/4}}{(1-iax)^{3/4}}dx}{4a}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2}-\frac{3i\left(\frac{3}{2}\int\frac{1}{(1-iax)^{3/4}\sqrt[4]{iax+1}}dx+\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)}{4a}$$

$\Big\downarrow$ 73

$$\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2}-\frac{3i\left(\frac{6i\int\frac{1}{\sqrt[4]{iax+1}}d\sqrt[4]{1-iax}}{a}+\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)}{4a}$$

$\Big\downarrow$ 770

$$\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2}-\frac{3i\left(\frac{6i\int\frac{1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{a}+\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)}{4a}$$

$\Big\downarrow$ 755

$$\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2}-\frac{3i\left(\frac{6i\left(\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int\frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)}{4a}$$

$\Big\downarrow$ 1476

$$\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2}-\frac{3i\left(\frac{6i\left(\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int\frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)\right)}{a}\right.}{4a}$$

$\Big\downarrow$ 1082

$$\frac{\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2}-3i\left(\frac{6i\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+\frac{i\sqrt[4]{1-iax}(1}{a}\right.}{4a}$$

$\downarrow$ 217

$$\frac{\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2}-3i\left(\frac{6i\left(\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)\right)}{a}+\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)}{4a}$$

$\downarrow$ 1479

$$\frac{\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2}-3i\left(\frac{6i\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right.\right.}{a}\right.}{4a}$$

$\downarrow$ 25

$$\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2}-$$

$$\frac{3i\left(\frac{6i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\text{ar}}{}\right.\right.}{a}\right.}{4a}$$

$\downarrow$ 27

$$\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2}-$$

$$\frac{3i\left(\frac{6i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\right.\right.}{a}\right.}{4a}$$

$\downarrow$ 1103

$$\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2a^2} - \frac{3i\left(\frac{6i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{2\sqrt{2}}\right)\right)}{a}\right)}{4a}$$

input
```
Int[E^(((3*I)/2)*ArcTan[a*x])*x,x]
```

output
```
((1 - I*a*x)^(1/4)*(1 + I*a*x)^(7/4))/(2*a^2) - (((3*I)/4)*((I*(1 - I*a*x)
^(1/4)*(1 + I*a*x)^(3/4))/a + ((6*I)*((-(ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(
1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))
/(1 + I*a*x)^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 + Sqrt[1 - I*a*x] - (Sqrt[2]*
(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - I*a*x] +
(Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]))/2))/a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} x dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.06

$$\int e^{\frac{3}{2}i\arctan(ax)}x\,dx =$$

$$-\frac{2\,a^2\sqrt{\frac{81i}{16\,a^4}}\log\left(\frac{4}{9}i\,a^2\sqrt{\frac{81i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2\,a^2\sqrt{\frac{81i}{16\,a^4}}\log\left(-\frac{4}{9}i\,a^2\sqrt{\frac{81i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2\,a^2\sqrt{-}}{}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x,x, algorithm="fricas")
```

output
```
-1/4*(2*a^2*sqrt(81/16*I/a^4)*log(4/9*I*a^2*sqrt(81/16*I/a^4) + sqrt(I*sqr
t(a^2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(81/16*I/a^4)*log(-4/9*I*a^2*sqrt(8
1/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(-81/16*I/a
^4)*log(4/9*I*a^2*sqrt(-81/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))
) + 2*a^2*sqrt(-81/16*I/a^4)*log(-4/9*I*a^2*sqrt(-81/16*I/a^4) + sqrt(I*sq
rt(a^2*x^2 + 1)/(a*x + I))) - sqrt(a^2*x^2 + 1)*(2*I*a*x + 5)*sqrt(I*sqrt(
a^2*x^2 + 1)/(a*x + I)))/a^2
```

Sympy [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}x\,dx = \int x\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)*x,x)
```

output
```
Integral(x*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}x\,dx = \int x\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x,x, algorithm="maxima")
```

output
```
integrate(x*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{3}{2}i\arctan(ax)}x\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}i\arctan(ax)}x\,dx = \int x\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}\,dx$$

input
```
int(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
```

output
```
int(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

Reduce [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}x\,dx = \int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} xdx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x,x)
```

3.88 $\int e^{\frac{3}{2}i\arctan(ax)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 195

$$\int e^{\frac{3}{2}i\arctan(ax)}\,dx=\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}-\frac{3i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}+\frac{3i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}+\frac{3i\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)}{\sqrt{2}a}$$

output
```
I*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/a-3/2*I*arctan(1-2^(1/2)*(1-I*a*x)^(1/4)
/(1+I*a*x)^(1/4))*2^(1/2)/a+3/2*I*arctan(1+2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*
x)^(1/4))*2^(1/2)/a+3/2*I*arctanh(2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)/
(1+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2)))*2^(1/2)/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.21

$$\int e^{\frac{3}{2}i\arctan(ax)}\,dx = -\frac{8ie^{\frac{7}{2}i\arctan(ax)}\,\text{Hypergeometric2F1}\left(\frac{7}{4},2,\frac{11}{4},-e^{2i\arctan(ax)}\right)}{7a}$$

input
```
Integrate[E^(((3*I)/2)*ArcTan[a*x]),x]
```

output
```
(((-8*I)/7)*E^(((7*I)/2)*ArcTan[a*x])*Hypergeometric2F1[7/4, 2, 11/4, -E^(
(2*I)*ArcTan[a*x])])/a
```

Rubi [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.23, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5584, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{3}{2}i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5584}$$

$$\int \frac{(1+iax)^{3/4}}{(1-iax)^{3/4}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{2}\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{iax+1}}dx + \frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}$$

$$\Big\downarrow \text{73}$$

$$\frac{6i\int \frac{1}{\sqrt[4]{iax+1}}d\sqrt[4]{1-iax}}{a} + \frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}$$

$$\Big\downarrow \text{ 770}$$

$$\frac{6i\int \frac{1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{a}+\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}$$

$$\Big\downarrow \text{ 755}$$

$$\frac{6i\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{6i\left(\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int\frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-i}}{\sqrt[4]{iax+}}\right)\right)}{a}$$

$$\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{6i\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+$$

$$\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{6i\left(\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)\right)}{a}+$$

$$\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{\frac{6i\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-i\cdots}}{\sqrt[4]{1+i\cdots}}\right.}{\sqrt{2}}\cdots\right.\right.}{a}}{\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{6i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\cdots\right.\right.}{a}}{\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{6i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia\cdots}}{\sqrt[4]{1+iax\cdots}}\right.}{\sqrt{2}}\cdots\right.\right.}{a}}{\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}}$$

$$\Big\downarrow \; 1103$$

$$\frac{6i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{2\sqrt{2}}\right.\right.}{a}$$

$$\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}$$

input
```
Int[E^(((3*I)/2)*ArcTan[a*x]),x]
```

output
```
(I*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/a + ((6*I)*((-(ArcTan[1 - (Sqrt[2]
*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 -
 I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 + Sqrt[1 - I*a*
x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt
[1 - I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]))/
2))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5584
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.10

$$\int e^{\frac{3}{2}i\arctan(ax)}\,dx = \frac{a\sqrt{\frac{9i}{a^2}}\log\left(\frac{1}{3}\,a\sqrt{\frac{9i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-a\sqrt{\frac{9i}{a^2}}\log\left(-\frac{1}{3}\,a\sqrt{\frac{9i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-a\sqrt{-\frac{9i}{a^2}}\log\left(\frac{1}{3}\,a\sqrt{-\frac{9i}{a^2}}+\right.}{2\,a}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
1/2*(a*sqrt(9*I/a^2)*log(1/3*a*sqrt(9*I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a
*x + I))) - a*sqrt(9*I/a^2)*log(-1/3*a*sqrt(9*I/a^2) + sqrt(I*sqrt(a^2*x^2
 + 1)/(a*x + I))) - a*sqrt(-9*I/a^2)*log(1/3*a*sqrt(-9*I/a^2) + sqrt(I*sqr
t(a^2*x^2 + 1)/(a*x + I))) + a*sqrt(-9*I/a^2)*log(-1/3*a*sqrt(-9*I/a^2) +
sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 2*I*sqrt(a^2*x^2 + 1)*sqrt(I*sqrt(a
^2*x^2 + 1)/(a*x + I)))/a
```

Sympy [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}\,dx = \int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2),x)
```

output
```
Integral(((I*a*x + 1)/sqrt(a**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}\,dx = \int \left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{3}{2}i\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}i\arctan(ax)}\,dx = \int \left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}\,dx$$

input
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
```

output
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

Reduce [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}\,dx = \int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

3.89 $\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 204

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x}\,dx = 2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)$$

output
```
2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+2^(1/2)*arctan(1-2^(1/2)*(1-I*a*
x)^(1/4)/(1+I*a*x)^(1/4))-2^(1/2)*arctan(1+2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*
x)^(1/4))-2*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-2^(1/2)*arctanh(2^(1/
2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)/(1+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.47

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x}\,dx = -22^{3/4}\sqrt[4]{1-iax}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{1}{2}(1-iax)\right) - \frac{4\sqrt[4]{1-iax}\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},-\frac{1-iax}{-1-iax}\right)}{\sqrt[4]{1+iax}}$$

input
```
Integrate[E^(((3*I)/2)*ArcTan[a*x])/x,x]
```

output
```
-2*2^(3/4)*(1 - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, (1 - I*a*x)/
2] - (4*(1 - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 1, 5/4, -((1 - I*a*x)/(-1
 - I*a*x))])/(1 + I*a*x)^(1/4)
```

Rubi [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.30, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.062$, Rules used = {5585, 140, 73, 104, 25, 770, 755, 827, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{(1+iax)^{3/4}}{x(1-iax)^{3/4}}dx$$

$$\Big\downarrow \text{140}$$

$$ia\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{iax+1}}dx+\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{iax+1}}dx$$

$$\Big\downarrow \ 73$$

$$\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-4\int \frac{1}{\sqrt[4]{iax+1}}d\sqrt[4]{1-iax}$$

$$\Big\downarrow \ 104$$

$$4\int -\frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-4\int \frac{1}{\sqrt[4]{iax+1}}d\sqrt[4]{1-iax}$$

$$\Big\downarrow \ 25$$

$$-4\int \frac{1}{\sqrt[4]{iax+1}}d\sqrt[4]{1-iax}-4\int \frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}$$

$$\Big\downarrow \ 770$$

$$-4\int \frac{1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}-4\int \frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}$$

$$\Big\downarrow \ 755$$

$$-4\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)-4\int \frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}$$

$$\Big\downarrow \ 827$$

$$4\left(\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-4\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)$$

$$\Big\downarrow \ 216$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-4\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)$$

$$\Big\downarrow \ 219$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-4\left(\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int\frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)$$

$\big\downarrow$ 1476

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-4\left(\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int\frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{iax}}}\right.\right.$$

$\big\downarrow$ 1082

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-4\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-ia}}{\sqrt[4]{iax+}}\right.$$

$\big\downarrow$ 217

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-4\left(\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)\right)$$

$\big\downarrow$ 1479

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-4\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\right.}{\sqrt{}}\right.\right.$$

$\big\downarrow$ 25

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-4\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{1+}}\right.}{\sqrt{2}}\right.\right.$$

$\downarrow$ 27

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-4\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1\right.}{}\right.\right.$$

$\downarrow$ 1103

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-4\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}}-\right.\right.$$

input
```
Int[E^(((3*I)/2)*ArcTan[a*x])/x,x]
```

output
```
4*(ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/2 - ArcTanh[(1 + I*a*x)^(1/
4)/(1 - I*a*x)^(1/4)]/2) - 4*((-(ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1
 + I*a*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*
a*x)^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 + Sqrt[1 - I*a*x] - (Sqrt[2]*(1 - I*a
*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - I*a*x] + (Sqrt[2]
*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.19

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x}\,dx = \frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - \frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - \frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}i\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) + \frac{1}{2}\sqrt{-4i}\log\left(-\frac{1}{2}i\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - \log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right) + i\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right) - i\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right) + \log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-1\right)$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="fricas")
```

output
```
1/2*sqrt(4*I)*log(1/2*I*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) -
 1/2*sqrt(4*I)*log(-1/2*I*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
 - 1/2*sqrt(-4*I)*log(1/2*I*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I
))) + 1/2*sqrt(-4*I)*log(-1/2*I*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x
 + I))) - log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + I*log(sqrt(I*sqrt
(a^2*x^2 + 1)/(a*x + I)) + I) - I*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))
- I) + log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1)
```

Sympy [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x}\,dx = \int \frac{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x,x)
```

output
```
Integral((I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x}\,dx = \int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x}\,dx = \int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}{x}\,dx$$

input
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x,x)
```

output
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x, x)
```

Reduce [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x}\,dx = \int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

3.90 $\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 92

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x} + 3ia\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 3ia\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

output
```
-(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x+3*I*a*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^
(1/4))-3*I*a*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.74

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = -\frac{i\sqrt[4]{1-iax}\left(-i+ax+6ax\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{i+ax}{i-ax}\right)\right)}{x\sqrt[4]{1+iax}}$$

input
```
Integrate[E^(((3*I)/2)*ArcTan[a*x])/x^2,x]
```

output
```
((-I)*(1 - I*a*x)^(1/4)*(-I + a*x + 6*a*x*Hypergeometric2F1[1/4, 1, 5/4, (
I + a*x)/(I - a*x)]))/(x*(1 + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5585, 105, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^2}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{(1+iax)^{3/4}}{x^2(1-iax)^{3/4}}dx$$

$$\Big\downarrow \text{105}$$

$$\frac{3}{2}ia\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}$$

$$\Big\downarrow \text{104}$$

$$6ia\int -\frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}$$

$$\Big\downarrow \text{25}$$

$$-6ia\int \frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}$$

$$\Big\downarrow \text{827}$$

$$6ia\left(\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}$$

$$\Big\downarrow \text{216}$$

$$6ia\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}$$

$\big\downarrow$ 219

$$6ia\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}$$

input
```
Int[E^(((3*I)/2)*ArcTan[a*x])/x^2,x]
```

output
```
-(((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x) + (6*I)*a*(ArcTan[(1 + I*a*x)^(
1/4)/(1 - I*a*x)^(1/4)]/2 - ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/2
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^2}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(64) = 128$.

Time = 0.16 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.71

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^2}\,dx$$
$$= \frac{-3i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-3\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+3\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)+3i\,ax\log\left(\sqrt{\right.}\right.}{2\,x}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
1/2*(-3*I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 3*a*x*log(sqr
t(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 3*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)
/(a*x + I)) - I) + 3*I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1) -
2*sqrt(a^2*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x
```

Sympy [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**2,x)
```

output
```
Integral((I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}{x^2}\,dx$$

input
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x^2,x)
```

output
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x^2, x)
```

Reduce [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = \frac{-2\sqrt{aix+1}\,(a^2x^2+1)^{\frac{1}{4}}+3\left(\int \frac{\sqrt{aix+1}\,(a^2x^2+1)^{\frac{1}{4}}}{a^2x^3+x}dx\right)aix}{2x}$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
```

output
```
( - 2*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(1/4) + 3*int((sqrt(a*i*x + 1)*(a**
2*x**2 + 1)**(1/4))/(a**2*x**3 + x),x)*a*i*x)/(2*x)
```

3.91 $\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 132

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = -\frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x} - \frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2x^2} - \frac{9}{4}a^2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{9}{4}a^2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

output
```
-3/4*I*a*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x-1/2*(1-I*a*x)^(1/4)*(1+I*a*x)^(
7/4)/x^2-9/4*a^2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+9/4*a^2*arctanh((
1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.61

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \frac{\sqrt[4]{1-iax}\left(-2-7iax+5a^2x^2+18a^2x^2\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{i+ax}{i-ax}\right)\right)}{4x^2\sqrt[4]{1+iax}}$$

input
```
Integrate[E^(((3*I)/2)*ArcTan[a*x])/x^3,x]
```

output
```
((1 - I*a*x)^(1/4)*(-2 - (7*I)*a*x + 5*a^2*x^2 + 18*a^2*x^2*Hypergeometric
2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]))/(4*x^2*(1 + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5585, 107, 105, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^3}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{(1+iax)^{3/4}}{x^3(1-iax)^{3/4}}dx$$

$$\Big\downarrow \text{107}$$

$$\frac{3}{4}ia\int \frac{(iax+1)^{3/4}}{x^2(1-iax)^{3/4}}dx-\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2x^2}$$

$$\Big\downarrow \text{105}$$

$$\frac{3}{4}ia\left(\frac{3}{2}ia\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2x^2}$$

$$\Big\downarrow \text{104}$$

$$\frac{3}{4}ia\left(6ia\int -\frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{4}ia\left(-6ia\int \frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2x^2}$$

$\big\downarrow$ 827

$$\frac{3}{4}ia\left(6ia\left(\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2x^2}$$

$\big\downarrow$ 216

$$\frac{3}{4}ia\left(6ia\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2x^2}$$

$\big\downarrow$ 219

$$\frac{3}{4}ia\left(6ia\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{7/4}}{2x^2}$$

input
```
Int[E^(((3*I)/2)*ArcTan[a*x])/x^3,x]
```

output
```
-1/2*((1 - I*a*x)^(1/4)*(1 + I*a*x)^(7/4))/x^2 + ((3*I)/4)*a*(-(((1 - I*a*
x)^(1/4)*(1 + I*a*x)^(3/4))/x) + (6*I)*a*(ArcTan[(1 + I*a*x)^(1/4)/(1 - I*
a*x)^(1/4)]/2 - ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^3}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.36

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \frac{9\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-9i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+9i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-9\,a^2x^2\log}{8\,x^2}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="fricas")
```

output
```
1/8*(9*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 9*I*a^2*x^2*
log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 9*I*a^2*x^2*log(sqrt(I*sqrt
(a^2*x^2 + 1)/(a*x + I)) - I) - 9*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*
x + I)) - 1) - 2*sqrt(a^2*x^2 + 1)*(5*I*a*x + 2)*sqrt(I*sqrt(a^2*x^2 + 1)/
(a*x + I)))/x^2
```

Sympy [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**3,x)
```

output
```
Integral((I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}{x^3}\,dx$$

input
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x^3,x)
```

output
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x^3, x)
```

Reduce [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \frac{-10\sqrt{aix+1}\,(a^2x^2+1)^{\frac{1}{4}}\,aix - 4\sqrt{aix+1}\,(a^2x^2+1)^{\frac{1}{4}} - 9\left(\int \frac{\sqrt{aix+1}\,(a^2x^2+1)^{\frac{1}{4}}}{a^2x^3+x}dx\right)a^2x^2}{8x^2}$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x)
```

output
```
( - 10*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(1/4)*a*i*x - 4*sqrt(a*i*x + 1)*(a
**2*x**2 + 1)**(1/4) - 9*int((sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(1/4))/(a**
2*x**3 + x),x)*a**2*x**2)/(8*x**2)
```

3.92 $\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 170

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3} - \frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2} + \frac{23a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x} - \frac{17}{8}ia^3\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{17}{8}ia^3\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

output

```
-1/3*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x^3-7/12*I*a*(1-I*a*x)^(1/4)*(1+I*a*x
)^(3/4)/x^2+23/24*a^2*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x-17/8*I*a^3*arctan(
(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+17/8*I*a^3*arctanh((1+I*a*x)^(1/4)/(1-I*a
*x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.55

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = \frac{\sqrt[4]{1-iax}\left(-8-22iax+37a^2x^2+23ia^3x^3+102ia^3x^3\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{i+ax}{i-ax}\right)\right)}{24x^3\sqrt[4]{1+iax}}$$

input
```
Integrate[E^(((3*I)/2)*ArcTan[a*x])/x^4,x]
```

output
```
((1 - I*a*x)^(1/4)*(-8 - (22*I)*a*x + 37*a^2*x^2 + (23*I)*a^3*x^3 + (102*I
)*a^3*x^3*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]))/(24*x^3*(1
 + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5585, 110, 27, 168, 27, 168, 27, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^4}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{(1+iax)^{3/4}}{x^4(1-iax)^{3/4}}dx$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{3}\int \frac{a(7i-4ax)}{2x^3(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}a\int \frac{7i-4ax}{x^3(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{6}a\left(-\frac{1}{2}\int \frac{a(14iax+23)}{2x^2(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{7i\sqrt[4]{1-iax}(1+iax)^{3/4}}{2x^2}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}a\left(-\frac{1}{4}a\int \frac{14iax+23}{x^2(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{7i\sqrt[4]{1-iax}(1+iax)^{3/4}}{2x^2}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{6}a\left(-\frac{1}{4}a\left(-\int -\frac{51ia}{2x(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{23\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{7i\sqrt[4]{1-iax}(1+iax)^{3/4}}{2x^2}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}a\left(-\frac{1}{4}a\left(\frac{51}{2}ia\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{23\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{7i\sqrt[4]{1-iax}(1+iax)^{3/4}}{2x^2}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}$$

$$\Big\downarrow \text{104}$$

$$\frac{1}{6}a\left(-\frac{1}{4}a\left(102ia\int -\frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{23\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{7i\sqrt[4]{1-iax}(1+iax)^{3/4}}{2x^2}\right.$$
$$\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{6}a\left(-\frac{1}{4}a\left(-102ia\int \frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{23\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{7i\sqrt[4]{1-iax}(1+iax)^{3/4}}{2x^2}\right.$$
$$\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{6}a\left(-\frac{1}{4}a\left(102ia\left(\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{23\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\right.$$
$$\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}$$

$\downarrow$ 216

$$\frac{1}{6}a\left(-\frac{1}{4}a\left(102ia\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{23\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{7i\sqrt[4]{1}\right.$$
$$\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}$$

$\downarrow$ 219

$$\frac{1}{6}a\left(-\frac{1}{4}a\left(102ia\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-\frac{23\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{7i\sqrt[4]{1-ia}\right.$$
$$\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}$$

input
```
Int[E^(((3*I)/2)*ArcTan[a*x])/x^4,x]
```

output
```
-1/3*((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x^3 + (a*((((-7*I)/2)*(1 - I*a*
x)^(1/4)*(1 + I*a*x)^(3/4))/x^2 - (a*((-23*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(
3/4))/x + (102*I)*a*(ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/2 - ArcTa
nh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/2)))/4))/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^4}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.10

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = \frac{51i\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+51\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-51\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-51i\,a^3}{48\,x^3}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="fricas")
```

output
```
1/48*(51*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 51*a^3*x
^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 51*a^3*x^3*log(sqrt(I*sq
rt(a^2*x^2 + 1)/(a*x + I)) - I) - 51*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1
)/(a*x + I)) - 1) + 2*(23*a^2*x^2 - 14*I*a*x - 8)*sqrt(a^2*x^2 + 1)*sqrt(I
*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^3
```

Sympy [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**4,x)
```

output
```
Integral((I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)/x^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}{x^4}\,dx$$

input
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x^4,x)
```

output
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x^4, x)
```

Reduce [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = \frac{46\sqrt{aix+1}\,(a^2x^2+1)^{\frac{1}{4}}\,a^2x^2 - 28\sqrt{aix+1}\,(a^2x^2+1)^{\frac{1}{4}}\,aix - 16\sqrt{aix+1}\,(a^2x^2+1)^{\frac{1}{4}} - 51\left(\int \frac{\sqrt{aix+1}\,(a}{a^2x^3}\right.}{48x^3}$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x)
```

output
```
(46*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(1/4)*a**2*x**2 - 28*sqrt(a*i*x + 1)*
(a**2*x**2 + 1)**(1/4)*a*i*x - 16*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(1/4) -
 51*int((sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(1/4))/(a**2*x**3 + x),x)*a**3*i
*x**3)/(48*x**3)
```

3.93 $\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 202

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^5}\,dx = -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4} - \frac{3ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{8x^3} + \frac{15a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{32x^2} + \frac{63ia^3\sqrt[4]{1-iax}(1+iax)^{3/4}}{64x} + \frac{123}{64}a^4\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{123}{64}a^4\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

output
```
-1/4*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x^4-3/8*I*a*(1-I*a*x)^(1/4)*(1+I*a*x)
^(3/4)/x^3+15/32*a^2*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x^2+63/64*I*a^3*(1-I*
a*x)^(1/4)*(1+I*a*x)^(3/4)/x+123/64*a^4*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(
1/4))-123/64*a^4*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.49

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^5}\,dx = -\frac{\sqrt[4]{1-iax}\left(16+40iax-54a^2x^2-93ia^3x^3+63a^4x^4+246a^4x^4\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{i+ax}{i-ax}\right)\right)}{64x^4\sqrt[4]{1+iax}}$$

input
```
Integrate[E^(((3*I)/2)*ArcTan[a*x])/x^5,x]
```

output
```
-1/64*((1 - I*a*x)^(1/4)*(16 + (40*I)*a*x - 54*a^2*x^2 - (93*I)*a^3*x^3 +
63*a^4*x^4 + 246*a^4*x^4*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x
)]))/(x^4*(1 + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {5585, 110, 27, 168, 27, 168, 27, 168, 27, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^5}dx$$

$$\Big\downarrow \text{ 5585}$$

$$\int \frac{(1+iax)^{3/4}}{x^5(1-iax)^{3/4}}dx$$

$$\Big\downarrow \text{ 110}$$

$$\frac{1}{4}\int \frac{3a(3i-2ax)}{2x^4(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{8}a\int \frac{3i-2ax}{x^4(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}$$

$\Big\downarrow$ 168

$$\frac{3}{8}a\left(-\frac{1}{3}\int \frac{3a(4iax+5)}{2x^3(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{x^3}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}$$

$\Big\downarrow$ 27

$$\frac{3}{8}a\left(-\frac{1}{2}a\int \frac{4iax+5}{x^3(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{x^3}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}$$

$\Big\downarrow$ 168

$$\frac{3}{8}a\left(-\frac{1}{2}a\left(-\frac{1}{2}\int -\frac{a(21i-10ax)}{2x^2(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{5\sqrt[4]{1-iax}(1+iax)^{3/4}}{2x^2}\right)-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{x^3}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}$$

$\Big\downarrow$ 27

$$\frac{3}{8}a\left(-\frac{1}{2}a\left(\frac{1}{4}a\int \frac{21i-10ax}{x^2(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{5\sqrt[4]{1-iax}(1+iax)^{3/4}}{2x^2}\right)-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{x^3}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}$$

$\Big\downarrow$ 168

$$\frac{3}{8}a\left(-\frac{1}{2}a\left(\frac{1}{4}a\left(-\int \frac{41a}{2x(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{21i\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{5\sqrt[4]{1-iax}(1+iax)^{3/4}}{2x^2}\right)-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{x^3}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}$$

$\Big\downarrow$ 27

$$\frac{3}{8}a\left(-\frac{1}{2}a\left(\frac{1}{4}a\left(-\frac{41}{2}a\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{21i\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{5\sqrt[4]{1-iax}(1+iax)^{3/4}}{2x^2}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}$$

$\Big\downarrow$ 104

$$\frac{3}{8}a\left(-\frac{1}{2}a\left(\frac{1}{4}a\left(-82a\int -\frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{21i\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{5\sqrt[4]{1-iax}(1+ia}{2x^2}\right.\right.$$
$$\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}$$

$\big\downarrow$ 25

$$\frac{3}{8}a\left(-\frac{1}{2}a\left(\frac{1}{4}a\left(82a\int \frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{21i\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{5\sqrt[4]{1-iax}(1+iax)^{3}}{2x^2}\right.\right.$$
$$\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}$$

$\big\downarrow$ 827

$$\frac{3}{8}a\left(-\frac{1}{2}a\left(\frac{1}{4}a\left(-82a\left(\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{21i\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right.\right.\right.$$
$$\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}$$

$\big\downarrow$ 216

$$\frac{3}{8}a\left(-\frac{1}{2}a\left(\frac{1}{4}a\left(-82a\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{21i\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\right.\right.$$
$$\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}$$

$\big\downarrow$ 219

$$\frac{3}{8}a\left(-\frac{1}{2}a\left(\frac{1}{4}a\left(-82a\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-\frac{21i\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{5\sqrt[4]{1}}{}\right.\right.$$
$$\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}$$

input

```
Int[E^(((3*I)/2)*ArcTan[a*x])/x^5,x]
```

output
```
-1/4*((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x^4 + (3*a*(((-I)*(1 - I*a*x)^(
1/4)*(1 + I*a*x)^(3/4))/x^3 - (a*((-5*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
/(2*x^2) + (a*(((-21*I)*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x - 82*a*(Arc
Tan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/2 - ArcTanh[(1 + I*a*x)^(1/4)/(1
- I*a*x)^(1/4)]/2)))/4))/2))/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^5}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.97

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^5}\,dx = -\frac{123\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-123i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+123i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-}{128\,x^4}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="fricas")
```

output
```
-1/128*(123*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 123*I*a
^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 123*I*a^4*x^4*log(sq
rt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 123*a^4*x^4*log(sqrt(I*sqrt(a^2*x
^2 + 1)/(a*x + I)) - 1) + 2*(-63*I*a^3*x^3 - 30*a^2*x^2 + 24*I*a*x + 16)*s
qrt(a^2*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^4
```

Sympy [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**5,x)
```

output
```
Integral((I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)/x**5, x)
```

Maxima [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)/x^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}{x^5}\,dx$$

input
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x^5,x)
```

output
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)/x^5, x)
```

Reduce [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(ax)}}{x^5}\,dx$$
$$= \frac{-60\sqrt{aix+1}\,(a^2x^2+1)^{\frac{1}{4}}\,a^3i\,x^3+30\sqrt{aix+1}\,(a^2x^2+1)^{\frac{1}{4}}\,a^2x^2-24\sqrt{aix+1}\,(a^2x^2+1)^{\frac{1}{4}}\,aix-16\sqrt{ai}}{64x^4}$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x)
```

output
```
( - 60*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(1/4)*a**3*i*x**3 + 30*sqrt(a*i*x
+ 1)*(a**2*x**2 + 1)**(1/4)*a**2*x**2 - 24*sqrt(a*i*x + 1)*(a**2*x**2 + 1)
**(1/4)*a*i*x - 16*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(1/4) - 123*int((sqrt(
a*i*x + 1)*(a**2*x**2 + 1)**(1/4))/(a**2*x**4 + x**2),x)*a**3*i*x**4)/(64*
x**4)
```

3.94 $\int e^{\frac{5}{2}i\arctan(ax)}x^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 325

$$\begin{aligned}\int e^{\frac{5}{2}i\arctan(ax)}x^3\,dx &= \frac{475(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4}-\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}\\&\quad+\frac{973(1-iax)^{3/4}(1+iax)^{5/4}}{96a^4}-\frac{17x^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a^2}\\&\quad-\frac{113(1-iax)^{7/4}(1+iax)^{5/4}}{24a^4}-\frac{475\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}\\&\quad+\frac{475\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}\\&\quad-\frac{475\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)}{64\sqrt{2}a^4}\end{aligned}$$

output
```
475/64*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a^4-4*I*x^3*(1+I*a*x)^(5/4)/a/(1-I*
a*x)^(1/4)+973/96*(1-I*a*x)^(3/4)*(1+I*a*x)^(5/4)/a^4-17/4*x^2*(1-I*a*x)^(
3/4)*(1+I*a*x)^(5/4)/a^2-113/24*(1-I*a*x)^(7/4)*(1+I*a*x)^(5/4)/a^4-475/12
8*arctan(1-2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))*2^(1/2)/a^4+475/128*ar
ctan(1+2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))*2^(1/2)/a^4-475/128*arctan
h(2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)/(1+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/
2)))*2^(1/2)/a^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.30

$$\int e^{\frac{5}{2}i\arctan(ax)}x^3\,dx$$
$$=\frac{-\sqrt[4]{1+iax}(-i+ax)^2\left(59-5iax+6a^2x^2\right)+380\sqrt[4]{2}(1-iax)\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-iax)\right)}{24a^4\sqrt[4]{1-iax}}$$

input
```
Integrate[E^(((5*I)/2)*ArcTan[a*x])*x^3,x]
```

output
```
(-((1 + I*a*x)^(1/4)*(-I + a*x)^2*(59 - (5*I)*a*x + 6*a^2*x^2)) + 380*2^(1
/4)*(1 - I*a*x)*Hypergeometric2F1[-5/4, 3/4, 7/4, (1 - I*a*x)/2])/(24*a^4*
(1 - I*a*x)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.87 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.14, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.062$, Rules used = {5585, 108, 27, 170, 27, 164, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{\frac{5}{2} i \arctan(ax)}\,dx$$

$$\Big\downarrow \; 5585$$

$$\int \frac{x^3 (1+iax)^{5/4}}{(1-iax)^{5/4}}dx$$

$$\Big\downarrow \; 108$$

$$\frac{4i\int \frac{x^2 \sqrt[4]{iax+1}(17iax+12)}{4\sqrt[4]{1-iax}}dx}{a}-\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

$$\Big\downarrow \; 27$$

$$\frac{i\int \frac{x^2 \sqrt[4]{iax+1}(17iax+12)}{\sqrt[4]{1-iax}}dx}{a}-\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

$$\Big\downarrow \; 170$$

$$\frac{i\left(\frac{\int -\frac{ax(68i-113ax)\sqrt[4]{iax+1}}{2\sqrt[4]{1-iax}}dx}{4a^2}+\frac{17ix^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a}\right)}{a}-\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

$$\Big\downarrow \; 27$$

$$\frac{i\left(\frac{17ix^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a}-\frac{\int \frac{x(68i-113ax)\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}dx}{8a}\right)}{a}-\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

$$\Big\downarrow \; 164$$

$$\frac{i\left(\frac{17ix^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a}-\frac{\frac{475\int \frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}dx}{8a}+\frac{(-452ax+521i)(1-iax)^{3/4}(1+iax)^{5/4}}{12a^2}}{8a}\right)}{a}-\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

$$\Big\downarrow \; 60$$

$$\frac{i\left(\frac{17ix^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a}-\frac{\frac{475\left(\frac{1}{2}\int\frac{1}{\sqrt[4]{1-iax}(iax+1)^{3/4}}dx+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)}{8a}+\frac{(-452ax+521i)(1-iax)^{3/4}(1+iax)^{5/4}}{12a^2}}{8a}\right)}{a}-\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

$\Big\downarrow$ 73

$$\frac{i\left(\frac{17ix^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a}-\frac{\frac{475\left(\frac{2i\int\frac{\sqrt{1-iax}}{(iax+1)^{3/4}}d\sqrt[4]{1-iax}}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)}{8a}+\frac{(-452ax+521i)(1-iax)^{3/4}(1+iax)^{5/4}}{12a^2}}{8a}\right)}{a}-\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

$\Big\downarrow$ 854

$$\frac{i\left(\frac{17ix^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a}-\frac{\frac{475\left(\frac{2i\int\frac{\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)}{8a}+\frac{(-452ax+521i)(1-iax)^{3/4}(1+iax)^{5/4}}{12a^2}}{8a}\right)}{a}-\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

$\Big\downarrow$ 826

$$\frac{i\left(\frac{17ix^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a}-\frac{\frac{475\left(\frac{2i\left(\frac{1}{2}\int\frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}-\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)}{8a}+\frac{(-452ax}{}}{8a}\right)}{a}$$

$$\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

$\downarrow$ 1476

$$\frac{i\left(\frac{17ix^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a}-\frac{\frac{475\left(\frac{2i\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int\frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)\right.}{a}\right.}{8a}}{8a}\right)}{a}$$

$$\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

$\downarrow$ 1082

$$\frac{i\left(\frac{17ix^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a}-\frac{\frac{475\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d}{a}}{8a}}{8a}\right)}{a}$$

$$\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

$\downarrow$ 217

$$\frac{i\left(\frac{17ix^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a}-\frac{\frac{475\left(\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+\frac{i(}{}}{8a}}{8a}\right)}{a}$$

$$\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

$\downarrow$ 1479

$$\frac{i\left(\frac{17ix^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a}-\frac{475\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right.\right.\right.}{a}}{8a}\right.}{a}$$

$$\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

$$\Big\downarrow \; 25$$

$$i\left(\frac{17ix^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a}-\frac{475\frac{2i\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\cdots\right)}{a}}{8a}\right)$$

$$\overline{a}$$

$$\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

$$\Big\downarrow \; 27$$

$$\frac{i\left(\frac{17ix^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a}-\frac{475\left(\frac{2i\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right.\right.\right.}{a}}{8a}\right)}{a}$$

$$\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{i\left(\frac{17ix^2(1-iax)^{3/4}(1+iax)^{5/4}}{4a}-\frac{(-452ax+521i)(1-iax)^{3/4}(1+iax)^{5/4}}{12a^2}+\frac{475\left(\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\cdots\right)\cdots\right)}{\cdots}\cdots\right)}{8\cdots}\right)}{a}$$

$$\frac{4ix^3(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

input
```
Int[E^(((5*I)/2)*ArcTan[a*x])*x^3,x]
```

output
```
((-4*I)*x^3*(1 + I*a*x)^(5/4))/(a*(1 - I*a*x)^(1/4)) + (I*((((17*I)/4)*x^2
*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(5/4))/a - (((521*I - 452*a*x)*(1 - I*a*x)^
(3/4)*(1 + I*a*x)^(5/4))/(12*a^2) + (475*((I*(1 - I*a*x)^(3/4)*(1 + I*a*x)
^(1/4))/a + ((2*I)*((-(ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^
(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)
]/Sqrt[2])/2 + (Log[1 + Sqrt[1 - I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 +
 I*a*x)^(1/4)]/(2*Sqrt[2]) - Log[1 + Sqrt[1 - I*a*x] + (Sqrt[2]*(1 - I*a*x
)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]))/2))/a))/(8*a))/(8*a)))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} x^3 dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^3,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.77

$$\int e^{\frac{5}{2}i\arctan(ax)}x^3\,dx = -\frac{96\,a^4\sqrt{\frac{225625i}{4096\,a^8}}\log\left(\frac{64}{475}\,a^4\sqrt{\frac{225625i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-96\,a^4\sqrt{\frac{225625i}{4096\,a^8}}\log\left(-\frac{64}{475}\,a^4\sqrt{\frac{225625i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right.}{}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^3,x, algorithm="fricas")
```

output
```
-1/192*(96*a^4*sqrt(225625/4096*I/a^8)*log(64/475*a^4*sqrt(225625/4096*I/a
^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 96*a^4*sqrt(225625/4096*I/a^8
)*log(-64/475*a^4*sqrt(225625/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x
+ I))) + 96*a^4*sqrt(-225625/4096*I/a^8)*log(64/475*a^4*sqrt(-225625/4096*
I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 96*a^4*sqrt(-225625/4096*I
/a^8)*log(-64/475*a^4*sqrt(-225625/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/
(a*x + I))) + (48*a^4*x^4 - 136*I*a^3*x^3 - 226*a^2*x^2 + 521*I*a*x - 2467
)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a^4
```

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{5}{2}i\arctan(ax)}x^3\,dx = \text{Timed out}$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)*x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\frac{5}{2}i\arctan(ax)}x^3\,dx = \int x^3\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^3,x, algorithm="maxima")
```

output
```
integrate(x^3*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{5}{2}i\arctan(ax)}x^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{5}{2}i\arctan(ax)}x^3\,dx = \int x^3\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}\,dx$$

input
```
int(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
```

output
```
int(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

Reduce [F]

$$\int e^{\frac{5}{2}i\arctan(ax)}x^3\,dx = \int\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}x^3dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^3,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^3,x)
```

3.95 $\int e^{\frac{5}{2}i\arctan(ax)}x^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 300

$$\int e^{\frac{5}{2}i\arctan(ax)}x^2\,dx=\frac{55i(1-iax)^{3/4}\sqrt[4]{1+iax}}{8a^3}+\frac{11i(1-iax)^{3/4}(1+iax)^{5/4}}{4a^3}+\frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}}+\frac{i(1-iax)^{3/4}(1+iax)^{9/4}}{3a^3}-\frac{55i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}+\frac{55i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}-\frac{55i\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)}{8\sqrt{2}a^3}$$

output

```
55/8*I*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a^3+11/4*I*(1-I*a*x)^(3/4)*(1+I*a*x
)^(5/4)/a^3+2*I*(1+I*a*x)^(9/4)/a^3/(1-I*a*x)^(1/4)+1/3*I*(1-I*a*x)^(3/4)*
(1+I*a*x)^(9/4)/a^3-55/16*I*arctan(1-2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/
4))*2^(1/2)/a^3+55/16*I*arctan(1+2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))*
2^(1/2)/a^3-55/16*I*arctanh(2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)/(1+(1-
I*a*x)^(1/2)/(1+I*a*x)^(1/2)))*2^(1/2)/a^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.29

$$\int e^{\frac{5}{2}i\arctan(ax)}x^2\,dx = \frac{-\sqrt[4]{1+iax}(-i+ax)^2(7i+ax)+44\sqrt[4]{2}(i+ax)\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-iax)\right)}{3a^3\sqrt[4]{1-iax}}$$

input
```
Integrate[E^(((5*I)/2)*ArcTan[a*x])*x^2,x]
```

output
```
(-((1 + I*a*x)^(1/4)*(-I + a*x)^2*(7*I + a*x)) + 44*2^(1/4)*(I + a*x)*Hype
rgeometric2F1[-5/4, 3/4, 7/4, (1 - I*a*x)/2])/(3*a^3*(1 - I*a*x)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.81 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.18, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5585, 100, 27, 90, 60, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{\frac{5}{2}i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{x^2(1+iax)^{5/4}}{(1-iax)^{5/4}}dx$$

$$\Big\downarrow \text{100}$$

$$\frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}}-\frac{2i\int \frac{a(5i-ax)(iax+1)^{5/4}}{2\sqrt[4]{1-iax}}dx}{a^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}}-\frac{i\int\frac{(5i-ax)(iax+1)^{5/4}}{\sqrt[4]{1-iax}}dx}{a^2}$$

$$\Big\downarrow \text{90}$$

$$\frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}}-\frac{i\left(\frac{11}{2}i\int\frac{(iax+1)^{5/4}}{\sqrt[4]{1-iax}}dx-\frac{(1-iax)^{3/4}(1+iax)^{9/4}}{3a}\right)}{a^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}}-\frac{i\left(\frac{11}{2}i\left(\frac{5}{4}\int\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}dx+\frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{2a}\right)-\frac{(1-iax)^{3/4}(1+iax)^{9/4}}{3a}\right)}{a^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}}-\frac{i\left(\frac{11}{2}i\left(\frac{5}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt[4]{1-iax}(iax+1)^{3/4}}dx+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)+\frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{2a}\right)-\frac{(1-iax)^{3/4}(1+iax)^{9/4}}{3a}\right)}{a^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}}-\frac{i\left(\frac{11}{2}i\left(\frac{5}{4}\left(\frac{2i\int\frac{\sqrt{1-iax}}{(iax+1)^{3/4}}d\sqrt[4]{1-iax}}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)+\frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{2a}\right)-\frac{(1-iax)^{3/4}(1+iax)^{9/4}}{3a}\right)}{a^2}$$

$$\Big\downarrow \text{854}$$

$$\frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}}-\frac{i\left(\frac{11}{2}i\left(\frac{5}{4}\left(\frac{2i\int\frac{\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)+\frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{2a}\right)-\frac{(1-iax)^{3/4}(1+iax)^{9/4}}{3a}\right)}{a^2}$$

$$\Big\downarrow \text{826}$$

$$\frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}}-\frac{i\left(\frac{11}{2}i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\int\frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}-\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)+\frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{2a}\right)\cdots}{a^2}$$

$\Big\downarrow$ 1476

$$\frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}}-\frac{i\left(\frac{11}{2}i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int\frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)-\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-\cdots}}{\sqrt[4]{ia\cdots}}\cdots}{a}\cdots}{a^2}$$

$\Big\downarrow$ 1082

$$\frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}}-\frac{i\left(\frac{11}{2}i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+\frac{i(1-\cdots}{\cdots}\cdots}{a^2}$$

$\Big\downarrow$ 217

$$\frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}}-\frac{i\left(\frac{11}{2}i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1+ia\cdots}}{a}\cdots}{a^2}$$

$\downarrow$ 1479

$$\frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}}-$$

$$i\left(\frac{11}{2}i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\cdots}}{\sqrt[4]{1+\cdots}}\right.}{\sqrt{2}}\cdots\right.\right.}{a}\cdots\right.\right.\right.}{a^2}\cdots$$

$\downarrow$ 25

$$\frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}}-$$

$$i\left(\frac{11}{2}i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\cdots}}{\sqrt[4]{1+\cdots}}\right.}{\sqrt{2}}\cdots\right.\right.}{a}\cdots\right.\right.\right.}{a^2}\cdots$$

$\downarrow$ 27

$$\frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}}-\frac{i\left(\frac{11}{2}i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1\cdots}}{\sqrt[4]{1+\cdots}}\right)}{\sqrt{2}}\cdots\right)}{a}\cdots\right)\right)\right)}{a^2}$$

$\downarrow$ 1103

$$\frac{2i(1+iax)^{9/4}}{a^3\sqrt[4]{1-iax}}-\frac{i\left(\frac{11}{2}i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{1\cdots}}\right)}{2\sqrt{2}}\cdots\right)}{a}\cdots\right)\right)\right)}{a^2}$$

input

```
Int[E^(((5*I)/2)*ArcTan[a*x])*x^2,x]
```

output

```
((2*I)*(1 + I*a*x)^(9/4))/(a^3*(1 - I*a*x)^(1/4)) - (I*(-1/3*((1 - I*a*x)^
(3/4)*(1 + I*a*x)^(9/4))/a + ((11*I)/2)*(((I/2)*(1 - I*a*x)^(3/4)*(1 + I*a
*x)^(5/4))/a + (5*((I*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a + ((2*I)*((-(
ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]) + ArcTa
n[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2])/2 + (Log[1 +
 Sqrt[1 - I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[
2]) - Log[1 + Sqrt[1 - I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1
/4)]/(2*Sqrt[2]))/2))/a))/4)))/a^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} x^2 dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^2,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.81

$$\int e^{\frac{5}{2}i\arctan(ax)}x^2\,dx$$
$$= \frac{12\,a^3\sqrt{\frac{3025i}{64\,a^6}}\log\left(\frac{8}{55}i\,a^3\sqrt{\frac{3025i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3\sqrt{\frac{3025i}{64\,a^6}}\log\left(-\frac{8}{55}i\,a^3\sqrt{\frac{3025i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+12\,a^3}{}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^2,x, algorithm="fricas")
```

output
```
1/24*(12*a^3*sqrt(3025/64*I/a^6)*log(8/55*I*a^3*sqrt(3025/64*I/a^6) + sqrt
(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(3025/64*I/a^6)*log(-8/55*I*
a^3*sqrt(3025/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 12*a^3*sq
rt(-3025/64*I/a^6)*log(8/55*I*a^3*sqrt(-3025/64*I/a^6) + sqrt(I*sqrt(a^2*x
^2 + 1)/(a*x + I))) - 12*a^3*sqrt(-3025/64*I/a^6)*log(-8/55*I*a^3*sqrt(-30
25/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - (8*a^3*x^3 - 26*I*a^
2*x^2 - 61*a*x - 287*I)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a^3
```

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{5}{2}i\arctan(ax)}x^2\,dx = \text{Timed out}$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)*x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\frac{5}{2}i\arctan(ax)}x^2\,dx = \int x^2\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^2,x, algorithm="maxima")
```

output
```
integrate(x^2*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{5}{2}i\arctan(ax)}x^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{5}{2}i\arctan(ax)}x^2\,dx = \int x^2\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}\,dx$$

input
```
int(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
```

output
```
int(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

Reduce [F]

$$\int e^{\frac{5}{2}i\arctan(ax)}x^2\,dx = \int\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}x^2dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^2,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^2,x)
```

3.96 $\int e^{\frac{5}{2}i\arctan(ax)}x\,dx$

Optimal result

Integrand size = 14, antiderivative size = 255

$$\begin{aligned}\int e^{\frac{5}{2}i\arctan(ax)}x\,dx = & -\frac{25(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}-\frac{5(1-iax)^{3/4}(1+iax)^{5/4}}{2a^2}\\ & -\frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}}+\frac{25\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}\\ & -\frac{25\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}\\ & +\frac{25\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)}{4\sqrt{2}a^2}\end{aligned}$$

output
```
-25/4*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a^2-5/2*(1-I*a*x)^(3/4)*(1+I*a*x)^(5
/4)/a^2-2*(1+I*a*x)^(9/4)/a^2/(1-I*a*x)^(1/4)+25/8*arctan(1-2^(1/2)*(1-I*a
*x)^(1/4)/(1+I*a*x)^(1/4))*2^(1/2)/a^2-25/8*arctan(1+2^(1/2)*(1-I*a*x)^(1/
4)/(1+I*a*x)^(1/4))*2^(1/2)/a^2+25/8*arctanh(2^(1/2)*(1-I*a*x)^(1/4)/(1+I*
a*x)^(1/4)/(1+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2)))*2^(1/2)/a^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.28

$$\int e^{\frac{5}{2}i\arctan(ax)}x\,dx = \frac{2\left(-3(1+iax)^{9/4}+20i\sqrt[4]{2}(i+ax)\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-iax)\right)\right)}{3a^2\sqrt[4]{1-iax}}$$

input
```
Integrate[E^(((5*I)/2)*ArcTan[a*x])*x,x]
```

output
```
(2*(-3*(1 + I*a*x)^(9/4) + (20*I)*2^(1/4)*(I + a*x)*Hypergeometric2F1[-5/4
, 3/4, 7/4, (1 - I*a*x)/2]))/(3*a^2*(1 - I*a*x)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.75 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.23, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5585, 87, 60, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\frac{5}{2}i\arctan(ax)}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{x(1+iax)^{5/4}}{(1-iax)^{5/4}}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{5i\int \frac{(iax+1)^{5/4}}{\sqrt[4]{1-iax}}dx}{a}-\frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{60}$$

$$\frac{5i\left(\frac{5}{4}\int \frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}dx+\frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{2a}\right)}{a}-\frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}}$$

$$\Big\downarrow \; 60$$

$$\frac{5i\left(\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt[4]{1-iax}(iax+1)^{3/4}}dx+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)+\frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{2a}\right)}{a}-\frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}}$$

$$\Big\downarrow \; 73$$

$$\frac{5i\left(\frac{5}{4}\left(\frac{2i\int \frac{\sqrt{1-iax}}{(iax+1)^{3/4}}d\sqrt[4]{1-iax}}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)+\frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{2a}\right)}{a}-\frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}}$$

$$\Big\downarrow \; 854$$

$$\frac{5i\left(\frac{5}{4}\left(\frac{2i\int \frac{\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)+\frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{2a}\right)}{a}-\frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}}$$

$$\Big\downarrow \; 826$$

$$\frac{5i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)+\frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{2a}\right)}{a}-\frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}}$$

$$\Big\downarrow \; 1476$$

$$5i\left(\frac{\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int\frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)-\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\cdots}{a}\right.}{a}\right.$$

$$\frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}}$$

$\downarrow$ 1082

$$5i\left(\frac{\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+\frac{i(1-iax)^{3/}\cdots}{}\right.}{a}\right.$$

$$\frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}}$$

$\downarrow$ 217

$$5i\left(\frac{\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)\cdots}{a}\right.$$

$$\frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}}$$

$\downarrow$ 1479

$$5i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right.}{\sqrt{2}}\right.\right.}{a}\right.}{a}\right.\right.$$

$$\frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}}$$

$\downarrow$ 25

$$5i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right.}{\sqrt{2}}\right.\right.}{a}\right.}{a}\right.\right.$$

$$\frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}}$$

$\downarrow$ 27

$$5i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia}}{\sqrt[4]{1+iax}}\right.}{\sqrt{2}}\right.\right.}{a}\right.\right.$$

$$\frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}}$$

$\Big\downarrow$ 1103

$$5i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{1+i}}\right.}{2\sqrt{2}}\right.\right.}{a}\right.\right.$$

$$\frac{2(1+iax)^{9/4}}{a^2\sqrt[4]{1-iax}}$$

input
```
Int[E^(((5*I)/2)*ArcTan[a*x])*x,x]
```

output
```
(-2*(1 + I*a*x)^(9/4))/(a^2*(1 - I*a*x)^(1/4)) + ((5*I)*(((I/2)*(1 - I*a*x
)^(3/4)*(1 + I*a*x)^(5/4))/a + (5*((I*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))
/a + ((2*I)*((-(ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/
Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[
2])/2 + (Log[1 + Sqrt[1 - I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)
^(1/4)]/(2*Sqrt[2]) - Log[1 + Sqrt[1 - I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4)
)/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]))/2))/a))/4))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} x dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x,x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.93

$$\int e^{\frac{5}{2}i\arctan(ax)}x\,dx = \frac{2\,a^2\sqrt{\frac{625i}{16\,a^4}}\log\left(\frac{4}{25}\,a^2\sqrt{\frac{625i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2\,a^2\sqrt{\frac{625i}{16\,a^4}}\log\left(-\frac{4}{25}\,a^2\sqrt{\frac{625i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+2\,a^2\sqrt{-\frac{62}{16}}}{}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x,x, algorithm="fricas")
```

output
```
1/4*(2*a^2*sqrt(625/16*I/a^4)*log(4/25*a^2*sqrt(625/16*I/a^4) + sqrt(I*sqr
t(a^2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(625/16*I/a^4)*log(-4/25*a^2*sqrt(6
25/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 2*a^2*sqrt(-625/16*I
/a^4)*log(4/25*a^2*sqrt(-625/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I
))) - 2*a^2*sqrt(-625/16*I/a^4)*log(-4/25*a^2*sqrt(-625/16*I/a^4) + sqrt(I
*sqrt(a^2*x^2 + 1)/(a*x + I))) - (2*a^2*x^2 - 9*I*a*x + 43)*sqrt(I*sqrt(a^
2*x^2 + 1)/(a*x + I)))/a^2
```

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{5}{2}i\arctan(ax)}x\,dx=\text{Timed out}$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)*x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\frac{5}{2}i\arctan(ax)}x\,dx=\int x\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x,x, algorithm="maxima")
```

output
```
integrate(x*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{5}{2}i\arctan(ax)}x\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{5}{2}i\arctan(ax)}x\,dx=\int x\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}\,dx$$

input
```
int(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
```

output
```
int(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

Reduce [F]

$$\int e^{\frac{5}{2}i\arctan(ax)}x\,dx=\int\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}xdx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x,x)
```

3.97 $\int e^{\frac{5}{2}i\arctan(ax)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 226

$$\int e^{\frac{5}{2}i\arctan(ax)}\,dx=-\frac{5i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}-\frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}+\frac{5i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}-\frac{5i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}+\frac{5i\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)}{\sqrt{2}a}$$

output
```
-5*I*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a-4*I*(1+I*a*x)^(5/4)/a/(1-I*a*x)^(1/
4)+5/2*I*arctan(1-2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))*2^(1/2)/a-5/2*I
*arctan(1+2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))*2^(1/2)/a+5/2*I*arctanh
(2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)/(1+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2
)))*2^(1/2)/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.18

$$\int e^{\frac{5}{2}i\arctan(ax)}\,dx = -\frac{8ie^{\frac{9}{2}i\arctan(ax)}\,\text{Hypergeometric2F1}\left(2,\frac{9}{4},\frac{13}{4},-e^{2i\arctan(ax)}\right)}{9a}$$

input
```
Integrate[E^(((5*I)/2)*ArcTan[a*x]),x]
```

output
```
(((-8*I)/9)*E^(((9*I)/2)*ArcTan[a*x])*Hypergeometric2F1[2, 9/4, 13/4, -E^(
(2*I)*ArcTan[a*x])])/a
```

Rubi [A] (warning: unable to verify)

Time = 0.67 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.21, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {5584, 57, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{5}{2}i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5584}$$

$$\int \frac{(1+iax)^{5/4}}{(1-iax)^{5/4}}dx$$

$$\Big\downarrow \text{57}$$

$$-5\int \frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}dx-\frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{60}$$

$$-5\left(\frac{1}{2}\int \frac{1}{\sqrt[4]{1-iax}(iax+1)^{3/4}}dx+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)-\frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{73}$$

$$-5\left(\frac{2i\int \frac{\sqrt{1-iax}}{(iax+1)^{3/4}}d\sqrt[4]{1-iax}}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)-\frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{854}$$

$$-5\left(\frac{2i\int \frac{\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)-\frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{826}$$

$$-5\left(\frac{2i\left(\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)-\frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{1476}$$

$$-5\left(\frac{2i\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\cdots\right.}{a}\right.$$

$$\frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{1082}$$

$$-5\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+\frac{i(\cdots}{\cdots}\right.$$

$$\frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

$$\Big\downarrow \; 217$$

$$-5\left(\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}+\frac{i(1-iax)^{3/4}\sqrt[4]{}}{a}\right.$$

$$\frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

$$\Big\downarrow \; 1479$$

$$-5\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{1-}}\right.}{\sqrt{2}}\right.\right.}{a}\right.$$

$$\frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

$$\Big\downarrow \; 25$$

$$-5\left(\frac{2i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\ldots}}{\sqrt[4]{1+\ldots}}\right.}{\sqrt{2}}\right.\right.}{a}\right.$$

$$\frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

$\downarrow$ 27

$$-5\left(\frac{2i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{\ldots}}{\sqrt[4]{\ldots}}\right.}{\sqrt{2}}\right.\right.}{a}\right.$$

$$\frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

$\downarrow$ 1103

$$-5\left(\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{1-iax}\right.}{}\right.\right.}{a}\right.$$

$$\frac{4i(1+iax)^{5/4}}{a\sqrt[4]{1-iax}}$$

input
```
Int[E^(((5*I)/2)*ArcTan[a*x]),x]
```

output
```
((-4*I)*(1 + I*a*x)^(5/4))/(a*(1 - I*a*x)^(1/4)) - 5*((I*(1 - I*a*x)^(3/4)
*(1 + I*a*x)^(1/4))/a + ((2*I)*((-(ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/
(1 + I*a*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 +
I*a*x)^(1/4)]/Sqrt[2])/2 + (Log[1 + Sqrt[1 - I*a*x] - (Sqrt[2]*(1 - I*a*x)
^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]) - Log[1 + Sqrt[1 - I*a*x] + (Sqrt[2
]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]))/2))/a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
 + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5584
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.92

$$\int e^{\frac{5}{2}i\arctan(ax)}\,dx = -\frac{a\sqrt{\frac{25i}{a^2}}\log\left(\frac{1}{5}i\,a\sqrt{\frac{25i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-a\sqrt{\frac{25i}{a^2}}\log\left(-\frac{1}{5}i\,a\sqrt{\frac{25i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+a\sqrt{-\frac{25i}{a^2}}\log\left(\frac{1}{5}i\,a\sqrt{\right.}$$

$$2\,a$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

output
```
-1/2*(a*sqrt(25*I/a^2)*log(1/5*I*a*sqrt(25*I/a^2) + sqrt(I*sqrt(a^2*x^2 +
1)/(a*x + I))) - a*sqrt(25*I/a^2)*log(-1/5*I*a*sqrt(25*I/a^2) + sqrt(I*sqr
t(a^2*x^2 + 1)/(a*x + I))) + a*sqrt(-25*I/a^2)*log(1/5*I*a*sqrt(-25*I/a^2)
 + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - a*sqrt(-25*I/a^2)*log(-1/5*I*a*s
qrt(-25*I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 2*(a*x + 9*I)*sqrt
(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a
```

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{5}{2}i\arctan(ax)}\,dx = \text{Timed out}$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\frac{5}{2}i\arctan(ax)}\,dx = \int \left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
```

output
```
integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{5}{2}i\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{5}{2}i\arctan(ax)}\,dx = \int \left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}\,dx$$

input
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
```

output
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

Reduce [F]

$$\int e^{\frac{5}{2}i\arctan(ax)}\,dx = \int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

3.98 $\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 230

$$\begin{aligned}\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x}\,dx &= \frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}-2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\\&-\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)+\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)\\&-2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)\end{aligned}$$

output
```
8*(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)-2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)
)-2^(1/2)*arctan(1-2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))+2^(1/2)*arctan
(1+2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))-2*arctanh((1+I*a*x)^(1/4)/(1-I
*a*x)^(1/4))-2^(1/2)*arctanh(2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)/(1+(1
-I*a*x)^(1/2)/(1+I*a*x)^(1/2)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.49

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x}\,dx = \frac{4\left(3+3iax+3\sqrt[4]{2}(1+iax)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},-\frac{1}{4},\frac{3}{4},\frac{1}{2}(1-iax)\right)+(-1+iax)\,\text{Hypergeomet}\right.}{3\sqrt[4]{1-iax}(1+iax)^{3/4}}$$

input
```
Integrate[E^(((5*I)/2)*ArcTan[a*x])/x,x]
```

output
```
(4*(3 + (3*I)*a*x + 3*2^(1/4)*(1 + I*a*x)^(3/4)*Hypergeometric2F1[-1/4, -1
/4, 3/4, (1 - I*a*x)/2] + (-1 + I*a*x)*Hypergeometric2F1[3/4, 1, 7/4, (I +
 a*x)/(I - a*x)]))/(3*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.78 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.27, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 1.188$, Rules used = {5585, 109, 27, 35, 140, 73, 104, 756, 216, 219, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x}\,dx$$

$$\Big\downarrow \ 5585$$

$$\int \frac{(1+iax)^{5/4}}{x(1-iax)^{5/4}}dx$$

$$\Big\downarrow \ 109$$

$$\frac{4i\int -\frac{a(ax+i)}{4x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx}{a}+\frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{27}$$

$$\frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}-i\int \frac{ax+i}{x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx$$

$$\Big\downarrow \text{35}$$

$$\int \frac{(1-iax)^{3/4}}{x(iax+1)^{3/4}}dx+\frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{140}$$

$$-ia\int \frac{1}{\sqrt[4]{1-iax}(iax+1)^{3/4}}dx+\int \frac{1}{x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx+\frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{73}$$

$$\int \frac{1}{x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx+4\int \frac{\sqrt{1-iax}}{(iax+1)^{3/4}}d\sqrt[4]{1-iax}+\frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{104}$$

$$4\int \frac{\sqrt{1-iax}}{(iax+1)^{3/4}}d\sqrt[4]{1-iax}+4\int \frac{1}{\frac{iax+1}{1-iax}-1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+\frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{756}$$

$$4\int \frac{\sqrt{1-iax}}{(iax+1)^{3/4}}d\sqrt[4]{1-iax}+$$
$$4\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)+\frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{216}$$

$$4\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+4\int \frac{\sqrt{1-iax}}{(iax+1)^{3/4}}d\sqrt[4]{1-iax}+$$
$$\frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{219}$$

$$4\int \frac{\sqrt{1-iax}}{(iax+1)^{3/4}}d\sqrt[4]{1-iax}+4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+$$
$$\frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{854}$$

$$4\int \frac{\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}$$

$\downarrow$ 826

$$4\left(\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}$$

$\downarrow$ 1476

$$4\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)-\frac{1}{2}\int \cdots\right.$$
$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}$$

$\downarrow$ 1082

$$4\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-ia\cdots}}{\sqrt[4]{iax+\cdots}}\right.$$
$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}$$

$\downarrow$ 217

$$4\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}$$

$\downarrow$ 1479

$$4\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{\sqrt{2}}\right.\right.$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}$$

$\downarrow$ 25

$$4\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{1}}\right.}{\sqrt{2}}\right.\right.$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}$$

$\downarrow$ 27

$$4\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\right.}{}\right.\right.$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}$$

$\downarrow$ 1103

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+$$
$$4\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}}-\right.\right.$$
$$\frac{8\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}$$

input
```
Int[E^(((5*I)/2)*ArcTan[a*x])/x,x]
```

output
```
(8*(1 + I*a*x)^(1/4))/(1 - I*a*x)^(1/4) + 4*(-1/2*ArcTan[(1 + I*a*x)^(1/4)
/(1 - I*a*x)^(1/4)] - ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/2) + 4*
((-(ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]) + A
rcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2])/2 + (Log
[1 + Sqrt[1 - I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(2*S
qrt[2]) - Log[1 + Sqrt[1 - I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x
)^(1/4)]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.16

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x}\,dx = -\frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) + \frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - \frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) + \frac{1}{2}\sqrt{-4i}\log\left(-\frac{1}{2}\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) + 8\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}} - \log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right) - i\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right) + i\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right) + \log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-1\right)$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="fricas")
```

output
```
-1/2*sqrt(4*I)*log(1/2*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) +
1/2*sqrt(4*I)*log(-1/2*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) -
1/2*sqrt(-4*I)*log(1/2*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) +
 1/2*sqrt(-4*I)*log(-1/2*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
 + 8*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - log(sqrt(I*sqrt(a^2*x^2 + 1)/(a
*x + I)) + 1) - I*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + I*log(sqr
t(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) + log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x
+ I)) - 1)
```

Sympy [F]

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x}\,dx = \int \frac{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x,x)
```

output
```
Integral((I*(a*x - I)/sqrt(a**2*x**2 + 1))**(5/2)/x, x)
```

Maxima [F]

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x}\,dx = \int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="maxima")
```

output
```
integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x}\,dx = \int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}{x}\,dx$$

input
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x,x)
```

output
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x, x)
```

Reduce [F]

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x}\,dx = \int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x)
```

3.99 $\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 121

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^2}\,dx = \frac{10ia\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}} - \frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}} - 5ia\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - 5ia\,\mathrm{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

output
```
10*I*a*(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)-(1+I*a*x)^(5/4)/x/(1-I*a*x)^(1/4)-5
*I*a*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-5*I*a*arctanh((1+I*a*x)^(1/4)
/(1-I*a*x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.72

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^2}\,dx = \frac{-3(1-8iax+9a^2x^2) - 10ax(i+ax)\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{i+ax}{i-ax}\right)}{3x\sqrt[4]{1-iax}(1+iax)^{3/4}}$$

input
```
Integrate[E^(((5*I)/2)*ArcTan[a*x])/x^2,x]
```

output
```
(-3*(1 - (8*I)*a*x + 9*a^2*x^2) - 10*a*x*(I + a*x)*Hypergeometric2F1[3/4,
1, 7/4, (I + a*x)/(I - a*x)])/(3*x*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5585, 105, 105, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{5}{2} i \arctan(ax)}}{x^2} dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{(1+iax)^{5/4}}{x^2(1-iax)^{5/4}}dx$$

$$\Big\downarrow \text{105}$$

$$\frac{5}{2}ia\int \frac{\sqrt[4]{iax+1}}{x(1-iax)^{5/4}}dx-\frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{105}$$

$$\frac{5}{2}ia\left(\int \frac{1}{x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx+\frac{4\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{104}$$

$$\frac{5}{2}ia\left(4\int \frac{1}{\frac{iax+1}{1-iax}-1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+\frac{4\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{756}$$

$$\frac{5}{2}ia\left(4\left(-\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int\frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)+\frac{4\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}}$$

$\Big\downarrow$ 216

$$\frac{5}{2}ia\left(4\left(-\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{4\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}}$$

$\Big\downarrow$ 219

$$\frac{5}{2}ia\left(4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{4\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}}$$

input
```
Int[E^(((5*I)/2)*ArcTan[a*x])/x^2,x]
```

output
```
-((1 + I*a*x)^(5/4)/(x*(1 - I*a*x)^(1/4))) + ((5*I)/2)*a*((4*(1 + I*a*x)^(
1/4))/(1 - I*a*x)^(1/4) + 4*(-1/2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/
4)] - ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/2))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^2}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.26

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^2}\,dx = \frac{-5i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+5\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-5\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)+5i\,ax\log\left(\sqrt{\right.}{2\,x}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="fricas")
```

output
```
1/2*(-5*I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 5*a*x*log(sqr
t(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 5*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)
/(a*x + I)) - I) + 5*I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1) -
2*(-9*I*a*x + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^2}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}{x^2}\,dx$$

input
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x^2,x)
```

output
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x^2, x)
```

Reduce [F]

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^2}\,dx$$

$$= \frac{8\sqrt{aix+1}\,(a^2x^2+1)^{\frac{3}{4}}\,aix - 2\sqrt{aix+1}\,(a^2x^2+1)^{\frac{3}{4}} + 5\left(\int \frac{\sqrt{aix+1}\,(a^2x^2+1)^{\frac{3}{4}}}{a^4x^5+2a^2x^3+x}dx\right)a^3i\,x^3 + 5\left(\int \frac{\sqrt{aix+1}\,(a^2x^2+1)^{\frac{3}{4}}}{a^4x^5+2a^2x^3+x}dx\right)aix}{2x\,(a^2x^2+1)}$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x)
```

output
```
(8*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4)*a*i*x - 2*sqrt(a*i*x + 1)*(a**2*
x**2 + 1)**(3/4) + 5*int((sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4))/(a**4*x*
*5 + 2*a**2*x**3 + x),x)*a**3*i*x**3 + 5*int((sqrt(a*i*x + 1)*(a**2*x**2 +
 1)**(3/4))/(a**4*x**5 + 2*a**2*x**3 + x),x)*a*i*x)/(2*x*(a**2*x**2 + 1))
```

3.100 $\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 163

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = -\frac{25a^2\sqrt[4]{1+iax}}{2\sqrt[4]{1-iax}} - \frac{5ia(1+iax)^{5/4}}{4x\sqrt[4]{1-iax}} - \frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}} + \frac{25}{4}a^2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{25}{4}a^2\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

output
```
-25/2*a^2*(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)-5/4*I*a*(1+I*a*x)^(5/4)/x/(1-I*a
*x)^(1/4)-1/2*(1+I*a*x)^(9/4)/x^2/(1-I*a*x)^(1/4)+25/4*a^2*arctan((1+I*a*x
)^(1/4)/(1-I*a*x)^(1/4))+25/4*a^2*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.61

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = \frac{-6-33iax-102a^2x^2-129ia^3x^3+50a^2x^2(1-iax)\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{i+ax}{i-ax}\right)}{12x^2\sqrt[4]{1-iax}(1+iax)^{3/4}}$$

input
```
Integrate[E^(((5*I)/2)*ArcTan[a*x])/x^3,x]
```

output
```
(-6 - (33*I)*a*x - 102*a^2*x^2 - (129*I)*a^3*x^3 + 50*a^2*x^2*(1 - I*a*x)*
Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)])/(12*x^2*(1 - I*a*x)^(
1/4)*(1 + I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5585, 107, 105, 105, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^3}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{(1+iax)^{5/4}}{x^3(1-iax)^{5/4}}dx$$

$$\Big\downarrow \text{107}$$

$$\frac{5}{4}ia\int \frac{(iax+1)^{5/4}}{x^2(1-iax)^{5/4}}dx-\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{105}$$

$$\frac{5}{4}ia\left(\frac{5}{2}ia\int \frac{\sqrt[4]{iax+1}}{x(1-iax)^{5/4}}dx-\frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}}\right)-\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{105}$$

$$\frac{5}{4}ia\left(\frac{5}{2}ia\left(\int \frac{1}{x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx+\frac{4\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}}\right)-\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{104}$$

$$\frac{5}{4}ia\left(\frac{5}{2}ia\left(4\int \frac{1}{\frac{iax+1}{1-iax}-1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+\frac{4\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}}\right)-\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{756}$$

$$\frac{5}{4}ia\left(\frac{5}{2}ia\left(4\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)+\frac{4\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}}\right)-\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{216}$$

$$\frac{5}{4}ia\left(\frac{5}{2}ia\left(4\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{4\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}}\right)-\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{4}ia\left(\frac{5}{2}ia\left(4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{4\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{(1+iax)^{5/4}}{x\sqrt[4]{1-iax}}\right)-\frac{(1+iax)^{9/4}}{2x^2\sqrt[4]{1-iax}}$$

input
```
Int[E^(((5*I)/2)*ArcTan[a*x])/x^3,x]
```

output
```
-1/2*(1 + I*a*x)^(9/4)/(x^2*(1 - I*a*x)^(1/4)) + ((5*I)/4)*a*(-((1 + I*a*x
)^(5/4)/(x*(1 - I*a*x)^(1/4))) + ((5*I)/2)*a*((4*(1 + I*a*x)^(1/4))/(1 - I
*a*x)^(1/4) + 4*(-1/2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - ArcTan
h[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/2)))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^3}\,dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.08

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = \frac{25\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+25i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-25i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-25\,a^2}{8\,x^2}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="fricas")
```

output
```
1/8*(25*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 25*I*a^2*x^
2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 25*I*a^2*x^2*log(sqrt(I*s
qrt(a^2*x^2 + 1)/(a*x + I)) - I) - 25*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)
/(a*x + I)) - 1) - 2*(43*a^2*x^2 + 9*I*a*x + 2)*sqrt(I*sqrt(a^2*x^2 + 1)/(
a*x + I)))/x^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^3}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}{x^3}\,dx$$

input
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x^3,x)
```

output
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x^3, x)
```

Reduce [F]

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^3}\,dx$$
$$= \frac{-36\sqrt{aix+1}\,(a^2x^2+1)^{\frac{3}{4}}\,a^2x^2 - 18\sqrt{aix+1}\,(a^2x^2+1)^{\frac{3}{4}}\,aix - 4\sqrt{aix+1}\,(a^2x^2+1)^{\frac{3}{4}} - 25\left(\int \frac{\sqrt{aix+1}\,(}{a^4x^5+2}\right.}{8x^2\,(a^2x^2+1)}$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x)
```

output
```
( - 36*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4)*a**2*x**2 - 18*sqrt(a*i*x +
1)*(a**2*x**2 + 1)**(3/4)*a*i*x - 4*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4)
 - 25*int((sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4))/(a**4*x**5 + 2*a**2*x**
3 + x),x)*a**4*x**4 - 25*int((sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4))/(a**
4*x**5 + 2*a**2*x**3 + x),x)*a**2*x**2)/(8*x**2*(a**2*x**2 + 1))
```

3.101 $\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 203

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^4}\,dx=-\frac{287ia^3\sqrt[4]{1+iax}}{24\sqrt[4]{1-iax}}-\frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}}-\frac{13ia\sqrt[4]{1+iax}}{12x^2\sqrt[4]{1-iax}}+\frac{61a^2\sqrt[4]{1+iax}}{24x\sqrt[4]{1-iax}}+\frac{55}{8}ia^3\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)+\frac{55}{8}ia^3\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

output
```
-287/24*I*a^3*(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)-1/3*(1+I*a*x)^(1/4)/x^3/(1-I
*a*x)^(1/4)-13/12*I*a*(1+I*a*x)^(1/4)/x^2/(1-I*a*x)^(1/4)+61/24*a^2*(1+I*a
*x)^(1/4)/x/(1-I*a*x)^(1/4)+55/8*I*a^3*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1
/4))+55/8*I*a^3*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.52

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^4}\,dx=\frac{-8-34iax+87a^2x^2-226ia^3x^3+287a^4x^4+110a^3x^3(i+ax)\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{i+ax}{i-ax}\right)}{24x^3\sqrt[4]{1-iax}(1+iax)^{3/4}}$$

input
```
Integrate[E^(((5*I)/2)*ArcTan[a*x])/x^4,x]
```

output
```
(-8 - (34*I)*a*x + 87*a^2*x^2 - (226*I)*a^3*x^3 + 287*a^4*x^4 + 110*a^3*x^
3*(I + a*x)*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)])/(24*x^3*(
1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {5585, 109, 27, 168, 27, 168, 27, 172, 27, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^4}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{(1+iax)^{5/4}}{x^4(1-iax)^{5/4}}dx$$

$$\Big\downarrow \text{109}$$

$$-\frac{1}{3}\int -\frac{a(13i-12ax)}{2x^3(1-iax)^{5/4}(iax+1)^{3/4}}dx-\frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}a\int \frac{13i-12ax}{x^3(1-iax)^{5/4}(iax+1)^{3/4}}dx-\frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{6}a\left(-\frac{1}{2}\int \frac{a(52iax+61)}{2x^2(1-iax)^{5/4}(iax+1)^{3/4}}dx-\frac{13i\sqrt[4]{1+iax}}{2x^2\sqrt[4]{1-iax}}\right)-\frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}a\left(-\frac{1}{4}a\int \frac{52iax+61}{x^2(1-iax)^{5/4}(iax+1)^{3/4}}dx-\frac{13i\sqrt[4]{1+iax}}{2x^2\sqrt[4]{1-iax}}\right)-\frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{6}a\left(-\frac{1}{4}a\left(-\int -\frac{a(165i-122ax)}{2x(1-iax)^{5/4}(iax+1)^{3/4}}dx-\frac{61\sqrt[4]{1+iax}}{x\sqrt[4]{1-iax}}\right)-\frac{13i\sqrt[4]{1+iax}}{2x^2\sqrt[4]{1-iax}}\right)-\frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}a\left(-\frac{1}{4}a\left(\frac{1}{2}a\int \frac{165i-122ax}{x(1-iax)^{5/4}(iax+1)^{3/4}}dx-\frac{61\sqrt[4]{1+iax}}{x\sqrt[4]{1-iax}}\right)-\frac{13i\sqrt[4]{1+iax}}{2x^2\sqrt[4]{1-iax}}\right)-\frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{172}$$

$$\frac{1}{6}a\left(-\frac{1}{4}a\left(\frac{1}{2}a\left(\frac{2i\int \frac{165a}{2x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx}{a}+\frac{574i\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{61\sqrt[4]{1+iax}}{x\sqrt[4]{1-iax}}\right)-\frac{13i\sqrt[4]{1+iax}}{2x^2\sqrt[4]{1-iax}}\right)-\frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}a\left(-\frac{1}{4}a\left(\frac{1}{2}a\left(165i\int \frac{1}{x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx+\frac{574i\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{61\sqrt[4]{1+iax}}{x\sqrt[4]{1-iax}}\right)-\frac{13i\sqrt[4]{1+iax}}{2x^2\sqrt[4]{1-iax}}\right)-\frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{104}$$

$$\frac{1}{6}a\left(-\frac{1}{4}a\left(\frac{1}{2}a\left(660i\int \frac{1}{\frac{iax+1}{1-iax}-1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+\frac{574i\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{61\sqrt[4]{1+iax}}{x\sqrt[4]{1-iax}}\right)-\frac{13i\sqrt[4]{1+iax}}{2x^2\sqrt[4]{1-iax}}\right)-\frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{6}a\left(-\frac{1}{4}a\left(\frac{1}{2}a\left(660i\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)+\frac{574i\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{61\sqrt[4]{}}{x\sqrt[4]{}}\right.\right.$$

$$\frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{6}a\left(-\frac{1}{4}a\left(\frac{1}{2}a\left(660i\left(-\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{574i\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{61\sqrt[4]{1+i}}{x\sqrt[4]{1-i}}\right.\right.$$
$$\frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{6}a\left(-\frac{1}{4}a\left(\frac{1}{2}a\left(660i\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{574i\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{61\sqrt[4]{1+iax}}{x\sqrt[4]{1-iax}}\right)\right.$$
$$\frac{\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}}$$

input
```
Int[E^(((5*I)/2)*ArcTan[a*x])/x^4,x]
```

output
```
-1/3*(1 + I*a*x)^(1/4)/(x^3*(1 - I*a*x)^(1/4)) + (a*((((-13*I)/2)*(1 + I*a
*x)^(1/4))/(x^2*(1 - I*a*x)^(1/4)) - (a*((-61*(1 + I*a*x)^(1/4))/(x*(1 - I
*a*x)^(1/4)) + (a*(((574*I)*(1 + I*a*x)^(1/4))/(1 - I*a*x)^(1/4) + (660*I)
*(-1/2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - ArcTanh[(1 + I*a*x)^(
1/4)/(1 - I*a*x)^(1/4)]/2)))/2))/4))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
]))))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^4}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.91

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \frac{165i\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-165\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+165\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-16}{48\,x^3}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="fricas")
```

output
```
1/48*(165*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 165*a^3
*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 165*a^3*x^3*log(sqrt(I
*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 165*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2
 + 1)/(a*x + I)) - 1) - 2*(287*I*a^3*x^3 - 61*a^2*x^2 + 26*I*a*x + 8)*sqrt
(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^4}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)/x^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}{x^4}\,dx$$

input
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x^4,x)
```

output
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x^4, x)
```

Reduce [F]

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^4}\,dx$$
$$= \frac{-244\sqrt{aix+1}\,(a^2x^2+1)^{\frac{3}{4}}\,a^3i\,x^3+122\sqrt{aix+1}\,(a^2x^2+1)^{\frac{3}{4}}\,a^2x^2-52\sqrt{aix+1}\,(a^2x^2+1)^{\frac{3}{4}}\,aix-16\sqrt{}}{48x^3\,(a^2x^2+}$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x)
```

output
```
( - 244*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4)*a**3*i*x**3 + 122*sqrt(a*i*
x + 1)*(a**2*x**2 + 1)**(3/4)*a**2*x**2 - 52*sqrt(a*i*x + 1)*(a**2*x**2 +
1)**(3/4)*a*i*x - 16*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4) - 165*int((sqr
t(a*i*x + 1)*(a**2*x**2 + 1)**(3/4))/(a**4*x**5 + 2*a**2*x**3 + x),x)*a**5
*i*x**5 - 165*int((sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4))/(a**4*x**5 + 2*
a**2*x**3 + x),x)*a**3*i*x**3)/(48*x**3*(a**2*x**2 + 1))
```

3.102 $\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 233

$$\begin{aligned}\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^5}\,dx &= \frac{2467a^4\sqrt[4]{1+iax}}{192\sqrt[4]{1-iax}}-\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}}-\frac{17ia\sqrt[4]{1+iax}}{24x^3\sqrt[4]{1-iax}}\\&\quad+\frac{113a^2\sqrt[4]{1+iax}}{96x^2\sqrt[4]{1-iax}}+\frac{521ia^3\sqrt[4]{1+iax}}{192x\sqrt[4]{1-iax}}\\&\quad-\frac{475}{64}a^4\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{475}{64}a^4\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\end{aligned}$$

output
```
2467/192*a^4*(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)-1/4*(1+I*a*x)^(1/4)/x^4/(1-I*
a*x)^(1/4)-17/24*I*a*(1+I*a*x)^(1/4)/x^3/(1-I*a*x)^(1/4)+113/96*a^2*(1+I*a
*x)^(1/4)/x^2/(1-I*a*x)^(1/4)+521/192*I*a^3*(1+I*a*x)^(1/4)/x/(1-I*a*x)^(1
/4)-475/64*a^4*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-475/64*a^4*arctanh(
(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.51

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = \frac{-48-184iax+362a^2x^2+747ia^3x^3+1946a^4x^4+2467ia^5x^5+950ia^4x^4(i+ax)\,\text{Hypergeometric2F1}\,(}{192x^4\sqrt[4]{1-iax}(1+iax)^{3/4}}$$

input
```
Integrate[E^(((5*I)/2)*ArcTan[a*x])/x^5,x]
```

output
```
(-48 - (184*I)*a*x + 362*a^2*x^2 + (747*I)*a^3*x^3 + 1946*a^4*x^4 + (2467*
I)*a^5*x^5 + (950*I)*a^4*x^4*(I + a*x)*Hypergeometric2F1[3/4, 1, 7/4, (I +
 a*x)/(I - a*x)])/(192*x^4*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.03, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.938$, Rules used = {5585, 109, 27, 168, 27, 168, 27, 168, 27, 172, 27, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^5}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{(1+iax)^{5/4}}{x^5(1-iax)^{5/4}}dx$$

$$\Big\downarrow \text{109}$$

$$-\frac{1}{4}\int -\frac{a(17i-16ax)}{2x^4(1-iax)^{5/4}(iax+1)^{3/4}}dx-\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}a\int \frac{17i-16ax}{x^4(1-iax)^{5/4}(iax+1)^{3/4}}dx-\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{8}a\left(-\frac{1}{3}\int \frac{a(102iax+113)}{2x^3(1-iax)^{5/4}(iax+1)^{3/4}}dx-\frac{17i\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}}\right)-\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}a\left(-\frac{1}{6}a\int \frac{102iax+113}{x^3(1-iax)^{5/4}(iax+1)^{3/4}}dx-\frac{17i\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}}\right)-\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{8}a\left(-\frac{1}{6}a\left(-\frac{1}{2}\int -\frac{a(521i-452ax)}{2x^2(1-iax)^{5/4}(iax+1)^{3/4}}dx-\frac{113\sqrt[4]{1+iax}}{2x^2\sqrt[4]{1-iax}}\right)-\frac{17i\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}}\right)-\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}a\left(-\frac{1}{6}a\left(\frac{1}{4}a\int \frac{521i-452ax}{x^2(1-iax)^{5/4}(iax+1)^{3/4}}dx-\frac{113\sqrt[4]{1+iax}}{2x^2\sqrt[4]{1-iax}}\right)-\frac{17i\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}}\right)-\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{8}a\left(-\frac{1}{6}a\left(\frac{1}{4}a\left(-\int \frac{a(1042iax+1425)}{2x(1-iax)^{5/4}(iax+1)^{3/4}}dx-\frac{521i\sqrt[4]{1+iax}}{x\sqrt[4]{1-iax}}\right)-\frac{113\sqrt[4]{1+iax}}{2x^2\sqrt[4]{1-iax}}\right)-\frac{17i\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}}\right)-\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}a\left(-\frac{1}{6}a\left(\frac{1}{4}a\left(-\frac{1}{2}a\int \frac{1042iax+1425}{x(1-iax)^{5/4}(iax+1)^{3/4}}dx-\frac{521i\sqrt[4]{1+iax}}{x\sqrt[4]{1-iax}}\right)-\frac{113\sqrt[4]{1+iax}}{2x^2\sqrt[4]{1-iax}}\right)-\frac{17i\sqrt[4]{1+iax}}{3x^3\sqrt[4]{1-iax}}\right)-\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{172}$$

$$\frac{1}{8}a\left(-\frac{1}{6}a\left(\frac{1}{4}a\left(-\frac{1}{2}a\left(\frac{2i\int -\frac{1425ia}{2x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx}{a}+\frac{4934\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{521i\sqrt[4]{1+iax}}{x\sqrt[4]{1-iax}}\right)-\frac{113\sqrt[4]{1+iax}}{2x^2\sqrt[4]{1-iax}}\right)\right.$$

$$\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{8}a\left(-\frac{1}{6}a\left(\frac{1}{4}a\left(-\frac{1}{2}a\left(1425\int \frac{1}{x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx+\frac{4934\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{521i\sqrt[4]{1+iax}}{x\sqrt[4]{1-iax}}\right)-\frac{113\sqrt[4]{1+iax}}{2x^2\sqrt[4]{1-iax}}\right.\right.$$

$$\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}}$$

$$\Big\downarrow \; 104$$

$$\frac{1}{8}a\left(-\frac{1}{6}a\left(\frac{1}{4}a\left(-\frac{1}{2}a\left(5700\int \frac{1}{\frac{iax+1}{1-iax}-1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+\frac{4934\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{521i\sqrt[4]{1+iax}}{x\sqrt[4]{1-iax}}\right)-\frac{113\sqrt[4]{1+iax}}{2x^2\sqrt[4]{1-iax}}\right)-\right.$$

$$\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}}$$

$$\Big\downarrow \; 756$$

$$\frac{1}{8}a\left(-\frac{1}{6}a\left(\frac{1}{4}a\left(-\frac{1}{2}a\left(5700\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)+\frac{4934\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right.\right.\right.$$

$$\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}}$$

$$\Big\downarrow \; 216$$

$$\frac{1}{8}a\left(-\frac{1}{6}a\left(\frac{1}{4}a\left(-\frac{1}{2}a\left(5700\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{4934\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\right.\right.\right.$$

$$\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{8}a\left(-\frac{1}{6}a\left(\frac{1}{4}a\left(-\frac{1}{2}a\left(5700\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{4934\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\right.\right.\right.$$

$$\frac{\sqrt[4]{1+iax}}{4x^4\sqrt[4]{1-iax}}$$

input
```
Int[E^(((5*I)/2)*ArcTan[a*x])/x^5,x]
```

output
```
-1/4*(1 + I*a*x)^(1/4)/(x^4*(1 - I*a*x)^(1/4)) + (a*((((-17*I)/3)*(1 + I*a
*x)^(1/4))/(x^3*(1 - I*a*x)^(1/4)) - (a*((-113*(1 + I*a*x)^(1/4))/(2*x^2*(
1 - I*a*x)^(1/4)) + (a*(((-521*I)*(1 + I*a*x)^(1/4))/(x*(1 - I*a*x)^(1/4))
 - (a*((4934*(1 + I*a*x)^(1/4))/(1 - I*a*x)^(1/4) + 5700*(-1/2*ArcTan[(1 +
 I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(
1/4)]/2)))/2))/4))/6))/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
]))))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^5}\,dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.82

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = -\frac{1425\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+1425i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-1425i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right.}{38}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="fricas")
```

output
```
-1/384*(1425*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 1425*I
*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 1425*I*a^4*x^4*log
(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 1425*a^4*x^4*log(sqrt(I*sqrt(a
^2*x^2 + 1)/(a*x + I)) - 1) - 2*(2467*a^4*x^4 + 521*I*a^3*x^3 + 226*a^2*x^
2 - 136*I*a*x - 48)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}{x^5}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)/x^5, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}{x^5}\,dx$$

input
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x^5,x)
```

output
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)/x^5, x)
```

Reduce [F]

$$\int \frac{e^{\frac{5}{2}i\arctan(ax)}}{x^5}\,dx$$
$$= \frac{-1808\sqrt{aix+1}\,(a^2x^2+1)^{\frac{3}{4}}\,a^4x^4 - 904\sqrt{aix+1}\,(a^2x^2+1)^{\frac{3}{4}}\,a^3i\,x^3 + 226\sqrt{aix+1}\,(a^2x^2+1)^{\frac{3}{4}}\,a^2x^2 -}{}$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x)
```

output
```
( - 1808*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4)*a**4*x**4 - 904*sqrt(a*i*x
 + 1)*(a**2*x**2 + 1)**(3/4)*a**3*i*x**3 + 226*sqrt(a*i*x + 1)*(a**2*x**2
+ 1)**(3/4)*a**2*x**2 - 136*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4)*a*i*x -
 48*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4) - 1425*int((sqrt(a*i*x + 1)*(a*
*2*x**2 + 1)**(3/4))/(a**4*x**6 + 2*a**2*x**4 + x**2),x)*a**5*i*x**6 - 142
5*int((sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4))/(a**4*x**6 + 2*a**2*x**4 +
x**2),x)*a**3*i*x**4)/(192*x**4*(a**2*x**2 + 1))
```

3.103 $\int e^{-\frac{1}{2}i\arctan(ax)}x^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 291

$$\begin{aligned}\int e^{-\frac{1}{2}i\arctan(ax)}x^3\,dx = &-\frac{11\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4}-\frac{7(1-iax)^{5/4}(1+iax)^{3/4}}{32a^4}\\&+\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}-\frac{(1-iax)^{9/4}(1+iax)^{3/4}}{24a^4}\\&-\frac{11\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}\\&+\frac{11\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}\\&+\frac{11\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)}{64\sqrt{2}a^4}\end{aligned}$$

output
```
-11/64*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/a^4-7/32*(1-I*a*x)^(5/4)*(1+I*a*x)^
(3/4)/a^4+1/4*x^2*(1-I*a*x)^(5/4)*(1+I*a*x)^(3/4)/a^2-1/24*(1-I*a*x)^(9/4)
*(1+I*a*x)^(3/4)/a^4-11/128*arctan(1-2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/
4))*2^(1/2)/a^4+11/128*arctan(1+2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))*2
^(1/2)/a^4+11/128*arctanh(2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)/(1+(1-I*
a*x)^(1/2)/(1+I*a*x)^(1/2)))*2^(1/2)/a^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.44

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^3\,dx = \frac{(1-iax)^{5/4}\left(5a^2x^2(1+iax)^{3/4}+4\,2^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{5}{4},\frac{9}{4},\frac{1}{2}(1-iax)\right)-12\,2^{3/4}\,\text{Hypergeon}\right.}{20a^4}$$

input
```
Integrate[x^3/E^((I/2)*ArcTan[a*x]),x]
```

output
```
((1 - I*a*x)^(5/4)*(5*a^2*x^2*(1 + I*a*x)^(3/4) + 4*2^(3/4)*Hypergeometric
2F1[-7/4, 5/4, 9/4, (1 - I*a*x)/2] - 12*2^(3/4)*Hypergeometric2F1[-3/4, 5/
4, 9/4, (1 - I*a*x)/2] + 5*2^(3/4)*Hypergeometric2F1[1/4, 5/4, 9/4, (1 - I
*a*x)/2]))/(20*a^4)
```

Rubi [A] (warning: unable to verify)

Time = 0.78 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.13, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.938$, Rules used = {5585, 111, 27, 164, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3e^{-\frac{1}{2}i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{x^3\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}dx$$

$$\Big\downarrow \text{111}$$

$$\frac{\int -\frac{x\sqrt[4]{1-iax}(4-iax)}{2\sqrt[4]{iax+1}}dx}{4a^2}+\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}-\frac{\int \frac{x\sqrt[4]{1-iax}(4-iax)}{\sqrt[4]{iax+1}}dx}{8a^2}$$

$$\Big\downarrow \text{164}$$

$$\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}-\frac{\frac{11i\int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}dx}{8a}+\frac{(1+iax)^{3/4}(25-4iax)(1-iax)^{5/4}}{12a^2}}{8a^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}-\frac{\frac{11i\left(\frac{1}{2}\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)}{8a}+\frac{(1+iax)^{3/4}(25-4iax)(1-iax)^{5/4}}{12a^2}}{8a^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}-\frac{\frac{11i\left(\frac{2i\int \frac{1}{\sqrt[4]{iax+1}}d\sqrt[4]{1-iax}}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)}{8a}+\frac{(1+iax)^{3/4}(25-4iax)(1-iax)^{5/4}}{12a^2}}{8a^2}$$

$$\Big\downarrow \text{770}$$

$$\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}-\frac{\frac{11i\left(\frac{2i\int \frac{1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)}{8a}+\frac{(1+iax)^{3/4}(25-4iax)(1-iax)^{5/4}}{12a^2}}{8a^2}$$

$$\Big\downarrow \text{755}$$

$$\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}-\frac{\frac{11i\left(\frac{2i\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)}{8a}+\frac{(1+iax)^{3/4}(25-4iax)(1-iax)^{5/4}}{12a^2}}{8a^2}$$

$\downarrow$ 1476

$$\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{11i\left(\frac{2i\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)\right)}{a} - \frac{i\sqrt[4]{}}{}\right)}{8a}}{8a^2}$$

$\downarrow$ 1082

$$\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{11i\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a} - \frac{i\sqrt[4]{1-iax}(1+iax}{a}\right)}{8a}}{8a^2}$$

$\downarrow$ 217

$$\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{\frac{11i\left(\frac{2i\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)\right)}{a} - \frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)}{8a} + \frac{(1+iax}{}}{8a^2}$$

$\downarrow$ 1479

$$\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{11i\left(\frac{2i\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}} - \frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \cdots\right.\right)}{a}\cdots\right)}{8a}\cdots}{8a^2}$$

$\downarrow$ 25

$$\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{11i\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}} + \frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \frac{\text{arcta}\cdots}{\cdots}\right.\right)}{a}\cdots\right)}{8a}\cdots}{8a^2}$$

$\downarrow$ 27

$$\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{11i\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\text{arc}}{}\right.}{a}\right.}{8a}}{8a^2}$$

$\Big\downarrow$ 1103

$$\frac{x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2} - \frac{\frac{(1+iax)^{3/4}(25-4iax)(1-iax)^{5/4}}{12a^2}+\frac{11i\left(\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right.}{2\sqrt{2}}\right.}{a}\right.}{8a}}{8a^2}$$

input
```
Int[x^3/E^((I/2)*ArcTan[a*x]),x]
```

output
```
(x^2*(1 - I*a*x)^(5/4)*(1 + I*a*x)^(3/4))/(4*a^2) - (((1 - I*a*x)^(5/4)*(1
 + I*a*x)^(3/4)*(25 - (4*I)*a*x))/(12*a^2) + (((11*I)/8)*(((-I)*(1 - I*a*x
)^(1/4)*(1 + I*a*x)^(3/4))/a + ((2*I)*((-(ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^
(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4)
)/(1 + I*a*x)^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 + Sqrt[1 - I*a*x] - (Sqrt[2]
*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - I*a*x] +
 (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]))/2))/a))/a)/(8
*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{x^3}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

input
```
int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

output
```
int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.88

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^3\,dx = -\frac{96\,a^4\sqrt{\frac{121i}{4096\,a^8}}\log\left(\frac{64}{11}i\,a^4\sqrt{\frac{121i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-96\,a^4\sqrt{\frac{121i}{4096\,a^8}}\log\left(-\frac{64}{11}i\,a^4\sqrt{\frac{121i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)}{}$$

input
```
integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-1/192*(96*a^4*sqrt(121/4096*I/a^8)*log(64/11*I*a^4*sqrt(121/4096*I/a^8) +
 sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 96*a^4*sqrt(121/4096*I/a^8)*log(-6
4/11*I*a^4*sqrt(121/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 9
6*a^4*sqrt(-121/4096*I/a^8)*log(64/11*I*a^4*sqrt(-121/4096*I/a^8) + sqrt(I
*sqrt(a^2*x^2 + 1)/(a*x + I))) + 96*a^4*sqrt(-121/4096*I/a^8)*log(-64/11*I
*a^4*sqrt(-121/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - (-48*I
*a^3*x^3 + 56*a^2*x^2 + 58*I*a*x - 83)*sqrt(a^2*x^2 + 1)*sqrt(I*sqrt(a^2*x
^2 + 1)/(a*x + I)))/a^4
```

Sympy [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^3\,dx = \int \frac{x^3}{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\,dx$$

input
```
integrate(x**3/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2),x)
```

output
```
Integral(x**3/sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^3\,dx = \int \frac{x^3}{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

input
```
integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^3\,dx = \int \frac{x^3}{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}\,dx$$

input
```
int(x^3/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)
```

output
```
int(x^3/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^3\,dx = \int \frac{x^3}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}dx$$

input
```
int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

output
```
int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

3.104 $\int e^{-\frac{1}{2}i\arctan(ax)}x^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 268

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^2\,dx=\frac{3i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3}+\frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{12a^3}+\frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2}+\frac{3i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}-\frac{3i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}-\frac{3i\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)}{8\sqrt{2}a^3}$$

output
```
3/8*I*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/a^3+1/12*I*(1-I*a*x)^(5/4)*(1+I*a*x)
^(3/4)/a^3+1/3*x*(1-I*a*x)^(5/4)*(1+I*a*x)^(3/4)/a^2+3/16*I*arctan(1-2^(1/
2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))*2^(1/2)/a^3-3/16*I*arctan(1+2^(1/2)*(1
-I*a*x)^(1/4)/(1+I*a*x)^(1/4))*2^(1/2)/a^3-3/16*I*arctanh(2^(1/2)*(1-I*a*x
)^(1/4)/(1+I*a*x)^(1/4)/(1+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2)))*2^(1/2)/a^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.27

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^2\,dx = \frac{(1-iax)^{5/4}\left(5(1+iax)^{3/4}(i+4ax)-9i2^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{5}{4},\frac{9}{4},\frac{1}{2}(1-iax)\right)\right)}{60a^3}$$

input

```
Integrate[x^2/E^((I/2)*ArcTan[a*x]),x]
```

output

```
((1 - I*a*x)^(5/4)*(5*(1 + I*a*x)^(3/4)*(I + 4*a*x) - (9*I)*2^(3/4)*Hyperg
eometric2F1[1/4, 5/4, 9/4, (1 - I*a*x)/2]))/(60*a^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.75 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.18, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.938$, Rules used = {5585, 101, 27, 90, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2e^{-\frac{1}{2}i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{x^2\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}dx$$

$$\Big\downarrow \text{101}$$

$$\frac{\int -\frac{\sqrt[4]{1-iax}(2-iax)}{2\sqrt[4]{iax+1}}dx}{3a^2}+\frac{x(1+iax)^{3/4}(1-iax)^{5/4}}{3a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2}-\frac{\int \frac{\sqrt[4]{1-iax}(2-iax)}{\sqrt[4]{iax+1}}dx}{6a^2}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2}-\frac{\frac{9}{4}\int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}dx-\frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{2a}}{6a^2}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2}-\frac{\frac{9}{4}\left(\frac{1}{2}\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)-\frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{2a}}{6a^2}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2}-\frac{\frac{9}{4}\left(\frac{2i\int \frac{1}{\sqrt[4]{iax+1}}d\sqrt[4]{1-iax}}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)-\frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{2a}}{6a^2}$$

$$\Big\downarrow \text{ 770}$$

$$\frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2}-\frac{\frac{9}{4}\left(\frac{2i\int \frac{1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)-\frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{2a}}{6a^2}$$

$$\Big\downarrow \text{ 755}$$

$$\frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2}-\frac{\frac{9}{4}\left(\frac{2i\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)-\frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{2a}}{6a^2}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2}-\frac{\frac{9}{4}\left(\frac{2i\left(\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int\frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)\right)}{a}-\right.}{6a^2}$$

$\downarrow$ 1082

$$\frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2}-\frac{\frac{9}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-\frac{i\sqrt[4]{1-iax}(1\cdots}{a}\right.}{6a^2}$$

$\downarrow$ 217

$$\frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2}-\frac{\frac{9}{4}\left(\frac{2i\left(\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)\right)}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)-\frac{i(1\cdots}{}}{6a^2}$$

$\downarrow$ 1479

$$\frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2}-\frac{\frac{9}{4}\left(\frac{2i\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\cdots\right.\right.}{a}\cdots\right.}{6a^2}$$

$$\Big\downarrow\ 25$$

$$\frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2}-\frac{\frac{9}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\cdots\right.\right.}{a}\cdots\right.}{6a^2}$$

$$\Big\downarrow\ 27$$

$$\frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2}-\frac{\frac{9}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\cdots\right.\right)}{a}\cdots\right)}{6a^2}$$

$\downarrow$ 1103

$$\frac{x(1-iax)^{5/4}(1+iax)^{3/4}}{3a^2}-\frac{\frac{9}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\cdots\right)}{2\sqrt{2}}\cdots\right.\right)}{a}\cdots\right)}{6a^2}$$

input
```
Int[x^2/E^((I/2)*ArcTan[a*x]),x]
```

output
```
(x*(1 - I*a*x)^(5/4)*(1 + I*a*x)^(3/4))/(3*a^2) - (((-1/2*I)*(1 - I*a*x)^(
5/4)*(1 + I*a*x)^(3/4))/a + (9*(((-I)*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))
/a + ((2*I)*((-(ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/
Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[
2])/2 + (-1/2*Log[1 + Sqrt[1 - I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I
*a*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4
))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]))/2))/a))/4)/(6*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}dx$$

input
```
int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

output
```
int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.92

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^2\,dx = -\frac{12\,a^3\sqrt{\frac{9i}{64\,a^6}}\log\left(\frac{8}{3}\,a^3\sqrt{\frac{9i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3\sqrt{\frac{9i}{64\,a^6}}\log\left(-\frac{8}{3}\,a^3\sqrt{\frac{9i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3\sqrt{}$$

input
```
integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-1/24*(12*a^3*sqrt(9/64*I/a^6)*log(8/3*a^3*sqrt(9/64*I/a^6) + sqrt(I*sqrt(
a^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(9/64*I/a^6)*log(-8/3*a^3*sqrt(9/64*
I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(-9/64*I/a^6)*l
og(8/3*a^3*sqrt(-9/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 12*a
^3*sqrt(-9/64*I/a^6)*log(-8/3*a^3*sqrt(-9/64*I/a^6) + sqrt(I*sqrt(a^2*x^2
+ 1)/(a*x + I))) - sqrt(a^2*x^2 + 1)*(-8*I*a^2*x^2 + 10*a*x + 11*I)*sqrt(I
*sqrt(a^2*x^2 + 1)/(a*x + I)))/a^3
```

Sympy [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^2\,dx = \int \frac{x^2}{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\,dx$$

input
```
integrate(x**2/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2),x)
```

output
```
Integral(x**2/sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^2\,dx = \int \frac{x^2}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

input
```
integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^2\,dx=\int\frac{x^2}{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}\,dx$$

input
```
int(x^2/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)
```

output
```
int(x^2/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^2\,dx=\int\frac{x^2}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}dx$$

input
```
int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

output
```
int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

3.105 $\int e^{-\frac{1}{2}i\arctan(ax)}x\,dx$

Optimal result

Integrand size = 14, antiderivative size = 226

$$\int e^{-\frac{1}{2}i\arctan(ax)}x\,dx = \frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2}+\frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2} + \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}-\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)}{4\sqrt{2}a^2}$$

output
```
1/4*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/a^2+1/2*(1-I*a*x)^(5/4)*(1+I*a*x)^(3/4
)/a^2+1/8*arctan(1-2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))*2^(1/2)/a^2-1/
8*arctan(1+2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))*2^(1/2)/a^2-1/8*arctan
h(2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)/(1+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/
2)))*2^(1/2)/a^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.28

$$\int e^{-\frac{1}{2}i\arctan(ax)}x\,dx = \frac{(1-iax)^{5/4}\left(5(1+iax)^{3/4}-2^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{5}{4},\frac{9}{4},\frac{1}{2}(1-iax)\right)\right)}{10a^2}$$

input
```
Integrate[x/E^((I/2)*ArcTan[a*x]),x]
```

output
```
((1 - I*a*x)^(5/4)*(5*(1 + I*a*x)^(3/4) - 2^(3/4)*Hypergeometric2F1[1/4, 5
/4, 9/4, (1 - I*a*x)/2]))/(10*a^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.69 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.24, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {5585, 90, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-\frac{1}{2}i\arctan(ax)}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{x\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}dx$$

$$\Big\downarrow \text{90}$$

$$\frac{i\int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}dx}{4a}+\frac{(1+iax)^{3/4}(1-iax)^{5/4}}{2a^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{i\left(\frac{1}{2}\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)}{4a}+\frac{(1+iax)^{3/4}(1-iax)^{5/4}}{2a^2}$$

$\downarrow$ 73

$$\frac{i\left(\frac{2i\int \frac{1}{\sqrt[4]{iax+1}}d\sqrt[4]{1-iax}}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)}{4a}+\frac{(1+iax)^{3/4}(1-iax)^{5/4}}{2a^2}$$

$\downarrow$ 770

$$\frac{i\left(\frac{2i\int \frac{1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)}{4a}+\frac{(1+iax)^{3/4}(1-iax)^{5/4}}{2a^2}$$

$\downarrow$ 755

$$\frac{i\left(\frac{2i\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)}{4a}+\frac{(1+iax)^{3/4}(1-iax)^{5/4}}{2a^2}$$

$\downarrow$ 1476

$$\frac{i\left(\frac{2i\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)\right)}{a}-\right.}{4a}$$

$$\frac{(1+iax)^{3/4}(1-iax)^{5/4}}{2a^2}$$

$\downarrow$ 1082

$$\frac{i\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-\frac{i\sqrt[4]{1-iax}(1+\cdots}{a}\right.}{4a}$$

$$\frac{(1+iax)^{3/4}(1-iax)^{5/4}}{2a^2}$$

$\downarrow$ 217

$$\frac{i\left(\frac{2i\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)\right)}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)}{4a}+$$

$$\frac{(1+iax)^{3/4}(1-iax)^{5/4}}{2a^2}$$

$\downarrow$ 1479

$$\frac{i\left(\frac{2i\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\cdots\right.\right.}{a}\right.}{4a}$$

$$\frac{(1+iax)^{3/4}(1-iax)^{5/4}}{2a^2}$$

$$\downarrow 25$$

$$i\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\text{arct}\ldots\right.\right.}{a}\right.$$

$$\overline{\qquad\qquad 4a \qquad\qquad}$$

$$\frac{(1+iax)^{3/4}(1-iax)^{5/4}}{2a^2}$$

$$\downarrow 27$$

$$i\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\text{ar}\ldots\right.\right.}{a}\right.$$

$$\overline{\qquad\qquad 4a \qquad\qquad}$$

$$\frac{(1+iax)^{3/4}(1-iax)^{5/4}}{2a^2}$$

$$\downarrow 1103$$

$$\frac{(1+iax)^{3/4}(1-iax)^{5/4}}{2a^2}+$$

$$\frac{i\left(\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+\right.}{2\sqrt{2}}\right.\right.}{a}\right.}{4a}$$

input
```
Int[x/E^((I/2)*ArcTan[a*x]),x]
```

output
```
((1 - I*a*x)^(5/4)*(1 + I*a*x)^(3/4))/(2*a^2) + ((I/4)*(((-I)*(1 - I*a*x)^
(1/4)*(1 + I*a*x)^(3/4))/a + ((2*I)*((-(ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1
/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/
(1 + I*a*x)^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 + Sqrt[1 - I*a*x] - (Sqrt[2]*(
1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - I*a*x] + (
Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]))/2))/a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{x}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}dx$$

input
```
int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

output
```
int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.05

$$\int e^{-\frac{1}{2}i\arctan(ax)}x\,dx$$
$$= \frac{2\,a^2\sqrt{\frac{i}{16\,a^4}}\log\left(4i\,a^2\sqrt{\frac{i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2\,a^2\sqrt{\frac{i}{16\,a^4}}\log\left(-4i\,a^2\sqrt{\frac{i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2\,a^2\sqrt{-\frac{i}{16}}}{}$$

input
```
integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/4*(2*a^2*sqrt(1/16*I/a^4)*log(4*I*a^2*sqrt(1/16*I/a^4) + sqrt(I*sqrt(a^2
*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(1/16*I/a^4)*log(-4*I*a^2*sqrt(1/16*I/a^
4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(-1/16*I/a^4)*log(4*
I*a^2*sqrt(-1/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 2*a^2*sqr
t(-1/16*I/a^4)*log(-4*I*a^2*sqrt(-1/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(
a*x + I))) + sqrt(a^2*x^2 + 1)*(-2*I*a*x + 3)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*
x + I)))/a^2
```

Sympy [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}x\,dx = \int \frac{x}{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\,dx$$

input
```
integrate(x/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2),x)
```

output
```
Integral(x/sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}x\,dx = \int \frac{x}{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

input
```
integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{1}{2}i\arctan(ax)}x\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{1}{2}i\arctan(ax)}x\,dx = \int \frac{x}{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}\,dx$$

input
```
int(x/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)
```

output
```
int(x/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}x\,dx = \int \frac{x}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}dx$$

input
```
int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

output
```
int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

3.106 $\int e^{-\frac{1}{2}i\arctan(ax)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 195

$$\int e^{-\frac{1}{2}i\arctan(ax)}\,dx=-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}-\frac{i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}+\frac{i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}+\frac{i\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)}{\sqrt{2}a}$$

output
```
-I*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/a-1/2*I*arctan(1-2^(1/2)*(1-I*a*x)^(1/4
)/(1+I*a*x)^(1/4))*2^(1/2)/a+1/2*I*arctan(1+2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a
*x)^(1/4))*2^(1/2)/a+1/2*I*arctanh(2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)
/(1+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2)))*2^(1/2)/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.21

$$\int e^{-\frac{1}{2}i\arctan(ax)}\,dx=-\frac{8ie^{\frac{3}{2}i\arctan(ax)}\,\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-e^{2i\arctan(ax)}\right)}{3a}$$

input
```
Integrate[E^((-1/2*I)*ArcTan[a*x]),x]
```

output
```
(((-8*I)/3)*E^(((3*I)/2)*ArcTan[a*x])*Hypergeometric2F1[3/4, 2, 7/4, -E^((
2*I)*ArcTan[a*x])])/a
```

Rubi [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.23, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5584, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\frac{1}{2}i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5584}$$

$$\int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}$$

$$\Big\downarrow \text{73}$$

$$\frac{2i\int \frac{1}{\sqrt[4]{iax+1}}d\sqrt[4]{1-iax}}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}$$

$$\Big\downarrow \text{770}$$

$$\frac{2i\int \frac{1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}$$

$$\Big\downarrow \text{755}$$

$$\frac{2i\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2i\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-ia}}{\sqrt[4]{iax+}}\right.\right.}{a}$$

$$\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-$$

$$\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}$$

$$\Big\downarrow \text{217}$$

$$\frac{2i\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)\right)}{a}-$$

$$\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}$$

$$\Big\downarrow \text{1479}$$

$$\frac{2i\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-i}}{\sqrt[4]{1+ia}}\right.}{\sqrt{2}}\right.}{a} \cdots$$

$$\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}$$

$\big\downarrow$ 25

$$\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right.}{a} \cdots$$

$$\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}$$

$\big\downarrow$ 27

$$\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia}}{\sqrt[4]{1+iax}}\right.}{\sqrt{2}}\right.}{a} \cdots$$

$$\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}$$

$\big\downarrow$ 1103

$$\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\cdots\right.}{2\sqrt{2}}\cdots\right.\cdots}{a}$$

$$\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}$$

input
```
Int[E^((-1/2*I)*ArcTan[a*x]),x]
```

output
```
((-I)*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/a + ((2*I)*((-(ArcTan[1 - (Sqrt
[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(
1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 + Sqrt[1 - I
*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2] + Log[1 + S
qrt[1 - I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]
))/2))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5584
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{1}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}dx$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

output
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.09

$$\int e^{-\frac{1}{2}i\arctan(ax)}\,dx = \frac{a\sqrt{\frac{i}{a^2}}\log\left(a\sqrt{\frac{i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-a\sqrt{\frac{i}{a^2}}\log\left(-a\sqrt{\frac{i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-a\sqrt{-\frac{i}{a^2}}\log\left(a\sqrt{-\frac{i}{a^2}}+\sqrt{\frac{i\sqrt{a}}{a}}\right.}{2a}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(a*sqrt(I/a^2)*log(a*sqrt(I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))
) - a*sqrt(I/a^2)*log(-a*sqrt(I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))
) - a*sqrt(-I/a^2)*log(a*sqrt(-I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)
)) + a*sqrt(-I/a^2)*log(-a*sqrt(-I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
I))) - 2*I*sqrt(a^2*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a
```

Sympy [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}\,dx = \int \frac{1}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2),x)
```

output
```
Integral(1/sqrt((I*a*x + 1)/sqrt(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}\,dx = \int \frac{1}{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{1}{2}i\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{1}{2}i\arctan(ax)}\,dx = \int \frac{1}{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}\,dx$$

input
```
int(1/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)
```

output
```
int(1/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}\,dx = \int \frac{1}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}dx$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

output
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

3.107 $\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 203

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x}\,dx = 2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) + \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)$$

output
```
2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-2^(1/2)*arctan(1-2^(1/2)*(1-I*a*
x)^(1/4)/(1+I*a*x)^(1/4))+2^(1/2)*arctan(1+2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*
x)^(1/4))-2*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+2^(1/2)*arctanh(2^(1/
2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)/(1+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.47

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x}\,dx = 2\ 2^{3/4}\sqrt[4]{1-iax}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{1}{2}(1-iax)\right) - \frac{4\sqrt[4]{1-iax}\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},-\frac{1-iax}{-1-iax}\right)}{\sqrt[4]{1+iax}}$$

input
```
Integrate[1/(E^((I/2)*ArcTan[a*x])*x),x]
```

output
```
2*2^(3/4)*(1 - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, (1 - I*a*x)/2
] - (4*(1 - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 1, 5/4, -((1 - I*a*x)/(-1
- I*a*x))])/(1 + I*a*x)^(1/4)
```

Rubi [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.31, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.062$, Rules used = {5585, 140, 73, 104, 25, 770, 755, 827, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x}dx$$

$$\Big\downarrow \text{ 5585}$$

$$\int \frac{\sqrt[4]{1-iax}}{x\sqrt[4]{1+iax}}dx$$

$$\Big\downarrow \text{ 140}$$

$$\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{iax+1}}dx - ia\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{iax+1}}dx$$

$$\Big\downarrow \; 73$$

$$4\int \frac{1}{\sqrt[4]{iax+1}}d\sqrt[4]{1-iax}+\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{iax+1}}dx$$

$$\Big\downarrow \; 104$$

$$4\int \frac{1}{\sqrt[4]{iax+1}}d\sqrt[4]{1-iax}+4\int -\frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}$$

$$\Big\downarrow \; 25$$

$$4\int \frac{1}{\sqrt[4]{iax+1}}d\sqrt[4]{1-iax}-4\int \frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}$$

$$\Big\downarrow \; 770$$

$$4\int \frac{1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}-4\int \frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}$$

$$\Big\downarrow \; 755$$

$$4\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)-4\int \frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}$$

$$\Big\downarrow \; 827$$

$$4\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+4\left(\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)$$

$$\Big\downarrow \; 216$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)+4\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)$$

$$\Big\downarrow \; 219$$

$$4\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+$$
$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)$$

$\downarrow$ 1476

$$4\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{iax}}}\right.\right.$$
$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)$$

$\downarrow$ 1082

$$4\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-ia}}{\sqrt[4]{iax+}}\right.$$
$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)$$

$\downarrow$ 217

$$4\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)\right)+$$
$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)$$

$\downarrow$ 1479

$$4\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{}}{}\right.}{\sqrt{}}\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)$$

$$\Big\downarrow \ 25$$

$$4\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{1+}}\right.}{\sqrt{2}}\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)$$

$$\Big\downarrow \ 27$$

$$4\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1\right.}{}\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)$$

$$\Big\downarrow \ 1103$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+$$

$$4\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}}-\right.\right.$$

input
```
Int[1/(E^((I/2)*ArcTan[a*x])*x),x]
```

output
```
4*(ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/2 - ArcTanh[(1 + I*a*x)^(1/
4)/(1 - I*a*x)^(1/4)]/2) + 4*((-(ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1
 + I*a*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*
a*x)^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 + Sqrt[1 - I*a*x] - (Sqrt[2]*(1 - I*a
*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - I*a*x] + (Sqrt[2]
*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{1}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,x}dx$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x)
```

output
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.20

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x}\,dx = -\frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) + \frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) + \frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}i\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - \frac{1}{2}\sqrt{-4i}\log\left(-\frac{1}{2}i\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - \log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right) + i\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right) - i\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right) + \log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-1\right)$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

output
```
-1/2*sqrt(4*I)*log(1/2*I*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
+ 1/2*sqrt(4*I)*log(-1/2*I*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))
) + 1/2*sqrt(-4*I)*log(1/2*I*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
I))) - 1/2*sqrt(-4*I)*log(-1/2*I*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*
x + I))) - log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + I*log(sqrt(I*sqr
t(a^2*x^2 + 1)/(a*x + I)) + I) - I*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))
 - I) + log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1)
```

Sympy [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x}\,dx = \int \frac{1}{x\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x,x)
```

output
```
Integral(1/(x*sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))), x)
```

Maxima [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x}\,dx = \int \frac{1}{x\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(1/(x*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x}\,dx = \int \frac{1}{x\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}\,dx$$

input
```
int(1/(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)),x)
```

output
```
int(1/(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x}\,dx = \int \frac{1}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,x}dx$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x)
```

output
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x,x)
```

3.108 $\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 92

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x} - ia\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + ia\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

output
```
-(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x-I*a*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1
/4))+I*a*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = \frac{i\sqrt[4]{1-iax}\left(i-ax+2ax\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{i+ax}{i-ax}\right)\right)}{x\sqrt[4]{1+iax}}$$

input
```
Integrate[1/(E^((I/2)*ArcTan[a*x])*x^2),x]
```

output
```
(I*(1 - I*a*x)^(1/4)*(I - a*x + 2*a*x*Hypergeometric2F1[1/4, 1, 5/4, (I +
a*x)/(I - a*x)]))/(x*(1 + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5585, 105, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^2}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{\sqrt[4]{1-iax}}{x^2\sqrt[4]{1+iax}}dx$$

$$\Big\downarrow \text{105}$$

$$-\frac{1}{2}ia\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}$$

$$\Big\downarrow \text{104}$$

$$-2ia\int -\frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}$$

$$\Big\downarrow \text{25}$$

$$2ia\int \frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}$$

$$\Big\downarrow \text{827}$$

$$-2ia\left(\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}$$

$$\Big\downarrow \text{216}$$

$$-2ia\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}$$

$$\Big\downarrow \text{219}$$

$$-2ia\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}$$

input
```
Int[1/(E^((I/2)*ArcTan[a*x])*x^2),x]
```

output
```
-(((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x) - (2*I)*a*(ArcTan[(1 + I*a*x)^(
1/4)/(1 - I*a*x)^(1/4)]/2 - ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/2
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{1}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,x^2}dx$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x)
```

output
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(64) = 128$.

Time = 0.15 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.70

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^2}\,dx$$
$$= \frac{i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+}}{ax+i}}\right.}{2\,x}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
1/2*(I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + a*x*log(sqrt(I*s
qrt(a^2*x^2 + 1)/(a*x + I)) + I) - a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
 I)) - I) - I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1) - 2*sqrt(a^
2*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x
```

Sympy [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{1}{x^2\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**2,x)
```

output
```
Integral(1/(x**2*sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))), x)
```

Maxima [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{1}{x^2\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(1/(x^2*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{1}{x^2\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}\,dx$$

input
```
int(1/(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)),x)
```

output
```
int(1/(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^2}\,dx = \frac{-2\sqrt{aix+1}\,(a^2x^2+1)^{\frac{1}{4}} - \left(\int \frac{\sqrt{aix+1}\,(a^2x^2+1)^{\frac{1}{4}}}{a^2x^3+x}dx\right)aix}{2x}$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^2,x)
```

output
```
( - 2*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(1/4) - int((sqrt(a*i*x + 1)*(a**2*
x**2 + 1)**(1/4))/(a**2*x**3 + x),x)*a*i*x)/(2*x)
```

3.109 $\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 132

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = \frac{ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x} - \frac{(1-iax)^{5/4}(1+iax)^{3/4}}{2x^2} - \frac{1}{4}a^2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{1}{4}a^2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

output
```
1/4*I*a*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x-1/2*(1-I*a*x)^(5/4)*(1+I*a*x)^(3
/4)/x^2-1/4*a^2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+1/4*a^2*arctanh((1
+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.61

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = \frac{\sqrt[4]{1-iax}\left(-2+iax-3a^2x^2+2a^2x^2\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{i+ax}{i-ax}\right)\right)}{4x^2\sqrt[4]{1+iax}}$$

input
```
Integrate[1/(E^((I/2)*ArcTan[a*x])*x^3),x]
```

output
```
((1 - I*a*x)^(1/4)*(-2 + I*a*x - 3*a^2*x^2 + 2*a^2*x^2*Hypergeometric2F1[1
/4, 1, 5/4, (I + a*x)/(I - a*x)]))/(4*x^2*(1 + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5585, 107, 105, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^3}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{\sqrt[4]{1-iax}}{x^3\sqrt[4]{1+iax}}dx$$

$$\Big\downarrow \text{107}$$

$$-\frac{1}{4}ia\int \frac{\sqrt[4]{1-iax}}{x^2\sqrt[4]{iax+1}}dx-\frac{(1+iax)^{3/4}(1-iax)^{5/4}}{2x^2}$$

$$\Big\downarrow \text{105}$$

$$-\frac{1}{4}ia\left(-\frac{1}{2}ia\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{(1+iax)^{3/4}(1-iax)^{5/4}}{2x^2}$$

$$\Big\downarrow \text{104}$$

$$-\frac{1}{4}ia\left(-2ia\int -\frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{(1+iax)^{3/4}(1-iax)^{5/4}}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{4}ia\left(2ia\int \frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{(1+iax)^{3/4}(1-iax)^{5/4}}{2x^2}$$

$\Big\downarrow$ 827

$$-\frac{1}{4}ia\left(-2ia\left(\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{(1+iax)^{3/4}(1-iax)^{5/4}}{2x^2}$$

$\Big\downarrow$ 216

$$-\frac{1}{4}ia\left(-2ia\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{(1+iax)^{3/4}(1-iax)^{5/4}}{2x^2}$$

$\Big\downarrow$ 219

$$-\frac{1}{4}ia\left(-2ia\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{(1+iax)^{3/4}(1-iax)^{5/4}}{2x^2}$$

input
```
Int[1/(E^((I/2)*ArcTan[a*x])*x^3),x]
```

output
```
-1/2*((1 - I*a*x)^(5/4)*(1 + I*a*x)^(3/4))/x^2 - (I/4)*a*(-(((1 - I*a*x)^(
1/4)*(1 + I*a*x)^(3/4))/x) - (2*I)*a*(ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)
^(1/4)]/2 - ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{1}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\, x^3} dx$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
```

output
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.35

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^3} dx = \frac{a^2x^2 \log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right) - i\,a^2x^2 \log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right) + i\,a^2x^2 \log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right) - a^2x^2 \log\left(\sqrt{\right.}{8\,x^2}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
1/8*(a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - I*a^2*x^2*log(
sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + I*a^2*x^2*log(sqrt(I*sqrt(a^2*x
^2 + 1)/(a*x + I)) - I) - a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))
- 1) - 2*sqrt(a^2*x^2 + 1)*(-3*I*a*x + 2)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
I)))/x^2
```

Sympy [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{1}{x^3\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**3,x)
```

output
```
Integral(1/(x**3*sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))), x)
```

Maxima [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{1}{x^3\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(1/(x^3*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{1}{x^3\,\sqrt{\frac{1+a\,x\,1i}{\sqrt{a^2\,x^2+1}}}}\,dx$$

input
```
int(1/(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)),x)
```

output
```
int(1/(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^3}\,dx$$
$$= \frac{6\sqrt{aix+1}\,(a^2x^2+1)^{\frac{1}{4}}\,aix - 4\sqrt{aix+1}\,(a^2x^2+1)^{\frac{1}{4}} - \left(\int \frac{\sqrt{aix+1}\,(a^2x^2+1)^{\frac{1}{4}}}{a^2x^3+x}dx\right)a^2x^2}{8x^2}$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^3,x)
```

output
```
(6*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(1/4)*a*i*x - 4*sqrt(a*i*x + 1)*(a**2*
x**2 + 1)**(1/4) - int((sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(1/4))/(a**2*x**3
 + x),x)*a**2*x**2)/(8*x**2)
```

3.110 $\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 170

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}+\frac{5ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{12x^2} + \frac{11a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x} + \frac{3}{8}ia^3\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{3}{8}ia^3\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

output
```
-1/3*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x^3+5/12*I*a*(1-I*a*x)^(1/4)*(1+I*a*x
)^(3/4)/x^2+11/24*a^2*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x+3/8*I*a^3*arctan((
1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-3/8*I*a^3*arctanh((1+I*a*x)^(1/4)/(1-I*a*x
)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.54

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = \frac{\sqrt[4]{1-iax}\left(-8+2iax+a^2x^2+11ia^3x^3-18ia^3x^3\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{i+ax}{i-ax}\right)\right)}{24x^3\sqrt[4]{1+iax}}$$

input
```
Integrate[1/(E^((I/2)*ArcTan[a*x])*x^4),x]
```

output
```
((1 - I*a*x)^(1/4)*(-8 + (2*I)*a*x + a^2*x^2 + (11*I)*a^3*x^3 - (18*I)*a^3
*x^3*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]))/(24*x^3*(1 + I*
a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5585, 110, 27, 168, 27, 168, 27, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^4}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{\sqrt[4]{1-iax}}{x^4\sqrt[4]{1+iax}}dx$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{3}\int -\frac{a(4ax+5i)}{2x^3(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{6}a\int \frac{4ax+5i}{x^3(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}$$

$$\Big\downarrow \text{168}$$

$$-\frac{1}{6}a\left(-\frac{1}{2}\int -\frac{a(11-10iax)}{2x^2(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{5i\sqrt[4]{1-iax}(1+iax)^{3/4}}{2x^2}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{6}a\left(\frac{1}{4}a\int \frac{11-10iax}{x^2(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{5i\sqrt[4]{1-iax}(1+iax)^{3/4}}{2x^2}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}$$

$$\Big\downarrow \text{168}$$

$$-\frac{1}{6}a\left(\frac{1}{4}a\left(-\int \frac{9ia}{2x(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{11\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{5i\sqrt[4]{1-iax}(1+iax)^{3/4}}{2x^2}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{6}a\left(\frac{1}{4}a\left(-\frac{9}{2}ia\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{11\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{5i\sqrt[4]{1-iax}(1+iax)^{3/4}}{2x^2}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}$$

$$\Big\downarrow \text{104}$$

$$-\frac{1}{6}a\left(\frac{1}{4}a\left(-18ia\int -\frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{11\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{5i\sqrt[4]{1-iax}(1+iax)^{3/4}}{2x^2}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{6}a\left(\frac{1}{4}a\left(18ia\int \frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{11\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{5i\sqrt[4]{1-iax}(1+iax)^{3/4}}{2x^2}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}$$

$$\Big\downarrow \; 827$$

$$-\frac{1}{6}a\left(\frac{1}{4}a\left(-18ia\left(\frac{1}{2}\int\frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{11\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\right.$$
$$\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}$$

$$\Big\downarrow \; 216$$

$$-\frac{1}{6}a\left(\frac{1}{4}a\left(-18ia\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{11\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{5i\sqrt[4]{}}{}\right.$$
$$\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}$$

$$\Big\downarrow \; 219$$

$$-\frac{1}{6}a\left(\frac{1}{4}a\left(-18ia\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-\frac{11\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{5i\sqrt[4]{1-ia}}{}\right.$$
$$\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}$$

input
```
Int[1/(E^((I/2)*ArcTan[a*x])*x^4),x]
```

output
```
-1/3*((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x^3 - (a*((((-5*I)/2)*(1 - I*a*
x)^(1/4)*(1 + I*a*x)^(3/4))/x^2 + (a*((-11*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(
3/4))/x - (18*I)*a*(ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/2 - ArcTan
h[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/2)))/4))/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{1}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,x^4}dx$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
```

output
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.10

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^4}dx = \frac{-9i\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-9\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+9\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)+9i\,a^3x^3}{48\,x^3}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="fricas")
```

output
```
1/48*(-9*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 9*a^3*x^
3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 9*a^3*x^3*log(sqrt(I*sqrt
(a^2*x^2 + 1)/(a*x + I)) - I) + 9*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(
a*x + I)) - 1) + 2*(11*a^2*x^2 + 10*I*a*x - 8)*sqrt(a^2*x^2 + 1)*sqrt(I*sq
rt(a^2*x^2 + 1)/(a*x + I)))/x^3
```

Sympy [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{1}{x^4\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**4,x)
```

output
```
Integral(1/(x**4*sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))), x)
```

Maxima [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{1}{x^4\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(1/(x^4*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{1}{x^4\,\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}\,dx$$

input
```
int(1/(x^4*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)),x)
```

output
```
int(1/(x^4*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^4}\,dx$$
$$= \frac{22\sqrt{aix+1}\,(a^2x^2+1)^{\frac{1}{4}}\,a^2x^2 + 20\sqrt{aix+1}\,(a^2x^2+1)^{\frac{1}{4}}\,aix - 16\sqrt{aix+1}\,(a^2x^2+1)^{\frac{1}{4}} + 9\left(\int \frac{\sqrt{aix+1}\,(a^2}{a^2x^3+}\right.}{48x^3}$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^4,x)
```

output
```
(22*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(1/4)*a**2*x**2 + 20*sqrt(a*i*x + 1)*
(a**2*x**2 + 1)**(1/4)*a*i*x - 16*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(1/4) +
 9*int((sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(1/4))/(a**2*x**3 + x),x)*a**3*i*
x**3)/(48*x**3)
```

3.111 $\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 202

$$\begin{aligned}\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^5}\,dx =& -\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}+\frac{7ia\sqrt[4]{1-iax}(1+iax)^{3/4}}{24x^3}\\&+\frac{29a^2\sqrt[4]{1-iax}(1+iax)^{3/4}}{96x^2}-\frac{83ia^3\sqrt[4]{1-iax}(1+iax)^{3/4}}{192x}\\&+\frac{11}{64}a^4\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{11}{64}a^4\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\end{aligned}$$

output
```
-1/4*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x^4+7/24*I*a*(1-I*a*x)^(1/4)*(1+I*a*x
)^(3/4)/x^3+29/96*a^2*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x^2-83/192*I*a^3*(1-
I*a*x)^(1/4)*(1+I*a*x)^(3/4)/x+11/64*a^4*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^
(1/4))-11/64*a^4*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.49

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = \frac{\sqrt[4]{1-iax}\left(-48+8iax+2a^2x^2-25ia^3x^3+83a^4x^4-66a^4x^4\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{i+ax}{i-ax}\right)\right)}{192x^4\sqrt[4]{1+iax}}$$

input
```
Integrate[1/(E^((I/2)*ArcTan[a*x])*x^5),x]
```

output
```
((1 - I*a*x)^(1/4)*(-48 + (8*I)*a*x + 2*a^2*x^2 - (25*I)*a^3*x^3 + 83*a^4*
x^4 - 66*a^4*x^4*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]))/(19
2*x^4*(1 + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {5585, 110, 27, 168, 27, 168, 27, 168, 27, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^5}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{\sqrt[4]{1-iax}}{x^5\sqrt[4]{1+iax}}dx$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{4}\int -\frac{a(6ax+7i)}{2x^4(1-iax)^{3/4}\sqrt[4]{iax+1}}dx - \frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{8}a\int \frac{6ax+7i}{x^4(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}$$

$\downarrow$ 168

$$-\frac{1}{8}a\left(-\frac{1}{3}\int -\frac{a(29-28iax)}{2x^3(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{7i\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}$$

$\downarrow$ 27

$$-\frac{1}{8}a\left(\frac{1}{6}a\int \frac{29-28iax}{x^3(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{7i\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}$$

$\downarrow$ 168

$$-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{2}\int \frac{a(58ax+83i)}{2x^2(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{29\sqrt[4]{1-iax}(1+iax)^{3/4}}{2x^2}\right)-\frac{7i\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}$$

$\downarrow$ 27

$$-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{4}a\int \frac{58ax+83i}{x^2(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{29\sqrt[4]{1-iax}(1+iax)^{3/4}}{2x^2}\right)-\frac{7i\sqrt[4]{1-iax}(1+iax)^{3/4}}{3x^3}\right)-\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}$$

$\downarrow$ 168

$$-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{4}a\left(-\int -\frac{33a}{2x(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{83i\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{29\sqrt[4]{1-iax}(1+iax)^{3/4}}{2x^2}\right)\right.$$
$$\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}$$

$\downarrow$ 27

$$-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{4}a\left(\frac{33}{2}a\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{83i\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{29\sqrt[4]{1-iax}(1+iax)^{3/4}}{2x^2}\right)-\right.$$
$$\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}$$

$\downarrow$ 104

$$-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{4}a\left(66a\int -\frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{83i\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{29\sqrt[4]{1-iax}(1+i}{2x^2}\right.\right.$$
$$\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}$$

$\downarrow$ 25

$$-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{4}a\left(-66a\int \frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{83i\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{29\sqrt[4]{1-iax}(1+i}{2x^2}\right.\right.$$
$$\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}$$

$\downarrow$ 827

$$-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{4}a\left(66a\left(\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{83i\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right.\right.\right.$$
$$\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}$$

$\downarrow$ 216

$$-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{4}a\left(66a\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{83i\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\right.\right.$$
$$\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}$$

$\downarrow$ 219

$$-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{4}a\left(66a\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-\frac{83i\sqrt[4]{1-iax}(1+iax)^{3/4}}{x}\right)-\frac{29\sqrt[4]{}}{}\right.\right.$$
$$\frac{\sqrt[4]{1-iax}(1+iax)^{3/4}}{4x^4}$$

input
```
Int[1/(E^((I/2)*ArcTan[a*x])*x^5),x]
```

output
```
-1/4*((1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x^4 - (a*((((-7*I)/3)*(1 - I*a*
x)^(1/4)*(1 + I*a*x)^(3/4))/x^3 + (a*((-29*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(
3/4))/(2*x^2) - (a*(((-83*I)*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/x + 66*a
*(ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/2 - ArcTanh[(1 + I*a*x)^(1/4
)/(1 - I*a*x)^(1/4)]/2)))/4))/6))/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{1}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\, x^5}dx$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x)
```

output
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.97

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = -\frac{33\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-33i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)+33i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-33}{384\,x^4}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="fricas")
```

output
```
-1/384*(33*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 33*I*a^4
*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 33*I*a^4*x^4*log(sqrt(
I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 33*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 +
 1)/(a*x + I)) - 1) + 2*(83*I*a^3*x^3 - 58*a^2*x^2 - 56*I*a*x + 48)*sqrt(a
^2*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^4
```

Sympy [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{1}{x^5\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)/x**5,x)
```

output
```
Integral(1/(x**5*sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1))), x)
```

Maxima [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{1}{x^5\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(1/(x^5*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{1}{x^5\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}\,dx$$

input
```
int(1/(x^5*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)),x)
```

output
```
int(1/(x^5*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(ax)}}{x^5}\,dx$$
$$= \frac{-116\sqrt{aix+1}\,(a^2x^2+1)^{\frac{1}{4}}\,a^3i\,x^3+58\sqrt{aix+1}\,(a^2x^2+1)^{\frac{1}{4}}\,a^2x^2+56\sqrt{aix+1}\,(a^2x^2+1)^{\frac{1}{4}}\,aix-48\sqrt{a}}{192x^4}$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)/x^5,x)
```

output
```
( - 116*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(1/4)*a**3*i*x**3 + 58*sqrt(a*i*x
 + 1)*(a**2*x**2 + 1)**(1/4)*a**2*x**2 + 56*sqrt(a*i*x + 1)*(a**2*x**2 + 1
)**(1/4)*a*i*x - 48*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(1/4) - 33*int((sqrt(
a*i*x + 1)*(a**2*x**2 + 1)**(1/4))/(a**2*x**4 + x**2),x)*a**3*i*x**4)/(192
*x**4)
```

3.112 $\int e^{-\frac{3}{2}i\arctan(ax)}x^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 291

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^3\,dx=-\frac{41(1-iax)^{3/4}\sqrt[4]{1+iax}}{64a^4}-\frac{7(1-iax)^{7/4}\sqrt[4]{1+iax}}{32a^4}+\frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2}-\frac{(1-iax)^{11/4}\sqrt[4]{1+iax}}{8a^4}-\frac{123\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}+\frac{123\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}-\frac{123\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)}{64\sqrt{2}a^4}$$

output

```
-41/64*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a^4-7/32*(1-I*a*x)^(7/4)*(1+I*a*x)^
(1/4)/a^4+1/4*x^2*(1-I*a*x)^(7/4)*(1+I*a*x)^(1/4)/a^2-1/8*(1-I*a*x)^(11/4)
*(1+I*a*x)^(1/4)/a^4-123/128*arctan(1-2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1
/4))*2^(1/2)/a^4+123/128*arctan(1+2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))
*2^(1/2)/a^4-123/128*arctanh(2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)/(1+(1
-I*a*x)^(1/2)/(1+I*a*x)^(1/2)))*2^(1/2)/a^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.44

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^3\,dx = \frac{(1-iax)^{7/4}\left(7a^2x^2\sqrt[4]{1+iax}+12\sqrt[4]{2}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{7}{4},\frac{11}{4},\frac{1}{2}(1-iax)\right)-20\sqrt[4]{2}\,\text{Hypergeomet}\right.}{28a^4}$$

input
```
Integrate[x^3/E^(((3*I)/2)*ArcTan[a*x]),x]
```

output
```
((1 - I*a*x)^(7/4)*(7*a^2*x^2*(1 + I*a*x)^(1/4) + 12*2^(1/4)*Hypergeometri
c2F1[-5/4, 7/4, 11/4, (1 - I*a*x)/2] - 20*2^(1/4)*Hypergeometric2F1[-1/4,
7/4, 11/4, (1 - I*a*x)/2] + 7*2^(1/4)*Hypergeometric2F1[3/4, 7/4, 11/4, (1
 - I*a*x)/2]))/(28*a^4)
```

Rubi [A] (warning: unable to verify)

Time = 0.81 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.13, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.938$, Rules used = {5585, 111, 27, 164, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3e^{-\frac{3}{2}i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{x^3(1-iax)^{3/4}}{(1+iax)^{3/4}}dx$$

$$\Big\downarrow \text{111}$$

$$\frac{\int -\frac{x(1-iax)^{3/4}(4-3iax)}{2(iax+1)^{3/4}}dx}{4a^2}+\frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2}-\frac{\int \frac{x(1-iax)^{3/4}(4-3iax)}{(iax+1)^{3/4}}dx}{8a^2}$$

$$\Big\downarrow \text{164}$$

$$\frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2}-\frac{\frac{41i\int \frac{(1-iax)^{3/4}}{(iax+1)^{3/4}}dx}{8a}+\frac{\sqrt[4]{1+iax}(11-4iax)(1-iax)^{7/4}}{4a^2}}{8a^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2}-\frac{\frac{41i\left(\frac{3}{2}\int \frac{1}{\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)}{8a}+\frac{\sqrt[4]{1+iax}(11-4iax)(1-iax)^{7/4}}{4a^2}}{8a^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2}-\frac{\frac{41i\left(\frac{6i\int \frac{\sqrt{1-iax}}{(iax+1)^{3/4}}d\sqrt[4]{1-iax}}{a}-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)}{8a}+\frac{\sqrt[4]{1+iax}(11-4iax)(1-iax)^{7/4}}{4a^2}}{8a^2}$$

$$\Big\downarrow \text{854}$$

$$\frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2}-\frac{\frac{41i\left(\frac{6i\int \frac{\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{a}-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)}{8a}+\frac{\sqrt[4]{1+iax}(11-4iax)(1-iax)^{7/4}}{4a^2}}{8a^2}$$

$$\Big\downarrow \text{826}$$

$$\frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2}-\frac{\frac{41i\left(\frac{6i\left(\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)}{8a}+\frac{\sqrt[4]{1+iax}(11-4iax)(1-iax)^{7/4}}{4a^2}}{8a^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2}-\frac{\frac{41i\left(\frac{6i\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int\frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)-\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-\frac{i(1\cdots}{\cdots}\right)}{8a}\cdots}{8a^2}$$

$\downarrow$ 1082

$$\frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2}-\frac{\frac{41i\left(\frac{6i\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-\frac{i(1-iax)^{3/4}\sqrt[4]{1+\cdots}}{a}\cdots\right)}{8a}\cdots}{8a^2}$$

$\downarrow$ 217

$$\frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2}-\frac{\frac{41i\left(\frac{6i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)}{8a}+\frac{\sqrt[4]{1+\cdots}}{\cdots}}{8a^2}$$

$\downarrow$ 1479

$$\frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2}-$$

$$\frac{41i\left(\frac{6i\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\cdots\right.\right)}{a}\cdots\right)}{8a}$$

$$\overline{\qquad 8a^2 \qquad}$$

$\Big\downarrow$ 25

$$\frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2}-$$

$$\frac{41i\left(\frac{6i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\cdots\right.\right)}{a}\cdots\right)}{8a}$$

$$\overline{\qquad 8a^2 \qquad}$$

$\Big\downarrow$ 27

$$\frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{\frac{41i\left(\frac{6i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\dots\right)\right)}{a}\dots\right)}{8a}}{8a^2}$$

$\downarrow$ 1103

$$\frac{x^2(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^2} - \frac{\frac{\sqrt[4]{1+iax}(11-4iax)(1-iax)^{7/4}}{4a^2}+\frac{41i\left(\frac{6i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\dots\right)}{2\sqrt{2}}\dots\right)\right)}{a}\dots\right)}{8a}}{8a^2}$$

input
```
Int[x^3/E^(((3*I)/2)*ArcTan[a*x]),x]
```

output
```
(x^2*(1 - I*a*x)^(7/4)*(1 + I*a*x)^(1/4))/(4*a^2) - (((1 - I*a*x)^(7/4)*(1
 + I*a*x)^(1/4)*(11 - (4*I)*a*x))/(4*a^2) + (((41*I)/8)*(((-I)*(1 - I*a*x)
^(3/4)*(1 + I*a*x)^(1/4))/a + ((6*I)*((-(ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(
1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))
/(1 + I*a*x)^(1/4)]/Sqrt[2])/2 + (Log[1 + Sqrt[1 - I*a*x] - (Sqrt[2]*(1 -
I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]) - Log[1 + Sqrt[1 - I*a*x] + (
Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]))/2))/a))/a)/(8*a
^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{x^3}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}dx$$

input
```
int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

output
```
int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.86

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^3\,dx =$$

$$-\frac{32\,a^4\sqrt{\frac{15129i}{4096\,a^8}}\log\left(\frac{64}{123}\,a^4\sqrt{\frac{15129i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-32\,a^4\sqrt{\frac{15129i}{4096\,a^8}}\log\left(-\frac{64}{123}\,a^4\sqrt{\frac{15129i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)}{}$$

input
```
integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
-1/64*(32*a^4*sqrt(15129/4096*I/a^8)*log(64/123*a^4*sqrt(15129/4096*I/a^8)
 + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 32*a^4*sqrt(15129/4096*I/a^8)*lo
g(-64/123*a^4*sqrt(15129/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))
) + 32*a^4*sqrt(-15129/4096*I/a^8)*log(64/123*a^4*sqrt(-15129/4096*I/a^8)
+ sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 32*a^4*sqrt(-15129/4096*I/a^8)*lo
g(-64/123*a^4*sqrt(-15129/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)
)) + (16*a^4*x^4 + 40*I*a^3*x^3 - 54*a^2*x^2 - 93*I*a*x + 63)*sqrt(I*sqrt(
a^2*x^2 + 1)/(a*x + I)))/a^4
```

Sympy [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^3\,dx = \int \frac{x^3}{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2),x)
```

output
```
Integral(x**3/(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^3\,dx = \int \frac{x^3}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^3\,dx = \int \frac{x^3}{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}\,dx$$

input
```
int(x^3/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
```

output
```
int(x^3/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

Reduce [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^3\,dx = \int \frac{x^3}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}dx$$

input
```
int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

output
```
int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

3.113 $\int e^{-\frac{3}{2}i\arctan(ax)}x^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 268

$$\begin{aligned}\int e^{-\frac{3}{2}i\arctan(ax)}x^2\,dx &= \frac{17i(1-iax)^{3/4}\sqrt[4]{1+iax}}{24a^3}+\frac{i(1-iax)^{7/4}\sqrt[4]{1+iax}}{4a^3}\\&\quad+\frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2}+\frac{17i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}\\&\quad-\frac{17i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}\\&\quad+\frac{17i\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)}{8\sqrt{2}a^3}\end{aligned}$$

output

```
17/24*I*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a^3+1/4*I*(1-I*a*x)^(7/4)*(1+I*a*x
)^(1/4)/a^3+1/3*x*(1-I*a*x)^(7/4)*(1+I*a*x)^(1/4)/a^2+17/16*I*arctan(1-2^(
1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))*2^(1/2)/a^3-17/16*I*arctan(1+2^(1/2)
*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))*2^(1/2)/a^3+17/16*I*arctanh(2^(1/2)*(1-I
*a*x)^(1/4)/(1+I*a*x)^(1/4)/(1+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2)))*2^(1/2)/a
^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.27

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^2\,dx$$
$$=\frac{(1-iax)^{7/4}\left(7\sqrt[4]{1+iax}(3i+4ax)-17i\sqrt[4]{2}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{4},\frac{11}{4},\frac{1}{2}(1-iax)\right)\right)}{84a^3}$$

input
```
Integrate[x^2/E^(((3*I)/2)*ArcTan[a*x]),x]
```

output
```
((1 - I*a*x)^(7/4)*(7*(1 + I*a*x)^(1/4)*(3*I + 4*a*x) - (17*I)*2^(1/4)*Hyp
ergeometric2F1[3/4, 7/4, 11/4, (1 - I*a*x)/2]))/(84*a^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.77 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.18, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.938, Rules used = {5585, 101, 27, 90, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2e^{-\frac{3}{2}i\arctan(ax)}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{x^2(1-iax)^{3/4}}{(1+iax)^{3/4}}dx$$

$$\Big\downarrow \text{101}$$

$$\frac{\int -\frac{(1-iax)^{3/4}(2-3iax)}{2(iax+1)^{3/4}}dx}{3a^2}+\frac{x\sqrt[4]{1+iax}(1-iax)^{7/4}}{3a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2}-\frac{\int \frac{(1-iax)^{3/4}(2-3iax)}{(iax+1)^{3/4}}dx}{6a^2}$$

$$\Big\downarrow \; 90$$

$$\frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2}-\frac{\frac{17}{4}\int \frac{(1-iax)^{3/4}}{(iax+1)^{3/4}}dx-\frac{3i(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a}}{6a^2}$$

$$\Big\downarrow \; 60$$

$$\frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2}-\frac{\frac{17}{4}\left(\frac{3}{2}\int \frac{1}{\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)-\frac{3i(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a}}{6a^2}$$

$$\Big\downarrow \; 73$$

$$\frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2}-\frac{\frac{17}{4}\left(\frac{6i\int \frac{\sqrt{1-iax}}{(iax+1)^{3/4}}d\sqrt[4]{1-iax}}{a}-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)-\frac{3i(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a}}{6a^2}$$

$$\Big\downarrow \; 854$$

$$\frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2}-\frac{\frac{17}{4}\left(\frac{6i\int \frac{\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{a}-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)-\frac{3i(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a}}{6a^2}$$

$$\Big\downarrow \; 826$$

$$\frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2}-\frac{\frac{17}{4}\left(\frac{6i\left(\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)-\frac{3i(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a}}{6a^2}$$

$$\Big\downarrow \; 1476$$

$$\frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2}-\frac{\frac{17}{4}\left(\frac{6i\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int\frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)-\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}\right.}{6a^2}$$

$\downarrow$ 1082

$$\frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2}-\frac{\frac{17}{4}\left(\frac{6i\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-\frac{i(1-iax)^{3/4}\sqrt[4]{\ }}{a}\right.}{6a^2}$$

$\downarrow$ 217

$$\frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2}-\frac{\frac{17}{4}\left(\frac{6i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)-3}{6a^2}$$

$\downarrow$ 1479

$$\frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2}-\frac{\frac{17}{4}\left(\frac{6i\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right.\cdots\right)}{a}\cdots\right)}{6a^2}$$

$\downarrow$ 25

$$\frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2}-\frac{\frac{17}{4}\left(\frac{6i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\cdots\right.\right)}{a}\cdots\right)}{6a^2}$$

$\downarrow$ 27

$$\frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2}-\frac{\frac{17}{4}\left(\frac{6i\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right.\right.}{a}\right.}{6a^2}$$

$\downarrow$ 1103

$$\frac{x(1-iax)^{7/4}\sqrt[4]{1+iax}}{3a^2}-\frac{\frac{17}{4}\left(\frac{6i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right.}{2\sqrt{2}}\right.\right.}{a}\right.}{6a^2}$$

input
```
Int[x^2/E^(((3*I)/2)*ArcTan[a*x]),x]
```

output
```
(x*(1 - I*a*x)^(7/4)*(1 + I*a*x)^(1/4))/(3*a^2) - ((((-3*I)/2)*(1 - I*a*x)
^(7/4)*(1 + I*a*x)^(1/4))/a + (17*(((-I)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/
4))/a + ((6*I)*((-(ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4
)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sq
rt[2])/2 + (Log[1 + Sqrt[1 - I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a
*x)^(1/4)]/(2*Sqrt[2]) - Log[1 + Sqrt[1 - I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1
/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]))/2))/a))/4)/(6*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{x^2}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

output
```
int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.91

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^2\,dx = -\frac{12\,a^3\sqrt{\frac{289i}{64\,a^6}}\log\left(\frac{8}{17}i\,a^3\sqrt{\frac{289i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,a^3\sqrt{\frac{289i}{64\,a^6}}\log\left(-\frac{8}{17}i\,a^3\sqrt{\frac{289i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+12\,a}{}$$

input
```
integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
-1/24*(12*a^3*sqrt(289/64*I/a^6)*log(8/17*I*a^3*sqrt(289/64*I/a^6) + sqrt(
I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 12*a^3*sqrt(289/64*I/a^6)*log(-8/17*I*a^
3*sqrt(289/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 12*a^3*sqrt(
-289/64*I/a^6)*log(8/17*I*a^3*sqrt(-289/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 +
1)/(a*x + I))) - 12*a^3*sqrt(-289/64*I/a^6)*log(-8/17*I*a^3*sqrt(-289/64*I
/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + (8*a^3*x^3 + 22*I*a^2*x^2 -
 37*a*x - 23*I)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/a^3
```

Sympy [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^2\,dx = \int \frac{x^2}{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2),x)
```

output
```
Integral(x**2/(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^2\,dx = \int \frac{x^2}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^2\,dx=\int \frac{x^2}{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}\,dx$$

input
```
int(x^2/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
```

output
```
int(x^2/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

Reduce [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^2\,dx=\int \frac{x^2}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

output
```
int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

3.114 $\int e^{-\frac{3}{2}i\arctan(ax)}x\,dx$

Optimal result

Integrand size = 14, antiderivative size = 226

$$\int e^{-\frac{3}{2}i\arctan(ax)}x\,dx=\frac{3(1-iax)^{3/4}\sqrt[4]{1+iax}}{4a^2}+\frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2a^2}+\frac{9\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}-\frac{9\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}+\frac{9\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)}{4\sqrt{2}a^2}$$

output
```
3/4*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a^2+1/2*(1-I*a*x)^(7/4)*(1+I*a*x)^(1/4
)/a^2+9/8*arctan(1-2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))*2^(1/2)/a^2-9/
8*arctan(1+2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))*2^(1/2)/a^2+9/8*arctan
h(2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)/(1+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/
2)))*2^(1/2)/a^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.28

$$\int e^{-\frac{3}{2}i\arctan(ax)}x\,dx = \frac{(1-iax)^{7/4}\left(7\sqrt[4]{1+iax}-3\sqrt[4]{2}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{4},\frac{11}{4},\frac{1}{2}(1-iax)\right)\right)}{14a^2}$$

input
```
Integrate[x/E^(((3*I)/2)*ArcTan[a*x]),x]
```

output
```
((1 - I*a*x)^(7/4)*(7*(1 + I*a*x)^(1/4) - 3*2^(1/4)*Hypergeometric2F1[3/4,
 7/4, 11/4, (1 - I*a*x)/2]))/(14*a^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.69 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.24, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {5585, 90, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-\frac{3}{2}i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{x(1-iax)^{3/4}}{(1+iax)^{3/4}}dx$$

$$\Big\downarrow \text{90}$$

$$\frac{3i\int \frac{(1-iax)^{3/4}}{(iax+1)^{3/4}}dx}{4a}+\frac{\sqrt[4]{1+iax}(1-iax)^{7/4}}{2a^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{3i\left(\frac{3}{2}\int \frac{1}{\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)}{4a}+\frac{\sqrt[4]{1+iax}(1-iax)^{7/4}}{2a^2}$$

$$\Big\downarrow \; 73$$

$$\frac{3i\left(\frac{6i\int \frac{\sqrt{1-iax}}{(iax+1)^{3/4}}d\sqrt[4]{1-iax}}{a}-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)}{4a}+\frac{\sqrt[4]{1+iax}(1-iax)^{7/4}}{2a^2}$$

$$\Big\downarrow \; 854$$

$$\frac{3i\left(\frac{6i\int \frac{\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{a}-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)}{4a}+\frac{\sqrt[4]{1+iax}(1-iax)^{7/4}}{2a^2}$$

$$\Big\downarrow \; 826$$

$$\frac{3i\left(\frac{6i\left(\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)}{4a}+\frac{\sqrt[4]{1+iax}(1-iax)^{7/4}}{2a^2}$$

$$\Big\downarrow \; 1476$$

$$\frac{3i\left(\frac{6i\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}\right.}{4a}$$

$$\frac{\sqrt[4]{1+iax}(1-iax)^{7/4}}{2a^2}$$

$$\Big\downarrow \; 1082$$

$$\frac{3i\left(\frac{6i\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-\frac{i(1-iax)^{3/4}\sqrt[4]{}}{a}\right.}{4a}$$

$$\frac{\sqrt[4]{1+iax}(1-iax)^{7/4}}{2a^2}$$

$\downarrow$ 217

$$\frac{3i\left(\frac{6i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}\right)}{4a}+$$

$$\frac{\sqrt[4]{1+iax}(1-iax)^{7/4}}{2a^2}$$

$\downarrow$ 1479

$$\frac{3i\left(\frac{6i\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\right.\right.}{a}\right.}{4a}$$

$$\frac{\sqrt[4]{1+iax}(1-iax)^{7/4}}{2a^2}$$

$\downarrow$ 25

$$\frac{3i\left(\frac{6i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\right.}{a}}{4a}$$

$$\frac{\sqrt[4]{1+iax}(1-iax)^{7/4}}{2a^2}$$

$\downarrow$ 27

$$\frac{3i\left(\frac{6i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right.}{a}}{4a}$$

$$\frac{\sqrt[4]{1+iax}(1-iax)^{7/4}}{2a^2}$$

$\downarrow$ 1103

$$\frac{\sqrt[4]{1+iax}(1-iax)^{7/4}}{2a^2}+\frac{3i\left(\frac{6i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right.}{2\sqrt{2}}\right.\right.}{a}\right.}{4a}$$

input
```
Int[x/E^(((3*I)/2)*ArcTan[a*x]),x]
```

output
```
((1 - I*a*x)^(7/4)*(1 + I*a*x)^(1/4))/(2*a^2) + (((3*I)/4)*(((-I)*(1 - I*a
*x)^(3/4)*(1 + I*a*x)^(1/4))/a + ((6*I)*((-(ArcTan[1 - (Sqrt[2]*(1 - I*a*x
)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/
4))/(1 + I*a*x)^(1/4)]/Sqrt[2])/2 + (Log[1 + Sqrt[1 - I*a*x] - (Sqrt[2]*(1
 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]) - Log[1 + Sqrt[1 - I*a*x]
+ (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]))/2))/a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{x}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}dx$$

input
```
int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

output
```
int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.04

$$\int e^{-\frac{3}{2}i\arctan(ax)}x\,dx$$
$$= \frac{2\,a^2\sqrt{\frac{81i}{16\,a^4}}\log\left(\frac{4}{9}\,a^2\sqrt{\frac{81i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-2\,a^2\sqrt{\frac{81i}{16\,a^4}}\log\left(-\frac{4}{9}\,a^2\sqrt{\frac{81i}{16\,a^4}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+2\,a^2\sqrt{-\frac{81i}{16\,a^4}}}{}$$

input
```
integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
1/4*(2*a^2*sqrt(81/16*I/a^4)*log(4/9*a^2*sqrt(81/16*I/a^4) + sqrt(I*sqrt(a
^2*x^2 + 1)/(a*x + I))) - 2*a^2*sqrt(81/16*I/a^4)*log(-4/9*a^2*sqrt(81/16*
I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 2*a^2*sqrt(-81/16*I/a^4)*l
og(4/9*a^2*sqrt(-81/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 2*a
^2*sqrt(-81/16*I/a^4)*log(-4/9*a^2*sqrt(-81/16*I/a^4) + sqrt(I*sqrt(a^2*x^
2 + 1)/(a*x + I))) - (2*a^2*x^2 + 7*I*a*x - 5)*sqrt(I*sqrt(a^2*x^2 + 1)/(a
*x + I)))/a^2
```

Sympy [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}x\,dx=\int\frac{x}{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2),x)
```

output
```
Integral(x/(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}x\,dx=\int\frac{x}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{3}{2}i\arctan(ax)}x\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{3}{2}i\arctan(ax)}x\,dx=\int\frac{x}{{\left(\frac{1+a\,x\,1{}\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)}^{3/2}}\,dx$$

input
```
int(x/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
```

output
```
int(x/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

Reduce [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}x\,dx = \int \frac{x}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}dx$$

input
```
int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

output
```
int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

3.115 $\int e^{-\frac{3}{2}i\arctan(ax)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 195

$$\int e^{-\frac{3}{2}i\arctan(ax)}\,dx = -\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}-\frac{3i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}+\frac{3i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}-\frac{3i\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)}{\sqrt{2}a}$$

output
```
-I*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/a-3/2*I*arctan(1-2^(1/2)*(1-I*a*x)^(1/4
)/(1+I*a*x)^(1/4))*2^(1/2)/a+3/2*I*arctan(1+2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a
*x)^(1/4))*2^(1/2)/a-3/2*I*arctanh(2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)
/(1+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2)))*2^(1/2)/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.20

$$\int e^{-\frac{3}{2}i\arctan(ax)}\,dx=-\frac{8ie^{\frac{1}{2}i\arctan(ax)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},2,\frac{5}{4},-e^{2i\arctan(ax)}\right)}{a}$$

input
```
Integrate[E^(((-3*I)/2)*ArcTan[a*x]),x]
```

output
```
((-8*I)*E^((I/2)*ArcTan[a*x])*Hypergeometric2F1[1/4, 2, 5/4, -E^((2*I)*Arc
Tan[a*x])])/a
```

Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.23, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5584, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\frac{3}{2}i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5584}$$

$$\int \frac{(1-iax)^{3/4}}{(1+iax)^{3/4}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{2}\int \frac{1}{\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}$$

$$\Big\downarrow \text{73}$$

$$\frac{6i\int \frac{\sqrt{1-iax}}{(iax+1)^{3/4}}d\sqrt[4]{1-iax}}{a}-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}$$

$$\Big\downarrow \text{854}$$

$$\frac{6i\int \frac{\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{a}-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}$$

$$\Big\downarrow \text{826}$$

$$\frac{6i\left(\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}$$

$$\Big\downarrow \text{1476}$$

$$\frac{6i\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-}}{\sqrt[4]{iax}}\right.}{a}$$

$$\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}$$

$$\Big\downarrow \text{1082}$$

$$\frac{6i\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-$$

$$\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}$$

$$\Big\downarrow \text{217}$$

$$\frac{6i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-$$

$$\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}$$

$$\Big\downarrow \text{1479}$$

$$\frac{\frac{6i\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right.}{a}}{\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}}$$

$\downarrow$ 25

$$\frac{\frac{6i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right.}{a}}{\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}}$$

$\downarrow$ 27

$$\frac{\frac{6i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{1+i}}\right.}{\sqrt{2}}\right.}{a}}{\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}}$$

$\downarrow$ 1103

$$\frac{6i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{2\sqrt{2}}\right.}{a}$$

$$\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{a}$$

input
```
Int[E^(((-3*I)/2)*ArcTan[a*x]),x]
```

output
```
((-I)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/a + ((6*I)*((-(ArcTan[1 - (Sqrt
[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(
1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2])/2 + (Log[1 + Sqrt[1 - I*a*x]
 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]) - Log[1 + Sq
rt[1 - I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2])
)/2))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5584
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] && !IntegerQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

output
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.07

$$\int e^{-\frac{3}{2}i\arctan(ax)}\,dx = \frac{a\sqrt{\frac{9i}{a^2}}\log\left(\frac{1}{3}i\,a\sqrt{\frac{9i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-a\sqrt{\frac{9i}{a^2}}\log\left(-\frac{1}{3}i\,a\sqrt{\frac{9i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)+a\sqrt{-\frac{9i}{a^2}}\log\left(\frac{1}{3}i\,a\sqrt{-\frac{9i}{a^2}}\right.}{2\,a}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
1/2*(a*sqrt(9*I/a^2)*log(1/3*I*a*sqrt(9*I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/
(a*x + I))) - a*sqrt(9*I/a^2)*log(-1/3*I*a*sqrt(9*I/a^2) + sqrt(I*sqrt(a^2
*x^2 + 1)/(a*x + I))) + a*sqrt(-9*I/a^2)*log(1/3*I*a*sqrt(-9*I/a^2) + sqrt
(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - a*sqrt(-9*I/a^2)*log(-1/3*I*a*sqrt(-9*I
/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 2*(a*x + I)*sqrt(I*sqrt(a^2
*x^2 + 1)/(a*x + I)))/a
```

Sympy [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}\,dx = \int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2),x)
```

output
```
Integral(((I*a*x + 1)/sqrt(a**2*x**2 + 1))**(-3/2), x)
```

Maxima [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}\,dx = \int \frac{1}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(-3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{3}{2}i\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{3}{2}i\arctan(ax)}\,dx = \int \frac{1}{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}\,dx$$

input
```
int(1/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
```

output
```
int(1/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

Reduce [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}\,dx = \int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

output
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

3.116 $\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 204

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x}\,dx = -2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) + \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)$$

output
```
-2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-2^(1/2)*arctan(1-2^(1/2)*(1-I*a
*x)^(1/4)/(1+I*a*x)^(1/4))+2^(1/2)*arctan(1+2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a
*x)^(1/4))-2*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-2^(1/2)*arctanh(2^(1
/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)/(1+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.48

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x}\,dx = \frac{2(1-iax)^{3/4}\left(\sqrt[4]{2}(1+iax)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},\frac{1}{2}(1-iax)\right)-2\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4}\right.\right.}{3(1+iax)^{3/4}}$$

input
```
Integrate[1/(E^(((3*I)/2)*ArcTan[a*x])*x),x]
```

output
```
(2*(1 - I*a*x)^(3/4)*(2^(1/4)*(1 + I*a*x)^(3/4)*Hypergeometric2F1[3/4, 3/4
, 7/4, (1 - I*a*x)/2] - 2*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*
x)]))/(3*(1 + I*a*x)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.30, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5585, 140, 73, 104, 756, 216, 219, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x}\,dx$$

$$\Big\downarrow \; 5585$$

$$\int \frac{(1-iax)^{3/4}}{x(1+iax)^{3/4}}\,dx$$

$$\Big\downarrow \; 140$$

$$\int \frac{1}{x\sqrt[4]{1-iax}(iax+1)^{3/4}}\,dx - ia\int \frac{1}{\sqrt[4]{1-iax}(iax+1)^{3/4}}\,dx$$

$$\Big\downarrow \; 73$$

$$\int \frac{1}{x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx+4\int \frac{\sqrt{1-iax}}{(iax+1)^{3/4}}d\sqrt[4]{1-iax}$$

$$\Big\downarrow \text{104}$$

$$4\int \frac{\sqrt{1-iax}}{(iax+1)^{3/4}}d\sqrt[4]{1-iax}+4\int \frac{1}{\frac{iax+1}{1-iax}-1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}$$

$$\Big\downarrow \text{756}$$

$$4\int \frac{\sqrt{1-iax}}{(iax+1)^{3/4}}d\sqrt[4]{1-iax}+$$
$$4\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)$$

$$\Big\downarrow \text{216}$$

$$4\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+4\int \frac{\sqrt{1-iax}}{(iax+1)^{3/4}}d\sqrt[4]{1-iax}$$

$$\Big\downarrow \text{219}$$

$$4\int \frac{\sqrt{1-iax}}{(iax+1)^{3/4}}d\sqrt[4]{1-iax}+4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)$$

$$\Big\downarrow \text{854}$$

$$4\int \frac{\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)$$

$$\Big\downarrow \text{826}$$

$$4\left(\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+$$
$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)$$

$$\Big\downarrow \text{1476}$$

$$4\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)-\frac{1}{2}\int \frac{1}{}\right.$$
$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)$$

$$\Big\downarrow \text{1082}$$

$$4\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-ia}}{\sqrt[4]{iax+}}\right.$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)$$

$$\Big\downarrow \text{217}$$

$$4\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)$$

$$\Big\downarrow \text{1479}$$

$$4\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{\sqrt{2}}\right.\right.$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$4\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{1}}\right.}{\sqrt{2}}\right.\right.$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)$$

$$\Big\downarrow 27$$

$$4\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\right.}{}\right.\right.$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)$$

$$\Big\downarrow 1103$$

$$4\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+$$

$$4\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}}-\right.\right.$$

input
```
Int[1/(E^(((3*I)/2)*ArcTan[a*x])*x),x]
```

output
```
4*(-1/2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - ArcTanh[(1 + I*a*x)^
(1/4)/(1 - I*a*x)^(1/4)]/2) + 4*((-(ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))
/(1 + I*a*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 +
 I*a*x)^(1/4)]/Sqrt[2])/2 + (Log[1 + Sqrt[1 - I*a*x] - (Sqrt[2]*(1 - I*a*x
)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]) - Log[1 + Sqrt[1 - I*a*x] + (Sqrt[
2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} x}dx$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

output
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.19

$$\begin{aligned}\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x}\,dx = & -\frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) \\ & +\frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) \\ & -\frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) \\ & +\frac{1}{2}\sqrt{-4i}\log\left(-\frac{1}{2}\sqrt{-4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) \\ & -\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)-i\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right) \\ & +i\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)+\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-1\right)\end{aligned}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="fricas")
```

output
```
-1/2*sqrt(4*I)*log(1/2*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) +
1/2*sqrt(4*I)*log(-1/2*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) -
1/2*sqrt(-4*I)*log(1/2*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) +
 1/2*sqrt(-4*I)*log(-1/2*sqrt(-4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))
 - log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - I*log(sqrt(I*sqrt(a^2*x^
2 + 1)/(a*x + I)) + I) + I*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) +
log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1)
```

Sympy [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x,x)
```

output
```
Integral(1/(x*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(1/(x*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}\,dx$$

input
```
int(1/(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)),x)
```

output
```
int(1/(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)), x)
```

Reduce [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x}\,dx = \int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}x}\,dx$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

output
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x,x)
```

3.117 $\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 92

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x} + 3ia\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + 3ia\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

output
```
-(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x+3*I*a*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^
(1/4))+3*I*a*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = \frac{i(1-iax)^{3/4}\left(i-ax+2ax\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{i+ax}{i-ax}\right)\right)}{x(1+iax)^{3/4}}$$

input
```
Integrate[1/(E^(((3*I)/2)*ArcTan[a*x])*x^2),x]
```

output
```
(I*(1 - I*a*x)^(3/4)*(I - a*x + 2*a*x*Hypergeometric2F1[3/4, 1, 7/4, (I +
a*x)/(I - a*x)]))/(x*(1 + I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5585, 105, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^2}dx$$

$$\Big\downarrow \text{ 5585}$$

$$\int \frac{(1-iax)^{3/4}}{x^2(1+iax)^{3/4}}dx$$

$$\Big\downarrow \text{ 105}$$

$$-\frac{3}{2}ia\int \frac{1}{x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}$$

$$\Big\downarrow \text{ 104}$$

$$-6ia\int \frac{1}{\frac{iax+1}{1-iax}-1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}$$

$$\Big\downarrow \text{ 756}$$

$$-6ia\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}$$

$$\Big\downarrow \text{ 216}$$

$$-6ia\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}$$

$$\Big\downarrow \text{ 219}$$

$$-6ia\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}$$

input
```
Int[1/(E^(((3*I)/2)*ArcTan[a*x])*x^2),x]
```

output
```
-(((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x) - (6*I)*a*(-1/2*ArcTan[(1 + I*a
*x)^(1/4)/(1 - I*a*x)^(1/4)] - ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)
]/2)
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} x^2}dx$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
```

output
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(64) = 128.

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.65

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = \frac{3i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right) - 3\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right) + 3\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right) - 3i\,ax\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}} ...\right.}{2\,x}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
1/2*(3*I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 3*a*x*log(sqrt
(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 3*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/
(a*x + I)) - I) - 3*I*a*x*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1) - 2
*(-I*a*x + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x
```

Sympy [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**2,x)
```

output
```
Integral(1/(x**2*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(1/(x^2*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}\,dx$$

input
```
int(1/(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)),x)
```

output
```
int(1/(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)), x)
```

Reduce [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}x^2}dx$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
```

output
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^2,x)
```

3.118 $\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 132

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x} - \frac{(1-iax)^{7/4}\sqrt[4]{1+iax}}{2x^2} + \frac{9}{4}a^2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{9}{4}a^2\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

output
```
3/4*I*a*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x-1/2*(1-I*a*x)^(7/4)*(1+I*a*x)^(1
/4)/x^2+9/4*a^2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+9/4*a^2*arctanh((1
+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.61

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \frac{(1-iax)^{3/4}\left(-2+3iax-5a^2x^2+6a^2x^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{i+ax}{i-ax}\right)\right)}{4x^2(1+iax)^{3/4}}$$

input
```
Integrate[1/(E^(((3*I)/2)*ArcTan[a*x])*x^3),x]
```

output
```
((1 - I*a*x)^(3/4)*(-2 + (3*I)*a*x - 5*a^2*x^2 + 6*a^2*x^2*Hypergeometric2
F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)]))/(4*x^2*(1 + I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5585, 107, 105, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^3}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{(1-iax)^{3/4}}{x^3(1+iax)^{3/4}}dx$$

$$\Big\downarrow \text{107}$$

$$-\frac{3}{4}ia\int \frac{(1-iax)^{3/4}}{x^2(iax+1)^{3/4}}dx-\frac{\sqrt[4]{1+iax}(1-iax)^{7/4}}{2x^2}$$

$$\Big\downarrow \text{105}$$

$$-\frac{3}{4}ia\left(-\frac{3}{2}ia\int \frac{1}{x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{\sqrt[4]{1+iax}(1-iax)^{7/4}}{2x^2}$$

$$\Big\downarrow \text{104}$$

$$-\frac{3}{4}ia\left(-6ia\int \frac{1}{\frac{iax+1}{1-iax}-1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{\sqrt[4]{1+iax}(1-iax)^{7/4}}{2x^2}$$

$$\Big\downarrow \text{756}$$

$$-\frac{3}{4}ia\left(-6ia\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{\sqrt[4]{1+iax}(1-iax)^{7/4}}{2x^2}$$

$$\Big\downarrow \ 216$$

$$-\frac{3}{4}ia\left(-6ia\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{\sqrt[4]{1+iax}(1-iax)^{7/4}}{2x^2}$$

$$\Big\downarrow \ 219$$

$$-\frac{3}{4}ia\left(-6ia\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{\sqrt[4]{1+iax}(1-iax)^{7/4}}{2x^2}$$

input
```
Int[1/(E^(((3*I)/2)*ArcTan[a*x])*x^3),x]
```

output
```
-1/2*((1 - I*a*x)^(7/4)*(1 + I*a*x)^(1/4))/x^2 - ((3*I)/4)*a*(-(((1 - I*a*
x)^(3/4)*(1 + I*a*x)^(1/4))/x) - (6*I)*a*(-1/2*ArcTan[(1 + I*a*x)^(1/4)/(1
 - I*a*x)^(1/4)] - ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/2))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} x^3}dx$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x)
```

output
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.33

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \frac{9\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+9i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-9i\,a^2x^2\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-9\,a^2x^2\,\mathrm{lo}}{8\,x^2}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="fricas")
```

output
```
1/8*(9*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 9*I*a^2*x^2*
log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 9*I*a^2*x^2*log(sqrt(I*sqrt
(a^2*x^2 + 1)/(a*x + I)) - I) - 9*a^2*x^2*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*
x + I)) - 1) + 2*(5*a^2*x^2 + 7*I*a*x - 2)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
 I)))/x^2
```

Sympy [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{1}{x^3\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**3,x)
```

output
```
Integral(1/(x**3*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{1}{x^3\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(1/(x^3*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{1}{x^3\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}\,dx$$

input
```
int(1/(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)),x)
```

output
```
int(1/(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)), x)
```

Reduce [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{\sqrt{aix+1}\,(a^2x^2+1)^{\frac{3}{4}}}{a^3i\,x^6+a^2x^5+ai\,x^4+x^3}dx - \left(\int \frac{\sqrt{aix+1}\,(a^2x^2+1)^{\frac{3}{4}}}{a^3i\,x^5+a^2x^4+ai\,x^3+x^2}dx\right)ai$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^3,x)
```

output
```
int((sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4))/(a**3*i*x**6 + a**2*x**5 + a*
i*x**4 + x**3),x) - int((sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4))/(a**3*i*x
**5 + a**2*x**4 + a*i*x**3 + x**2),x)*a*i
```

3.119 $\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 170

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}+\frac{7ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{12x^2}+\frac{23a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{24x}-\frac{17}{8}ia^3\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{17}{8}ia^3\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

output
```
-1/3*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^3+7/12*I*a*(1-I*a*x)^(3/4)*(1+I*a*x
)^(1/4)/x^2+23/24*a^2*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x-17/8*I*a^3*arctan(
(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-17/8*I*a^3*arctanh((1+I*a*x)^(1/4)/(1-I*a
*x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.55

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = \frac{(1-iax)^{3/4}\left(-8+6iax+9a^2x^2+23ia^3x^3-34ia^3x^3\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{i+ax}{i-ax}\right)\right)}{24x^3(1+iax)^{3/4}}$$

input
```
Integrate[1/(E^(((3*I)/2)*ArcTan[a*x])*x^4),x]
```

output
```
((1 - I*a*x)^(3/4)*(-8 + (6*I)*a*x + 9*a^2*x^2 + (23*I)*a^3*x^3 - (34*I)*a
^3*x^3*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)]))/(24*x^3*(1 +
I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {5585, 110, 27, 168, 27, 168, 27, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^4}\,dx$$

$$\Big\downarrow \text{ 5585}$$

$$\int \frac{(1-iax)^{3/4}}{x^4(1+iax)^{3/4}}dx$$

$$\Big\downarrow \text{ 110}$$

$$\frac{1}{3}\int -\frac{a(4ax+7i)}{2x^3\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{6}a\int \frac{4ax+7i}{x^3\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}$$

$$\Big\downarrow \text{168}$$

$$-\frac{1}{6}a\left(-\frac{1}{2}\int -\frac{a(23-14iax)}{2x^2\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{7i(1-iax)^{3/4}\sqrt[4]{1+iax}}{2x^2}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{6}a\left(\frac{1}{4}a\int \frac{23-14iax}{x^2\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{7i(1-iax)^{3/4}\sqrt[4]{1+iax}}{2x^2}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}$$

$$\Big\downarrow \text{168}$$

$$-\frac{1}{6}a\left(\frac{1}{4}a\left(-\int \frac{51ia}{2x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{23(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{7i(1-iax)^{3/4}\sqrt[4]{1+iax}}{2x^2}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{6}a\left(\frac{1}{4}a\left(-\frac{51}{2}ia\int \frac{1}{x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{23(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{7i(1-iax)^{3/4}\sqrt[4]{1+iax}}{2x^2}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}$$

$$\Big\downarrow \text{104}$$

$$-\frac{1}{6}a\left(\frac{1}{4}a\left(-102ia\int \frac{1}{\frac{iax+1}{1-iax}-1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{23(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{7i(1-iax)^{3/4}\sqrt[4]{1+iax}}{2x^2}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}$$

$$\Big\downarrow \text{756}$$

$$-\frac{1}{6}a\left(\frac{1}{4}a\left(-102ia\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{23(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)\right.$$
$$\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}$$

$$\Big\downarrow \text{216}$$

$$-\frac{1}{6}a\left(\frac{1}{4}a\left(-102ia\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-\frac{23(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{7}{}\right.$$
$$\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}$$

$$\Big\downarrow \text{219}$$

$$-\frac{1}{6}a\left(\frac{1}{4}a\left(-102ia\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-\frac{23(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{7i(1-}{}\right.$$
$$\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{3x^3}$$

input
```
Int[1/(E^(((3*I)/2)*ArcTan[a*x])*x^4),x]
```

output
```
-1/3*((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x^3 - (a*(((((-7*I)/2)*(1 - I*a*
x)^(3/4)*(1 + I*a*x)^(1/4))/x^2 + (a*((-23*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(
1/4))/x - (102*I)*a*(-1/2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - Ar
cTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/2)))/4))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} x^4} dx$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x)
```

output
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.08

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^4} dx = \frac{-51i\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+51\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-51\,a^3x^3\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)+51i}{48\,x^3}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="fricas")
```

output
```
1/48*(-51*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 51*a^3*
x^3*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 51*a^3*x^3*log(sqrt(I*s
qrt(a^2*x^2 + 1)/(a*x + I)) - I) + 51*I*a^3*x^3*log(sqrt(I*sqrt(a^2*x^2 +
1)/(a*x + I)) - 1) - 2*(23*I*a^3*x^3 - 37*a^2*x^2 - 22*I*a*x + 8)*sqrt(I*s
qrt(a^2*x^2 + 1)/(a*x + I)))/x^3
```

Sympy [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^4} dx = \int \frac{1}{x^4\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}} dx$$

input
```
integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**4,x)
```

output
```
Integral(1/(x**4*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^4}\,dx=\int \frac{1}{x^4\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(1/(x^4*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^4}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{1}{x^4\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}\,dx$$

input
```
int(1/(x^4*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)),x)
```

output
```
int(1/(x^4*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)), x)
```

Reduce [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^4}\,dx = -3\left(\int \frac{\sqrt{aix+1}\,(a^2x^2+1)^{\frac{3}{4}}}{3a^3x^7-3a^2i\,x^6+3a\,x^5-3i\,x^4}dx\right)i - 3\left(\int \frac{\sqrt{aix+1}\,(a^2x^2+1)^{\frac{3}{4}}}{3a^3x^6-3a^2i\,x^5+3a\,x^4-3i\,x^3}dx\right)a$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^4,x)
```

output
```
- 3*(int((sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4))/(3*a**3*x**7 - 3*a**2*i
*x**6 + 3*a*x**5 - 3*i*x**4),x)*i + int((sqrt(a*i*x + 1)*(a**2*x**2 + 1)**
(3/4))/(3*a**3*x**6 - 3*a**2*i*x**5 + 3*a*x**4 - 3*i*x**3),x)*a)
```

3.120 $\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 202

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^5}\,dx = -\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}+\frac{3ia(1-iax)^{3/4}\sqrt[4]{1+iax}}{8x^3}+\frac{15a^2(1-iax)^{3/4}\sqrt[4]{1+iax}}{32x^2}-\frac{63ia^3(1-iax)^{3/4}\sqrt[4]{1+iax}}{64x}-\frac{123}{64}a^4\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{123}{64}a^4\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

output
```
-1/4*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^4+3/8*I*a*(1-I*a*x)^(3/4)*(1+I*a*x)
^(1/4)/x^3+15/32*a^2*(1-I*a*x)^(3/4)*(1+I*a*x)^(1/4)/x^2-63/64*I*a^3*(1-I*
a*x)^(3/4)*(1+I*a*x)^(1/4)/x-123/64*a^4*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(
1/4))-123/64*a^4*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.49

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^5}\,dx = \frac{(1-iax)^{3/4}\left(-16+8iax+6a^2x^2-33ia^3x^3+63a^4x^4-82a^4x^4\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{i+ax}{i-ax}\right)\right)}{64x^4(1+iax)^{3/4}}$$

input
```
Integrate[1/(E^(((3*I)/2)*ArcTan[a*x])*x^5),x]
```

output
```
((1 - I*a*x)^(3/4)*(-16 + (8*I)*a*x + 6*a^2*x^2 - (33*I)*a^3*x^3 + 63*a^4*
x^4 - 82*a^4*x^4*Hypergeometric2F1[3/4, 1, 7/4, (I + a*x)/(I - a*x)]))/(64
*x^4*(1 + I*a*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {5585, 110, 27, 168, 27, 168, 27, 168, 27, 104, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^5}\,dx$$

$$\Big\downarrow \text{ 5585}$$

$$\int \frac{(1-iax)^{3/4}}{x^5(1+iax)^{3/4}}dx$$

$$\Big\downarrow \text{ 110}$$

$$\frac{1}{4}\int -\frac{3a(2ax+3i)}{2x^4\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3}{8}a\int \frac{2ax+3i}{x^4\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}$$

$$\Big\downarrow \text{ 168}$$

$$-\frac{3}{8}a\left(-\frac{1}{3}\int -\frac{3a(5-4iax)}{2x^3\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{x^3}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3}{8}a\left(\frac{1}{2}a\int \frac{5-4iax}{x^3\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{x^3}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}$$

$$\Big\downarrow \text{ 168}$$

$$-\frac{3}{8}a\left(\frac{1}{2}a\left(-\frac{1}{2}\int \frac{a(10ax+21i)}{2x^2\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{5(1-iax)^{3/4}\sqrt[4]{1+iax}}{2x^2}\right)-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{x^3}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3}{8}a\left(\frac{1}{2}a\left(-\frac{1}{4}a\int \frac{10ax+21i}{x^2\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{5(1-iax)^{3/4}\sqrt[4]{1+iax}}{2x^2}\right)-\frac{i(1-iax)^{3/4}\sqrt[4]{1+iax}}{x^3}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}$$

$$\Big\downarrow \text{ 168}$$

$$-\frac{3}{8}a\left(\frac{1}{2}a\left(-\frac{1}{4}a\left(-\int -\frac{41a}{2x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{21i(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{5(1-iax)^{3/4}\sqrt[4]{1+iax}}{2x^2}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3}{8}a\left(\frac{1}{2}a\left(-\frac{1}{4}a\left(\frac{41}{2}a\int \frac{1}{x\sqrt[4]{1-iax}(iax+1)^{3/4}}dx-\frac{21i(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{5(1-iax)^{3/4}\sqrt[4]{1+iax}}{2x^2}\right)-\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}$$

$$\Big\downarrow \text{ 104}$$

$$-\frac{3}{8}a\left(\frac{1}{2}a\left(-\frac{1}{4}a\left(82a\int \frac{1}{\frac{iax+1}{1-iax}-1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{21i(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{5(1-iax)^{3/4}\sqrt[4]{1+iax}}{2x^2}\right)-\frac{i(1-\right.$$
$$\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}$$

$$\Big\downarrow \; 756$$

$$-\frac{3}{8}a\left(\frac{1}{2}a\left(-\frac{1}{4}a\left(82a\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{21i(1-iax)^{3/4}\sqrt[4]{1+ia}}{x}\right.\right.\right.$$
$$\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}$$

$$\Big\downarrow \; 216$$

$$-\frac{3}{8}a\left(\frac{1}{2}a\left(-\frac{1}{4}a\left(82a\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-\frac{21i(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)\right.\right.$$
$$\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}$$

$$\Big\downarrow \; 219$$

$$-\frac{3}{8}a\left(\frac{1}{2}a\left(-\frac{1}{4}a\left(82a\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-\frac{21i(1-iax)^{3/4}\sqrt[4]{1+iax}}{x}\right)-\frac{5(\right.\right.$$
$$\frac{(1-iax)^{3/4}\sqrt[4]{1+iax}}{4x^4}$$

input

```
Int[1/(E^(((3*I)/2)*ArcTan[a*x])*x^5),x]
```

output

```
-1/4*((1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x^4 - (3*a*(((-I)*(1 - I*a*x)^(
3/4)*(1 + I*a*x)^(1/4))/x^3 + (a*((-5*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))
/(2*x^2) - (a*(((-21*I)*(1 - I*a*x)^(3/4)*(1 + I*a*x)^(1/4))/x + 82*a*(-1/
2*ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)] - ArcTanh[(1 + I*a*x)^(1/4)/
(1 - I*a*x)^(1/4)]/2)))/4))/2))/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}} x^5}dx$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x)
```

output
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.95

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^5}\,dx = -\frac{123\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+123i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+i\right)-123i\,a^4x^4\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-i\right)-}{128\,x^4}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="fricas")
```

output
```
-1/128*(123*a^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 123*I*a
^4*x^4*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) - 123*I*a^4*x^4*log(sq
rt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 123*a^4*x^4*log(sqrt(I*sqrt(a^2*x
^2 + 1)/(a*x + I)) - 1) + 2*(63*a^4*x^4 + 93*I*a^3*x^3 - 54*a^2*x^2 - 40*I
*a*x + 16)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/x^4
```

Sympy [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{1}{x^5\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)/x**5,x)
```

output
```
Integral(1/(x**5*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{1}{x^5\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(1/(x^5*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{1}{x^5\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}\,dx$$

input
```
int(1/(x^5*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)),x)
```

output
```
int(1/(x^5*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2)), x)
```

Reduce [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{\sqrt{aix+1}\,(a^2x^2+1)^{\frac{3}{4}}}{a^3i\,x^8+a^2x^7+ai\,x^6+x^5}dx - \left(\int \frac{\sqrt{aix+1}\,(a^2x^2+1)^{\frac{3}{4}}}{a^3i\,x^7+a^2x^6+ai\,x^5+x^4}dx\right)ai$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)/x^5,x)
```

output

```
int((sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4))/(a**3*i*x**8 + a**2*x**7 + a*
i*x**6 + x**5),x) - int((sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4))/(a**3*i*x
**7 + a**2*x**6 + a*i*x**5 + x**4),x)*a*i
```

3.121 $\int e^{-\frac{5}{2}i\arctan(ax)}x^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 325

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^3\,dx = \frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}+\frac{475\sqrt[4]{1-iax}(1+iax)^{3/4}}{64a^4}+\frac{23(1-iax)^{5/4}(1+iax)^{3/4}}{32a^4}-\frac{17x^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a^2}+\frac{113(1-iax)^{9/4}(1+iax)^{3/4}}{24a^4}+\frac{475\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}-\frac{475\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{64\sqrt{2}a^4}-\frac{475\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)}{64\sqrt{2}a^4}$$

output
```
4*I*x^3*(1-I*a*x)^(5/4)/a/(1+I*a*x)^(1/4)+475/64*(1-I*a*x)^(1/4)*(1+I*a*x)
^(3/4)/a^4+23/32*(1-I*a*x)^(5/4)*(1+I*a*x)^(3/4)/a^4-17/4*x^2*(1-I*a*x)^(5
/4)*(1+I*a*x)^(3/4)/a^2+113/24*(1-I*a*x)^(9/4)*(1+I*a*x)^(3/4)/a^4+475/128
*arctan(1-2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))*2^(1/2)/a^4-475/128*arc
tan(1+2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))*2^(1/2)/a^4-475/128*arctanh
(2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)/(1+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2
)))*2^(1/2)/a^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.31

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^3\,dx = -\frac{\sqrt[4]{1-iax}(i+ax)^2\left(3(59+5iax+6a^2x^2)-95\,2^{3/4}\sqrt[4]{1+iax}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{9}{4},\frac{13}{4},\frac{1}{2}(1-iax\right.\right.}{72a^4\sqrt[4]{1+iax}}$$

input
```
Integrate[x^3/E^(((5*I)/2)*ArcTan[a*x]),x]
```

output
```
-1/72*((1 - I*a*x)^(1/4)*(I + a*x)^2*(3*(59 + (5*I)*a*x + 6*a^2*x^2) - 95*
2^(3/4)*(1 + I*a*x)^(1/4)*Hypergeometric2F1[1/4, 9/4, 13/4, (1 - I*a*x)/2]
))/(a^4*(1 + I*a*x)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.14, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.062$, Rules used = {5585, 108, 27, 170, 27, 164, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{-\frac{5}{2} i \arctan(ax)} \, dx$$

$$\Big\downarrow \text{ 5585}$$

$$\int \frac{x^3(1-iax)^{5/4}}{(1+iax)^{5/4}}dx$$

$$\Big\downarrow \text{ 108}$$

$$\frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}-\frac{4i\int \frac{x^2\sqrt[4]{1-iax}(12-17iax)}{4\sqrt[4]{iax+1}}dx}{a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}-\frac{i\int \frac{x^2\sqrt[4]{1-iax}(12-17iax)}{\sqrt[4]{iax+1}}dx}{a}$$

$$\Big\downarrow \text{ 170}$$

$$\frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}-\frac{i\left(\frac{\int \frac{ax\sqrt[4]{1-iax}(113ax+68i)}{2\sqrt[4]{iax+1}}dx}{4a^2}-\frac{17ix^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a}\right)}{a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}-\frac{i\left(\frac{\int \frac{x\sqrt[4]{1-iax}(113ax+68i)}{\sqrt[4]{iax+1}}dx}{8a}-\frac{17ix^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a}\right)}{a}$$

$$\Big\downarrow \text{ 164}$$

$$\frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}-\frac{i\left(\frac{\frac{(1-iax)^{5/4}(1+iax)^{3/4}(452ax+521i)}{12a^2}-\frac{475\int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}dx}{8a}}{8a}-\frac{17ix^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a}\right)}{a}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{\frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}-i\left(\frac{\frac{(1-iax)^{5/4}(1+iax)^{3/4}(452ax+521i)}{12a^2}-\frac{475\left(\frac{1}{2}\int\frac{1}{(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)}{8a}}{8a}-\frac{17ix^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a}\right)}{a}$$

$\Big\downarrow$ 73

$$\frac{\frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}-i\left(\frac{\frac{(1-iax)^{5/4}(1+iax)^{3/4}(452ax+521i)}{12a^2}-\frac{475\left(\frac{2i\int\frac{1}{\sqrt[4]{iax+1}}d\sqrt[4]{1-iax}}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)}{8a}}{8a}-\frac{17ix^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a}\right)}{a}$$

$\Big\downarrow$ 770

$$\frac{\frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}-i\left(\frac{\frac{(1-iax)^{5/4}(1+iax)^{3/4}(452ax+521i)}{12a^2}-\frac{475\left(\frac{2i\int\frac{1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)}{8a}}{8a}-\frac{17ix^2(1-iax)^{5/4}(1+iax)^{3/4}}{4a}\right)}{a}$$

$\Big\downarrow$ 755

$$i\left(\frac{\frac{(1-iax)^{5/4}(1+iax)^{3/4}(452ax+521i)}{12a^2}-\frac{475\left(\frac{2i\left(\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int\frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)}{8a}}{8a}-\frac{17a\cdots}{}\right.$$

$$\frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}-\cdots$$

$$\overline{\qquad a \qquad}$$

$\downarrow$ 1476

$$i\left(\frac{\frac{(1-iax)^{5/4}(1+iax)^{3/4}(452ax+521i)}{12a^2}-\frac{475\left(\frac{2i\left(\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int\frac{\cdots}{\sqrt{1-iax}+\sqrt{\cdots}}\cdots\right.\right.}{a}\cdots\right.}{8a}}{8a}\cdots\right.$$

$$\frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}-\cdots$$

$$\overline{\qquad a \qquad}$$

$\downarrow$ 1082

$$\frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}-$$

$$i\left(\frac{\frac{(1-iax)^{5/4}(1+iax)^{3/4}(452ax+521i)}{12a^2}-\frac{475\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int\frac{1-\sqrt{1-ia}}{2-iax}\right.}{a}\right.}{8a}}{8a}\right.$$

$$\overline{\qquad\qquad a\qquad\qquad}$$

$\downarrow$ 217

$$\frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}-$$

$$i\left(\frac{\frac{(1-iax)^{5/4}(1+iax)^{3/4}(452ax+521i)}{12a^2}-\frac{475\left(\frac{2i\left(\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)\right)}{a}-\right.}{8a}}{8a}\right.$$

$$\overline{\qquad\qquad a\qquad\qquad}$$

$\downarrow$ 1479

$$\frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}-$$

$$\frac{i\left(\frac{\frac{(1-iax)^{5/4}(1+iax)^{3/4}(452ax+521i)}{12a^2}-\frac{475\left(\frac{2i\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+\cdots}}}{2\sqrt{2}}\cdots\right.\right.}{a}\cdots\right.}{8a}\cdots}{8a}\cdots\right.}{a}$$

$\downarrow$ 25

$$\frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}-$$

$$\frac{i\left(\frac{\frac{(1-iax)^{5/4}(1+iax)^{3/4}(452ax+521i)}{12a^2}-\frac{475\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)\right)}{a}\right)}{8a}}{8a}\right)}{a}$$

$$\Big\downarrow \; 27$$

$$\frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} -$$

$$i\left(\frac{\frac{(1-iax)^{5/4}(1+iax)^{3/4}(452ax+521i)}{12a^2} - \frac{475\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right.\right.\right.}{a}}{8a}}{8a}\right.$$

$$\overline{\qquad a \qquad}$$

↓ 1103

$$\frac{4ix^3(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} - \frac{i\left(\frac{\frac{(1-iax)^{5/4}(1+iax)^{3/4}(452ax+521i)}{12a^2} - \frac{475\left(\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right.}{2\sqrt{2}}\right.\right.}{a}}{8a}}{8a}}{a}$$

input
```
Int[x^3/E^(((5*I)/2)*ArcTan[a*x]),x]
```

output
```
((4*I)*x^3*(1 - I*a*x)^(5/4))/(a*(1 + I*a*x)^(1/4)) - (I*((((-17*I)/4)*x^2
*(1 - I*a*x)^(5/4)*(1 + I*a*x)^(3/4))/a + (((1 - I*a*x)^(5/4)*(1 + I*a*x)^
(3/4)*(521*I + 452*a*x))/(12*a^2) - (475*(((-I)*(1 - I*a*x)^(1/4)*(1 + I*a
*x)^(3/4))/a + ((2*I)*((-(ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*
x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1
/4)]/Sqrt[2])/2 + (-1/2*Log[1 + Sqrt[1 - I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/
4))/(1 + I*a*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - I*a*x] + (Sqrt[2]*(1 - I
*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]))/2))/a))/(8*a))/(8*a)))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 108

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{x^3}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}dx$$

input
```
int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

output
```
int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.94

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^3\,dx = \frac{96\left(a^5x-i\,a^4\right)\sqrt{\frac{225625i}{4096\,a^8}}\log\left(\frac{64}{475}i\,a^4\sqrt{\frac{225625i}{4096\,a^8}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-96\left(a^5x-i\,a^4\right)\sqrt{\frac{225625i}{4096\,a^8}}\log\left(-\frac{64}{475}i\,a^4\sqrt{\frac{22}{40}}\right.}{}$$

input
```
integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

output

```
1/192*(96*(a^5*x - I*a^4)*sqrt(225625/4096*I/a^8)*log(64/475*I*a^4*sqrt(22
5625/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 96*(a^5*x - I*a^
4)*sqrt(225625/4096*I/a^8)*log(-64/475*I*a^4*sqrt(225625/4096*I/a^8) + sqr
t(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 96*(a^5*x - I*a^4)*sqrt(-225625/4096*I
/a^8)*log(64/475*I*a^4*sqrt(-225625/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)
/(a*x + I))) + 96*(a^5*x - I*a^4)*sqrt(-225625/4096*I/a^8)*log(-64/475*I*a
^4*sqrt(-225625/4096*I/a^8) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + (48*I
*a^4*x^4 - 136*a^3*x^3 - 226*I*a^2*x^2 + 521*a*x - 2467*I)*sqrt(a^2*x^2 +
1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/(a^5*x - I*a^4)
```

Sympy [F]

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^3\,dx = \int \frac{x^3}{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

input

```
integrate(x**3/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2),x)
```

output

```
Integral(x**3/(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(5/2), x)
```

Maxima [F]

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^3\,dx = \int \frac{x^3}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

input

```
integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
```

output

```
integrate(x^3/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^3\,dx = \int \frac{x^3}{\left(\frac{1+a\,x\,1i}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}\,dx$$

input
```
int(x^3/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
```

output
```
int(x^3/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

Reduce [F]

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^3\,dx = -\left(\int \frac{(a^2x^2+1)^{\frac{1}{4}}\,x^5}{\sqrt{aix+1}\,a^2x^2-2\sqrt{aix+1}\,aix-\sqrt{aix+1}}dx\right)a^2 - \left(\int \frac{(a^2x^2+1)^{\frac{1}{4}}\,x^3}{\sqrt{aix+1}\,a^2x^2-2\sqrt{aix+1}\,aix-\sqrt{aix+1}}dx\right)$$

input
```
int(x^3/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

output

```
 - (int(((a**2*x**2 + 1)**(1/4)*x**5)/(sqrt(a*i*x + 1)*a**2*x**2 - 2*sqrt(
a*i*x + 1)*a*i*x - sqrt(a*i*x + 1)),x)*a**2 + int(((a**2*x**2 + 1)**(1/4)*
x**3)/(sqrt(a*i*x + 1)*a**2*x**2 - 2*sqrt(a*i*x + 1)*a*i*x - sqrt(a*i*x +
1)),x))
```

3.122 $\int e^{-\frac{5}{2}i\arctan(ax)}x^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 300

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^2\,dx = -\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}}-\frac{55i\sqrt[4]{1-iax}(1+iax)^{3/4}}{8a^3}-\frac{11i(1-iax)^{5/4}(1+iax)^{3/4}}{4a^3}-\frac{i(1-iax)^{9/4}(1+iax)^{3/4}}{3a^3}-\frac{55i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}+\frac{55i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{8\sqrt{2}a^3}+\frac{55i\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)}{8\sqrt{2}a^3}$$

output
```
-2*I*(1-I*a*x)^(9/4)/a^3/(1+I*a*x)^(1/4)-55/8*I*(1-I*a*x)^(1/4)*(1+I*a*x)^
(3/4)/a^3-11/4*I*(1-I*a*x)^(5/4)*(1+I*a*x)^(3/4)/a^3-1/3*I*(1-I*a*x)^(9/4)
*(1+I*a*x)^(3/4)/a^3-55/16*I*arctan(1-2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1
/4))*2^(1/2)/a^3+55/16*I*arctan(1+2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))
*2^(1/2)/a^3+55/16*I*arctanh(2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)/(1+(1
-I*a*x)^(1/2)/(1+I*a*x)^(1/2)))*2^(1/2)/a^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.30

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^2\,dx = -\frac{\sqrt[4]{1-iax}(i+ax)^2\left(-21i+3ax+11i2^{3/4}\sqrt[4]{1+iax}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{9}{4},\frac{13}{4},\frac{1}{2}(1-iax)\right)\right)}{9a^3\sqrt[4]{1+iax}}$$

input
```
Integrate[x^2/E^(((5*I)/2)*ArcTan[a*x]),x]
```

output
```
-1/9*((1 - I*a*x)^(1/4)*(I + a*x)^2*(-21*I + 3*a*x + (11*I)*2^(3/4)*(1 + I
*a*x)^(1/4)*Hypergeometric2F1[1/4, 9/4, 13/4, (1 - I*a*x)/2]))/(a^3*(1 + I
*a*x)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.81 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.18, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5585, 100, 27, 90, 60, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2e^{-\frac{5}{2}i\arctan(ax)}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{x^2(1-iax)^{5/4}}{(1+iax)^{5/4}}dx$$

$$\Big\downarrow \text{100}$$

$$\frac{2i\int -\frac{a(1-iax)^{5/4}(ax+5i)}{2\sqrt[4]{iax+1}}dx}{a^3}-\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{i\int \frac{(1-iax)^{5/4}(ax+5i)}{\sqrt[4]{iax+1}}dx}{a^2}-\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}}$$

$$\Big\downarrow \text{ 90}$$

$$-\frac{i\left(\frac{11}{2}i\int \frac{(1-iax)^{5/4}}{\sqrt[4]{iax+1}}dx+\frac{(1+iax)^{3/4}(1-iax)^{9/4}}{3a}\right)}{a^2}-\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}}$$

$$\Big\downarrow \text{ 60}$$

$$-\frac{i\left(\frac{11}{2}i\left(\frac{5}{4}\int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}dx-\frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{2a}\right)+\frac{(1+iax)^{3/4}(1-iax)^{9/4}}{3a}\right)}{a^2}-\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}}$$

$$\Big\downarrow \text{ 60}$$

$$-\frac{i\left(\frac{11}{2}i\left(\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)-\frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{2a}\right)+\frac{(1+iax)^{3/4}(1-iax)^{9/4}}{3a}\right)}{a^2}-\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{i\left(\frac{11}{2}i\left(\frac{5}{4}\left(\frac{2i\int \frac{1}{\sqrt[4]{iax+1}}d\sqrt[4]{1-iax}}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)-\frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{2a}\right)+\frac{(1+iax)^{3/4}(1-iax)^{9/4}}{3a}\right)}{a^2}-\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}}$$

$$\Big\downarrow \text{ 770}$$

$$-\frac{i\left(\frac{11}{2}i\left(\frac{5}{4}\left(\frac{2i\int \frac{1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)-\frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{2a}\right)+\frac{(1+iax)^{3/4}(1-iax)^{9/4}}{3a}\right)}{a^2}-\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}}$$

$$\Big\downarrow \text{ 755}$$

$$-\frac{i\left(\frac{11}{2}i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int\frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)-\frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{2a}\right.\right.}{a^2}$$

$$\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}}$$

$$\Big\downarrow \; 1476$$

$$-\frac{i\left(\frac{11}{2}i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int\frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-\right.\right.}{\sqrt[4]{ia}}\right.\right.}{a}\right.\right.\right.}{a^2}$$

$$\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}}$$

$$\Big\downarrow \; 1082$$

$$-\frac{i\left(\frac{11}{2}i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-\frac{i\sqrt[4]{\ }}{}\right.\right.\right.}{a^2}$$

$$\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}}$$

$$\Big\downarrow \; 217$$

$$-\frac{i\left(\frac{11}{2}i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)\right)}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)}{a}\right.\right.\right.}{a^2}$$

$$\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}}$$

$$\Big\downarrow \ 1479$$

$$-\frac{i\left(\frac{11}{2}i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{\ldots}}{\sqrt[4]{\ldots}}\right.}{\sqrt{2}}\right.\right.}{a}\right.\right.\right.}{a^2}$$

$$\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}}$$

$$\Big\downarrow \ 25$$

$$-\frac{i\left(\frac{11}{2}i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\cdots}}{\sqrt[4]{1+\cdots}}\right.}{\sqrt{2}}\right.\cdots\right)}{a}\cdots\right.\cdots\right.\cdots\right)}{a^2\cdots}$$

$$\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}}$$

$\downarrow$ 27

$$-\frac{i\left(\frac{11}{2}i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1\cdots}}{\sqrt[4]{1+\cdots}}\right.}{\sqrt{2}}\right.\cdots\right)}{a}\cdots\right.\cdots\right.\cdots\right)}{a^2\cdots}$$

$$\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}}$$

$\downarrow$ 1103

$$-\frac{2i(1-iax)^{9/4}}{a^3\sqrt[4]{1+iax}}-\frac{i\left(\frac{11}{2}i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{1}}\right.}{2\sqrt{2}}\right.\right.}{a}\right.\right.\right.}{a^2}$$

input
```
Int[x^2/E^(((5*I)/2)*ArcTan[a*x]),x]
```

output
```
((-2*I)*(1 - I*a*x)^(9/4))/(a^3*(1 + I*a*x)^(1/4)) - (I*(((1 - I*a*x)^(9/4
)*(1 + I*a*x)^(3/4))/(3*a) + ((11*I)/2)*(((-1/2*I)*(1 - I*a*x)^(5/4)*(1 +
I*a*x)^(3/4))/a + (5*(((-I)*(1 - I*a*x)^(1/4)*(1 + I*a*x)^(3/4))/a + ((2*I
)*((-(ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]) +
 ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2])/2 + (-
1/2*Log[1 + Sqrt[1 - I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4
)]/Sqrt[2] + Log[1 + Sqrt[1 - I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*
a*x)^(1/4)]/(2*Sqrt[2]))/2))/a))/4)))/a^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{x^2}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}dx$$

input
```
int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

output
```
int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.99

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^2\,dx = \frac{12\,(a^4x-i\,a^3)\sqrt{\frac{3025i}{64\,a^6}}\log\left(\frac{8}{55}\,a^3\sqrt{\frac{3025i}{64\,a^6}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-12\,(a^4x-i\,a^3)\sqrt{\frac{3025i}{64\,a^6}}\log\left(-\frac{8}{55}\,a^3\sqrt{\frac{3025i}{64\,a^6}}+\sqrt{\cdots}\right.}{\cdots}$$

input
```
integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

output
```
1/24*(12*(a^4*x - I*a^3)*sqrt(3025/64*I/a^6)*log(8/55*a^3*sqrt(3025/64*I/a
^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 12*(a^4*x - I*a^3)*sqrt(3025/
64*I/a^6)*log(-8/55*a^3*sqrt(3025/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*
x + I))) - 12*(a^4*x - I*a^3)*sqrt(-3025/64*I/a^6)*log(8/55*a^3*sqrt(-3025
/64*I/a^6) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 12*(a^4*x - I*a^3)*sqr
t(-3025/64*I/a^6)*log(-8/55*a^3*sqrt(-3025/64*I/a^6) + sqrt(I*sqrt(a^2*x^2
 + 1)/(a*x + I))) + (8*I*a^3*x^3 - 26*a^2*x^2 - 61*I*a*x - 287)*sqrt(a^2*x
^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/(a^4*x - I*a^3)
```

Sympy [F]

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^2\,dx = \int \frac{x^2}{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2),x)
```

output
```
Integral(x**2/(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(5/2), x)
```

Maxima [F]

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^2\,dx = \int \frac{x^2}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^2\,dx = \int \frac{x^2}{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}\,dx$$

input
```
int(x^2/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
```

output
```
int(x^2/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

Reduce [F]

$$\int e^{-\frac{5}{2}i\arctan(ax)}x^2\,dx = -\left(\int \frac{(a^2x^2+1)^{\frac{1}{4}}\,x^4}{\sqrt{aix+1}\,a^2x^2-2\sqrt{aix+1}\,aix-\sqrt{aix+1}}dx\right)a^2 - \left(\int \frac{(a^2x^2+1)^{\frac{1}{4}}\,x^2}{\sqrt{aix+1}\,a^2x^2-2\sqrt{aix+1}\,aix-\sqrt{aix+1}}dx\right)$$

input
```
int(x^2/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

output
```
- (int(((a**2*x**2 + 1)**(1/4)*x**4)/(sqrt(a*i*x + 1)*a**2*x**2 - 2*sqrt(
a*i*x + 1)*a*i*x - sqrt(a*i*x + 1)),x)*a**2 + int(((a**2*x**2 + 1)**(1/4)*
x**2)/(sqrt(a*i*x + 1)*a**2*x**2 - 2*sqrt(a*i*x + 1)*a*i*x - sqrt(a*i*x +
1)),x))
```

3.123 $\int e^{-\frac{5}{2}i\arctan(ax)}x\,dx$

Optimal result

Integrand size = 14, antiderivative size = 255

$$\begin{aligned}\int e^{-\frac{5}{2}i\arctan(ax)}x\,dx &= -\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}}-\frac{25\sqrt[4]{1-iax}(1+iax)^{3/4}}{4a^2}\\ &\quad-\frac{5(1-iax)^{5/4}(1+iax)^{3/4}}{2a^2}-\frac{25\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}\\ &\quad+\frac{25\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{4\sqrt{2}a^2}\\ &\quad+\frac{25\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)}{4\sqrt{2}a^2}\end{aligned}$$

output
```
-2*(1-I*a*x)^(9/4)/a^2/(1+I*a*x)^(1/4)-25/4*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4
)/a^2-5/2*(1-I*a*x)^(5/4)*(1+I*a*x)^(3/4)/a^2-25/8*arctan(1-2^(1/2)*(1-I*a
*x)^(1/4)/(1+I*a*x)^(1/4))*2^(1/2)/a^2+25/8*arctan(1+2^(1/2)*(1-I*a*x)^(1/
4)/(1+I*a*x)^(1/4))*2^(1/2)/a^2+25/8*arctanh(2^(1/2)*(1-I*a*x)^(1/4)/(1+I*
a*x)^(1/4)/(1+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2)))*2^(1/2)/a^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.25

$$\int e^{-\frac{5}{2}i\arctan(ax)}x\,dx = \frac{2(1-iax)^{9/4}\left(-\frac{9}{\sqrt[4]{1+iax}}+5\,2^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{9}{4},\frac{13}{4},\frac{1}{2}(1-iax)\right)\right)}{9a^2}$$

input
```
Integrate[x/E^(((5*I)/2)*ArcTan[a*x]),x]
```

output
```
(2*(1 - I*a*x)^(9/4)*(-9/(1 + I*a*x)^(1/4) + 5*2^(3/4)*Hypergeometric2F1[1
/4, 9/4, 13/4, (1 - I*a*x)/2]))/(9*a^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.23, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5585, 87, 60, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-\frac{5}{2}i\arctan(ax)}dx$$

$$\Big\downarrow \text{ 5585}$$

$$\int \frac{x(1-iax)^{5/4}}{(1+iax)^{5/4}}dx$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{5i\int \frac{(1-iax)^{5/4}}{\sqrt[4]{iax+1}}dx}{a}-\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}}$$

$$\Big\downarrow \text{ 60}$$

$$-\frac{5i\left(\frac{5}{4}\int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}dx-\frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{2a}\right)}{a}-\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}}$$

$\downarrow$ 60

$$-\frac{5i\left(\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)-\frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{2a}\right)}{a}-\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}}$$

$\downarrow$ 73

$$-\frac{5i\left(\frac{5}{4}\left(\frac{2i\int \frac{1}{\sqrt[4]{iax+1}}d\sqrt[4]{1-iax}}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)-\frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{2a}\right)}{a}-\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}}$$

$\downarrow$ 770

$$-\frac{5i\left(\frac{5}{4}\left(\frac{2i\int \frac{1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)-\frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{2a}\right)}{a}-\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}}$$

$\downarrow$ 755

$$-\frac{5i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)-\frac{i(1-iax)^{5/4}(1+iax)^{3/4}}{2a}\right)}{a}-\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}}$$

$\downarrow$ 1476

$$-\frac{5i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-ia\cdots}}{\sqrt[4]{iax+\cdots}}\cdots\right)\right)}{a}\cdots\right)\right)}{a}\cdots$$

$$\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}}$$

$\downarrow$ 1082

$$-\frac{5i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-\frac{i\sqrt[4]{1-\cdots}}{\cdots}\right)\right)}{a}\cdots$$

$$\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}}$$

$\downarrow$ 217

$$-\frac{5i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)\right)}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)\right)}{a}\cdots$$

$$\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}}$$

$\downarrow$ 1479

$$-\frac{5i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-\cdots}}{\sqrt[4]{1+\cdots}}\right.}{\sqrt{2}}\cdots\right.\right.}{a}\cdots\right.\right.}{a}\cdots$$

$$\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}}$$

$\downarrow$ 25

$$-\frac{5i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right.}{\sqrt{2}}\cdots\right.\right.}{a}\cdots\right.\right.}{a}\cdots$$

$$\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}}$$

$\downarrow$ 27

$$-\frac{5i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia}}{\sqrt[4]{1+ia}}\right.}{\sqrt{2}}\right.}{a}\right.}{a}$$

$$\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}}$$

$$\Big\downarrow 1103$$

$$-\frac{2(1-iax)^{9/4}}{a^2\sqrt[4]{1+iax}}-$$

$$\frac{5i\left(\frac{5}{4}\left(\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{1+i}}\right.}{2\sqrt{2}}\right.}{a}\right.}{a}$$

input
```
Int[x/E^(((5*I)/2)*ArcTan[a*x]),x]
```

output
```
(-2*(1 - I*a*x)^(9/4))/(a^2*(1 + I*a*x)^(1/4)) - ((5*I)*(((-1/2*I)*(1 - I*
a*x)^(5/4)*(1 + I*a*x)^(3/4))/a + (5*(((-I)*(1 - I*a*x)^(1/4)*(1 + I*a*x)^
(3/4))/a + ((2*I)*((-(ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(
1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]
/Sqrt[2])/2 + (-1/2*Log[1 + Sqrt[1 - I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/
(1 + I*a*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - I*a*x] + (Sqrt[2]*(1 - I*a*x
)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]))/2))/a))/4))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{x}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

input
```
int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

output
```
int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.13

$$\int e^{-\frac{5}{2} i \arctan(ax)} x\,dx = -\frac{2\,(a^3x - i\,a^2)\sqrt{\frac{625i}{16\,a^4}}\log\left(\frac{4}{25} i\,a^2\sqrt{\frac{625i}{16\,a^4}} + \sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right) - 2\,(a^3x - i\,a^2)\sqrt{\frac{625i}{16\,a^4}}\log\left(-\frac{4}{25} i\,a^2\sqrt{\frac{625i}{16\,a^4}} + \sqrt{}\right.}{}$$

input
```
integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

output
```
-1/4*(2*(a^3*x - I*a^2)*sqrt(625/16*I/a^4)*log(4/25*I*a^2*sqrt(625/16*I/a^
4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 2*(a^3*x - I*a^2)*sqrt(625/16*
I/a^4)*log(-4/25*I*a^2*sqrt(625/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x
+ I))) - 2*(a^3*x - I*a^2)*sqrt(-625/16*I/a^4)*log(4/25*I*a^2*sqrt(-625/16
*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + 2*(a^3*x - I*a^2)*sqrt(-6
25/16*I/a^4)*log(-4/25*I*a^2*sqrt(-625/16*I/a^4) + sqrt(I*sqrt(a^2*x^2 + 1
)/(a*x + I))) - sqrt(a^2*x^2 + 1)*(2*I*a^2*x^2 - 9*a*x + 43*I)*sqrt(I*sqrt
(a^2*x^2 + 1)/(a*x + I)))/(a^3*x - I*a^2)
```

Sympy [F]

$$\int e^{-\frac{5}{2}i\arctan(ax)}x\,dx = \int \frac{x}{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2),x)
```

output
```
Integral(x/(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(5/2), x)
```

Maxima [F]

$$\int e^{-\frac{5}{2}i\arctan(ax)}x\,dx = \int \frac{x}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
```

output
```
integrate(x/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{5}{2}i\arctan(ax)}x\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{5}{2}i\arctan(ax)}x\,dx=\int\frac{x}{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}\,dx$$

input
```
int(x/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
```

output
```
int(x/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

Reduce [F]

$$\int e^{-\frac{5}{2}i\arctan(ax)}x\,dx=-\left(\int\frac{(a^2x^2+1)^{\frac{1}{4}}x^3}{\sqrt{aix+1}\,a^2x^2-2\sqrt{aix+1}\,aix-\sqrt{aix+1}}dx\right)a^2-\left(\int\frac{(a^2x^2+1)^{\frac{1}{4}}x}{\sqrt{aix+1}\,a^2x^2-2\sqrt{aix+1}\,aix-\sqrt{aix+1}}dx\right)$$

input
```
int(x/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

output
```
- (int(((a**2*x**2 + 1)**(1/4)*x**3)/(sqrt(a*i*x + 1)*a**2*x**2 - 2*sqrt(
a*i*x + 1)*a*i*x - sqrt(a*i*x + 1)),x)*a**2 + int(((a**2*x**2 + 1)**(1/4)*
x)/(sqrt(a*i*x + 1)*a**2*x**2 - 2*sqrt(a*i*x + 1)*a*i*x - sqrt(a*i*x + 1))
,x))
```

3.124 $\int e^{-\frac{5}{2}i\arctan(ax)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 226

$$\int e^{-\frac{5}{2}i\arctan(ax)}\,dx=\frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}+\frac{5i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}+\frac{5i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}-\frac{5i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}a}-\frac{5i\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)}{\sqrt{2}a}$$

output
```
4*I*(1-I*a*x)^(5/4)/a/(1+I*a*x)^(1/4)+5*I*(1-I*a*x)^(1/4)*(1+I*a*x)^(3/4)/
a+5/2*I*arctan(1-2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))*2^(1/2)/a-5/2*I*
arctan(1+2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))*2^(1/2)/a-5/2*I*arctanh(
2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)/(1+(1-I*a*x)^(1/2)/(1+I*a*x)^(1/2)
))*2^(1/2)/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.17

$$\int e^{-\frac{5}{2}i\arctan(ax)}\,dx=\frac{8ie^{-\frac{1}{2}i\arctan(ax)}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},2,\frac{3}{4},-e^{2i\arctan(ax)}\right)}{a}$$

input
```
Integrate[E^(((-5*I)/2)*ArcTan[a*x]),x]
```

output
```
((8*I)*Hypergeometric2F1[-1/4, 2, 3/4, -E^((2*I)*ArcTan[a*x])])/(a*E^((I/2
)*ArcTan[a*x]))
```

Rubi [A] (warning: unable to verify)

Time = 0.67 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.21, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {5584, 57, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\frac{5}{2}i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5584}$$

$$\int \frac{(1-iax)^{5/4}}{(1+iax)^{5/4}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}-5\int \frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}-5\left(\frac{1}{2}\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{iax+1}}dx-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)$$

$\downarrow$ 73

$$\frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}-5\left(\frac{2i\int \frac{1}{\sqrt[4]{iax+1}}d\sqrt[4]{1-iax}}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)$$

$\downarrow$ 770

$$\frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}-5\left(\frac{2i\int \frac{1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)$$

$\downarrow$ 755

$$\frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}-5\left(\frac{2i\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-\frac{i\sqrt[4]{1-iax}(1+iax)^{3/4}}{a}\right)$$

$\downarrow$ 1476

$$\frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}-5\left(\frac{2i\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1}}{\sqrt[4]{ia}}\right.\right.}{a}\right.$$

$\downarrow$ 1082

$$\frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}-5\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{a}-\frac{i\sqrt[4]{1}}{}\right.$$

$\downarrow$ 217

$$5\left(\frac{\frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}-2i\left(\frac{1}{2}\int\frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)\right)}{a}-\frac{i\sqrt[4]{1-iax}(1+ia}{a}\right.$$

$$\downarrow \text{1479}$$

$$5\left(\frac{\frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}-2i\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{1-}}\right.}{\sqrt{2}}\right.\right.}{a}\right.$$

$$\downarrow \text{25}$$

$$5\left(\frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}-\frac{2i\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia}}{\sqrt[4]{1+iax}}\right.}{\sqrt{2}}\right.\cdots}{a}\right.$$

$\downarrow$ 27

$$5\left(\frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}}-\frac{2i\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{1+}}\right.}{\sqrt{2}}\right.\cdots}{a}\right.$$

$\downarrow$ 1103

$$\frac{4i(1-iax)^{5/4}}{a\sqrt[4]{1+iax}} - 5\left(\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right) + \frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}} - \frac{\log\left(\sqrt{1-iax}-\cdots\right)}{\cdots}\right)\right)}{a}\right.$$

input
```
Int[E^(((-5*I)/2)*ArcTan[a*x]),x]
```

output
```
((4*I)*(1 - I*a*x)^(5/4))/(a*(1 + I*a*x)^(1/4)) - 5*(((-I)*(1 - I*a*x)^(1/
4)*(1 + I*a*x)^(3/4))/a + ((2*I)*((-(ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4)
)/(1 + I*a*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1
+ I*a*x)^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 + Sqrt[1 - I*a*x] - (Sqrt[2]*(1 -
 I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - I*a*x] + (Sqr
t[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/(2*Sqrt[2]))/2))/a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5584
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}dx$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

output
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.15

$$\int e^{-\frac{5}{2}i\arctan(ax)}\,dx =$$

$$-\frac{(a^2x-i\,a)\sqrt{\frac{25i}{a^2}}\log\left(\frac{1}{5}\,a\sqrt{\frac{25i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-(a^2x-i\,a)\sqrt{\frac{25i}{a^2}}\log\left(-\frac{1}{5}\,a\sqrt{\frac{25i}{a^2}}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-(a^2}{}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="fricas")
```

output
```
-1/2*((a^2*x - I*a)*sqrt(25*I/a^2)*log(1/5*a*sqrt(25*I/a^2) + sqrt(I*sqrt(
a^2*x^2 + 1)/(a*x + I))) - (a^2*x - I*a)*sqrt(25*I/a^2)*log(-1/5*a*sqrt(25
*I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - (a^2*x - I*a)*sqrt(-25*I/
a^2)*log(1/5*a*sqrt(-25*I/a^2) + sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) + (a
^2*x - I*a)*sqrt(-25*I/a^2)*log(-1/5*a*sqrt(-25*I/a^2) + sqrt(I*sqrt(a^2*x
^2 + 1)/(a*x + I))) + 2*sqrt(a^2*x^2 + 1)*(-I*a*x - 9)*sqrt(I*sqrt(a^2*x^2
 + 1)/(a*x + I)))/(a^2*x - I*a)
```

Sympy [F]

$$\int e^{-\frac{5}{2}i\arctan(ax)}\,dx = \int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2),x)
```

output
```
Integral(((I*a*x + 1)/sqrt(a**2*x**2 + 1))**(-5/2), x)
```

Maxima [F]

$$\int e^{-\frac{5}{2}i\arctan(ax)}\,dx=\int \frac{1}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="maxima")
```

output
```
integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(-5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{5}{2}i\arctan(ax)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{5}{2}i\arctan(ax)}\,dx=\int \frac{1}{{\left(\frac{1+a\,x\,1{}\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)}^{5/2}}\,dx$$

input
```
int(1/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
```

output
```
int(1/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

Reduce [F]

$$\int e^{-\frac{5}{2}i\arctan(ax)}\,dx = -\left(\int \frac{(a^2x^2+1)^{\frac{1}{4}}}{\sqrt{aix+1}\,a^2x^2-2\sqrt{aix+1}\,aix-\sqrt{aix+1}}dx\right) - \left(\int \frac{(a^2x^2+1)^{\frac{1}{4}}\,x^2}{\sqrt{aix+1}\,a^2x^2-2\sqrt{aix+1}\,aix-\sqrt{aix+1}}dx\right)a^2$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2),x)
```

output
```
- (int((a**2*x**2 + 1)**(1/4)/(sqrt(a*i*x + 1)*a**2*x**2 - 2*sqrt(a*i*x +
 1)*a*i*x - sqrt(a*i*x + 1)),x) + int(((a**2*x**2 + 1)**(1/4)*x**2)/(sqrt(
a*i*x + 1)*a**2*x**2 - 2*sqrt(a*i*x + 1)*a*i*x - sqrt(a*i*x + 1)),x)*a**2)
```

3.125 $\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 230

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x}\,dx = \frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} + 2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right) - 2\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}\left(1+\frac{\sqrt{1-iax}}{\sqrt{1+iax}}\right)}\right)$$

output
```
8*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)+2*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)
)+2^(1/2)*arctan(1-2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))-2^(1/2)*arctan
(1+2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4))-2*arctanh((1+I*a*x)^(1/4)/(1-I
*a*x)^(1/4))-2^(1/2)*arctanh(2^(1/2)*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)/(1+(1
-I*a*x)^(1/2)/(1+I*a*x)^(1/2)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.46

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x}\,dx = \frac{\sqrt[4]{1-iax}\left(20-20\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{i+ax}{i-ax}\right)+2^{3/4}(1-iax)\sqrt[4]{1+iax}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\right.\right.}{5\sqrt[4]{1+iax}}$$

input
```
Integrate[1/(E^(((5*I)/2)*ArcTan[a*x])*x),x]
```

output
```
((1 - I*a*x)^(1/4)*(20 - 20*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I -
a*x)] + 2^(3/4)*(1 - I*a*x)*(1 + I*a*x)^(1/4)*Hypergeometric2F1[5/4, 5/4,
9/4, (1 - I*a*x)/2]))/(5*(1 + I*a*x)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.80 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.27, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {5585, 109, 27, 35, 140, 73, 104, 25, 770, 755, 827, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{(1-iax)^{5/4}}{x(1+iax)^{5/4}}dx$$

$$\Big\downarrow \text{109}$$

$$\frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{4i\int \frac{a(i-ax)}{4x(1-iax)^{3/4}\sqrt[4]{iax+1}}dx}{a}$$

$$\Big\downarrow \ 27$$

$$\frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-i\int \frac{i-ax}{x(1-iax)^{3/4}\sqrt[4]{iax+1}}dx$$

$$\Big\downarrow \ 35$$

$$\int \frac{(iax+1)^{3/4}}{x(1-iax)^{3/4}}dx+\frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}$$

$$\Big\downarrow \ 140$$

$$ia\int \frac{1}{(1-iax)^{3/4}\sqrt[4]{iax+1}}dx+\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{iax+1}}dx+\frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}$$

$$\Big\downarrow \ 73$$

$$-4\int \frac{1}{\sqrt[4]{iax+1}}d\sqrt[4]{1-iax}+\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{iax+1}}dx+\frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}$$

$$\Big\downarrow \ 104$$

$$-4\int \frac{1}{\sqrt[4]{iax+1}}d\sqrt[4]{1-iax}+4\int -\frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+\frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}$$

$$\Big\downarrow \ 25$$

$$-4\int \frac{1}{\sqrt[4]{iax+1}}d\sqrt[4]{1-iax}-4\int \frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+\frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}$$

$$\Big\downarrow \ 770$$

$$-4\int \frac{1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}-4\int \frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+\frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}$$

$$\Big\downarrow \ 755$$

$$-4\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)-4\int \frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+\frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}$$

$$\Big\downarrow \ 827$$

$$-4\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+$$
$$4\left(\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)+\frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}$$

$\downarrow$ 216

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-$$
$$4\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}$$

$\downarrow$ 219

$$-4\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{\sqrt{1-iax}+1}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+$$
$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}$$

$\downarrow$ 1476

$$-4\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\int \frac{1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{\ldots}}{\sqrt[4]{i\ldots}}}\right.\right.$$
$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}$$

$\downarrow$ 1082

$$-4\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{1-iax}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-\ldots}}{\sqrt[4]{iax\ldots}}\right.$$
$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}$$

$\downarrow$ 217

$$-4\left(\frac{1}{2}\int \frac{1-\sqrt{1-iax}}{2-iax}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)\right)+$$
$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}$$

$\downarrow$ 1479

$$-4\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\right.}{}\right.\right.$$
$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}$$

$\downarrow$ 25

$$-4\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1\right)}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{1}}\right.}{\sqrt{2}}\right.\right.$$
$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}$$

$\downarrow$ 27

$$-4\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{\sqrt{1-iax}-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}{\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}+1}d\frac{\sqrt[4]{1-iax}}{\sqrt[4]{iax+1}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\right.}{}\right.\right.$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}$$

$$\Big\downarrow \text{1103}$$

$$4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)-$$

$$4\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{1-iax}+\frac{\sqrt{2}\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}+1\right)}{2\sqrt{2}}-\right.\right.$$

$$\frac{8\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}$$

input
```
Int[1/(E^(((5*I)/2)*ArcTan[a*x])*x),x]
```

output
```
(8*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4) + 4*(ArcTan[(1 + I*a*x)^(1/4)/(1 -
 I*a*x)^(1/4)]/2 - ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/2) - 4*((-
(ArcTan[1 - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2]) + ArcT
an[1 + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sqrt[2])/2 + (-1/2*L
og[1 + Sqrt[1 - I*a*x] - (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4)]/Sq
rt[2] + Log[1 + Sqrt[1 - I*a*x] + (Sqrt[2]*(1 - I*a*x)^(1/4))/(1 + I*a*x)^
(1/4)]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 35
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}
, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] &&  !(IntegerQ[n] && SimplerQ[a +
b*x, c + d*x])
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} x}dx$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x)
```

output
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(162) = 324.

Time = 0.12 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.43

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x}\,dx$$
$$= \frac{\sqrt{4i}(ax-i)\log\left(\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-\sqrt{4i}(ax-i)\log\left(-\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}\right)-\sqrt{-4i}(ax-i)\log}{}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="fricas")
```

output
```
1/2*(sqrt(4*I)*(a*x - I)*log(1/2*I*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2 + 1)/(a
*x + I))) - sqrt(4*I)*(a*x - I)*log(-1/2*I*sqrt(4*I) + sqrt(I*sqrt(a^2*x^2
 + 1)/(a*x + I))) - sqrt(-4*I)*(a*x - I)*log(1/2*I*sqrt(-4*I) + sqrt(I*sqr
t(a^2*x^2 + 1)/(a*x + I))) + sqrt(-4*I)*(a*x - I)*log(-1/2*I*sqrt(-4*I) +
sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))) - 2*(a*x - I)*log(sqrt(I*sqrt(a^2*x^2
 + 1)/(a*x + I)) + 1) - 2*(-I*a*x - 1)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x +
 I)) + I) - 2*(I*a*x + 1)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) + 2
*(a*x - I)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1) - 16*I*sqrt(a^2*x^
2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)))/(a*x - I)
```

Sympy [F]

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x,x)
```

output
```
Integral(1/(x*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(5/2)), x)
```

Maxima [F]

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="maxima")
```

output
```
integrate(1/(x*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x}\,dx = \int \frac{1}{x\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}\,dx$$

input
```
int(1/(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)),x)
```

output
```
int(1/(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)), x)
```

Reduce [F]

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x}\,dx = -\left(\int \frac{(a^2x^2+1)^{\frac{1}{4}}}{\sqrt{aix+1}\,a^2x^3-2\sqrt{aix+1}\,ai\,x^2-\sqrt{aix+1}\,x}dx\right) - \left(\int \frac{(a^2x^2+1)^{\frac{1}{4}}\,x}{\sqrt{aix+1}\,a^2x^2-2\sqrt{aix+1}\,aix-\sqrt{aix+1}}dx\right)a^2$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x,x)
```

output
```
- (int((a**2*x**2 + 1)**(1/4)/(sqrt(a*i*x + 1)*a**2*x**3 - 2*sqrt(a*i*x +
 1)*a*i*x**2 - sqrt(a*i*x + 1)*x),x) + int(((a**2*x**2 + 1)**(1/4)*x)/(sqr
t(a*i*x + 1)*a**2*x**2 - 2*sqrt(a*i*x + 1)*a*i*x - sqrt(a*i*x + 1)),x)*a**
2)
```

3.126 $\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 121

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^2}\,dx = -\frac{10ia\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}} - \frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}} - 5ia\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + 5ia\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

output
```
-10*I*a*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)-(1-I*a*x)^(5/4)/x/(1+I*a*x)^(1/4)-
5*I*a*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))+5*I*a*arctanh((1+I*a*x)^(1/4
)/(1-I*a*x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.57

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^2}\,dx = \frac{i\sqrt[4]{1-iax}\left(i-9ax+10ax\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{i+ax}{i-ax}\right)\right)}{x\sqrt[4]{1+iax}}$$

input
```
Integrate[1/(E^(((5*I)/2)*ArcTan[a*x])*x^2),x]
```

output
```
(I*(1 - I*a*x)^(1/4)*(I - 9*a*x + 10*a*x*Hypergeometric2F1[1/4, 1, 5/4, (I
 + a*x)/(I - a*x)]))/(x*(1 + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5585, 105, 105, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^2}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{(1-iax)^{5/4}}{x^2(1+iax)^{5/4}}dx$$

$$\Big\downarrow \text{105}$$

$$-\frac{5}{2}ia\int \frac{\sqrt[4]{1-iax}}{x(iax+1)^{5/4}}dx-\frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{5}{2}ia\left(\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{iax+1}}dx+\frac{4\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-\frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{5}{2}ia\left(4\int -\frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+\frac{4\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-\frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{5}{2}ia\left(\frac{4\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-4\int \frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}}$$

$$\Big\downarrow \text{827}$$

$$-\frac{5}{2}ia\left(4\left(\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)+\frac{4\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-\frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}}$$

$\Big\downarrow$ 216

$$-\frac{5}{2}ia\left(4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)+\frac{4\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-\frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}}$$

$\Big\downarrow$ 219

$$-\frac{5}{2}ia\left(4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{4\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-\frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}}$$

input
```
Int[1/(E^(((5*I)/2)*ArcTan[a*x])*x^2),x]
```

output
```
-((1 - I*a*x)^(5/4)/(x*(1 + I*a*x)^(1/4))) - ((5*I)/2)*a*((4*(1 - I*a*x)^(
1/4))/(1 + I*a*x)^(1/4) + 4*(ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/2
 - ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} x^2}dx$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x)
```

output
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(83) = 166$.

Time = 0.13 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.75

$$\int \frac{e^{-\frac{5}{2} i \arctan(ax)}}{x^2} \, dx =$$

$$-\frac{2 \sqrt{a^2x^2+1}(9\,ax - i)\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}} + 5\left(-i\,a^2x^2 - ax\right)\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}} + 1\right) - 5\left(a^2x^2 - i\,ax\right)\log\left(\sqrt{i \cdots}\right.}{2\left(ax^2 - i \cdots\right.}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(a^2*x^2 + 1)*(9*a*x - I)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))
+ 5*(-I*a^2*x^2 - a*x)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) - 5*(a
^2*x^2 - I*a*x)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 5*(a^2*x^2
- I*a*x)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) + 5*(I*a^2*x^2 + a*x
)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1))/(a*x^2 - I*x)
```

Sympy [F]

$$\int \frac{e^{-\frac{5}{2} i \arctan(ax)}}{x^2} \, dx = \int \frac{1}{x^2 \left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}} \, dx$$

input
```
integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**2,x)
```

output
```
Integral(1/(x**2*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(5/2)), x)
```

Maxima [F]

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(1/(x^2*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{1+a\,x\,1i}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}\,dx$$

input
```
int(1/(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)),x)
```

output
```
int(1/(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)), x)
```

Reduce [F]

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^2}\,dx = -\left(\int \frac{(a^2x^2+1)^{\frac{1}{4}}}{\sqrt{aix+1}\,a^2x^4-2\sqrt{aix+1}\,ai\,x^3-\sqrt{aix+1}\,x^2}dx\right) - \left(\int \frac{(a^2x^2+1)^{\frac{1}{4}}}{\sqrt{aix+1}\,a^2x^2-2\sqrt{aix+1}\,aix-\sqrt{aix+1}}dx\right)a^2$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^2,x)
```

output
```
- (int((a**2*x**2 + 1)**(1/4)/(sqrt(a*i*x + 1)*a**2*x**4 - 2*sqrt(a*i*x +
 1)*a*i*x**3 - sqrt(a*i*x + 1)*x**2),x) + int((a**2*x**2 + 1)**(1/4)/(sqrt
(a*i*x + 1)*a**2*x**2 - 2*sqrt(a*i*x + 1)*a*i*x - sqrt(a*i*x + 1)),x)*a**2
)
```

3.127 $\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 163

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = -\frac{25a^2\sqrt[4]{1-iax}}{2\sqrt[4]{1+iax}} + \frac{5ia(1-iax)^{5/4}}{4x\sqrt[4]{1+iax}} - \frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}} - \frac{25}{4}a^2\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) + \frac{25}{4}a^2\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

output
```
-25/2*a^2*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)+5/4*I*a*(1-I*a*x)^(5/4)/x/(1+I*a
*x)^(1/4)-1/2*(1-I*a*x)^(9/4)/x^2/(1+I*a*x)^(1/4)-25/4*a^2*arctan((1+I*a*x
)^(1/4)/(1-I*a*x)^(1/4))+25/4*a^2*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.50

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = \frac{\sqrt[4]{1-iax}\left(-2+9iax-43a^2x^2+50a^2x^2\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{i+ax}{i-ax}\right)\right)}{4x^2\sqrt[4]{1+iax}}$$

input
```
Integrate[1/(E^(((5*I)/2)*ArcTan[a*x])*x^3),x]
```

output
```
((1 - I*a*x)^(1/4)*(-2 + (9*I)*a*x - 43*a^2*x^2 + 50*a^2*x^2*Hypergeometri
c2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]))/(4*x^2*(1 + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5585, 107, 105, 105, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{5}{2} i \arctan (a x)}}{x^3} \, dx$$

$$\Big\downarrow \text{ 5585}$$

$$\int \frac{(1-i a x)^{5/4}}{x^3 (1+i a x)^{5/4}}dx$$

$$\Big\downarrow \text{ 107}$$

$$-\frac{5}{4} i a \int \frac{(1-i a x)^{5/4}}{x^2 (i a x+1)^{5/4}}dx-\frac{(1-i a x)^{9/4}}{2 x^2 \sqrt[4]{1+i a x}}$$

$$\Big\downarrow \text{ 105}$$

$$-\frac{5}{4} i a \left(-\frac{5}{2} i a \int \frac{\sqrt[4]{1-i a x}}{x (i a x+1)^{5/4}}dx-\frac{(1-i a x)^{5/4}}{x \sqrt[4]{1+i a x}}\right)-\frac{(1-i a x)^{9/4}}{2 x^2 \sqrt[4]{1+i a x}}$$

$$\Big\downarrow \text{ 105}$$

$$-\frac{5}{4} i a \left(-\frac{5}{2} i a \left(\int \frac{1}{x (1-i a x)^{3/4} \sqrt[4]{i a x+1}}dx+\frac{4 \sqrt[4]{1-i a x}}{\sqrt[4]{1+i a x}}\right)-\frac{(1-i a x)^{5/4}}{x \sqrt[4]{1+i a x}}\right)-\frac{(1-i a x)^{9/4}}{2 x^2 \sqrt[4]{1+i a x}}$$

$$\Big\downarrow \text{ 104}$$

$$-\frac{5}{4}ia\left(-\frac{5}{2}ia\left(4\int -\frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+\frac{4\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-\frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}}\right)-\frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}}$$

$$\Big\downarrow \ 25$$

$$-\frac{5}{4}ia\left(-\frac{5}{2}ia\left(\frac{4\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-4\int \frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}}\right)-\frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}}$$

$$\Big\downarrow \ 827$$

$$-\frac{5}{4}ia\left(-\frac{5}{2}ia\left(4\left(\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)+\frac{4\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-\frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}}\right)-\frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}}$$

$$\Big\downarrow \ 216$$

$$-\frac{5}{4}ia\left(-\frac{5}{2}ia\left(4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)+\frac{4\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-\frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}}\right)-\frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}}$$

$$\Big\downarrow \ 219$$

$$-\frac{5}{4}ia\left(-\frac{5}{2}ia\left(4\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{4\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-\frac{(1-iax)^{5/4}}{x\sqrt[4]{1+iax}}\right)-\frac{(1-iax)^{9/4}}{2x^2\sqrt[4]{1+iax}}$$

input
```
Int[1/(E^(((5*I)/2)*ArcTan[a*x])*x^3),x]
```

output
```
-1/2*(1 - I*a*x)^(9/4)/(x^2*(1 + I*a*x)^(1/4)) - ((5*I)/4)*a*(-((1 - I*a*x
)^(5/4)/(x*(1 + I*a*x)^(1/4))) - ((5*I)/2)*a*((4*(1 - I*a*x)^(1/4))/(1 + I
*a*x)^(1/4) + 4*(ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/2 - ArcTanh[(
1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/2)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} x^3}dx$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x)
```

output
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(111) = 222$.

Time = 0.11 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.46

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = -\frac{2\sqrt{a^2x^2+1}(-43i\,a^2x^2-9\,ax-2i)\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}-25\left(a^3x^3-i\,a^2x^2\right)\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1\right)+25\left(i\,a^3x\right.}{}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="fricas")
```

output
```
-1/8*(2*sqrt(a^2*x^2 + 1)*(-43*I*a^2*x^2 - 9*a*x - 2*I)*sqrt(I*sqrt(a^2*x^
2 + 1)/(a*x + I)) - 25*(a^3*x^3 - I*a^2*x^2)*log(sqrt(I*sqrt(a^2*x^2 + 1)/
(a*x + I)) + 1) + 25*(I*a^3*x^3 + a^2*x^2)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a
*x + I)) + I) + 25*(-I*a^3*x^3 - a^2*x^2)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*
x + I)) - I) + 25*(a^3*x^3 - I*a^2*x^2)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x
+ I)) - 1))/(a*x^3 - I*x^2)
```

Sympy [F]

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{1}{x^3\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**3,x)
```

output
```
Integral(1/(x**3*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(5/2)), x)
```

Maxima [F]

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{1}{x^3\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(1/(x^3*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = \int \frac{1}{x^3\left(\frac{1+a\,x\,1\text{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}\,dx$$

input
```
int(1/(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)),x)
```

output
```
int(1/(x^3*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)), x)
```

Reduce [F]

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^3}\,dx = -\left(\int \frac{(a^2x^2+1)^{\frac{1}{4}}}{\sqrt{aix+1}\,a^2x^5-2\sqrt{aix+1}\,ai\,x^4-\sqrt{aix+1}\,x^3}dx\right) - \left(\int \frac{(a^2x^2+1)^{\frac{1}{4}}}{\sqrt{aix+1}\,a^2x^3-2\sqrt{aix+1}\,ai\,x^2-\sqrt{aix+1}\,x}dx\right)a^2$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^3,x)
```

output
```
- (int((a**2*x**2 + 1)**(1/4)/(sqrt(a*i*x + 1)*a**2*x**5 - 2*sqrt(a*i*x +
 1)*a*i*x**4 - sqrt(a*i*x + 1)*x**3),x) + int((a**2*x**2 + 1)**(1/4)/(sqrt
(a*i*x + 1)*a**2*x**3 - 2*sqrt(a*i*x + 1)*a*i*x**2 - sqrt(a*i*x + 1)*x),x)
*a**2)
```

3.128 $\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 203

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \frac{287ia^3\sqrt[4]{1-iax}}{24\sqrt[4]{1+iax}} - \frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}} + \frac{13ia\sqrt[4]{1-iax}}{12x^2\sqrt[4]{1+iax}} + \frac{61a^2\sqrt[4]{1-iax}}{24x\sqrt[4]{1+iax}} + \frac{55}{8}ia^3\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right) - \frac{55}{8}ia^3\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)$$

output
```
287/24*I*a^3*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)-1/3*(1-I*a*x)^(1/4)/x^3/(1+I*
a*x)^(1/4)+13/12*I*a*(1-I*a*x)^(1/4)/x^2/(1+I*a*x)^(1/4)+61/24*a^2*(1-I*a*
x)^(1/4)/x/(1+I*a*x)^(1/4)+55/8*I*a^3*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/
4))-55/8*I*a^3*arctanh((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.46

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \frac{\sqrt[4]{1-iax}\left(-8+26iax+61a^2x^2+287ia^3x^3-330ia^3x^3\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{i+ax}{i-ax}\right)\right)}{24x^3\sqrt[4]{1+iax}}$$

input
```
Integrate[1/(E^(((5*I)/2)*ArcTan[a*x])*x^4),x]
```

output
```
((1 - I*a*x)^(1/4)*(-8 + (26*I)*a*x + 61*a^2*x^2 + (287*I)*a^3*x^3 - (330*
I)*a^3*x^3*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a*x)]))/(24*x^3*(
1 + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {5585, 109, 27, 168, 27, 168, 27, 172, 27, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^4}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{(1-iax)^{5/4}}{x^4(1+iax)^{5/4}}dx$$

$$\Big\downarrow \text{109}$$

$$-\frac{1}{3}\int \frac{a(12ax+13i)}{2x^3(1-iax)^{3/4}(iax+1)^{5/4}}dx-\frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{6}a\int \frac{12ax+13i}{x^3(1-iax)^{3/4}(iax+1)^{5/4}}dx-\frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}}$$

$$\Big\downarrow \text{168}$$

$$-\frac{1}{6}a\left(-\frac{1}{2}\int -\frac{a(61-52iax)}{2x^2(1-iax)^{3/4}(iax+1)^{5/4}}dx-\frac{13i\sqrt[4]{1-iax}}{2x^2\sqrt[4]{1+iax}}\right)-\frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{6}a\left(\frac{1}{4}a\int \frac{61-52iax}{x^2(1-iax)^{3/4}(iax+1)^{5/4}}dx-\frac{13i\sqrt[4]{1-iax}}{2x^2\sqrt[4]{1+iax}}\right)-\frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}}$$

$$\Big\downarrow \text{168}$$

$$-\frac{1}{6}a\left(\frac{1}{4}a\left(-\int \frac{a(122ax+165i)}{2x(1-iax)^{3/4}(iax+1)^{5/4}}dx-\frac{61\sqrt[4]{1-iax}}{x\sqrt[4]{1+iax}}\right)-\frac{13i\sqrt[4]{1-iax}}{2x^2\sqrt[4]{1+iax}}\right)-\frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{6}a\left(\frac{1}{4}a\left(-\frac{1}{2}a\int \frac{122ax+165i}{x(1-iax)^{3/4}(iax+1)^{5/4}}dx-\frac{61\sqrt[4]{1-iax}}{x\sqrt[4]{1+iax}}\right)-\frac{13i\sqrt[4]{1-iax}}{2x^2\sqrt[4]{1+iax}}\right)-\frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}}$$

$$\Big\downarrow \text{172}$$

$$-\frac{1}{6}a\left(\frac{1}{4}a\left(-\frac{1}{2}a\left(\frac{574i\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{2i\int -\frac{165a}{2x(1-iax)^{3/4}\sqrt[4]{iax+1}}dx}{a}\right)-\frac{61\sqrt[4]{1-iax}}{x\sqrt[4]{1+iax}}\right)-\frac{13i\sqrt[4]{1-iax}}{2x^2\sqrt[4]{1+iax}}\right)-\frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{6}a\left(\frac{1}{4}a\left(-\frac{1}{2}a\left(165i\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{iax+1}}dx+\frac{574i\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-\frac{61\sqrt[4]{1-iax}}{x\sqrt[4]{1+iax}}\right)-\frac{13i\sqrt[4]{1-iax}}{2x^2\sqrt[4]{1+iax}}\right)-\frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{1}{6}a\left(\frac{1}{4}a\left(-\frac{1}{2}a\left(660i\int -\frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+\frac{574i\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-\frac{61\sqrt[4]{1-iax}}{x\sqrt[4]{1+iax}}\right)-\frac{13i\sqrt[4]{1-i}}{2x^2\sqrt[4]{1+i}}\right.$$
$$\frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{6}a\left(\frac{1}{4}a\left(-\frac{1}{2}a\left(\frac{574i\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-660i\int\frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{61\sqrt[4]{1-iax}}{x\sqrt[4]{1+iax}}\right)-\frac{13i\sqrt[4]{1-iax}}{2x^2\sqrt[4]{1+iax}}\right)-\frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}}$$

$\Big\downarrow$ 827

$$-\frac{1}{6}a\left(\frac{1}{4}a\left(-\frac{1}{2}a\left(660i\left(\frac{1}{2}\int\frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)+\frac{574i\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-\frac{61\sqrt[4]{1-iax}}{x\sqrt[4]{1+iax}}\right)-\frac{13i\sqrt[4]{1-iax}}{2x^2\sqrt[4]{1+iax}}\right)-\frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}}$$

$\Big\downarrow$ 216

$$-\frac{1}{6}a\left(\frac{1}{4}a\left(-\frac{1}{2}a\left(660i\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)+\frac{574i\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-\frac{61\sqrt[4]{1-iax}}{x\sqrt[4]{1+iax}}\right)-\frac{13i\sqrt[4]{1-iax}}{2x^2\sqrt[4]{1+iax}}\right)-\frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}}$$

$\Big\downarrow$ 219

$$-\frac{1}{6}a\left(\frac{1}{4}a\left(-\frac{1}{2}a\left(660i\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{574i\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-\frac{61\sqrt[4]{1-iax}}{x\sqrt[4]{1+iax}}\right)-\frac{13i\sqrt[4]{1-iax}}{2x^2\sqrt[4]{1+iax}}\right)-\frac{\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}}$$

input
```
Int[1/(E^(((5*I)/2)*ArcTan[a*x])*x^4),x]
```

output
```
-1/3*(1 - I*a*x)^(1/4)/(x^3*(1 + I*a*x)^(1/4)) - (a*((((-13*I)/2)*(1 - I*a
*x)^(1/4))/(x^2*(1 + I*a*x)^(1/4)) + (a*((-61*(1 - I*a*x)^(1/4))/(x*(1 + I
*a*x)^(1/4)) - (a*(((574*I)*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4) + (660*I)
*(ArcTan[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/2 - ArcTanh[(1 + I*a*x)^(1/4
)/(1 - I*a*x)^(1/4)]/2)))/2))/4))/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
]))))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} x^4}dx$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x)
```

output
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.21

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \frac{2\,(287\,a^3x^3 - 61i\,a^2x^2 + 26\,ax + 8i)\sqrt{a^2x^2+1}\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}} - 165\,(i\,a^4x^4 + a^3x^3)\log\left(\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}} + 1\right) -$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="fricas")
```

output
```
1/48*(2*(287*a^3*x^3 - 61*I*a^2*x^2 + 26*a*x + 8*I)*sqrt(a^2*x^2 + 1)*sqrt
(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 165*(I*a^4*x^4 + a^3*x^3)*log(sqrt(I*sqr
t(a^2*x^2 + 1)/(a*x + I)) + 1) - 165*(a^4*x^4 - I*a^3*x^3)*log(sqrt(I*sqrt
(a^2*x^2 + 1)/(a*x + I)) + I) + 165*(a^4*x^4 - I*a^3*x^3)*log(sqrt(I*sqrt(
a^2*x^2 + 1)/(a*x + I)) - I) - 165*(-I*a^4*x^4 - a^3*x^3)*log(sqrt(I*sqrt(
a^2*x^2 + 1)/(a*x + I)) - 1))/(a*x^4 - I*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{1}{x^4\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(1/(x^4*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = \int \frac{1}{x^4\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}\,dx$$

input
```
int(1/(x^4*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)),x)
```

output
```
int(1/(x^4*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)), x)
```

Reduce [F]

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^4}\,dx = -\left(\int \frac{(a^2x^2+1)^{\frac{1}{4}}}{\sqrt{aix+1}\,a^2x^6-2\sqrt{aix+1}\,ai\,x^5-\sqrt{aix+1}\,x^4}dx\right) - \left(\int \frac{(a^2x^2+1)^{\frac{1}{4}}}{\sqrt{aix+1}\,a^2x^4-2\sqrt{aix+1}\,ai\,x^3-\sqrt{aix+1}\,x^2}dx\right)a^2$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^4,x)
```

output
```
 - (int((a**2*x**2 + 1)**(1/4)/(sqrt(a*i*x + 1)*a**2*x**6 - 2*sqrt(a*i*x +
 1)*a*i*x**5 - sqrt(a*i*x + 1)*x**4),x) + int((a**2*x**2 + 1)**(1/4)/(sqrt
(a*i*x + 1)*a**2*x**4 - 2*sqrt(a*i*x + 1)*a*i*x**3 - sqrt(a*i*x + 1)*x**2)
,x)*a**2)
```

3.129 $\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 233

$$\begin{aligned}\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^5}\,dx &= \frac{2467a^4\sqrt[4]{1-iax}}{192\sqrt[4]{1+iax}}-\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}}+\frac{17ia\sqrt[4]{1-iax}}{24x^3\sqrt[4]{1+iax}}\\&\quad+\frac{113a^2\sqrt[4]{1-iax}}{96x^2\sqrt[4]{1+iax}}-\frac{521ia^3\sqrt[4]{1-iax}}{192x\sqrt[4]{1+iax}}\\&\quad+\frac{475}{64}a^4\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{475}{64}a^4\operatorname{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\end{aligned}$$

output
```
2467/192*a^4*(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)-1/4*(1-I*a*x)^(1/4)/x^4/(1+I*
a*x)^(1/4)+17/24*I*a*(1-I*a*x)^(1/4)/x^3/(1+I*a*x)^(1/4)+113/96*a^2*(1-I*a
*x)^(1/4)/x^2/(1+I*a*x)^(1/4)-521/192*I*a^3*(1-I*a*x)^(1/4)/x/(1+I*a*x)^(1
/4)+475/64*a^4*arctan((1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))-475/64*a^4*arctanh(
(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.42

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = \frac{\sqrt[4]{1-iax}\left(-48+136iax+226a^2x^2-521ia^3x^3+2467a^4x^4-2850a^4x^4\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{i+ax}{i-ax}\right)\right)}{192x^4\sqrt[4]{1+iax}}$$

input
```
Integrate[1/(E^(((5*I)/2)*ArcTan[a*x])*x^5),x]
```

output
```
((1 - I*a*x)^(1/4)*(-48 + (136*I)*a*x + 226*a^2*x^2 - (521*I)*a^3*x^3 + 24
67*a^4*x^4 - 2850*a^4*x^4*Hypergeometric2F1[1/4, 1, 5/4, (I + a*x)/(I - a*
x)]))/(192*x^4*(1 + I*a*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.03, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5585, 109, 27, 168, 27, 168, 27, 168, 27, 172, 27, 104, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^5}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{(1-iax)^{5/4}}{x^5(1+iax)^{5/4}}dx$$

$$\Big\downarrow \text{109}$$

$$-\frac{1}{4}\int \frac{a(16ax+17i)}{2x^4(1-iax)^{3/4}(iax+1)^{5/4}}dx-\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{8}a\int \frac{16ax+17i}{x^4(1-iax)^{3/4}(iax+1)^{5/4}}dx-\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}}$$

$\Big\downarrow$ 168

$$-\frac{1}{8}a\left(-\frac{1}{3}\int -\frac{a(113-102iax)}{2x^3(1-iax)^{3/4}(iax+1)^{5/4}}dx-\frac{17i\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}}\right)-\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}}$$

$\Big\downarrow$ 27

$$-\frac{1}{8}a\left(\frac{1}{6}a\int \frac{113-102iax}{x^3(1-iax)^{3/4}(iax+1)^{5/4}}dx-\frac{17i\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}}\right)-\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}}$$

$\Big\downarrow$ 168

$$-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{2}\int \frac{a(452ax+521i)}{2x^2(1-iax)^{3/4}(iax+1)^{5/4}}dx-\frac{113\sqrt[4]{1-iax}}{2x^2\sqrt[4]{1+iax}}\right)-\frac{17i\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}}\right)-\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}}$$

$\Big\downarrow$ 27

$$-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{4}a\int \frac{452ax+521i}{x^2(1-iax)^{3/4}(iax+1)^{5/4}}dx-\frac{113\sqrt[4]{1-iax}}{2x^2\sqrt[4]{1+iax}}\right)-\frac{17i\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}}\right)-\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}}$$

$\Big\downarrow$ 168

$$-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{4}a\left(-\int -\frac{a(1425-1042iax)}{2x(1-iax)^{3/4}(iax+1)^{5/4}}dx-\frac{521i\sqrt[4]{1-iax}}{x\sqrt[4]{1+iax}}\right)-\frac{113\sqrt[4]{1-iax}}{2x^2\sqrt[4]{1+iax}}\right)-\frac{17i\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}}\right)-\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}}$$

$\Big\downarrow$ 27

$$-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{4}a\left(\frac{1}{2}a\int \frac{1425-1042iax}{x(1-iax)^{3/4}(iax+1)^{5/4}}dx-\frac{521i\sqrt[4]{1-iax}}{x\sqrt[4]{1+iax}}\right)-\frac{113\sqrt[4]{1-iax}}{2x^2\sqrt[4]{1+iax}}\right)-\frac{17i\sqrt[4]{1-iax}}{3x^3\sqrt[4]{1+iax}}\right)-\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}}$$

$\Big\downarrow$ 172

$$-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{4}a\left(\frac{1}{2}a\left(\frac{4934\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-\frac{2i\int \frac{1425ia}{2x(1-iax)^{3/4}\sqrt[4]{iax+1}}dx}{a}\right)-\frac{521i\sqrt[4]{1-iax}}{x\sqrt[4]{1+iax}}\right)-\frac{113\sqrt[4]{1-iax}}{2x^2\sqrt[4]{1+iax}}\right)-\right.\right.$$
$$\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}}$$

$$\Big\downarrow \ 27$$

$$-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{4}a\left(\frac{1}{2}a\left(1425\int \frac{1}{x(1-iax)^{3/4}\sqrt[4]{iax+1}}dx+\frac{4934\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-\frac{521i\sqrt[4]{1-iax}}{x\sqrt[4]{1+iax}}\right)-\frac{113\sqrt[4]{1-iax}}{2x^2\sqrt[4]{1+iax}}\right.\right.\right.$$
$$\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}}$$

$$\Big\downarrow \ 104$$

$$-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{4}a\left(\frac{1}{2}a\left(5700\int -\frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+\frac{4934\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-\frac{521i\sqrt[4]{1-iax}}{x\sqrt[4]{1+iax}}\right)-\frac{113}{2x}\right.\right.\right.$$
$$\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}}$$

$$\Big\downarrow \ 25$$

$$-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{4}a\left(\frac{1}{2}a\left(\frac{4934\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}-5700\int \frac{\sqrt{iax+1}}{\sqrt{1-iax}\left(1-\frac{iax+1}{1-iax}\right)}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)-\frac{521i\sqrt[4]{1-iax}}{x\sqrt[4]{1+iax}}\right)-\frac{113}{2x^2}\right.\right.\right.$$
$$\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}}$$

$$\Big\downarrow \ 827$$

$$-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{4}a\left(\frac{1}{2}a\left(5700\left(\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)+\frac{4934\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-\right.\right.\right.$$
$$\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}}$$

$$\Big\downarrow \ 216$$

$$-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{4}a\left(\frac{1}{2}a\left(5700\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}\right)+\frac{4934\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-\frac{521}{x}\right.\right.\right.\right.$$

$$\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{1}{8}a\left(\frac{1}{6}a\left(-\frac{1}{4}a\left(\frac{1}{2}a\left(5700\left(\frac{1}{2}\arctan\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)\right)+\frac{4934\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)-\frac{521i\sqrt[4]{1-}}{x\sqrt[4]{1+}}\right.\right.\right.\right.$$

$$\frac{\sqrt[4]{1-iax}}{4x^4\sqrt[4]{1+iax}}$$

input
```
Int[1/(E^(((5*I)/2)*ArcTan[a*x])*x^5),x]
```

output
```
-1/4*(1 - I*a*x)^(1/4)/(x^4*(1 + I*a*x)^(1/4)) - (a*((((-17*I)/3)*(1 - I*a
*x)^(1/4))/(x^3*(1 + I*a*x)^(1/4)) + (a*((-113*(1 - I*a*x)^(1/4))/(2*x^2*(
1 + I*a*x)^(1/4)) - (a*(((-521*I)*(1 - I*a*x)^(1/4))/(x*(1 + I*a*x)^(1/4))
 + (a*((4934*(1 - I*a*x)^(1/4))/(1 + I*a*x)^(1/4) + 5700*(ArcTan[(1 + I*a*
x)^(1/4)/(1 - I*a*x)^(1/4)]/2 - ArcTanh[(1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4
)]/2)))/2))/4))/6))/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
]))))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{1}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} x^5}dx$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x)
```

output
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.09

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = -\frac{2\left(2467i\,a^4x^4+521\,a^3x^3+226i\,a^2x^2-136\,ax-48i\right)\sqrt{a^2x^2+1}\sqrt{\frac{i\sqrt{a^2x^2+1}}{ax+i}}+1425\left(a^5x^5-i\,a^4x^4\right)\log}{}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="fricas")
```

output
```
-1/384*(2*(2467*I*a^4*x^4 + 521*a^3*x^3 + 226*I*a^2*x^2 - 136*a*x - 48*I)*
sqrt(a^2*x^2 + 1)*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1425*(a^5*x^5 - I*
a^4*x^4)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + 1) + 1425*(-I*a^5*x^5 -
 a^4*x^4)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) + I) + 1425*(I*a^5*x^5 +
 a^4*x^4)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - I) - 1425*(a^5*x^5 - I
*a^4*x^4)*log(sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)) - 1))/(a*x^5 - I*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate(1/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)/x**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{1}{x^5\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(1/(x^5*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = \int \frac{1}{x^5\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}}\,dx$$

input
```
int(1/(x^5*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)),x)
```

output
```
int(1/(x^5*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2)), x)
```

Reduce [F]

$$\int \frac{e^{-\frac{5}{2}i\arctan(ax)}}{x^5}\,dx = -\left(\int \frac{(a^2x^2+1)^{\frac{1}{4}}}{\sqrt{aix+1}\,a^2x^7-2\sqrt{aix+1}\,ai\,x^6-\sqrt{aix+1}\,x^5}dx\right) - \left(\int \frac{(a^2x^2+1)^{\frac{1}{4}}}{\sqrt{aix+1}\,a^2x^5-2\sqrt{aix+1}\,ai\,x^4-\sqrt{aix+1}\,x^3}dx\right)a^2$$

input
```
int(1/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)/x^5,x)
```

output

```
- (int((a**2*x**2 + 1)**(1/4)/(sqrt(a*i*x + 1)*a**2*x**7 - 2*sqrt(a*i*x +
 1)*a*i*x**6 - sqrt(a*i*x + 1)*x**5),x) + int((a**2*x**2 + 1)**(1/4)/(sqrt
(a*i*x + 1)*a**2*x**5 - 2*sqrt(a*i*x + 1)*a*i*x**4 - sqrt(a*i*x + 1)*x**3)
,x)*a**2)
```

3.130 $\int e^{\frac{1}{3}i\arctan(x)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 255

$$\int e^{\frac{1}{3}i\arctan(x)}x^2\,dx = -\frac{19}{54}i(1-ix)^{5/6}\sqrt[6]{1+ix}-\frac{1}{18}i(1-ix)^{5/6}(1+ix)^{7/6}$$

$$+\frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x+\frac{19}{162}i\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)-\frac{19}{162}i\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)-\frac{19}{81}i\,\text{arcta}$$

output
```
-19/54*I*(1-I*x)^(5/6)*(1+I*x)^(1/6)-1/18*I*(1-I*x)^(5/6)*(1+I*x)^(7/6)+1/
3*(1-I*x)^(5/6)*(1+I*x)^(7/6)*x-19/162*I*arctan(-3^(1/2)+2*(1-I*x)^(1/6)/(
1+I*x)^(1/6))-19/162*I*arctan(3^(1/2)+2*(1-I*x)^(1/6)/(1+I*x)^(1/6))-19/81
*I*arctan((1-I*x)^(1/6)/(1+I*x)^(1/6))+19/162*I*arctanh(3^(1/2)*(1-I*x)^(1
/6)/(1+(1-I*x)^(1/3)/(1+I*x)^(1/3))/(1+I*x)^(1/6))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.29

$$\int e^{\frac{1}{3}i\arctan(x)}x^2\,dx=\frac{1}{90}(1-ix)^{5/6}\left(5\sqrt[6]{1+ix}(-i+7x+6ix^2)-38i\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}-\frac{ix}{2}\right)\right)$$

input
```
Integrate[E^((I/3)*ArcTan[x])*x^2,x]
```

output
```
((1 - I*x)^(5/6)*(5*(1 + I*x)^(1/6)*(-I + 7*x + (6*I)*x^2) - (38*I)*2^(1/6
)*Hypergeometric2F1[-1/6, 5/6, 11/6, 1/2 - (I/2)*x]))/90
```

Rubi [A] (warning: unable to verify)

Time = 0.71 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.18, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.071, Rules used = {5585, 101, 27, 90, 60, 73, 854, 824, 27, 216, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2e^{\frac{1}{3}i\arctan(x)}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{\sqrt[6]{1+ix}x^2}{\sqrt[6]{1-ix}}dx$$

$$\Big\downarrow \text{101}$$

$$\frac{1}{3}\int -\frac{\sqrt[6]{ix+1}(ix+3)}{3\sqrt[6]{1-ix}}dx+\frac{1}{3}(1-ix)^{5/6}x(1+ix)^{7/6}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}(1-ix)^{5/6}(1+ix)^{7/6}x-\frac{1}{9}\int \frac{\sqrt[6]{ix+1}(ix+3)}{\sqrt[6]{1-ix}}dx$$

$\big\downarrow$ 90

$$\frac{1}{9}\left(-\frac{19}{6}\int \frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}dx-\frac{1}{2}i(1-ix)^{5/6}(1+ix)^{7/6}\right)+\frac{1}{3}(1-ix)^{5/6}x(1+ix)^{7/6}$$

$\big\downarrow$ 60

$$\frac{1}{9}\left(-\frac{19}{6}\left(\frac{1}{3}\int \frac{1}{\sqrt[6]{1-ix}(ix+1)^{5/6}}dx+i(1-ix)^{5/6}\sqrt[6]{1+ix}\right)-\frac{1}{2}i(1-ix)^{5/6}(1+ix)^{7/6}\right)+\frac{1}{3}(1-ix)^{5/6}x(1+ix)^{7/6}$$

$\big\downarrow$ 73

$$\frac{1}{9}\left(-\frac{19}{6}\left(2i\int \frac{(1-ix)^{2/3}}{(ix+1)^{5/6}}d\sqrt[6]{1-ix}+i(1-ix)^{5/6}\sqrt[6]{1+ix}\right)-\frac{1}{2}i(1-ix)^{5/6}(1+ix)^{7/6}\right)+\frac{1}{3}(1-ix)^{5/6}x(1+ix)^{7/6}$$

$\big\downarrow$ 854

$$\frac{1}{9}\left(-\frac{19}{6}\left(2i\int \frac{(1-ix)^{2/3}}{2-ix}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+i(1-ix)^{5/6}\sqrt[6]{1+ix}\right)-\frac{1}{2}i(1-ix)^{5/6}(1+ix)^{7/6}\right)+\frac{1}{3}(1-ix)^{5/6}x(1+ix)^{7/6}$$

$\big\downarrow$ 824

$$\frac{1}{9}\left(-\frac{19}{6}\left(2i\left(\frac{1}{3}\int \frac{1}{\sqrt[3]{1-ix}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+\frac{1}{3}\int -\frac{1-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{2\left(\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1\right)}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+\frac{1}{3}\int -\frac{}{2\left(\sqrt[3]{1-\right.}} \right.\right.\right.$$
$$\frac{1}{3}(1-ix)^{5/6}x(1+ix)^{7/6}$$

$\big\downarrow$ 27

$$\frac{1}{9}\left(-\frac{19}{6}\left(2i\left(\frac{1}{3}\int \frac{1}{\sqrt[3]{1-ix}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}-\frac{1}{6}\int \frac{1-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}-\frac{1}{6}\int \frac{\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{\sqrt[3]{1-ix}+\frac{\sqrt{3}\sqrt[6]{1}}{\sqrt[6]{ix}}} \right.\right.\right.$$
$$\frac{1}{3}(1-ix)^{5/6}x(1+ix)^{7/6}$$

$\big\downarrow$ 216

$$\frac{1}{9}\left(-\frac{19}{6}\left(2i\left(-\frac{1}{6}\int \frac{1-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}-\frac{1}{6}\int \frac{\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}{\sqrt[3]{1-ix}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+\frac{1}{3}\text{arct}\right.\right.\right.$$

$$\frac{1}{3}(1-ix)^{5/6}x(1+ix)^{7/6}$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{9}\left(-\frac{19}{6}\left(2i\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}\right)\right.\right.\right.$$

$$\frac{1}{3}(1-ix)^{5/6}x(1+ix)^{7/6}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{9}\left(-\frac{19}{6}\left(2i\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}\right)+\right.\right.\right.$$

$$\frac{1}{3}(1-ix)^{5/6}x(1+ix)^{7/6}$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{9}\left(-\frac{19}{6}\left(2i\left(\frac{1}{6}\left(-\int \frac{1}{-\sqrt[3]{1-ix}-1}d\left(\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}-\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}\right)+\right.\right.\right.$$

$$\frac{1}{3}(1-ix)^{5/6}x(1+ix)^{7/6}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{9}\left(-\frac{19}{6}\left(2i\left(\frac{1}{6}\left(-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}-\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)\right)+\frac{1}{6}\left(\arctan\left(\right.\right.\right.\right.\right.$$

$$\frac{1}{3}(1-ix)^{5/6}x(1+ix)^{7/6}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{9}\left(-\frac{19}{6}\left(2i\left(\frac{1}{3}\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\log\left(\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)-\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)\right.\right.\right.\right.$$
$$\frac{1}{3}(1-ix)^{5/6}x(1+ix)^{7/6}$$

input
```
Int[E^((I/3)*ArcTan[x])*x^2,x]
```

output
```
((1 - I*x)^(5/6)*(1 + I*x)^(7/6)*x)/3 + ((-1/2*I)*(1 - I*x)^(5/6)*(1 + I*x
)^(7/6) - (19*(I*(1 - I*x)^(5/6)*(1 + I*x)^(1/6) + (2*I)*(ArcTan[(1 - I*x)
^(1/6)/(1 + I*x)^(1/6)]/3 + (-ArcTan[Sqrt[3] - (2*(1 - I*x)^(1/6))/(1 + I*
x)^(1/6)] + (Sqrt[3]*Log[1 + (1 - I*x)^(1/3) - (Sqrt[3]*(1 - I*x)^(1/6))/(
1 + I*x)^(1/6)])/2)/6 + (ArcTan[Sqrt[3] + (2*(1 - I*x)^(1/6))/(1 + I*x)^(1
/6)] - (Sqrt[3]*Log[1 + (1 - I*x)^(1/3) + (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I
*x)^(1/6)])/2)/6)))/6)/9
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}} x^2 dx$$

input
```
int(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x^2,x)
```

output
```
int(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.82

$$\int e^{\frac{1}{3}i\arctan(x)}x^2\,dx = -\frac{19}{324}\left(-i\sqrt{3}+1\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}i\right) - \frac{19}{324}\left(-i\sqrt{3}-1\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}i\right) - \frac{19}{324}\left(i\sqrt{3}+1\right)\log\left(-\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}i\right) - \frac{19}{324}\left(i\sqrt{3}-1\right)\log\left(-\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}i\right) + \frac{1}{54}\left(18\,x^3-3i\,x^2-x-22i\right)\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}} - \frac{19}{162}\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+i\right) + \frac{19}{162}\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-i\right)$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x^2,x, algorithm="fricas")
```

output
```
-19/324*(-I*sqrt(3) + 1)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3)
 + 1/2*I) - 19/324*(-I*sqrt(3) - 1)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x
+ I))^(1/3) - 1/2*I) - 19/324*(I*sqrt(3) + 1)*log(-1/2*sqrt(3) + (I*sqrt(x
^2 + 1)/(x + I))^(1/3) + 1/2*I) - 19/324*(I*sqrt(3) - 1)*log(-1/2*sqrt(3)
+ (I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1/2*I) + 1/54*(18*x^3 - 3*I*x^2 - x -
22*I)*(I*sqrt(x^2 + 1)/(x + I))^(1/3) - 19/162*log((I*sqrt(x^2 + 1)/(x + I
))^(1/3) + I) + 19/162*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) - I)
```

Sympy [F]

$$\int e^{\frac{1}{3}i\arctan(x)}x^2\,dx = \int x^2\sqrt[3]{\frac{i(x-i)}{\sqrt{x^2+1}}}\,dx$$

input
```
integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3)*x**2,x)
```

output
```
Integral(x**2*(I*(x - I)/sqrt(x**2 + 1))**(1/3), x)
```

Maxima [F]

$$\int e^{\frac{1}{3}i\arctan(x)}x^2\,dx = \int x^2\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}\,dx$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x^2,x, algorithm="maxima")
```

output
```
integrate(x^2*((I*x + 1)/sqrt(x^2 + 1))^(1/3), x)
```

Giac [F]

$$\int e^{\frac{1}{3}i\arctan(x)}x^2\,dx = \int x^2\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}\,dx$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x^2,x, algorithm="giac")
```

output
```
integrate(x^2*((I*x + 1)/sqrt(x^2 + 1))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{3}i\arctan(x)}x^2\,dx = \int x^2\left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{1/3}\,dx$$

input
```
int(x^2*((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3),x)
```

output
```
int(x^2*((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3), x)
```

Reduce [F]

$$\int e^{\frac{1}{3}i\arctan(x)}x^2\,dx = \int \frac{(ix+1)^{\frac{1}{3}}\,x^2}{(x^2+1)^{\frac{1}{6}}}dx$$

input
```
int(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x^2,x)
```

output
```
int(((i*x + 1)**(1/3)*x**2)/(x**2 + 1)**(1/6),x)
```

3.131 $\int e^{\frac{1}{3}i \arctan(x)} x \, dx$

Optimal result

Integrand size = 12, antiderivative size = 216

$$\int e^{\frac{1}{3}i\arctan(x)}x\,dx = \frac{1}{6}(1-ix)^{5/6}\sqrt[6]{1+ix}+\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}$$

$$-\frac{1}{18}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{1}{18}\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{1}{9}\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)-\frac{\operatorname{arctanh}\left(\frac{\quad}{\left(1+\right.}\right.}{}$$

output
```
1/6*(1-I*x)^(5/6)*(1+I*x)^(1/6)+1/2*(1-I*x)^(5/6)*(1+I*x)^(7/6)+1/18*arcta
n(-3^(1/2)+2*(1-I*x)^(1/6)/(1+I*x)^(1/6))+1/18*arctan(3^(1/2)+2*(1-I*x)^(1
/6)/(1+I*x)^(1/6))+1/9*arctan((1-I*x)^(1/6)/(1+I*x)^(1/6))-1/18*arctanh(3^
(1/2)*(1-I*x)^(1/6)/(1+(1-I*x)^(1/3)/(1+I*x)^(1/3))/(1+I*x)^(1/6))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.26

$$\int e^{\frac{1}{3}i\arctan(x)}x\,dx=\frac{1}{10}(1-ix)^{5/6}\left(5(1+ix)^{7/6}+2\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}-\frac{ix}{2}\right)\right)$$

input
```
Integrate[E^((I/3)*ArcTan[x])*x,x]
```

output
```
((1 - I*x)^(5/6)*(5*(1 + I*x)^(7/6) + 2*2^(1/6)*Hypergeometric2F1[-1/6, 5/
6, 11/6, 1/2 - (I/2)*x]))/10
```

Rubi [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.25, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {5585, 90, 60, 73, 854, 824, 27, 216, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\frac{1}{3}i\arctan(x)}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{\sqrt[6]{1+ix}x}{\sqrt[6]{1-ix}}dx$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}-\frac{1}{6}i\int \frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}-\frac{1}{6}i\left(\frac{1}{3}\int \frac{1}{\sqrt[6]{1-ix}(ix+1)^{5/6}}dx+i(1-ix)^{5/6}\sqrt[6]{1+ix}\right)$$

$\Big\downarrow$ 73

$$\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}-\frac{1}{6}i\left(2i\int \frac{(1-ix)^{2/3}}{(ix+1)^{5/6}}d\sqrt[6]{1-ix}+i(1-ix)^{5/6}\sqrt[6]{1+ix}\right)$$

$\Big\downarrow$ 854

$$\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}-\frac{1}{6}i\left(2i\int \frac{(1-ix)^{2/3}}{2-ix}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+i(1-ix)^{5/6}\sqrt[6]{1+ix}\right)$$

$\Big\downarrow$ 824

$$\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}-$$
$$\frac{1}{6}i\left(2i\left(\frac{1}{3}\int \frac{1}{\sqrt[3]{1-ix}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+\frac{1}{3}\int -\frac{1-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{2\left(\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1\right)}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+\frac{1}{3}\int -\frac{\frac{\sqrt{3}\sqrt[6]{1}}{\sqrt[6]{ix}}}{2\left(\sqrt[3]{1-ix}+\right.}\right.\right.$$

$\Big\downarrow$ 27

$$\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}-$$
$$\frac{1}{6}i\left(2i\left(\frac{1}{3}\int \frac{1}{\sqrt[3]{1-ix}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}-\frac{1}{6}\int \frac{1-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}-\frac{1}{6}\int \frac{\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}{\sqrt[3]{1-ix}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}\right.\right.$$

$\Big\downarrow$ 216

$$\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}-$$
$$\frac{1}{6}i\left(2i\left(-\frac{1}{6}\int \frac{1-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}-\frac{1}{6}\int \frac{\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}{\sqrt[3]{1-ix}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+\frac{1}{3}\arctan\left(\right.\right.\right.$$

$\Big\downarrow$ 1142

$$\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}-$$
$$\frac{1}{6}i\left(2i\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}\right)+\frac{1}{6}\left(\right.\right.\right.$$

$\Big\downarrow$ 25

$$\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}- \frac{1}{6}i\left(2i\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}-\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}\right)+\frac{1}{6}\left(\frac{1}{2}\right.\right.\right.$$

$\downarrow$ 1083

$$\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}- \frac{1}{6}i\left(2i\left(\frac{1}{6}\left(-\int\frac{1}{-\sqrt[3]{1-ix}-1}d\left(\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}-\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}\right)+\frac{1}{6}\left(-\right.\right.\right.$$

$\downarrow$ 217

$$\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}- \frac{1}{6}i\left(2i\left(\frac{1}{6}\left(-\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}-\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)\right)+\frac{1}{6}\left(\arctan\left(\sqrt{3}+\right.\right.\right.\right.$$

$\downarrow$ 1103

$$\frac{1}{2}(1-ix)^{5/6}(1+ix)^{7/6}- \frac{1}{6}i\left(2i\left(\frac{1}{3}\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\log\left(\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)-\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)\right)+\frac{1}{6}\right.\right.$$

input
```
Int[E^((I/3)*ArcTan[x])*x,x]
```

output
```
((1 - I*x)^(5/6)*(1 + I*x)^(7/6))/2 - (I/6)*(I*(1 - I*x)^(5/6)*(1 + I*x)^(
1/6) + (2*I)*(ArcTan[(1 - I*x)^(1/6)/(1 + I*x)^(1/6)]/3 + (-ArcTan[Sqrt[3]
 - (2*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)] + (Sqrt[3]*Log[1 + (1 - I*x)^(1/3)
 - (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)])/2)/6 + (ArcTan[Sqrt[3] + (2
*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)] - (Sqrt[3]*Log[1 + (1 - I*x)^(1/3) + (S
qrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)])/2)/6))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}} x dx$$

input
```
int(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x,x)
```

output
```
int(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.90

$$\int e^{\frac{1}{3}i\arctan(x)}x\,dx = -\frac{1}{36}\left(\sqrt{3}+i\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}i\right) - \frac{1}{36}\left(\sqrt{3}-i\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}i\right) + \frac{1}{36}\left(\sqrt{3}-i\right)\log\left(-\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}i\right) + \frac{1}{36}\left(\sqrt{3}+i\right)\log\left(-\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}i\right) + \frac{1}{6}\left(3x^2-ix+4\right)\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}} - \frac{1}{18}i\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+i\right) + \frac{1}{18}i\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-i\right)$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x,x, algorithm="fricas")
```

output
```
-1/36*(sqrt(3) + I)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/
2*I) - 1/36*(sqrt(3) - I)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3
) - 1/2*I) + 1/36*(sqrt(3) - I)*log(-1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I
))^(1/3) + 1/2*I) + 1/36*(sqrt(3) + I)*log(-1/2*sqrt(3) + (I*sqrt(x^2 + 1)
/(x + I))^(1/3) - 1/2*I) + 1/6*(3*x^2 - I*x + 4)*(I*sqrt(x^2 + 1)/(x + I))
^(1/3) - 1/18*I*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) + I) + 1/18*I*log((I*s
qrt(x^2 + 1)/(x + I))^(1/3) - I)
```

Sympy [F]

$$\int e^{\frac{1}{3}i\arctan(x)}x\,dx = \int x\sqrt[3]{\frac{i(x-i)}{\sqrt{x^2+1}}}\,dx$$

input
```
integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3)*x,x)
```

output
```
Integral(x*(I*(x - I)/sqrt(x**2 + 1))**(1/3), x)
```

Maxima [F]

$$\int e^{\frac{1}{3}i\arctan(x)}x\,dx = \int x\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}\,dx$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x,x, algorithm="maxima")
```

output
```
integrate(x*((I*x + 1)/sqrt(x^2 + 1))^(1/3), x)
```

Giac [F]

$$\int e^{\frac{1}{3}i\arctan(x)}x\,dx=\int x\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}\,dx$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x,x, algorithm="giac")
```

output
```
integrate(x*((I*x + 1)/sqrt(x^2 + 1))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{3}i\arctan(x)}x\,dx=\int x\left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{1/3}\,dx$$

input
```
int(x*((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3),x)
```

output
```
int(x*((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3), x)
```

Reduce [F]

$$\int e^{\frac{1}{3}i\arctan(x)}x\,dx=\int \frac{(ix+1)^{\frac{1}{3}}\,x}{(x^2+1)^{\frac{1}{6}}}dx$$

input
```
int(((1+I*x)/(x^2+1)^(1/2))^(1/3)*x,x)
```

output
```
int(((i*x + 1)**(1/3)*x)/(x**2 + 1)**(1/6),x)
```

3.132 $\int e^{\frac{1}{3}i\arctan(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 196

$$\int e^{\frac{1}{3}i\arctan(x)}\,dx = i(1-ix)^{5/6}\sqrt[6]{1+ix} - \frac{1}{3}i\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{1}{3}i\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) + \frac{2}{3}i\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right) - \frac{i\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{1-ix}}{\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)\sqrt[6]{1+ix}}\right)}{\sqrt{3}}$$

output
```
I*(1-I*x)^(5/6)*(1+I*x)^(1/6)+1/3*I*arctan(-3^(1/2)+2*(1-I*x)^(1/6)/(1+I*x
)^(1/6))+1/3*I*arctan(3^(1/2)+2*(1-I*x)^(1/6)/(1+I*x)^(1/6))+2/3*I*arctan(
(1-I*x)^(1/6)/(1+I*x)^(1/6))-1/3*I*arctanh(3^(1/2)*(1-I*x)^(1/6)/(1+(1-I*x
)^(1/3)/(1+I*x)^(1/3))/(1+I*x)^(1/6))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.17

$$\int e^{\frac{1}{3}i\arctan(x)}\,dx=-\frac{12}{7}ie^{\frac{7}{3}i\arctan(x)}\,\text{Hypergeometric2F1}\left(\frac{7}{6},2,\frac{13}{6},-e^{2i\arctan(x)}\right)$$

input
```
Integrate[E^((I/3)*ArcTan[x]),x]
```

output
```
((-12*I)/7)*E^(((7*I)/3)*ArcTan[x])*Hypergeometric2F1[7/6, 2, 13/6, -E^((2
*I)*ArcTan[x])]
```

Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.21, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}=1.200$, Rules used = {5584, 60, 73, 854, 824, 27, 216, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{1}{3}i\arctan(x)}\,dx$$

$$\Big\downarrow \text{5584}$$

$$\int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\int \frac{1}{\sqrt[6]{1-ix}(ix+1)^{5/6}}dx+i(1-ix)^{5/6}\sqrt[6]{1+ix}$$

$$\Big\downarrow \text{73}$$

$$2i\int \frac{(1-ix)^{2/3}}{(ix+1)^{5/6}}d\sqrt[6]{1-ix}+i(1-ix)^{5/6}\sqrt[6]{1+ix}$$

$$\Big\downarrow \text{854}$$

$$2i\int \frac{(1-ix)^{2/3}}{2-ix}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+i(1-ix)^{5/6}\sqrt[6]{1+ix}$$

$\downarrow$ 824

$$2i\left(\frac{1}{3}\int \frac{1}{\sqrt[3]{1-ix}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+\frac{1}{3}\int -\frac{1-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{2\left(\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1\right)}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+\frac{1}{3}\int -\frac{\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{2\left(\sqrt[3]{1-ix}+\frac{\sqrt{3}\sqrt[6]{}}{\sqrt[6]{i}}\right.}\right.$$
$$i(1-ix)^{5/6}\sqrt[6]{1+ix}$$

$\downarrow$ 27

$$2i\left(\frac{1}{3}\int \frac{1}{\sqrt[3]{1-ix}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}-\frac{1}{6}\int \frac{1-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}-\frac{1}{6}\int \frac{\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}{\sqrt[3]{1-ix}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\right.$$
$$i(1-ix)^{5/6}\sqrt[6]{1+ix}$$

$\downarrow$ 216

$$2i\left(-\frac{1}{6}\int \frac{1-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}-\frac{1}{6}\int \frac{\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}{\sqrt[3]{1-ix}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+\frac{1}{3}\arctan\left(\frac{\sqrt[6]{1-}}{\sqrt[6]{1+}}\right.\right.$$
$$i(1-ix)^{5/6}\sqrt[6]{1+ix}$$

$\downarrow$ 1142

$$2i\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}\right)+\frac{1}{6}\left(\frac{1}{2}\int\right.\right.$$
$$i(1-ix)^{5/6}\sqrt[6]{1+ix}$$

$\downarrow$ 25

$$2i\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}\right)+\frac{1}{6}\left(\frac{1}{2}\int\right.\right.$$
$$i(1-ix)^{5/6}\sqrt[6]{1+ix}$$

$\downarrow$ 1083

$$2i\left(\frac{1}{6}\left(-\int \frac{1}{-\sqrt[3]{1-ix}-1}d\left(\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}-\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}\right)+\frac{1}{6}\left(-\int \frac{}{-}\right.\right.$$
$$i(1-ix)^{5/6}\sqrt[6]{1+ix}$$

$$\Big\downarrow \textbf{217}$$

$$2i\left(\frac{1}{6}\left(-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}-\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)\right)+\frac{1}{6}\left(\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{1}}{\sqrt[6]{1}}\right.\right.\right.$$
$$i(1-ix)^{5/6}\sqrt[6]{1+ix}$$

$$\Big\downarrow \textbf{1103}$$

$$2i\left(\frac{1}{3}\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\log\left(\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)-\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)\right)+\frac{1}{6}\left(\text{arc}\right.\right.$$
$$i(1-ix)^{5/6}\sqrt[6]{1+ix}$$

input
```
Int[E^((I/3)*ArcTan[x]),x]
```

output
```
I*(1 - I*x)^(5/6)*(1 + I*x)^(1/6) + (2*I)*(ArcTan[(1 - I*x)^(1/6)/(1 + I*x
)^(1/6)]/3 + (-ArcTan[Sqrt[3] - (2*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)] + (Sq
rt[3]*Log[1 + (1 - I*x)^(1/3) - (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)]
)/2)/6 + (ArcTan[Sqrt[3] + (2*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)] - (Sqrt[3]
*Log[1 + (1 - I*x)^(1/3) + (Sqrt[3]*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)])/2)/
6)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5584
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Maple [F]

$$\int \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}} dx$$

input
```
int(((1+I*x)/(x^2+1)^(1/2))^(1/3),x)
```

output
```
int(((1+I*x)/(x^2+1)^(1/2))^(1/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.99

$$\int e^{\frac{1}{3}i\arctan(x)}\,dx = \frac{1}{6}\left(-i\sqrt{3}+1\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}i\right) + \frac{1}{6}\left(-i\sqrt{3}-1\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}i\right) + \frac{1}{6}\left(i\sqrt{3}+1\right)\log\left(-\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}i\right) + \frac{1}{6}\left(i\sqrt{3}-1\right)\log\left(-\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}i\right) + (x+i)\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}} + \frac{1}{3}\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+i\right) - \frac{1}{3}\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-i\right)$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3),x, algorithm="fricas")
```

output
```
1/6*(-I*sqrt(3) + 1)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1
/2*I) + 1/6*(-I*sqrt(3) - 1)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(
1/3) - 1/2*I) + 1/6*(I*sqrt(3) + 1)*log(-1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x
 + I))^(1/3) + 1/2*I) + 1/6*(I*sqrt(3) - 1)*log(-1/2*sqrt(3) + (I*sqrt(x^2
 + 1)/(x + I))^(1/3) - 1/2*I) + (x + I)*(I*sqrt(x^2 + 1)/(x + I))^(1/3) +
1/3*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) + I) - 1/3*log((I*sqrt(x^2 + 1)/(x
 + I))^(1/3) - I)
```

Sympy [F]

$$\int e^{\frac{1}{3}i\arctan(x)}\,dx = \int \sqrt[3]{\frac{ix+1}{\sqrt{x^2+1}}}\,dx$$

input
```
integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3),x)
```

output
```
Integral(((I*x + 1)/sqrt(x**2 + 1))**(1/3), x)
```

Maxima [F]

$$\int e^{\frac{1}{3}i\arctan(x)}\,dx = \int \left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}\,dx$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3),x, algorithm="maxima")
```

output
```
integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3), x)
```

Giac [F]

$$\int e^{\frac{1}{3}i\arctan(x)}\,dx = \int \left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}\,dx$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3),x, algorithm="giac")
```

output
```
integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{3}i\arctan(x)}\,dx = \int \left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{1/3}\,dx$$

input
```
int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3),x)
```

output
```
int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3), x)
```

Reduce [F]

$$\int e^{\frac{1}{3}i\arctan(x)}\,dx = \int \frac{(ix+1)^{\frac{1}{3}}}{(x^2+1)^{\frac{1}{6}}}dx$$

input
```
int(((1+I*x)/(x^2+1)^(1/2))^(1/3),x)
```

output
```
int((i*x + 1)**(1/3)/(x**2 + 1)**(1/6),x)
```

3.133 $\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 311

$$\begin{aligned}\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x}\,dx &= \arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)-\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)\\
&\quad+\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)-\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)\\
&\quad-2\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\sqrt{3}\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{1-ix}}{\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)\sqrt[6]{1+ix}}\right)\\
&\quad-2\operatorname{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)-\operatorname{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\left(1+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right)\sqrt[6]{1-ix}}\right)\end{aligned}$$

output
```
-arctan(-3^(1/2)+2*(1-I*x)^(1/6)/(1+I*x)^(1/6))-arctan(3^(1/2)+2*(1-I*x)^(
1/6)/(1+I*x)^(1/6))+3^(1/2)*arctan(1/3*(1-2*(1+I*x)^(1/6)/(1-I*x)^(1/6))*3
^(1/2))-3^(1/2)*arctan(1/3*(1+2*(1+I*x)^(1/6)/(1-I*x)^(1/6))*3^(1/2))-2*ar
ctan((1-I*x)^(1/6)/(1+I*x)^(1/6))+arctanh(3^(1/2)*(1-I*x)^(1/6)/(1+(1-I*x)
^(1/3)/(1+I*x)^(1/3))/(1+I*x)^(1/6))*3^(1/2)-2*arctanh((1+I*x)^(1/6)/(1-I*
x)^(1/6))-arctanh((1+I*x)^(1/6)/(1+(1+I*x)^(1/3)/(1-I*x)^(1/3))/(1-I*x)^(1
/6))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.29

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x}\,dx = -\frac{3(1-ix)^{5/6}\left(\sqrt[6]{2}(1+ix)^{5/6}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}-\frac{ix}{2}\right)+2\,\text{Hypergeometric2F1}\left(\frac{5}{6},1,\frac{11}{6},\frac{i+}{i-}\right.\right.}{5(1+ix)^{5/6}}$$

input
```
Integrate[E^((I/3)*ArcTan[x])/x,x]
```

output
```
(-3*(1 - I*x)^(5/6)*(2^(1/6)*(1 + I*x)^(5/6)*Hypergeometric2F1[5/6, 5/6, 1
1/6, 1/2 - (I/2)*x] + 2*Hypergeometric2F1[5/6, 1, 11/6, (I + x)/(I - x)]))
/(5*(1 + I*x)^(5/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.84 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.41, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$, Rules used = {5585, 140, 73, 104, 754, 27, 219, 854, 824, 27, 216, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{3} i \arctan(x)}}{x} \, dx$$

$$\Big\downarrow \; 5585$$

$$\int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}x}dx$$

$$\Big\downarrow \; 140$$

$$i \int \frac{1}{\sqrt[6]{1-ix}(ix+1)^{5/6}}dx + \int \frac{1}{\sqrt[6]{1-ix}(ix+1)^{5/6}x}dx$$

$$\Big\downarrow \; 73$$

$$\int \frac{1}{\sqrt[6]{1-ix}(ix+1)^{5/6}x}dx - 6\int \frac{(1-ix)^{2/3}}{(ix+1)^{5/6}}d\sqrt[6]{1-ix}$$

$$\Big\downarrow \; 104$$

$$6\int \frac{1}{\frac{ix+1}{1-ix}-1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}} - 6\int \frac{(1-ix)^{2/3}}{(ix+1)^{5/6}}d\sqrt[6]{1-ix}$$

$$\Big\downarrow \; 754$$

$$6\left(-\frac{1}{3}\int \frac{1}{1-\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}} - \frac{1}{3}\int \frac{2-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{2\left(\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1\right)}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}} - \frac{1}{3}\int \frac{\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+2}{2\left(\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}+\frac{\sqrt[6]{ix+}}{\sqrt[6]{1-i}}\right.}\right.$$

$$6\int \frac{(1-ix)^{2/3}}{(ix+1)^{5/6}}d\sqrt[6]{1-ix}$$

$$\Big\downarrow \; 27$$

$$6\left(-\frac{1}{3}\int \frac{1}{1-\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}} - \frac{1}{6}\int \frac{2-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}} - \frac{1}{6}\int \frac{\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+2}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}+\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\right.$$

$$6\int \frac{(1-ix)^{2/3}}{(ix+1)^{5/6}}d\sqrt[6]{1-ix}$$

$$\Big\downarrow \; 219$$

$$6\left(-\frac{1}{6}\int \frac{2-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{6}\int \frac{\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+2}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}+\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{3}\text{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right.\right.$$

$$6\int \frac{(1-ix)^{2/3}}{(ix+1)^{5/6}}d\sqrt[6]{1-ix}$$

$$\Big\downarrow \; 854$$

$$6\left(-\frac{1}{6}\int \frac{2-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{6}\int \frac{\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+2}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}+\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{3}\text{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right.\right.$$

$$6\int \frac{(1-ix)^{2/3}}{2-ix}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}$$

$$\Big\downarrow \; 824$$

$$6\left(-\frac{1}{6}\int \frac{2-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{6}\int \frac{\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+2}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}+\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{3}\text{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right.\right.$$

$$6\left(\frac{1}{3}\int \frac{1}{\sqrt[3]{1-ix}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+\frac{1}{3}\int -\frac{1-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{2\left(\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1\right)}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+\frac{1}{3}\int -\frac{\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{2\left(\sqrt[3]{1-ix}+\frac{\sqrt{3}\sqrt[6]{\cdots}}{\sqrt[6]{i\cdots}}\right.}\right.$$

$$\Big\downarrow \; 27$$

$$6\left(-\frac{1}{6}\int \frac{2-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{6}\int \frac{\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+2}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}+\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{3}\text{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right.\right.$$

$$6\left(\frac{1}{3}\int \frac{1}{\sqrt[3]{1-ix}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}-\frac{1}{6}\int \frac{1-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}-\frac{1}{6}\int \frac{\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}{\sqrt[3]{1-ix}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\right.$$

$$\Big\downarrow \; 216$$

$$6\left(-\frac{1}{6}\int \frac{2-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{6}\int \frac{\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+2}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}+\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{3}\text{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right.\right.$$
$$6\left(-\frac{1}{6}\int \frac{1-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}-\frac{1}{6}\int \frac{\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}{\sqrt[3]{1-ix}+\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+\frac{1}{3}\arctan\left(\frac{\sqrt[6]{1-}}{\sqrt[6]{1+}}\right.\right.$$

$\downarrow$ 1142

$$6\left(-\frac{1}{3}\text{arctanh}\left(\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}\right)+\frac{1}{6}\left(\frac{1}{2}\int -\frac{1-\frac{2\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{3}{2}\int \frac{1}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\right.\right.$$
$$6\left(\frac{1}{3}\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+}\right.\right.$$

$\downarrow$ 25

$$6\left(-\frac{1}{3}\text{arctanh}\left(\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}\right)+\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\right.\right.$$
$$6\left(\frac{1}{3}\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\right.\right.$$

$\downarrow$ 1083

$$6\left(\frac{1}{6}\left(3\int \frac{1}{-\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-3}d\left(\frac{2\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-1\right)-\frac{1}{2}\int \frac{1-\frac{2\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}\right)+\frac{1}{6}\left(3\int \frac{1}{-\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}}\right.\right.$$
$$6\left(\frac{1}{6}\left(-\int \frac{1}{-\sqrt[3]{1-ix}-1}d\left(\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}-\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}\right)+\frac{1}{6}\left(-\int \frac{}{-}\right.\right.$$

$\downarrow$ 217

$$6\left(\frac{1}{6}\left(-\frac{1}{2}\int \frac{1-\frac{2\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\sqrt{3}\arctan\left(\frac{-1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(-\frac{1}{2}\int \frac{\frac{2\sqrt[6]{ix+}}{\sqrt[6]{1-i}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}+\frac{\sqrt[6]{}}{\sqrt[6]{}}}\right.\right.$$

$$6\left(\frac{1}{6}\left(-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}}{\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}+1}d\frac{\sqrt[6]{1-ix}}{\sqrt[6]{ix+1}}-\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)\right)+\frac{1}{6}\left(\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{1}}{\sqrt[6]{1}}\right.\right.\right.$$

$\downarrow$ 1103

$$6\left(\frac{1}{6}\left(\frac{1}{2}\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)-\sqrt{3}\arctan\left(\frac{-1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right.\right.\right.$$

$$6\left(\frac{1}{3}\arctan\left(\frac{\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\log\left(\sqrt[3]{1-ix}-\frac{\sqrt{3}\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}+1\right)-\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{1-ix}}{\sqrt[6]{1+ix}}\right)\right)+\frac{1}{6}\left(\text{arct}\right.\right.$$

input
```
Int[E^((I/3)*ArcTan[x])/x,x]
```

output
```
-6*(ArcTan[(1 - I*x)^(1/6)/(1 + I*x)^(1/6)]/3 + (-ArcTan[Sqrt[3] - (2*(1 -
 I*x)^(1/6))/(1 + I*x)^(1/6)] + (Sqrt[3]*Log[1 + (1 - I*x)^(1/3) - (Sqrt[3
]*(1 - I*x)^(1/6))/(1 + I*x)^(1/6)])/2)/6 + (ArcTan[Sqrt[3] + (2*(1 - I*x)
^(1/6))/(1 + I*x)^(1/6)] - (Sqrt[3]*Log[1 + (1 - I*x)^(1/3) + (Sqrt[3]*(1
- I*x)^(1/6))/(1 + I*x)^(1/6)])/2)/6) + 6*(-1/3*ArcTanh[(1 + I*x)^(1/6)/(1
 - I*x)^(1/6)] + (-(Sqrt[3]*ArcTan[(-1 + (2*(1 + I*x)^(1/6))/(1 - I*x)^(1/
6))/Sqrt[3]]) + Log[1 - (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/
(1 - I*x)^(1/3)]/2)/6 + (-(Sqrt[3]*ArcTan[(1 + (2*(1 + I*x)^(1/6))/(1 - I*
x)^(1/6))/Sqrt[3]]) - Log[1 + (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^
(1/3)/(1 - I*x)^(1/3)]/2)/6)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x}dx$$

input
```
int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x,x)
```

output
```
int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.09

$$\begin{aligned}\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x}\,dx =& \frac{1}{2}\left(\sqrt{3}+i\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)\\ &+\frac{1}{2}\left(\sqrt{3}-i\right)\log\left(\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)\\ &+\frac{1}{2}\left(-i\sqrt{3}-1\right)\log\left(\frac{1}{2}i\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}\right)\\ &+\frac{1}{2}\left(-i\sqrt{3}+1\right)\log\left(\frac{1}{2}i\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}\right)\\ &+\frac{1}{2}\left(i\sqrt{3}-1\right)\log\left(-\frac{1}{2}i\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}\right)\\ &+\frac{1}{2}\left(i\sqrt{3}+1\right)\log\left(-\frac{1}{2}i\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}\right)\\ &-\frac{1}{2}\left(\sqrt{3}-i\right)\log\left(-\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}i\right)\\ &-\frac{1}{2}\left(\sqrt{3}+i\right)\log\left(-\frac{1}{2}\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-\frac{1}{2}i\right)\\ &-\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+1\right)+i\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+i\right)\\ &-i\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-i\right)+\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-1\right)\end{aligned}$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x,x, algorithm="fricas")
```

output
```
1/2*(sqrt(3) + I)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/2*
I) + 1/2*(sqrt(3) - I)*log(1/2*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) -
 1/2*I) + 1/2*(-I*sqrt(3) - 1)*log(1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I
))^(1/3) + 1/2) + 1/2*(-I*sqrt(3) + 1)*log(1/2*I*sqrt(3) + (I*sqrt(x^2 + 1
)/(x + I))^(1/3) - 1/2) + 1/2*(I*sqrt(3) - 1)*log(-1/2*I*sqrt(3) + (I*sqrt
(x^2 + 1)/(x + I))^(1/3) + 1/2) + 1/2*(I*sqrt(3) + 1)*log(-1/2*I*sqrt(3) +
 (I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1/2) - 1/2*(sqrt(3) - I)*log(-1/2*sqrt(
3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/2*I) - 1/2*(sqrt(3) + I)*log(-1/2
*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1/2*I) - log((I*sqrt(x^2 + 1)
/(x + I))^(1/3) + 1) + I*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) + I) - I*log(
(I*sqrt(x^2 + 1)/(x + I))^(1/3) - I) + log((I*sqrt(x^2 + 1)/(x + I))^(1/3)
 - 1)
```

Sympy [F]

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x}\,dx = \int \frac{\sqrt[3]{\frac{i(x-i)}{\sqrt{x^2+1}}}}{x}\,dx$$

input
```
integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3)/x,x)
```

output
```
Integral((I*(x - I)/sqrt(x**2 + 1))**(1/3)/x, x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x}\,dx = \int \frac{\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x}\,dx$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x,x, algorithm="maxima")
```

output
```
integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x, x)
```

Giac [F]

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x}\,dx = \int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x}\,dx$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x,x, algorithm="giac")
```

output
```
integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x}\,dx = \int \frac{\left(\frac{1+x\,1i}{\sqrt{x^2+1}}\right)^{1/3}}{x}\,dx$$

input
```
int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3)/x,x)
```

output
```
int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3)/x, x)
```

Reduce [F]

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x}\,dx = \int \frac{(ix+1)^{\frac{1}{3}}}{(x^2+1)^{\frac{1}{6}}\,x}dx$$

input
```
int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x,x)
```

output
```
int((i*x + 1)**(1/3)/((x**2 + 1)**(1/6)*x),x)
```

3.134 $\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 199

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^2}\,dx = -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}+\frac{i\arctan\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{i\arctan\left(\frac{1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3}i\text{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) - \frac{1}{3}i\text{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\left(1+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right)\sqrt[6]{1-ix}}\right)$$

output
```
-(1-I*x)^(5/6)*(1+I*x)^(1/6)/x+1/3*I*arctan(1/3*(1-2*(1+I*x)^(1/6)/(1-I*x)
^(1/6))*3^(1/2))*3^(1/2)-1/3*I*arctan(1/3*(1+2*(1+I*x)^(1/6)/(1-I*x)^(1/6)
)*3^(1/2))*3^(1/2)-2/3*I*arctanh((1+I*x)^(1/6)/(1-I*x)^(1/6))-1/3*I*arctan
h((1+I*x)^(1/6)/(1+(1+I*x)^(1/3)/(1-I*x)^(1/3))/(1-I*x)^(1/6))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.32

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^2}\,dx = -\frac{i(1-ix)^{5/6}\left(-5i+5x+2x\,\text{Hypergeometric2F1}\left(\frac{5}{6},1,\frac{11}{6},\frac{i+x}{i-x}\right)\right)}{5(1+ix)^{5/6}x}$$

input
```
Integrate[E^((I/3)*ArcTan[x])/x^2,x]
```

output
```
((-1/5*I)*(1 - I*x)^(5/6)*(-5*I + 5*x + 2*x*Hypergeometric2F1[5/6, 1, 11/6
, (I + x)/(I - x)]))/((1 + I*x)^(5/6)*x)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.30, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5585, 105, 104, 754, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^2}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}x^2}dx$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{3}i\int \frac{1}{\sqrt[6]{1-ix}(ix+1)^{5/6}x}dx-\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}$$

$$\Big\downarrow \text{104}$$

$$2i\int \frac{1}{\frac{ix+1}{1-ix}-1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}$$

$$\Big\downarrow \text{754}$$

$$2i\left(-\frac{1}{3}\int\frac{1}{1-\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{3}\int\frac{2-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{2\left(\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1\right)}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{3}\int\frac{\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+2}{2\left(\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}+\frac{\sqrt[6]{ix+}}{\sqrt[6]{1-}}\right.}\right.$$
$$\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}$$

$$\Big\downarrow \text{27}$$

$$2i\left(-\frac{1}{3}\int\frac{1}{1-\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{6}\int\frac{2-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{6}\int\frac{\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+2}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}+\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\right.$$
$$\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}$$

$$\Big\downarrow \text{219}$$

$$2i\left(-\frac{1}{6}\int\frac{2-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{6}\int\frac{\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+2}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}+\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{3}\text{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right.\right.$$
$$\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}$$

$$\Big\downarrow \text{1142}$$

$$2i\left(\frac{1}{6}\left(\frac{1}{2}\int-\frac{1-\frac{2\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{3}{2}\int\frac{1}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}\right)+\frac{1}{6}\left(-\frac{3}{2}\int\frac{}{\frac{\sqrt[3]{i}}{\sqrt[3]{1}}}\right.\right.$$
$$\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}$$

$$\Big\downarrow \text{25}$$

$$2i\left(\frac{1}{6}\left(-\frac{3}{2}\int\frac{1}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{2}\int\frac{1-\frac{2\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}\right)+\frac{1}{6}\left(-\frac{3}{2}\int\frac{}{\frac{\sqrt[3]{i}}{\sqrt[3]{1}}}\right.\right.$$
$$\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}$$

$$\Big\downarrow \text{1083}$$

$$2i\left(\frac{1}{6}\left(3\int \frac{1}{-\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-3}d\left(\frac{2\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-1\right)-\frac{1}{2}\int \frac{1-\frac{2\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}\right)+\frac{1}{6}\left(3\int \frac{1}{-\frac{\sqrt[3]{ix+}}{\sqrt[3]{1-i}}\right.\right.$$
$$\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}$$

$$\Big\downarrow \text{217}$$

$$2i\left(\frac{1}{6}\left(-\frac{1}{2}\int \frac{1-\frac{2\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\sqrt{3}\arctan\left(\frac{-1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(-\frac{1}{2}\int \frac{\frac{2\sqrt[6]{ix+}}{\sqrt[6]{1-i}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}+\frac{\sqrt[6]{}}{\sqrt[6]{}}}\right.\right.$$
$$\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}$$

$$\Big\downarrow \text{1103}$$

$$2i\left(\frac{1}{6}\left(\frac{1}{2}\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)-\sqrt{3}\arctan\left(\frac{-1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right.\right.\right.$$
$$\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}$$

input
```
Int[E^((I/3)*ArcTan[x])/x^2,x]
```

output
```
-(((1 - I*x)^(5/6)*(1 + I*x)^(1/6))/x) + (2*I)*(-1/3*ArcTanh[(1 + I*x)^(1/
6)/(1 - I*x)^(1/6)] + (-(Sqrt[3]*ArcTan[(-1 + (2*(1 + I*x)^(1/6))/(1 - I*x
)^(1/6))/Sqrt[3]]) + Log[1 - (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(
1/3)/(1 - I*x)^(1/3)]/2)/6 + (-(Sqrt[3]*ArcTan[(1 + (2*(1 + I*x)^(1/6))/(1
 - I*x)^(1/6))/Sqrt[3]]) - Log[1 + (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 +
I*x)^(1/3)/(1 - I*x)^(1/3)]/2)/6)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^2}dx$$

input
```
int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^2,x)
```

output
```
int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.06

$$\int \frac{e^{\frac{1}{3} i \arctan(x)}}{x^2}\,dx$$
$$= \frac{\left(\sqrt{3}x - i\,x\right)\log\left(\frac{1}{2}i\,\sqrt{3} + \left(\frac{i\,\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}} + \frac{1}{2}\right) + \left(\sqrt{3}x + i\,x\right)\log\left(\frac{1}{2}i\,\sqrt{3} + \left(\frac{i\,\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}} - \frac{1}{2}\right) - \left(\sqrt{3}x + i\,x\right.}{}$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^2,x, algorithm="fricas")
```

output
```
1/6*((sqrt(3)*x - I*x)*log(1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3)
 + 1/2) + (sqrt(3)*x + I*x)*log(1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^
(1/3) - 1/2) - (sqrt(3)*x + I*x)*log(-1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x
+ I))^(1/3) + 1/2) - (sqrt(3)*x - I*x)*log(-1/2*I*sqrt(3) + (I*sqrt(x^2 +
1)/(x + I))^(1/3) - 1/2) - 2*I*x*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1)
+ 2*I*x*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1) - 6*(-I*x + 1)*(I*sqrt(x^
2 + 1)/(x + I))^(1/3))/x
```

Sympy [F]

$$\int \frac{e^{\frac{1}{3} i \arctan(x)}}{x^2}\,dx = \int \frac{\sqrt[3]{\frac{i\,(x-i)}{\sqrt{x^2+1}}}}{x^2}\,dx$$

input
```
integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3)/x**2,x)
```

output
```
Integral((I*(x - I)/sqrt(x**2 + 1))**(1/3)/x**2, x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^2}\,dx = \int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^2}\,dx$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^2,x, algorithm="maxima")
```

output
```
integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x^2, x)
```

Giac [F]

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^2}\,dx = \int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^2}\,dx$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^2,x, algorithm="giac")
```

output
```
integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^2}\,dx = \int \frac{\left(\frac{1+x\,1i}{\sqrt{x^2+1}}\right)^{1/3}}{x^2}\,dx$$

input
```
int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3)/x^2,x)
```

output
```
int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3)/x^2, x)
```

Reduce [F]

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^2}\,dx = \int \frac{(ix+1)^{\frac{1}{3}}}{(x^2+1)^{\frac{1}{6}}\,x^2}dx$$

input
```
int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^2,x)
```

output
```
int((i*x + 1)**(1/3)/((x**2 + 1)**(1/6)*x**2),x)
```

3.135 $\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 228

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^3}\,dx = -\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2} - \frac{i(1-ix)^{5/6}\sqrt[6]{1+ix}}{6x} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{6\sqrt{3}} + \frac{\arctan\left(\frac{1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{6\sqrt{3}} + \frac{1}{9}\text{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right) + \frac{1}{18}\text{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\left(1+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right)\sqrt[6]{1-ix}}\right)$$

output
```
-1/2*(1-I*x)^(5/6)*(1+I*x)^(7/6)/x^2-1/6*I*(1-I*x)^(5/6)*(1+I*x)^(1/6)/x-1
/18*3^(1/2)*arctan(1/3*(1-2*(1+I*x)^(1/6)/(1-I*x)^(1/6))*3^(1/2))+1/18*3^(
1/2)*arctan(1/3*(1+2*(1+I*x)^(1/6)/(1-I*x)^(1/6))*3^(1/2))+1/9*arctanh((1+
I*x)^(1/6)/(1-I*x)^(1/6))+1/18*arctanh((1+I*x)^(1/6)/(1+(1+I*x)^(1/3)/(1-I
*x)^(1/3))/(1-I*x)^(1/6))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.32

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^3}\,dx = \frac{(1-ix)^{5/6}\left(5(-3-7ix+4x^2)+2x^2\,\text{Hypergeometric2F1}\left(\frac{5}{6},1,\frac{11}{6},\frac{i+x}{i-x}\right)\right)}{30(1+ix)^{5/6}x^2}$$

input
```
Integrate[E^((I/3)*ArcTan[x])/x^3,x]
```

output
```
((1 - I*x)^(5/6)*(5*(-3 - (7*I)*x + 4*x^2) + 2*x^2*Hypergeometric2F1[5/6,
1, 11/6, (I + x)/(I - x)]))/(30*(1 + I*x)^(5/6)*x^2)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.29, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {5585, 107, 105, 104, 754, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^3}dx$$

$$\Big\downarrow \ 5585$$

$$\int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}x^3}dx$$

$$\Big\downarrow \ 107$$

$$\frac{1}{6}i\int \frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}x^2}dx-\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2}$$

$$\Big\downarrow \ 105$$

$$\frac{1}{6}i\left(\frac{1}{3}i\int \frac{1}{\sqrt[6]{1-ix}(ix+1)^{5/6}x}dx-\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}\right)-\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2}$$

$$\Big\downarrow 104$$

$$\frac{1}{6}i\left(2i\int \frac{1}{\frac{ix+1}{1-ix}-1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}\right)-\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2}$$

$$\Big\downarrow 754$$

$$\frac{1}{6}i\left(2i\left(-\frac{1}{3}\int \frac{1}{1-\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{3}\int \frac{2-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{2\left(\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1\right)}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{3}\int \frac{\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{2\left(\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}+\frac{\sqrt[6]{}}{\sqrt[6]{}}\right.}\right.\right.$$
$$\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2}$$

$$\Big\downarrow 27$$

$$\frac{1}{6}i\left(2i\left(-\frac{1}{3}\int \frac{1}{1-\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{6}\int \frac{2-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{6}\int \frac{\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+2}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}+\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-}\right.\right.$$
$$\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2}$$

$$\Big\downarrow 219$$

$$\frac{1}{6}i\left(2i\left(-\frac{1}{6}\int \frac{2-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{6}\int \frac{\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+2}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}+\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{3}\text{arctanh}\left(\frac{\sqrt[6]{1}}{\sqrt[6]{1}}\right.\right.\right.$$
$$\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2}$$

$$\Big\downarrow 1142$$

$$\frac{1}{6}i\left(2i\left(\frac{1}{6}\left(\frac{1}{2}\int -\frac{1-\frac{2\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{3}{2}\int \frac{1}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}\right)+\frac{1}{6}\left(-\frac{3}{2}\int\right.\right.\right.$$
$$\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2}$$

$$\Big\downarrow 25$$

$$\frac{\frac{1}{6}i\left(2i\left(\frac{1}{6}\left(-\frac{3}{2}\int \frac{1}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}\right)+\frac{1}{6}\left(-\frac{3}{2}\int\right.\right.\right.}{\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2}}$$

$$\Big\downarrow \; 1083$$

$$\frac{\frac{1}{6}i\left(2i\left(\frac{1}{6}\left(3\int \frac{1}{-\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-3}d\left(\frac{2\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-1\right)-\frac{1}{2}\int \frac{1-\frac{2\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}\right)+\frac{1}{6}\left(3\int \frac{}{-\frac{\sqrt[3]{}}{\sqrt[3]{}}}\right.\right.\right.}{\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2}}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{1}{6}i\left(2i\left(\frac{1}{6}\left(-\frac{1}{2}\int \frac{1-\frac{2\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\sqrt{3}\arctan\left(\frac{-1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(-\frac{1}{2}\int \frac{\frac{2\sqrt[6]{}}{\sqrt[6]{}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}}\right.\right.\right.}{\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2}}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{1}{6}i\left(2i\left(\frac{1}{6}\left(\frac{1}{2}\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)-\sqrt{3}\arctan\left(\frac{-1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[6]{1}}{\sqrt[6]{1}}}{\sqrt{3}}\right.\right.\right.\right.}{\frac{(1-ix)^{5/6}(1+ix)^{7/6}}{2x^2}}$$

input
```
Int[E^((I/3)*ArcTan[x])/x^3,x]
```

output
```
-1/2*((1 - I*x)^(5/6)*(1 + I*x)^(7/6))/x^2 + (I/6)*(-(((1 - I*x)^(5/6)*(1
+ I*x)^(1/6))/x) + (2*I)*(-1/3*ArcTanh[(1 + I*x)^(1/6)/(1 - I*x)^(1/6)] +
(-(Sqrt[3]*ArcTan[(-1 + (2*(1 + I*x)^(1/6))/(1 - I*x)^(1/6))/Sqrt[3]]) + L
og[1 - (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1 - I*x)^(1/3)]/
2)/6 + (-(Sqrt[3]*ArcTan[(1 + (2*(1 + I*x)^(1/6))/(1 - I*x)^(1/6))/Sqrt[3]
]) - Log[1 + (1 + I*x)^(1/6)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1 - I*x)^(
1/3)]/2)/6))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^3}\,dx$$

input
```
int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^3,x)
```

output
```
int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.03

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^3}\,dx = \frac{2\,x^2\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+1\right)-2\,x^2\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-1\right)+\left(i\sqrt{3}x^2+x^2\right)\log\left(\frac{1}{2}i\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+\frac{1}{2}\right)}{}$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^3,x, algorithm="fricas")
```

output
```
1/36*(2*x^2*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1) - 2*x^2*log((I*sqrt(x
^2 + 1)/(x + I))^(1/3) - 1) + (I*sqrt(3)*x^2 + x^2)*log(1/2*I*sqrt(3) + (I
*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/2) + (I*sqrt(3)*x^2 - x^2)*log(1/2*I*sqr
t(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1/2) + (-I*sqrt(3)*x^2 + x^2)*log
(-1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/2) + (-I*sqrt(3)*x^2
 - x^2)*log(-1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1/2) - 6*(4
*x^2 + I*x + 3)*(I*sqrt(x^2 + 1)/(x + I))^(1/3))/x^2
```

Sympy [F]

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^3}\,dx = \int \frac{\sqrt[3]{\frac{i(x-i)}{\sqrt{x^2+1}}}}{x^3}\,dx$$

input
```
integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3)/x**3,x)
```

output
```
Integral((I*(x - I)/sqrt(x**2 + 1))**(1/3)/x**3, x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^3}\,dx = \int \frac{\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^3}\,dx$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^3,x, algorithm="maxima")
```

output
```
integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x^3, x)
```

Giac [F]

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^3}\,dx = \int \frac{\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^3}\,dx$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^3,x, algorithm="giac")
```

output
```
integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^3}\,dx = \int \frac{\left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{1/3}}{x^3}\,dx$$

input
```
int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3)/x^3,x)
```

output
```
int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3)/x^3, x)
```

Reduce [F]

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^3}\,dx = \int \frac{(ix+1)^{\frac{1}{3}}}{(x^2+1)^{\frac{1}{6}}\,x^3}dx$$

input
```
int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^3,x)
```

output
```
int((i*x + 1)**(1/3)/((x**2 + 1)**(1/6)*x**3),x)
```

3.136 $\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 265

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^4}\,dx = -\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3}-\frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{18x^2}+\frac{11(1-ix)^{5/6}\sqrt[6]{1+ix}}{27x} - \frac{19i\arctan\left(\frac{1-\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{54\sqrt{3}}+\frac{19i\arctan\left(\frac{1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)}{54\sqrt{3}} + \frac{19}{81}i\operatorname{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}\right)+\frac{19}{162}i\operatorname{arctanh}\left(\frac{\sqrt[6]{1+ix}}{\left(1+\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}\right)\sqrt[6]{1-ix}}\right)$$

output
```
-1/3*(1-I*x)^(5/6)*(1+I*x)^(1/6)/x^3-7/18*I*(1-I*x)^(5/6)*(1+I*x)^(1/6)/x^
2+11/27*(1-I*x)^(5/6)*(1+I*x)^(1/6)/x-19/162*I*arctan(1/3*(1-2*(1+I*x)^(1/
6)/(1-I*x)^(1/6))*3^(1/2))*3^(1/2)+19/162*I*arctan(1/3*(1+2*(1+I*x)^(1/6)/
(1-I*x)^(1/6))*3^(1/2))*3^(1/2)+19/81*I*arctanh((1+I*x)^(1/6)/(1-I*x)^(1/6
))+19/162*I*arctanh((1+I*x)^(1/6)/(1+(1+I*x)^(1/3)/(1-I*x)^(1/3))/(1-I*x)^
(1/6))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.31

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^4}\,dx = \frac{(1-ix)^{5/6}\left(5(-18-39ix+43x^2+22ix^3)+38ix^3\,\text{Hypergeometric2F1}\left(\frac{5}{6},1,\frac{11}{6},\frac{i+x}{i-x}\right)\right)}{270(1+ix)^{5/6}x^3}$$

input
```
Integrate[E^((I/3)*ArcTan[x])/x^4,x]
```

output
```
((1 - I*x)^(5/6)*(5*(-18 - (39*I)*x + 43*x^2 + (22*I)*x^3) + (38*I)*x^3*Hy
pergeometric2F1[5/6, 1, 11/6, (I + x)/(I - x)]))/(270*(1 + I*x)^(5/6)*x^3)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.24, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$, Rules used = {5585, 110, 27, 168, 27, 168, 27, 104, 754, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^4}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}x^4}dx$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{3}\int \frac{7i-6x}{3\sqrt[6]{1-ix}(ix+1)^{5/6}x^3}dx-\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int \frac{7i-6x}{\sqrt[6]{1-ix}(ix+1)^{5/6}x^3}dx-\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3}$$

$$\Big\downarrow \ 168$$

$$\frac{1}{9}\left(-\frac{1}{2}\int \frac{21ix+22}{3\sqrt[6]{1-ix}(ix+1)^{5/6}x^2}dx-\frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{2x^2}\right)-\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{9}\left(-\frac{1}{6}\int \frac{21ix+22}{\sqrt[6]{1-ix}(ix+1)^{5/6}x^2}dx-\frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{2x^2}\right)-\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3}$$

$$\Big\downarrow \ 168$$

$$\frac{1}{9}\left(\frac{1}{6}\left(\int -\frac{19i}{3\sqrt[6]{1-ix}(ix+1)^{5/6}x}dx+\frac{22(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}\right)-\frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{2x^2}\right)-\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{9}\left(\frac{1}{6}\left(\frac{22(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}-\frac{19}{3}i\int \frac{1}{\sqrt[6]{1-ix}(ix+1)^{5/6}x}dx\right)-\frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{2x^2}\right)-\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3}$$

$$\Big\downarrow \ 104$$

$$\frac{1}{9}\left(\frac{1}{6}\left(\frac{22(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}-38i\int \frac{1}{\frac{ix+1}{1-ix}-1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}\right)-\frac{7i(1-ix)^{5/6}\sqrt[6]{1+ix}}{2x^2}\right)-\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3}$$

$$\Big\downarrow \ 754$$

$$\frac{1}{9}\left(\frac{1}{6}\left(\frac{22(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}-38i\left(-\frac{1}{3}\int \frac{1}{1-\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{3}\int \frac{2-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{2\left(\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1\right)}d\frac{\sqrt[6]{ix}}{\sqrt[6]{1-}}\right.\right.\right.$$
$$-\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{9}\left(\frac{1}{6}\left(\frac{22(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}-38i\left(-\frac{1}{3}\int\frac{1}{1-\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{6}\int\frac{2-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}\right.\right.\right.$$
$$\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3}$$

$\Big\downarrow$ 219

$$\frac{1}{9}\left(\frac{1}{6}\left(\frac{22(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}-38i\left(-\frac{1}{6}\int\frac{2-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{6}\int\frac{\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+2}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}+\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}\right.\right.\right.$$
$$\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3}$$

$\Big\downarrow$ 1142

$$\frac{1}{9}\left(\frac{1}{6}\left(\frac{22(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}-38i\left(\frac{1}{6}\left(\frac{1}{2}\int-\frac{1-\frac{2\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{3}{2}\int\frac{1}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}\right.\right.\right.\right.$$
$$\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3}$$

$\Big\downarrow$ 25

$$\frac{1}{9}\left(\frac{1}{6}\left(\frac{22(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}-38i\left(\frac{1}{6}\left(-\frac{3}{2}\int\frac{1}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\frac{1}{2}\int\frac{1-\frac{2\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}\right.\right.\right.\right.$$
$$\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3}$$

$\Big\downarrow$ 1083

$$\frac{1}{9}\left(\frac{1}{6}\left(\frac{22(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}-38i\left(\frac{1}{6}\left(3\int\frac{1}{-\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-3}d\left(\frac{2\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-1\right)-\frac{1}{2}\int\frac{1-\frac{2\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+}\right.\right.\right.\right.$$
$$\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3}$$

$\Big\downarrow$ 217

$$\frac{1}{9}\left(\frac{1}{6}\left(\frac{22(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}-38i\left(\frac{1}{6}\left(-\frac{1}{2}\int\frac{1-\frac{2\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}}{\frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}+1}d\frac{\sqrt[6]{ix+1}}{\sqrt[6]{1-ix}}-\sqrt{3}\arctan\left(\frac{-1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right.\right.\right.\right.\right.$$

$$\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{9}\left(\frac{1}{6}\left(\frac{22(1-ix)^{5/6}\sqrt[6]{1+ix}}{x}-38i\left(\frac{1}{6}\left(\frac{1}{2}\log\left(\frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}-\frac{\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}+1\right)-\sqrt{3}\arctan\left(\frac{-1+\frac{2\sqrt[6]{1+ix}}{\sqrt[6]{1-ix}}}{\sqrt{3}}\right)\right)\right.\right.\right.$$

$$\frac{(1-ix)^{5/6}\sqrt[6]{1+ix}}{3x^3}$$

input
```
Int[E^((I/3)*ArcTan[x])/x^4,x]
```

output
```
-1/3*((1 - I*x)^(5/6)*(1 + I*x)^(1/6))/x^3 + ((((-7*I)/2)*(1 - I*x)^(5/6)*
(1 + I*x)^(1/6))/x^2 + ((22*(1 - I*x)^(5/6)*(1 + I*x)^(1/6))/x - (38*I)*(-
1/3*ArcTanh[(1 + I*x)^(1/6)/(1 - I*x)^(1/6)] + (-(Sqrt[3]*ArcTan[(-1 + (2*
(1 + I*x)^(1/6))/(1 - I*x)^(1/6))/Sqrt[3]]) + Log[1 - (1 + I*x)^(1/6)/(1 -
 I*x)^(1/6) + (1 + I*x)^(1/3)/(1 - I*x)^(1/3)]/2)/6 + (-(Sqrt[3]*ArcTan[(1
 + (2*(1 + I*x)^(1/6))/(1 - I*x)^(1/6))/Sqrt[3]]) - Log[1 + (1 + I*x)^(1/6
)/(1 - I*x)^(1/6) + (1 + I*x)^(1/3)/(1 - I*x)^(1/3)]/2)/6))/6)/9
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^4}dx$$

input
```
int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^4,x)
```

output
```
int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.92

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^4}\,dx$$
$$= \frac{38i\,x^3\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+1\right)-38i\,x^3\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}-1\right)-19\left(\sqrt{3}x^3-i\,x^3\right)\log\left(\frac{1}{2}i\sqrt{3}+\left(\frac{i\sqrt{x^2+1}}{x+i}\right)\right.}{}$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^4,x, algorithm="fricas")
```

output
```
1/324*(38*I*x^3*log((I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1) - 38*I*x^3*log((I
*sqrt(x^2 + 1)/(x + I))^(1/3) - 1) - 19*(sqrt(3)*x^3 - I*x^3)*log(1/2*I*sq
rt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/2) - 19*(sqrt(3)*x^3 + I*x^3)*
log(1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) - 1/2) + 19*(sqrt(3)*x
^3 + I*x^3)*log(-1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1/3) + 1/2) +
19*(sqrt(3)*x^3 - I*x^3)*log(-1/2*I*sqrt(3) + (I*sqrt(x^2 + 1)/(x + I))^(1
/3) - 1/2) - 6*(22*I*x^3 - x^2 + 3*I*x + 18)*(I*sqrt(x^2 + 1)/(x + I))^(1/
3))/x^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate(((1+I*x)/(x**2+1)**(1/2))**(1/3)/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^4}\,dx = \int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^4}\,dx$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^4,x, algorithm="maxima")
```

output
```
integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x^4, x)
```

Giac [F]

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^4}\,dx = \int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{1}{3}}}{x^4}\,dx$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^4,x, algorithm="giac")
```

output
```
integrate(((I*x + 1)/sqrt(x^2 + 1))^(1/3)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^4}\,dx = \int \frac{\left(\frac{1+x\,1i}{\sqrt{x^2+1}}\right)^{1/3}}{x^4}\,dx$$

input
```
int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3)/x^4,x)
```

output
```
int(((x*1i + 1)/(x^2 + 1)^(1/2))^(1/3)/x^4, x)
```

Reduce [F]

$$\int \frac{e^{\frac{1}{3}i\arctan(x)}}{x^4}\,dx = \int \frac{(ix+1)^{\frac{1}{3}}}{(x^2+1)^{\frac{1}{6}}\,x^4}dx$$

input
```
int(((1+I*x)/(x^2+1)^(1/2))^(1/3)/x^4,x)
```

output
```
int((i*x + 1)**(1/3)/((x**2 + 1)**(1/6)*x**4),x)
```

3.137 $\int e^{\frac{2}{3}i\arctan(x)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 177

$$\int e^{\frac{2}{3}i\arctan(x)}x^2\,dx = -\frac{11}{27}i(1-ix)^{2/3}\sqrt[3]{1+ix}-\frac{1}{9}i(1-ix)^{2/3}(1+ix)^{4/3} +\frac{1}{3}(1-ix)^{2/3}(1+ix)^{4/3}x+\frac{22i\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{27\sqrt{3}}+\frac{11}{27}i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)+\frac{11}{81}i\log(1+ix)$$

output
```
-11/27*I*(1-I*x)^(2/3)*(1+I*x)^(1/3)-1/9*I*(1-I*x)^(2/3)*(1+I*x)^(4/3)+1/3
*(1-I*x)^(2/3)*(1+I*x)^(4/3)*x+22/81*I*arctan(1/3*3^(1/2)-2/3*(1-I*x)^(1/3
)*3^(1/2)/(1+I*x)^(1/3))*3^(1/2)+11/27*I*ln(1+(1-I*x)^(1/3)/(1+I*x)^(1/3))
+11/81*I*ln(1+I*x)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.41

$$\int e^{\frac{2}{3}i\arctan(x)}x^2\,dx = \frac{1}{18}(1-ix)^{2/3}\left(2\sqrt[3]{1+ix}(-i+4x+3ix^2) - 11i\sqrt[3]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{1}{2}-\frac{ix}{2}\right)\right)$$

input
```
Integrate[E^(((2*I)/3)*ArcTan[x])*x^2,x]
```

output
```
((1 - I*x)^(2/3)*(2*(1 + I*x)^(1/3)*(-I + 4*x + (3*I)*x^2) - (11*I)*2^(1/3
)*Hypergeometric2F1[-1/3, 2/3, 5/3, 1/2 - (I/2)*x]))/18
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5585, 101, 27, 90, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{\frac{2}{3} i \arctan(x)} \, dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{\sqrt[3]{1+ix}x^2}{\sqrt[3]{1-ix}}dx$$

$$\Big\downarrow \text{101}$$

$$\frac{1}{3}\int -\frac{\sqrt[3]{ix+1}(2ix+3)}{3\sqrt[3]{1-ix}}dx+\frac{1}{3}(1-ix)^{2/3}x(1+ix)^{4/3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}(1-ix)^{2/3}(1+ix)^{4/3}x-\frac{1}{9}\int \frac{\sqrt[3]{ix+1}(2ix+3)}{\sqrt[3]{1-ix}}dx$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{9}\left(-\frac{11}{3}\int \frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}dx-i(1-ix)^{2/3}(1+ix)^{4/3}\right)+\frac{1}{3}(1-ix)^{2/3}x(1+ix)^{4/3}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{9}\left(-\frac{11}{3}\left(\frac{2}{3}\int \frac{1}{\sqrt[3]{1-ix}(ix+1)^{2/3}}dx+i(1-ix)^{2/3}\sqrt[3]{1+ix}\right)-i(1-ix)^{2/3}(1+ix)^{4/3}\right)+\frac{1}{3}(1-ix)^{2/3}x(1+ix)^{4/3}$$

$\downarrow$ 72

$$\frac{1}{9}\left(-\frac{11}{3}\left(\frac{2}{3}\left(-i\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)-\frac{3}{2}i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)-\frac{1}{2}i\log(1+ix)\right)+i(1-ix)^{2/3}\sqrt[3]{1\cdots}\right.\right.$$
$$\frac{1}{3}(1-ix)^{2/3}x(1+ix)^{4/3}$$

input
```
Int[E^(((2*I)/3)*ArcTan[x])*x^2,x]
```

output
```
((1 - I*x)^(2/3)*(1 + I*x)^(4/3)*x)/3 + ((-I)*(1 - I*x)^(2/3)*(1 + I*x)^(4
/3) - (11*(I*(1 - I*x)^(2/3)*(1 + I*x)^(1/3) + (2*((-I)*Sqrt[3]*ArcTan[1/S
qrt[3] - (2*(1 - I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))] - ((3*I)/2)*Log[1
+ (1 - I*x)^(1/3)/(1 + I*x)^(1/3)] - (I/2)*Log[1 + I*x]))/3))/3)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}} x^2 dx$$

input
```
int(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x^2,x)
```

output
```
int(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.66

$$\int e^{\frac{2}{3}i\arctan(x)}x^2\,dx = -\frac{11}{81}\left(\sqrt{3}+i\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}+\frac{1}{2}i\sqrt{3}-\frac{1}{2}\right) + \frac{11}{81}\left(\sqrt{3}-i\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}-\frac{1}{2}i\sqrt{3}-\frac{1}{2}\right) + \frac{1}{27}\left(9\,x^3-3i\,x^2-2\,x-14i\right)\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}} + \frac{22}{81}i\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}+1\right)$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x^2,x, algorithm="fricas")
```

output
```
-11/81*(sqrt(3) + I)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1/2*I*sqrt(3) -
 1/2) + 11/81*(sqrt(3) - I)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) - 1/2*I*sq
rt(3) - 1/2) + 1/27*(9*x^3 - 3*I*x^2 - 2*x - 14*I)*(I*sqrt(x^2 + 1)/(x + I
))^(2/3) + 22/81*I*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{2}{3}i\arctan(x)}x^2\,dx = \text{Timed out}$$

input
```
integrate(((1+I*x)/(x**2+1)**(1/2))**(2/3)*x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\frac{2}{3}i\arctan(x)}x^2\,dx=\int x^2\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}\,dx$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x^2,x, algorithm="maxima")
```

output
```
integrate(x^2*((I*x + 1)/sqrt(x^2 + 1))^(2/3), x)
```

Giac [F]

$$\int e^{\frac{2}{3}i\arctan(x)}x^2\,dx=\int x^2\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}\,dx$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x^2,x, algorithm="giac")
```

output
```
integrate(x^2*((I*x + 1)/sqrt(x^2 + 1))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{2}{3}i\arctan(x)}x^2\,dx=\int x^2\left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{2/3}\,dx$$

input
```
int(x^2*((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3),x)
```

output
```
int(x^2*((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3), x)
```

Reduce [F]

$$\int e^{\frac{2}{3}i\arctan(x)}x^2\,dx = \int \frac{(ix+1)^{\frac{2}{3}}\,x^2}{(x^2+1)^{\frac{1}{3}}}dx$$

input
```
int(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x^2,x)
```

output
```
int(((i*x + 1)**(2/3)*x***2)/(x**2 + 1)**(1/3),x)
```

3.138 $\int e^{\frac{2}{3}i\arctan(x)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 140

$$\int e^{\frac{2}{3}i\arctan(x)}x\,dx=\frac{1}{3}(1-ix)^{2/3}\sqrt[3]{1+ix}+\frac{1}{2}(1-ix)^{2/3}(1+ix)^{4/3}-\frac{2\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{3\sqrt{3}}-\frac{1}{3}\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)-\frac{1}{9}\log(1+ix)$$

output
```
1/3*(1-I*x)^(2/3)*(1+I*x)^(1/3)+1/2*(1-I*x)^(2/3)*(1+I*x)^(4/3)-2/9*arctan
(1/3*3^(1/2)-2/3*(1-I*x)^(1/3)*3^(1/2)/(1+I*x)^(1/3))*3^(1/2)-1/3*ln(1+(1-
I*x)^(1/3)/(1+I*x)^(1/3))-1/9*ln(1+I*x)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.39

$$\int e^{\frac{2}{3}i\arctan(x)}x\,dx=\frac{1}{2}(1-ix)^{2/3}\left((1+ix)^{4/3}+\sqrt[3]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{2}{3},\frac{5}{3},\frac{1}{2}-\frac{ix}{2}\right)\right)$$

input
```
Integrate[E^(((2*I)/3)*ArcTan[x])*x,x]
```

output
```
((1 - I*x)^(2/3)*((1 + I*x)^(4/3) + 2^(1/3)*Hypergeometric2F1[-1/3, 2/3, 5
/3, 1/2 - (I/2)*x]))/2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5585, 90, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\frac{2}{3}i\arctan(x)}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{\sqrt[3]{1+ix}x}{\sqrt[3]{1-ix}}dx$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}(1-ix)^{2/3}(1+ix)^{4/3}-\frac{1}{3}i\int \frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}(1-ix)^{2/3}(1+ix)^{4/3}-\frac{1}{3}i\left(\frac{2}{3}\int\frac{1}{\sqrt[3]{1-ix}(ix+1)^{2/3}}dx+i(1-ix)^{2/3}\sqrt[3]{1+ix}\right)$$

$$\Big\downarrow \text{72}$$

$$\frac{1}{2}(1-ix)^{2/3}(1+ix)^{4/3}-$$
$$\frac{1}{3}i\left(\frac{2}{3}\left(-i\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)-\frac{3}{2}i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)-\frac{1}{2}i\log(1+ix)\right)+i(1-ix)^{2/3}\sqrt[3]{1+ix}\right)$$

input
```
Int[E^(((2*I)/3)*ArcTan[x])*x,x]
```

output
```
((1 - I*x)^(2/3)*(1 + I*x)^(4/3))/2 - (I/3)*(I*(1 - I*x)^(2/3)*(1 + I*x)^(
1/3) + (2*((-I)*Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(1 - I*x)^(1/3))/(Sqrt[3]*(1
 + I*x)^(1/3))] - ((3*I)/2)*Log[1 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3)] - (I/
2)*Log[1 + I*x]))/3)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}} x dx$$

input
```
int(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x,x)
```

output
```
int(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x,x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.83

$$\begin{aligned}\int e^{\frac{2}{3}i\arctan(x)}x\,dx = &-\frac{1}{9}\left(i\sqrt{3}-1\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}+\frac{1}{2}i\sqrt{3}-\frac{1}{2}\right)\\&-\frac{1}{9}\left(-i\sqrt{3}-1\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}-\frac{1}{2}i\sqrt{3}-\frac{1}{2}\right)\\&+\frac{1}{6}\left(3x^2-2i\,x+5\right)\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}-\frac{2}{9}\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}+1\right)\end{aligned}$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x,x, algorithm="fricas")
```

output
```
-1/9*(I*sqrt(3) - 1)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1/2*I*sqrt(3) -
 1/2) - 1/9*(-I*sqrt(3) - 1)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) - 1/2*I*s
qrt(3) - 1/2) + 1/6*(3*x^2 - 2*I*x + 5)*(I*sqrt(x^2 + 1)/(x + I))^(2/3) -
2/9*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{2}{3}i\arctan(x)}x\,dx = \text{Timed out}$$

input
```
integrate(((1+I*x)/(x**2+1)**(1/2))**(2/3)*x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\frac{2}{3}i\arctan(x)}x\,dx = \int x\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}\,dx$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x,x, algorithm="maxima")
```

output
```
integrate(x*((I*x + 1)/sqrt(x^2 + 1))^(2/3), x)
```

Giac [F]

$$\int e^{\frac{2}{3}i\arctan(x)}x\,dx = \int x\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}\,dx$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x,x, algorithm="giac")
```

output
```
integrate(x*((I*x + 1)/sqrt(x^2 + 1))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{2}{3}i\arctan(x)}x\,dx = \int x\left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{2/3}\,dx$$

input
```
int(x*((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3),x)
```

output
```
int(x*((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3), x)
```

Reduce [F]

$$\int e^{\frac{2}{3}i\arctan(x)}x\,dx = \int \frac{(ix+1)^{\frac{2}{3}}\,x}{(x^2+1)^{\frac{1}{3}}}dx$$

input
```
int(((1+I*x)/(x^2+1)^(1/2))^(2/3)*x,x)
```

output
```
int(((i*x + 1)**(2/3)*x)/(x**2 + 1)**(1/3),x)
```

3.139 $\int e^{\frac{2}{3}i\arctan(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 116

$$\int e^{\frac{2}{3}i\arctan(x)}\,dx = i(1-ix)^{2/3}\sqrt[3]{1+ix} - \frac{2i\arctan\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{\sqrt{3}} - i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right) - \frac{1}{3}i\log(1+ix)$$

output
```
I*(1-I*x)^(2/3)*(1+I*x)^(1/3)-2/3*I*arctan(1/3*3^(1/2)-2/3*(1-I*x)^(1/3)*3
^(1/2)/(1+I*x)^(1/3))*3^(1/2)-I*ln(1+(1-I*x)^(1/3)/(1+I*x)^(1/3))-1/3*I*ln
(1+I*x)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.29

$$\int e^{\frac{2}{3}i\arctan(x)}\,dx = -\frac{3}{2}ie^{\frac{8}{3}i\arctan(x)}\,\text{Hypergeometric2F1}\left(\frac{4}{3},2,\frac{7}{3},-e^{2i\arctan(x)}\right)$$

input
```
Integrate[E^(((2*I)/3)*ArcTan[x]),x]
```

output
```
((-3*I)/2)*E^(((8*I)/3)*ArcTan[x])*Hypergeometric2F1[4/3, 2, 7/3, -E^((2*I
)*ArcTan[x])]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5584, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{2}{3}i\arctan(x)}\,dx$$

$$\Big\downarrow \text{ 5584}$$

$$\int \frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{2}{3}\int \frac{1}{\sqrt[3]{1-ix}(ix+1)^{2/3}}dx+i(1-ix)^{2/3}\sqrt[3]{1+ix}$$

$$\Big\downarrow \text{ 72}$$

$$\frac{2}{3}\left(-i\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)-\frac{3}{2}i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)-\frac{1}{2}i\log(1+ix)\right)+i(1-ix)^{2/3}\sqrt[3]{1+ix}$$

input
```
Int[E^(((2*I)/3)*ArcTan[x]),x]
```

output
```
I*(1 - I*x)^(2/3)*(1 + I*x)^(1/3) + (2*((-I)*Sqrt[3]*ArcTan[1/Sqrt[3] - (2
*(1 - I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))] - ((3*I)/2)*Log[1 + (1 - I*x)
^(1/3)/(1 + I*x)^(1/3)] - (I/2)*Log[1 + I*x]))/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

rule 5584
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Maple [F]

$$\int \left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}} dx$$

input
```
int(((1+I*x)/(x^2+1)^(1/2))^(2/3),x)
```

output
```
int(((1+I*x)/(x^2+1)^(1/2))^(2/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int e^{\frac{2}{3}i\arctan(x)}\,dx = \frac{1}{3}\left(\sqrt{3}+i\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}+\frac{1}{2}i\sqrt{3}-\frac{1}{2}\right) - \frac{1}{3}\left(\sqrt{3}-i\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}-\frac{1}{2}i\sqrt{3}-\frac{1}{2}\right) + (x+i)\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}} - \frac{2}{3}i\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}+1\right)$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3),x, algorithm="fricas")
```

output
```
1/3*(sqrt(3) + I)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1/2*I*sqrt(3) - 1/
2) - 1/3*(sqrt(3) - I)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) - 1/2*I*sqrt(3)
 - 1/2) + (x + I)*(I*sqrt(x^2 + 1)/(x + I))^(2/3) - 2/3*I*log((I*sqrt(x^2
+ 1)/(x + I))^(2/3) + 1)
```

Sympy [F]

$$\int e^{\frac{2}{3}i\arctan(x)}\,dx = \int\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}\,dx$$

input
```
integrate(((1+I*x)/(x**2+1)**(1/2))**(2/3),x)
```

output
```
Integral(((I*x + 1)/sqrt(x**2 + 1))**(2/3), x)
```

Maxima [F]

$$\int e^{\frac{2}{3}i\arctan(x)}\,dx=\int\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}\,dx$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3),x, algorithm="maxima")
```

output
```
integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3), x)
```

Giac [F]

$$\int e^{\frac{2}{3}i\arctan(x)}\,dx=\int\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}\,dx$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3),x, algorithm="giac")
```

output
```
integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{2}{3}i\arctan(x)}\,dx=\int\left(\frac{1+x\,1\mathrm{i}}{\sqrt{x^2+1}}\right)^{2/3}\,dx$$

input
```
int(((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3),x)
```

output
```
int(((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3), x)
```

Reduce [F]

$$\int e^{\frac{2}{3}i\arctan(x)}\,dx = \int \frac{(ix+1)^{\frac{2}{3}}}{(x^2+1)^{\frac{1}{3}}}dx$$

input
```
int(((1+I*x)/(x^2+1)^(1/2))^(2/3),x)
```

output
```
int((i*x + 1)**(2/3)/(x**2 + 1)**(1/3),x)
```

3.140 $\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 163

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x}\,dx = \sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)+\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)+\frac{3}{2}\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)+\frac{3}{2}\log\left(\sqrt[3]{1-ix}-\sqrt[3]{1+ix}\right)+\frac{1}{2}\log(1+ix)-\frac{\log(x)}{2}$$

output
```
arctan(1/3*3^(1/2)-2/3*(1-I*x)^(1/3)*3^(1/2)/(1+I*x)^(1/3))*3^(1/2)+3^(1/2
)*arctan(1/3*3^(1/2)+2/3*(1-I*x)^(1/3)*3^(1/2)/(1+I*x)^(1/3))+3/2*ln(1+(1-
I*x)^(1/3)/(1+I*x)^(1/3))+3/2*ln((1-I*x)^(1/3)-(1+I*x)^(1/3))+1/2*ln(1+I*x
)-1/2*ln(x)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.55

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x}\,dx =$$
$$-\frac{3(1-ix)^{2/3}\left(\sqrt[3]{2}(1+ix)^{2/3}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{1}{2}-\frac{ix}{2}\right)+2\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},\frac{i+x}{i-x}\right.}{4(1+ix)^{2/3}}$$

input
```
Integrate[E^(((2*I)/3)*ArcTan[x])/x,x]
```

output
```
(-3*(1 - I*x)^(2/3)*(2^(1/3)*(1 + I*x)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5
/3, 1/2 - (I/2)*x] + 2*Hypergeometric2F1[2/3, 1, 5/3, (I + x)/(I - x)]))/(
4*(1 + I*x)^(2/3))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5585, 140, 72, 102}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x}dx$$
$$\Big\downarrow \; 5585$$
$$\int \frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}x}dx$$
$$\Big\downarrow \; 140$$
$$i\int \frac{1}{\sqrt[3]{1-ix}(ix+1)^{2/3}}dx+\int \frac{1}{\sqrt[3]{1-ix}(ix+1)^{2/3}x}dx$$
$$\Big\downarrow \; 72$$

$$\int \frac{1}{\sqrt[3]{1-ix}(ix+1)^{2/3}x}dx+$$
$$i\left(-i\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)-\frac{3}{2}i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)-\frac{1}{2}i\log(1+ix)\right)$$

$$\Big\downarrow \ 102$$

$$\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)+$$
$$i\left(-i\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)-\frac{3}{2}i\log\left(1+\frac{\sqrt[3]{1-ix}}{\sqrt[3]{1+ix}}\right)-\frac{1}{2}i\log(1+ix)\right)+$$
$$\frac{3}{2}\log\left(\sqrt[3]{1-ix}-\sqrt[3]{1+ix}\right)-\frac{\log(x)}{2}$$

input
```
Int[E^(((2*I)/3)*ArcTan[x])/x,x]
```

output
```
Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(1 - I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))]
+ (3*Log[(1 - I*x)^(1/3) - (1 + I*x)^(1/3)])/2 + I*((-I)*Sqrt[3]*ArcTan[1/
Sqrt[3] - (2*(1 - I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))] - ((3*I)/2)*Log[1
 + (1 - I*x)^(1/3)/(1 + I*x)^(1/3)] - (I/2)*Log[1 + I*x]) - Log[x]/2
```

Defintions of rubi rules used

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

rule 102
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, Simp[(-Sqrt[3])*
q*(ArcTan[1/Sqrt[3] + 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))]/(d*e
 - c*f)), x] + (Simp[q*(Log[e + f*x]/(2*(d*e - c*f))), x] - Simp[3*q*(Log[q
*(a + b*x)^(1/3) - (c + d*x)^(1/3)]/(2*(d*e - c*f))), x])] /; FreeQ[{a, b,
c, d, e, f}, x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x}dx$$

input
```
int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x,x)
```

output
```
int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.89

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x}\,dx = \frac{1}{2}\left(i\sqrt{3}-1\right)\log\left(\frac{\sqrt{3}(ix-1)+x+2i\sqrt{x^2+1}\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+i}{2(x+i)}\right) + \frac{1}{2}\left(-i\sqrt{3}-1\right)\log\left(\frac{\sqrt{3}(-ix+1)+x+2i\sqrt{x^2+1}\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+i}{2(x+i)}\right) + \log\left(-\frac{x-i\sqrt{x^2+1}\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{1}{3}}+i}{x+i}\right)$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x,x, algorithm="fricas")
```

output
```
1/2*(I*sqrt(3) - 1)*log(1/2*(sqrt(3)*(I*x - 1) + x + 2*I*sqrt(x^2 + 1)*(I*
sqrt(x^2 + 1)/(x + I))^(1/3) + I)/(x + I)) + 1/2*(-I*sqrt(3) - 1)*log(1/2*
(sqrt(3)*(-I*x + 1) + x + 2*I*sqrt(x^2 + 1)*(I*sqrt(x^2 + 1)/(x + I))^(1/3
) + I)/(x + I)) + log(-(x - I*sqrt(x^2 + 1)*(I*sqrt(x^2 + 1)/(x + I))^(1/3
) + I)/(x + I))
```

Sympy [F]

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x}\,dx = \int \frac{\left(\frac{i(x-i)}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x}\,dx$$

input
```
integrate(((1+I*x)/(x**2+1)**(1/2))**(2/3)/x,x)
```

output
```
Integral((I*(x - I)/sqrt(x**2 + 1))**(2/3)/x, x)
```

Maxima [F]

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x}\,dx = \int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x}\,dx$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x,x, algorithm="maxima")
```

output
```
integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3)/x, x)
```

Giac [F]

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x}\,dx = \int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x}\,dx$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x,x, algorithm="giac")
```

output
```
integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x}\,dx = \int \frac{\left(\frac{1+x\,1i}{\sqrt{x^2+1}}\right)^{2/3}}{x}\,dx$$

input
```
int(((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3)/x,x)
```

output
```
int(((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3)/x, x)
```

Reduce [F]

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x}\,dx = \int \frac{(ix+1)^{\frac{2}{3}}}{(x^2+1)^{\frac{1}{3}}\,x}dx$$

input
```
int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x,x)
```

output
```
int((i*x + 1)**(2/3)/((x**2 + 1)**(1/3)*x),x)
```

3.141 $\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 111

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^2}\,dx = -\frac{(1-ix)^{2/3}\sqrt[3]{1+ix}}{x} + \frac{2i\arctan\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{\sqrt{3}} + i\log\left(\sqrt[3]{1-ix} - \sqrt[3]{1+ix}\right) - \frac{1}{3}i\log(x)$$

output
```
-(1-I*x)^(2/3)*(1+I*x)^(1/3)/x+2/3*I*arctan(1/3*3^(1/2)+2/3*(1-I*x)^(1/3)*
3^(1/2)/(1+I*x)^(1/3))*3^(1/2)+I*ln((1-I*x)^(1/3)-(1+I*x)^(1/3))-1/3*I*ln(
x)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.53

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^2}\,dx = -\frac{i(1-ix)^{2/3}\left(-i+x+x\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},\frac{i+x}{i-x}\right)\right)}{(1+ix)^{2/3}x}$$

input
```
Integrate[E^(((2*I)/3)*ArcTan[x])/x^2,x]
```

output
```
((-I)*(1 - I*x)^(2/3)*(-I + x + x*Hypergeometric2F1[2/3, 1, 5/3, (I + x)/(
I - x)]))/((1 + I*x)^(2/3)*x)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5585, 105, 102}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^2}dx$$
$$\Big\downarrow \text{5585}$$
$$\int \frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}x^2}dx$$
$$\Big\downarrow \text{105}$$
$$\frac{2}{3}i\int \frac{1}{\sqrt[3]{1-ix}(ix+1)^{2/3}x}dx-\frac{(1-ix)^{2/3}\sqrt[3]{1+ix}}{x}$$
$$\Big\downarrow \text{102}$$
$$\frac{2}{3}i\left(\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)+\frac{3}{2}\log\left(\sqrt[3]{1-ix}-\sqrt[3]{1+ix}\right)-\frac{\log(x)}{2}\right)-\frac{(1-ix)^{2/3}\sqrt[3]{1+ix}}{x}$$

input
```
Int[E^(((2*I)/3)*ArcTan[x])/x^2,x]
```

output
```
-(((1 - I*x)^(2/3)*(1 + I*x)^(1/3))/x) + ((2*I)/3)*(Sqrt[3]*ArcTan[1/Sqrt[
3] + (2*(1 - I*x)^(1/3))/(Sqrt[3]*(1 + I*x)^(1/3))] + (3*Log[(1 - I*x)^(1/
3) - (1 + I*x)^(1/3)])/2 - Log[x]/2)
```

Defintions of rubi rules used

rule 102
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, Simp[(-Sqrt[3])*
q*(ArcTan[1/Sqrt[3] + 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))]/(d*e
 - c*f)), x] + (Simp[q*(Log[e + f*x]/(2*(d*e - c*f))), x] - Simp[3*q*(Log[q
*(a + b*x)^(1/3) - (c + d*x)^(1/3)]/(2*(d*e - c*f))), x])] /; FreeQ[{a, b,
c, d, e, f}, x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x^2}dx$$

input
```
int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^2,x)
```

output
```
int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.08

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^2}\,dx = \frac{\left(\sqrt{3}x - i\,x\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}} + \frac{1}{2}i\sqrt{3} + \frac{1}{2}\right) - \left(\sqrt{3}x + i\,x\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}} - \frac{1}{2}i\sqrt{3} + \frac{1}{2}\right) + 2i\,x\log\left(\left(\right.\right.}{3\,x}$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^2,x, algorithm="fricas")
```

output
```
1/3*((sqrt(3)*x - I*x)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1/2*I*sqrt(3)
 + 1/2) - (sqrt(3)*x + I*x)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) - 1/2*I*sq
rt(3) + 1/2) + 2*I*x*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) - 1) - 3*(-I*x +
1)*(I*sqrt(x^2 + 1)/(x + I))^(2/3))/x
```

Sympy [F]

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^2}\,dx = \int \frac{\left(\frac{i(x-i)}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x^2}\,dx$$

input
```
integrate(((1+I*x)/(x**2+1)**(1/2))**(2/3)/x**2,x)
```

output
```
Integral((I*(x - I)/sqrt(x**2 + 1))**(2/3)/x**2, x)
```

Maxima [F]

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^2}\,dx = \int \frac{\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x^2}\,dx$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^2,x, algorithm="maxima")
```

output
```
integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3)/x^2, x)
```

Giac [F]

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^2}\,dx = \int \frac{\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x^2}\,dx$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^2,x, algorithm="giac")
```

output
```
integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^2}\,dx = \int \frac{\left(\frac{1+x\,1i}{\sqrt{x^2+1}}\right)^{2/3}}{x^2}\,dx$$

input
```
int(((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3)/x^2,x)
```

output
```
int(((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3)/x^2, x)
```

Reduce [F]

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^2}\,dx = \int \frac{(ix+1)^{\frac{2}{3}}}{(x^2+1)^{\frac{1}{3}}\,x^2}dx$$

input
```
int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^2,x)
```

output
```
int((i*x + 1)**(2/3)/((x**2 + 1)**(1/3)*x**2),x)
```

3.142 $\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 142

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^3}\,dx = -\frac{(1-ix)^{2/3}(1+ix)^{4/3}}{2x^2} - \frac{i(1-ix)^{2/3}\sqrt[3]{1+ix}}{3x} - \frac{2\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)}{3\sqrt{3}} - \frac{1}{3}\log\left(\sqrt[3]{1-ix}-\sqrt[3]{1+ix}\right) + \frac{\log(x)}{9}$$

output
```
-1/2*(1-I*x)^(2/3)*(1+I*x)^(4/3)/x^2-1/3*I*(1-I*x)^(2/3)*(1+I*x)^(1/3)/x-2
/9*3^(1/2)*arctan(1/3*3^(1/2)+2/3*(1-I*x)^(1/3)*3^(1/2)/(1+I*x)^(1/3))-1/3
*ln((1-I*x)^(1/3)-(1+I*x)^(1/3))+1/9*ln(x)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.49

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^3}\,dx=\frac{(1-ix)^{2/3}\left(-3-8ix+5x^2+2x^2\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},\frac{i+x}{i-x}\right)\right)}{6(1+ix)^{2/3}x^2}$$

input
```
Integrate[E^(((2*I)/3)*ArcTan[x])/x^3,x]
```

output
```
((1 - I*x)^(2/3)*(-3 - (8*I)*x + 5*x^2 + 2*x^2*Hypergeometric2F1[2/3, 1, 5
/3, (I + x)/(I - x)]))/(6*(1 + I*x)^(2/3)*x^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5585, 107, 105, 102}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^3}dx$$
$$\Big\downarrow \text{ 5585}$$
$$\int \frac{\sqrt[3]{1+ix}}{\sqrt[3]{1-ix}x^3}dx$$
$$\Big\downarrow \text{ 107}$$
$$\frac{1}{3}i\int \frac{\sqrt[3]{ix+1}}{\sqrt[3]{1-ix}x^2}dx-\frac{(1-ix)^{2/3}(1+ix)^{4/3}}{2x^2}$$
$$\Big\downarrow \text{ 105}$$
$$\frac{1}{3}i\left(\frac{2}{3}i\int \frac{1}{\sqrt[3]{1-ix}(ix+1)^{2/3}x}dx-\frac{(1-ix)^{2/3}\sqrt[3]{1+ix}}{x}\right)-\frac{(1-ix)^{2/3}(1+ix)^{4/3}}{2x^2}$$

$$\Big\downarrow \text{102}$$

$$\frac{1}{3}i\left(\frac{2}{3}i\left(\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1-ix}}{\sqrt{3}\sqrt[3]{1+ix}}\right)+\frac{3}{2}\log\left(\sqrt[3]{1-ix}-\sqrt[3]{1+ix}\right)-\frac{\log(x)}{2}\right)-\frac{(1-ix)^{2/3}\sqrt[3]{1+ix}}{x}\right)-\frac{(1-ix)^{2/3}(1+ix)^{4/3}}{2x^2}$$

input
```
Int[E^(((2*I)/3)*ArcTan[x])/x^3,x]
```

output
```
-1/2*((1 - I*x)^(2/3)*(1 + I*x)^(4/3))/x^2 + (I/3)*(-(((1 - I*x)^(2/3)*(1
+ I*x)^(1/3))/x) + ((2*I)/3)*(Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(1 - I*x)^(1/3
))/(Sqrt[3]*(1 + I*x)^(1/3))] + (3*Log[(1 - I*x)^(1/3) - (1 + I*x)^(1/3)])
/2 - Log[x]/2))
```

Defintions of rubi rules used

rule 102
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, Simp[(-Sqrt[3])*
q*(ArcTan[1/Sqrt[3] + 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))]/(d*e
 - c*f)), x] + (Simp[q*(Log[e + f*x]/(2*(d*e - c*f))), x] - Simp[3*q*(Log[q
*(a + b*x)^(1/3) - (c + d*x)^(1/3)]/(2*(d*e - c*f))), x])] /; FreeQ[{a, b,
c, d, e, f}, x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\left(\frac{ix+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x^3}dx$$

input
```
int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^3,x)
```

output
```
int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^3}\,dx = -\frac{4\,x^2\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}-1\right)+2\left(-i\sqrt{3}x^2-x^2\right)\log\left(\left(\frac{i\sqrt{x^2+1}}{x+i}\right)^{\frac{2}{3}}+\frac{1}{2}i\sqrt{3}+\frac{1}{2}\right)+2\left(i\sqrt{3}x^2-x^2\right)\log}{18\,x^2}$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^3,x, algorithm="fricas")
```

output
```
-1/18*(4*x^2*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) - 1) + 2*(-I*sqrt(3)*x^2
- x^2)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) + 1/2*I*sqrt(3) + 1/2) + 2*(I*s
qrt(3)*x^2 - x^2)*log((I*sqrt(x^2 + 1)/(x + I))^(2/3) - 1/2*I*sqrt(3) + 1/
2) + 3*(5*x^2 + 2*I*x + 3)*(I*sqrt(x^2 + 1)/(x + I))^(2/3))/x^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate(((1+I*x)/(x**2+1)**(1/2))**(2/3)/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^3}\,dx = \int \frac{\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x^3}\,dx$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^3,x, algorithm="maxima")
```

output
```
integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3)/x^3, x)
```

Giac [F]

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^3}\,dx = \int \frac{\left(\frac{i\,x+1}{\sqrt{x^2+1}}\right)^{\frac{2}{3}}}{x^3}\,dx$$

input
```
integrate(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^3,x, algorithm="giac")
```

output
```
integrate(((I*x + 1)/sqrt(x^2 + 1))^(2/3)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^3}\,dx = \int \frac{\left(\frac{1+x\,1i}{\sqrt{x^2+1}}\right)^{2/3}}{x^3}\,dx$$

input
```
int(((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3)/x^3,x)
```

output
```
int(((x*1i + 1)/(x^2 + 1)^(1/2))^(2/3)/x^3, x)
```

Reduce [F]

$$\int \frac{e^{\frac{2}{3}i\arctan(x)}}{x^3}\,dx = \int \frac{(ix+1)^{\frac{2}{3}}}{(x^2+1)^{\frac{1}{3}}\,x^3}dx$$

input
```
int(((1+I*x)/(x^2+1)^(1/2))^(2/3)/x^3,x)
```

output
```
int((i*x + 1)**(2/3)/((x**2 + 1)**(1/3)*x**3),x)
```

3.143 $\int e^{\frac{1}{4}i\arctan(ax)}x^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 571

$$
\begin{aligned}
\int e^{\frac{1}{4}i\arctan(ax)}x^2\,dx = & -\frac{11i(1-iax)^{7/8}\sqrt[8]{1+iax}}{32a^3}-\frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{24a^3} \\
& +\frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2} \\
& +\frac{11i\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{128a^3} \\
& +\frac{11i\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)}{128a^3} \\
& -\frac{11i\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{128a^3} \\
& -\frac{11i\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)}{128a^3} \\
& +\frac{11i\sqrt{2-\sqrt{2}}\,\text{arctanh}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}\right)}{128a^3} \\
& +\frac{11i\sqrt{2+\sqrt{2}}\,\text{arctanh}\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}\right)}{128a^3}
\end{aligned}
$$

output
```
-11/32*I*(1-I*a*x)^(7/8)*(1+I*a*x)^(1/8)/a^3-1/24*I*(1-I*a*x)^(7/8)*(1+I*a
*x)^(9/8)/a^3+1/3*x*(1-I*a*x)^(7/8)*(1+I*a*x)^(9/8)/a^2+11/128*I*(2+2^(1/2
))^(1/2)*arctan(((2-2^(1/2))^(1/2)-2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8))/(2+2
^(1/2))^(1/2))/a^3+11/128*I*(2-2^(1/2))^(1/2)*arctan(((2+2^(1/2))^(1/2)-2*
(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8))/(2-2^(1/2))^(1/2))/a^3-11/128*I*(2+2^(1/2
))^(1/2)*arctan(((2-2^(1/2))^(1/2)+2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8))/(2+2
^(1/2))^(1/2))/a^3-11/128*I*(2-2^(1/2))^(1/2)*arctan(((2+2^(1/2))^(1/2)+2*
(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8))/(2-2^(1/2))^(1/2))/a^3+11/128*I*(2-2^(1/2
))^(1/2)*arctanh((2-2^(1/2))^(1/2)*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)/(1+(1-I
*a*x)^(1/4)/(1+I*a*x)^(1/4)))/a^3+11/128*I*(2+2^(1/2))^(1/2)*arctanh((2+2^
(1/2))^(1/2)*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)/(1+(1-I*a*x)^(1/4)/(1+I*a*x)^
(1/4)))/a^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.15

$$\int e^{\frac{1}{4}i\arctan(ax)}x^2\,dx = \frac{(1-iax)^{7/8}\left(7\sqrt[8]{1+iax}(-i+9ax+8ia^2x^2)-66i\sqrt[8]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{7}{8},\frac{15}{8},\frac{1}{2}(1-iax)\right)\right)}{168a^3}$$

input
```
Integrate[E^((I/4)*ArcTan[a*x])*x^2,x]
```

output
```
((1 - I*a*x)^(7/8)*(7*(1 + I*a*x)^(1/8)*(-I + 9*a*x + (8*I)*a^2*x^2) - (66
*I)*2^(1/8)*Hypergeometric2F1[-1/8, 7/8, 15/8, (1 - I*a*x)/2]))/(168*a^3)
```

Rubi [A] (warning: unable to verify)

Time = 1.40 (sec) , antiderivative size = 734, normalized size of antiderivative = 1.29, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.938$, Rules used = {5585, 101, 27, 90, 60, 73, 854, 828, 1442, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{\frac{1}{4} i \arctan (a x)} \, dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{x^2 \sqrt[8]{1+i a x}}{\sqrt[8]{1-i a x}}dx$$

$$\Big\downarrow \text{101}$$

$$\frac{\int -\frac{\sqrt[8]{i a x+1} (i a x+4)}{4 \sqrt[8]{1-i a x}}dx}{3 a^2}+\frac{x (1-i a x)^{7/8} (1+i a x)^{9/8}}{3 a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x (1-i a x)^{7/8} (1+i a x)^{9/8}}{3 a^2}-\frac{\int \frac{\sqrt[8]{i a x+1} (i a x+4)}{\sqrt[8]{1-i a x}}dx}{12 a^2}$$

$$\Big\downarrow \text{90}$$

$$\frac{x (1-i a x)^{7/8} (1+i a x)^{9/8}}{3 a^2}-\frac{\frac{33}{8} \int \frac{\sqrt[8]{i a x+1}}{\sqrt[8]{1-i a x}}dx+\frac{i (1-i a x)^{7/8} (1+i a x)^{9/8}}{2 a}}{12 a^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{x (1-i a x)^{7/8} (1+i a x)^{9/8}}{3 a^2}-\frac{\frac{33}{8} \left(\frac{1}{4} \int \frac{1}{\sqrt[8]{1-i a x} (i a x+1)^{7/8}}dx+\frac{i (1-i a x)^{7/8} \sqrt[8]{1+i a x}}{a}\right)+\frac{i (1-i a x)^{7/8} (1+i a x)^{9/8}}{2 a}}{12 a^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2}-\frac{\frac{33}{8}\left(\frac{2i\int\frac{(1-iax)^{3/4}}{(iax+1)^{7/8}}d\sqrt[8]{1-iax}}{a}+\frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a}\right)+\frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{2a}}{12a^2}$$

$\downarrow$ 854

$$\frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2}-\frac{\frac{33}{8}\left(\frac{2i\int\frac{(1-iax)^{3/4}}{2-iax}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{a}+\frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a}\right)+\frac{i(1-iax)^{7/8}(1+iax)^{9/8}}{2a}}{12a^2}$$

$\downarrow$ 828

$$\frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2}-\frac{\frac{33}{8}\left(\frac{2i\left(\frac{\int\frac{\sqrt{1-iax}}{\sqrt{1-iax}-\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{1-iax}}{\sqrt{1-iax}+\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}\right)}{a}+\frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a}\right)+\frac{i(1-ia}{}}{12a^2}$$

$\downarrow$ 1442

$$\frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2}-\frac{\frac{33}{8}\left(\frac{2i\left(\frac{\frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}-\int\frac{1-\sqrt{2}\sqrt[4]{1-iax}}{\sqrt{1-iax}-\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}-\int\frac{\sqrt{2}\sqrt[4]{1-iax}+1}{\sqrt{1-iax}+\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}\right)}{a}+\frac{i(1-iax)}{}\right.}{12a^2}$$

$\downarrow$ 1483

$$\frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2}-\frac{33}{8}\left(\frac{2i\left(\frac{-\frac{\int \frac{\sqrt{2+\sqrt{2}}-\frac{\left(1+\sqrt{2}\right)\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2+\sqrt{2}}}-\frac{\int \frac{\frac{\left(1+\sqrt{2}\right)\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+\sqrt{2+\sqrt{2}}}{\sqrt[4]{1-iax}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2+\sqrt{2}}}+\frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{2\sqrt{2}}-\cdots\right)}{a}\cdots\right)$$

$\downarrow$ 1142

$$\frac{x(1-iax)^{7/8}(iax+1)^{9/8}}{3a^2}-\frac{i(1-iax)^{7/8}(iax+1)^{9/8}}{2a}+\frac{33}{8}\left(\frac{2i\left(-\frac{-\frac{1}{2}\sqrt{2-\sqrt{2}}\int \frac{1}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}-\frac{1}{2}\left(1+\sqrt{2}\right)\int -\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\cdots}}{\sqrt[8]{iax\cdots}}}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{\cdots}}{\sqrt[8]{iax\cdots}}}\cdots}{2\sqrt{2+\sqrt{2}}}\cdots\right)}{\cdots}\cdots\right)$$

$\big\downarrow$ 25

$$\frac{i(1-iax)^{7/8}(iax+1)^{9/8}}{2a}+\frac{33}{8}\left(\frac{x(1-iax)^{7/8}(iax+1)^{9/8}}{3a^2}-\frac{2i\left(-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}-\frac{1}{2}\sqrt{2-\sqrt{2}}\int\frac{1}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\cdots}}{\sqrt[8]{iax+\cdots}}}\cdots}{2\sqrt{2+\sqrt{2}}}\cdots\right)}{\cdots}\cdots\right)$$

$\big\downarrow$ 1083

$$\frac{i(1-iax)^{7/8}(iax+1)^{9/8}}{2a}+\frac{33}{8}\left(\frac{x(1-iax)^{7/8}(iax+1)^{9/8}}{3a^2}-\frac{2i\left(-\frac{\sqrt{2-\sqrt{2}}\int\frac{1}{-\sqrt[4]{1-iax}+\sqrt{2}-2}d\left(\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}-\sqrt{2+\sqrt{2}}\right)+\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-ia\cdots}}{\sqrt[8]{iax+1}}\cdots}\cdots}{2\sqrt{2+\sqrt{2}}}\cdots\right)}{\cdots}\cdots\right)$$

$$\Big\downarrow \; 217$$

$$\frac{i(1-iax)^{7/8}(iax+1)^{9/8}}{2a}+\frac{33}{8}\left(\frac{2i\left(-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}-\arctan\left(\frac{\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1\cdots\right.}{\cdots}\right)}{2\cdots}\right.$$

$$\frac{x(1-iax)^{7/8}(iax+1)^{9/8}}{3a^2}-$$

$$\Big\downarrow \; 1103$$

$$\frac{33}{8}\left(\frac{2i\left(-\frac{-\arctan\left(\frac{-\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{2}\left(1+\sqrt{2}\right)\log\left(\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}+1\right)}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\log\left(\sqrt[4]{1-iax}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1\cdots}}{\sqrt[8]{1+\cdots}}\right.}{\cdots}\right)}{2\sqrt{2}}\right.$$

$$\frac{x(1-iax)^{7/8}(1+iax)^{9/8}}{3a^2}-$$

input
```
Int[E^((I/4)*ArcTan[a*x])*x^2,x]
```

output
```
(x*(1 - I*a*x)^(7/8)*(1 + I*a*x)^(9/8))/(3*a^2) - (((I/2)*(1 - I*a*x)^(7/8
)*(1 + I*a*x)^(9/8))/a + (33*((I*(1 - I*a*x)^(7/8)*(1 + I*a*x)^(1/8))/a +
((2*I)*(-1/2*((1 - I*a*x)^(1/8)/(1 + I*a*x)^(1/8) - (ArcTan[(-Sqrt[2 - Sqr
t[2]] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqrt[2]]] - ((1
- Sqrt[2])*Log[1 + (1 - I*a*x)^(1/4) - (Sqrt[2 - Sqrt[2]]*(1 - I*a*x)^(1/8
))/(1 + I*a*x)^(1/8)])/2)/(2*Sqrt[2 - Sqrt[2]]) - (ArcTan[(Sqrt[2 - Sqrt[2
]] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqrt[2]]] + ((1 - S
qrt[2])*Log[1 + (1 - I*a*x)^(1/4) + (Sqrt[2 - Sqrt[2]]*(1 - I*a*x)^(1/8))/
(1 + I*a*x)^(1/8)])/2)/(2*Sqrt[2 - Sqrt[2]]))/Sqrt[2] + ((1 - I*a*x)^(1/8)
/(1 + I*a*x)^(1/8) - (-ArcTan[(-Sqrt[2 + Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/
(1 + I*a*x)^(1/8))/Sqrt[2 - Sqrt[2]]] - ((1 + Sqrt[2])*Log[1 + (1 - I*a*x)
^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/2)/(2*S
qrt[2 + Sqrt[2]]) - (-ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1
 + I*a*x)^(1/8))/Sqrt[2 - Sqrt[2]]] + ((1 + Sqrt[2])*Log[1 + (1 - I*a*x)^(
1/4) + (Sqrt[2 + Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/2)/(2*Sqr
t[2 + Sqrt[2]]))/(2*Sqrt[2])))/a))/8)/(12*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 828
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Simp[s^3/(2*Sqrt[2]*b*r)   Int[x^
(m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Simp[s^3/(2*S
qrt[2]*b*r)   Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] &&
 GtQ[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}} x^2 dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^2,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 435, normalized size of antiderivative = 0.76

$$\int e^{\frac{1}{4}i\arctan(ax)} x^2 \, dx = \text{Too large to display}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^2,x, algorithm="fricas")
```

output
```
1/96*(96*I*a^3*(14641/268435456*I/a^12)^(1/4)*log(128/11*a^3*(14641/268435
456*I/a^12)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - 96*a^3*(14641
/268435456*I/a^12)^(1/4)*log(128/11*I*a^3*(14641/268435456*I/a^12)^(1/4) +
 (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + 96*a^3*(14641/268435456*I/a^12)^
(1/4)*log(-128/11*I*a^3*(14641/268435456*I/a^12)^(1/4) + (I*sqrt(a^2*x^2 +
 1)/(a*x + I))^(1/4)) - 96*I*a^3*(14641/268435456*I/a^12)^(1/4)*log(-128/1
1*a^3*(14641/268435456*I/a^12)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/
4)) + 96*I*a^3*(-14641/268435456*I/a^12)^(1/4)*log(128/11*a^3*(-14641/2684
35456*I/a^12)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - 96*a^3*(-14
641/268435456*I/a^12)^(1/4)*log(128/11*I*a^3*(-14641/268435456*I/a^12)^(1/
4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + 96*a^3*(-14641/268435456*I/a
^12)^(1/4)*log(-128/11*I*a^3*(-14641/268435456*I/a^12)^(1/4) + (I*sqrt(a^2
*x^2 + 1)/(a*x + I))^(1/4)) - 96*I*a^3*(-14641/268435456*I/a^12)^(1/4)*log
(-128/11*a^3*(-14641/268435456*I/a^12)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x +
 I))^(1/4)) + (32*a^3*x^3 - 4*I*a^2*x^2 - a*x - 37*I)*(I*sqrt(a^2*x^2 + 1)
/(a*x + I))^(1/4))/a^3
```

Sympy [F]

$$\int e^{\frac{1}{4}i\arctan(ax)}x^2\,dx = \int x^2\sqrt[4]{\frac{i\,(ax-i)}{\sqrt{a^2x^2+1}}}\,dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4)*x**2,x)
```

output
```
Integral(x**2*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(1/4), x)
```

Maxima [F]

$$\int e^{\frac{1}{4}i\arctan(ax)}x^2\,dx = \int x^2\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^2,x, algorithm="maxima")
```

output
```
integrate(x^2*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{1}{4}i\arctan(ax)}x^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{4}i\arctan(ax)}x^2\,dx = \int x^2\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{1/4}\,dx$$

input
```
int(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4),x)
```

output
```
int(x^2*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4), x)
```

Reduce [F]

$$\int e^{\frac{1}{4}i\arctan(ax)}x^2\,dx = \int \frac{(aix+1)^{\frac{1}{4}}\,x^2}{(a^2x^2+1)^{\frac{1}{8}}}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^2,x)
```

output
```
int(((a*i*x + 1)**(1/4)*x**2)/(a**2*x**2 + 1)**(1/8),x)
```

3.144 $\int e^{\frac{1}{4}i\arctan(ax)}x\,dx$

Optimal result

Integrand size = 14, antiderivative size = 523

$$\int e^{\frac{1}{4}i\arctan(ax)}x\,dx = \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{8a^2}+\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} - \frac{\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{32a^2} - \frac{\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)}{32a^2} + \frac{\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{32a^2} + \frac{\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)}{32a^2} - \frac{\sqrt{2-\sqrt{2}}\operatorname{arctanh}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}\right)}{32a^2} - \frac{\sqrt{2+\sqrt{2}}\operatorname{arctanh}\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}\right)}{32a^2}$$

output
```
1/8*(1-I*a*x)^(7/8)*(1+I*a*x)^(1/8)/a^2+1/2*(1-I*a*x)^(7/8)*(1+I*a*x)^(9/8
)/a^2-1/32*(2+2^(1/2))^(1/2)*arctan(((2-2^(1/2))^(1/2)-2*(1-I*a*x)^(1/8)/(
1+I*a*x)^(1/8))/(2+2^(1/2))^(1/2))/a^2-1/32*(2-2^(1/2))^(1/2)*arctan(((2+2
^(1/2))^(1/2)-2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8))/(2-2^(1/2))^(1/2))/a^2+1/
32*(2+2^(1/2))^(1/2)*arctan(((2-2^(1/2))^(1/2)+2*(1-I*a*x)^(1/8)/(1+I*a*x)
^(1/8))/(2+2^(1/2))^(1/2))/a^2+1/32*(2-2^(1/2))^(1/2)*arctan(((2+2^(1/2))^
(1/2)+2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8))/(2-2^(1/2))^(1/2))/a^2-1/32*(2-2^
(1/2))^(1/2)*arctanh((2-2^(1/2))^(1/2)*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)/(1+
(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)))/a^2-1/32*(2+2^(1/2))^(1/2)*arctanh((2+2^
(1/2))^(1/2)*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)/(1+(1-I*a*x)^(1/4)/(1+I*a*x)^
(1/4)))/a^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.12

$$\int e^{\frac{1}{4}i\arctan(ax)}x\,dx = \frac{(1-iax)^{7/8}\left(7(1+iax)^{9/8}+2\sqrt[8]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{7}{8},\frac{15}{8},\frac{1}{2}(1-iax)\right)\right)}{14a^2}$$

input
```
Integrate[E^((I/4)*ArcTan[a*x])*x,x]
```

output
```
((1 - I*a*x)^(7/8)*(7*(1 + I*a*x)^(9/8) + 2*2^(1/8)*Hypergeometric2F1[-1/8
, 7/8, 15/8, (1 - I*a*x)/2]))/(14*a^2)
```

Rubi [A] (warning: unable to verify)

Time = 1.27 (sec) , antiderivative size = 697, normalized size of antiderivative = 1.33, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {5585, 90, 60, 73, 854, 828, 1442, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\frac{1}{4}i\arctan(ax)}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{x\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}dx$$

$$\Big\downarrow \text{90}$$

$$\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2}-\frac{i\int \frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}dx}{8a}$$

$$\Big\downarrow \text{60}$$

$$\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2}-\frac{i\left(\frac{1}{4}\int \frac{1}{\sqrt[8]{1-iax}(iax+1)^{7/8}}dx+\frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a}\right)}{8a}$$

$$\Big\downarrow \text{73}$$

$$\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2}-\frac{i\left(\frac{2i\int \frac{(1-iax)^{3/4}}{(iax+1)^{7/8}}d\sqrt[8]{1-iax}}{a}+\frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a}\right)}{8a}$$

$$\Big\downarrow \text{854}$$

$$\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2}-\frac{i\left(\frac{2i\int \frac{(1-iax)^{3/4}}{2-iax}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{a}+\frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a}\right)}{8a}$$

$$\Big\downarrow \text{828}$$

$$\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2}-\frac{i\left(\frac{2i\left(\frac{\int \frac{\sqrt{1-iax}}{\sqrt{1-iax}-\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{1-iax}}{\sqrt{1-iax}+\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}\right)}{a}+\frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a}\right)}{8a}$$

$$\Big\downarrow \text{1442}$$

$$\frac{i\left(\frac{2i\left(\frac{\frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}-\int \frac{1-\sqrt{2}\sqrt[4]{1-iax}}{\sqrt{1-iax}-\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}-\int \frac{\sqrt{2}\sqrt[4]{1-iax}+1}{\sqrt{1-iax}+\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}\right)}{a}+\frac{i(1-iax)^{7}}{}\right)}{8a}$$

$$\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2}-$$

$\downarrow$ 1483

$$\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2}-$$

$$i\left(\frac{2i\left(\frac{-\frac{\int \frac{\sqrt{2+\sqrt{2}}-\frac{(1+\sqrt{2})\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2+\sqrt{2}}}-\frac{\int \frac{\frac{(1+\sqrt{2})\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+\sqrt{2+\sqrt{2}}}{\sqrt[4]{1-iax}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2+\sqrt{2}}}+\frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{2\sqrt{2}}-\frac{-\int \frac{}{\sqrt[4]{}}}{}\right.}{a}\right.$$

$\downarrow$ 1142

$$\frac{(1-iax)^{7/8}(iax+1)^{9/8}}{2a^2}-$$

$$i\left(2i\left(-\frac{-\frac{1}{2}\sqrt{2-\sqrt{2}}\int\frac{1}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}-\frac{1}{2}\left(1+\sqrt{2}\right)\int-\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2+\sqrt{2}}}-\right.\right.$$

$\downarrow$ 25

$$\frac{(1-iax)^{7/8}(iax+1)^{9/8}}{2a^2}-$$

$$i\left(2i\left(-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}-\frac{1}{2}\sqrt{2-\sqrt{2}}\int\frac{1}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2+\sqrt{2}}}-\right.\right.$$

$\downarrow$ 1083

$$\frac{(1-iax)^{7/8}(iax+1)^{9/8}}{2a^2}-$$

$$i\left(2i\left(-\frac{\sqrt{2-\sqrt{2}}\int\frac{1}{-\sqrt[4]{1-iax}+\sqrt{2}-2}d\left(\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}-\sqrt{2+\sqrt{2}}\right)+\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2+\sqrt{2}}}-\frac{\sqrt{2-\sqrt{2}}\int}{2\sqrt{2}}\right.\right.$$

$\downarrow$ 217

$$\frac{(1-iax)^{7/8}(iax+1)^{9/8}}{2a^2}-$$

$$i\left(2i\left(-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}-\arctan\left(\frac{\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+}{\sqrt[4]{1-iax}+\frac{\sqrt{2+\sqrt{2}}}{\sqrt[8]{i}}}}{2\sqrt{2}}\right.\right.$$

$\downarrow$ 1103

$$\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2a^2} - \frac{i\left(\frac{2i\left(-\frac{-\arctan\left(\frac{-\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{2}\left(1+\sqrt{2}\right)\log\left(\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}+1\right)}{2\sqrt{2+\sqrt{2}}} - \frac{\frac{1}{2}\left(1+\sqrt{2}\right)\log\left(\sqrt[4]{1-iax}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-}}{\sqrt[8]{1+i}}\right.}{2\sqrt{}}\right.}{2\sqrt{2}}\right.}{}$$

input
```
Int[E^((I/4)*ArcTan[a*x])*x,x]
```

output
```
((1 - I*a*x)^(7/8)*(1 + I*a*x)^(9/8))/(2*a^2) - ((I/8)*((I*(1 - I*a*x)^(7/
8)*(1 + I*a*x)^(1/8))/a + ((2*I)*(-1/2*((1 - I*a*x)^(1/8)/(1 + I*a*x)^(1/8
) - (ArcTan[(-Sqrt[2 - Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))
/Sqrt[2 + Sqrt[2]]] - ((1 - Sqrt[2])*Log[1 + (1 - I*a*x)^(1/4) - (Sqrt[2 -
 Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/2)/(2*Sqrt[2 - Sqrt[2]])
- (ArcTan[(Sqrt[2 - Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sq
rt[2 + Sqrt[2]]] + ((1 - Sqrt[2])*Log[1 + (1 - I*a*x)^(1/4) + (Sqrt[2 - Sq
rt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/2)/(2*Sqrt[2 - Sqrt[2]]))/Sq
rt[2] + ((1 - I*a*x)^(1/8)/(1 + I*a*x)^(1/8) - (-ArcTan[(-Sqrt[2 + Sqrt[2]
] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 - Sqrt[2]]] - ((1 + Sq
rt[2])*Log[1 + (1 - I*a*x)^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - I*a*x)^(1/8))/(
1 + I*a*x)^(1/8)])/2)/(2*Sqrt[2 + Sqrt[2]]) - (-ArcTan[(Sqrt[2 + Sqrt[2]]
+ (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 - Sqrt[2]]] + ((1 + Sqrt
[2])*Log[1 + (1 - I*a*x)^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - I*a*x)^(1/8))/(1
+ I*a*x)^(1/8)])/2)/(2*Sqrt[2 + Sqrt[2]]))/(2*Sqrt[2])))/a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 828
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Simp[s^3/(2*Sqrt[2]*b*r)   Int[x^
(m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Simp[s^3/(2*S
qrt[2]*b*r)   Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] &&
 GtQ[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}} x dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x,x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.82

$$\int e^{\frac{1}{4}i\arctan(ax)}x\,dx = -\frac{8\,a^2\left(\frac{i}{1048576\,a^8}\right)^{\frac{1}{4}}\log\left(32\,a^2\left(\frac{i}{1048576\,a^8}\right)^{\frac{1}{4}}+\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)+8i\,a^2\left(\frac{i}{1048576\,a^8}\right)^{\frac{1}{4}}\log\left(32i\,a^2\left(\frac{i}{1048576\,a^8}\right)^{\frac{1}{4}}+\right.}{}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x,x, algorithm="fricas")
```

output
```
-1/8*(8*a^2*(1/1048576*I/a^8)^(1/4)*log(32*a^2*(1/1048576*I/a^8)^(1/4) + (
I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + 8*I*a^2*(1/1048576*I/a^8)^(1/4)*lo
g(32*I*a^2*(1/1048576*I/a^8)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)
) - 8*I*a^2*(1/1048576*I/a^8)^(1/4)*log(-32*I*a^2*(1/1048576*I/a^8)^(1/4)
+ (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - 8*a^2*(1/1048576*I/a^8)^(1/4)*l
og(-32*a^2*(1/1048576*I/a^8)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)
) + 8*a^2*(-1/1048576*I/a^8)^(1/4)*log(32*a^2*(-1/1048576*I/a^8)^(1/4) + (
I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + 8*I*a^2*(-1/1048576*I/a^8)^(1/4)*l
og(32*I*a^2*(-1/1048576*I/a^8)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/
4)) - 8*I*a^2*(-1/1048576*I/a^8)^(1/4)*log(-32*I*a^2*(-1/1048576*I/a^8)^(1
/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - 8*a^2*(-1/1048576*I/a^8)^(1
/4)*log(-32*a^2*(-1/1048576*I/a^8)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))
^(1/4)) - (4*a^2*x^2 - I*a*x + 5)*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4))/a
^2
```

Sympy [F]

$$\int e^{\frac{1}{4}i\arctan(ax)}x\,dx = \int x\sqrt[4]{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}\,dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4)*x,x)
```

output
```
Integral(x*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(1/4), x)
```

Maxima [F]

$$\int e^{\frac{1}{4}i\arctan(ax)}x\,dx = \int x\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x,x, algorithm="maxima")
```

output
```
integrate(x*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{1}{4}i\arctan(ax)}x\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{4}i\arctan(ax)}x\,dx=\int x\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{1/4}\,dx$$

input
```
int(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4),x)
```

output
```
int(x*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.06

$$\int e^{\frac{1}{4}i\arctan(ax)}x\,dx=\frac{4(aix+1)^{\frac{1}{8}}\,(a^2x^2+1)}{9\,(-aix+1)^{\frac{1}{8}}\,a^2}$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x,x)
```

output
```
(4*(a*i*x + 1)**(1/8)*( - a*i*x + 1)**(1/8)*(a**2*x**2 + 1))/(9*( - a*i*x
+ 1)**(1/4)*a**2)
```

3.145 $\int e^{\frac{1}{4}i\arctan(ax)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 504

$$\int e^{\frac{1}{4}i\arctan(ax)}\,dx = \frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a} - \frac{i\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{4a} - \frac{i\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)}{4a} + \frac{i\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right)}{4a} + \frac{i\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)}{4a} - \frac{i\sqrt{2-\sqrt{2}}\operatorname{arctanh}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}\right)}{4a} - \frac{i\sqrt{2+\sqrt{2}}\operatorname{arctanh}\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}\right)}{4a}$$

output
```
I*(1-I*a*x)^(7/8)*(1+I*a*x)^(1/8)/a-1/4*I*(2+2^(1/2))^(1/2)*arctan(((2-2^(
1/2))^(1/2)-2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8))/(2+2^(1/2))^(1/2))/a-1/4*I*
(2-2^(1/2))^(1/2)*arctan(((2+2^(1/2))^(1/2)-2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1
/8))/(2-2^(1/2))^(1/2))/a+1/4*I*(2+2^(1/2))^(1/2)*arctan(((2-2^(1/2))^(1/2
)+2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8))/(2+2^(1/2))^(1/2))/a+1/4*I*(2-2^(1/2)
)^(1/2)*arctan(((2+2^(1/2))^(1/2)+2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8))/(2-2^
(1/2))^(1/2))/a-1/4*I*(2-2^(1/2))^(1/2)*arctanh((2-2^(1/2))^(1/2)*(1-I*a*x
)^(1/8)/(1+I*a*x)^(1/8)/(1+(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)))/a-1/4*I*(2+2^
(1/2))^(1/2)*arctanh((2+2^(1/2))^(1/2)*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)/(1+
(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)))/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.08

$$\int e^{\frac{1}{4}i\arctan(ax)}\,dx=-\frac{16ie^{\frac{9}{4}i\arctan(ax)}\,\text{Hypergeometric2F1}\left(\frac{9}{8},2,\frac{17}{8},-e^{2i\arctan(ax)}\right)}{9a}$$

input
```
Integrate[E^((I/4)*ArcTan[a*x]),x]
```

output
```
(((-16*I)/9)*E^(((9*I)/4)*ArcTan[a*x])*Hypergeometric2F1[9/8, 2, 17/8, -E^
((2*I)*ArcTan[a*x])])/a
```

Rubi [A] (warning: unable to verify)

Time = 1.20 (sec) , antiderivative size = 656, normalized size of antiderivative = 1.30, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5584, 60, 73, 854, 828, 1442, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{1}{4}i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5584}$$

$$\int \frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\int \frac{1}{\sqrt[8]{1-iax}(iax+1)^{7/8}}dx+\frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a}$$

$$\Big\downarrow \text{73}$$

$$\frac{2i\int \frac{(1-iax)^{3/4}}{(iax+1)^{7/8}}d\sqrt[8]{1-iax}}{a}+\frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a}$$

$$\Big\downarrow \text{854}$$

$$\frac{2i\int \frac{(1-iax)^{3/4}}{2-iax}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{a}+\frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a}$$

$$\Big\downarrow \text{828}$$

$$\frac{2i\left(\frac{\int \frac{\sqrt{1-iax}}{\sqrt{1-iax}-\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{1-iax}}{\sqrt{1-iax}+\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}\right)}{a}+\frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a}$$

$$\Big\downarrow \text{1442}$$

$$\frac{2i\left(\frac{\frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}-\int \frac{1-\sqrt{2}\sqrt[4]{1-iax}}{\sqrt{1-iax}-\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}-\int \frac{\sqrt{2}\sqrt[4]{1-iax}+1}{\sqrt{1-iax}+\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}\right)}{a}+\frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a}$$

$$\Big\downarrow \text{1483}$$

$$\frac{2i\left(\frac{-\frac{\int \frac{\sqrt{2+\sqrt{2}}-\frac{\left(1+\sqrt{2}\right)\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2+\sqrt{2}}}-\frac{\int \frac{\frac{\left(1+\sqrt{2}\right)\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+\sqrt{2+\sqrt{2}}}{\sqrt[4]{1-iax}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2+\sqrt{2}}}+\frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{2\sqrt{2}}-\frac{-\frac{\int \frac{\cdots}{\sqrt[4]{\cdots}}}{\cdots}}{\cdots}\cdots\right)}{a}$$

$$\frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a}$$

$\Big\downarrow$ 1142

$$\frac{2i\left(\frac{-\frac{-\frac{1}{2}\sqrt{2-\sqrt{2}}\int \frac{1}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}-\frac{1}{2}\left(1+\sqrt{2}\right)\int -\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(\cdots\right.}{\cdots}}{2\cdots}\cdots\right)}{\cdots}$$

$$\frac{i(1-iax)^{7/8}\sqrt[8]{iax+1}}{a}$$

$\Big\downarrow$ 25

$$\frac{2i\left(\frac{-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}-\frac{1}{2}\sqrt{2-\sqrt{2}}\int \frac{1}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\cdots\right.}{\cdots}}{2\sqrt{\cdots}}\cdots\right)}{\cdots}$$

$$\frac{i(1-iax)^{7/8}\sqrt[8]{iax+1}}{a}$$

$\Big\downarrow$ 1083

$$\frac{2i\left(\frac{-\frac{\sqrt{2-\sqrt{2}}\int \frac{1}{-\sqrt[4]{1-iax}+\sqrt{2}-2}d\left(\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}-\sqrt{2+\sqrt{2}}\right)+\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2+\sqrt{2}}}-\frac{\sqrt{2-\sqrt{2}}\int \cdots}{\cdots}}{2\sqrt{2}}\cdots\right)}{\frac{i(1-iax)^{7/8}\sqrt[8]{iax+1}}{a}}$$

$$\Big\downarrow \ 217$$

$$\frac{2i\left(\frac{-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}-\arctan\left(\frac{\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int \frac{\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+\cdots}{\sqrt[4]{1-iax}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{\cdots}}{\sqrt[8]{iax\cdots}}\cdots}\cdots}{\cdots}}{2\sqrt{2}}\cdots\right)}{\frac{i(1-iax)^{7/8}\sqrt[8]{iax+1}}{a}}$$

$$\Big\downarrow \ 1103$$

$$\frac{2i\left(\frac{-\frac{-\arctan\left(\frac{-\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{2}\left(1+\sqrt{2}\right)\log\left(\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}+1\right)}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\log\left(\sqrt[4]{1-iax}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\cdots}}{\sqrt[8]{1+iax}}\cdots\right)\cdots}{2\sqrt{2-\cdots}}}{2\sqrt{2}}\cdots\right)}{\frac{i(1-iax)^{7/8}\sqrt[8]{1+iax}}{a}}$$

input
```
Int[E^((I/4)*ArcTan[a*x]),x]
```

output
```
(I*(1 - I*a*x)^(7/8)*(1 + I*a*x)^(1/8))/a + ((2*I)*(-1/2*((1 - I*a*x)^(1/8
)/(1 + I*a*x)^(1/8) - (ArcTan[(-Sqrt[2 - Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/
(1 + I*a*x)^(1/8))/Sqrt[2 + Sqrt[2]]] - ((1 - Sqrt[2])*Log[1 + (1 - I*a*x)
^(1/4) - (Sqrt[2 - Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/2)/(2*S
qrt[2 - Sqrt[2]]) - (ArcTan[(Sqrt[2 - Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1
+ I*a*x)^(1/8))/Sqrt[2 + Sqrt[2]]] + ((1 - Sqrt[2])*Log[1 + (1 - I*a*x)^(1
/4) + (Sqrt[2 - Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/2)/(2*Sqrt
[2 - Sqrt[2]]))/Sqrt[2] + ((1 - I*a*x)^(1/8)/(1 + I*a*x)^(1/8) - (-ArcTan[
(-Sqrt[2 + Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 - Sq
rt[2]]] - ((1 + Sqrt[2])*Log[1 + (1 - I*a*x)^(1/4) - (Sqrt[2 + Sqrt[2]]*(1
 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/2)/(2*Sqrt[2 + Sqrt[2]]) - (-ArcTan[(
Sqrt[2 + Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 - Sqrt
[2]]] + ((1 + Sqrt[2])*Log[1 + (1 - I*a*x)^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 -
 I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/2)/(2*Sqrt[2 + Sqrt[2]]))/(2*Sqrt[2])))
/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 828
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Simp[s^3/(2*Sqrt[2]*b*r)   Int[x^
(m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Simp[s^3/(2*S
qrt[2]*b*r)   Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] &&
 GtQ[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 5584
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4),x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.76

$$\int e^{\frac{1}{4}i\arctan(ax)}\,dx = \frac{-i\,a\left(\frac{i}{256\,a^4}\right)^{\frac{1}{4}}\log\left(4\,a\left(\frac{i}{256\,a^4}\right)^{\frac{1}{4}}+\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)+a\left(\frac{i}{256\,a^4}\right)^{\frac{1}{4}}\log\left(4i\,a\left(\frac{i}{256\,a^4}\right)^{\frac{1}{4}}+\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}\right)-a\left(\frac{i}{256}\right.}{}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4),x, algorithm="fricas")
```

output
```
(-I*a*(1/256*I/a^4)^(1/4)*log(4*a*(1/256*I/a^4)^(1/4) + (I*sqrt(a^2*x^2 +
1)/(a*x + I))^(1/4)) + a*(1/256*I/a^4)^(1/4)*log(4*I*a*(1/256*I/a^4)^(1/4)
 + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - a*(1/256*I/a^4)^(1/4)*log(-4*I
*a*(1/256*I/a^4)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + I*a*(1/2
56*I/a^4)^(1/4)*log(-4*a*(1/256*I/a^4)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x +
 I))^(1/4)) - I*a*(-1/256*I/a^4)^(1/4)*log(4*a*(-1/256*I/a^4)^(1/4) + (I*s
qrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + a*(-1/256*I/a^4)^(1/4)*log(4*I*a*(-1/
256*I/a^4)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - a*(-1/256*I/a^
4)^(1/4)*log(-4*I*a*(-1/256*I/a^4)^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))
^(1/4)) + I*a*(-1/256*I/a^4)^(1/4)*log(-4*a*(-1/256*I/a^4)^(1/4) + (I*sqrt
(a^2*x^2 + 1)/(a*x + I))^(1/4)) + (a*x + I)*(I*sqrt(a^2*x^2 + 1)/(a*x + I)
)^(1/4))/a
```

Sympy [F]

$$\int e^{\frac{1}{4}i\arctan(ax)}\,dx=\int\sqrt[4]{\frac{iax+1}{\sqrt{a^2x^2+1}}}\,dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4),x)
```

output
```
Integral(((I*a*x + 1)/sqrt(a**2*x**2 + 1))**(1/4), x)
```

Maxima [F]

$$\int e^{\frac{1}{4}i\arctan(ax)}\,dx=\int\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4),x, algorithm="maxima")
```

output
```
integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{1}{4}i\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{4}i\arctan(ax)}\,dx = \int \left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{1/4}\,dx$$

input
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4),x)
```

output
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4), x)
```

Reduce [F]

$$\int e^{\frac{1}{4}i\arctan(ax)}\,dx = \int \frac{(aix+1)^{\frac{1}{4}}}{(a^2x^2+1)^{\frac{1}{8}}}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4),x)
```

output
```
int((a*i*x + 1)**(1/4)/(a**2*x**2 + 1)**(1/8),x)
```

3.146 $\int \frac{e^{\frac{1}{4} i \arctan(ax)}}{x} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 630

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x}\,dx = -2\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right) + \sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right) - \sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2+\sqrt{2}}}\right) - \sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}+\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}}{\sqrt{2-\sqrt{2}}}\right) + \sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - 2\text{arctanh}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) + \sqrt{2-\sqrt{2}}\text{arctanh}\left(\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}\right) + \sqrt{2+\sqrt{2}}\text{arctanh}\left(\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}\left(1+\frac{\sqrt[4]{1-iax}}{\sqrt[4]{1+iax}}\right)}\right) - \sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}\left(1+\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)}\right)$$

output
```
-2*arctan((1+I*a*x)^(1/8)/(1-I*a*x)^(1/8))+(2+2^(1/2))^(1/2)*arctan(((2-2^
(1/2))^(1/2)-2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8))/(2+2^(1/2))^(1/2))+(2-2^(1
/2))^(1/2)*arctan(((2+2^(1/2))^(1/2)-2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8))/(2
-2^(1/2))^(1/2))-(2+2^(1/2))^(1/2)*arctan(((2-2^(1/2))^(1/2)+2*(1-I*a*x)^(
1/8)/(1+I*a*x)^(1/8))/(2+2^(1/2))^(1/2))-(2-2^(1/2))^(1/2)*arctan(((2+2^(1
/2))^(1/2)+2*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8))/(2-2^(1/2))^(1/2))+2^(1/2)*a
rctan(1-2^(1/2)*(1+I*a*x)^(1/8)/(1-I*a*x)^(1/8))-2^(1/2)*arctan(1+2^(1/2)*
(1+I*a*x)^(1/8)/(1-I*a*x)^(1/8))-2*arctanh((1+I*a*x)^(1/8)/(1-I*a*x)^(1/8)
)+(2-2^(1/2))^(1/2)*arctanh((2-2^(1/2))^(1/2)*(1-I*a*x)^(1/8)/(1+I*a*x)^(1
/8)/(1+(1-I*a*x)^(1/4)/(1+I*a*x)^(1/4)))+(2+2^(1/2))^(1/2)*arctanh((2+2^(1
/2))^(1/2)*(1-I*a*x)^(1/8)/(1+I*a*x)^(1/8)/(1+(1-I*a*x)^(1/4)/(1+I*a*x)^(1
/4)))-2^(1/2)*arctanh(2^(1/2)*(1+I*a*x)^(1/8)/(1-I*a*x)^(1/8)/(1+(1+I*a*x)
^(1/4)/(1-I*a*x)^(1/4)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.15

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x}\,dx = -\frac{4(1-iax)^{7/8}\left(\sqrt[8]{2}(1+iax)^{7/8}\,\text{Hypergeometric2F1}\left(\frac{7}{8},\frac{7}{8},\frac{15}{8},\frac{1}{2}(1-iax)\right)+2\,\text{Hypergeometric2F1}\left(\frac{7}{8},\right.\right.}{7(1+iax)^{7/8}}$$

input
```
Integrate[E^((I/4)*ArcTan[a*x])/x,x]
```

output
```
(-4*(1 - I*a*x)^(7/8)*(2^(1/8)*(1 + I*a*x)^(7/8)*Hypergeometric2F1[7/8, 7/
8, 15/8, (1 - I*a*x)/2] + 2*Hypergeometric2F1[7/8, 1, 15/8, (I + a*x)/(I -
 a*x)]))/(7*(1 + I*a*x)^(7/8))
```

Rubi [A] (warning: unable to verify)

Time = 1.78 (sec) , antiderivative size = 918, normalized size of antiderivative = 1.46, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 1.562$, Rules used = {5585, 140, 73, 104, 758, 755, 756, 216, 219, 854, 828, 1442, 1476, 1082, 217, 1479, 25, 27, 1103, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{\sqrt[8]{1+iax}}{x\sqrt[8]{1-iax}}dx$$

$$\Big\downarrow \text{140}$$

$$ia\int \frac{1}{\sqrt[8]{1-iax}(iax+1)^{7/8}}dx+\int \frac{1}{x\sqrt[8]{1-iax}(iax+1)^{7/8}}dx$$

$$\Big\downarrow \text{73}$$

$$\int \frac{1}{x\sqrt[8]{1-iax}(iax+1)^{7/8}}dx-8\int \frac{(1-iax)^{3/4}}{(iax+1)^{7/8}}d\sqrt[8]{1-iax}$$

$$\Big\downarrow \text{104}$$

$$8\int \frac{1}{\frac{iax+1}{1-iax}-1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}-8\int \frac{(1-iax)^{3/4}}{(iax+1)^{7/8}}d\sqrt[8]{1-iax}$$

$$\Big\downarrow \text{758}$$

$$8\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}-\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)-8\int \frac{(1-iax)^{3/4}}{(iax+1)^{7/8}}d\sqrt[8]{1-iax}$$

$$\Big\downarrow \text{755}$$

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}-\frac{1}{2}\int \frac{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)- 8\int \frac{(1-iax)^{3/4}}{(iax+1)^{7/8}}d\sqrt[8]{1-iax}$$

$$\Big\downarrow \text{756}$$

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}-\frac{1}{2}\int \frac{1}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+}}{\sqrt[8]{1-i}}\right.\right. \quad 8\int \frac{(1-iax)^{3/4}}{(iax+1)^{7/8}}d\sqrt[8]{1-iax}$$

$$\Big\downarrow \text{216}$$

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}-\frac{1}{2}\right.\right. \quad 8\int \frac{(1-iax)^{3/4}}{(iax+1)^{7/8}}d\sqrt[8]{1-iax}$$

$$\Big\downarrow \text{219}$$

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}-\frac{1}{2}\int \frac{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)-\frac{1}{2}a\right.\right. \quad 8\int \frac{(1-iax)^{3/4}}{(iax+1)^{7/8}}d\sqrt[8]{1-iax}$$

$$\Big\downarrow \text{854}$$

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}-\frac{1}{2}\int \frac{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)-\frac{1}{2}a\right.\right. \quad 8\int \frac{(1-iax)^{3/4}}{2-iax}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}$$

$$\Big\downarrow \text{828}$$

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}-\frac{1}{2}\int \frac{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)-\frac{1}{2}a\right.\right.$$
$$8\left(\frac{\int \frac{\sqrt{1-iax}}{\sqrt{1-iax}-\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{1-iax}}{\sqrt{1-iax}+\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}\right)$$

$\downarrow$ 1442

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}-\frac{1}{2}\int \frac{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)-\frac{1}{2}a\right.\right.$$
$$8\left(\frac{\frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}-\int \frac{1-\sqrt{2}\sqrt[4]{1-iax}}{\sqrt{1-iax}-\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}-\int \frac{\sqrt{2}\sqrt[4]{1-iax}+1}{\sqrt{1-iax}+\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}\right)$$

$\downarrow$ 1476

$$8\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}-\frac{1}{2}\int \frac{1}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)-\right.\right.$$
$$8\left(\frac{\frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}-\int \frac{1-\sqrt{2}\sqrt[4]{1-iax}}{\sqrt{1-iax}-\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}-\int \frac{\sqrt{2}\sqrt[4]{1-iax}+1}{\sqrt{1-iax}+\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}\right)$$

$\downarrow$ 1082

$$8\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-1}d\left(\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{\sqrt[4]{iax+}}{\sqrt[4]{1-i}}}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+}\right.\right.$$
$$8\left(\frac{\frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}-\int \frac{1-\sqrt{2}\sqrt[4]{1-iax}}{\sqrt{1-iax}-\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}-\int \frac{\sqrt{2}\sqrt[4]{1-iax}+1}{\sqrt{1-iax}+\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}\right)$$

$\downarrow$ 217

$$8\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)+\frac{1}{2}\left(\right.\right.$$

$$8\left(\frac{\frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}-\int\frac{1-\sqrt{2}\sqrt[4]{1-iax}}{\sqrt{1-iax}-\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}-\int\frac{\sqrt{2}\sqrt[4]{1-iax}+1}{\sqrt{1-iax}+\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}\right)$$

$\big\downarrow$ 1479

$$8\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1\right)}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\text{arcta}}{}\right.\right.\right.$$

$$8\left(\frac{\frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}-\int\frac{1-\sqrt{2}\sqrt[4]{1-iax}}{\sqrt{1-iax}-\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}-\int\frac{\sqrt{2}\sqrt[4]{1-iax}+1}{\sqrt{1-iax}+\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}\right)$$

$\big\downarrow$ 25

$$8\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1\right)}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\text{arctan}}{}\right.\right.\right.$$

$$8\left(\frac{\frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}-\int\frac{1-\sqrt{2}\sqrt[4]{1-iax}}{\sqrt{1-iax}-\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}-\int\frac{\sqrt{2}\sqrt[4]{1-iax}+1}{\sqrt{1-iax}+\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}\right)$$

$\big\downarrow$ 27

$$8\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)+\frac{1}{2}\left(\right.\right.\right.$$

$$8\left(\frac{\frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}-\int \frac{1-\sqrt{2}\sqrt[4]{1-iax}}{\sqrt{1-iax}-\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}-\int \frac{\sqrt{2}\sqrt[4]{1-iax}+1}{\sqrt{1-iax}+\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}\right)$$

$\downarrow$ 1103

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1+\right.}{}\right.\right.\right.$$

$$8\left(\frac{\frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}-\int \frac{1-\sqrt{2}\sqrt[4]{1-iax}}{\sqrt{1-iax}-\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}-\frac{\frac{\sqrt[8]{1-iax}}{\sqrt[8]{1+iax}}-\int \frac{\sqrt{2}\sqrt[4]{1-iax}+1}{\sqrt{1-iax}+\sqrt{2}\sqrt[4]{1-iax}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}\right)$$

$\downarrow$ 1483

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}}{\sqrt[8]{}}\right.}{}\right.\right.\right.$$

$$8\left(\frac{-\frac{\int \frac{\sqrt{2+\sqrt{2}}-\frac{(1+\sqrt{2})\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2+\sqrt{2}}}-\frac{\int \frac{\frac{(1+\sqrt{2})\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+\sqrt{2+\sqrt{2}}}{\sqrt[4]{1-iax}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2+\sqrt{2}}}+\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2}}-\right.$$

$\downarrow$ 1142

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}}{\sqrt[8]{}}\right.}{}\right.\right.\right.$$

$$8\left(\frac{-\frac{-\frac{1}{2}\sqrt{2-\sqrt{2}}\int\frac{1}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}-\frac{1}{2}\left(1+\sqrt{2}\right)\int-\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2+\sqrt{2}}}-}{}\right.$$

$$\Big\downarrow \; 25$$

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}}{\sqrt[8]{}}\right.}{}\right.\right.\right.$$

$$8\left(\frac{-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}-\frac{1}{2}\sqrt{2-\sqrt{2}}\int\frac{1}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}}{}}{}\right.$$

$$\Big\downarrow \; 1083$$

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}}{\cdots}\right.}{\cdots}\right.\right.\right.$$

$$8\left(\frac{-\frac{\sqrt{2-\sqrt{2}}\int\frac{1}{-\sqrt[4]{1-iax}+\sqrt{2}-2}d\left(\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}-\sqrt{2+\sqrt{2}}\right)+\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{2\sqrt{2+\sqrt{2}}}-\frac{\sqrt{2\cdots}}{\cdots}}{2\cdots}\right.$$

$\Big\downarrow$ 217

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}}{\cdots}\right.}{\cdots}\right.\right.\right.$$

$$8\left(\frac{-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}}{\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1}d\frac{\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}-\arctan\left(\frac{\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\int\frac{\frac{2\sqrt[8]{1-i\cdots}}{\sqrt[8]{iax+\cdots}}}{\sqrt[4]{1-iax}+\sqrt{2+\cdots}}}{\cdots}}{2\sqrt{2}}\right.$$

$\Big\downarrow$ 1103

$$8\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}}{\sqrt[8]{}}\right.}{}\right.\right.\right.$$

$$8\left(\frac{-\frac{-\arctan\left(\frac{\frac{2\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{2}\left(1+\sqrt{2}\right)\log\left(\sqrt[4]{1-iax}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-iax}}{\sqrt[8]{iax+1}}+1\right)}{2\sqrt{2+\sqrt{2}}}-\frac{\frac{1}{2}\left(1+\sqrt{2}\right)\log\left(\sqrt[4]{1-iax}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{}}{\sqrt[8]{ia}}\right.}{}}{2\sqrt{2}}\right.$$

input
```
Int[E^((I/4)*ArcTan[a*x])/x,x]
```

output
```
-8*(-1/2*((1 - I*a*x)^(1/8)/(1 + I*a*x)^(1/8) - (ArcTan[(-Sqrt[2 - Sqrt[2]
] + (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqrt[2]]] - ((1 - Sq
rt[2])*Log[1 + (1 - I*a*x)^(1/4) - (Sqrt[2 - Sqrt[2]]*(1 - I*a*x)^(1/8))/(
1 + I*a*x)^(1/8)])/2)/(2*Sqrt[2 - Sqrt[2]]) - (ArcTan[(Sqrt[2 - Sqrt[2]] +
 (2*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8))/Sqrt[2 + Sqrt[2]]] + ((1 - Sqrt[
2])*Log[1 + (1 - I*a*x)^(1/4) + (Sqrt[2 - Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 +
 I*a*x)^(1/8)])/2)/(2*Sqrt[2 - Sqrt[2]]))/Sqrt[2] + ((1 - I*a*x)^(1/8)/(1
+ I*a*x)^(1/8) - (-ArcTan[(-Sqrt[2 + Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1 +
 I*a*x)^(1/8))/Sqrt[2 - Sqrt[2]]] - ((1 + Sqrt[2])*Log[1 + (1 - I*a*x)^(1/
4) - (Sqrt[2 + Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/2)/(2*Sqrt[
2 + Sqrt[2]]) - (-ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - I*a*x)^(1/8))/(1 + I
*a*x)^(1/8))/Sqrt[2 - Sqrt[2]]] + ((1 + Sqrt[2])*Log[1 + (1 - I*a*x)^(1/4)
 + (Sqrt[2 + Sqrt[2]]*(1 - I*a*x)^(1/8))/(1 + I*a*x)^(1/8)])/2)/(2*Sqrt[2
+ Sqrt[2]]))/(2*Sqrt[2])) + 8*((-1/2*ArcTan[(1 + I*a*x)^(1/8)/(1 - I*a*x)^
(1/8)] - ArcTanh[(1 + I*a*x)^(1/8)/(1 - I*a*x)^(1/8)]/2)/2 + ((ArcTan[1 -
(Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8)]/Sqrt[2] - ArcTan[1 + (Sqrt[
2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8)]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*(1
 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8) + (1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/
(2*Sqrt[2]) - Log[1 + (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8) + (1 +
 I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/(2*Sqrt[2]))/2)/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 758
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^(n/2)),
 x], x] + Simp[r/(2*a)   Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x]
 && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

rule 828
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Simp[s^3/(2*Sqrt[2]*b*r)   Int[x^
(m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Simp[s^3/(2*S
qrt[2]*b*r)   Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] &&
 GtQ[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}}{x}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 509, normalized size of antiderivative = 0.81

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x}\,dx = \text{Too large to display}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x,x, algorithm="fricas")
```

output
```
-1/2*sqrt(4*I)*log(1/2*sqrt(4*I) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4))
+ 1/2*sqrt(4*I)*log(-1/2*sqrt(4*I) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)
) - 1/2*sqrt(-4*I)*log(1/2*sqrt(-4*I) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1
/4)) + 1/2*sqrt(-4*I)*log(-1/2*sqrt(-4*I) + (I*sqrt(a^2*x^2 + 1)/(a*x + I)
)^(1/4)) + I^(1/4)*log(I^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) +
I*I^(1/4)*log(I*I^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - I*I^(1/
4)*log(-I*I^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - I^(1/4)*log(-
I^(1/4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + (-I)^(1/4)*log((-I)^(1/
4) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) + I*(-I)^(1/4)*log(I*(-I)^(1/4
) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - I*(-I)^(1/4)*log(-I*(-I)^(1/4
) + (I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - (-I)^(1/4)*log(-(-I)^(1/4) +
(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)) - log((I*sqrt(a^2*x^2 + 1)/(a*x + I
))^(1/4) + 1) - I*log((I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) + I) + I*log((
I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) - I) + log((I*sqrt(a^2*x^2 + 1)/(a*x
+ I))^(1/4) - 1)
```

Sympy [F]

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x}\,dx = \int \frac{\sqrt[4]{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x}\,dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4)/x,x)
```

output
```
Integral((I*(a*x - I)/sqrt(a**2*x**2 + 1))**(1/4)/x, x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x}\,dx = \int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}}{x}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x,x, algorithm="maxima")
```

output
```
integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x}\,dx = \int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{1/4}}{x}\,dx$$

input
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4)/x,x)
```

output
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4)/x, x)
```

Reduce [F]

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x}\,dx = \int \frac{(aix+1)^{\frac{1}{4}}}{(a^2x^2+1)^{\frac{1}{8}}\,x}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x,x)
```

output
```
int((a*i*x + 1)**(1/4)/((a**2*x**2 + 1)**(1/8)*x),x)
```

3.147 $\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 259

$$\begin{aligned}\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^2}\,dx = &-\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x} \\ &-\frac{1}{2}ia\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)+\frac{ia\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{2\sqrt{2}} \\ &-\frac{ia\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{2\sqrt{2}}-\frac{1}{2}ia\text{arctanh}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) \\ &-\frac{ia\text{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}\left(1+\frac{\sqrt[4]{1+iax}}{\sqrt[4]{1-iax}}\right)}\right)}{2\sqrt{2}}\end{aligned}$$

output
```
-(1-I*a*x)^(7/8)*(1+I*a*x)^(1/8)/x-1/2*I*a*arctan((1+I*a*x)^(1/8)/(1-I*a*x
)^(1/8))+1/4*I*a*arctan(1-2^(1/2)*(1+I*a*x)^(1/8)/(1-I*a*x)^(1/8))*2^(1/2)
-1/4*I*a*arctan(1+2^(1/2)*(1+I*a*x)^(1/8)/(1-I*a*x)^(1/8))*2^(1/2)-1/2*I*a
*arctanh((1+I*a*x)^(1/8)/(1-I*a*x)^(1/8))-1/4*I*a*arctanh(2^(1/2)*(1+I*a*x
)^(1/8)/(1-I*a*x)^(1/8)/(1+(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)))*2^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.27

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^2}\,dx = -\frac{i(1-iax)^{7/8}\left(-7i+7ax+2ax\,\text{Hypergeometric2F1}\left(\frac{7}{8},1,\frac{15}{8},\frac{i+ax}{i-ax}\right)\right)}{7x(1+iax)^{7/8}}$$

input
```
Integrate[E^((I/4)*ArcTan[a*x])/x^2,x]
```

output
```
((-1/7*I)*(1 - I*a*x)^(7/8)*(-7*I + 7*a*x + 2*a*x*Hypergeometric2F1[7/8, 1
, 15/8, (I + a*x)/(I - a*x)]))/(x*(1 + I*a*x)^(7/8))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.28, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.938$, Rules used = {5585, 105, 104, 758, 755, 756, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^2}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{\sqrt[8]{1+iax}}{x^2\sqrt[8]{1-iax}}dx$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{4}ia\int \frac{1}{x\sqrt[8]{1-iax}(iax+1)^{7/8}}dx-\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x}$$

$$\Big\downarrow \text{104}$$

$$2ia\int \frac{1}{\frac{iax+1}{1-iax}-1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}-\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x}$$

$\downarrow$ 758

$$2ia\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}-\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)-\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x}$$

$\downarrow$ 755

$$2ia\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}-\frac{1}{2}\int \frac{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)- \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x}$$

$\downarrow$ 756

$$2ia\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}-\frac{1}{2}\int \frac{1}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax\ldots}}{\sqrt[8]{1-\ldots}}\right.\right. \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x}$$

$\downarrow$ 216

$$2ia\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}-\ldots\right.\right. \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x}$$

$\downarrow$ 219

$$2ia\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}-\frac{1}{2}\int \frac{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)-\ldots\right.\right. \frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x}$$

$\downarrow$ 1476

$$\frac{2ia\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}-\frac{1}{2}\int\frac{1}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)\right.\right.}{\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x}}$$

$$\Big\downarrow \ 1082$$

$$\frac{2ia\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-1}d\left(\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{\sqrt[4]{iax}}{\sqrt[4]{1-}}}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}\right.\right.}{\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x}}$$

$$\Big\downarrow \ 217$$

$$\frac{2ia\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)+\right.}{\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x}}$$

$$\Big\downarrow \ 1479$$

$$\frac{2ia\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1\right)}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\text{ar}}{}\right.\right.\right.}{\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x}}$$

$$\Big\downarrow \ 25$$

$$2ia\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1\right)}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\text{arct}}{}\right.\right.\right.$$

$$\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x}$$

$$\Big\downarrow 27$$

$$2ia\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)+\frac{1}{2}\left(\right.\right.\right.$$

$$\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x}$$

$$\Big\downarrow 1103$$

$$2ia\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1\right.}{}\right.\right.\right.$$

$$\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x}$$

input
```
Int[E^((I/4)*ArcTan[a*x])/x^2,x]
```

output
```
-(((1 - I*a*x)^(7/8)*(1 + I*a*x)^(1/8))/x) + (2*I)*a*((-1/2*ArcTan[(1 + I*
a*x)^(1/8)/(1 - I*a*x)^(1/8)] - ArcTanh[(1 + I*a*x)^(1/8)/(1 - I*a*x)^(1/8
)]/2)/2 + ((ArcTan[1 - (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8)]/Sqrt
[2] - ArcTan[1 + (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8)]/Sqrt[2])/2
 + (Log[1 - (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8) + (1 + I*a*x)^(1
/4)/(1 - I*a*x)^(1/4)]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*(1 + I*a*x)^(1/8))/(
1 - I*a*x)^(1/8) + (1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/(2*Sqrt[2]))/2)/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 758
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^(n/2)),
 x], x] + Simp[r/(2*a)   Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x]
 && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}}{x^2}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^2,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(171) = 342.

Time = 0.09 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.33

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^2}\,dx$$
$$= \frac{-i\,ax\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}+1\right)+ax\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}+i\right)-ax\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}-i\right)+i\,ax\log\left(\left(\frac{i\cdots}{}\right.\right.}{}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^2,x, algorithm="fricas")
```

output
```
1/4*(-I*a*x*log((I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) + 1) + a*x*log((I*sq
rt(a^2*x^2 + 1)/(a*x + I))^(1/4) + I) - a*x*log((I*sqrt(a^2*x^2 + 1)/(a*x
+ I))^(1/4) - I) + I*a*x*log((I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) - 1) +
sqrt(I*a^2)*x*log((a*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) + I*sqrt(I*a^2)
)/a) - sqrt(I*a^2)*x*log((a*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) - I*sqrt
(I*a^2))/a) + sqrt(-I*a^2)*x*log((a*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4)
+ I*sqrt(-I*a^2))/a) - sqrt(-I*a^2)*x*log((a*(I*sqrt(a^2*x^2 + 1)/(a*x + I
))^(1/4) - I*sqrt(-I*a^2))/a) - 4*(-I*a*x + 1)*(I*sqrt(a^2*x^2 + 1)/(a*x +
 I))^(1/4))/x
```

Sympy [F]

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^2}\,dx = \int \frac{\sqrt[4]{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x^2}\,dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4)/x**2,x)
```

output
```
Integral((I*(a*x - I)/sqrt(a**2*x**2 + 1))**(1/4)/x**2, x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^2}\,dx = \int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}}{x^2}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^2,x, algorithm="maxima")
```

output
```
integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4)/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^2}\,dx = \int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{1/4}}{x^2}\,dx$$

input
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4)/x^2,x)
```

output
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4)/x^2, x)
```

Reduce [F]

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^2}\,dx = \int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}}{x^2}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^2,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^2,x)
```

3.148 $\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 295

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^3}\,dx = -\frac{ia(1-iax)^{7/8}\sqrt[8]{1+iax}}{8x} - \frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2}$$

$$+\frac{1}{16}a^2\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right) - \frac{a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{16\sqrt{2}} + \frac{a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{16\sqrt{2}} + \frac{1}{16}a^2\mathrm{arct}$$

output
```
-1/8*I*a*(1-I*a*x)^(7/8)*(1+I*a*x)^(1/8)/x-1/2*(1-I*a*x)^(7/8)*(1+I*a*x)^(
9/8)/x^2+1/16*a^2*arctan((1+I*a*x)^(1/8)/(1-I*a*x)^(1/8))-1/32*a^2*arctan(
1-2^(1/2)*(1+I*a*x)^(1/8)/(1-I*a*x)^(1/8))*2^(1/2)+1/32*a^2*arctan(1+2^(1/
2)*(1+I*a*x)^(1/8)/(1-I*a*x)^(1/8))*2^(1/2)+1/16*a^2*arctanh((1+I*a*x)^(1/
8)/(1-I*a*x)^(1/8))+1/32*a^2*arctanh(2^(1/2)*(1+I*a*x)^(1/8)/(1-I*a*x)^(1/
8)/(1+(1+I*a*x)^(1/4)/(1-I*a*x)^(1/4)))*2^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.28

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^3}\,dx = \frac{(1-iax)^{7/8}\left(7(-4-9iax+5a^2x^2)+2a^2x^2\,\text{Hypergeometric2F1}\left(\frac{7}{8},1,\frac{15}{8},\frac{i+ax}{i-ax}\right)\right)}{56x^2(1+iax)^{7/8}}$$

input
```
Integrate[E^((I/4)*ArcTan[a*x])/x^3,x]
```

output
```
((1 - I*a*x)^(7/8)*(7*(-4 - (9*I)*a*x + 5*a^2*x^2) + 2*a^2*x^2*Hypergeomet
ric2F1[7/8, 1, 15/8, (I + a*x)/(I - a*x)]))/(56*x^2*(1 + I*a*x)^(7/8))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.25, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5585, 107, 105, 104, 758, 755, 756, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^3}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{\sqrt[8]{1+iax}}{x^3\sqrt[8]{1-iax}}dx$$

$$\Big\downarrow \text{107}$$

$$\frac{1}{8}ia\int \frac{\sqrt[8]{iax+1}}{x^2\sqrt[8]{1-iax}}dx-\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2}$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{8}ia\left(\frac{1}{4}ia\int \frac{1}{x\sqrt[8]{1-iax}(iax+1)^{7/8}}dx-\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x}\right)-\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2}$$

$$\Big\downarrow \text{104}$$

$$\frac{1}{8}ia\left(2ia\int \frac{1}{\frac{iax+1}{1-iax}-1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}-\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x}\right)-\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2}$$

$$\Big\downarrow \text{758}$$

$$\frac{1}{8}ia\left(2ia\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}-\frac{1}{2}\int \frac{1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)-\frac{(1-iax)^{7/8}\sqrt[8]{1+iax}}{x}\right)-\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2}$$

$$\Big\downarrow \text{755}$$

$$\frac{1}{8}ia\left(2ia\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}-\frac{1}{2}\int \frac{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}}d\frac{\sqrt[8]{iax+}}{\sqrt[8]{1-ia}} \right.\right.$$
$$\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2}$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{8}ia\left(2ia\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}-\frac{1}{2}\int \frac{1}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d \right.\right.\right.$$
$$\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{8}ia\left(2ia\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{1-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)\right)+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+}}{\sqrt[8]{1-ia}} \right.\right.\right.$$
$$\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{8}ia\left(2ia\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}-\frac{1}{2}\int\frac{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+1}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)+\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right.\right.\right.\right.$$
$$\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{1}{8}ia\left(2ia\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}-\frac{1}{2}\int\frac{1}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}d\frac{\sqrt[8]{iax}}{\sqrt[8]{1-}}\right.\right.\right.\right.$$
$$\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{1}{8}ia\left(2ia\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-1}d\left(\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-}{\frac{\sqrt{i}}{\sqrt{1}}}\right.\right.\right.$$
$$\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{8}ia\left(2ia\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{\sqrt{2}}-\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}}{\frac{\sqrt{iax+1}}{\sqrt{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right.\right.\right.$$
$$\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{1}{8}ia\left(2ia\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1\right)}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}}{2\sqrt{2}}\right)+\right.\right.\right.$$

$$\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2}$$

$\downarrow$ 25

$$\frac{1}{8}ia\left(2ia\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1\right)}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}}{2\sqrt{2}}\right)+\frac{1}{2}\right.\right.\right.$$

$$\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2}$$

$\downarrow$ 27

$$\frac{1}{8}ia\left(2ia\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}-\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}{\frac{\sqrt[4]{iax+1}}{\sqrt[4]{1-iax}}+\frac{\sqrt{2}\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}+1}d\frac{\sqrt[8]{iax+1}}{\sqrt[8]{1-iax}}\right)\right.\right.\right.$$

$$\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2}$$

$\downarrow$ 1103

$$\frac{1}{8}ia\left(2ia\left(\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{1+iax}}{\sqrt[8]{1-iax}}\right)}{\sqrt{2}}-\frac{\text{arct}}{}\right.\right.\right.\right.$$

$$\frac{(1-iax)^{7/8}(1+iax)^{9/8}}{2x^2}$$

input
```
Int[E^((I/4)*ArcTan[a*x])/x^3,x]
```

output
```
-1/2*((1 - I*a*x)^(7/8)*(1 + I*a*x)^(9/8))/x^2 + (I/8)*a*(-(((1 - I*a*x)^(
7/8)*(1 + I*a*x)^(1/8))/x) + (2*I)*a*((-1/2*ArcTan[(1 + I*a*x)^(1/8)/(1 -
I*a*x)^(1/8)] - ArcTanh[(1 + I*a*x)^(1/8)/(1 - I*a*x)^(1/8)]/2)/2 + ((ArcT
an[1 - (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8)]/Sqrt[2] - ArcTan[1 +
 (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8)]/Sqrt[2])/2 + (Log[1 - (Sqr
t[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8) + (1 + I*a*x)^(1/4)/(1 - I*a*x)^
(1/4)]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*(1 + I*a*x)^(1/8))/(1 - I*a*x)^(1/8)
 + (1 + I*a*x)^(1/4)/(1 - I*a*x)^(1/4)]/(2*Sqrt[2]))/2)/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 758
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^(n/2)),
 x], x] + Simp[r/(2*a)   Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x]
 && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}}{x^3}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^3,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.29

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^3}\,dx$$
$$= \frac{a^2x^2\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}+1\right)+i\,a^2x^2\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}+i\right)-i\,a^2x^2\log\left(\left(\frac{i\sqrt{a^2x^2+1}}{ax+i}\right)^{\frac{1}{4}}-i\right)-a^2x^2\log}{}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^3,x, algorithm="fricas")
```

output
```
1/32*(a^2*x^2*log((I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) + 1) + I*a^2*x^2*l
og((I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) + I) - I*a^2*x^2*log((I*sqrt(a^2*
x^2 + 1)/(a*x + I))^(1/4) - I) - a^2*x^2*log((I*sqrt(a^2*x^2 + 1)/(a*x + I
))^(1/4) - 1) + sqrt(I*a^4)*x^2*log((a^2*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(
1/4) + sqrt(I*a^4))/a^2) - sqrt(I*a^4)*x^2*log((a^2*(I*sqrt(a^2*x^2 + 1)/(
a*x + I))^(1/4) - sqrt(I*a^4))/a^2) + sqrt(-I*a^4)*x^2*log((a^2*(I*sqrt(a^
2*x^2 + 1)/(a*x + I))^(1/4) + sqrt(-I*a^4))/a^2) - sqrt(-I*a^4)*x^2*log((a
^2*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4) - sqrt(-I*a^4))/a^2) - 4*(5*a^2*x
^2 + I*a*x + 4)*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4))/x^2
```

Sympy [F]

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^3}\,dx = \int \frac{\sqrt[4]{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}{x^3}\,dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4)/x**3,x)
```

output
```
Integral((I*(a*x - I)/sqrt(a**2*x**2 + 1))**(1/4)/x**3, x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^3}\,dx = \int \frac{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}}{x^3}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^3,x, algorithm="maxima")
```

output
```
integrate(((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4)/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^3}\,dx = \int \frac{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{1/4}}{x^3}\,dx$$

input
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4)/x^3,x)
```

output
```
int(((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4)/x^3, x)
```

Reduce [F]

$$\int \frac{e^{\frac{1}{4}i\arctan(ax)}}{x^3}\,dx = \int \frac{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}}{x^3}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^3,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)/x^3,x)
```

3.149 $\int e^{6i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 124

$$\int e^{6i\arctan(ax)}x^m\,dx = \frac{4(2+m)x^{1+m}}{(1+m)(1-iax)^2}-\frac{4(3+3m+m^2)\,x^{1+m}}{(1+m)(1-iax)}-\frac{x^{1+m}(1+iax)^2}{(1+m)(1-iax)^2}+\frac{2(3+4m+2m^2)\,x^{1+m}\operatorname{Hypergeometric2F1}(1,1+m,2+m,iax)}{1+m}$$

output
```
4*(2+m)*x^(1+m)/(1+m)/(1-I*a*x)^2-4*(m^2+3*m+3)*x^(1+m)/(1+m)/(1-I*a*x)-x^
(1+m)*(1+I*a*x)^2/(1+m)/(1-I*a*x)^2+2*(2*m^2+4*m+3)*x^(1+m)*hypergeom([1,
1+m],[2+m],I*a*x)/(1+m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.76

$$\int e^{6i\arctan(ax)}x^m\,dx = \frac{x^{1+m}(5-10iax-a^2x^2+4m(2-3iax)+m^2(4-4iax)+2(3+4m+2m^2)\,(i+ax)^2\operatorname{Hypergeometric2}}{(1+m)(i+ax)^2}$$

input
```
Integrate[E^((6*I)*ArcTan[a*x])*x^m,x]
```

output
```
(x^(1 + m)*(5 - (10*I)*a*x - a^2*x^2 + 4*m*(2 - (3*I)*a*x) + m^2*(4 - (4*I
)*a*x) + 2*(3 + 4*m + 2*m^2)*(I + a*x)^2*Hypergeometric2F1[1, 1 + m, 2 + m
, I*a*x]))/((1 + m)*(I + a*x)^2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5585, 111, 27, 162, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{6i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{(1+iax)^3x^m}{(1-iax)^3}dx$$

$$\Big\downarrow \text{111}$$

$$\frac{i\int -\frac{2ax^m(iax+1)(i(m+1)-a(m+3)x)}{(1-iax)^3}dx}{a(m+1)}-\frac{(1+iax)^2x^{m+1}}{(m+1)(1-iax)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2i\int \frac{x^m(iax+1)(i(m+1)-a(m+3)x)}{(1-iax)^3}dx}{m+1}-\frac{(1+iax)^2x^{m+1}}{(m+1)(1-iax)^2}$$

$$\Big\downarrow \text{162}$$

$$-\frac{2i\left(i(m+1)\left(2m^2+4m+3\right)\int \frac{x^m}{1-iax}dx-\frac{2x^{m+1}\left(a\left(m^2+3m+3\right)x+i(m+1)^2\right)}{(1-iax)^2}\right)}{m+1}-\frac{(1+iax)^2x^{m+1}}{(m+1)(1-iax)^2}$$

$$\Big\downarrow \text{74}$$

$$-\frac{2i\left(i\left(2m^2+4m+3\right)x^{m+1}\,\text{Hypergeometric2F1}(1,m+1,m+2,iax)-\frac{2x^{m+1}\left(a\left(m^2+3m+3\right)x+i(m+1)^2\right)}{(1-iax)^2}\right)}{m+1}-\frac{(1+iax)^2x^{m+1}}{(m+1)(1-iax)^2}$$

input
```
Int[E^((6*I)*ArcTan[a*x])*x^m,x]
```

output
```
-((x^(1 + m)*(1 + I*a*x)^2)/((1 + m)*(1 - I*a*x)^2)) - ((2*I)*((-2*x^(1 +
m)*(I*(1 + m)^2 + a*(3 + 3*m + m^2)*x))/(1 - I*a*x)^2 + I*(3 + 4*m + 2*m^2
)*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, I*a*x]))/(1 + m)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.68 (sec) , antiderivative size = 748, normalized size of antiderivative = 6.03

method	result
meijerg	$\frac{\left(a^2\right)^{-\frac{1}{2}-\frac{m}{2}}\left(\frac{x^{1+m}\left(a^2\right)^{\frac{m}{2}+\frac{1}{2}}\left(-a^2m^2x^2+2a^2m\,x^2+3a^2x^2-m^2+4m+5\right)}{2(1+m)\left(a^2x^2+1\right)^2}+\frac{4x^{1+m}\left(a^2\right)^{\frac{m}{2}+\frac{1}{2}}\left(\frac{1}{16}m^3-\frac{3}{16}m^2-\frac{1}{16}m+\frac{3}{16}\right)\mathrm{LerchPhi}\left(-a^2x\right.}{1+m}}{4}$

input
```
int((1+I*a*x)^6/(a^2*x^2+1)^3*x^m,x,method=_RETURNVERBOSE)
```

output
```
1/4*(a^2)^(-1/2-1/2*m)*(1/2/(1+m)*x^(1+m)*(a^2)^(1/2*m+1/2)*(-a^2*m^2*x^2+
2*a^2*m*x^2+3*a^2*x^2-m^2+4*m+5)/(a^2*x^2+1)^2+4/(1+m)*x^(1+m)*(a^2)^(1/2*
m+1/2)*(1/16*m^3-3/16*m^2-1/16*m+3/16)*LerchPhi(-a^2*x^2,1,1/2*m+1/2))+3/2
*I/a*(a^2)^(-1/2*m)*(1/2*x^m*(a^2)^(1/2*m)*(a^2*m*x^2+m-2)/(a^2*x^2+1)^2-1
/4*x^m*(a^2)^(1/2*m)*(-2+m)*m*LerchPhi(-a^2*x^2,1,1/2*m))-15/4*(a^2)^(-1/2
-1/2*m)*(1/2*x^(1+m)*(a^2)^(3/2+1/2*m)*(a^2*m*x^2+a^2*x^2+m-1)/(a^2*x^2+1)
^2/a^2-1/4*x^(1+m)*(a^2)^(3/2+1/2*m)*(1+m)*(m-1)/a^2*LerchPhi(-a^2*x^2,1,1
/2*m+1/2))-5*I/a*(a^2)^(-1/2*m)*(-1/2*x^m*(a^2)^(1/2*m)*(a^2*m*x^2+4*a^2*x
^2+m+2)/(a^2*x^2+1)^2+1/4*x^m*(a^2)^(1/2*m)*m*(2+m)*LerchPhi(-a^2*x^2,1,1/
2*m))+15/4*(a^2)^(-1/2-1/2*m)*(-1/2*x^(1+m)*(a^2)^(5/2+1/2*m)*(a^2*m*x^2+5
*a^2*x^2+m+3)/a^4/(a^2*x^2+1)^2+1/4*x^(1+m)*(a^2)^(5/2+1/2*m)*(m^2+4*m+3)/
a^4*LerchPhi(-a^2*x^2,1,1/2*m+1/2))+3/2*I/a*(a^2)^(-1/2*m)*(1/2*x^m*(a^2)^
(1/2*m)*(8*a^4*x^4+a^2*m^2*x^2+8*a^2*m*x^2+16*a^2*x^2+m^2+6*m+8)/(a^2*x^2+
1)^2/m-1/4*x^m*(a^2)^(1/2*m)*(m^2+6*m+8)*LerchPhi(-a^2*x^2,1,1/2*m))-1/4*(
a^2)^(-1/2-1/2*m)*(1/2*x^(1+m)*(a^2)^(7/2+1/2*m)*(8*a^4*x^4+a^2*m^2*x^2+10
*a^2*m*x^2+25*a^2*x^2+m^2+8*m+15)/(a^2*x^2+1)^2/(1+m)/a^6-1/4*x^(1+m)*(a^2
)^(7/2+1/2*m)*(m^2+8*m+15)/a^6*LerchPhi(-a^2*x^2,1,1/2*m+1/2))
```

Fricas [F]

$$\int e^{6i\arctan(ax)}x^m\,dx = \int \frac{(i\,ax+1)^6x^m}{(a^2x^2+1)^3}\,dx$$

input
```
integrate((1+I*a*x)^6/(a^2*x^2+1)^3*x^m,x, algorithm="fricas")
```

output
```
integral(-(a^3*x^3 - 3*I*a^2*x^2 - 3*a*x + I)*x^m/(a^3*x^3 + 3*I*a^2*x^2 -
 3*a*x - I), x)
```

Sympy [F]

$$\int e^{6i\arctan(ax)}x^m\,dx = -\int\left(-\frac{x^m}{a^6x^6+3a^4x^4+3a^2x^2+1}\right)dx - \int\frac{15a^2x^2x^m}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx - \int\left(-\frac{15a^4x^4x^m}{a^6x^6+3a^4x^4+3a^2x^2+1}\right)dx - \int\frac{a^6x^6x^m}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx - \int\left(-\frac{6iaxx^m}{a^6x^6+3a^4x^4+3a^2x^2+1}\right)dx - \int\frac{20ia^3x^3x^m}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx - \int\left(-\frac{6ia^5x^5x^m}{a^6x^6+3a^4x^4+3a^2x^2+1}\right)dx$$

input
```
integrate((1+I*a*x)**6/(a**2*x**2+1)**3*x**m,x)
```

output
```
-Integral(-x**m/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x) - Integral
(15*a**2*x**2*x**m/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x) - Integ
ral(-15*a**4*x**4*x**m/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x) - I
ntegral(a**6*x**6*x**m/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x) - I
ntegral(-6*I*a*x*x**m/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x) - In
tegral(20*I*a**3*x**3*x**m/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)
 - Integral(-6*I*a**5*x**5*x**m/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1
), x)
```

Maxima [F]

$$\int e^{6i\arctan(ax)}x^m\,dx = \int\frac{(i\,ax+1)^6x^m}{(a^2x^2+1)^3}\,dx$$

input
```
integrate((1+I*a*x)^6/(a^2*x^2+1)^3*x^m,x, algorithm="maxima")
```

output
```
integrate((I*a*x + 1)^6*x^m/(a^2*x^2 + 1)^3, x)
```

Giac [F]

$$\int e^{6i\arctan(ax)}x^m\,dx = \int \frac{(i\,ax+1)^6 x^m}{(a^2x^2+1)^3}\,dx$$

input
```
integrate((1+I*a*x)^6/(a^2*x^2+1)^3*x^m,x, algorithm="giac")
```

output
```
integrate((I*a*x + 1)^6*x^m/(a^2*x^2 + 1)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{6i\arctan(ax)}x^m\,dx = \int \frac{x^m\,(1+a\,x\,1\mathrm{i})^6}{(a^2\,x^2+1)^3}\,dx$$

input
```
int((x^m*(a*x*1i + 1)^6)/(a^2*x^2 + 1)^3,x)
```

output
```
int((x^m*(a*x*1i + 1)^6)/(a^2*x^2 + 1)^3, x)
```

Reduce [F]

$$\int e^{6i\arctan(ax)}x^m\,dx = \text{too large to display}$$

input
```
int((1+I*a*x)^6/(a^2*x^2+1)^3*x^m,x)
```

output

```
( - x**m*a**5*m**5*x**5 + 10*x**m*a**5*m**4*x**5 - 35*x**m*a**5*m**3*x**5
+ 50*x**m*a**5*m**2*x**5 - 24*x**m*a**5*m*x**5 + 6*x**m*a**4*i*m**5*x**4 -
 54*x**m*a**4*i*m**4*x**4 + 150*x**m*a**4*i*m**3*x**4 - 90*x**m*a**4*i*m**
2*x**4 - 156*x**m*a**4*i*m*x**4 + 144*x**m*a**4*i*x**4 + 16*x**m*a**3*m**5
*x**3 - 124*x**m*a**3*m**4*x**3 + 236*x**m*a**3*m**3*x**3 + 136*x**m*a**3*
m**2*x**3 - 480*x**m*a**3*m*x**3 - 26*x**m*a**2*i*m**5*x**2 + 158*x**m*a**
2*i*m**4*x**2 - 118*x**m*a**2*i*m**3*x**2 - 446*x**m*a**2*i*m**2*x**2 + 14
4*x**m*a**2*i*m*x**2 + 288*x**m*a**2*i*x**2 - 31*x**m*a*m**5*x + 118*x**m*
a*m**4*x + 115*x**m*a*m**3*x - 274*x**m*a*m**2*x - 360*x**m*a*m*x + 32*x**
m*i*m**5 - 32*x**m*i*m**4 - 176*x**m*i*m**3 - 112*x**m*i*m**2 + 144*x**m*i
*m + 144*x**m*i - 32*int(x**m/(a**6*m**4*x**7 - 10*a**6*m**3*x**7 + 35*a**
6*m**2*x**7 - 50*a**6*m*x**7 + 24*a**6*x**7 + 3*a**4*m**4*x**5 - 30*a**4*m
**3*x**5 + 105*a**4*m**2*x**5 - 150*a**4*m*x**5 + 72*a**4*x**5 + 3*a**2*m*
*4*x**3 - 30*a**2*m**3*x**3 + 105*a**2*m**2*x**3 - 150*a**2*m*x**3 + 72*a*
*2*x**3 + m**4*x - 10*m**3*x + 35*m**2*x - 50*m*x + 24*x),x)*a**4*i*m**10*
x**4 + 352*int(x**m/(a**6*m**4*x**7 - 10*a**6*m**3*x**7 + 35*a**6*m**2*x**
7 - 50*a**6*m*x**7 + 24*a**6*x**7 + 3*a**4*m**4*x**5 - 30*a**4*m**3*x**5 +
 105*a**4*m**2*x**5 - 150*a**4*m*x**5 + 72*a**4*x**5 + 3*a**2*m**4*x**3 -
30*a**2*m**3*x**3 + 105*a**2*m**2*x**3 - 150*a**2*m*x**3 + 72*a**2*x**3 +
m**4*x - 10*m**3*x + 35*m**2*x - 50*m*x + 24*x),x)*a**4*i*m**9*x**4 - 1...
```

3.150 $\int e^{4i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int e^{4i\arctan(ax)}x^m\,dx=\frac{x^{1+m}}{1+m}+\frac{4x^{1+m}}{1-iax}-4x^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,iax)$$

output
```
x^(1+m)/(1+m)+4*x^(1+m)/(1-I*a*x)-4*x^(1+m)*hypergeom([1, 1+m],[2+m],I*a*x
)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.16

$$\int e^{4i\arctan(ax)}x^m\,dx =\frac{x^{1+m}(5i+4im+ax-4(1+m)(i+ax)\,\text{Hypergeometric2F1}(1,1+m,2+m,iax))}{(1+m)(i+ax)}$$

input
```
Integrate[E^((4*I)*ArcTan[a*x])*x^m,x]
```

output
```
(x^(1 + m)*(5*I + (4*I)*m + a*x - 4*(1 + m)*(I + a*x)*Hypergeometric2F1[1,
 1 + m, 2 + m, I*a*x]))/((1 + m)*(I + a*x))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5585, 100, 25, 27, 90, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{4 i \arctan (a x)} \, dx$$

$$\Big\downarrow \text{ 5585}$$

$$\int \frac{(1+i a x)^2 x^m}{(1-i a x)^2}dx$$

$$\Big\downarrow \text{ 100}$$

$$\frac{\int -\frac{a^2 x^m (4 m+i a x+3)}{1-i a x}dx}{a^2}+\frac{4 x^{m+1}}{1-i a x}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{4 x^{m+1}}{1-i a x}-\frac{\int \frac{a^2 x^m (4 m+i a x+3)}{1-i a x}dx}{a^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4 x^{m+1}}{1-i a x}-\int \frac{x^m (4 m+i a x+3)}{1-i a x}dx$$

$$\Big\downarrow \text{ 90}$$

$$-4 (m+1) \int \frac{x^m}{1-i a x}dx+\frac{4 x^{m+1}}{1-i a x}+\frac{x^{m+1}}{m+1}$$

$$\Big\downarrow \text{ 74}$$

$$-4 x^{m+1} \text{Hypergeometric2F1}(1,m+1,m+2,i a x)+\frac{4 x^{m+1}}{1-i a x}+\frac{x^{m+1}}{m+1}$$

input
```
Int[E^((4*I)*ArcTan[a*x])*x^m,x]
```

output
```
x^(1 + m)/(1 + m) + (4*x^(1 + m))/(1 - I*a*x) - 4*x^(1 + m)*Hypergeometric
2F1[1, 1 + m, 2 + m, I*a*x]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.33 (sec) , antiderivative size = 417, normalized size of antiderivative = 8.34

method	result
meijerg	$\frac{\left(a^{2}\right)^{-\frac{1}{2}-\frac{m}{2}} \left(\frac{2 x^{1+m} \left(a^{2}\right)^{\frac{m}{2}+\frac{1}{2}}}{2 a^{2} x^{2}+2}+\frac{2 x^{1+m} \left(a^{2}\right)^{\frac{m}{2}+\frac{1}{2}} \left(-\frac{m^{2}}{4}+\frac{1}{4}\right) \mathrm{LerchPhi}\left(-a^{2} x^{2}, 1, \frac{m}{2}+\frac{1}{2}\right)}{1+m}\right)}{2}+\frac{2 i \left(a^{2}\right)^{-\frac{m}{2}} \left(\frac{x^{m} \left(a^{2}\right)^{\frac{m}{2}} \left(-2-m\right)}{\left(2+m\right) \left(a^{2} x^{2}+1\right)}+\frac{x^{m}\cdots}{\cdots}\right.}{a\cdots}$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^2*x^m,x,method=_RETURNVERBOSE)
```

output
```
1/2*(a^2)^(-1/2-1/2*m)*(2*x^(1+m)*(a^2)^(1/2*m+1/2)/(2*a^2*x^2+2)+2/(1+m)*
x^(1+m)*(a^2)^(1/2*m+1/2)*(-1/4*m^2+1/4)*LerchPhi(-a^2*x^2,1,1/2*m+1/2))+2
*I/a*(a^2)^(-1/2*m)*(1/(2+m)*x^m*(a^2)^(1/2*m)*(-2-m)/(a^2*x^2+1)+1/2*x^m*
(a^2)^(1/2*m)*m*LerchPhi(-a^2*x^2,1,1/2*m))-3*(a^2)^(-1/2-1/2*m)*(1/(3+m)*
x^(1+m)*(a^2)^(3/2+1/2*m)*(-3-m)/a^2/(a^2*x^2+1)+1/2*x^(1+m)*(a^2)^(3/2+1/
2*m)*(1+m)/a^2*LerchPhi(-a^2*x^2,1,1/2*m+1/2))-2*I/a*(a^2)^(-1/2*m)*(x^m*(
a^2)^(1/2*m)*(2*a^2*x^2+m+2)/(a^2*x^2+1)/m-1/2*x^m*(a^2)^(1/2*m)*(2+m)*Ler
chPhi(-a^2*x^2,1,1/2*m))+1/2*(a^2)^(-1/2-1/2*m)*(x^(1+m)*(a^2)^(5/2+1/2*m)
*(2*a^2*x^2+m+3)/(a^2*x^2+1)/a^4/(1+m)-1/2*x^(1+m)*(a^2)^(5/2+1/2*m)*(3+m)
/a^4*LerchPhi(-a^2*x^2,1,1/2*m+1/2))
```

Fricas [F]

$$\int e^{4 i \arctan(ax)} x^{m}\, dx = \int \frac{\left(i\, a x+1\right)^{4} x^{m}}{\left(a^{2} x^{2}+1\right)^{2}}\, dx$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^m,x, algorithm="fricas")
```

output
```
integral((a^2*x^2 - 2*I*a*x - 1)*x^m/(a^2*x^2 + 2*I*a*x - 1), x)
```

Sympy [F]

$$\int e^{4i\arctan(ax)}x^m\,dx = \int \frac{x^m(ax-i)^4}{(a^2x^2+1)^2}\,dx$$

input
```
integrate((1+I*a*x)**4/(a**2*x**2+1)**2*x**m,x)
```

output
```
Integral(x**m*(a*x - I)**4/(a**2*x**2 + 1)**2, x)
```

Maxima [F]

$$\int e^{4i\arctan(ax)}x^m\,dx = \int \frac{(i\,ax+1)^4x^m}{(a^2x^2+1)^2}\,dx$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^m,x, algorithm="maxima")
```

output
```
integrate((I*a*x + 1)^4*x^m/(a^2*x^2 + 1)^2, x)
```

Giac [F]

$$\int e^{4i\arctan(ax)}x^m\,dx = \int \frac{(i\,ax+1)^4x^m}{(a^2x^2+1)^2}\,dx$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^m,x, algorithm="giac")
```

output
```
integrate((I*a*x + 1)^4*x^m/(a^2*x^2 + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{4i\arctan(ax)}x^m\,dx=\int \frac{x^m\,(1+a\,x\,1\mathrm{i})^4}{(a^2\,x^2+1)^2}\,dx$$

input
```
int((x^m*(a*x*1i + 1)^4)/(a^2*x^2 + 1)^2,x)
```

output
```
int((x^m*(a*x*1i + 1)^4)/(a^2*x^2 + 1)^2, x)
```

Reduce [F]

$$\int e^{4i\arctan(ax)}x^m\,dx=\text{too large to display}$$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^2*x^m,x)
```

output
```
(x**m*a**3*m**3*x**3 - 3*x**m*a**3*m**2*x**3 + 2*x**m*a**3*m*x**3 - 4*x**m
*a**2*i*m**3*x**2 + 8*x**m*a**2*i*m**2*x**2 + 4*x**m*a**2*i*m*x**2 - 8*x**
m*a**2*i*x**2 - 7*x**m*a*m**3*x + 5*x**m*a*m**2*x + 18*x**m*a*m*x + 8*x**m
*i*m**3 + 8*x**m*i*m**2 - 8*x**m*i*m - 8*x**m*i - 8*int(x**m/(a**4*m**2*x*
*5 - 3*a**4*m*x**5 + 2*a**4*x**5 + 2*a**2*m**2*x**3 - 6*a**2*m*x**3 + 4*a*
*2*x**3 + m**2*x - 3*m*x + 2*x),x)*a**2*i*m**6*x**2 + 16*int(x**m/(a**4*m*
*2*x**5 - 3*a**4*m*x**5 + 2*a**4*x**5 + 2*a**2*m**2*x**3 - 6*a**2*m*x**3 +
 4*a**2*x**3 + m**2*x - 3*m*x + 2*x),x)*a**2*i*m**5*x**2 + 16*int(x**m/(a*
*4*m**2*x**5 - 3*a**4*m*x**5 + 2*a**4*x**5 + 2*a**2*m**2*x**3 - 6*a**2*m*x
**3 + 4*a**2*x**3 + m**2*x - 3*m*x + 2*x),x)*a**2*i*m**4*x**2 - 32*int(x**
m/(a**4*m**2*x**5 - 3*a**4*m*x**5 + 2*a**4*x**5 + 2*a**2*m**2*x**3 - 6*a**
2*m*x**3 + 4*a**2*x**3 + m**2*x - 3*m*x + 2*x),x)*a**2*i*m**3*x**2 - 8*int
(x**m/(a**4*m**2*x**5 - 3*a**4*m*x**5 + 2*a**4*x**5 + 2*a**2*m**2*x**3 - 6
*a**2*m*x**3 + 4*a**2*x**3 + m**2*x - 3*m*x + 2*x),x)*a**2*i*m**2*x**2 + 1
6*int(x**m/(a**4*m**2*x**5 - 3*a**4*m*x**5 + 2*a**4*x**5 + 2*a**2*m**2*x**
3 - 6*a**2*m*x**3 + 4*a**2*x**3 + m**2*x - 3*m*x + 2*x),x)*a**2*i*m*x**2 -
 8*int(x**m/(a**4*m**2*x**5 - 3*a**4*m*x**5 + 2*a**4*x**5 + 2*a**2*m**2*x*
*3 - 6*a**2*m*x**3 + 4*a**2*x**3 + m**2*x - 3*m*x + 2*x),x)*i*m**6 + 16*in
t(x**m/(a**4*m**2*x**5 - 3*a**4*m*x**5 + 2*a**4*x**5 + 2*a**2*m**2*x**3 -
6*a**2*m*x**3 + 4*a**2*x**3 + m**2*x - 3*m*x + 2*x),x)*i*m**5 + 16*int(...
```

3.151 $\int e^{2i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int e^{2i\arctan(ax)}x^m\,dx = -\frac{x^{1+m}}{1+m}+\frac{2x^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,iax)}{1+m}$$

output
```
-x^(1+m)/(1+m)+2*x^(1+m)*hypergeom([1, 1+m],[2+m],I*a*x)/(1+m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int e^{2i\arctan(ax)}x^m\,dx = \frac{x^{1+m}(-1+2\,\text{Hypergeometric2F1}(1,1+m,2+m,iax))}{1+m}$$

input
```
Integrate[E^((2*I)*ArcTan[a*x])*x^m,x]
```

output
```
(x^(1 + m)*(-1 + 2*Hypergeometric2F1[1, 1 + m, 2 + m, I*a*x]))/(1 + m)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5585, 90, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{2i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{(1+iax)x^m}{1-iax}dx$$

$$\Big\downarrow \text{90}$$

$$-\frac{x^{m+1}}{m+1}+2\int \frac{x^m}{1-iax}dx$$

$$\Big\downarrow \text{74}$$

$$-\frac{x^{m+1}}{m+1}+\frac{2x^{m+1}\,\text{Hypergeometric2F1}(1,m+1,m+2,iax)}{m+1}$$

input
```
Int[E^((2*I)*ArcTan[a*x])*x^m,x]
```

output
```
-(x^(1 + m)/(1 + m)) + (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, I*a
*x])/(1 + m)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.18 (sec) , antiderivative size = 175, normalized size of antiderivative = 4.49

method	result
meijerg	$\frac{x^{1+m}\left(\frac{m}{2}+\frac{1}{2}\right)\mathrm{LerchPhi}\left(-a^2x^2,1,\frac{m}{2}+\frac{1}{2}\right)}{1+m}+\frac{i(a^2)^{-\frac{m}{2}}\left(\frac{2x^m(a^2)^{\frac{m}{2}}}{m}+\frac{x^m(a^2)^{\frac{m}{2}}(-2-m)\mathrm{LerchPhi}\left(-a^2x^2,1,\frac{m}{2}\right)}{2+m}\right)}{a}-\frac{(a^2)^{-\frac{1}{2}-\frac{m}{2}}}{}$

input
```
int((1+I*a*x)^2/(a^2*x^2+1)*x^m,x,method=_RETURNVERBOSE)
```

output
```
1/(1+m)*x^(1+m)*(1/2*m+1/2)*LerchPhi(-a^2*x^2,1,1/2*m+1/2)+I/a*(a^2)^(-1/2
*m)*(2*x^m*(a^2)^(1/2*m)/m+1/(2+m)*x^m*(a^2)^(1/2*m)*(-2-m)*LerchPhi(-a^2*
x^2,1,1/2*m))-1/2*(a^2)^(-1/2-1/2*m)*(2*x^(1+m)*(a^2)^(3/2+1/2*m)/(1+m)/a^
2+1/(3+m)*x^(1+m)*(a^2)^(3/2+1/2*m)*(-3-m)/a^2*LerchPhi(-a^2*x^2,1,1/2*m+1
/2))
```

Fricas [F]

$$\int e^{2i\arctan(ax)}x^m\,dx = \int \frac{(i\,ax+1)^2x^m}{a^2x^2+1}\,dx$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)*x^m,x, algorithm="fricas")
```

output
```
integral(-(a*x - I)*x^m/(a*x + I), x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(27) = 54$.

Time = 1.66 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.23

$$\int e^{2i\arctan(ax)}x^m\,dx = \frac{iamx^{m+2}\Phi\left(axe^{\frac{i\pi}{2}},1,m+2\right)\Gamma(m+2)}{\Gamma(m+3)} + \frac{2iax^{m+2}\Phi\left(axe^{\frac{i\pi}{2}},1,m+2\right)\Gamma(m+2)}{\Gamma(m+3)} + \frac{mx^{m+1}\Phi\left(axe^{\frac{i\pi}{2}},1,m+1\right)\Gamma(m+1)}{\Gamma(m+2)} + \frac{x^{m+1}\Phi\left(axe^{\frac{i\pi}{2}},1,m+1\right)\Gamma(m+1)}{\Gamma(m+2)}$$

input
```
integrate((1+I*a*x)**2/(a**2*x**2+1)*x***m,x)
```

output
```
I*a*m*x**(m + 2)*lerchphi(a*x*exp_polar(I*pi/2), 1, m + 2)*gamma(m + 2)/ga
mma(m + 3) + 2*I*a*x**(m + 2)*lerchphi(a*x*exp_polar(I*pi/2), 1, m + 2)*ga
mma(m + 2)/gamma(m + 3) + m*x**(m + 1)*lerchphi(a*x*exp_polar(I*pi/2), 1,
m + 1)*gamma(m + 1)/gamma(m + 2) + x**(m + 1)*lerchphi(a*x*exp_polar(I*pi/
2), 1, m + 1)*gamma(m + 1)/gamma(m + 2)
```

Maxima [F]

$$\int e^{2i\arctan(ax)}x^m\,dx = \int \frac{(i\,ax+1)^2x^m}{a^2x^2+1}\,dx$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)*x^m,x, algorithm="maxima")
```

output
```
integrate((I*a*x + 1)^2*x^m/(a^2*x^2 + 1), x)
```

Giac [F]

$$\int e^{2i\arctan(ax)}x^m\,dx = \int \frac{(i\,ax+1)^2x^m}{a^2x^2+1}\,dx$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)*x^m,x, algorithm="giac")
```

output
```
integrate((I*a*x + 1)^2*x^m/(a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2i\arctan(ax)}x^m\,dx = \int \frac{x^m\,(1+a\,x\,1\mathrm{i})^2}{a^2\,x^2+1}\,dx$$

input
```
int((x^m*(a*x*1i + 1)^2)/(a^2*x^2 + 1),x)
```

output
```
int((x^m*(a*x*1i + 1)^2)/(a^2*x^2 + 1), x)
```

Reduce [F]

$$\int e^{2i\arctan(ax)}x^m\,dx$$
$$= \frac{-x^m amx + 2x^m im + 2x^m i - 2\left(\int \frac{x^m}{a^2x^3+x}dx\right) i\,m^2 - 2\left(\int \frac{x^m}{a^2x^3+x}dx\right) im + 2\left(\int \frac{x^m}{a^2x^2+1}dx\right) a\,m^2 + 2\left(\int \frac{x^m}{a^2x^2+}\right.}{am\,(m+1)}$$

input
```
int((1+I*a*x)^2/(a^2*x^2+1)*x^m,x)
```

output
```
( - x**m*a*m*x + 2*x**m*i*m + 2*x**m*i - 2*int(x**m/(a**2*x**3 + x),x)*i*m
**2 - 2*int(x**m/(a**2*x**3 + x),x)*i*m + 2*int(x**m/(a**2*x**2 + 1),x)*a*
m**2 + 2*int(x**m/(a**2*x**2 + 1),x)*a*m)/(a*m*(m + 1))
```

3.152 $\int e^{-2i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int e^{-2i\arctan(ax)}x^m\,dx = -\frac{x^{1+m}}{1+m}+\frac{2x^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,-iax)}{1+m}$$

output
```
-x^(1+m)/(1+m)+2*x^(1+m)*hypergeom([1, 1+m],[2+m],-I*a*x)/(1+m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int e^{-2i\arctan(ax)}x^m\,dx = \frac{x^{1+m}(-1+2\,\text{Hypergeometric2F1}(1,1+m,2+m,-iax))}{1+m}$$

input
```
Integrate[x^m/E^((2*I)*ArcTan[a*x]),x]
```

output
```
(x^(1 + m)*(-1 + 2*Hypergeometric2F1[1, 1 + m, 2 + m, (-I)*a*x]))/(1 + m)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5585, 90, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{-2i\arctan(ax)}\,dx$$

$$\Big\downarrow \ 5585$$

$$\int \frac{(1-iax)x^m}{1+iax}dx$$

$$\Big\downarrow \ 90$$

$$-\frac{x^{m+1}}{m+1}+2\int \frac{x^m}{iax+1}dx$$

$$\Big\downarrow \ 74$$

$$-\frac{x^{m+1}}{m+1}+\frac{2x^{m+1}\,\text{Hypergeometric2F1}(1,m+1,m+2,-iax)}{m+1}$$

input
```
Int[x^m/E^((2*I)*ArcTan[a*x]),x]
```

output
```
-(x^(1 + m)/(1 + m)) + (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-I
)*a*x])/(1 + m)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.19 (sec) , antiderivative size = 158, normalized size of antiderivative = 4.05

method	result
meijerg	$\frac{i(ia)^{-m}\left(\frac{x^m(ia)^m\left(-a^2 m\,x^2-iamx-2iax-m^2-3m-2\right)}{(1+m)m(iax+1)}+x^m(ia)^m(2+m)\operatorname{LerchPhi}(-iax,1,m)\right)}{a}-\frac{i(ia)^{-m}\left(\frac{x^m(ia)^m(-1-m)}{(1+m)(iax+1)}+x^r\right.}{}$

input
```
int(x^m/(1+I*a*x)^2*(a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
I*(I*a)^(-m)/a*(x^m*(I*a)^m*(-a^2*m*x^2-I*a*m*x-m^2-2*I*a*x-3*m-2)/(1+m)/m
/(1+I*a*x)+x^m*(I*a)^m*(2+m)*LerchPhi(-I*a*x,1,m))-I*(I*a)^(-m)/a*(1/(1+m)
*x^m*(I*a)^m*(-1-m)/(1+I*a*x)+x^m*(I*a)^m*m*LerchPhi(-I*a*x,1,m))
```

Fricas [F]

$$\int e^{-2i\arctan(ax)}x^m\,dx=\int\frac{(a^2x^2+1)x^m}{(i\,ax+1)^2}\,dx$$

input
```
integrate(x^m/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="fricas")
```

output
```
integral(-(a*x + I)*x^m/(a*x - I), x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(29) = 58$.

Time = 2.36 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.41

$$\int e^{-2i\arctan(ax)}x^m\,dx = -\frac{iamx^{m+2}\Phi\left(axe^{\frac{3i\pi}{2}},1,m+2\right)\Gamma(m+2)}{\Gamma(m+3)} - \frac{2iax^{m+2}\Phi\left(axe^{\frac{3i\pi}{2}},1,m+2\right)\Gamma(m+2)}{\Gamma(m+3)} + \frac{mx^{m+1}\Phi\left(axe^{\frac{3i\pi}{2}},1,m+1\right)\Gamma(m+1)}{\Gamma(m+2)} + \frac{x^{m+1}\Phi\left(axe^{\frac{3i\pi}{2}},1,m+1\right)\Gamma(m+1)}{\Gamma(m+2)}$$

input
```
integrate(x**m/(1+I*a*x)**2*(a**2*x**2+1),x)
```

output
```
-I*a*m*x**(m + 2)*lerchphi(a*x*exp_polar(3*I*pi/2), 1, m + 2)*gamma(m + 2)
/gamma(m + 3) - 2*I*a*x**(m + 2)*lerchphi(a*x*exp_polar(3*I*pi/2), 1, m +
2)*gamma(m + 2)/gamma(m + 3) + m*x**(m + 1)*lerchphi(a*x*exp_polar(3*I*pi/
2), 1, m + 1)*gamma(m + 1)/gamma(m + 2) + x**(m + 1)*lerchphi(a*x*exp_pola
r(3*I*pi/2), 1, m + 1)*gamma(m + 1)/gamma(m + 2)
```

Maxima [F]

$$\int e^{-2i\arctan(ax)}x^m\,dx = \int \frac{(a^2x^2+1)x^m}{(i\,ax+1)^2}\,dx$$

input
```
integrate(x^m/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="maxima")
```

output
```
integrate((a^2*x^2 + 1)*x^m/(I*a*x + 1)^2, x)
```

Giac [F]

$$\int e^{-2i\arctan(ax)}x^m\,dx = \int \frac{(a^2x^2+1)x^m}{(i\,ax+1)^2}\,dx$$

input
```
integrate(x^m/(1+I*a*x)^2*(a^2*x^2+1),x, algorithm="giac")
```

output
```
integrate((a^2*x^2 + 1)*x^m/(I*a*x + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2i\arctan(ax)}x^m\,dx = \int \frac{x^m\,(a^2\,x^2+1)}{(1+a\,x\,1\mathrm{i})^2}\,dx$$

input
```
int((x^m*(a^2*x^2 + 1))/(a*x*1i + 1)^2,x)
```

output
```
int((x^m*(a^2*x^2 + 1))/(a*x*1i + 1)^2, x)
```

Reduce [F]

$$\int e^{-2i\arctan(ax)}x^m\,dx = -\left(\int \frac{x^m}{a^2x^2-2aix-1}dx\right)-\left(\int \frac{x^m x^2}{a^2x^2-2aix-1}dx\right)a^2$$

input
```
int(x^m/(1+I*a*x)^2*(a^2*x^2+1),x)
```

output
```
 - (int(x**m/(a**2*x**2 - 2*a*i*x - 1),x) + int((x**m*x**2)/(a**2*x**2 - 2
*a*i*x - 1),x)*a**2)
```

3.153 $\int e^{-4i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int e^{-4i\arctan(ax)}x^m\,dx=\frac{x^{1+m}}{1+m}+\frac{4x^{1+m}}{1+iax}-4x^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,-iax)$$

output
```
x^(1+m)/(1+m)+4*x^(1+m)/(1+I*a*x)-4*x^(1+m)*hypergeom([1, 1+m],[2+m],-I*a*
x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.16

$$\int e^{-4i\arctan(ax)}x^m\,dx=\frac{x^{1+m}(-5i-4im+ax-4(1+m)(-i+ax)\,\text{Hypergeometric2F1}(1,1+m,2+m,-iax))}{(1+m)(-i+ax)}$$

input
```
Integrate[x^m/E^((4*I)*ArcTan[a*x]),x]
```

output
```
(x^(1 + m)*(-5*I - (4*I)*m + a*x - 4*(1 + m)*(-I + a*x)*Hypergeometric2F1[
1, 1 + m, 2 + m, (-I)*a*x]))/((1 + m)*(-I + a*x))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5585, 100, 25, 27, 90, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{-4i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{(1-iax)^2x^m}{(1+iax)^2}dx$$

$$\Big\downarrow \text{100}$$

$$\frac{\int -\frac{a^2x^m(4m-iax+3)}{iax+1}dx}{a^2}+\frac{4x^{m+1}}{1+iax}$$

$$\Big\downarrow \text{25}$$

$$\frac{4x^{m+1}}{1+iax}-\frac{\int \frac{a^2x^m(4m-iax+3)}{iax+1}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{4x^{m+1}}{1+iax}-\int \frac{x^m(4m-iax+3)}{iax+1}dx$$

$$\Big\downarrow \text{90}$$

$$-4(m+1)\int \frac{x^m}{iax+1}dx+\frac{4x^{m+1}}{1+iax}+\frac{x^{m+1}}{m+1}$$

$$\Big\downarrow \text{74}$$

$$-4x^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,-iax)+\frac{4x^{m+1}}{1+iax}+\frac{x^{m+1}}{m+1}$$

input
```
Int[x^m/E^((4*I)*ArcTan[a*x]),x]
```

output
```
x^(1 + m)/(1 + m) + (4*x^(1 + m))/(1 + I*a*x) - 4*x^(1 + m)*Hypergeometric
2F1[1, 1 + m, 2 + m, (-I)*a*x]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.47 (sec) , antiderivative size = 428, normalized size of antiderivative = 8.56

method	result
meijerg	$-\frac{i(ia)^{-m}\left(\frac{x^m (ia)^m \left(6a^4x^4m+6ia^3m\,x^3+a^2x^2m^4+24ia^3x^3+11a^2x^2m^3-2iax\,m^4+46a^2m^2x^2-21iax\,m^3+90a^2m\,x^2-79iax\,m^2+72a^2x^2-\right.}{(1+m)m(iax+1)^3}\right.}{6a}$

input
```
int(x^m/(1+I*a*x)^4*(a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*I*(I*a)^(-m)/a*(x^m*(I*a)^m*(a^2*x^2*m^4+6*a^4*x^4*m+11*a^2*x^2*m^3-1
26*I*a*m*x-79*I*a*x*m^2+46*a^2*m^2*x^2+6*I*a^3*m*x^3-m^4-21*I*a*x*m^3+90*a
^2*m*x^2+24*I*a^3*x^3-10*m^3+72*a^2*x^2-72*I*a*x-35*m^2-2*I*a*x*m^4-50*m-2
4)/(1+m)/m/(1+I*a*x)^3+x^m*(I*a)^m*(m^3+9*m^2+26*m+24)*LerchPhi(-I*a*x,1,m
))+1/3*I*(I*a)^(-m)/a*(-x^m*(I*a)^m*(-a^2*m^2*x^2-4*a^2*m*x^2+2*I*a*x*m^2-
6*a^2*x^2+7*I*a*m*x+m^2+6*I*a*x+3*m+2)/(1+I*a*x)^3+x^m*(I*a)^m*m*(m^2+3*m+
2)*LerchPhi(-I*a*x,1,m))-1/6*I*(I*a)^(-m)/a*(-x^m*(I*a)^m*(-a^2*m^2*x^2+2*
a^2*m*x^2+2*I*a*x*m^2-5*I*a*m*x+m^2-3*m+2)/(1+I*a*x)^3+x^m*(I*a)^m*(m^2-3*
m+2)*m*LerchPhi(-I*a*x,1,m))
```

Fricas [F]

$$\int e^{-4i\arctan(ax)}x^m\,dx=\int \frac{\left(a^2x^2+1\right)^2x^m}{\left(i\,ax+1\right)^4}\,dx$$

input
```
integrate(x^m/(1+I*a*x)^4*(a^2*x^2+1)^2,x, algorithm="fricas")
```

output
```
integral((a^2*x^2 + 2*I*a*x - 1)*x^m/(a^2*x^2 - 2*I*a*x - 1), x)
```

Sympy [F]

$$\int e^{-4i\arctan(ax)}x^m\,dx = \int \frac{x^m(a^2x^2+1)^2}{(ax-i)^4}\,dx$$

input
```
integrate(x**m/(1+I*a*x)**4*(a**2*x**2+1)**2,x)
```

output
```
Integral(x**m*(a**2*x**2 + 1)**2/(a*x - I)**4, x)
```

Maxima [F]

$$\int e^{-4i\arctan(ax)}x^m\,dx = \int \frac{(a^2x^2+1)^2x^m}{(i\,ax+1)^4}\,dx$$

input
```
integrate(x^m/(1+I*a*x)^4*(a^2*x^2+1)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*x^2 + 1)^2*x^m/(I*a*x + 1)^4, x)
```

Giac [F]

$$\int e^{-4i\arctan(ax)}x^m\,dx = \int \frac{(a^2x^2+1)^2x^m}{(i\,ax+1)^4}\,dx$$

input
```
integrate(x^m/(1+I*a*x)^4*(a^2*x^2+1)^2,x, algorithm="giac")
```

output
```
integrate((a^2*x^2 + 1)^2*x^m/(I*a*x + 1)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-4i\arctan(ax)}x^m\,dx = \int \frac{x^m\,(a^2\,x^2+1)^2}{(1+a\,x\,1\mathrm{i})^4}\,dx$$

input
```
int((x^m*(a^2*x^2 + 1)^2)/(a*x*1i + 1)^4,x)
```

output
```
int((x^m*(a^2*x^2 + 1)^2)/(a*x*1i + 1)^4, x)
```

Reduce [F]

$$\int e^{-4i\arctan(ax)}x^m\,dx$$

$$= \frac{x^m amx + 2x^m im + 2x^m i + 2\left(\int \frac{x^m}{a^4i\,x^5+4a^3x^4-6a^2i\,x^3-4a\,x^2+ix}dx\right)m^2 + 2\left(\int \frac{x^m}{a^4i\,x^5+4a^3x^4-6a^2i\,x^3-4a\,x^2+ix}dx\right)m}{}$$

input
```
int(x^m/(1+I*a*x)^4*(a^2*x^2+1)^2,x)
```

output
```
(x**m*a*m*x + 2*x**m*i*m + 2*x**m*i + 2*int(x**m/(a**4*i*x**5 + 4*a**3*x**
4 - 6*a**2*i*x**3 - 4*a*x**2 + i*x),x)*m**2 + 2*int(x**m/(a**4*i*x**5 + 4*
a**3*x**4 - 6*a**2*i*x**3 - 4*a*x**2 + i*x),x)*m + 8*int(x**m/(a**4*i*x**4
 + 4*a**3*x**3 - 6*a**2*i*x**2 - 4*a*x + i),x)*a*i*m**2 + 8*int(x**m/(a**4
*i*x**4 + 4*a**3*x**3 - 6*a**2*i*x**2 - 4*a*x + i),x)*a*i*m + 8*int(( - x*
*m*x)/(a**4*i*x**4 + 4*a**3*x**3 - 6*a**2*i*x**2 - 4*a*x + i),x)*a**2*m**2
 + 8*int(( - x**m*x)/(a**4*i*x**4 + 4*a**3*x**3 - 6*a**2*i*x**2 - 4*a*x +
i),x)*a**2*m - 2*int((x**m*x**3)/(a**4*i*x**4 + 4*a**3*x**3 - 6*a**2*i*x**
2 - 4*a*x + i),x)*a**4*m**2 - 2*int((x**m*x**3)/(a**4*i*x**4 + 4*a**3*x**3
 - 6*a**2*i*x**2 - 4*a*x + i),x)*a**4*m)/(a*m*(m + 1))
```

3.154 $\int e^{-6i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 124

$$\int e^{-6i\arctan(ax)}x^m\,dx = \frac{4(2+m)x^{1+m}}{(1+m)(1+iax)^2}-\frac{x^{1+m}(1-iax)^2}{(1+m)(1+iax)^2}-\frac{4(3+3m+m^2)\,x^{1+m}}{(1+m)(1+iax)} + \frac{2(3+4m+2m^2)\,x^{1+m}\,\text{Hypergeometric2F1}(1,1+m,2+m,-iax)}{1+m}$$

output
```
4*(2+m)*x^(1+m)/(1+m)/(1+I*a*x)^2-x^(1+m)*(1-I*a*x)^2/(1+m)/(1+I*a*x)^2-4*
(m^2+3*m+3)*x^(1+m)/(1+m)/(1+I*a*x)+2*(2*m^2+4*m+3)*x^(1+m)*hypergeom([1,
1+m],[2+m],-I*a*x)/(1+m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.76

$$\int e^{-6i\arctan(ax)}x^m\,dx = \frac{x^{1+m}(5+10iax-a^2x^2+4m(2+3iax)+m^2(4+4iax)+2(3+4m+2m^2)\,(-i+ax)^2\,\text{Hypergeometri}}{(1+m)(-i+ax)^2}$$

input
```
Integrate[x^m/E^((6*I)*ArcTan[a*x]),x]
```

output
```
(x^(1 + m)*(5 + (10*I)*a*x - a^2*x^2 + 4*m*(2 + (3*I)*a*x) + m^2*(4 + (4*I
)*a*x) + 2*(3 + 4*m + 2*m^2)*(-I + a*x)^2*Hypergeometric2F1[1, 1 + m, 2 +
m, (-I)*a*x]))/((1 + m)*(-I + a*x)^2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5585, 111, 27, 162, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{-6i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{(1-iax)^3x^m}{(1+iax)^3}dx$$

$$\Big\downarrow \text{111}$$

$$-\frac{i\int \frac{2ax^m(1-iax)(i(m+1)+a(m+3)x)}{(iax+1)^3}dx}{a(m+1)}-\frac{(1-iax)^2x^{m+1}}{(m+1)(1+iax)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2i\int \frac{x^m(1-iax)(i(m+1)+a(m+3)x)}{(iax+1)^3}dx}{m+1}-\frac{(1-iax)^2x^{m+1}}{(m+1)(1+iax)^2}$$

$$\Big\downarrow \text{162}$$

$$-\frac{2i\left(i(m+1)\left(2m^2+4m+3\right)\int \frac{x^m}{iax+1}dx-\frac{2x^{m+1}\left(-a\left(m^2+3m+3\right)x+i(m+1)^2\right)}{(1+iax)^2}\right)}{m+1}-\frac{(1-iax)^2x^{m+1}}{(m+1)(1+iax)^2}$$

$$\Big\downarrow \text{74}$$

$$-\frac{2i\left(i\left(2m^2+4m+3\right)x^{m+1}\,\text{Hypergeometric2F1}(1,m+1,m+2,-iax)-\frac{2x^{m+1}\left(-a\left(m^2+3m+3\right)x+i(m+1)^2\right)}{(1+iax)^2}\right)}{m+1}-\frac{(1-iax)^2x^{m+1}}{(m+1)(1+iax)^2}$$

input
```
Int[x^m/E^((6*I)*ArcTan[a*x]),x]
```

output
```
-((x^(1 + m)*(1 - I*a*x)^2)/((1 + m)*(1 + I*a*x)^2)) - ((2*I)*((-2*x^(1 +
m)*(I*(1 + m)^2 - a*(3 + 3*m + m^2)*x))/(1 + I*a*x)^2 + I*(3 + 4*m + 2*m^2
)*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (-I)*a*x]))/(1 + m)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 1.33 (sec) , antiderivative size = 1196, normalized size of antiderivative = 9.65

method	result	size
meijerg	Expression too large to display	1196

input
```
int(x^m/(1+I*a*x)^6*(a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/120*I*(I*a)^(-m)/a*(x^m*(I*a)^m*(-720-1624*m^2-1764*m-3600*a^4*x^4+14400
*a^2*m*x^2+11722*a^2*m^2*x^2-a^4*x^4*m^6-22*a^4*x^4*m^5-197*a^4*x^4*m^4-93
2*a^4*x^4*m^3-2556*a^4*x^4*m^2+4*I*a^3*x^3*m^6+87*I*a^3*x^3*m^5+764*I*a^3*
x^3*m^4-4*I*a*x*m^6-120*I*a^5*m*x^5+3483*I*a^3*x^3*m^3-85*I*a*x*m^5+8802*I
*a^3*x^3*m^2-720*I*a*x*m^4+12000*I*a^3*m*x^3-3095*I*a*x*m^3-7076*I*a*x*m^2
-8100*I*a*m*x+1112*a^2*x^2*m^4+4911*a^2*x^2*m^3-4200*a^4*x^4*m-175*m^4+6*a
^2*x^2*m^6-120*a^6*x^6*m+129*a^2*x^2*m^5-735*m^3-m^6-21*m^5+7200*a^2*x^2-7
20*I*a^5*x^5+7200*I*a^3*x^3-3600*I*a*x)/(1+m)/m/(1+I*a*x)^5+x^m*(I*a)^m*(m
^5+20*m^4+155*m^3+580*m^2+1044*m+720)*LerchPhi(-I*a*x,1,m))-1/40*I*(I*a)^(
-m)/a*(-x^m*(I*a)^m*(24+35*m^2+50*m+120*a^4*x^4-392*a^2*m*x^2-239*a^2*m^2*
x^2+a^4*x^4*m^4+11*a^4*x^4*m^3+46*a^4*x^4*m^2-6*a^2*x^2*m^4-63*a^2*x^2*m^3
+96*a^4*x^4*m+m^4-4*I*a^3*x^3*m^4-43*I*a^3*x^3*m^3-171*I*a^3*x^3*m^2+4*I*a
*x*m^4-312*I*a^3*m*x^3+41*I*a*x*m^3+149*I*a*x*m^2+226*I*a*m*x+10*m^3-240*a
^2*x^2-240*I*a^3*x^3+120*I*a*x)/(1+I*a*x)^5+x^m*(I*a)^m*m*(m^4+10*m^3+35*m
^2+50*m+24)*LerchPhi(-I*a*x,1,m))+1/40*I*(I*a)^(-m)/a*(-x^m*(I*a)^m*(a^4*x
^4*m^4+a^4*x^4*m^3+I*a*x*m^3-4*a^4*x^4*m^2-4*I*a*m*x-6*a^2*x^2*m^4-4*a^4*x
^4*m-21*I*a*x*m^2-3*a^2*x^2*m^3-4*I*a^3*x^3*m^4+4*I*a*x*m^4+31*a^2*m^2*x^2
+18*I*a^3*m*x^3+m^4+18*a^2*m*x^2+19*I*a^3*x^3*m^2-40*a^2*x^2-3*I*a^3*x^3*m
^3-5*m^2+20*I*a*x+4)/(1+I*a*x)^5+x^m*(I*a)^m*(m^2-3*m+2)*m*(m^2+3*m+2)*Ler
chPhi(-I*a*x,1,m))-1/120*I*(I*a)^(-m)/a*(-x^m*(I*a)^m*(a^4*x^4*m^4-9*a^...
```

Fricas [F]

$$\int e^{-6i\arctan(ax)}x^m\,dx=\int\frac{\left(a^2x^2+1\right)^3x^m}{\left(i\,ax+1\right)^6}\,dx$$

input
```
integrate(x^m/(1+I*a*x)^6*(a^2*x^2+1)^3,x, algorithm="fricas")
```

output
```
integral(-(a^3*x^3 + 3*I*a^2*x^2 - 3*a*x - I)*x^m/(a^3*x^3 - 3*I*a^2*x^2 -
 3*a*x + I), x)
```

Sympy [F]

$$\int e^{-6i\arctan(ax)}x^m\,dx = -\int \frac{x^m}{a^6x^6 - 6ia^5x^5 - 15a^4x^4 + 20ia^3x^3 + 15a^2x^2 - 6iax - 1}\,dx - \int \frac{3a^2x^2x^m}{a^6x^6 - 6ia^5x^5 - 15a^4x^4 + 20ia^3x^3 + 15a^2x^2 - 6iax - 1}\,dx - \int \frac{3a^4x^4x^m}{a^6x^6 - 6ia^5x^5 - 15a^4x^4 + 20ia^3x^3 + 15a^2x^2 - 6iax - 1}\,dx - \int \frac{a^6x^6x^m}{a^6x^6 - 6ia^5x^5 - 15a^4x^4 + 20ia^3x^3 + 15a^2x^2 - 6iax - 1}\,dx$$

input
```
integrate(x**m/(1+I*a*x)**6*(a**2*x**2+1)**3,x)
```

output
```
-Integral(x**m/(a**6*x**6 - 6*I*a**5*x**5 - 15*a**4*x**4 + 20*I*a**3*x**3
+ 15*a**2*x**2 - 6*I*a*x - 1), x) - Integral(3*a**2*x**2*x**m/(a**6*x**6 -
 6*I*a**5*x**5 - 15*a**4*x**4 + 20*I*a**3*x**3 + 15*a**2*x**2 - 6*I*a*x -
1), x) - Integral(3*a**4*x**4*x**m/(a**6*x**6 - 6*I*a**5*x**5 - 15*a**4*x*
*4 + 20*I*a**3*x**3 + 15*a**2*x**2 - 6*I*a*x - 1), x) - Integral(a**6*x**6
*x**m/(a**6*x**6 - 6*I*a**5*x**5 - 15*a**4*x**4 + 20*I*a**3*x**3 + 15*a**2
*x**2 - 6*I*a*x - 1), x)
```

Maxima [F]

$$\int e^{-6i\arctan(ax)}x^m\,dx = \int \frac{(a^2x^2+1)^3x^m}{(i\,ax+1)^6}\,dx$$

input
```
integrate(x^m/(1+I*a*x)^6*(a^2*x^2+1)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*x^2 + 1)^3*x^m/(I*a*x + 1)^6, x)
```

Giac [F]

$$\int e^{-6i\arctan(ax)}x^m\,dx = \int \frac{(a^2x^2+1)^3x^m}{(i\,ax+1)^6}\,dx$$

input
```
integrate(x^m/(1+I*a*x)^6*(a^2*x^2+1)^3,x, algorithm="giac")
```

output
```
integrate((a^2*x^2 + 1)^3*x^m/(I*a*x + 1)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-6i\arctan(ax)}x^m\,dx = \int \frac{x^m\,(a^2\,x^2+1)^3}{(1+a\,x\,1\mathrm{i})^6}\,dx$$

input
```
int((x^m*(a^2*x^2 + 1)^3)/(a*x*1i + 1)^6,x)
```

output
```
int((x^m*(a^2*x^2 + 1)^3)/(a*x*1i + 1)^6, x)
```

Reduce [F]

$$\int e^{-6i\arctan(ax)}x^m\,dx$$
$$= \frac{-x^m amx - 3x^m im - 3x^m i + 3\left(\int \frac{x^m}{a^6i\,x^7+6a^5x^6-15a^4i\,x^5-20a^3x^4+15a^2i\,x^3+6a\,x^2-ix}dx\right)m^2 + 3\left(\int \frac{}{a^6i\,x^7+6a^5x^6-15a^4}\right.}{}$$

input
```
int(x^m/(1+I*a*x)^6*(a^2*x^2+1)^3,x)
```

output

```
( - x**m*a*m*x - 3*x**m*i*m - 3*x**m*i + 3*int(x**m/(a**6*i*x**7 + 6*a**5*
x**6 - 15*a**4*i*x**5 - 20*a**3*x**4 + 15*a**2*i*x**3 + 6*a*x**2 - i*x),x)
*m**2 + 3*int(x**m/(a**6*i*x**7 + 6*a**5*x**6 - 15*a**4*i*x**5 - 20*a**3*x
**4 + 15*a**2*i*x**3 + 6*a*x**2 - i*x),x)*m + 16*int(x**m/(a**6*i*x**6 + 6
*a**5*x**5 - 15*a**4*i*x**4 - 20*a**3*x**3 + 15*a**2*i*x**2 + 6*a*x - i),x
)*a*i*m**2 + 16*int(x**m/(a**6*i*x**6 + 6*a**5*x**5 - 15*a**4*i*x**4 - 20*
a**3*x**3 + 15*a**2*i*x**2 + 6*a*x - i),x)*a*i*m - 25*int(( - x**m*x**3)/(
a**6*i*x**6 + 6*a**5*x**5 - 15*a**4*i*x**4 - 20*a**3*x**3 + 15*a**2*i*x**2
 + 6*a*x - i),x)*a**4*m**2 - 25*int(( - x**m*x**3)/(a**6*i*x**6 + 6*a**5*x
**5 - 15*a**4*i*x**4 - 20*a**3*x**3 + 15*a**2*i*x**2 + 6*a*x - i),x)*a**4*
m + 39*int(( - x**m*x)/(a**6*i*x**6 + 6*a**5*x**5 - 15*a**4*i*x**4 - 20*a*
*3*x**3 + 15*a**2*i*x**2 + 6*a*x - i),x)*a**2*m**2 + 39*int(( - x**m*x)/(a
**6*i*x**6 + 6*a**5*x**5 - 15*a**4*i*x**4 - 20*a**3*x**3 + 15*a**2*i*x**2
+ 6*a*x - i),x)*a**2*m + 3*int((x**m*x**5)/(a**6*i*x**6 + 6*a**5*x**5 - 15
*a**4*i*x**4 - 20*a**3*x**3 + 15*a**2*i*x**2 + 6*a*x - i),x)*a**6*m**2 + 3
*int((x**m*x**5)/(a**6*i*x**6 + 6*a**5*x**5 - 15*a**4*i*x**4 - 20*a**3*x**
3 + 15*a**2*i*x**2 + 6*a*x - i),x)*a**6*m - 48*int((x**m*x**2)/(a**6*i*x**
6 + 6*a**5*x**5 - 15*a**4*i*x**4 - 20*a**3*x**3 + 15*a**2*i*x**2 + 6*a*x -
 i),x)*a**3*i*m**2 - 48*int((x**m*x**2)/(a**6*i*x**6 + 6*a**5*x**5 - 15*a*
*4*i*x**4 - 20*a**3*x**3 + 15*a**2*i*x**2 + 6*a*x - i),x)*a**3*i*m)/(a*...
```

3.155 $\int e^{3i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 118

$$\int e^{3i\arctan(ax)}x^m\,dx=\frac{4x^{1+m}(1+iax)}{\sqrt{1+a^2x^2}}-\frac{(3+4m)x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-a^2x^2\right)}{1+m}-\frac{ia(5+4m)x^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+m}$$

output
```
4*x^(1+m)*(1+I*a*x)/(a^2*x^2+1)^(1/2)-(3+4*m)*x^(1+m)*hypergeom([1/2, 1/2+
1/2*m],[3/2+1/2*m],-a^2*x^2)/(1+m)-I*a*(5+4*m)*x^(2+m)*hypergeom([1/2, 1+1
/2*m],[2+1/2*m],-a^2*x^2)/(2+m)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96

$$\int e^{3i\arctan(ax)}x^m\,dx=-\frac{ix^{1+m}\sqrt{1-iax}\sqrt{-i+ax}\left(\text{AppellF1}\left(1+m,-\frac{1}{2},\frac{1}{2},2+m,-iax,iax\right)-2\,\text{AppellF1}\left(1+m,-\frac{1}{2},\frac{3}{2},2\right.\right.}{(1+m)\sqrt{1+iax}\sqrt{i+ax}}$$

input
```
Integrate[E^((3*I)*ArcTan[a*x])*x^m,x]
```

output
```
((-I)*x^(1 + m)*Sqrt[1 - I*a*x]*Sqrt[-I + a*x]*(AppellF1[1 + m, -1/2, 1/2,
 2 + m, (-I)*a*x, I*a*x] - 2*AppellF1[1 + m, -1/2, 3/2, 2 + m, (-I)*a*x, I
*a*x]))/((1 + m)*Sqrt[1 + I*a*x]*Sqrt[I + a*x])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.36, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5583, 2355, 557, 278, 583, 557, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{3i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{(1+iax)^2x^m}{(1-iax)\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{2355}$$

$$\int \frac{x^m(-iax-3)}{\sqrt{a^2x^2+1}}dx+4\int \frac{x^m}{(1-iax)\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{557}$$

$$-ia\int \frac{x^{m+1}}{\sqrt{a^2x^2+1}}dx-3\int \frac{x^m}{\sqrt{a^2x^2+1}}dx+4\int \frac{x^m}{(1-iax)\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{278}$$

$$4\int \frac{x^m}{(1-iax)\sqrt{a^2x^2+1}}dx-\frac{3x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1}-\frac{iax^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m+2}$$

$$\Big\downarrow \text{583}$$

$$4\int \frac{x^m(iax+1)}{(a^2x^2+1)^{3/2}}dx-\frac{3x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1}-\frac{iax^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m+2}$$

$$\Big\downarrow \text{557}$$

$$4\left(\int \frac{x^m}{(a^2x^2+1)^{3/2}}dx+ia\int \frac{x^{m+1}}{(a^2x^2+1)^{3/2}}dx\right)-\frac{3x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1}-\frac{iax^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m+2}$$

$$\Big\downarrow \text{278}$$

$$-\frac{3x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1}-\frac{iax^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m+2}+4\left(\frac{x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1}+\frac{iax^{m+2}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m+2}\right)$$

input
```
Int[E^((3*I)*ArcTan[a*x])*x^m,x]
```

output
```
(-3*x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(a^2*x^2)])/(1
 + m) - (I*a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*
x^2)])/(2 + m) + 4*((x^(1 + m)*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2
, -(a^2*x^2)])/(1 + m) + (I*a*x^(2 + m)*Hypergeometric2F1[3/2, (2 + m)/2,
(4 + m)/2, -(a^2*x^2)])/(2 + m))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 583
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, 0]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.24

method	result
meijerg	$\frac{x^{1+m}\operatorname{hypergeom}\left(\left[\frac{3}{2},\frac{m}{2}+\frac{1}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-a^{2}x^{2}\right)}{1+m}+\frac{3ia\,x^{2+m}\operatorname{hypergeom}\left(\left[\frac{3}{2},\frac{m}{2}+1\right],\left[2+\frac{m}{2}\right],-a^{2}x^{2}\right)}{2+m}-\frac{3a^{2}x^{3+m}\operatorname{hypergeom}\left(\left[\frac{3}{2},\right.\right.}{3+m}$ [illegible]

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^m,x,method=_RETURNVERBOSE)
```

output
```
1/(1+m)*x^(1+m)*hypergeom([3/2,1/2*m+1/2],[3/2+1/2*m],-a^2*x^2)+3*I*a/(2+m
)*x^(2+m)*hypergeom([3/2,1/2*m+1],[2+1/2*m],-a^2*x^2)-3*a^2/(3+m)*x^(3+m)*
hypergeom([3/2,3/2+1/2*m],[5/2+1/2*m],-a^2*x^2)-I*a^3/(4+m)*x^(4+m)*hyperg
eom([3/2,2+1/2*m],[1/2*m+3],-a^2*x^2)
```

Fricas [F]

$$\int e^{3i\arctan(ax)}x^m\,dx = \int \frac{(i\,ax+1)^3x^m}{(a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^m,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*x^2 + 1)*(-I*a*x - 1)*x^m/(a^2*x^2 + 2*I*a*x - 1), x)
```

Sympy [F]

$$\int e^{3i\arctan(ax)}x^m\,dx = -i\left(\int \frac{ix^m}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx + \int\left(-\frac{3axx^m}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx + \int \frac{a^3x^3x^m}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx + \int\left(-\frac{3ia^2x^2x^m}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx\right)$$

input
```
integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)*x**m,x)
```

output
```
-I*(Integral(I*x**m/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)),
 x) + Integral(-3*a*x*x**m/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2
 + 1)), x) + Integral(a**3*x**3*x**m/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt
(a**2*x**2 + 1)), x) + Integral(-3*I*a**2*x**2*x**m/(a**2*x**2*sqrt(a**2*x
**2 + 1) + sqrt(a**2*x**2 + 1)), x))
```

Maxima [F]

$$\int e^{3i\arctan(ax)}x^m\,dx = \int \frac{(i\,ax+1)^3x^m}{(a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^m,x, algorithm="maxima")
```

output
```
integrate((I*a*x + 1)^3*x^m/(a^2*x^2 + 1)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{3i\arctan(ax)}x^m\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^m,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{3i\arctan(ax)}x^m\,dx = \int \frac{x^m\,(1+a\,x\,1\mathrm{i})^3}{(a^2\,x^2+1)^{3/2}}\,dx$$

input
```
int((x^m*(a*x*1i + 1)^3)/(a^2*x^2 + 1)^(3/2),x)
```

output
```
int((x^m*(a*x*1i + 1)^3)/(a^2*x^2 + 1)^(3/2), x)
```

Reduce [F]

$$\int e^{3i\arctan(ax)}x^m\,dx = \int \frac{x^m}{\sqrt{a^2x^2+1}\,a^2x^2+\sqrt{a^2x^2+1}}dx - \left(\int \frac{x^m x^3}{\sqrt{a^2x^2+1}\,a^2x^2+\sqrt{a^2x^2+1}}dx\right)a^3i - 3\left(\int \frac{x^m x^2}{\sqrt{a^2x^2+1}\,a^2x^2+\sqrt{a^2x^2+1}}dx\right)a^2 + 3\left(\int \frac{x^m x}{\sqrt{a^2x^2+1}\,a^2x^2+\sqrt{a^2x^2+1}}dx\right)ai$$

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^m,x)
```

output
```
int(x**m/(sqrt(a**2*x**2 + 1)*a**2*x**2 + sqrt(a**2*x**2 + 1)),x) - int((x
**m*x**3)/(sqrt(a**2*x**2 + 1)*a**2*x**2 + sqrt(a**2*x**2 + 1)),x)*a**3*i
- 3*int((x**m*x**2)/(sqrt(a**2*x**2 + 1)*a**2*x**2 + sqrt(a**2*x**2 + 1)),
x)*a**2 + 3*int((x**m*x)/(sqrt(a**2*x**2 + 1)*a**2*x**2 + sqrt(a**2*x**2 +
 1)),x)*a*i
```

3.156 $\int e^{i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 79

$$\int e^{i\arctan(ax)}x^m\,dx = \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-a^2x^2\right)}{1+m} + \frac{iax^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+m}$$

output
```
x^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-a^2*x^2)/(1+m)+I*a*x^(2+m)
*hypergeom([1/2, 1+1/2*m],[2+1/2*m],-a^2*x^2)/(2+m)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.08

$$\int e^{i\arctan(ax)}x^m\,dx = \frac{ix^{1+m}\sqrt{1-iax}\sqrt{-i+ax}\,\text{AppellF1}\left(1+m,-\frac{1}{2},\frac{1}{2},2+m,-iax,iax\right)}{(1+m)\sqrt{1+iax}\sqrt{i+ax}}$$

input
```
Integrate[E^(I*ArcTan[a*x])*x^m,x]
```

output
```
(I*x^(1 + m)*Sqrt[1 - I*a*x]*Sqrt[-I + a*x]*AppellF1[1 + m, -1/2, 1/2, 2 +
 m, (-I)*a*x, I*a*x])/((1 + m)*Sqrt[1 + I*a*x]*Sqrt[I + a*x])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5583, 557, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{(1+iax)x^m}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{557}$$

$$\int \frac{x^m}{\sqrt{a^2x^2+1}}dx+ia\int \frac{x^{m+1}}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{278}$$

$$\frac{x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1}+\frac{iax^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m+2}$$

input
```
Int[E^(I*ArcTan[a*x])*x^m,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(a^2*x^2)])/(1 +
m) + (I*a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^2
)])/(2 + m)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

method	result	size
meijerg	$\frac{x^{1+m} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{m}{2}+\frac{1}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-a^{2} x^{2}\right)}{1+m}+\frac{i a\, x^{2+m} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{m}{2}+1\right],\left[2+\frac{m}{2}\right],-a^{2} x^{2}\right)}{2+m}$	71

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^m,x,method=_RETURNVERBOSE)
```

output
```
x^(1+m)*hypergeom([1/2,1/2*m+1/2],[3/2+1/2*m],-a^2*x^2)/(1+m)+I*a*x^(2+m)*
hypergeom([1/2,1/2*m+1],[2+1/2*m],-a^2*x^2)/(2+m)
```

Fricas [F]

$$\int e^{i\arctan(ax)}x^m\,dx = \int \frac{(i\,ax+1)x^m}{\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^m,x, algorithm="fricas")
```

output
```
integral(I*sqrt(a^2*x^2 + 1)*x^m/(a*x + I), x)
```

Sympy [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.19

$$\int e^{i\arctan(ax)}x^m\,dx = \frac{iax^{m+2}\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|a^2x^2e^{i\pi}\right)}{2\Gamma\left(\frac{m}{2}+2\right)} + \frac{x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|a^2x^2e^{i\pi}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

input
```
integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)*x**m,x)
```

output
```
I*a*x**(m + 2)*gamma(m/2 + 1)*hyper((1/2, m/2 + 1), (m/2 + 2,), a**2*x**2*
exp_polar(I*pi))/(2*gamma(m/2 + 2)) + x**(m + 1)*gamma(m/2 + 1/2)*hyper((1
/2, m/2 + 1/2), (m/2 + 3/2,), a**2*x**2*exp_polar(I*pi))/(2*gamma(m/2 + 3/
2))
```

Maxima [F]

$$\int e^{i \arctan(ax)} x^m \, dx = \int \frac{(i\, ax + 1)x^m}{\sqrt{a^2x^2 + 1}} \, dx$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^m,x, algorithm="maxima")
```

output
```
integrate((I*a*x + 1)*x^m/sqrt(a^2*x^2 + 1), x)
```

Giac [F]

$$\int e^{i \arctan(ax)} x^m \, dx = \int \frac{(i\, ax + 1)x^m}{\sqrt{a^2x^2 + 1}} \, dx$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^m,x, algorithm="giac")
```

output
```
integrate((I*a*x + 1)*x^m/sqrt(a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{i \arctan(ax)} x^m \, dx = \int \frac{x^m \left(1 + a\, x\, 1\mathrm{i}\right)}{\sqrt{a^2\, x^2 + 1}} \, dx$$

input
```
int((x^m*(a*x*1i + 1))/(a^2*x^2 + 1)^(1/2),x)
```

output
```
int((x^m*(a*x*1i + 1))/(a^2*x^2 + 1)^(1/2), x)
```

Reduce [F]

$$\int e^{i\arctan(ax)}x^m\,dx = \int \frac{x^m}{\sqrt{a^2x^2+1}}dx + \left(\int \frac{x^m x}{\sqrt{a^2x^2+1}}dx\right)ai$$

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^m,x)
```

output
```
int(x**m/sqrt(a**2*x**2 + 1),x) + int((x**m*x)/sqrt(a**2*x**2 + 1),x)*a*i
```

3.157 $\int e^{-i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 79

$$\int e^{-i\arctan(ax)}x^m\,dx = \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-a^2x^2\right)}{1+m} - \frac{iax^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+m}$$

output
```
x^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-a^2*x^2)/(1+m)-I*a*x^(2+m)
*hypergeom([1/2, 1+1/2*m],[2+1/2*m],-a^2*x^2)/(2+m)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.08

$$\int e^{-i\arctan(ax)}x^m\,dx = -\frac{ix^{1+m}\sqrt{1+iax}\sqrt{i+ax}\,\text{AppellF1}\left(1+m,\frac{1}{2},-\frac{1}{2},2+m,-iax,iax\right)}{(1+m)\sqrt{1-iax}\sqrt{-i+ax}}$$

input
```
Integrate[x^m/E^(I*ArcTan[a*x]),x]
```

output
```
((-I)*x^(1 + m)*Sqrt[1 + I*a*x]*Sqrt[I + a*x]*AppellF1[1 + m, 1/2, -1/2, 2
 + m, (-I)*a*x, I*a*x])/((1 + m)*Sqrt[1 - I*a*x]*Sqrt[-I + a*x])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5583, 557, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{-i \arctan(ax)} \, dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{(1-iax)x^m}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{557}$$

$$\int \frac{x^m}{\sqrt{a^2x^2+1}}dx - ia\int \frac{x^{m+1}}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{278}$$

$$\frac{x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1} - \frac{iax^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m+2}$$

input
```
Int[x^m/E^(I*ArcTan[a*x]),x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(a^2*x^2)])/(1 +
m) - (I*a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^2
)])/(2 + m)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{x^m \sqrt{a^2x^2+1}}{iax+1}dx$$

input
```
int(x^m/(1+I*a*x)*(a^2*x^2+1)^(1/2),x)
```

output
```
int(x^m/(1+I*a*x)*(a^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int e^{-i\arctan(ax)}x^m\,dx = \int \frac{\sqrt{a^2x^2+1}x^m}{i\,ax+1}\,dx$$

input
```
integrate(x^m/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-I*sqrt(a^2*x^2 + 1)*x^m/(a*x - I), x)
```

Sympy [F]

$$\int e^{-i\arctan(ax)}x^m\,dx = -i\int \frac{x^m\sqrt{a^2x^2+1}}{ax-i}\,dx$$

input
```
integrate(x**m/(1+I*a*x)*(a**2*x**2+1)**(1/2),x)
```

output
```
-I*Integral(x**m*sqrt(a**2*x**2 + 1)/(a*x - I), x)
```

Maxima [F]

$$\int e^{-i\arctan(ax)}x^m\,dx = \int \frac{\sqrt{a^2x^2+1}x^m}{i\,ax+1}\,dx$$

input
```
integrate(x^m/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*x^2 + 1)*x^m/(I*a*x + 1), x)
```

Giac [F]

$$\int e^{-i\arctan(ax)}x^m\,dx = \int \frac{\sqrt{a^2x^2+1}x^m}{i\,ax+1}\,dx$$

input
```
integrate(x^m/(1+I*a*x)*(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*x^2 + 1)*x^m/(I*a*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-i\arctan(ax)}x^m\,dx = \int \frac{x^m\,\sqrt{a^2\,x^2+1}}{1+a\,x\,1\mathrm{i}}\,dx$$

input
```
int((x^m*(a^2*x^2 + 1)^(1/2))/(a*x*1i + 1),x)
```

output
```
int((x^m*(a^2*x^2 + 1)^(1/2))/(a*x*1i + 1), x)
```

Reduce [F]

$$\int e^{-i\arctan(ax)}x^m\,dx = \int \frac{x^m\sqrt{a^2x^2+1}}{aix+1}dx$$

input
```
int(x^m/(1+I*a*x)*(a^2*x^2+1)^(1/2),x)
```

output
```
int((x**m*sqrt(a**2*x**2 + 1))/(a*i*x + 1),x)
```

3.158 $\int e^{-3i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 118

$$\int e^{-3i\arctan(ax)}x^m\,dx = \frac{4x^{1+m}(1-iax)}{\sqrt{1+a^2x^2}} - \frac{(3+4m)x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-a^2x^2\right)}{1+m} + \frac{ia(5+4m)x^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+m}$$

output
```
4*x^(1+m)*(1-I*a*x)/(a^2*x^2+1)^(1/2)-(3+4*m)*x^(1+m)*hypergeom([1/2, 1/2+
1/2*m],[3/2+1/2*m],-a^2*x^2)/(1+m)+I*a*(5+4*m)*x^(2+m)*hypergeom([1/2, 1+1
/2*m],[2+1/2*m],-a^2*x^2)/(2+m)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96

$$\int e^{-3i\arctan(ax)}x^m\,dx = \frac{ix^{1+m}\sqrt{1+iax}\sqrt{i+ax}\left(\text{AppellF1}\left(1+m,\frac{1}{2},-\frac{1}{2},2+m,-iax,iax\right)-2\,\text{AppellF1}\left(1+m,\frac{3}{2},-\frac{1}{2},2+m\right.\right.}{(1+m)\sqrt{1-iax}\sqrt{-i+ax}}$$

input
```
Integrate[x^m/E^((3*I)*ArcTan[a*x]),x]
```

output
```
(I*x^(1 + m)*Sqrt[1 + I*a*x]*Sqrt[I + a*x]*(AppellF1[1 + m, 1/2, -1/2, 2 +
 m, (-I)*a*x, I*a*x] - 2*AppellF1[1 + m, 3/2, -1/2, 2 + m, (-I)*a*x, I*a*x
]))/((1 + m)*Sqrt[1 - I*a*x]*Sqrt[-I + a*x])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.36, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5583, 2355, 557, 278, 583, 557, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{-3i\arctan(ax)}dx$$

$$\Big\downarrow \text{5583}$$

$$\int \frac{(1-iax)^2x^m}{(1+iax)\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{2355}$$

$$\int \frac{x^m(iax-3)}{\sqrt{a^2x^2+1}}dx+4\int \frac{x^m}{(iax+1)\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{557}$$

$$ia\int \frac{x^{m+1}}{\sqrt{a^2x^2+1}}dx-3\int \frac{x^m}{\sqrt{a^2x^2+1}}dx+4\int \frac{x^m}{(iax+1)\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{278}$$

$$4\int \frac{x^m}{(iax+1)\sqrt{a^2x^2+1}}dx-\frac{3x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1}+\frac{iax^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m+2}$$

$$\Big\downarrow \text{583}$$

$$4\int \frac{x^m(1-iax)}{(a^2x^2+1)^{3/2}}dx - \frac{3x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1} + \frac{iax^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m+2}$$

$$\Big\downarrow \text{557}$$

$$4\left(\int \frac{x^m}{(a^2x^2+1)^{3/2}}dx - ia\int \frac{x^{m+1}}{(a^2x^2+1)^{3/2}}dx\right) - \frac{3x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1} + \frac{iax^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m+2}$$

$$\Big\downarrow \text{278}$$

$$-\frac{3x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1} + \frac{iax^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m+2} + 4\left(\frac{x^{m+1}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},-a^2x^2\right)}{m+1} - \frac{iax^{m+2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m+2}\right)$$

input
```
Int[x^m/E^((3*I)*ArcTan[a*x]),x]
```

output
```
(-3*x^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(a^2*x^2)])/(1
 + m) + (I*a*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*
x^2)])/(2 + m) + 4*((x^(1 + m)*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2
, -(a^2*x^2)])/(1 + m) - (I*a*x^(2 + m)*Hypergeometric2F1[3/2, (2 + m)/2,
(4 + m)/2, -(a^2*x^2)])/(2 + m))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

rule 583
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
x_Symbol] :> Simp[c^(2*n)/a^n   Int[(e*x)^m*((a + b*x^2)^(n + p)/(c - d*x)^
n), x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b*c^2 + a*d^2, 0] && I
LtQ[n, 0]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

rule 5583
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a*
x)^((I*n + 1)/2)/((1 + I*a*x)^((I*n - 1)/2)*Sqrt[1 + a^2*x^2])), x] /; Free
Q[{a, m}, x] && IntegerQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{x^m(a^2x^2+1)^{\frac{3}{2}}}{(iax+1)^3}dx$$

input
```
int(x^m/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x)
```

output
```
int(x^m/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x)
```

Fricas [F]

$$\int e^{-3i\arctan(ax)}x^m\,dx=\int\frac{(a^2x^2+1)^{\frac{3}{2}}x^m}{(i\,ax+1)^3}\,dx$$

input
```
integrate(x^m/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*x^2 + 1)*(I*a*x - 1)*x^m/(a^2*x^2 - 2*I*a*x - 1), x)
```

Sympy [F]

$$\int e^{-3i\arctan(ax)}x^m\,dx$$
$$=i\left(\int\frac{x^m\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx+\int\frac{a^2x^2x^m\sqrt{a^2x^2+1}}{a^3x^3-3ia^2x^2-3ax+i}\,dx\right)$$

input
```
integrate(x**m/(1+I*a*x)**3*(a**2*x**2+1)**(3/2),x)
```

output
```
I*(Integral(x**m*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2*x**2 - 3*a*x +
I), x) + Integral(a**2*x**2*x**m*sqrt(a**2*x**2 + 1)/(a**3*x**3 - 3*I*a**2
*x**2 - 3*a*x + I), x))
```

Maxima [F]

$$\int e^{-3i\arctan(ax)}x^m\,dx=\int\frac{(a^2x^2+1)^{\frac{3}{2}}x^m}{(i\,ax+1)^3}\,dx$$

input
```
integrate(x^m/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a^2*x^2 + 1)^(3/2)*x^m/(I*a*x + 1)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-3i\arctan(ax)}x^m\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3i\arctan(ax)}x^m\,dx = \int \frac{x^m\,(a^2\,x^2+1)^{3/2}}{(1+a\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((x^m*(a^2*x^2 + 1)^(3/2))/(a*x*1i + 1)^3,x)
```

output
```
int((x^m*(a^2*x^2 + 1)^(3/2))/(a*x*1i + 1)^3, x)
```

Reduce [F]

$$\int e^{-3i\arctan(ax)}x^m\,dx = -\left(\int \frac{x^m\sqrt{a^2x^2+1}\,x^2}{a^3i\,x^3+3a^2x^2-3aix-1}dx\right)a^2 - \left(\int \frac{x^m\sqrt{a^2x^2+1}}{a^3i\,x^3+3a^2x^2-3aix-1}dx\right)$$

input
```
int(x^m/(1+I*a*x)^3*(a^2*x^2+1)^(3/2),x)
```

output
```
- (int((x**m*sqrt(a**2*x**2 + 1)*x**2)/(a**3*i*x**3 + 3*a**2*x**2 - 3*a*i
*x - 1),x)*a**2 + int((x**m*sqrt(a**2*x**2 + 1))/(a**3*i*x**3 + 3*a**2*x**
2 - 3*a*i*x - 1),x))
```

3.159 $\int e^{\frac{5}{2}i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 16, antiderivative size = 36

$$\int e^{\frac{5}{2}i\arctan(ax)}x^m\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(1+m,\frac{5}{4},-\frac{5}{4},2+m,iax,-iax\right)}{1+m}$$

output
```
x^(1+m)*AppellF1(1+m,-5/4,5/4,2+m,-I*a*x,I*a*x)/(1+m)
```

Mathematica [F]

$$\int e^{\frac{5}{2}i\arctan(ax)}x^m\,dx=\int e^{\frac{5}{2}i\arctan(ax)}x^m\,dx$$

input
```
Integrate[E^(((5*I)/2)*ArcTan[a*x])*x^m,x]
```

output
```
Integrate[E^(((5*I)/2)*ArcTan[a*x])*x^m, x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5585, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{\frac{5}{2} i \arctan(ax)} \, dx$$

$$\Big\downarrow \text{ 5585}$$

$$\int \frac{(1+iax)^{5/4} x^m}{(1-iax)^{5/4}} dx$$

$$\Big\downarrow \text{ 150}$$

$$\frac{x^{m+1} \operatorname{AppellF1}\left(m+1,\frac{5}{4},-\frac{5}{4},m+2,iax,-iax\right)}{m+1}$$

input
```
Int[E^(((5*I)/2)*ArcTan[a*x])*x^m,x]
```

output
```
(x^(1 + m)*AppellF1[1 + m, 5/4, -5/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}} x^m dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^m,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^m,x)
```

Fricas [F]

$$\int e^{\frac{5}{2}i\arctan(ax)}x^m\,dx = \int x^m\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^m,x, algorithm="fricas")
```

output
```
integral(-(a*x - I)*x^m*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))/(a*x + I), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{5}{2}i\arctan(ax)}x^m\,dx = \text{Timed out}$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(5/2)*x**m,x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\frac{5}{2}i\arctan(ax)}x^m\,dx=\int x^m\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^m,x, algorithm="maxima")
```

output
```
integrate(x^m*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{5}{2}i\arctan(ax)}x^m\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^m,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{5}{2}i\arctan(ax)}x^m\,dx=\int x^m\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{5/2}\,dx$$

input
```
int(x^m*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2),x)
```

output
```
int(x^m*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(5/2), x)
```

Reduce [F]

$$\int e^{\frac{5}{2}i\arctan(ax)}x^m\,dx = \text{Too large to display}$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(5/2)*x^m,x)
```

output
```
( - 2*x**m*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4)*a*x - 8*x**m*sqrt(a*i*x
+ 1)*(a**2*x**2 + 1)**(3/4)*i*m - 8*x**m*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**
(3/4)*i + 8*int((x**m*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4)*x)/(a**4*m*x*
*4 + a**4*x**4 + 2*a**2*m*x**2 + 2*a**2*x**2 + m + 1),x)*a**4*i*m**3*x**2
+ 20*int((x**m*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4)*x)/(a**4*m*x**4 + a*
*4*x**4 + 2*a**2*m*x**2 + 2*a**2*x**2 + m + 1),x)*a**4*i*m**2*x**2 + 17*in
t((x**m*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4)*x)/(a**4*m*x**4 + a**4*x**4
 + 2*a**2*m*x**2 + 2*a**2*x**2 + m + 1),x)*a**4*i*m*x**2 + 5*int((x**m*sqr
t(a*i*x + 1)*(a**2*x**2 + 1)**(3/4)*x)/(a**4*m*x**4 + a**4*x**4 + 2*a**2*m
*x**2 + 2*a**2*x**2 + m + 1),x)*a**4*i*x**2 + 8*int((x**m*sqrt(a*i*x + 1)*
(a**2*x**2 + 1)**(3/4)*x)/(a**4*m*x**4 + a**4*x**4 + 2*a**2*m*x**2 + 2*a**
2*x**2 + m + 1),x)*a**2*i*m**3 + 20*int((x**m*sqrt(a*i*x + 1)*(a**2*x**2 +
 1)**(3/4)*x)/(a**4*m*x**4 + a**4*x**4 + 2*a**2*m*x**2 + 2*a**2*x**2 + m +
 1),x)*a**2*i*m**2 + 17*int((x**m*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4)*x
)/(a**4*m*x**4 + a**4*x**4 + 2*a**2*m*x**2 + 2*a**2*x**2 + m + 1),x)*a**2*
i*m + 5*int((x**m*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4)*x)/(a**4*m*x**4 +
 a**4*x**4 + 2*a**2*m*x**2 + 2*a**2*x**2 + m + 1),x)*a**2*i + 8*int((x**m*
sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4))/(a**4*m*x**5 + a**4*x**5 + 2*a**2*
m*x**3 + 2*a**2*x**3 + m*x + x),x)*a**2*i*m**3*x**2 + 16*int((x**m*sqrt(a*
i*x + 1)*(a**2*x**2 + 1)**(3/4))/(a**4*m*x**5 + a**4*x**5 + 2*a**2*m*x*...
```

3.160 $\int e^{\frac{3}{2}i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 16, antiderivative size = 36

$$\int e^{\frac{3}{2}i\arctan(ax)}x^m\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(1+m,\frac{3}{4},-\frac{3}{4},2+m,iax,-iax\right)}{1+m}$$

output
```
x^(1+m)*AppellF1(1+m,-3/4,3/4,2+m,-I*a*x,I*a*x)/(1+m)
```

Mathematica [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}x^m\,dx=\int e^{\frac{3}{2}i\arctan(ax)}x^m\,dx$$

input
```
Integrate[E^(((3*I)/2)*ArcTan[a*x])*x^m,x]
```

output
```
Integrate[E^(((3*I)/2)*ArcTan[a*x])*x^m, x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5585, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{\frac{3}{2}i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{(1+iax)^{3/4}x^m}{(1-iax)^{3/4}}dx$$

$$\Big\downarrow \text{150}$$

$$\frac{x^{m+1}\operatorname{AppellF1}\left(m+1,\frac{3}{4},-\frac{3}{4},m+2,iax,-iax\right)}{m+1}$$

input
```
Int[E^(((3*I)/2)*ArcTan[a*x])*x^m,x]
```

output
```
(x^(1 + m)*AppellF1[1 + m, 3/4, -3/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \left(\frac{iax + 1}{\sqrt{a^2x^2 + 1}}\right)^{\frac{3}{2}} x^m dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^m,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^m,x)
```

Fricas [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}x^m\,dx = \int x^m\left(\frac{i\,ax + 1}{\sqrt{a^2x^2 + 1}}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^m,x, algorithm="fricas")
```

output
```
integral(I*sqrt(a^2*x^2 + 1)*x^m*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))/(a*x
+ I), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}i\arctan(ax)}x^m\,dx = \text{Timed out}$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2)*x**m,x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}x^m\,dx=\int x^m\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^m,x, algorithm="maxima")
```

output
```
integrate(x^m*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{3}{2}i\arctan(ax)}x^m\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^m,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}i\arctan(ax)}x^m\,dx=\int x^m\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}\,dx$$

input
```
int(x^m*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
```

output
```
int(x^m*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

Reduce [F]

$$\int e^{\frac{3}{2}i\arctan(ax)}x^m\,dx$$
$$= \frac{i\left(-2x^m\sqrt{aix+1}\,(a^2x^2+1)^{\frac{1}{4}}+2\left(\int\frac{x^m\sqrt{aix+1}\,x}{(a^2x^2+1)^{\frac{3}{4}}}dx\right)a^2m+3\left(\int\frac{x^m\sqrt{aix+1}\,x}{(a^2x^2+1)^{\frac{3}{4}}}dx\right)a^2+2\left(\int\frac{x^m\sqrt{aix+1}\,(a^2x^2}{a^2x^3+x}\right.\right.}{a}$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2)*x^m,x)
```

output
```
(i*( - 2*x**m*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(1/4) + 2*int((x**m*sqrt(a*
i*x + 1)*(a**2*x**2 + 1)**(1/4)*x)/(a**2*x**2 + 1),x)*a**2*m + 3*int((x**m
*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(1/4)*x)/(a**2*x**2 + 1),x)*a**2 + 2*int
((x**m*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(1/4))/(a**2*x**3 + x),x)*m))/a
```

3.161 $\int e^{\frac{1}{2}i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 16, antiderivative size = 36

$$\int e^{\frac{1}{2}i\arctan(ax)}x^m\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(1+m,\frac{1}{4},-\frac{1}{4},2+m,iax,-iax\right)}{1+m}$$

output
```
x^(1+m)*AppellF1(1+m,-1/4,1/4,2+m,-I*a*x,I*a*x)/(1+m)
```

Mathematica [F]

$$\int e^{\frac{1}{2}i\arctan(ax)}x^m\,dx=\int e^{\frac{1}{2}i\arctan(ax)}x^m\,dx$$

input
```
Integrate[E^((I/2)*ArcTan[a*x])*x^m,x]
```

output
```
Integrate[E^((I/2)*ArcTan[a*x])*x^m, x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5585, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{\frac{1}{2}i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{\sqrt[4]{1+iax}\,x^m}{\sqrt[4]{1-iax}}dx$$

$$\Big\downarrow \text{150}$$

$$\frac{x^{m+1}\operatorname{AppellF1}\left(m+1,\frac{1}{4},-\frac{1}{4},m+2,iax,-iax\right)}{m+1}$$

input
```
Int[E^((I/2)*ArcTan[a*x])*x^m,x]
```

output
```
(x^(1 + m)*AppellF1[1 + m, 1/4, -1/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}\, x^m dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^m,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^m,x)
```

Fricas [F]

$$\int e^{\frac{1}{2}i\arctan(ax)}x^m\, dx = \int x^m \sqrt{\frac{i\, ax+1}{\sqrt{a^2x^2+1}}}\, dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^m,x, algorithm="fricas")
```

output
```
integral(x^m*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I)), x)
```

Sympy [F]

$$\int e^{\frac{1}{2}i\arctan(ax)}x^m\, dx = \int x^m \sqrt{\frac{i\,(ax-i)}{\sqrt{a^2x^2+1}}}\, dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2)*x**m,x)
```

output
```
Integral(x**m*sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int e^{\frac{1}{2}i\arctan(ax)}x^m\,dx = \int x^m\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}\,dx$$

input
```
integrate((((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^m,x, algorithm="maxima")
```

output
```
integrate(x^m*sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{1}{2}i\arctan(ax)}x^m\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^m,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{2}i\arctan(ax)}x^m\,dx = \int x^m\,\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}\,dx$$

input
```
int(x^m*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)
```

output
```
int(x^m*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int e^{\frac{1}{2}i\arctan(ax)}x^m\,dx = \int \frac{x^m\sqrt{aix+1}}{(a^2x^2+1)^{\frac{1}{4}}}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2)*x^m,x)
```

output
```
int((x**m*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(3/4))/(a**2*x**2 + 1),x)
```

3.162 $\int e^{-\frac{1}{2}i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 16, antiderivative size = 36

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^m\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(1+m,-\frac{1}{4},\frac{1}{4},2+m,iax,-iax\right)}{1+m}$$

output
```
x^(1+m)*AppellF1(1+m,1/4,-1/4,2+m,-I*a*x,I*a*x)/(1+m)
```

Mathematica [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^m\,dx=\int e^{-\frac{1}{2}i\arctan(ax)}x^m\,dx$$

input
```
Integrate[x^m/E^((I/2)*ArcTan[a*x]),x]
```

output
```
Integrate[x^m/E^((I/2)*ArcTan[a*x]), x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5585, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{-\frac{1}{2}i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{\sqrt[4]{1-iax}x^m}{\sqrt[4]{1+iax}}dx$$

$$\Big\downarrow \text{150}$$

$$\frac{x^{m+1}\operatorname{AppellF1}\left(m+1,-\frac{1}{4},\frac{1}{4},m+2,iax,-iax\right)}{m+1}$$

input
```
Int[x^m/E^((I/2)*ArcTan[a*x]),x]
```

output
```
(x^(1 + m)*AppellF1[1 + m, -1/4, 1/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{x^m}{\sqrt{\frac{iax+1}{\sqrt{a^2x^2+1}}}}dx$$

input
```
int(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

output
```
int(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

Fricas [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^m\,dx = \int \frac{x^m}{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

input
```
integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
integral(-I*sqrt(a^2*x^2 + 1)*x^m*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))/(a*x
 - I), x)
```

Sympy [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^m\,dx = \int \frac{x^m}{\sqrt{\frac{i(ax-i)}{\sqrt{a^2x^2+1}}}}\,dx$$

input
```
integrate(x**m/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/2),x)
```

output
```
Integral(x**m/sqrt(I*(a*x - I)/sqrt(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^m\,dx = \int \frac{x^m}{\sqrt{\frac{i\,ax+1}{\sqrt{a^2x^2+1}}}}\,dx$$

input
```
integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m/sqrt((I*a*x + 1)/sqrt(a^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^m\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^m\,dx = \int \frac{x^m}{\sqrt{\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}}}\,dx$$

input
```
int(x^m/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2),x)
```

output
```
int(x^m/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int e^{-\frac{1}{2}i\arctan(ax)}x^m\,dx$$
$$= \frac{i\left(-2x^m\sqrt{aix+1}\,(a^2x^2+1)^{\frac{1}{4}}+2\left(\int \frac{x^m\sqrt{aix+1}\,x}{(a^2x^2+1)^{\frac{3}{4}}}dx\right)a^2m+\left(\int \frac{x^m\sqrt{aix+1}\,x}{(a^2x^2+1)^{\frac{3}{4}}}dx\right)a^2+2\left(\int \frac{x^m\sqrt{aix+1}\,(a^2x^2+}{a^2x^3+x}\right.\right.}{a}$$

input
```
int(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/2),x)
```

output
```
(i*( - 2*x**m*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(1/4) + 2*int((x**m*sqrt(a*
i*x + 1)*(a**2*x**2 + 1)**(1/4)*x)/(a**2*x**2 + 1),x)*a**2*m + int((x**m*s
qrt(a*i*x + 1)*(a**2*x**2 + 1)**(1/4)*x)/(a**2*x**2 + 1),x)*a**2 + 2*int((
x**m*sqrt(a*i*x + 1)*(a**2*x**2 + 1)**(1/4))/(a**2*x**3 + x),x)*m))/a
```

3.163 $\int e^{-\frac{3}{2}i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 16, antiderivative size = 36

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^m\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(1+m,-\frac{3}{4},\frac{3}{4},2+m,iax,-iax\right)}{1+m}$$

output
```
x^(1+m)*AppellF1(1+m,3/4,-3/4,2+m,-I*a*x,I*a*x)/(1+m)
```

Mathematica [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^m\,dx=\int e^{-\frac{3}{2}i\arctan(ax)}x^m\,dx$$

input
```
Integrate[x^m/E^(((3*I)/2)*ArcTan[a*x]),x]
```

output
```
Integrate[x^m/E^(((3*I)/2)*ArcTan[a*x]), x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5585, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{-\frac{3}{2}i\arctan(ax)}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{(1-iax)^{3/4}x^m}{(1+iax)^{3/4}}dx$$

$$\Big\downarrow \text{150}$$

$$\frac{x^{m+1}\text{AppellF1}\left(m+1,-\frac{3}{4},\frac{3}{4},m+2,iax,-iax\right)}{m+1}$$

input
```
Int[x^m/E^(((3*I)/2)*ArcTan[a*x]),x]
```

output
```
(x^(1 + m)*AppellF1[1 + m, -3/4, 3/4, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{x^m}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

output
```
int(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

Fricas [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^m\,dx=\int \frac{x^m}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
integral(-(a*x + I)*x^m*sqrt(I*sqrt(a^2*x^2 + 1)/(a*x + I))/(a*x - I), x)
```

Sympy [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^m\,dx=\int \frac{x^m}{\left(\frac{i(ax-i)}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**m/((1+I*a*x)/(a**2*x**2+1)**(1/2))**(3/2),x)
```

output
```
Integral(x**m/(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^m/((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^m\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{3/2}}\,dx$$

input
```
int(x^m/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2),x)
```

output
```
int(x^m/((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(3/2), x)
```

Reduce [F]

$$\int e^{-\frac{3}{2}i\arctan(ax)}x^m\,dx = \int \frac{x^m}{\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

output
```
int(x^m/((1+I*a*x)/(a^2*x^2+1)^(1/2))^(3/2),x)
```

3.164 $\int e^{\frac{2\arctan(x)}{3}} x^m \, dx$

Optimal result

Integrand size = 12, antiderivative size = 38

$$\int e^{\frac{2\arctan(x)}{3}} x^m \, dx = \frac{x^{1+m} \operatorname{AppellF1}\left(1+m, -\frac{i}{3}, \frac{i}{3}, 2+m, ix, -ix\right)}{1+m}$$

output
```
x^(1+m)*AppellF1(1+m,1/3*I,-1/3*I,2+m,-I*x,I*x)/(1+m)
```

Mathematica [F]

$$\int e^{\frac{2\arctan(x)}{3}} x^m \, dx = \int e^{\frac{2\arctan(x)}{3}} x^m \, dx$$

input
```
Integrate[E^((2*ArcTan[x])/3)*x^m,x]
```

output
```
Integrate[E^((2*ArcTan[x])/3)*x^m, x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5585, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{2\arctan(x)}{3}} x^m \, dx$$

$$\Big\downarrow \text{5585}$$

$$\int (1-ix)^{\frac{i}{3}} (1+ix)^{-\frac{i}{3}} x^m dx$$

$$\Big\downarrow \text{150}$$

$$\frac{x^{m+1} \text{AppellF1}\left(m+1,-\frac{i}{3},\frac{i}{3},m+2,ix,-ix\right)}{m+1}$$

input
```
Int[E^((2*ArcTan[x])/3)*x^m,x]
```

output
```
(x^(1 + m)*AppellF1[1 + m, -1/3*I, I/3, 2 + m, I*x, (-I)*x])/(1 + m)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] && !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int e^{\frac{2\arctan(x)}{3}} x^m dx$$

input
```
int(exp(2/3*arctan(x))*x^m,x)
```

output
```
int(exp(2/3*arctan(x))*x^m,x)
```

Fricas [F]

$$\int e^{\frac{2\arctan(x)}{3}} x^m\,dx = \int x^m e^{\left(\frac{2}{3}\arctan(x)\right)}\,dx$$

input
```
integrate(exp(2/3*arctan(x))*x^m,x, algorithm="fricas")
```

output
```
integral(x^m*e^(2/3*arctan(x)), x)
```

Sympy [F]

$$\int e^{\frac{2\arctan(x)}{3}} x^m\,dx = \int x^m e^{\frac{2\operatorname{atan}(x)}{3}}\,dx$$

input
```
integrate(exp(2/3*atan(x))*x**m,x)
```

output
```
Integral(x**m*exp(2*atan(x)/3), x)
```

Maxima [F]

$$\int e^{\frac{2\arctan(x)}{3}}x^m\,dx=\int x^m e^{\left(\frac{2}{3}\arctan(x)\right)}\,dx$$

input
```
integrate(exp(2/3*arctan(x))*x^m,x, algorithm="maxima")
```

output
```
integrate(x^m*e^(2/3*arctan(x)), x)
```

Giac [F]

$$\int e^{\frac{2\arctan(x)}{3}}x^m\,dx=\int x^m e^{\left(\frac{2}{3}\arctan(x)\right)}\,dx$$

input
```
integrate(exp(2/3*arctan(x))*x^m,x, algorithm="giac")
```

output
```
integrate(x^m*e^(2/3*arctan(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{2\arctan(x)}{3}}x^m\,dx=\int x^m\,\mathrm{e}^{\frac{2\,\mathrm{atan}(x)}{3}}\,dx$$

input
```
int(x^m*exp((2*atan(x))/3),x)
```

output
```
int(x^m*exp((2*atan(x))/3), x)
```

Reduce [F]

$$\int e^{\frac{2\arctan(x)}{3}} x^m\,dx = \int x^m e^{\frac{2atan(x)}{3}}\,dx$$

input
```
int(exp(2/3*atan(x))*x^m,x)
```

output
```
int(x**m*e**((2*atan(x))/3),x)
```

3.165 $\int e^{\frac{\arctan(x)}{3}} x^m\,dx$

Optimal result

Integrand size = 12, antiderivative size = 38

$$\int e^{\frac{\arctan(x)}{3}} x^m\,dx = \frac{x^{1+m}\,\text{AppellF1}\left(1+m,-\frac{i}{6},\frac{i}{6},2+m,ix,-ix\right)}{1+m}$$

output
```
x^(1+m)*AppellF1(1+m,1/6*I,-1/6*I,2+m,-I*x,I*x)/(1+m)
```

Mathematica [F]

$$\int e^{\frac{\arctan(x)}{3}} x^m\,dx = \int e^{\frac{\arctan(x)}{3}} x^m\,dx$$

input
```
Integrate[E^(ArcTan[x]/3)*x^m,x]
```

output
```
Integrate[E^(ArcTan[x]/3)*x^m, x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5585, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{\arctan(x)}{3}} x^m \, dx$$
$$\Big\downarrow \ 5585$$
$$\int (1-ix)^{\frac{i}{6}}(1+ix)^{-\frac{i}{6}} x^m dx$$
$$\Big\downarrow \ 150$$
$$\frac{x^{m+1} \text{AppellF1}\left(m+1,-\frac{i}{6},\frac{i}{6},m+2,ix,-ix\right)}{m+1}$$

input
```
Int[E^(ArcTan[x]/3)*x^m,x]
```

output
```
(x^(1 + m)*AppellF1[1 + m, -1/6*I, I/6, 2 + m, I*x, (-I)*x])/(1 + m)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int e^{\frac{\arctan(x)}{3}} x^m dx$$

input
```
int(exp(1/3*arctan(x))*x^m,x)
```

output
```
int(exp(1/3*arctan(x))*x^m,x)
```

Fricas [F]

$$\int e^{\frac{\arctan(x)}{3}} x^m\, dx = \int x^m e^{\left(\frac{1}{3}\arctan(x)\right)}\, dx$$

input
```
integrate(exp(1/3*arctan(x))*x^m,x, algorithm="fricas")
```

output
```
integral(x^m*e^(1/3*arctan(x)), x)
```

Sympy [F]

$$\int e^{\frac{\arctan(x)}{3}} x^m\, dx = \int x^m e^{\frac{\operatorname{atan}(x)}{3}}\, dx$$

input
```
integrate(exp(1/3*atan(x))*x**m,x)
```

output
```
Integral(x**m*exp(atan(x)/3), x)
```

Maxima [F]

$$\int e^{\frac{\arctan(x)}{3}} x^m \, dx = \int x^m e^{\left(\frac{1}{3} \arctan(x)\right)} \, dx$$

input
```
integrate(exp(1/3*arctan(x))*x^m,x, algorithm="maxima")
```

output
```
integrate(x^m*e^(1/3*arctan(x)), x)
```

Giac [F]

$$\int e^{\frac{\arctan(x)}{3}} x^m \, dx = \int x^m e^{\left(\frac{1}{3} \arctan(x)\right)} \, dx$$

input
```
integrate(exp(1/3*arctan(x))*x^m,x, algorithm="giac")
```

output
```
integrate(x^m*e^(1/3*arctan(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{\arctan(x)}{3}} x^m \, dx = \int x^m \, \mathrm{e}^{\frac{\mathrm{atan}(x)}{3}} \, dx$$

input
```
int(x^m*exp(atan(x)/3),x)
```

output
```
int(x^m*exp(atan(x)/3), x)
```

Reduce [F]

$$\int e^{\frac{\arctan(x)}{3}} x^m \, dx = \int x^m e^{\frac{atan(x)}{3}} dx$$

input
```
int(exp(1/3*atan(x))*x^m,x)
```

output
```
int(x**m*e**(atan(x)/3),x)
```

3.166 $\int e^{\frac{1}{4}i\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 16, antiderivative size = 36

$$\int e^{\frac{1}{4}i\arctan(ax)}x^m\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(1+m,\frac{1}{8},-\frac{1}{8},2+m,iax,-iax\right)}{1+m}$$

output
```
x^(1+m)*AppellF1(1+m,-1/8,1/8,2+m,-I*a*x,I*a*x)/(1+m)
```

Mathematica [F]

$$\int e^{\frac{1}{4}i\arctan(ax)}x^m\,dx=\int e^{\frac{1}{4}i\arctan(ax)}x^m\,dx$$

input
```
Integrate[E^((I/4)*ArcTan[a*x])*x^m,x]
```

output
```
Integrate[E^((I/4)*ArcTan[a*x])*x^m, x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5585, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{\frac{1}{4}i\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{\sqrt[8]{1+iax}x^m}{\sqrt[8]{1-iax}}dx$$

$$\Big\downarrow \text{150}$$

$$\frac{x^{m+1}\operatorname{AppellF1}\left(m+1,\frac{1}{8},-\frac{1}{8},m+2,iax,-iax\right)}{m+1}$$

input
```
Int[E^((I/4)*ArcTan[a*x])*x^m,x]
```

output
```
(x^(1 + m)*AppellF1[1 + m, 1/8, -1/8, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}} x^m dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^m,x)
```

output
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^m,x)
```

Fricas [F]

$$\int e^{\frac{1}{4}i\arctan(ax)}x^m\,dx = \int x^m\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^m,x, algorithm="fricas")
```

output
```
integral(x^m*(I*sqrt(a^2*x^2 + 1)/(a*x + I))^(1/4), x)
```

Sympy [F]

$$\int e^{\frac{1}{4}i\arctan(ax)}x^m\,dx = \int x^m\sqrt[4]{\frac{i\,(ax-i)}{\sqrt{a^2x^2+1}}}\,dx$$

input
```
integrate(((1+I*a*x)/(a**2*x**2+1)**(1/2))**(1/4)*x**m,x)
```

output
```
Integral(x**m*(I*(a*x - I)/sqrt(a**2*x**2 + 1))**(1/4), x)
```

Maxima [F]

$$\int e^{\frac{1}{4}i\arctan(ax)}x^m\,dx = \int x^m\left(\frac{i\,ax+1}{\sqrt{a^2x^2+1}}\right)^{\frac{1}{4}}\,dx$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^m,x, algorithm="maxima")
```

output
```
integrate(x^m*((I*a*x + 1)/sqrt(a^2*x^2 + 1))^(1/4), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{1}{4}i\arctan(ax)}x^m\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^m,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{4}i\arctan(ax)}x^m\,dx = \int x^m\left(\frac{1+a\,x\,1\mathrm{i}}{\sqrt{a^2\,x^2+1}}\right)^{1/4}\,dx$$

input
```
int(x^m*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4),x)
```

output
```
int(x^m*((a*x*1i + 1)/(a^2*x^2 + 1)^(1/2))^(1/4), x)
```

Reduce [F]

$$\int e^{\frac{1}{4}i\arctan(ax)}x^m\,dx = \int \frac{x^m(aix+1)^{\frac{1}{4}}}{(a^2x^2+1)^{\frac{1}{8}}}dx$$

input
```
int(((1+I*a*x)/(a^2*x^2+1)^(1/2))^(1/4)*x^m,x)
```

output
```
int((x**m*(a*i*x + 1)**(1/4))/(a**2*x**2 + 1)**(1/8),x)
```

3.167 $\int e^{in\arctan(ax)}x^m\,dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int e^{in\arctan(ax)}x^m\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(1+m,\frac{n}{2},-\frac{n}{2},2+m,iax,-iax\right)}{1+m}$$

output
```
x^(1+m)*AppellF1(1+m,-1/2*n,1/2*n,2+m,-I*a*x,I*a*x)/(1+m)
```

Mathematica [F]

$$\int e^{in\arctan(ax)}x^m\,dx=\int e^{in\arctan(ax)}x^m\,dx$$

input
```
Integrate[E^(I*n*ArcTan[a*x])*x^m,x]
```

output
```
Integrate[E^(I*n*ArcTan[a*x])*x^m, x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5585, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{in \arctan(ax)} \, dx$$

$$\Big\downarrow \text{5585}$$

$$\int x^m (1-iax)^{-n/2}(1+iax)^{n/2} dx$$

$$\Big\downarrow \text{150}$$

$$\frac{x^{m+1} \operatorname{AppellF1}\left(m+1,\frac{n}{2},-\frac{n}{2},m+2,iax,-iax\right)}{m+1}$$

input
```
Int[E^(I*n*ArcTan[a*x])*x^m,x]
```

output
```
(x^(1 + m)*AppellF1[1 + m, n/2, -1/2*n, 2 + m, I*a*x, (-I)*a*x])/(1 + m)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int e^{in \arctan(ax)} x^m dx$$

input
```
int(exp(I*n*arctan(a*x))*x^m,x)
```

output
```
int(exp(I*n*arctan(a*x))*x^m,x)
```

Fricas [F]

$$\int e^{in \arctan(ax)} x^m \, dx = \int x^m e^{(i \, n \arctan(ax))} \, dx$$

input
```
integrate(exp(I*n*arctan(a*x))*x^m,x, algorithm="fricas")
```

output
```
integral(x^m/(-(a*x + I)/(a*x - I))^(1/2*n), x)
```

Sympy [F]

$$\int e^{in \arctan(ax)} x^m \, dx = \int x^m e^{in \operatorname{atan}(ax)} \, dx$$

input
```
integrate(exp(I*n*atan(a*x))*x**m,x)
```

output
```
Integral(x**m*exp(I*n*atan(a*x)), x)
```

Maxima [F]

$$\int e^{in\arctan(ax)}x^m\,dx=\int x^m e^{(i\,n\arctan(ax))}\,dx$$

input
```
integrate(exp(I*n*arctan(a*x))*x^m,x, algorithm="maxima")
```

output
```
integrate(x^m*e^(I*n*arctan(a*x)), x)
```

Giac [F]

$$\int e^{in\arctan(ax)}x^m\,dx=\int x^m e^{(i\,n\arctan(ax))}\,dx$$

input
```
integrate(exp(I*n*arctan(a*x))*x^m,x, algorithm="giac")
```

output
```
integrate(x^m*e^(I*n*arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{in\arctan(ax)}x^m\,dx=\int x^m\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)\,1\mathrm{i}}\,dx$$

input
```
int(x^m*exp(n*atan(a*x)*1i),x)
```

output
```
int(x^m*exp(n*atan(a*x)*1i), x)
```

Reduce [F]

$$\int e^{in\arctan(ax)}x^m\,dx=\int x^m e^{atan(ax)in}dx$$

input
```
int(exp(I*n*atan(a*x))*x^m,x)
```

output
```
int(x**m*e**(atan(a*x)*i*n),x)
```

3.168 $\int e^{in\arctan(ax)}x^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 207

$$\int e^{in\arctan(ax)}x^3\,dx = -\frac{(6+2n+n^2)(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{24a^4} + \frac{x^2(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{4a^2} + \frac{n(1-iax)^{2-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{12a^4} - \frac{2^{-2+\frac{n}{2}}n(8+n^2)(1-iax)^{1-\frac{n}{2}}\,\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-iax)\right)}{3a^4(2-n)}$$

output
```
-1/24*(n^2+2*n+6)*(1-I*a*x)^(1-1/2*n)*(1+I*a*x)^(1+1/2*n)/a^4+1/4*x^2*(1-I
*a*x)^(1-1/2*n)*(1+I*a*x)^(1+1/2*n)/a^2+1/12*n*(1-I*a*x)^(2-1/2*n)*(1+I*a*
x)^(1+1/2*n)/a^4-1/3*2^(-2+1/2*n)*n*(n^2+8)*(1-I*a*x)^(1-1/2*n)*hypergeom(
[-1/2*n, 1-1/2*n],[2-1/2*n],1/2-1/2*I*a*x)/a^4/(2-n)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.01

$$\int e^{in\arctan(ax)}x^3\,dx = \frac{(1-iax)^{-n/2}(i+ax)\left(-i2^{3+\frac{n}{2}}n\,\text{Hypergeometric2F1}\left(-2-\frac{n}{2},1-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-iax)\right)+i2^{3+\frac{n}{2}}(-1+\right.}{}$$

input
```
Integrate[E^(I*n*ArcTan[a*x])*x^3,x]
```

output
```
((I + a*x)*((-I)*2^(3 + n/2)*n*Hypergeometric2F1[-2 - n/2, 1 - n/2, 2 - n/
2, (1 - I*a*x)/2] + I*2^(3 + n/2)*(-1 + n)*Hypergeometric2F1[-1 - n/2, 1 -
 n/2, 2 - n/2, (1 - I*a*x)/2] + (-2 + n)*(a^2*x^2*(1 + I*a*x)^(n/2)*(-I +
a*x) - I*2^(1 + n/2)*Hypergeometric2F1[1 - n/2, -1/2*n, 2 - n/2, (1 - I*a*
x)/2])))/(4*a^4*(-2 + n)*(1 - I*a*x)^(n/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5585, 111, 25, 164, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3e^{in\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int x^3(1-iax)^{-n/2}(1+iax)^{n/2}dx$$

$$\Big\downarrow \text{111}$$

$$\frac{\int -x(1-iax)^{-n/2}(iax+1)^{n/2}(ianx+2)dx}{4a^2}+\frac{x^2(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{4a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^2(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n+2}{2}}}{4a^2}-\frac{\int x(1-iax)^{-n/2}(iax+1)^{n/2}(ianx+2)dx}{4a^2}$$

$$\Big\downarrow 164$$

$$\frac{x^2(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n+2}{2}}}{4a^2}-\frac{\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n+2}{2}}(2ianx+n^2+6)}{6a^2}-\frac{in(n^2+8)\int(1-iax)^{-n/2}(iax+1)^{n/2}dx}{6a}}{4a^2}$$

$$\Big\downarrow 79$$

$$\frac{x^2(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n+2}{2}}}{4a^2}-\frac{\frac{2^{n/2}n(n^2+8)(1-iax)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-iax)\right)}{3a^2(2-n)}+\frac{(1+iax)^{\frac{n+2}{2}}(2ianx+n^2+6)(1-iax)^{1-\frac{n}{2}}}{6a^2}}{4a^2}$$

input
```
Int[E^(I*n*ArcTan[a*x])*x^3,x]
```

output
```
(x^2*(1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/(4*a^2) - (((1 - I*a*x
)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2)*(6 + n^2 + (2*I)*a*n*x))/(6*a^2) + (2^
(n/2)*n*(8 + n^2)*(1 - I*a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -1/2*n,
 2 - n/2, (1 - I*a*x)/2])/(3*a^2*(2 - n)))/(4*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int e^{in \arctan(ax)} x^3 dx$$

input
```
int(exp(I*n*arctan(a*x))*x^3,x)
```

output
```
int(exp(I*n*arctan(a*x))*x^3,x)
```

Fricas [F]

$$\int e^{in\arctan(ax)}x^3\,dx = \int x^3 e^{(i\,n\arctan(ax))}\,dx$$

input
```
integrate(exp(I*n*arctan(a*x))*x^3,x, algorithm="fricas")
```

output
```
integral(x^3/(-(a*x + I)/(a*x - I))^(1/2*n), x)
```

Sympy [F]

$$\int e^{in\arctan(ax)}x^3\,dx = \int x^3 e^{in\operatorname{atan}(ax)}\,dx$$

input
```
integrate(exp(I*n*atan(a*x))*x**3,x)
```

output
```
Integral(x**3*exp(I*n*atan(a*x)), x)
```

Maxima [F]

$$\int e^{in\arctan(ax)}x^3\,dx = \int x^3 e^{(i\,n\arctan(ax))}\,dx$$

input
```
integrate(exp(I*n*arctan(a*x))*x^3,x, algorithm="maxima")
```

output
```
integrate(x^3*e^(I*n*arctan(a*x)), x)
```

Giac [F]

$$\int e^{in \arctan(ax)} x^3 \, dx = \int x^3 e^{(i n \arctan(ax))} \, dx$$

input
```
integrate(exp(I*n*arctan(a*x))*x^3,x, algorithm="giac")
```

output
```
integrate(x^3*e^(I*n*arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{in \arctan(ax)} x^3 \, dx = \int x^3 \, e^{n \operatorname{atan}(a x) \, 1i} \, dx$$

input
```
int(x^3*exp(n*atan(a*x)*1i),x)
```

output
```
int(x^3*exp(n*atan(a*x)*1i), x)
```

Reduce [F]

$$\int e^{in \arctan(ax)} x^3 \, dx = \int e^{atan(ax)in} x^3 dx$$

input
```
int(exp(I*n*atan(a*x))*x^3,x)
```

output
```
int(e**(atan(a*x)*i*n)*x**3,x)
```

3.169 $\int e^{in \arctan(ax)} x^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 159

$$\int e^{in \arctan(ax)} x^2 \, dx = -\frac{in(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{6a^3} + \frac{x(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{3a^2} - \frac{i2^{n/2}(2+n^2)(1-iax)^{1-\frac{n}{2}} \text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-iax)\right)}{3a^3(2-n)}$$

output
```
-1/6*I*n*(1-I*a*x)^(1-1/2*n)*(1+I*a*x)^(1+1/2*n)/a^3+1/3*x*(1-I*a*x)^(1-1/
2*n)*(1+I*a*x)^(1+1/2*n)/a^2-1/3*I*2^(1/2*n)*(n^2+2)*(1-I*a*x)^(1-1/2*n)*h
ypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2-1/2*I*a*x)/a^3/(2-n)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.73

$$\int e^{in \arctan(ax)} x^2 \, dx = \frac{(1-iax)^{-n/2}(i+ax)\left((-2+n)(1+iax)^{n/2}(-i+ax)(-in+2ax)+2^{1+\frac{n}{2}}(2+n^2)\,\text{Hypergeometric2F1}\right.}{6a^3(-2+n)}$$

input
```
Integrate[E^(I*n*ArcTan[a*x])*x^2,x]
```

output
```
((I + a*x)*((-2 + n)*(1 + I*a*x)^(n/2)*(-I + a*x)*((-I)*n + 2*a*x) + 2^(1
+ n/2)*(2 + n^2)*Hypergeometric2F1[1 - n/2, -1/2*n, 2 - n/2, (1 - I*a*x)/2
]))/(6*a^3*(-2 + n)*(1 - I*a*x)^(n/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5585, 101, 25, 90, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{in\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int x^2(1-iax)^{-n/2}(1+iax)^{n/2}dx$$

$$\Big\downarrow \text{101}$$

$$\frac{\int -(1-iax)^{-n/2}(iax+1)^{n/2}(ianx+1)dx}{3a^2}+\frac{x(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{3a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{x(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n+2}{2}}}{3a^2}-\frac{\int (1-iax)^{-n/2}(iax+1)^{n/2}(ianx+1)dx}{3a^2}$$

$$\Big\downarrow \text{90}$$

$$\frac{x(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n+2}{2}}}{3a^2}-\frac{\frac{1}{2}\left(n^2+2\right)\int (1-iax)^{-n/2}(iax+1)^{n/2}dx+\frac{in(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{2a}}{3a^2}$$

$$\Big\downarrow \text{79}$$

$$\frac{x(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n+2}{2}}}{3a^2}-\frac{\frac{i2^{n/2}(n^2+2)(1-iax)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-iax)\right)}{a(2-n)}+\frac{in(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{2a}}{3a^2}$$

input
```
Int[E^(I*n*ArcTan[a*x])*x^2,x]
```

output
```
(x*(1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/(3*a^2) - (((I/2)*n*(1 -
 I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/a + (I*2^(n/2)*(2 + n^2)*(1 - I
*a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -1/2*n, 2 - n/2, (1 - I*a*x)/2]
)/(a*(2 - n)))/(3*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int e^{in\arctan(ax)}x^2dx$$

input
```
int(exp(I*n*arctan(a*x))*x^2,x)
```

output
```
int(exp(I*n*arctan(a*x))*x^2,x)
```

Fricas [F]

$$\int e^{in\arctan(ax)}x^2\,dx = \int x^2e^{(i\,n\arctan(ax))}\,dx$$

input
```
integrate(exp(I*n*arctan(a*x))*x^2,x, algorithm="fricas")
```

output
```
integral(x^2/(-(a*x + I)/(a*x - I))^(1/2*n), x)
```

Sympy [F]

$$\int e^{in\arctan(ax)}x^2\,dx = \int x^2e^{in\,\mathrm{atan}\,(ax)}\,dx$$

input
```
integrate(exp(I*n*atan(a*x))*x**2,x)
```

output
```
Integral(x**2*exp(I*n*atan(a*x)), x)
```

Maxima [F]

$$\int e^{in \arctan(ax)} x^2 \, dx = \int x^2 e^{(i\, n \arctan(ax))} \, dx$$

input
```
integrate(exp(I*n*arctan(a*x))*x^2,x, algorithm="maxima")
```

output
```
integrate(x^2*e^(I*n*arctan(a*x)), x)
```

Giac [F]

$$\int e^{in \arctan(ax)} x^2 \, dx = \int x^2 e^{(i\, n \arctan(ax))} \, dx$$

input
```
integrate(exp(I*n*arctan(a*x))*x^2,x, algorithm="giac")
```

output
```
integrate(x^2*e^(I*n*arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{in \arctan(ax)} x^2 \, dx = \int x^2 \, \mathrm{e}^{n \operatorname{atan}(a\, x)\, 1\mathrm{i}} \, dx$$

input
```
int(x^2*exp(n*atan(a*x)*1i),x)
```

output
```
int(x^2*exp(n*atan(a*x)*1i), x)
```

Reduce [F]

$$\int e^{in\arctan(ax)}x^2\,dx=\int e^{atan(ax)in}x^2dx$$

input
```
int(exp(I*n*atan(a*x))*x^2,x)
```

output
```
int(e**(atan(a*x)*i*n)*x**2,x)
```

3.170 $\int e^{in \arctan(ax)} x \, dx$

Optimal result

Integrand size = 13, antiderivative size = 107

$$\int e^{in \arctan(ax)} x \, dx = \frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{2a^2} + \frac{2^{n/2} n (1-iax)^{1-\frac{n}{2}} \text{Hypergeometric2F1}\left(1-\frac{n}{2}, -\frac{n}{2}, 2-\frac{n}{2}, \frac{1}{2}(1-iax)\right)}{a^2(2-n)}$$

output
```
1/2*(1-I*a*x)^(1-1/2*n)*(1+I*a*x)^(1+1/2*n)/a^2+2^(1/2*n)*n*(1-I*a*x)^(1-1
/2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2-1/2*I*a*x)/a^2/(2-n)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.98

$$\int e^{in \arctan(ax)} x \, dx = \frac{(1-iax)^{-n/2}(i+ax)\left((-2+n)(1+iax)^{n/2}(-i+ax) + i2^{1+\frac{n}{2}} n \, \text{Hypergeometric2F1}\left(1-\frac{n}{2}, -\frac{n}{2}, 2-\frac{n}{2}\right.\right.}{2a^2(-2+n)}$$

input
```
Integrate[E^(I*n*ArcTan[a*x])*x,x]
```

output
```
((I + a*x)*((-2 + n)*(1 + I*a*x)^(n/2)*(-I + a*x) + I*2^(1 + n/2)*n*Hyperg
eometric2F1[1 - n/2, -1/2*n, 2 - n/2, (1 - I*a*x)/2]))/(2*a^2*(-2 + n)*(1
- I*a*x)^(n/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = $\{5585, 90, 79\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{in\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5585}$$

$$\int x(1-iax)^{-n/2}(1+iax)^{n/2}dx$$

$$\Big\downarrow \text{90}$$

$$\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n+2}{2}}}{2a^2}-\frac{in\int(1-iax)^{-n/2}(iax+1)^{n/2}dx}{2a}$$

$$\Big\downarrow \text{79}$$

$$\frac{2^{n/2}n(1-iax)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-iax)\right)}{a^2(2-n)}+\frac{(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{2a^2}$$

input
```
Int[E^(I*n*ArcTan[a*x])*x,x]
```

output
```
((1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/(2*a^2) + (2^(n/2)*n*(1 -
I*a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -1/2*n, 2 - n/2, (1 - I*a*x)/2
])/(a^2*(2 - n))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \mathrm{e}^{in\arctan(ax)}x dx$$

input
```
int(exp(I*n*arctan(a*x))*x,x)
```

output
```
int(exp(I*n*arctan(a*x))*x,x)
```

Fricas [F]

$$\int e^{in\arctan(ax)}x\,dx = \int xe^{(i\,n\arctan(ax))}\,dx$$

input
```
integrate(exp(I*n*arctan(a*x))*x,x, algorithm="fricas")
```

output
```
integral(x/(-(a*x + I)/(a*x - I))^(1/2*n), x)
```

Sympy [F]

$$\int e^{in\arctan(ax)}x\,dx = \int xe^{in\operatorname{atan}(ax)}\,dx$$

input
```
integrate(exp(I*n*atan(a*x))*x,x)
```

output
```
Integral(x*exp(I*n*atan(a*x)), x)
```

Maxima [F]

$$\int e^{in\arctan(ax)}x\,dx = \int xe^{(i\,n\arctan(ax))}\,dx$$

input
```
integrate(exp(I*n*arctan(a*x))*x,x, algorithm="maxima")
```

output
```
integrate(x*e^(I*n*arctan(a*x)), x)
```

Giac [F]

$$\int e^{in\arctan(ax)}x\,dx = \int xe^{(i\,n\arctan(ax))}\,dx$$

input
```
integrate(exp(I*n*arctan(a*x))*x,x, algorithm="giac")
```

output
```
integrate(x*e^(I*n*arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{in\arctan(ax)}x\,dx = \int x\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)\,1\mathrm{i}}\,dx$$

input
```
int(x*exp(n*atan(a*x)*1i),x)
```

output
```
int(x*exp(n*atan(a*x)*1i), x)
```

Reduce [F]

$$\int e^{in\arctan(ax)}x\,dx = \int e^{atan(ax)in}xdx$$

input
```
int(exp(I*n*atan(a*x))*x,x)
```

output
```
int(e**(atan(a*x)*i*n)*x,x)
```

3.171 $\int e^{in\arctan(ax)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 71

$$\int e^{in\arctan(ax)}\,dx = \frac{i2^{1+\frac{n}{2}}(1-iax)^{1-\frac{n}{2}}\,\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-iax)\right)}{a(2-n)}$$

output
```
I*2^(1+1/2*n)*(1-I*a*x)^(1-1/2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/
2-1/2*I*a*x)/a/(2-n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.75

$$\int e^{in\arctan(ax)}\,dx = -\frac{4ie^{i(2+n)\arctan(ax)}\,\text{Hypergeometric2F1}\left(2,1+\frac{n}{2},2+\frac{n}{2},-e^{2i\arctan(ax)}\right)}{a(2+n)}$$

input
```
Integrate[E^(I*n*ArcTan[a*x]),x]
```

output
```
((-4*I)*E^(I*(2 + n)*ArcTan[a*x])*Hypergeometric2F1[2, 1 + n/2, 2 + n/2, -
E^((2*I)*ArcTan[a*x])])/(a*(2 + n))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = $\{5584, 79\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{in\arctan(ax)}dx$$

$$\Big\downarrow \ 5584$$

$$\int (1-iax)^{-n/2}(1+iax)^{n/2}dx$$

$$\Big\downarrow \ 79$$

$$\frac{i2^{\frac{n}{2}+1}(1-iax)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left(1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-iax)\right)}{a(2-n)}$$

input
```
Int[E^(I*n*ArcTan[a*x]),x]
```

output
```
(I*2^(1 + n/2)*(1 - I*a*x)^(1 - n/2)*Hypergeometric2F1[1 - n/2, -1/2*n, 2
- n/2, (1 - I*a*x)/2])/(a*(2 - n))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5584
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Maple [F]

$$\int e^{in\arctan(ax)}dx$$

input
```
int(exp(I*n*arctan(a*x)),x)
```

output
```
int(exp(I*n*arctan(a*x)),x)
```

Fricas [F]

$$\int e^{in\arctan(ax)}\,dx = \int e^{(i\,n\arctan(ax))}\,dx$$

input
```
integrate(exp(I*n*arctan(a*x)),x, algorithm="fricas")
```

output
```
integral(1/((-(a*x + I)/(a*x - I))^(1/2*n)), x)
```

Sympy [F]

$$\int e^{in\arctan(ax)}\,dx = \int e^{in\operatorname{atan}(ax)}\,dx$$

input
```
integrate(exp(I*n*atan(a*x)),x)
```

output
```
Integral(exp(I*n*atan(a*x)), x)
```

Maxima [F]

$$\int e^{in\arctan(ax)}\,dx=\int e^{(i\,n\arctan(ax))}\,dx$$

input
```
integrate(exp(I*n*arctan(a*x)),x, algorithm="maxima")
```

output
```
integrate(e^(I*n*arctan(a*x)), x)
```

Giac [F]

$$\int e^{in\arctan(ax)}\,dx=\int e^{(i\,n\arctan(ax))}\,dx$$

input
```
integrate(exp(I*n*arctan(a*x)),x, algorithm="giac")
```

output
```
integrate(e^(I*n*arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{in\arctan(ax)}\,dx=\int e^{n\,\mathrm{atan}(a\,x)\,1\mathrm{i}}\,dx$$

input
```
int(exp(n*atan(a*x)*1i),x)
```

output
```
int(exp(n*atan(a*x)*1i), x)
```

Reduce [F]

$$\int e^{in\arctan(ax)}\,dx = \int e^{atan(ax)in}dx$$

input
```
int(exp(I*n*atan(a*x)),x)
```

output
```
int(e**(atan(a*x)*i*n),x)
```

3.172 $\int \frac{e^{in \arctan(ax)}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 125

$$\int \frac{e^{in \arctan(ax)}}{x} \, dx = \frac{2(1-iax)^{-n/2}(1+iax)^{n/2} \text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{1-iax}{1+iax}\right)}{n} - \frac{2^{1+\frac{n}{2}}(1-iax)^{-n/2} \text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{1}{2}(1-iax)\right)}{n}$$

output
```
2*(1+I*a*x)^(1/2*n)*hypergeom([1, -1/2*n],[1-1/2*n],(1-I*a*x)/(1+I*a*x))/n
/((1-I*a*x)^(1/2*n))-2^(1+1/2*n)*hypergeom([-1/2*n, -1/2*n],[1-1/2*n],1/2-
1/2*I*a*x)/n/((1-I*a*x)^(1/2*n))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.85

$$\int \frac{e^{in \arctan(ax)}}{x} \, dx = \frac{2(1-iax)^{-n/2}\left((1+iax)^{n/2} \text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{i+ax}{i-ax}\right)-2^{n/2} \text{Hypergeometric2F1}\left(-\frac{n}{2},-\right.\right.}{n}$$

input
```
Integrate[E^(I*n*ArcTan[a*x])/x,x]
```

output
```
(2*((1 + I*a*x)^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (I + a*x)/(I -
 a*x)] - 2^(n/2)*Hypergeometric2F1[-1/2*n, -1/2*n, 1 - n/2, (1 - I*a*x)/2]
))/(n*(1 - I*a*x)^(n/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {5585, 140, 79, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{in\arctan(ax)}}{x}dx$$
$$\Big\downarrow \text{5585}$$
$$\int \frac{(1-iax)^{-n/2}(1+iax)^{n/2}}{x}dx$$
$$\Big\downarrow \text{140}$$
$$\int \frac{(1-iax)^{-\frac{n}{2}-1}(iax+1)^{n/2}}{x}dx-ia\int (1-iax)^{-\frac{n}{2}-1}(iax+1)^{n/2}dx$$
$$\Big\downarrow \text{79}$$
$$\int \frac{(1-iax)^{-\frac{n}{2}-1}(iax+1)^{n/2}}{x}dx-\frac{2^{\frac{n}{2}+1}(1-iax)^{-n/2}\text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{1}{2}(1-iax)\right)}{n}$$
$$\Big\downarrow \text{141}$$
$$\frac{2(1-iax)^{-n/2}(1+iax)^{n/2}\text{Hypergeometric2F1}\left(1,-\frac{n}{2},1-\frac{n}{2},\frac{1-iax}{iax+1}\right)}{n}-\frac{2^{\frac{n}{2}+1}(1-iax)^{-n/2}\text{Hypergeometric2F1}\left(-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{1}{2}(1-iax)\right)}{n}$$

input
```
Int[E^(I*n*ArcTan[a*x])/x,x]
```

output
```
(2*(1 + I*a*x)^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (1 - I*a*x)/(1
+ I*a*x)])/(n*(1 - I*a*x)^(n/2)) - (2^(1 + n/2)*Hypergeometric2F1[-1/2*n,
-1/2*n, 1 - n/2, (1 - I*a*x)/2])/(n*(1 - I*a*x)^(n/2))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{e^{in\arctan(ax)}}{x}dx$$

input
```
int(exp(I*n*arctan(a*x))/x,x)
```

output
```
int(exp(I*n*arctan(a*x))/x,x)
```

Fricas [F]

$$\int \frac{e^{in\arctan(ax)}}{x}\,dx = \int \frac{e^{(i\,n\arctan(ax))}}{x}\,dx$$

input
```
integrate(exp(I*n*arctan(a*x))/x,x, algorithm="fricas")
```

output
```
integral(1/(x*(-(a*x + I)/(a*x - I))^(1/2*n)), x)
```

Sympy [F]

$$\int \frac{e^{in\arctan(ax)}}{x}\,dx = \int \frac{e^{in\operatorname{atan}(ax)}}{x}\,dx$$

input
```
integrate(exp(I*n*atan(a*x))/x,x)
```

output
```
Integral(exp(I*n*atan(a*x))/x, x)
```

Maxima [F]

$$\int \frac{e^{in\arctan(ax)}}{x}\,dx = \int \frac{e^{(i\,n\arctan(ax))}}{x}\,dx$$

input
```
integrate(exp(I*n*arctan(a*x))/x,x, algorithm="maxima")
```

output
```
integrate(e^(I*n*arctan(a*x))/x, x)
```

Giac [F]

$$\int \frac{e^{in\arctan(ax)}}{x}\,dx = \int \frac{e^{(i\,n\arctan(ax))}}{x}\,dx$$

input
```
integrate(exp(I*n*arctan(a*x))/x,x, algorithm="giac")
```

output
```
integrate(e^(I*n*arctan(a*x))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{in\arctan(ax)}}{x}\,dx = \int \frac{e^{n\,\mathrm{atan}(a\,x)\,1\mathrm{i}}}{x}\,dx$$

input
```
int(exp(n*atan(a*x)*1i)/x,x)
```

output
```
int(exp(n*atan(a*x)*1i)/x, x)
```

Reduce [F]

$$\int \frac{e^{in\arctan(ax)}}{x}\,dx = \int \frac{e^{atan(ax)in}}{x}dx$$

input
```
int(exp(I*n*atan(a*x))/x,x)
```

output
```
int(e**(atan(a*x)*i*n)/x,x)
```

3.173 $\int \frac{e^{in\arctan(ax)}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \frac{e^{in\arctan(ax)}}{x^2} dx = -\frac{4ia(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{1}{2}(-2+n)}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{1-iax}{1+iax}\right)}{2-n}$$

output
```
-4*I*a*(1-I*a*x)^(1-1/2*n)*(1+I*a*x)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-
1/2*n],(1-I*a*x)/(1+I*a*x))/(2-n)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.04

$$\int \frac{e^{in\arctan(ax)}}{x^2} dx = -\frac{2ia(1-iax)^{1-\frac{n}{2}}(1+iax)^{-1+\frac{n}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},-\frac{1-iax}{-1-iax}\right)}{1-\frac{n}{2}}$$

input
```
Integrate[E^(I*n*ArcTan[a*x])/x^2,x]
```

output
```
((-2*I)*a*(1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^(-1 + n/2)*Hypergeometric2F1[2
, 1 - n/2, 2 - n/2, -((1 - I*a*x)/(-1 - I*a*x))])/(1 - n/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5585, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{in\arctan(ax)}}{x^2}dx$$

$$\Big\downarrow \text{ 5585}$$

$$\int \frac{(1-iax)^{-n/2}(1+iax)^{n/2}}{x^2}dx$$

$$\Big\downarrow \text{ 141}$$

$$-\frac{4ia(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n-2}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{1-iax}{iax+1}\right)}{2-n}$$

input
```
Int[E^(I*n*ArcTan[a*x])/x^2,x]
```

output
```
((-4*I)*a*(1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((-2 + n)/2)*Hypergeometric2F1
[2, 1 - n/2, 2 - n/2, (1 - I*a*x)/(1 + I*a*x)])/(2 - n)
```

Defintions of rubi rules used

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{\mathrm{e}^{in\arctan(ax)}}{x^2}dx$$

input
```
int(exp(I*n*arctan(a*x))/x^2,x)
```

output
```
int(exp(I*n*arctan(a*x))/x^2,x)
```

Fricas [F]

$$\int \frac{e^{in\arctan(ax)}}{x^2}\,dx = \int \frac{e^{(i\,n\arctan(ax))}}{x^2}\,dx$$

input
```
integrate(exp(I*n*arctan(a*x))/x^2,x, algorithm="fricas")
```

output
```
integral(1/(x^2*(-(a*x + I)/(a*x - I))^(1/2*n)), x)
```

Sympy [F]

$$\int \frac{e^{in\arctan(ax)}}{x^2}\,dx = \int \frac{e^{in\operatorname{atan}(ax)}}{x^2}\,dx$$

input
```
integrate(exp(I*n*atan(a*x))/x**2,x)
```

output
```
Integral(exp(I*n*atan(a*x))/x**2, x)
```

Maxima [F]

$$\int \frac{e^{in\arctan(ax)}}{x^2}\,dx = \int \frac{e^{(i\,n\arctan(ax))}}{x^2}\,dx$$

input
```
integrate(exp(I*n*arctan(a*x))/x^2,x, algorithm="maxima")
```

output
```
integrate(e^(I*n*arctan(a*x))/x^2, x)
```

Giac [F]

$$\int \frac{e^{in\arctan(ax)}}{x^2}\,dx = \int \frac{e^{(i\,n\arctan(ax))}}{x^2}\,dx$$

input
```
integrate(exp(I*n*arctan(a*x))/x^2,x, algorithm="giac")
```

output
```
integrate(e^(I*n*arctan(a*x))/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{in\arctan(ax)}}{x^2}\,dx = \int \frac{e^{n\,\mathrm{atan}(a\,x)\,1i}}{x^2}\,dx$$

input
```
int(exp(n*atan(a*x)*1i)/x^2,x)
```

output
```
int(exp(n*atan(a*x)*1i)/x^2, x)
```

Reduce [F]

$$\int \frac{e^{in\arctan(ax)}}{x^2}\,dx = \frac{-e^{atan(ax)in} + \left(\int \frac{e^{atan(ax)in}}{a^2x^3+x}dx\right)ainx}{x}$$

input
```
int(exp(I*n*atan(a*x))/x^2,x)
```

output
```
( - e**(atan(a*x)*i*n) + int(e**(atan(a*x)*i*n)/(a**2*x**3 + x),x)*a*i*n*x
)/x
```

3.174 $\int \frac{e^{in\arctan(ax)}}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 120

$$\int \frac{e^{in\arctan(ax)}}{x^3}\,dx = -\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{2x^2} + \frac{2a^2n(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{1}{2}(-2+n)}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{1-iax}{1+iax}\right)}{2-n}$$

output
```
-1/2*(1-I*a*x)^(1-1/2*n)*(1+I*a*x)^(1+1/2*n)/x^2+2*a^2*n*(1-I*a*x)^(1-1/2*
n)*(1+I*a*x)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*n],(1-I*a*x)/(1+I*a*
x))/(2-n)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.95

$$\int \frac{e^{in\arctan(ax)}}{x^3}\,dx = \frac{(1-iax)^{-n/2}(1+iax)^{n/2}(i+ax)\left(-((-2+n)(-i+ax)^2)+4a^2nx^2\,\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\right.\right.}{2(-2+n)x^2(-i+ax)}$$

input
```
Integrate[E^(I*n*ArcTan[a*x])/x^3,x]
```

output
```
((1 + I*a*x)^(n/2)*(I + a*x)*(-((-2 + n)*(-I + a*x)^2) + 4*a^2*n*x^2*Hyper
geometric2F1[2, 1 - n/2, 2 - n/2, (I + a*x)/(I - a*x)]))/(2*(-2 + n)*x^2*(
1 - I*a*x)^(n/2)*(-I + a*x))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5585, 107, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{in\arctan(ax)}}{x^3}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{(1-iax)^{-n/2}(1+iax)^{n/2}}{x^3}dx$$

$$\Big\downarrow \text{107}$$

$$\frac{1}{2}ian\int \frac{(1-iax)^{-n/2}(iax+1)^{n/2}}{x^2}dx-\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n+2}{2}}}{2x^2}$$

$$\Big\downarrow \text{141}$$

$$\frac{2a^2n(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n-2}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{1-iax}{iax+1}\right)}{2-n}-\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n+2}{2}}}{2x^2}$$

input
```
Int[E^(I*n*ArcTan[a*x])/x^3,x]
```

output
```
-1/2*((1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/x^2 + (2*a^2*n*(1 - I
*a*x)^(1 - n/2)*(1 + I*a*x)^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 -
 n/2, (1 - I*a*x)/(1 + I*a*x)])/(2 - n)
```

Defintions of rubi rules used

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{e^{in \arctan(ax)}}{x^3}dx$$

input
```
int(exp(I*n*arctan(a*x))/x^3,x)
```

output
```
int(exp(I*n*arctan(a*x))/x^3,x)
```

Fricas [F]

$$\int \frac{e^{in \arctan(ax)}}{x^3} \, dx = \int \frac{e^{(i\,n \arctan(ax))}}{x^3} \, dx$$

input
```
integrate(exp(I*n*arctan(a*x))/x^3,x, algorithm="fricas")
```

output
```
integral(1/(x^3*(-(a*x + I)/(a*x - I))^(1/2*n)), x)
```

Sympy [F]

$$\int \frac{e^{in \arctan(ax)}}{x^3} \, dx = \int \frac{e^{in \operatorname{atan}(ax)}}{x^3} \, dx$$

input
```
integrate(exp(I*n*atan(a*x))/x**3,x)
```

output
```
Integral(exp(I*n*atan(a*x))/x**3, x)
```

Maxima [F]

$$\int \frac{e^{in \arctan(ax)}}{x^3} \, dx = \int \frac{e^{(i\,n \arctan(ax))}}{x^3} \, dx$$

input
```
integrate(exp(I*n*arctan(a*x))/x^3,x, algorithm="maxima")
```

output
```
integrate(e^(I*n*arctan(a*x))/x^3, x)
```

Giac [F]

$$\int \frac{e^{in\arctan(ax)}}{x^3}\,dx = \int \frac{e^{(i\,n\arctan(ax))}}{x^3}\,dx$$

input
```
integrate(exp(I*n*arctan(a*x))/x^3,x, algorithm="giac")
```

output
```
integrate(e^(I*n*arctan(a*x))/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{in\arctan(ax)}}{x^3}\,dx = \int \frac{\mathrm{e}^{n\,\mathrm{atan}(a\,x)\,1\mathrm{i}}}{x^3}\,dx$$

input
```
int(exp(n*atan(a*x)*1i)/x^3,x)
```

output
```
int(exp(n*atan(a*x)*1i)/x^3, x)
```

Reduce [F]

$$\int \frac{e^{in\arctan(ax)}}{x^3}\,dx = \frac{-e^{atan(ax)in}a^2x^2 - e^{atan(ax)in}ainx - e^{atan(ax)in} - \left(\int \frac{e^{atan(ax)in}}{a^2x^3+x}dx\right)a^2n^2x^2}{2x^2}$$

input
```
int(exp(I*n*atan(a*x))/x^3,x)
```

output
```
( - (e**(atan(a*x)*i*n)*a**2*x**2 + e**(atan(a*x)*i*n)*a*i*n*x + e**(atan(
a*x)*i*n) + int(e**(atan(a*x)*i*n)/(a**2*x**3 + x),x)*a**2*n**2*x**2))/(2*
x**2)
```

3.175 $\int \frac{e^{in\arctan(ax)}}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 171

$$\int \frac{e^{in\arctan(ax)}}{x^4}\,dx = -\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{3x^3} - \frac{ian(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{2+n}{2}}}{6x^2} + \frac{2ia^3(2+n^2)(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{1}{2}(-2+n)}\,\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{1-iax}{1+iax}\right)}{3(2-n)}$$

output
```
-1/3*(1-I*a*x)^(1-1/2*n)*(1+I*a*x)^(1+1/2*n)/x^3-1/6*I*a*n*(1-I*a*x)^(1-1/
2*n)*(1+I*a*x)^(1+1/2*n)/x^2+2/3*I*a^3*(n^2+2)*(1-I*a*x)^(1-1/2*n)*(1+I*a*
x)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*n],(1-I*a*x)/(1+I*a*x))/(2-n)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.70

$$\int \frac{e^{in\arctan(ax)}}{x^4}\,dx = -\frac{(1-iax)^{-n/2}(1+iax)^{\frac{1}{2}(-2+n)}(i+ax)\left(-((-2+n)(-i+ax)^2(-2i+anx))+4a^3(2+n^2)\,x^3\,\text{Hyperg}\right.}{6(-2+n)x^3}$$

input
```
Integrate[E^(I*n*ArcTan[a*x])/x^4,x]
```

output
```
-1/6*((1 + I*a*x)^((-2 + n)/2)*(I + a*x)*(-((-2 + n)*(-I + a*x)^2*(-2*I +
a*n*x)) + 4*a^3*(2 + n^2)*x^3*Hypergeometric2F1[2, 1 - n/2, 2 - n/2, (I +
a*x)/(I - a*x)]))/((-2 + n)*x^3*(1 - I*a*x)^(n/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5585, 114, 25, 27, 168, 27, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{in\arctan(ax)}}{x^4}dx$$

$$\Big\downarrow \text{5585}$$

$$\int \frac{(1-iax)^{-n/2}(1+iax)^{n/2}}{x^4}dx$$

$$\Big\downarrow \text{114}$$

$$-\frac{1}{3}\int -\frac{a(in-ax)(1-iax)^{-n/2}(iax+1)^{n/2}}{x^3}dx-\frac{(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \frac{a(in-ax)(1-iax)^{-n/2}(iax+1)^{n/2}}{x^3}dx-\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n+2}{2}}}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}a\int \frac{(in-ax)(1-iax)^{-n/2}(iax+1)^{n/2}}{x^3}dx-\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n+2}{2}}}{3x^3}$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{3}a\left(-\frac{1}{2}\int \frac{a\left(n^2+2\right)(1-iax)^{-n/2}(iax+1)^{n/2}}{x^2}dx-\frac{in(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{2x^2}\right)-\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n+2}{2}}}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}a\left(-\frac{1}{2}a\left(n^2+2\right)\int\frac{(1-iax)^{-n/2}(iax+1)^{n/2}}{x^2}dx-\frac{in(1+iax)^{\frac{n+2}{2}}(1-iax)^{1-\frac{n}{2}}}{2x^2}\right)-\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n+2}{2}}}{3x^3}$$

$$\Big\downarrow \text{141}$$

$$\frac{1}{3}a\left(\frac{2ia^2\left(n^2+2\right)(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n-2}{2}}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},\frac{1-iax}{iax+1}\right)}{2-n}-\frac{in(1-iax)^{1-\frac{n}{2}}(1+}{2x^2}\right.$$
$$\frac{(1-iax)^{1-\frac{n}{2}}(1+iax)^{\frac{n+2}{2}}}{3x^3}$$

input
```
Int[E^(I*n*ArcTan[a*x])/x^4,x]
```

output
```
-1/3*((1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/x^3 + (a*(((-1/2*I)*n
*(1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((2 + n)/2))/x^2 + ((2*I)*a^2*(2 + n^2)
*(1 - I*a*x)^(1 - n/2)*(1 + I*a*x)^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n
/2, 2 - n/2, (1 - I*a*x)/(1 + I*a*x)])/(2 - n)))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 5585
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 - I*a
*x)^(I*(n/2))/(1 + I*a*x)^(I*(n/2))), x] /; FreeQ[{a, m, n}, x] &&  !Intege
rQ[(I*n - 1)/2]
```

Maple [F]

$$\int \frac{e^{in \arctan(ax)}}{x^4} dx$$

input
```
int(exp(I*n*arctan(a*x))/x^4,x)
```

output
```
int(exp(I*n*arctan(a*x))/x^4,x)
```

Fricas [F]

$$\int \frac{e^{in\arctan(ax)}}{x^4}\,dx = \int \frac{e^{(i\,n\arctan(ax))}}{x^4}\,dx$$

input
```
integrate(exp(I*n*arctan(a*x))/x^4,x, algorithm="fricas")
```

output
```
integral(1/(x^4*(-(a*x + I)/(a*x - I))^(1/2*n)), x)
```

Sympy [F]

$$\int \frac{e^{in\arctan(ax)}}{x^4}\,dx = \int \frac{e^{in\operatorname{atan}(ax)}}{x^4}\,dx$$

input
```
integrate(exp(I*n*atan(a*x))/x**4,x)
```

output
```
Integral(exp(I*n*atan(a*x))/x**4, x)
```

Maxima [F]

$$\int \frac{e^{in\arctan(ax)}}{x^4}\,dx = \int \frac{e^{(i\,n\arctan(ax))}}{x^4}\,dx$$

input
```
integrate(exp(I*n*arctan(a*x))/x^4,x, algorithm="maxima")
```

output
```
integrate(e^(I*n*arctan(a*x))/x^4, x)
```

Giac [F]

$$\int \frac{e^{in\arctan(ax)}}{x^4}\,dx = \int \frac{e^{(i\,n\arctan(ax))}}{x^4}\,dx$$

input
```
integrate(exp(I*n*arctan(a*x))/x^4,x, algorithm="giac")
```

output
```
integrate(e^(I*n*arctan(a*x))/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{in\arctan(ax)}}{x^4}\,dx = \int \frac{e^{n\,\mathrm{atan}(a\,x)\,1i}}{x^4}\,dx$$

input
```
int(exp(n*atan(a*x)*1i)/x^4,x)
```

output
```
int(exp(n*atan(a*x)*1i)/x^4, x)
```

Reduce [F]

$$\int \frac{e^{in\arctan(ax)}}{x^4}\,dx = \frac{-e^{atan(ax)in}a^3in\,x^3 + e^{atan(ax)in}a^2n^2x^2 - e^{atan(ax)in}ainx - 2e^{atan(ax)in} - \left(\int \frac{e^{atan(ax)in}}{a^2x^3+x}dx\right)a^3i\,n^3x^3 - 2\left(\int \right.}{6x^3}$$

input
```
int(exp(I*n*atan(a*x))/x^4,x)
```

output
```
( - e**(atan(a*x)*i*n)*a**3*i*n*x**3 + e**(atan(a*x)*i*n)*a**2*n**2*x**2 -
 e**(atan(a*x)*i*n)*a*i*n*x - 2*e**(atan(a*x)*i*n) - int(e**(atan(a*x)*i*n
)/(a**2*x**3 + x),x)*a**3*i*n**3*x**3 - 2*int(e**(atan(a*x)*i*n)/(a**2*x**
3 + x),x)*a**3*i*n*x**3)/(6*x**3)
```

3.176 $\int e^{i\arctan(a+bx)}x^4\,dx$

Optimal result

Integrand size = 16, antiderivative size = 315

$$\begin{aligned}\int e^{i\arctan(a+bx)}x^4\,dx &= \frac{(3i+12a-24ia^2-16a^3+8ia^4)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5}\\&+\frac{(15i+56a-62ia^2-56a^3)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{40b^5}\\&-\frac{(i+8a)x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{20b^3}\\&+\frac{x^3\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{5b^2}\\&-\frac{(13i+14a-36ia^2)(1-ia-ibx)^{3/2}(1+ia+ibx)^{3/2}}{60b^5}\\&+\frac{(3-12ia-24a^2+16ia^3+8a^4)\operatorname{arcsinh}(a+bx)}{8b^5}\end{aligned}$$

output
```
1/8*(3*I+12*a-24*I*a^2-16*a^3+8*I*a^4)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(
1/2)/b^5+1/40*(15*I+56*a-62*I*a^2-56*a^3)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x
)^(3/2)/b^5-1/20*(I+8*a)*x^2*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(3/2)/b^3+1
/5*x^3*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(3/2)/b^2-1/60*(13*I+14*a-36*I*a^
2)*(1-I*a-I*b*x)^(3/2)*(1+I*a+I*b*x)^(3/2)/b^5+1/8*(3-12*I*a-24*a^2+16*I*a
^3+8*a^4)*arcsinh(b*x+a)/b^5
```

Mathematica [A] (warning: unable to verify)

Time = 1.11 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.69

$$\int e^{i\arctan(a+bx)}x^4\,dx$$
$$= \frac{i\sqrt{1+a^2+2abx+b^2x^2}(64+24a^4+45ibx-32b^2x^2-30ib^3x^3+24b^4x^4+a^3(250i-24bx)+2a^2(-166}{120b^5}$$
$$+ \frac{\sqrt[4]{-1}(3-12ia-24a^2+16ia^3+8a^4)\sqrt{-ib}\operatorname{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{4b^{11/2}}$$

input
```
Integrate[E^(I*ArcTan[a + b*x])*x^4,x]
```

output
```
((I/120)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(64 + 24*a^4 + (45*I)*b*x - 32*
b^2*x^2 - (30*I)*b^3*x^3 + 24*b^4*x^4 + a^3*(250*I - 24*b*x) + 2*a^2*(-166
 - (65*I)*b*x + 12*b^2*x^2) + a*(-275*I + 116*b*x + (70*I)*b^2*x^2 - 24*b^
3*x^3)))/b^5 + ((-1)^(1/4)*(3 - (12*I)*a - 24*a^2 + (16*I)*a^3 + 8*a^4)*Sq
rt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I
)*b]])/(4*b^(11/2))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.90, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5618, 111, 25, 170, 27, 164, 60, 62, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \ 5618$$

$$\int \frac{x^4\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}dx$$

$$\Big\downarrow \ 111$$

$$\frac{\int -\frac{x^2\sqrt{ia+ibx+1}(3(a^2+1)+(8a+i)bx)}{\sqrt{-ia-ibx+1}}dx}{5b^2}+\frac{x^3\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{5b^2}$$

$\Big\downarrow$ 25

$$\frac{x^3\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{5b^2}-\frac{\int \frac{x^2\sqrt{ia+ibx+1}(3(a^2+1)+(8a+i)bx)}{\sqrt{-ia-ibx+1}}dx}{5b^2}$$

$\Big\downarrow$ 170

$$\frac{x^3\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{5b^2}-\frac{\frac{\int \frac{bx\sqrt{ia+ibx+1}\left(2(i-a)(a+i)(8a+i)+\left(-36a^2-14ia+13\right)bx\right)}{\sqrt{-ia-ibx+1}}dx}{4b^2}+\frac{(8a+i)x^2\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{4b}}{5b^2}$$

$\Big\downarrow$ 27

$$\frac{x^3\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{5b^2}-\frac{\frac{\int \frac{x\sqrt{ia+ibx+1}\left(2(i-a)(a+i)(8a+i)+\left(-36a^2-14ia+13\right)bx\right)}{\sqrt{-ia-ibx+1}}dx}{4b}+\frac{(8a+i)x^2\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{4b}}{5b^2}$$

$\Big\downarrow$ 164

$$\frac{x^3\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{5b^2}-\frac{\frac{-\frac{5\left(8a^4+16ia^3-24a^2-12ia+3\right)\int \frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}dx}{2b}-\frac{\sqrt{-ia-ibx+1}\left(-96a^3-2\left(-36a^2-14ia+13\right)bx-86ia^2+114a+19i\right)(ia+ibx+1)^{3/2}}{6b^2}}{4b}+\frac{(8a+i)x^2\sqrt{-ia-i}}{}}{5b^2}$$

$\Big\downarrow$ 60

$$\frac{x^3\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{5b^2}-\frac{\frac{-\frac{5\left(8a^4+16ia^3-24a^2-12ia+3\right)\left(\int \frac{1}{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}-\frac{\sqrt{-ia-ibx+1}\left(-96a^3-2\left(-36a^2-14ia+13\right)bx-86ia^2+114a+19\right)}{6b^2}}{4b}}{5b^2}$$

$\Big\downarrow$ 62

$$\frac{x^3\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{5b^2}-\frac{\frac{-\frac{5\left(8a^4+16ia^3-24a^2-12ia+3\right)\left(\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}-\frac{\sqrt{-ia-ibx+1}\left(-96a^3-2\left(-36a^2-14ia+13\right)bx-86ia^2+114a\right)}{6b^2}}{4b}}{5b^2}$$

$\Big\downarrow$ 1090

$$\frac{x^3\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{5b^2}-\frac{-\frac{5\left(8a^4+16ia^3-24a^2-12ia+3\right)\left(\frac{\int \frac{1}{\sqrt{\frac{\left(2xb^2+2ab\right)^2}{4b^2}+1}}d\left(2xb^2+2ab\right)}{2b^2}+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}-\frac{\sqrt{-ia-ibx+1}\left(-96a^3-2\left(-36a^2-14ia+13\right)bx-86ia^2+1\right.}{6b^2}}{4b}}{5b^2}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{x^3\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{5b^2}-\frac{-\frac{\sqrt{-ia-ibx+1}\left(-96a^3-2\left(-36a^2-14ia+13\right)bx-86ia^2+114a+19i\right)(ia+ibx+1)^{3/2}}{6b^2}-\frac{5\left(8a^4+16ia^3-24a^2-12ia+3\right)\left(\frac{\text{arcsinh}\left(\frac{2ab+2b^2x}{2b}\right)}{b}+\frac{i\sqrt{-ia-ibx+1}}{b}\right.}{2b}}{4b}}{5b^2}$$

input
```
Int[E^(I*ArcTan[a + b*x])*x^4,x]
```

output
```
(x^3*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(5*b^2) - (((I + 8*a)*
x^2*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(4*b) + (-1/6*(Sqrt[1 -
 I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2)*(19*I + 114*a - (86*I)*a^2 - 96*a^3
- 2*(13 - (14*I)*a - 36*a^2)*b*x))/b^2 - (5*(3 - (12*I)*a - 24*a^2 + (16*I
)*a^3 + 8*a^4)*((I*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b + ArcSin
h[(2*a*b + 2*b^2*x)/(2*b)]/b))/(2*b))/(4*b))/(5*b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 111

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 164

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.63

method	result
risch	$\frac{i\left(24b^4x^4-24a\,b^3x^3-30ib^3x^3+24a^2b^2x^2+70ia\,b^2x^2-24a^3bx-130ia^2bx+24a^4+250ia^3-32b^2x^2+116abx+45ibx-332a^2-275ia+6\right.}{120b^5}$
default	Expression too large to display

input
```
int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/120*I*(24*b^4*x^4-30*I*b^3*x^3-24*a*b^3*x^3+70*I*a*b^2*x^2+24*a^2*b^2*x^
2-130*I*a^2*b*x-24*a^3*b*x+250*I*a^3+24*a^4-32*b^2*x^2+45*I*b*x+116*a*b*x-
275*I*a-332*a^2+64)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^5+1/8*(3-12*I*a-24*a^2
+16*I*a^3+8*a^4)/b^4*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1
/2))/(b^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.56

$$\int e^{i\arctan(a+bx)}x^4\,dx = \frac{186i\,a^5 - 1345\,a^4 - 1730i\,a^3 + 1320\,a^2 - 120\,(8\,a^4 + 16i\,a^3 - 24\,a^2 - 12i\,a + 3)\log\left(-bx - a + \sqrt{b^2x^2 + \cdots}\right)\cdots}{\cdots}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^4,x, algorithm="fricas")
```

output
```
1/960*(186*I*a^5 - 1345*a^4 - 1730*I*a^3 + 1320*a^2 - 120*(8*a^4 + 16*I*a^
3 - 24*a^2 - 12*I*a + 3)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))
 - 8*(-24*I*b^4*x^4 + 6*(4*I*a - 5)*b^3*x^3 + 2*(-12*I*a^2 + 35*a + 16*I)*
b^2*x^2 - 24*I*a^4 + 250*a^3 + (24*I*a^3 - 130*a^2 - 116*I*a + 45)*b*x + 3
32*I*a^2 - 275*a - 64*I)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 300*I*a)/b^5
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1222 vs. 2(275) = 550.

Time = 1.86 (sec) , antiderivative size = 1222, normalized size of antiderivative = 3.88

$$\int e^{i\arctan(a+bx)}x^4\,dx = \text{Too large to display}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)*x**4,x)
```

output
```
Piecewise(((-a*(-3*a*(-5*a*(-7*a*(-4*I*a/5 + 1)/(4*b) - I*(4*a**2 + 4)/(5*
b))/(3*b) - (3*a**2 + 3)*(-4*I*a/5 + 1)/(4*b**2))/(2*b) - (2*a**2 + 2)*(-7
*a*(-4*I*a/5 + 1)/(4*b) - I*(4*a**2 + 4)/(5*b))/(3*b**2))/b - (a**2 + 1)*(
-5*a*(-7*a*(-4*I*a/5 + 1)/(4*b) - I*(4*a**2 + 4)/(5*b))/(3*b) - (3*a**2 +
3)*(-4*I*a/5 + 1)/(4*b**2))/(2*b**2))*log(2*a*b + 2*b**2*x + 2*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1)*sqrt(b**2))/sqrt(b**2) + sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1)*(I*x**4/(5*b) + x**3*(-4*I*a/5 + 1)/(4*b**2) + x**2*(-7*a*(-
4*I*a/5 + 1)/(4*b) - I*(4*a**2 + 4)/(5*b))/(3*b**2) + x*(-5*a*(-7*a*(-4*I*
a/5 + 1)/(4*b) - I*(4*a**2 + 4)/(5*b))/(3*b) - (3*a**2 + 3)*(-4*I*a/5 + 1)
/(4*b**2))/(2*b**2) + (-3*a*(-5*a*(-7*a*(-4*I*a/5 + 1)/(4*b) - I*(4*a**2 +
 4)/(5*b))/(3*b) - (3*a**2 + 3)*(-4*I*a/5 + 1)/(4*b**2))/(2*b) - (2*a**2 +
 2)*(-7*a*(-4*I*a/5 + 1)/(4*b) - I*(4*a**2 + 4)/(5*b))/(3*b**2))/b**2), Ne
(b**2, 0)), ((I*(a**8*sqrt(a**2 + 2*a*b*x + 1) + 4*a**6*sqrt(a**2 + 2*a*b*
x + 1) + 6*a**4*sqrt(a**2 + 2*a*b*x + 1) + 4*a**2*sqrt(a**2 + 2*a*b*x + 1)
 + (-4*a**2 - 4)*(a**2 + 2*a*b*x + 1)**(7/2)/7 + (a**2 + 2*a*b*x + 1)**(9/
2)/9 + (a**2 + 2*a*b*x + 1)**(5/2)*(6*a**4 + 12*a**2 + 6)/5 + (a**2 + 2*a*
b*x + 1)**(3/2)*(-4*a**6 - 12*a**4 - 12*a**2 - 4)/3 + sqrt(a**2 + 2*a*b*x
+ 1))/(8*a**3*b**4) + (a**8*sqrt(a**2 + 2*a*b*x + 1) + 4*a**6*sqrt(a**2 +
2*a*b*x + 1) + 6*a**4*sqrt(a**2 + 2*a*b*x + 1) + 4*a**2*sqrt(a**2 + 2*a*b*
x + 1) + (-4*a**2 - 4)*(a**2 + 2*a*b*x + 1)**(7/2)/7 + (a**2 + 2*a*b*x ...
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(223) = 446$.

Time = 0.04 (sec) , antiderivative size = 749, normalized size of antiderivative = 2.38

$$\int e^{i\arctan(a+bx)}x^4\,dx = \text{Too large to display}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^4,x, algorithm="maxima")
```

output
```
1/5*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x^4/b - 9/20*I*sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1)*a*x^3/b^2 - 1/4*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-I*a - 1
)*x^3/b^2 + 21/20*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*x^2/b^3 - 7/12*s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*(I*a + 1)*x^2/b^3 - 63/8*I*a^5*arcsinh(
2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 + 35/8*a^4*(I*a +
1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 - 21/8*
I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3*x/b^4 + 35/24*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1)*a^2*(I*a + 1)*x/b^4 - 4/15*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
*(I*a^2 + I)*x^2/b^3 + 35/4*I*(a^2 + 1)*a^3*arcsinh(2*(b^2*x + a*b)/sqrt(-
4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 - 15/4*(a^2 + 1)*a^2*(I*a + 1)*arcsinh(2
*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 + 63/8*I*sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)*a^4/b^5 - 35/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a
^3*(I*a + 1)/b^5 + 161/120*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*a
*x/b^4 - 3/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*(I*a + 1)*x/b^4 -
 15/8*I*(a^2 + 1)^2*a*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1
)*b^2))/b^5 - 3/8*(a^2 + 1)^2*(-I*a - 1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a
^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 - 49/8*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*
(a^2 + 1)*a^2/b^5 + 55/24*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*a*(I
*a + 1)/b^5 + 8/15*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2/b^5
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.65

$$\int e^{i\arctan(a+bx)}x^4\,dx = -\frac{1}{120}\sqrt{(bx+a)^2+1}\left(\left(2\left(3\,x\left(-\frac{4i\,x}{b}-\frac{-4i\,ab^7+5\,b^7}{b^9}\right)-\frac{12i\,a^2b^6-35\,ab^6-16i\,b^6}{b^9}\right)x-\frac{-24i\,a^3b^5}{}\right.\right.$$
$$-\frac{(8\,a^4+16i\,a^3-24\,a^2-12i\,a+3)\log\left(\left|-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right|\right)}{8\,b^4|b|}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^4,x, algorithm="giac")
```

output
```
-1/120*sqrt((b*x + a)^2 + 1)*((2*(3*x*(-4*I*x/b - (-4*I*a*b^7 + 5*b^7)/b^9
) - (12*I*a^2*b^6 - 35*a*b^6 - 16*I*b^6)/b^9)*x - (-24*I*a^3*b^5 + 130*a^2
*b^5 + 116*I*a*b^5 - 45*b^5)/b^9)*x - (24*I*a^4*b^4 - 250*a^3*b^4 - 332*I*
a^2*b^4 + 275*a*b^4 + 64*I*b^4)/b^9) - 1/8*(8*a^4 + 16*I*a^3 - 24*a^2 - 12
*I*a + 3)*log(abs(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b)))/(b^4*
abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{i\arctan(a+bx)}x^4\,dx=\int \frac{x^4\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})}{\sqrt{{(a+b\,x)}^2+1}}\,dx$$

input
```
int((x^4*(a*1i + b*x*1i + 1))/((a + b*x)^2 + 1)^(1/2),x)
```

output
```
int((x^4*(a*1i + b*x*1i + 1))/((a + b*x)^2 + 1)^(1/2), x)
```

Reduce [F]

$$\int e^{i\arctan(a+bx)}x^4\,dx=\int \frac{(1+i(bx+a))\,x^4}{\sqrt{1+(bx+a)^2}}dx$$

input
```
int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^4,x)
```

output
```
int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^4,x)
```

3.177 $\int e^{i\arctan(a+bx)}x^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 238

$$\begin{aligned}\int e^{i\arctan(a+bx)}x^3\,dx =& -\frac{(3-12ia-12a^2+8ia^3)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4}\\ &-\frac{(3-8ia-10a^2)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{8b^4}\\ &+\frac{x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{4b^2}\\ &+\frac{(1-6ia)(1-ia-ibx)^{3/2}(1+ia+ibx)^{3/2}}{12b^4}\\ &+\frac{(3i+12a-12ia^2-8a^3)\operatorname{arcsinh}(a+bx)}{8b^4}\end{aligned}$$

output
```
-1/8*(3-12*I*a-12*a^2+8*I*a^3)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^4
-1/8*(3-8*I*a-10*a^2)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(3/2)/b^4+1/4*x^2*
(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(3/2)/b^2+1/12*(1-6*I*a)*(1-I*a-I*b*x)^(
3/2)*(1+I*a+I*b*x)^(3/2)/b^4+1/8*(3*I+12*a-12*I*a^2-8*a^3)*arcsinh(b*x+a)/
b^4
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.74

$$\int e^{i\arctan(a+bx)}x^3\,dx$$
$$=\frac{\sqrt{b}\sqrt{1+a^2+2abx+b^2x^2}(-16-6ia^3-9ibx+8b^2x^2+6ib^3x^3+a^2(44+6ibx)+a(39i-20bx-6ib^2x}{24b^{9/2}}$$

input
```
Integrate[E^(I*ArcTan[a + b*x])*x^3,x]
```

output
```
(Sqrt[b]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(-16 - (6*I)*a^3 - (9*I)*b*x +
8*b^2*x^2 + (6*I)*b^3*x^3 + a^2*(44 + (6*I)*b*x) + a*(39*I - 20*b*x - (6*I
)*b^2*x^2)) - 6*(-1)^(1/4)*(-3*I - 12*a + (12*I)*a^2 + 8*a^3)*Sqrt[(-I)*b]
*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/(24
*b^(9/2))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.87, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5618, 111, 25, 164, 60, 62, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{i\arctan(a+bx)}\,dx$$
$$\Big\downarrow \text{5618}$$
$$\int \frac{x^3\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}dx$$
$$\Big\downarrow \text{111}$$
$$\frac{\int -\frac{x\sqrt{ia+ibx+1}(2(a^2+1)+(6a+i)bx)}{\sqrt{-ia-ibx+1}}dx}{4b^2}+\frac{x^2\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{4b^2}$$
$$\Big\downarrow \text{25}$$

$$\frac{x^2\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{4b^2}-\frac{\int \frac{x\sqrt{ia+ibx+1}(2(a^2+1)+(6a+i)bx)}{\sqrt{-ia-ibx+1}}dx}{4b^2}$$

$\Big\downarrow$ 164

$$\frac{x^2\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{4b^2}-\frac{\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}(-18a^2+2(6a+i)bx-10ia+7)}{6b^2}-\frac{(-8a^3-12ia^2+12a+3i)\int \frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}dx}{2b}}{4b^2}$$

$\Big\downarrow$ 60

$$\frac{x^2\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{4b^2}-\frac{\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}(-18a^2+2(6a+i)bx-10ia+7)}{6b^2}-\frac{(-8a^3-12ia^2+12a+3i)\left(\int \frac{1}{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}}{4b^2}$$

$\Big\downarrow$ 62

$$\frac{x^2\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{4b^2}-\frac{\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}(-18a^2+2(6a+i)bx-10ia+7)}{6b^2}-\frac{(-8a^3-12ia^2+12a+3i)\left(\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}}{4b^2}$$

$\Big\downarrow$ 1090

$$\frac{x^2\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{4b^2}-\frac{\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}(-18a^2+2(6a+i)bx-10ia+7)}{6b^2}-\frac{(-8a^3-12ia^2+12a+3i)\left(\frac{\int \frac{1}{\sqrt{\frac{(2xb^2+2ab)^2}{4b^2}+1}}d(2xb^2+2ab)}{2b^2}+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}}{4b^2}$$

$\Big\downarrow$ 222

$$\frac{x^2\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{4b^2}-\frac{\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}(-18a^2+2(6a+i)bx-10ia+7)}{6b^2}-\frac{(-8a^3-12ia^2+12a+3i)\left(\frac{\operatorname{arcsinh}\left(\frac{2ab+2b^2x}{2b}\right)}{b}+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}}{4b^2}$$

input

```
Int[E^(I*ArcTan[a + b*x])*x^3,x]
```

output
```
(x^2*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(4*b^2) - ((Sqrt[1 - I
*a - I*b*x]*(1 + I*a + I*b*x)^(3/2)*(7 - (10*I)*a - 18*a^2 + 2*(I + 6*a)*b
*x))/(6*b^2) - ((3*I + 12*a - (12*I)*a^2 - 8*a^3)*((I*Sqrt[1 - I*a - I*b*x
]*Sqrt[1 + I*a + I*b*x])/b + ArcSinh[(2*a*b + 2*b^2*x)/(2*b)]/b))/(2*b))/(
4*b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.63

method	result
risch	$-\frac{i(-6b^3x^3+6a\,b^2x^2+8ib^2x^2-6a^2bx-20iabx+6a^3+44ia^2+9bx-39a-16i)\sqrt{b^2x^2+2abx+a^2+1}}{24b^4}-\frac{(8a^3+12ia^2-12a-3i)\ln\left(\frac{b^2}{\sqrt{}}\right.}{8b^3}$
default	$(ia+1)\left(\frac{x^2\sqrt{b^2x^2+2abx+a^2+1}}{3b^2}-\frac{5a\left(\frac{x\sqrt{b^2x^2+2abx+a^2+1}}{2b^2}-\frac{3a\left(\frac{\sqrt{b^2x^2+2abx+a^2+1}}{b^2}-\frac{a\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{b\sqrt{b^2}}\right)}{2b}\right.}{3b}\right.$

input
```
int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/24*I*(-6*b^3*x^3+8*I*b^2*x^2+6*a*b^2*x^2-20*I*a*b*x-6*a^2*b*x+44*I*a^2+
6*a^3+9*b*x-16*I-39*a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^4-1/8*(12*I*a^2+8*a
^3-3*I-12*a)/b^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))
/(b^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.58

$$\int e^{i\arctan(a+bx)}x^3\,dx$$
$$=\frac{-45i\,a^4+224\,a^3+192i\,a^2+24\left(8\,a^3+12i\,a^2-12\,a-3i\right)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-8}{}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^3,x, algorithm="fricas")
```

output
```
1/192*(-45*I*a^4 + 224*a^3 + 192*I*a^2 + 24*(8*a^3 + 12*I*a^2 - 12*a - 3*I
)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 8*(-6*I*b^3*x^3 + 2*
(3*I*a - 4)*b^2*x^2 + 6*I*a^3 + (-6*I*a^2 + 20*a + 9*I)*b*x - 44*a^2 - 39*
I*a + 16)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 72*a)/b^4
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 857 vs. $2(202) = 404$.

Time = 1.58 (sec) , antiderivative size = 857, normalized size of antiderivative = 3.60

$$\int e^{i\arctan(a+bx)}x^3\,dx = \text{Too large to display}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)*x**3,x)
```

output
```
Piecewise(((-a*(-3*a*(-5*a*(-3*I*a/4 + 1)/(3*b) - I*(3*a**2 + 3)/(4*b))/(2
*b) - (2*a**2 + 2)*(-3*I*a/4 + 1)/(3*b**2))/b - (a**2 + 1)*(-5*a*(-3*I*a/4
 + 1)/(3*b) - I*(3*a**2 + 3)/(4*b))/(2*b**2))*log(2*a*b + 2*b**2*x + 2*sqr
t(a**2 + 2*a*b*x + b**2*x**2 + 1)*sqrt(b**2))/sqrt(b**2) + sqrt(a**2 + 2*a
*b*x + b**2*x**2 + 1)*(I*x**3/(4*b) + x**2*(-3*I*a/4 + 1)/(3*b**2) + x*(-5
*a*(-3*I*a/4 + 1)/(3*b) - I*(3*a**2 + 3)/(4*b))/(2*b**2) + (-3*a*(-5*a*(-3
*I*a/4 + 1)/(3*b) - I*(3*a**2 + 3)/(4*b))/(2*b) - (2*a**2 + 2)*(-3*I*a/4 +
 1)/(3*b**2))/b**2), Ne(b**2, 0)), ((I*(-a**6*sqrt(a**2 + 2*a*b*x + 1) - 3
*a**4*sqrt(a**2 + 2*a*b*x + 1) - 3*a**2*sqrt(a**2 + 2*a*b*x + 1) + (-3*a**
2 - 3)*(a**2 + 2*a*b*x + 1)**(5/2)/5 + (a**2 + 2*a*b*x + 1)**(7/2)/7 + (a*
*2 + 2*a*b*x + 1)**(3/2)*(3*a**4 + 6*a**2 + 3)/3 - sqrt(a**2 + 2*a*b*x + 1
))/(4*a**2*b**3) + (-a**6*sqrt(a**2 + 2*a*b*x + 1) - 3*a**4*sqrt(a**2 + 2*
a*b*x + 1) - 3*a**2*sqrt(a**2 + 2*a*b*x + 1) + (-3*a**2 - 3)*(a**2 + 2*a*b
*x + 1)**(5/2)/5 + (a**2 + 2*a*b*x + 1)**(7/2)/7 + (a**2 + 2*a*b*x + 1)**(
3/2)*(3*a**4 + 6*a**2 + 3)/3 - sqrt(a**2 + 2*a*b*x + 1))/(4*a**3*b**3) + I
*(a**8*sqrt(a**2 + 2*a*b*x + 1) + 4*a**6*sqrt(a**2 + 2*a*b*x + 1) + 6*a**4
*sqrt(a**2 + 2*a*b*x + 1) + 4*a**2*sqrt(a**2 + 2*a*b*x + 1) + (-4*a**2 - 4
)*(a**2 + 2*a*b*x + 1)**(7/2)/7 + (a**2 + 2*a*b*x + 1)**(9/2)/9 + (a**2 +
2*a*b*x + 1)**(5/2)*(6*a**4 + 12*a**2 + 6)/5 + (a**2 + 2*a*b*x + 1)**(3/2)
*(-4*a**6 - 12*a**4 - 12*a**2 - 4)/3 + sqrt(a**2 + 2*a*b*x + 1))/(8*a**...
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(168) = 336$.

Time = 0.04 (sec) , antiderivative size = 529, normalized size of antiderivative = 2.22

$$\int e^{i\arctan(a+bx)}x^3\,dx = \frac{i\sqrt{b^2x^2+2\,abx+a^2+1}x^3}{4\,b} - \frac{7i\sqrt{b^2x^2+2\,abx+a^2+1}ax^2}{12\,b^2} - \frac{\sqrt{b^2x^2+2\,abx+a^2+1}(-i\,a-1)x^2}{3\,b^2} + \frac{35i\,a^4\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{8\,b^4} - \frac{5\,a^3(i\,a+1)\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^4} + \frac{35i\sqrt{b^2x^2+2\,abx+a^2+1}a^2x}{24\,b^3} - \frac{5\sqrt{b^2x^2+2\,abx+a^2+1}a(i\,a+1)x}{6\,b^3} - \frac{15i\,(a^2+1)a^2\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{4\,b^4} + \frac{3\,(a^2+1)a(i\,a+1)\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^4} - \frac{35i\sqrt{b^2x^2+2\,abx+a^2+1}a^3}{8\,b^4} + \frac{5\sqrt{b^2x^2+2\,abx+a^2+1}a^2(i\,a+1)}{2\,b^4} - \frac{3\sqrt{b^2x^2+2\,abx+a^2+1}(i\,a^2+i)x}{8\,b^3} + \frac{3i\,(a^2+1)^2\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{8\,b^4} + \frac{55i\sqrt{b^2x^2+2\,abx+a^2+1}(a^2+1)a}{24\,b^4} - \frac{2\sqrt{b^2x^2+2\,abx+a^2+1}(a^2+1)(i\,a+1)}{3\,b^4}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^3,x, algorithm="maxima")
```

output
```
1/4*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x^3/b - 7/12*I*sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1)*a*x^2/b^2 - 1/3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-I*a - 1
)*x^2/b^2 + 35/8*I*a^4*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 +
1)*b^2))/b^4 - 5/2*a^3*(I*a + 1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 +
 4*(a^2 + 1)*b^2))/b^4 + 35/24*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*x/b
^3 - 5/6*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*(I*a + 1)*x/b^3 - 15/4*I*(a^2
 + 1)*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^4
+ 3/2*(a^2 + 1)*a*(I*a + 1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a
^2 + 1)*b^2))/b^4 - 35/8*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3/b^4 + 5/2
*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*(I*a + 1)/b^4 - 3/8*sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)*(I*a^2 + I)*x/b^3 + 3/8*I*(a^2 + 1)^2*arcsinh(2*(b^2*x
+ a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^4 + 55/24*I*sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)*(a^2 + 1)*a/b^4 - 2/3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(
a^2 + 1)*(I*a + 1)/b^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.66

$$\int e^{i\arctan(a+bx)}x^3\,dx = -\frac{1}{24}\sqrt{(bx+a)^2+1}\left(\left(2\,x\left(-\frac{3i\,x}{b}-\frac{-3i\,ab^5+4\,b^5}{b^7}\right)-\frac{6i\,a^2b^4-20\,ab^4-9i\,b^4}{b^7}\right)x-\frac{-6i\,a^3b^3+44\,a^2\cdots}{\cdots}\right) + \frac{(8\,a^3+12i\,a^2-12\,a-3i)\log\left(\left|-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right|\right)}{8\,b^3|b|}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^3,x, algorithm="giac")
```

output
```
-1/24*sqrt((b*x + a)^2 + 1)*((2*x*(-3*I*x/b - (-3*I*a*b^5 + 4*b^5)/b^7) -
(6*I*a^2*b^4 - 20*a*b^4 - 9*I*b^4)/b^7)*x - (-6*I*a^3*b^3 + 44*a^2*b^3 + 3
9*I*a*b^3 - 16*b^3)/b^7) + 1/8*(8*a^3 + 12*I*a^2 - 12*a - 3*I)*log(abs(-a*
b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b)))/(b^3*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{i\arctan(a+bx)}x^3\,dx=\int \frac{x^3\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})}{\sqrt{(a+b\,x)^2+1}}\,dx$$

input
```
int((x^3*(a*1i + b*x*1i + 1))/((a + b*x)^2 + 1)^(1/2),x)
```

output
```
int((x^3*(a*1i + b*x*1i + 1))/((a + b*x)^2 + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.52

$$\int e^{i\arctan(a+bx)}x^3\,dx$$
$$=\frac{-6\sqrt{b^2x^2+2abx+a^2+1}\,a^3i+6\sqrt{b^2x^2+2abx+a^2+1}\,a^2bix+44\sqrt{b^2x^2+2abx+a^2+1}\,a^2-6\sqrt{b^2x^2}}{}$$

input
```
int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^3,x)
```

output
```
( - 6*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**3*i + 6*sqrt(a**2 + 2*a*b*x
+ b**2*x**2 + 1)*a**2*b*i*x + 44*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**2
 - 6*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a*b**2*i*x**2 - 20*sqrt(a**2 + 2
*a*b*x + b**2*x**2 + 1)*a*b*x + 39*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a*
i + 6*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*b**3*i*x**3 + 8*sqrt(a**2 + 2*a
*b*x + b**2*x**2 + 1)*b**2*x**2 - 9*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*b
*i*x - 16*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) - 24*log(sqrt(a**2 + 2*a*b*
x + b**2*x**2 + 1) + a + b*x)*a**3 - 36*log(sqrt(a**2 + 2*a*b*x + b**2*x**
2 + 1) + a + b*x)*a**2*i + 36*log(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + a
 + b*x)*a + 9*log(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + a + b*x)*i)/(24*b
**4)
```

3.178 $\int e^{i\arctan(a+bx)}x^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 171

$$\begin{aligned}\int e^{i\arctan(a+bx)}x^2\,dx = &-\frac{(i+2a-2ia^2)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3}\\ &-\frac{(i+4a)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{6b^3}\\ &+\frac{x\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{3b^2}\\ &-\frac{(1-2ia-2a^2)\operatorname{arcsinh}(a+bx)}{2b^3}\end{aligned}$$

output
```
-1/2*(I+2*a-2*I*a^2)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^3-1/6*(I+4*
a)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(3/2)/b^3+1/3*x*(1-I*a-I*b*x)^(1/2)*(
1+I*a+I*b*x)^(3/2)/b^2-1/2*(1-2*I*a-2*a^2)*arcsinh(b*x+a)/b^3
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.79

$$\int e^{i\arctan(a+bx)}x^2\,dx = \frac{\sqrt{1+a^2+2abx+b^2x^2}(-4i+2ia^2+3bx+2ib^2x^2+a(-9-2ibx))}{6b^3} + \frac{\sqrt[4]{-1}(-1+2ia+2a^2)\sqrt{-ib}\text{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{b^{7/2}}$$

input
```
Integrate[E^(I*ArcTan[a + b*x])*x^2,x]
```

output
```
(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(-4*I + (2*I)*a^2 + 3*b*x + (2*I)*b^2*x
^2 + a*(-9 - (2*I)*b*x)))/(6*b^3) + ((-1)^(1/4)*(-1 + (2*I)*a + 2*a^2)*Sqr
t[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)
*b]])/b^(7/2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5618, 101, 25, 90, 60, 62, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2e^{i\arctan(a+bx)}dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{x^2\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}dx$$

$$\Big\downarrow \text{101}$$

$$\frac{\int -\frac{\sqrt{ia+ibx+1}(a^2+(4a+i)bx+1)}{\sqrt{-ia-ibx+1}}dx}{3b^2}+\frac{x\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{3b^2}$$

$$\Big\downarrow \; 25$$

$$\frac{x\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{3b^2}-\frac{\int \frac{\sqrt{ia+ibx+1}\left(a^2+(4a+i)bx+1\right)}{\sqrt{-ia-ibx+1}}dx}{3b^2}$$

$$\Big\downarrow \; 90$$

$$\frac{x\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{3b^2}-\frac{\frac{3}{2}\left(-2a^2-2ia+1\right)\int \frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}dx+\frac{(4a+i)\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2b}}{3b^2}$$

$$\Big\downarrow \; 60$$

$$\frac{x\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{3b^2}-\frac{\frac{3}{2}\left(-2a^2-2ia+1\right)\left(\int \frac{1}{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)+\frac{(4a+i)\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2b}}{3b^2}$$

$$\Big\downarrow \; 62$$

$$\frac{x\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{3b^2}-\frac{\frac{3}{2}\left(-2a^2-2ia+1\right)\left(\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)+\frac{(4a+i)\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2b}}{3b^2}$$

$$\Big\downarrow \; 1090$$

$$\frac{x\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{3b^2}-\frac{\frac{3}{2}\left(-2a^2-2ia+1\right)\left(\frac{\int \frac{1}{\sqrt{\frac{\left(2xb^2+2ab\right)^2}{4b^2}+1}}d\left(2xb^2+2ab\right)}{2b^2}+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)+\frac{(4a+i)\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2b}}{3b^2}$$

$$\Big\downarrow \; 222$$

$$\frac{x\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{3b^2}-\frac{\frac{3}{2}\left(-2a^2-2ia+1\right)\left(\frac{\operatorname{arcsinh}\left(\frac{2ab+2b^2x}{2b}\right)}{b}+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)+\frac{(4a+i)\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2b}}{3b^2}$$

input

```
Int[E^(I*ArcTan[a + b*x])*x^2,x]
```

output
```
(x*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(3*b^2) - (((I + 4*a)*Sq
rt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(2*b) + (3*(1 - (2*I)*a - 2*a
^2)*((I*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b + ArcSinh[(2*a*b +
2*b^2*x)/(2*b)]/b))/2)/(3*b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.66

method	result
risch	$\frac{i(2b^2x^2-2abx-3ibx+2a^2+9ia-4)\sqrt{b^2x^2+2abx+a^2+1}}{6b^3}+\frac{(2a^2+2ia-1)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{2b^2\sqrt{b^2}}$
default	$(ia+1)\left(\frac{x\sqrt{b^2x^2+2abx+a^2+1}}{2b^2}-\frac{3a\left(\frac{\sqrt{b^2x^2+2abx+a^2+1}}{b^2}-\frac{a\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{b\sqrt{b^2}}\right)}{2b}-\frac{(a^2+1)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{\right.}{2b^2\sqrt{}}\right.$

input
```
int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*I*(2*b^2*x^2-3*I*b*x-2*a*b*x+9*I*a+2*a^2-4)*(b^2*x^2+2*a*b*x+a^2+1)^(1
/2)/b^3+1/2*(2*I*a+2*a^2-1)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*
x+a^2+1)^(1/2))/(b^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.62

$$\int e^{i\arctan(a+bx)}x^2\,dx$$
$$= \frac{7i\,a^3 - 21\,a^2 - 12\,(2\,a^2 + 2i\,a - 1)\log\left(-bx - a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right) - 4\,\sqrt{b^2x^2 + 2\,abx + a^2 + 1}\cdots}{24\,b^3}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^2,x, algorithm="fricas")
```

output
```
1/24*(7*I*a^3 - 21*a^2 - 12*(2*a^2 + 2*I*a - 1)*log(-b*x - a + sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)) - 4*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-2*I*b^2*x^
2 + (2*I*a - 3)*b*x - 2*I*a^2 + 9*a + 4*I) - 9*I*a)/b^3
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 585 vs. 2(133) = 266.

Time = 1.35 (sec) , antiderivative size = 585, normalized size of antiderivative = 3.42

$$\int e^{i\arctan(a+bx)}x^2\,dx$$
$$= \begin{cases} \frac{\left(-\frac{a\left(-\frac{3a\left(-\frac{2ia}{3}+1\right)}{2b}-\frac{i\left(2a^2+2\right)}{3b}\right)}{b}-\frac{\left(a^2+1\right)\left(-\frac{2ia}{3}+1\right)}{2b^2}\right)\log\left(2ab+2b^2x+2\sqrt{a^2+2abx+b^2x^2+1}\sqrt{b^2}\right)}{\sqrt{b^2}} + \left(\frac{ix^2}{3b}+\frac{x\left(-\frac{2ia}{3}+1\right)}{2b^2}+\frac{-\frac{3a\left(-\cdots\right.}{\cdots}}{\cdots}\cdots\right. \\ \frac{\frac{i\left(a^4\sqrt{a^2+2abx+1}+2a^2\sqrt{a^2+2abx+1}+\frac{\left(-2a^2-2\right)\left(a^2+2abx+1\right)^{\frac{3}{2}}}{3}+\frac{\left(a^2+2abx+1\right)^{\frac{5}{2}}}{5}+\sqrt{a^2+2abx+1}\right)}{2ab^2}+\frac{a^4\sqrt{a^2+2abx+1}+2a^2\sqrt{a^2+2abx+1}+\frac{\left(-2a^2-2\right)\cdots}{\cdots}}{2\cdots}}{\cdots} \\ \frac{\frac{iax^3}{3}+\frac{ibx^4}{4}+\frac{x^3}{3}}{\sqrt{a^2+1}} \end{cases}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)*x**2,x)
```

output

```
Piecewise(((-a*(-3*a*(-2*I*a/3 + 1)/(2*b) - I*(2*a**2 + 2)/(3*b))/b - (a**
2 + 1)*(-2*I*a/3 + 1)/(2*b**2))*log(2*a*b + 2*b**2*x + 2*sqrt(a**2 + 2*a*b
*x + b**2*x**2 + 1)*sqrt(b**2))/sqrt(b**2) + (I*x**2/(3*b) + x*(-2*I*a/3 +
 1)/(2*b**2) + (-3*a*(-2*I*a/3 + 1)/(2*b) - I*(2*a**2 + 2)/(3*b))/b**2)*sq
rt(a**2 + 2*a*b*x + b**2*x**2 + 1), Ne(b**2, 0)), ((I*(a**4*sqrt(a**2 + 2*
a*b*x + 1) + 2*a**2*sqrt(a**2 + 2*a*b*x + 1) + (-2*a**2 - 2)*(a**2 + 2*a*b
*x + 1)**(3/2)/3 + (a**2 + 2*a*b*x + 1)**(5/2)/5 + sqrt(a**2 + 2*a*b*x + 1
))/(2*a*b**2) + (a**4*sqrt(a**2 + 2*a*b*x + 1) + 2*a**2*sqrt(a**2 + 2*a*b*
x + 1) + (-2*a**2 - 2)*(a**2 + 2*a*b*x + 1)**(3/2)/3 + (a**2 + 2*a*b*x + 1
)**(5/2)/5 + sqrt(a**2 + 2*a*b*x + 1))/(2*a**2*b**2) + I*(-a**6*sqrt(a**2
+ 2*a*b*x + 1) - 3*a**4*sqrt(a**2 + 2*a*b*x + 1) - 3*a**2*sqrt(a**2 + 2*a*
b*x + 1) + (-3*a**2 - 3)*(a**2 + 2*a*b*x + 1)**(5/2)/5 + (a**2 + 2*a*b*x +
 1)**(7/2)/7 + (a**2 + 2*a*b*x + 1)**(3/2)*(3*a**4 + 6*a**2 + 3)/3 - sqrt(
a**2 + 2*a*b*x + 1))/(4*a**3*b**2))/(2*a*b), Ne(a*b, 0)), ((I*a*x**3/3 + I
*b*x**4/4 + x**3/3)/sqrt(a**2 + 1), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(119) = 238$.

Time = 0.04 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.05

$$\int e^{i\arctan(a+bx)}x^2\,dx = \frac{i\sqrt{b^2x^2+2\,abx+a^2+1}x^2}{3\,b} - \frac{5i\,a^3\,\text{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^3} + \frac{3\,a^2(i\,a+1)\,\text{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^3} - \frac{5i\sqrt{b^2x^2+2\,abx+a^2+1}ax}{6\,b^2} - \frac{\sqrt{b^2x^2+2\,abx+a^2+1}(-i\,a-1)x}{2\,b^2} + \frac{3i\,(a^2+1)a\,\text{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^3} - \frac{(a^2+1)(i\,a+1)\,\text{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^3} + \frac{5i\sqrt{b^2x^2+2\,abx+a^2+1}a^2}{2\,b^3} - \frac{3\sqrt{b^2x^2+2\,abx+a^2+1}a(i\,a+1)}{2\,b^3} - \frac{2\sqrt{b^2x^2+2\,abx+a^2+1}(i\,a^2+i)}{3\,b^3}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^2,x, algorithm="maxima")
```

output
```
1/3*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x^2/b - 5/2*I*a^3*arcsinh(2*(b^2*x
 + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 + 3/2*a^2*(I*a + 1)*arcsin
h(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 - 5/6*I*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)*a*x/b^2 - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*
(-I*a - 1)*x/b^2 + 3/2*I*(a^2 + 1)*a*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b
^2 + 4*(a^2 + 1)*b^2))/b^3 - 1/2*(a^2 + 1)*(I*a + 1)*arcsinh(2*(b^2*x + a*
b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 + 5/2*I*sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)*a^2/b^3 - 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*(I*a + 1)/b^3
 - 2/3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(I*a^2 + I)/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.67

$$\int e^{i\arctan(a+bx)}x^2\,dx = -\frac{1}{6}\sqrt{(bx+a)^2+1}\left(x\left(-\frac{2i\,x}{b}-\frac{-2i\,ab^3+3\,b^3}{b^5}\right)-\frac{2i\,a^2b^2-9\,ab^2-4i\,b^2}{b^5}\right) - \frac{(2\,a^2+2i\,a-1)\log\left(\left|-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right|\right)}{2\,b^2|b|}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^2,x, algorithm="giac")
```

output
```
-1/6*sqrt((b*x + a)^2 + 1)*(x*(-2*I*x/b - (-2*I*a*b^3 + 3*b^3)/b^5) - (2*I
*a^2*b^2 - 9*a*b^2 - 4*I*b^2)/b^5) - 1/2*(2*a^2 + 2*I*a - 1)*log(abs(-a*b
- (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b)))/(b^2*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{i\arctan(a+bx)}x^2\,dx = \int \frac{x^2\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})}{\sqrt{(a+b\,x)^2+1}}\,dx$$

input
```
int((x^2*(a*1i + b*x*1i + 1))/((a + b*x)^2 + 1)^(1/2),x)
```

output
```
int((x^2*(a*1i + b*x*1i + 1))/((a + b*x)^2 + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.33

$$\int e^{i\arctan(a+bx)}x^2\,dx$$
$$= \frac{2\sqrt{b^2x^2+2abx+a^2+1}\,a^2i-2\sqrt{b^2x^2+2abx+a^2+1}\,abix-9\sqrt{b^2x^2+2abx+a^2+1}\,a+2\sqrt{b^2x^2+2a}$$

input
```
int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x^2,x)
```

output
```
(2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**2*i - 2*sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1)*a*b*i*x - 9*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a + 2*sqrt(
a**2 + 2*a*b*x + b**2*x**2 + 1)*b**2*i*x**2 + 3*sqrt(a**2 + 2*a*b*x + b**2
*x**2 + 1)*b*x - 4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + 6*log(sqrt(a**
2 + 2*a*b*x + b**2*x**2 + 1) + a + b*x)*a**2 + 6*log(sqrt(a**2 + 2*a*b*x +
 b**2*x**2 + 1) + a + b*x)*a*i - 3*log(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1
) + a + b*x))/(6*b**3)
```

3.179 $\int e^{i\arctan(a+bx)}x\,dx$

Optimal result

Integrand size = 14, antiderivative size = 110

$$\int e^{i\arctan(a+bx)}x\,dx=\frac{(1-2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2}+\frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2b^2}-\frac{(i+2a)\text{arcsinh}(a+bx)}{2b^2}$$

output
```
1/2*(1-2*I*a)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^2+1/2*(1-I*a-I*b*x
)^(1/2)*(1+I*a+I*b*x)^(3/2)/b^2-1/2*(I+2*a)*arcsinh(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.98

$$\int e^{i\arctan(a+bx)}x\,dx=\frac{(2-ia+ibx)\sqrt{1+a^2+2abx+b^2x^2}}{2b^2}+\frac{(-1)^{3/4}(i+2a)\text{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{\sqrt{-ib}b^{3/2}}$$

input
```
Integrate[E^(I*ArcTan[a + b*x])*x,x]
```

output
```
((2 - I*a + I*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/(2*b^2) + ((-1)^(3/4
)*(I + 2*a)*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-
I)*b]])/(Sqrt[(-I)*b]*b^(3/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5618, 90, 60, 62, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{x\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}dx$$

$$\Big\downarrow \text{90}$$

$$\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2b^2}-\frac{(2a+i)\int \frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}dx}{2b}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2b^2}-\frac{(2a+i)\left(\int \frac{1}{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}$$

$$\Big\downarrow \text{62}$$

$$\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2b^2}-\frac{(2a+i)\left(\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}$$

$$\Big\downarrow \text{1090}$$

$$\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2b^2}-\frac{(2a+i)\left(\frac{\int \frac{1}{\sqrt{\frac{\left(2xb^2+2ab\right)^2}{4b^2}+1}}d(2xb^2+2ab)}{2b^2}+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}$$

$$\Big\downarrow \; 222$$

$$\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2b^2}-\frac{(2a+i)\left(\frac{\operatorname{arcsinh}\left(\frac{2ab+2b^2x}{2b}\right)}{b}+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}$$

input
```
Int[E^(I*ArcTan[a + b*x])*x,x]
```

output
```
(Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/(2*b^2) - ((I + 2*a)*((I*S
qrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b + ArcSinh[(2*a*b + 2*b^2*x)/
(2*b)]/b))/(2*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{i(-bx+a+2i)\sqrt{b^{2}x^{2}+2abx+a^{2}+1}}{2b^{2}}-\frac{(i+2a)\ln\left(\frac{b^{2}x+ab}{\sqrt{b^{2}}}+\sqrt{b^{2}x^{2}+2abx+a^{2}+1}\right)}{2b\sqrt{b^{2}}}$
default	$(ia+1)\left(\frac{\sqrt{b^{2}x^{2}+2abx+a^{2}+1}}{b^{2}}-\frac{a\ln\left(\frac{b^{2}x+ab}{\sqrt{b^{2}}}+\sqrt{b^{2}x^{2}+2abx+a^{2}+1}\right)}{b\sqrt{b^{2}}}\right)+ib\left(\frac{x\sqrt{b^{2}x^{2}+2abx+a^{2}+1}}{2b^{2}}-\frac{3a\left(\frac{\sqrt{b^{2}x^{2}+2a}}{b^{2}}\right.}{}\right.$

input
```
int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*(-b*x+a+2*I)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^2-1/2*(I+2*a)/b*ln((b^
2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int e^{i\arctan(a+bx)}x\,dx$$
$$= \frac{-3i\,a^2 + 4\,(2\,a+i)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right) - 4\,\sqrt{b^2x^2+2\,abx+a^2+1}(-i\,bx+i\,a-2}{8\,b^2}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x,x, algorithm="fricas")
```

output
```
1/8*(-3*I*a^2 + 4*(2*a + I)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1)) - 4*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-I*b*x + I*a - 2) + 4*a)/b^2
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(83) = 166.

Time = 0.91 (sec) , antiderivative size = 362, normalized size of antiderivative = 3.29

$$\int e^{i\arctan(a+bx)}x\,dx$$
$$= \begin{cases} \left(\frac{ix}{2b}+\frac{-\frac{ia}{2}+1}{b^2}\right)\sqrt{a^2+2abx+b^2x^2+1}+\frac{\left(-\frac{a\left(-\frac{ia}{2}+1\right)}{b}-\frac{i\left(a^2+1\right)}{2b}\right)\log\left(2ab+2b^2x+2\sqrt{a^2+2abx+b^2x^2+1}\sqrt{b^2}\right)}{\sqrt{b^2}} \\ \frac{\frac{i\left(-a^2\sqrt{a^2+2abx+1}+\frac{\left(a^2+2abx+1\right)^{\frac{3}{2}}}{3}-\sqrt{a^2+2abx+1}\right)}{b}+\frac{-a^2\sqrt{a^2+2abx+1}+\frac{\left(a^2+2abx+1\right)^{\frac{3}{2}}}{3}-\sqrt{a^2+2abx+1}}{ab}+\frac{i\left(a^4\sqrt{a^2+2abx+1}+2a^2\sqrt{a^2+2abx+1}+\cdots\right.}{\cdots}}{2ab} \\ \frac{\frac{iax^2}{2}+\frac{ibx^3}{3}+\frac{x^2}{2}}{\sqrt{a^2+1}} \end{cases}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)*x,x)
```

output
```
Piecewise(((I*x/(2*b) + (-I*a/2 + 1)/b**2)*sqrt(a**2 + 2*a*b*x + b**2*x**2
 + 1) + (-a*(-I*a/2 + 1)/b - I*(a**2 + 1)/(2*b))*log(2*a*b + 2*b**2*x + 2*
sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*sqrt(b**2))/sqrt(b**2), Ne(b**2, 0)),
 ((I*(-a**2*sqrt(a**2 + 2*a*b*x + 1) + (a**2 + 2*a*b*x + 1)**(3/2)/3 - sqr
t(a**2 + 2*a*b*x + 1))/b + (-a**2*sqrt(a**2 + 2*a*b*x + 1) + (a**2 + 2*a*b
*x + 1)**(3/2)/3 - sqrt(a**2 + 2*a*b*x + 1))/(a*b) + I*(a**4*sqrt(a**2 + 2
*a*b*x + 1) + 2*a**2*sqrt(a**2 + 2*a*b*x + 1) + (-2*a**2 - 2)*(a**2 + 2*a*
b*x + 1)**(3/2)/3 + (a**2 + 2*a*b*x + 1)**(5/2)/5 + sqrt(a**2 + 2*a*b*x +
1))/(2*a**2*b))/(2*a*b), Ne(a*b, 0)), ((I*a*x**2/2 + I*b*x**3/3 + x**2/2)/
sqrt(a**2 + 1), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(76) = 152$.

Time = 0.03 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.90

$$\int e^{i\arctan(a+bx)}x\,dx = \frac{3i\,a^2\,\mathrm{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^2} - \frac{a(i\,a+1)\,\mathrm{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{b^2} + \frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}x}{2\,b} - \frac{(i\,a^2+i)\,\mathrm{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^2} - \frac{3i\,\sqrt{b^2x^2+2\,abx+a^2+1}a}{2\,b^2} + \frac{\sqrt{b^2x^2+2\,abx+a^2+1}(i\,a+1)}{b^2}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x,x, algorithm="maxima")
```

output
```
3/2*I*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^2
- a*(I*a + 1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/
b^2 + 1/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b - 1/2*(I*a^2 + I)*arcsin
h(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^2 - 3/2*I*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)*a/b^2 + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(I*a +
 1)/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.69

$$\int e^{i\arctan(a+bx)}x\,dx = -\frac{1}{2}\sqrt{(bx+a)^2+1}\left(-\frac{ix}{b}+\frac{iab-2b}{b^3}\right)+\frac{(2a+i)\log\left(\left|-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right|\right)}{2b|b|}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x,x, algorithm="giac")
```

output
```
-1/2*sqrt((b*x + a)^2 + 1)*(-I*x/b + (I*a*b - 2*b)/b^3) + 1/2*(2*a + I)*lo
g(abs(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b)))/(b*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{i\arctan(a+bx)}x\,dx = \int \frac{x\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})}{\sqrt{(a+b\,x)^2+1}}\,dx$$

input
```
int((x*(a*1i + b*x*1i + 1))/((a + b*x)^2 + 1)^(1/2),x)
```

output
```
int((x*(a*1i + b*x*1i + 1))/((a + b*x)^2 + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.13

$$\int e^{i\arctan(a+bx)}x\,dx = \frac{-\sqrt{b^2x^2+2abx+a^2+1}\,ai+\sqrt{b^2x^2+2abx+a^2+1}\,bix+2\sqrt{b^2x^2+2abx+a^2+1}-2\log\left(\sqrt{b^2x^2+2a\right.}{2b^2}$$

input
```
int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)*x,x)
```

output
```
( - sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a*i + sqrt(a**2 + 2*a*b*x + b**2*
x**2 + 1)*b*i*x + 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) - 2*log(sqrt(a**2
 + 2*a*b*x + b**2*x**2 + 1) + a + b*x)*a - log(sqrt(a**2 + 2*a*b*x + b**2*
x**2 + 1) + a + b*x)*i)/(2*b**2)
```

3.180 $\int e^{i \arctan(a+bx)}\, dx$

Optimal result

Integrand size = 12, antiderivative size = 52

$$\int e^{i \arctan(a+bx)}\, dx = \frac{i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} + \frac{\operatorname{arcsinh}(a+bx)}{b}$$

output
```
I*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b+arcsinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.54

$$\int e^{i \arctan(a+bx)}\, dx = \frac{i\sqrt{1+(a+bx)^2} + \operatorname{arcsinh}(a+bx)}{b}$$

input
```
Integrate[E^(I*ArcTan[a + b*x]),x]
```

output
```
(I*Sqrt[1 + (a + b*x)^2] + ArcSinh[a + b*x])/b
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5616, 60, 62, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{i\arctan(a+bx)}dx$$

$$\Big\downarrow \text{5616}$$

$$\int \frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}dx$$

$$\Big\downarrow \text{60}$$

$$\int \frac{1}{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}$$

$$\Big\downarrow \text{62}$$

$$\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}$$

$$\Big\downarrow \text{1090}$$

$$\frac{\int \frac{1}{\sqrt{\frac{(2xb^2+2ab)^2}{4b^2}+1}}d(2xb^2+2ab)}{2b^2}+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}$$

$$\Big\downarrow \text{222}$$

$$\frac{\text{arcsinh}\left(\frac{2ab+2b^2x}{2b}\right)}{b}+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}$$

input
```
Int[E^(I*ArcTan[a + b*x]),x]
```

output
```
(I*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b + ArcSinh[(2*a*b + 2*b^2
*x)/(2*b)]/b
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5616
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^(I*(n/2))/(1 + I*a*c + I*b*c*x)^(I*(n/2)), x] /; FreeQ[{a, b,
c, n}, x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33

method	result
risch	$\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{b}+\frac{\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}$
default	$\frac{\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}+\frac{ia\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}+ib\left(\frac{\sqrt{b^2x^2+2abx+a^2+1}}{b^2}-\frac{a\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b\right.}{b\sqrt{}}\right.$

input
```
int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I/b*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*
b*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

$$\int e^{i\arctan(a+bx)}\,dx = \frac{i\,a + 2i\,\sqrt{b^2x^2 + 2\,abx + a^2 + 1} - 2\,\log\left(-bx - a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)}{2\,b}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(I*a + 2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*log(-b*x - a + sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int e^{i\arctan(a+bx)}\,dx = \begin{cases} \frac{i\sqrt{(a+bx)^2+1}+\operatorname{asinh}(a+bx)}{b} & \text{for } b \neq 0 \\ \frac{x(ia+1)}{\sqrt{a^2+1}} & \text{otherwise} \end{cases}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2),x)
```

output
```
Piecewise((((I*sqrt((a + b*x)**2 + 1) + asinh(a + b*x))/b, Ne(b, 0)), (x*(I
*a + 1)/sqrt(a**2 + 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.19

$$\int e^{i\arctan(a+bx)}\,dx=\frac{\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{b}+\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{b}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b + I*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int e^{i\arctan(a+bx)}\,dx=-\frac{\log\left(\left|-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right|\right)}{|b|}+\frac{i\,\sqrt{(bx+a)^2+1}}{b}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-log(abs(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b)))/abs(b) + I*sqr
t((b*x + a)^2 + 1)/b
```

Mupad [B] (verification not implemented)

Time = 23.84 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.87

$$\int e^{i\arctan(a+bx)}\,dx=\frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}\ 1\mathrm{i}}{b}+\frac{\operatorname{asinh}(a+b\,x)}{b}+\frac{a\operatorname{asinh}(a+b\,x)\ 1\mathrm{i}}{b}-\frac{a\,b^2\,\ln\left(\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}+\frac{x\,b^2+a\,b}{\sqrt{b^2}}\right)\ 1\mathrm{i}}{\left(b^2\right)^{3/2}}$$

input
```
int((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2),x)
```

output
```
((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)*1i)/b + asinh(a + b*x)/b + (a*asinh(a
 + b*x)*1i)/b - (a*b^2*log((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2) + (a*b + b^
2*x)/(b^2)^(1/2))*1i)/(b^2)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int e^{i\arctan(a+bx)}\,dx = \frac{\sqrt{b^2x^2+2abx+a^2+1}\,i+\log\left(\sqrt{b^2x^2+2abx+a^2+1}+a+bx\right)}{b}$$

input
```
int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2),x)
```

output
```
(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + log(sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1) + a + b*x))/b
```

3.181 $\int \frac{e^{i \arctan(a+bx)}}{x} dx$

Optimal result

Integrand size = 16, antiderivative size = 89

$$\int \frac{e^{i \arctan(a+bx)}}{x} dx = i\operatorname{arcsinh}(a+bx) - \frac{2\sqrt{i-a}\operatorname{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{\sqrt{i+a}}$$

output
```
I*arcsinh(b*x+a)-2*(I-a)^(1/2)*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-
a)^(1/2)/(1-I*a-I*b*x)^(1/2))/(I+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.60

$$\int \frac{e^{i \arctan(a+bx)}}{x} dx = \frac{2(-1)^{3/4}\sqrt{-ib}\operatorname{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{\sqrt{b}} - \frac{2\sqrt{-1-ia}\operatorname{arctanh}\left(\frac{\sqrt{-1-ia}\sqrt{-i(i+a+bx)}}{\sqrt{-1+ia}\sqrt{1+ia+ibx}}\right)}{\sqrt{-1+ia}}$$

input
```
Integrate[E^(I*ArcTan[a + b*x])/x,x]
```

output
```
(2*(-1)^(3/4)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a +
 b*x)])/Sqrt[(-I)*b]])/Sqrt[b] - (2*Sqrt[-1 - I*a]*ArcTanh[(Sqrt[-1 - I*a]
*Sqrt[(-I)*(I + a + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/Sqrt[-
1 + I*a]
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.25, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5618, 140, 27, 62, 104, 221, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{i\arctan(a+bx)}}{x}dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{\sqrt{ia+ibx+1}}{x\sqrt{-ia-ibx+1}}dx$$

$$\Big\downarrow \text{140}$$

$$ib\int \frac{1}{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx+\int \frac{ia+1}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx$$

$$\Big\downarrow \text{27}$$

$$ib\int \frac{1}{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx+(1+ia)\int \frac{1}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx$$

$$\Big\downarrow \text{62}$$

$$ib\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx+(1+ia)\int \frac{1}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx$$

$$\Big\downarrow \text{104}$$

$$ib\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx+2(1+ia)\int \frac{1}{-ia+\frac{(1-ia)(ia+ibx+1)}{-ia-ibx+1}-1}d\frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \text{221}$$

$$ib\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx-\frac{2i(1+ia)\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}\sqrt{a+i}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{i\int \frac{1}{\sqrt{\frac{\left(2xb^2+2ab\right)^2}{4b^2}+1}}d\left(2xb^2+2ab\right)}{2b}-\frac{2i(1+ia)\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}\sqrt{a+i}}$$

$$\Big\downarrow \text{222}$$

$$i\text{arcsinh}\left(\frac{2ab+2b^2x}{2b}\right)-\frac{2i(1+ia)\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}\sqrt{a+i}}$$

input
```
Int[E^(I*ArcTan[a + b*x])/x,x]
```

output
```
I*ArcSinh[(2*a*b + 2*b^2*x)/(2*b)] - ((2*I)*(1 + I*a)*ArcTanh[(Sqrt[I + a]
*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/(Sqrt[I - a]
*Sqrt[I + a])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.20

method	result	size
default	$\frac{ib\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}-\frac{(ia+1)\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{x}\right)}{\sqrt{a^2+1}}$	107

input
```
int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
I*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-
(1+I*a)/(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x
+a^2+1)^(1/2))/x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(59) = 118$.

Time = 0.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.62

$$\int \frac{e^{i\arctan(a+bx)}}{x}\,dx = \sqrt{-\frac{a-i}{a+i}}\log\left(-bx+(i\,a-1)\sqrt{-\frac{a-i}{a+i}}+\sqrt{b^2x^2+2\,abx+a^2+1}\right) - \sqrt{-\frac{a-i}{a+i}}\log\left(-bx+(-i\,a+1)\sqrt{-\frac{a-i}{a+i}} + \sqrt{b^2x^2+2\,abx+a^2+1}\right) - i\,\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
sqrt(-(a - I)/(a + I))*log(-b*x + (I*a - 1)*sqrt(-(a - I)/(a + I)) + sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)) - sqrt(-(a - I)/(a + I))*log(-b*x + (-I*a +
1)*sqrt(-(a - I)/(a + I)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - I*log(-b*
x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))
```

Sympy [F]

$$\int \frac{e^{i\arctan(a+bx)}}{x}\,dx = i\left(\int \frac{b}{\sqrt{a^2+2abx+b^2x^2+1}}\,dx + \int \left(-\frac{i}{x\sqrt{a^2+2abx+b^2x^2+1}}\right)dx + \int \frac{a}{x\sqrt{a^2+2abx+b^2x^2+1}}\,dx\right)$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)/x,x)
```

output
```
I*(Integral(b/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x) + Integral(-I/(x*sq
rt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a/(x*sqrt(a**2 + 2*a*b*
x + b**2*x**2 + 1)), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(59) = 118$.

Time = 0.03 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.62

$$\int \frac{e^{i\arctan(a+bx)}}{x}\,dx = -\frac{i\,a\,\mathrm{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{\sqrt{a^2+1}} - \frac{\mathrm{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{\sqrt{a^2+1}} + i\,\mathrm{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
-I*a*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(
sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 +
 1)*b^2)*abs(x)))/sqrt(a^2 + 1) - arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^
2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) +
2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/sqrt(a^2 + 1) + I*arcsinh(2
*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.27

$$\int \frac{e^{i\arctan(a+bx)}}{x}\,dx = -\frac{(-ia-1)\log\left(\frac{\left|-2\,x|b|+2\,\sqrt{(bx+a)^2+1}-2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\,\sqrt{(bx+a)^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{\sqrt{a^2+1}} - \frac{i\,b\log\left(\left|-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right|\right)}{|b|}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x,x, algorithm="giac")
```

output
```
-(-I*a - 1)*log(abs(-2*x*abs(b) + 2*sqrt((b*x + a)^2 + 1) - 2*sqrt(a^2 + 1
))/abs(-2*x*abs(b) + 2*sqrt((b*x + a)^2 + 1) + 2*sqrt(a^2 + 1)))/sqrt(a^2
+ 1) - I*b*log(abs(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b)))/abs(
b)
```

Mupad [B] (verification not implemented)

Time = 23.89 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.33

$$\int \frac{e^{i\arctan(a+bx)}}{x}\,dx = \operatorname{asinh}(a+b\,x)\ 1\mathrm{i} - \frac{\ln\left(a\,b+\frac{a^2+1}{x}+\frac{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x}\right)}{\sqrt{a^2+1}} - \frac{a\,\ln\left(a\,b+\frac{a^2+1}{x}+\frac{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x}\right)\ 1\mathrm{i}}{\sqrt{a^2+1}}$$

input
```
int((a*1i + b*x*1i + 1)/(x*((a + b*x)^2 + 1)^(1/2)),x)
```

output
```
asinh(a + b*x)*1i - log(a*b + (a^2 + 1)/x + ((a^2 + 1)^(1/2)*(a^2 + b^2*x^
2 + 2*a*b*x + 1)^(1/2))/x)/(a^2 + 1)^(1/2) - (a*log(a*b + (a^2 + 1)/x + ((
a^2 + 1)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2))/x)*1i)/(a^2 + 1)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.71

$$\int \frac{e^{i\arctan(a+bx)}}{x}\,dx$$
$$= \frac{-2\sqrt{a^2+1}\,atan\left(\frac{\sqrt{b^2x^2+2abx+a^2+1}\,i+bix}{\sqrt{a^2+1}}\right)a+2\sqrt{a^2+1}\,atan\left(\frac{\sqrt{b^2x^2+2abx+a^2+1}\,i+bix}{\sqrt{a^2+1}}\right)i+\log\left(\sqrt{b^2x^2+2abx\right.}{a^2+1}$$

input
```
int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x,x)
```

output
```
( - 2*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)
/sqrt(a**2 + 1))*a + 2*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**
2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*i + log(sqrt(a**2 + 2*a*b*x + b**2*x**2
+ 1) + a + b*x)*a**2*i + log(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + a + b*
x)*i)/(a**2 + 1)
```

3.182 $\int \frac{e^{i\arctan(a+bx)}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 130

$$\int \frac{e^{i\arctan(a+bx)}}{x^2}\,dx=-\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{(1-ia)x}+\frac{2ib\,\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{\sqrt{i-a}(i+a)^{3/2}}$$

output
```
-(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/(1-I*a)/x+2*I*b*arctanh((I+a)^(1/
2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-I*b*x)^(1/2))/(I-a)^(1/2)/(I+a)^
(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.92

$$\int \frac{e^{i\arctan(a+bx)}}{x^2}\,dx=-i\left(\frac{\sqrt{1+a^2+2abx+b^2x^2}}{ix+ax}+\frac{2b\,\text{arctanh}\left(\frac{\sqrt{-1-ia}\sqrt{-i(i+a+bx)}}{\sqrt{-1+ia}\sqrt{1+ia+ibx}}\right)}{\sqrt{-1-ia}(-1+ia)^{3/2}}\right)$$

input
```
Integrate[E^(I*ArcTan[a + b*x])/x^2,x]
```

output
```
(-I)*(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/(I*x + a*x) + (2*b*ArcTanh[(Sqrt[-
1 - I*a]*Sqrt[(-I)*(I + a + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])]
)/(Sqrt[-1 - I*a]*(-1 + I*a)^(3/2)))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5618, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{i\arctan(a+bx)}}{x^2}\,dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{\sqrt{ia+ibx+1}}{x^2\sqrt{-ia-ibx+1}}dx$$

$$\Big\downarrow \text{105}$$

$$-\frac{b\int \frac{1}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{a+i}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{(1-ia)x}$$

$$\Big\downarrow \text{104}$$

$$-\frac{2b\int \frac{1}{-ia+\frac{(1-ia)(ia+ibx+1)}{-ia-ibx+1}-1}d\frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}}{a+i}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{(1-ia)x}$$

$$\Big\downarrow \text{221}$$

$$\frac{2ib\,\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}(a+i)^{3/2}}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{(1-ia)x}$$

input
```
Int[E^(I*ArcTan[a + b*x])/x^2,x]
```

output
```
-((Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/((1 - I*a)*x)) + ((2*I)*b*
ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*
b*x])])/(Sqrt[I - a]*(I + a)^(3/2))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{(i+a)x}+\frac{b\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{x}\right)}{(i+a)\sqrt{a^2+1}}$
default	$(ia+1)\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{(a^2+1)x}+\frac{ab\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{x}\right)}{(a^2+1)^{\frac{3}{2}}}\right)-\frac{ib\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x}}{x}\right.}{\sqrt{a^2+1}}$

input
```
int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-I*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/(I+a)/x+1/(I+a)*b/(a^2+1)^(1/2)*ln((2*a^2
+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(86) = 172.

Time = 0.09 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.72

$$\int \frac{e^{i\arctan(a+bx)}}{x^2}\,dx =$$

$$-\frac{(a+i)\sqrt{\frac{b^2}{a^4+2i\,a^3+2i\,a-1}}\,x\log\left(-\frac{b^2x-\sqrt{b^2x^2+2\,abx+a^2+1}b+(a^3+i\,a^2+a+i)\sqrt{\frac{b^2}{a^4+2i\,a^3+2i\,a-1}}}{b}\right)-(a+i)\sqrt{\frac{b^2}{a^4+2i\,a^3+2}}}{(a+i)}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-((a + I)*sqrt(b^2/(a^4 + 2*I*a^3 + 2*I*a - 1))*x*log(-(b^2*x - sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)*b + (a^3 + I*a^2 + a + I)*sqrt(b^2/(a^4 + 2*I*a^3
+ 2*I*a - 1)))/b) - (a + I)*sqrt(b^2/(a^4 + 2*I*a^3 + 2*I*a - 1))*x*log(-(
b^2*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b - (a^3 + I*a^2 + a + I)*sqrt(b
^2/(a^4 + 2*I*a^3 + 2*I*a - 1)))/b) + I*b*x + I*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1))/((a + I)*x)
```

Sympy [F]

$$\int \frac{e^{i\arctan(a+bx)}}{x^2}\,dx = i\left(\int \left(-\frac{i}{x^2\sqrt{a^2+2abx+b^2x^2+1}}\right)dx + \int \frac{a}{x^2\sqrt{a^2+2abx+b^2x^2+1}}\,dx + \int \frac{b}{x\sqrt{a^2+2abx+b^2x^2+1}}\,dx\right)$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)/x**2,x)
```

output
```
I*(Integral(-I/(x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(
a/(x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(b/(x*sqrt(a**
2 + 2*a*b*x + b**2*x**2 + 1)), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(86) = 172$.

Time = 0.04 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.84

$$\int \frac{e^{i\arctan(a+bx)}}{x^2}\,dx = \frac{a(i\,a+1)b\,\mathrm{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{(a^2+1)^{\frac{3}{2}}} - \frac{i\,b\,\mathrm{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{\sqrt{a^2+1}} + \frac{\sqrt{b^2x^2+2\,abx+a^2+1}(-i\,a-1)}{(a^2+1)x}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
a*(I*a + 1)*b*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x))
+ 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 +
 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3/2) - I*b*arcsinh(2*a*b*x/(sqrt(-4*
a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*
b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/sqrt(a^2 + 1
) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-I*a - 1)/((a^2 + 1)*x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12

$$\int \frac{e^{i\arctan(a+bx)}}{x^2}\,dx = \frac{b\log\left(\frac{\left|2\,x|b|-2\,\sqrt{(bx+a)^2+1}-2\,\sqrt{a^2+1}\right|}{\left|2\,x|b|-2\,\sqrt{(bx+a)^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{\sqrt{a^2+1}(a+i)} - \frac{2\left(\left(x|b|-\sqrt{(bx+a)^2+1}\right)ab+a^2|b|+|b|\right)}{\left(\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2-a^2-1\right)(i\,a-1)}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
b*log(abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) - 2*sqrt(a^2 + 1))/abs(2*x*
abs(b) - 2*sqrt((b*x + a)^2 + 1) + 2*sqrt(a^2 + 1)))/(sqrt(a^2 + 1)*(a + I
)) - 2*((x*abs(b) - sqrt((b*x + a)^2 + 1))*a*b + a^2*abs(b) + abs(b))/(((x
*abs(b) - sqrt((b*x + a)^2 + 1))^2 - a^2 - 1)*(I*a - 1))
```

Mupad [B] (verification not implemented)

Time = 24.48 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.68

$$\int \frac{e^{i\arctan(a+bx)}}{x^2}\,dx = \frac{a\,b\,\mathrm{atanh}\left(\frac{a^2+b\,x\,a+1}{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}\right)}{\left(a^2+1\right)^{3/2}} - \frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x\,\left(a^2+1\right)} - \frac{b\,\ln\left(a\,b+\frac{a^2+1}{x}+\frac{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x}\right)\,1{}\mathrm{i}}{\sqrt{a^2+1}} + \frac{a^2\,b\,\mathrm{atanh}\left(\frac{a^2+b\,x\,a+1}{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}\right)\,1{}\mathrm{i}}{\left(a^2+1\right)^{3/2}} - \frac{a\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}\,1{}\mathrm{i}}{x\,\left(a^2+1\right)}$$

input
```
int((a*1i + b*x*1i + 1)/(x^2*((a + b*x)^2 + 1)^(1/2)),x)
```

output
```
(a^2*b*atanh((a^2 + a*b*x + 1)/((a^2 + 1)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x +
 1)^(1/2)))*1i)/(a^2 + 1)^(3/2) - (a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/(x*(
a^2 + 1)) - (b*log(a*b + (a^2 + 1)/x + ((a^2 + 1)^(1/2)*(a^2 + b^2*x^2 + 2
*a*b*x + 1)^(1/2))/x)*1i)/(a^2 + 1)^(1/2) - (a*(a^2 + b^2*x^2 + 2*a*b*x +
1)^(1/2)*1i)/(x*(a^2 + 1)) + (a*b*atanh((a^2 + a*b*x + 1)/((a^2 + 1)^(1/2)
*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2))))/(a^2 + 1)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.52

$$\int \frac{e^{i\arctan(a+bx)}}{x^2}\,dx$$
$$= \frac{-2\sqrt{a^2+1}\,atan\left(\frac{\sqrt{b^2x^2+2abx+a^2+1}\,i+bix}{\sqrt{a^2+1}}\right)abix-2\sqrt{a^2+1}\,atan\left(\frac{\sqrt{b^2x^2+2abx+a^2+1}\,i+bix}{\sqrt{a^2+1}}\right)bx-\sqrt{b^2x^2+2abx}}{x\,(a^4+2a^2}$$

input
```
int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^2,x)
```

output
```
( - 2*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)
/sqrt(a**2 + 1))*a*b*i*x - 2*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*b*x - sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1)*a**3*i - sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**2 - sqrt(a**2 +
2*a*b*x + b**2*x**2 + 1)*a*i - sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/(x*(a
**4 + 2*a**2 + 1))
```

3.183 $\int \frac{e^{i\arctan(a+bx)}}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 201

$$\int \frac{e^{i\arctan(a+bx)}}{x^3}\,dx = -\frac{(1+2ia)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2(i-a)(i+a)^2x} - \frac{\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2(1+a^2)x^2} + \frac{(1+2ia)b^2\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{3/2}(i+a)^{5/2}}$$

output
```
-1/2*(1+2*I*a)*b*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)/(I+a)^2/x-1
/2*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(3/2)/(a^2+1)/x^2+(1+2*I*a)*b^2*arcta
nh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-I*b*x)^(1/2))/(I-a)^
(3/2)/(I+a)^(5/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.77

$$\int \frac{e^{i\arctan(a+bx)}}{x^3}\,dx = \frac{-\frac{i(1+a^2+2ibx-abx)\sqrt{1+a^2+2abx+b^2x^2}}{x^2}+\frac{2(-i+2a)b^2\operatorname{arctanh}\left(\frac{\sqrt{-1-ia}\sqrt{-i(i+a+bx)}}{\sqrt{-1+ia}\sqrt{1+ia+ibx}}\right)}{\sqrt{-1-ia}\sqrt{-1+ia}}}{2(-i+a)(i+a)^2}$$

input

```
Integrate[E^(I*ArcTan[a + b*x])/x^3,x]
```

output

```
(((-I)*(1 + a^2 + (2*I)*b*x - a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/x^
2 + (2*(-I + 2*a)*b^2*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-I)*(I + a + b*x)])/(S
qrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/(Sqrt[-1 - I*a]*Sqrt[-1 + I*a]))/(2
*(-I + a)*(I + a)^2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5618, 107, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{i\arctan(a+bx)}}{x^3}\,dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{\sqrt{ia+ibx+1}}{x^3\sqrt{-ia-ibx+1}}dx$$

$$\Big\downarrow \text{107}$$

$$\frac{(-2a+i)b\int \frac{\sqrt{ia+ibx+1}}{x^2\sqrt{-ia-ibx+1}}dx}{2\left(a^2+1\right)}-\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2\left(a^2+1\right)x^2}$$

$$\Big\downarrow \text{105}$$

$$\frac{(-2a+i)b\left(-\frac{b\int \frac{1}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{a+i}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{(1-ia)x}\right)}{2\left(a^2+1\right)}-\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2\left(a^2+1\right)x^2}$$

$$\Big\downarrow \text{ 104}$$

$$\frac{(-2a+i)b\left(-\frac{2b\int \frac{1}{-ia+\frac{(1-ia)(ia+ibx+1)}{-ia-ibx+1}-1}d\frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}}{a+i}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{(1-ia)x}\right)}{2\left(a^2+1\right)}-\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2\left(a^2+1\right)x^2}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{(-2a+i)b\left(\frac{2ib\,\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}(a+i)^{3/2}}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{(1-ia)x}\right)}{2\left(a^2+1\right)}-\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2\left(a^2+1\right)x^2}$$

input
```
Int[E^(I*ArcTan[a + b*x])/x^3,x]
```

output
```
-1/2*(Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/((1 + a^2)*x^2) + ((I
 - 2*a)*b*(-((Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/((1 - I*a)*x))
+ ((2*I)*b*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1
 - I*a - I*b*x])])/(Sqrt[I - a]*(I + a)^(3/2))))/(2*(1 + a^2))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{i\left(-a\,b^{3} x^{3}+2 i b^{3} x^{3}-a^{2} b^{2} x^{2}+4 i a\,b^{2} x^{2}+a^{3} b x+2 i a^{2} b x+a^{4}+b^{2} x^{2}+a b x+2 i b x+2 a^{2}+1\right)}{2 x^{2} \left(i+a\right)^{2} \left(-i+a\right) \sqrt{b^{2} x^{2}+2 a b x+a^{2}+1}}-\frac{b^{2} \left(-i+2 a\right) \ln \left(\frac{2 a^{2}+2+2 a b x+2 \sqrt{a^{2}+1}\,\sqrt{}}{x}\right.}{2 \left(a^{2}+1\right)^{\frac{3}{2}} \left(i+a\right)}$
default	$\left(i a+1\right) \left(-\frac{\sqrt{b^{2} x^{2}+2 a b x+a^{2}+1}}{2 \left(a^{2}+1\right) x^{2}}-\frac{3 a b \left(-\frac{\sqrt{b^{2} x^{2}+2 a b x+a^{2}+1}}{\left(a^{2}+1\right) x}+\frac{a b \ln \left(\frac{2 a^{2}+2+2 a b x+2 \sqrt{a^{2}+1}\,\sqrt{b^{2} x^{2}+2 a b x+a^{2}+1}}{x}\right)}{\left(a^{2}+1\right)^{\frac{3}{2}}}\right)}{2 \left(a^{2}+1\right)}+\frac{b^{2} \ln}{}\right.$

input
```
int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*(-a*b^3*x^3-a^2*b^2*x^2+a^3*b*x+2*I*b^3*x^3+a^4+b^2*x^2+4*I*a*b^2*x
^2+a*b*x+2*I*a^2*b*x+2*a^2+2*I*b*x+1)/x^2/(I+a)^2/(-I+a)/(b^2*x^2+2*a*b*x+
a^2+1)^(1/2)-1/2*b^2*(-I+2*a)/(a^2+1)^(3/2)/(I+a)*ln((2*a^2+2+2*a*b*x+2*(a
^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(135) = 270$.

Time = 0.08 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.25

$$\int \frac{e^{i\arctan(a+bx)}}{x^3}\,dx = \frac{(i\,a+2)b^2x^2+\sqrt{\frac{(4\,a^2-4i\,a-1)b^4}{a^8+2i\,a^7+2\,a^6+6i\,a^5+6i\,a^3-2\,a^2+2i\,a-1}}(a^3+i\,a^2+a+i)x^2\log\left(-\frac{(2\,a-i)b^3x-\sqrt{b^2x^2+2\,abx+a^2+1}(2}{}\right.}{}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
1/2*((I*a + 2)*b^2*x^2 + sqrt((4*a^2 - 4*I*a - 1)*b^4/(a^8 + 2*I*a^7 + 2*a
^6 + 6*I*a^5 + 6*I*a^3 - 2*a^2 + 2*I*a - 1))*(a^3 + I*a^2 + a + I)*x^2*log
(-((2*a - I)*b^3*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*a - I)*b^2 + (a^
5 + I*a^4 + 2*a^3 + 2*I*a^2 + a + I)*sqrt((4*a^2 - 4*I*a - 1)*b^4/(a^8 + 2
*I*a^7 + 2*a^6 + 6*I*a^5 + 6*I*a^3 - 2*a^2 + 2*I*a - 1)))/((2*a - I)*b^2))
 - sqrt((4*a^2 - 4*I*a - 1)*b^4/(a^8 + 2*I*a^7 + 2*a^6 + 6*I*a^5 + 6*I*a^3
 - 2*a^2 + 2*I*a - 1))*(a^3 + I*a^2 + a + I)*x^2*log(-((2*a - I)*b^3*x - s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*a - I)*b^2 - (a^5 + I*a^4 + 2*a^3 + 2*
I*a^2 + a + I)*sqrt((4*a^2 - 4*I*a - 1)*b^4/(a^8 + 2*I*a^7 + 2*a^6 + 6*I*a
^5 + 6*I*a^3 - 2*a^2 + 2*I*a - 1)))/((2*a - I)*b^2)) + sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1)*((I*a + 2)*b*x - I*a^2 - I))/((a^3 + I*a^2 + a + I)*x^2)
```

Sympy [F]

$$\int \frac{e^{i\arctan(a+bx)}}{x^3}\,dx = i\left(\int\left(-\frac{i}{x^3\sqrt{a^2+2abx+b^2x^2+1}}\right)dx + \int\frac{a}{x^3\sqrt{a^2+2abx+b^2x^2+1}}\,dx + \int\frac{b}{x^2\sqrt{a^2+2abx+b^2x^2+1}}\,dx\right)$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)/x**3,x)
```

output
```
I*(Integral(-I/(x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(
a/(x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(b/(x**2*sqrt(
a**2 + 2*a*b*x + b**2*x**2 + 1)), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(135) = 270$.

Time = 0.04 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.11

$$\int \frac{e^{i\arctan(a+bx)}}{x^3}\,dx = -\frac{3\,a^2(i\,a+1)b^2\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{2\,(a^2+1)^{\frac{5}{2}}} + \frac{i\,ab^2\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{(a^2+1)^{\frac{3}{2}}} - \frac{(-i\,a-1)b^2\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}|x|}\right)}{2\,(a^2+1)^{\frac{3}{2}}} + \frac{3\sqrt{b^2x^2+2\,abx+a^2+1}a(i\,a+1)b}{2\,(a^2+1)^2x} - \frac{i\sqrt{b^2x^2+2\,abx+a^2+1}b}{(a^2+1)x} - \frac{\sqrt{b^2x^2+2\,abx+a^2+1}(i\,a+1)}{2\,(a^2+1)x^2}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-3/2*a^2*(I*a + 1)*b^2*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)
*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*
a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(5/2) + I*a*b^2*arcsinh(2*a*
b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 +
 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))
/(a^2 + 1)^(3/2) - 1/2*(-I*a - 1)*b^2*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4
*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)
) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3/2) + 3/2*s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*(I*a + 1)*b/((a^2 + 1)^2*x) - I*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)*b/((a^2 + 1)*x) - 1/2*sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)*(I*a + 1)/((a^2 + 1)*x^2)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(135) = 270$.

Time = 0.16 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.34

$$\int \frac{e^{i\arctan(a+bx)}}{x^3}\,dx = -\frac{(2\,ab^2 - i\,b^2)\log\left(\frac{\left|2\,x|b| - 2\,\sqrt{(bx+a)^2+1} - 2\,\sqrt{a^2+1}\right|}{\left|2\,x|b| - 2\,\sqrt{(bx+a)^2+1} + 2\,\sqrt{a^2+1}\right|}\right)}{2\,(a^3 + i\,a^2 + a + i)\sqrt{a^2+1}} - \frac{4\left(-i\,x|b| + i\,\sqrt{(bx+a)^2+1}\right)a^4b^2 - 2i\left(x|b| - \sqrt{(bx+a)^2+1}\right)^2 a^3b|b| - 2i\,a^5b|b| + 2\left(x|b| - \sqrt{(}\right.}{}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

output
```
-1/2*(2*a*b^2 - I*b^2)*log(abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) - 2*sq
rt(a^2 + 1))/abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) + 2*sqrt(a^2 + 1)))/
((a^3 + I*a^2 + a + I)*sqrt(a^2 + 1)) - (4*(-I*x*abs(b) + I*sqrt((b*x + a)
^2 + 1))*a^4*b^2 - 2*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a^3*b*abs(b) -
 2*I*a^5*b*abs(b) + 2*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a*b^2 - 2*(x*ab
s(b) - sqrt((b*x + a)^2 + 1))*a^3*b^2 + 2*(x*abs(b) - sqrt((b*x + a)^2 + 1
))^2*a^2*b*abs(b) - 2*a^4*b*abs(b) - I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^
3*b^2 + 5*(-I*x*abs(b) + I*sqrt((b*x + a)^2 + 1))*a^2*b^2 - 2*I*(x*abs(b)
- sqrt((b*x + a)^2 + 1))^2*a*b*abs(b) - 4*I*a^3*b*abs(b) - 2*(x*abs(b) - s
qrt((b*x + a)^2 + 1))*a*b^2 + 2*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b*abs
(b) - 4*a^2*b*abs(b) - (I*x*abs(b) - I*sqrt((b*x + a)^2 + 1))*b^2 - 2*I*a*
b*abs(b) - 2*b*abs(b))/((a^3 + I*a^2 + a + I)*((x*abs(b) - sqrt((b*x + a)^
2 + 1))^2 - a^2 - 1)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{i\arctan(a+bx)}}{x^3}\,dx = \int \frac{1 + a\,1\mathrm{i} + b\,x\,1\mathrm{i}}{x^3\,\sqrt{(a+b\,x)^2+1}}\,dx$$

input
```
int((a*1i + b*x*1i + 1)/(x^3*((a + b*x)^2 + 1)^(1/2)),x)
```

output
```
int((a*1i + b*x*1i + 1)/(x^3*((a + b*x)^2 + 1)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 510, normalized size of antiderivative = 2.54

$$\int \frac{e^{i\arctan(a+bx)}}{x^3}\,dx = \frac{8\sqrt{a^2+1}\,atan\left(\frac{\sqrt{b^2x^2+2abx+a^2+1}\,i+bix}{\sqrt{a^2+1}}\right)a^3b^2i\,x^2 + 12\sqrt{a^2+1}\,atan\left(\frac{\sqrt{b^2x^2+2abx+a^2+1}\,i+bix}{\sqrt{a^2+1}}\right)a^2b^2x^2 - 4\sqrt{a^2+}$$

input
```
int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^3,x)
```

output

```
(8*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/sq
rt(a**2 + 1))*a**3*b**2*i*x**2 + 12*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b
*x + b**2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*a**2*b**2*x**2 - 4*sqrt(a**
2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1
))*a*b**2*i*x**2 - 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**6*i + 2*sqrt(
a**2 + 2*a*b*x + b**2*x**2 + 1)*a**5*b*i*x - 2*sqrt(a**2 + 2*a*b*x + b**2*
x**2 + 1)*a**5 + 6*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**4*b*x - 4*sqrt(
a**2 + 2*a*b*x + b**2*x**2 + 1)*a**4*i - 2*sqrt(a**2 + 2*a*b*x + b**2*x**2
 + 1)*a**3*b*i*x - 4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**3 + 6*sqrt(a*
*2 + 2*a*b*x + b**2*x**2 + 1)*a**2*b*x - 2*sqrt(a**2 + 2*a*b*x + b**2*x**2
 + 1)*a**2*i - 4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a*b*i*x - 2*sqrt(a**
2 + 2*a*b*x + b**2*x**2 + 1)*a - 2*a**5*b**2*i*x**2 - 4*a**4*b**2*x**2 - a
**3*b**2*i*x**2 - 5*a**2*b**2*x**2 + a*b**2*i*x**2 - b**2*x**2)/(4*a*x**2*
(a**6 + 3*a**4 + 3*a**2 + 1))
```

3.184 $\int \frac{e^{i\arctan(a+bx)}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 283

$$\begin{aligned}\int \frac{e^{i\arctan(a+bx)}}{x^4}\,dx =& -\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1-ia)x^3} \\ & -\frac{(3i-2a)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(1-ia)\,(1+a^2)\,x^2} \\ & +\frac{(4+9ia-2a^2)\,b^2\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(1-ia)\,(1+a^2)^2\,x} \\ & +\frac{(2a-i(1-2a^2))\,b^3\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{5/2}(i+a)^{7/2}}\end{aligned}$$

output
```
-1/3*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/(1-I*a)/x^3-1/6*(3*I-2*a)*b*(
1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/(1-I*a)/(a^2+1)/x^2+1/6*(4+9*I*a-2*
a^2)*b^2*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/(1-I*a)/(a^2+1)^2/x+(2*a-
I*(-2*a^2+1))*b^3*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I
*a-I*b*x)^(1/2))/(I-a)^(5/2)/(I+a)^(7/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.83

$$\int \frac{e^{i\arctan(a+bx)}}{x^4}\,dx$$

$$= \frac{\frac{2(1-ia)(-i+a)(-i+a+bx)\sqrt{1+a^2+2abx+b^2x^2}}{x^3} + \frac{(1+4ia)b(-i+a+bx)\sqrt{1+a^2+2abx+b^2x^2}}{x^2} + 3i(1+2ia-2a^2)\,b^2\left(\frac{\sqrt{1+a^2+2abx}}{ix+ax}\right.}{6\,(1+a^2)^2}$$

input
```
Integrate[E^(I*ArcTan[a + b*x])/x^4,x]
```

output
```
((2*(1 - I*a)*(-I + a)*(-I + a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/x
^3 + ((1 + (4*I)*a)*b*(-I + a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/x^
2 + (3*I)*(1 + (2*I)*a - 2*a^2)*b^2*(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/(I*
x + a*x) + (2*b*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-I)*(I + a + b*x)])/(Sqrt[-1
 + I*a]*Sqrt[1 + I*a + I*b*x])])/(Sqrt[-1 - I*a]*(-1 + I*a)^(3/2))))/(6*(1
 + a^2)^2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5618, 110, 27, 168, 27, 168, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{i\arctan(a+bx)}}{x^4}dx$$

$$\Big\downarrow \text{ 5618}$$

$$\int \frac{\sqrt{ia+ibx+1}}{x^4\sqrt{-ia-ibx+1}}dx$$

$$\Big\downarrow \text{ 110}$$

$$\frac{\int \frac{b(-2a-2bx+3i)}{x^3\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{3(1-ia)}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{3(1-ia)x^3}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{b\int \frac{-2a-2bx+3i}{x^3\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{3(1-ia)}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{3(1-ia)x^3}$$

$$\Big\downarrow \textbf{168}$$

$$\frac{b\left(-\frac{\int \frac{b\left(-2a^2+9ia+(3i-2a)bx+4\right)}{x^2\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{2(a^2+1)}-\frac{(-2a+3i)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2(a^2+1)x^2}\right)}{3(1-ia)}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{3(1-ia)x^3}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{b\left(-\frac{b\int \frac{-2a^2+9ia+(3i-2a)bx+4}{x^2\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{2(a^2+1)}-\frac{(-2a+3i)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2(a^2+1)x^2}\right)}{3(1-ia)}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{3(1-ia)x^3}$$

$$\Big\downarrow \textbf{168}$$

$$\frac{b\left(-\frac{b\left(-\frac{\int -\frac{3\left(-2ia^2-2a+i\right)b}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{a^2+1}-\frac{\left(-2a^2+9ia+4\right)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{\left(a^2+1\right)x}\right)}{2(a^2+1)}-\frac{(-2a+3i)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2(a^2+1)x^2}\right)}{3(1-ia)}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{3(1-ia)x^3}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{b\left(-\frac{b\left(\frac{3\left(-2ia^2-2a+i\right)b\int \frac{1}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{a^2+1}-\frac{\left(-2a^2+9ia+4\right)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{\left(a^2+1\right)x}\right)}{2(a^2+1)}-\frac{(-2a+3i)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2(a^2+1)x^2}\right)}{3(1-ia)}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{3(1-ia)x^3}$$

$\downarrow$ 104

$$\frac{b\left(-\frac{b\left(\frac{6\left(-2ia^2-2a+i\right)b\int \frac{1}{-ia+\frac{(1-ia)(ia+ibx+1)}{-ia-ibx+1}-1}d\frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}}{a^2+1}-\frac{\left(-2a^2+9ia+4\right)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{\left(a^2+1\right)x}\right)}{2(a^2+1)}-\frac{(-2a+3i)\sqrt{-ia-ibx+1}\sqrt{ia+ib}}{2(a^2+1)x^2}}{3(1-ia)}$$

$$\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{3(1-ia)x^3}$$

$\downarrow$ 221

$$\frac{b\left(-\frac{b\left(-\frac{6i\left(-2ia^2-2a+i\right)b\,\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}\sqrt{a+i}\left(a^2+1\right)}-\frac{\left(-2a^2+9ia+4\right)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{\left(a^2+1\right)x}\right)}{2(a^2+1)}-\frac{(-2a+3i)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2(a^2+1)x^2}\right)}{3(1-ia)}$$

$$\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{3(1-ia)x^3}$$

input
```
Int[E^(I*ArcTan[a + b*x])/x^4,x]
```

output
```
-1/3*(Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/((1 - I*a)*x^3) + (b*(-
1/2*((3*I - 2*a)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/((1 + a^2)*x
^2) - (b*(-(((4 + (9*I)*a - 2*a^2)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*
b*x])/((1 + a^2)*x)) - ((6*I)*(I - 2*a - (2*I)*a^2)*b*ArcTanh[(Sqrt[I + a]
*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/(Sqrt[I - a]
*Sqrt[I + a]*(1 + a^2))))/(2*(1 + a^2))))/(3*(1 - I*a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.99

method	result
risch	$-\frac{i(2a^2b^4x^4-9iab^4x^4+2a^3b^3x^3-15ia^2b^3x^3-3ia^3b^2x^2-4b^4x^4+2a^5bx+3ia^4bx-10ab^3x^3+3ib^3x^3+2a^6-2a^2b^2x^2-3iab^2x^2+4a^3}{6x^3(-i+a)^2(i+a)^3\sqrt{b^2x^2+2abx+a^2+1}}$
default	$(ia+1)\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{3(a^2+1)x^3}-\frac{5ab\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{2(a^2+1)x^2}-\frac{3ab\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{(a^2+1)x}+\frac{ab\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\sqrt{b^2x}}{x}\right.}{(a^2+1)^{\frac{3}{2}}}\right.}{2(a^2+1)}\right.}{3(a^2+1)}\right.$

input
```
int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*I*(-3*I*a^3*b^2*x^2+2*a^2*b^4*x^4-3*I*a*b^2*x^2+2*a^3*b^3*x^3+3*I*b^3
*x^3-4*b^4*x^4+3*I*b*x+3*I*a^4*b*x+2*a^5*b*x-10*a*b^3*x^3+6*I*a^2*b*x+2*a^
6-2*a^2*b^2*x^2-15*I*a^2*b^3*x^3+4*a^3*b*x+6*a^4-2*b^2*x^2-9*I*a*b^4*x^4+2
*a*b*x+6*a^2+2)/x^3/(-I+a)^2/(I+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/2*b^3
*(-2*I*a+2*a^2-1)/(a^2+1)^(5/2)/(I+a)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*
(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 690 vs. 2(198) = 396.

Time = 0.09 (sec) , antiderivative size = 690, normalized size of antiderivative = 2.44

$$\int \frac{e^{i\arctan(a+bx)}}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
1/6*((-2*I*a^2 - 9*a + 4*I)*b^3*x^3 - 3*sqrt((4*a^4 - 8*I*a^3 - 8*a^2 + 4*
I*a + 1)*b^6/(a^12 + 2*I*a^11 + 4*a^10 + 10*I*a^9 + 5*a^8 + 20*I*a^7 + 20*
I*a^5 - 5*a^4 + 10*I*a^3 - 4*a^2 + 2*I*a - 1))*(a^5 + I*a^4 + 2*a^3 + 2*I*
a^2 + a + I)*x^3*log(-((2*a^2 - 2*I*a - 1)*b^4*x - sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)*(2*a^2 - 2*I*a - 1)*b^3 + (a^7 + I*a^6 + 3*a^5 + 3*I*a^4 + 3*a^
3 + 3*I*a^2 + a + I)*sqrt((4*a^4 - 8*I*a^3 - 8*a^2 + 4*I*a + 1)*b^6/(a^12
+ 2*I*a^11 + 4*a^10 + 10*I*a^9 + 5*a^8 + 20*I*a^7 + 20*I*a^5 - 5*a^4 + 10*
I*a^3 - 4*a^2 + 2*I*a - 1)))/((2*a^2 - 2*I*a - 1)*b^3)) + 3*sqrt((4*a^4 -
8*I*a^3 - 8*a^2 + 4*I*a + 1)*b^6/(a^12 + 2*I*a^11 + 4*a^10 + 10*I*a^9 + 5*
a^8 + 20*I*a^7 + 20*I*a^5 - 5*a^4 + 10*I*a^3 - 4*a^2 + 2*I*a - 1))*(a^5 +
I*a^4 + 2*a^3 + 2*I*a^2 + a + I)*x^3*log(-((2*a^2 - 2*I*a - 1)*b^4*x - sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*a^2 - 2*I*a - 1)*b^3 - (a^7 + I*a^6 + 3*
a^5 + 3*I*a^4 + 3*a^3 + 3*I*a^2 + a + I)*sqrt((4*a^4 - 8*I*a^3 - 8*a^2 + 4
*I*a + 1)*b^6/(a^12 + 2*I*a^11 + 4*a^10 + 10*I*a^9 + 5*a^8 + 20*I*a^7 + 20
*I*a^5 - 5*a^4 + 10*I*a^3 - 4*a^2 + 2*I*a - 1)))/((2*a^2 - 2*I*a - 1)*b^3)
) + ((-2*I*a^2 - 9*a + 4*I)*b^2*x^2 - 2*I*a^4 + (2*I*a^3 + 3*a^2 + 2*I*a +
 3)*b*x - 4*I*a^2 - 2*I)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((a^5 + I*a^4
+ 2*a^3 + 2*I*a^2 + a + I)*x^3)
```

Sympy [F]

$$\int \frac{e^{i\arctan(a+bx)}}{x^4}\,dx = i\left(\int\left(-\frac{i}{x^4\sqrt{a^2+2abx+b^2x^2+1}}\right)dx + \int\frac{a}{x^4\sqrt{a^2+2abx+b^2x^2+1}}\,dx + \int\frac{b}{x^3\sqrt{a^2+2abx+b^2x^2+1}}\,dx\right)$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2)/x**4,x)
```

output
```
I*(Integral(-I/(x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(
a/(x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(b/(x**3*sqrt(
a**2 + 2*a*b*x + b**2*x**2 + 1)), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 644 vs. 2(198) = 396.

Time = 0.04 (sec) , antiderivative size = 644, normalized size of antiderivative = 2.28

$$\int \frac{e^{i\arctan(a+bx)}}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
5/2*a^3*(I*a + 1)*b^3*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*
abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a
^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(7/2) - 3/2*I*a^2*b^3*arcsinh
(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*
b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs
(x)))/(a^2 + 1)^(5/2) - 3/2*a*(I*a + 1)*b^3*arcsinh(2*a*b*x/(sqrt(-4*a^2*b
^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*
abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(5/2) +
 1/2*I*b^3*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2
*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*
(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3/2) - 5/2*sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1)*a^2*(I*a + 1)*b^2/((a^2 + 1)^3*x) + 3/2*I*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)*a*b^2/((a^2 + 1)^2*x) - 2/3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-I*
a - 1)*b^2/((a^2 + 1)^2*x) + 5/6*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*(I*a
+ 1)*b/((a^2 + 1)^2*x^2) - 1/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b/((a^2
 + 1)*x^2) - 1/3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(I*a + 1)/((a^2 + 1)*x^
3)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 884 vs. 2(198) = 396.

Time = 0.17 (sec) , antiderivative size = 884, normalized size of antiderivative = 3.12

$$\int \frac{e^{i\arctan(a+bx)}}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

output
```
1/2*(2*a^2*b^3 - 2*I*a*b^3 - b^3)*log(abs(2*x*abs(b) - 2*sqrt((b*x + a)^2
+ 1) - 2*sqrt(a^2 + 1))/abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) + 2*sqrt(
a^2 + 1)))/((a^5 + I*a^4 + 2*a^3 + 2*I*a^2 + a + I)*sqrt(a^2 + 1)) + 1/3*(
8*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a^5*b^3 + 24*(I*x*abs(b) - I*sqrt
((b*x + a)^2 + 1))*a^7*b^3 + 24*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a^6
*b^2*abs(b) + 8*I*a^8*b^2*abs(b) + 6*(x*abs(b) - sqrt((b*x + a)^2 + 1))^5*
a^2*b^3 - 24*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a^4*b^3 + 18*(x*abs(b) -
 sqrt((b*x + a)^2 + 1))*a^6*b^3 - 12*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*
a^5*b^2*abs(b) + 12*a^7*b^2*abs(b) - 6*I*(x*abs(b) - sqrt((b*x + a)^2 + 1)
)^5*a*b^3 + 32*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a^3*b^3 + 54*(I*x*ab
s(b) - I*sqrt((b*x + a)^2 + 1))*a^5*b^3 + 60*I*(x*abs(b) - sqrt((b*x + a)^
2 + 1))^2*a^4*b^2*abs(b) + 20*I*a^6*b^2*abs(b) - 3*(x*abs(b) - sqrt((b*x +
 a)^2 + 1))^5*b^3 - 24*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a^2*b^3 + 39*(
x*abs(b) - sqrt((b*x + a)^2 + 1))*a^4*b^3 - 24*(x*abs(b) - sqrt((b*x + a)^
2 + 1))^2*a^3*b^2*abs(b) + 36*a^5*b^2*abs(b) + 24*I*(x*abs(b) - sqrt((b*x
+ a)^2 + 1))^3*a*b^3 + 36*(I*x*abs(b) - I*sqrt((b*x + a)^2 + 1))*a^3*b^3 +
 48*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a^2*b^2*abs(b) + 12*I*a^4*b^2*a
bs(b) + 24*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*b^3 - 12*(x*abs(b) - sqr
t((b*x + a)^2 + 1))^2*a*b^2*abs(b) + 36*a^3*b^2*abs(b) + 6*(I*x*abs(b) - I
*sqrt((b*x + a)^2 + 1))*a*b^3 + 12*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{i\arctan(a+bx)}}{x^4}\,dx = \int \frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{x^4\,\sqrt{{(a+b\,x)}^2+1}}\,dx$$

input
```
int((a*1i + b*x*1i + 1)/(x^4*((a + b*x)^2 + 1)^(1/2)),x)
```

output
```
int((a*1i + b*x*1i + 1)/(x^4*((a + b*x)^2 + 1)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 852, normalized size of antiderivative = 3.01

$$\int \frac{e^{i\arctan(a+bx)}}{x^4}\,dx$$
$$= \frac{-2\sqrt{b^2x^2+2abx+a^2+1}\,a^2i - a\,b^3x^3 - b^3i\,x^3 + 5\sqrt{b^2x^2+2abx+a^2+1}\,a^6bx - 11\sqrt{b^2x^2+2abx+a^2}}{}$$

input
```
int((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2)/x^4,x)
```

output
```
( - 12*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x
)/sqrt(a**2 + 1))*a**4*b**3*i*x**3 - 24*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2
*a*b*x + b**2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*a**3*b**3*x**3 + 18*sqr
t(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/sqrt(a**
2 + 1))*a**2*b**3*i*x**3 + 6*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*a*b**3*x**3 - 2*sqrt(a**2 + 2*a*b*
x + b**2*x**2 + 1)*a**8*i + 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**7*b*
i*x - 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**7 - 2*sqrt(a**2 + 2*a*b*x
+ b**2*x**2 + 1)*a**6*b**2*i*x**2 + 5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
*a**6*b*x - 6*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**6*i - 11*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1)*a**5*b**2*x**2 + sqrt(a**2 + 2*a*b*x + b**2*x**2
 + 1)*a**5*b*i*x - 6*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**5 + 11*sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1)*a**4*b**2*i*x**2 + 10*sqrt(a**2 + 2*a*b*x +
 b**2*x**2 + 1)*a**4*b*x - 6*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**4*i -
 7*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**3*b**2*x**2 - 4*sqrt(a**2 + 2*a
*b*x + b**2*x**2 + 1)*a**3*b*i*x - 6*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*
a**3 + 13*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**2*b**2*i*x**2 + 5*sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1)*a**2*b*x - 2*sqrt(a**2 + 2*a*b*x + b**2*x**
2 + 1)*a**2*i + 4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a*b**2*x**2 - 3*sqr
t(a**2 + 2*a*b*x + b**2*x**2 + 1)*a*b*i*x - 2*sqrt(a**2 + 2*a*b*x + b**...
```

3.185 $\int e^{2i\arctan(a+bx)}x^4\,dx$

Optimal result

Integrand size = 16, antiderivative size = 92

$$\int e^{2i\arctan(a+bx)}x^4\,dx = -\frac{2(1-ia)^3x}{b^4}+\frac{i(i+a)^2x^2}{b^3}+\frac{2(1-ia)x^3}{3b^2}+\frac{ix^4}{2b}-\frac{x^5}{5}+\frac{2i(i+a)^4\log(i+a+bx)}{b^5}$$

output
```
-2*(1-I*a)^3*x/b^4+I*(I+a)^2*x^2/b^3+2/3*(1-I*a)*x^3/b^2+1/2*I*x^4/b-1/5*x
^5+2*I*(I+a)^4*ln(I+a+b*x)/b^5
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int e^{2i\arctan(a+bx)}x^4\,dx = -\frac{2(1-ia)^3x}{b^4}+\frac{i(i+a)^2x^2}{b^3}+\frac{2(1-ia)x^3}{3b^2}+\frac{ix^4}{2b}-\frac{x^5}{5}+\frac{2i(i+a)^4\log(i+a+bx)}{b^5}$$

input
```
Integrate[E^((2*I)*ArcTan[a + b*x])*x^4,x]
```

output
```
(-2*(1 - I*a)^3*x)/b^4 + (I*(I + a)^2*x^2)/b^3 + (2*(1 - I*a)*x^3)/(3*b^2)
 + ((I/2)*x^4)/b - x^5/5 + ((2*I)*(I + a)^4*Log[I + a + b*x])/b^5
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5618, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{2i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \; 5618$$

$$\int \frac{x^4(ia+ibx+1)}{-ia-ibx+1}dx$$

$$\Big\downarrow \; 86$$

$$\int \left(\frac{2i(a+i)^4}{b^4(a+bx+i)}+\frac{2(-1+ia)^3}{b^4}+\frac{2i(a+i)^2x}{b^3}+\frac{2(1-ia)x^2}{b^2}+\frac{2ix^3}{b}-x^4\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2i(a+i)^4\log(a+bx+i)}{b^5}-\frac{2(1-ia)^3x}{b^4}+\frac{i(a+i)^2x^2}{b^3}+\frac{2(1-ia)x^3}{3b^2}+\frac{ix^4}{2b}-\frac{x^5}{5}$$

input
```
Int[E^((2*I)*ArcTan[a + b*x])*x^4,x]
```

output

```
(-2*(1 - I*a)^3*x)/b^4 + (I*(I + a)^2*x^2)/b^3 + (2*(1 - I*a)*x^3)/(3*b^2)
 + ((I/2)*x^4)/b - x^5/5 + ((2*I)*(I + a)^4*Log[I + a + b*x])/b^5
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(78) = 156$.

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.74

method	result
parallelrisch	$\frac{-6x^5b^5-30ib^2x^2+60i\ln(bx+a+i)a^4+180iabx-240\ln(bx+a+i)a^3+30ix^2a^2b^2-60ix\,a^3b+20b^3x^3-20ix^3a\,b^3-60a\,b^2x^2+24}{30b^5}$
default	$-\frac{i\left(-\frac{1}{5}ib^4x^5-\frac{1}{2}b^3x^4+\frac{2}{3}ib^2x^3+\frac{2}{3}a\,b^2x^3-2iab\,x^2-a^2b\,x^2+6ix\,a^2+2a^3x+b\,x^2-2ix-6ax\right)}{b^4}+\frac{\left(2ia^4b-8a^3b-12ia^2b+8ab+2ib\right)}{2b^2}$
risch	$-\frac{x^5}{5}-\frac{6i\ln\left(b^2x^2+2abx+a^2+1\right)a^2}{b^5}+\frac{2x^3}{3b^2}+\frac{8i\arctan(bx+a)a^3}{b^5}-\frac{2a\,x^2}{b^3}-\frac{8i\arctan(bx+a)a}{b^5}+\frac{6x\,a^2}{b^4}+\frac{ia^2x^2}{b^3}-$

input
```
int((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/30*(-6*x^5*b^5-30*I*x^2*b^2+60*I*ln(I+a+b*x)*a^4+180*I*x*a*b-240*ln(I+a+
b*x)*a^3+30*I*x^2*a^2*b^2-60*I*x*a^3*b+20*b^3*x^3-20*I*x^3*a*b^3-60*a*b^2*
x^2+240*ln(I+a+b*x)*a+60*I*ln(I+a+b*x)+15*I*x^4*b^4+180*a^2*b*x-360*I*ln(I
+a+b*x)*a^2-60*b*x)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.14

$$\int e^{2i\arctan(a+bx)}x^4\,dx = -\frac{6\,b^5x^5 - 15i\,b^4x^4 + 20\,(i\,a-1)b^3x^3 + 30\,(-i\,a^2+2\,a+i)b^2x^2 + 60\,(i\,a^3-3\,a^2-3i\,a+1)bx + 60\,(-i}{30\,b^5}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^4,x, algorithm="fricas")
```

output
```
-1/30*(6*b^5*x^5 - 15*I*b^4*x^4 + 20*(I*a - 1)*b^3*x^3 + 30*(-I*a^2 + 2*a
+ I)*b^2*x^2 + 60*(I*a^3 - 3*a^2 - 3*I*a + 1)*b*x + 60*(-I*a^4 + 4*a^3 + 6
*I*a^2 - 4*a - I)*log((b*x + a + I)/b))/b^5
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.20

$$\int e^{2i\arctan(a+bx)}x^4\,dx = -\frac{x^5}{5} - x^3\cdot\left(\frac{2ia}{3b^2} - \frac{2}{3b^2}\right) - x^2\left(-\frac{ia^2}{b^3} + \frac{2a}{b^3} + \frac{i}{b^3}\right) - x\left(\frac{2ia^3}{b^4} - \frac{6a^2}{b^4} - \frac{6ia}{b^4} + \frac{2}{b^4}\right) + \frac{ix^4}{2b} + \frac{2i(a+i)^4\log(a+bx+i)}{b^5}$$

input
```
integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)*x**4,x)
```

output
```
-x**5/5 - x**3*(2*I*a/(3*b**2) - 2/(3*b**2)) - x**2*(-I*a**2/b**3 + 2*a/b*
*3 + I/b**3) - x*(2*I*a**3/b**4 - 6*a**2/b**4 - 6*I*a/b**4 + 2/b**4) + I*x
**4/(2*b) + 2*I*(a + I)**4*log(a + b*x + I)/b**5
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(70) = 140$.

Time = 0.10 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.62

$$\int e^{2i\arctan(a+bx)}x^4\,dx = -\frac{6\,b^4x^5 - 15i\,b^3x^4 + 20\,(i\,a-1)b^2x^3 + 30\,(-i\,a^2 + 2\,a + i)bx^2 + 60\,(i\,a^3 - 3\,a^2 - 3i\,a + 1)x}{30\,b^4} + \frac{2\,(a^4 + 4i\,a^3 - 6\,a^2 - 4i\,a + 1)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^5} + \frac{(i\,a^4 - 4\,a^3 - 6i\,a^2 + 4\,a + i)\log\left(b^2x^2 + 2\,abx + a^2 + 1\right)}{b^5}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^4,x, algorithm="maxima")
```

output
```
-1/30*(6*b^4*x^5 - 15*I*b^3*x^4 + 20*(I*a - 1)*b^2*x^3 + 30*(-I*a^2 + 2*a
+ I)*b*x^2 + 60*(I*a^3 - 3*a^2 - 3*I*a + 1)*x)/b^4 + 2*(a^4 + 4*I*a^3 - 6*
a^2 - 4*I*a + 1)*arctan((b^2*x + a*b)/b)/b^5 + (I*a^4 - 4*a^3 - 6*I*a^2 +
4*a + I)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.34

$$\int e^{2i\arctan(a+bx)}x^4\,dx = -\frac{2\,(-i\,a^4 + 4\,a^3 + 6i\,a^2 - 4\,a - i)\log\left(bx + a + i\right)}{b^5} - \frac{6\,b^5x^5 - 15i\,b^4x^4 + 20i\,ab^3x^3 - 30i\,a^2b^2x^2 - 20\,b^3x^3 + 60i\,a^3bx + 60\,ab^2x^2 - 180\,a^2bx + 30i\,b^2x^2 - 18}{30\,b^5}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^4,x, algorithm="giac")
```

output
```
-2*(-I*a^4 + 4*a^3 + 6*I*a^2 - 4*a - I)*log(b*x + a + I)/b^5 - 1/30*(6*b^5
*x^5 - 15*I*b^4*x^4 + 20*I*a*b^3*x^3 - 30*I*a^2*b^2*x^2 - 20*b^3*x^3 + 60*
I*a^3*b*x + 60*a*b^2*x^2 - 180*a^2*b*x + 30*I*b^2*x^2 - 180*I*a*b*x + 60*b
*x)/b^5
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.18

$$\int e^{2i\arctan(a+bx)}x^4\,dx = \ln\left(x+\frac{a+1\mathrm{i}}{b}\right)\left(\frac{8\,a-8\,a^3}{b^5}+\frac{(2\,a^4-12\,a^2+2)\ 1\mathrm{i}}{b^5}\right) - x^4\left(\frac{(-1+a\,1\mathrm{i})\ 1\mathrm{i}}{4\,b}-\frac{(1+a\,1\mathrm{i})\ 1\mathrm{i}}{4\,b}\right)-\frac{x^5}{5} + \frac{x^2\,(-1+a\,1\mathrm{i})^2\left(\frac{(-1+a\,1\mathrm{i})\,1\mathrm{i}}{b}-\frac{(1+a\,1\mathrm{i})\,1\mathrm{i}}{b}\right)}{2\,b^2} - \frac{x^3\,(-1+a\,1\mathrm{i})\left(\frac{(-1+a\,1\mathrm{i})\,1\mathrm{i}}{b}-\frac{(1+a\,1\mathrm{i})\,1\mathrm{i}}{b}\right)\ 1\mathrm{i}}{3\,b} + \frac{x\,(-1+a\,1\mathrm{i})^3\left(\frac{(-1+a\,1\mathrm{i})\,1\mathrm{i}}{b}-\frac{(1+a\,1\mathrm{i})\,1\mathrm{i}}{b}\right)\ 1\mathrm{i}}{b^3}$$

input
```
int((x^4*(a*1i + b*x*1i + 1)^2)/((a + b*x)^2 + 1),x)
```

output
```
log(x + (a + 1i)/b)*((8*a - 8*a^3)/b^5 + ((2*a^4 - 12*a^2 + 2)*1i)/b^5) -
x^4*(((a*1i - 1)*1i)/(4*b) - ((a*1i + 1)*1i)/(4*b)) - x^5/5 + (x^2*(a*1i -
 1)^2*(((a*1i - 1)*1i)/b - ((a*1i + 1)*1i)/b))/(2*b^2) - (x^3*(a*1i - 1)*(
((a*1i - 1)*1i)/b - ((a*1i + 1)*1i)/b)*1i)/(3*b) + (x*(a*1i - 1)^3*(((a*1i
 - 1)*1i)/b - ((a*1i + 1)*1i)/b)*1i)/b^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.84

$$\int e^{2i\arctan(a+bx)}x^4\,dx = \frac{60atan(bx+a)\,a^4+240atan(bx+a)\,a^3i-360atan(bx+a)\,a^2-240atan(bx+a)\,ai+60atan(bx+a)}{}$$

input
```
int((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^4,x)
```

output
```
(60*atan(a + b*x)*a**4 + 240*atan(a + b*x)*a**3*i - 360*atan(a + b*x)*a**2
 - 240*atan(a + b*x)*a*i + 60*atan(a + b*x) + 30*log(a**2 + 2*a*b*x + b**2
*x**2 + 1)*a**4*i - 120*log(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**3 - 180*log
(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**2*i + 120*log(a**2 + 2*a*b*x + b**2*x*
*2 + 1)*a + 30*log(a**2 + 2*a*b*x + b**2*x**2 + 1)*i - 60*a**3*b*i*x + 30*
a**2*b**2*i*x**2 + 180*a**2*b*x - 20*a*b**3*i*x**3 - 60*a*b**2*x**2 + 180*
a*b*i*x - 6*b**5*x**5 + 15*b**4*i*x**4 + 20*b**3*x**3 - 30*b**2*i*x**2 - 6
0*b*x)/(30*b**5)
```

3.186 $\int e^{2i\arctan(a+bx)}x^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 72

$$\int e^{2i\arctan(a+bx)}x^3\,dx=\frac{2i(i+a)^2x}{b^3}+\frac{(1-ia)x^2}{b^2}+\frac{2ix^3}{3b}-\frac{x^4}{4}-\frac{2(1-ia)^3\log(i+a+bx)}{b^4}$$

output
```
2*I*(I+a)^2*x/b^3+(1-I*a)*x^2/b^2+2/3*I*x^3/b-1/4*x^4-2*(1-I*a)^3*ln(I+a+b
*x)/b^4
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int e^{2i\arctan(a+bx)}x^3\,dx=\frac{2i(i+a)^2x}{b^3}+\frac{(1-ia)x^2}{b^2}+\frac{2ix^3}{3b}-\frac{x^4}{4}-\frac{2(1-ia)^3\log(i+a+bx)}{b^4}$$

input
```
Integrate[E^((2*I)*ArcTan[a + b*x])*x^3,x]
```

output
```
((2*I)*(I + a)^2*x)/b^3 + ((1 - I*a)*x^2)/b^2 + (((2*I)/3)*x^3)/b - x^4/4
- (2*(1 - I*a)^3*Log[I + a + b*x])/b^4
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5618, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{2i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{x^3(ia+ibx+1)}{-ia-ibx+1}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{2(-1+ia)^3}{b^3(a+bx+i)}+\frac{2i(a+i)^2}{b^3}+\frac{2(1-ia)x}{b^2}+\frac{2ix^2}{b}-x^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(1-ia)^3\log(a+bx+i)}{b^4}+\frac{2i(a+i)^2x}{b^3}+\frac{(1-ia)x^2}{b^2}+\frac{2ix^3}{3b}-\frac{x^4}{4}$$

input
```
Int[E^((2*I)*ArcTan[a + b*x])*x^3,x]
```

output
```
((2*I)*(I + a)^2*x)/b^3 + ((1 - I*a)*x^2)/b^2 + (((2*I)/3)*x^3)/b - x^4/4
- (2*(1 - I*a)^3*Log[I + a + b*x])/b^4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.51

method	result
parallelrisch	$-\frac{3b^4x^4-8ib^3x^3+12ia\,b^2x^2-72\ln(bx+a+i)a^2+24i\ln(bx+a+i)a^3-24ia^2bx-12b^2x^2+24\ln(bx+a+i)-72i\ln(bx+a+i)a+24}{12b^4}$
default	$\frac{i\left(\frac{1}{4}ib^3x^4+\frac{2}{3}b^2x^3-ib\,x^2-ab\,x^2+4iax+2a^2x-2x\right)}{b^3}+\frac{\frac{\left(-2ia^3b+6a^2b+6iab-2b\right)\ln\left(b^2x^2+2abx+a^2+1\right)}{2b^2}+\frac{\left(-2ia^4+4a^3+2i+4a-\left(\right.\right.}{}}{b^3}$
risch	$-\frac{x^4}{4}+\frac{2ix^3}{3b}+\frac{x^2}{b^2}-\frac{ia\,x^2}{b^2}-\frac{4ax}{b^3}+\frac{2ia^2x}{b^3}-\frac{2ix}{b^3}+\frac{3\ln\left(b^2x^2+2abx+a^2+1\right)a^2}{b^4}-\frac{i\ln\left(b^2x^2+2abx+a^2+1\right)a^3}{b^4}-$

input
```
int((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(3*b^4*x^4-8*I*x^3*b^3+12*I*a*b^2*x^2-72*ln(I+a+b*x)*a^2+24*I*ln(I+a
+b*x)*a^3-24*I*x*a^2*b-12*b^2*x^2+24*ln(I+a+b*x)-72*I*ln(I+a+b*x)*a+24*I*x
*b+48*a*b*x)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

$$\int e^{2i\arctan(a+bx)}x^3\,dx = -\frac{3\,b^4x^4-8i\,b^3x^3+12\,(i\,a-1)b^2x^2+24\,(-i\,a^2+2\,a+i)bx+24\,(i\,a^3-3\,a^2-3i\,a+1)\log\left(\frac{bx+a+i}{b}\right)}{12\,b^4}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^3,x, algorithm="fricas")
```

output
```
-1/12*(3*b^4*x^4 - 8*I*b^3*x^3 + 12*(I*a - 1)*b^2*x^2 + 24*(-I*a^2 + 2*a +
 I)*b*x + 24*(I*a^3 - 3*a^2 - 3*I*a + 1)*log((b*x + a + I)/b))/b^4
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int e^{2i\arctan(a+bx)}x^3\,dx = -\frac{x^4}{4} - x^2\left(\frac{ia}{b^2} - \frac{1}{b^2}\right) - x\left(-\frac{2ia^2}{b^3} + \frac{4a}{b^3} + \frac{2i}{b^3}\right) + \frac{2ix^3}{3b} - \frac{2i(a+i)^3\log(a+bx+i)}{b^4}$$

input
```
integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)*x**3,x)
```

output
```
-x**4/4 - x**2*(I*a/b**2 - 1/b**2) - x*(-2*I*a**2/b**3 + 4*a/b**3 + 2*I/b*
*3) + 2*I*x**3/(3*b) - 2*I*(a + I)**3*log(a + b*x + I)/b**4
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(56) = 112.

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.61

$$\int e^{2i\arctan(a+bx)}x^3\,dx = -\frac{3\,b^3x^4 - 8i\,b^2x^3 + 12\,(i\,a-1)bx^2 + 24\,(-i\,a^2 + 2\,a + i)x}{12\,b^3} - \frac{2\,(a^3 + 3i\,a^2 - 3\,a - i)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^4} + \frac{(-i\,a^3 + 3\,a^2 + 3i\,a - 1)\log(b^2x^2 + 2\,abx + a^2 + 1)}{b^4}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^3,x, algorithm="maxima")
```

output
```
-1/12*(3*b^3*x^4 - 8*I*b^2*x^3 + 12*(I*a - 1)*b*x^2 + 24*(-I*a^2 + 2*a + I
)*x)/b^3 - 2*(a^3 + 3*I*a^2 - 3*a - I)*arctan((b^2*x + a*b)/b)/b^4 + (-I*a
^3 + 3*a^2 + 3*I*a - 1)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.15

$$\int e^{2i\arctan(a+bx)}x^3\,dx = -\frac{2\,(i\,a^3-3\,a^2-3i\,a+1)\log{(bx+a+i)}}{b^4} - \frac{3\,b^4x^4-8i\,b^3x^3+12i\,ab^2x^2-24i\,a^2bx-12\,b^2x^2+48\,abx+24i\,bx}{12\,b^4}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^3,x, algorithm="giac")
```

output
```
-2*(I*a^3 - 3*a^2 - 3*I*a + 1)*log(b*x + a + I)/b^4 - 1/12*(3*b^4*x^4 - 8*
I*b^3*x^3 + 12*I*a*b^2*x^2 - 24*I*a^2*b*x - 12*b^2*x^2 + 48*a*b*x + 24*I*b
*x)/b^4
```

Mupad [B] (verification not implemented)

Time = 23.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.12

$$\int e^{2i\arctan(a+bx)}x^3\,dx = -x^3\left(\frac{(-1+a\,1\mathrm{i})\,1\mathrm{i}}{3\,b}-\frac{(1+a\,1\mathrm{i})\,1\mathrm{i}}{3\,b}\right)-\frac{x^4}{4} + \ln\left(x+\frac{a+1\mathrm{i}}{b}\right)\left(\frac{6\,a^2-2}{b^4}+\frac{(6\,a-2\,a^3)\,1\mathrm{i}}{b^4}\right) - \frac{x^2\,(-1+a\,1\mathrm{i})\,\left(\frac{(-1+a\,1\mathrm{i})\,1\mathrm{i}}{b}-\frac{(1+a\,1\mathrm{i})\,1\mathrm{i}}{b}\right)\,1\mathrm{i}}{2\,b} + \frac{x\,(-1+a\,1\mathrm{i})^2\,\left(\frac{(-1+a\,1\mathrm{i})\,1\mathrm{i}}{b}-\frac{(1+a\,1\mathrm{i})\,1\mathrm{i}}{b}\right)}{b^2}$$

input
```
int((x^3*(a*1i + b*x*1i + 1)^2)/((a + b*x)^2 + 1),x)
```

output
```
log(x + (a + 1i)/b)*(((6*a - 2*a^3)*1i)/b^4 + (6*a^2 - 2)/b^4) - x^4/4 - x
^3*(((a*1i - 1)*1i)/(3*b) - ((a*1i + 1)*1i)/(3*b)) - (x^2*(a*1i - 1)*(((a*
1i - 1)*1i)/b - ((a*1i + 1)*1i)/b)*1i)/(2*b) + (x*(a*1i - 1)^2*(((a*1i - 1
)*1i)/b - ((a*1i + 1)*1i)/b))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.62

$$\int e^{2i\arctan(a+bx)}x^3\,dx$$
$$= \frac{-24atan(bx+a)\,a^3-72atan(bx+a)\,a^2i+72atan(bx+a)\,a+24atan(bx+a)\,i-12\log(b^2x^2+2abx+\cdots}{}$$

input
```
int((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^3,x)
```

output
```
( - 24*atan(a + b*x)*a**3 - 72*atan(a + b*x)*a**2*i + 72*atan(a + b*x)*a +
 24*atan(a + b*x)*i - 12*log(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**3*i + 36*l
og(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**2 + 36*log(a**2 + 2*a*b*x + b**2*x**
2 + 1)*a*i - 12*log(a**2 + 2*a*b*x + b**2*x**2 + 1) + 24*a**2*b*i*x - 12*a
*b**2*i*x**2 - 48*a*b*x - 3*b**4*x**4 + 8*b**3*i*x**3 + 12*b**2*x**2 - 24*
b*i*x)/(12*b**4)
```

3.187 $\int e^{2i\arctan(a+bx)}x^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 54

$$\int e^{2i\arctan(a+bx)}x^2\,dx=\frac{2(1-ia)x}{b^2}+\frac{ix^2}{b}-\frac{x^3}{3}+\frac{2i(i+a)^2\log(i+a+bx)}{b^3}$$

output
```
2*(1-I*a)*x/b^2+I*x^2/b-1/3*x^3+2*I*(I+a)^2*ln(I+a+b*x)/b^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int e^{2i\arctan(a+bx)}x^2\,dx=\frac{2(1-ia)x}{b^2}+\frac{ix^2}{b}-\frac{x^3}{3}+\frac{2i(i+a)^2\log(i+a+bx)}{b^3}$$

input
```
Integrate[E^((2*I)*ArcTan[a + b*x])*x^2,x]
```

output
```
(2*(1 - I*a)*x)/b^2 + (I*x^2)/b - x^3/3 + ((2*I)*(I + a)^2*Log[I + a + b*x
])/b^3
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5618, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{2i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{x^2(ia+ibx+1)}{-ia-ibx+1}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{2i(a+i)^2}{b^2(a+bx+i)}-\frac{2i(a+i)}{b^2}+\frac{2ix}{b}-x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2i(a+i)^2\log(a+bx+i)}{b^3}+\frac{2(1-ia)x}{b^2}+\frac{ix^2}{b}-\frac{x^3}{3}$$

input

```
Int[E^((2*I)*ArcTan[a + b*x])*x^2,x]
```

output

```
(2*(1 - I*a)*x)/b^2 + (I*x^2)/b - x^3/3 + ((2*I)*(I + a)^2*Log[I + a + b*x
])/b^3
```

Defintions of rubi rules used

rule 86

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

method	result
parallelrisch	$\frac{-b^{3} x^{3}+3 i b^{2} x^{2}-12 \ln \left(b x+a+i\right) a+6 i \ln \left(b x+a+i\right) a^{2}-6 i a b x-6 i \ln \left(b x+a+i\right)+6 b x}{3 b^{3}}$
default	$\frac{i \left(\frac{1}{3} i b^{2} x^{3}+b \,x^{2}-2 i x-2 a x\right)}{b^{2}}+\frac{\left(2 i a^{2} b-4 a b-2 i b\right) \ln \left(b^{2} x^{2}+2 a b x+a^{2}+1\right)}{2 b^{2}}+\frac{\left(2 i a^{3}+2 i a-2 a^{2}-2-\frac{\left(2 i a^{2} b-4 a b-2 i b\right) a}{b}\right) \arctan \left(\frac{2 b^{2} x+}{2 b}\right.}{b^{2}}$
risch	$-\frac{x^{3}}{3}+\frac{i x^{2}}{b}+\frac{2 x}{b^{2}}-\frac{2 i a x}{b^{2}}-\frac{2 \ln \left(b^{2} x^{2}+2 a b x+a^{2}+1\right) a}{b^{3}}+\frac{i \ln \left(b^{2} x^{2}+2 a b x+a^{2}+1\right) a^{2}}{b^{3}}-\frac{i \ln \left(b^{2} x^{2}+2 a b x+a^{2}+1\right)}{b^{3}}+$

input
```
int((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-b^3*x^3+3*I*b^2*x^2-12*ln(I+a+b*x)*a+6*I*ln(I+a+b*x)*a^2-6*I*x*a*b-6
*I*ln(I+a+b*x)+6*b*x)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int e^{2 i \arctan (a+b x)} x^2 \, dx=-\frac{b^{3} x^{3}-3 i \, b^{2} x^{2}+6 \, {\left(i \, a-1\right)} b x+6 \, {\left(-i \, a^{2}+2 \, a+i\right)} \log \left(\frac{b x+a+i}{b}\right)}{3 \, b^{3}}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^2,x, algorithm="fricas")
```

output
```
-1/3*(b^3*x^3 - 3*I*b^2*x^2 + 6*(I*a - 1)*b*x + 6*(-I*a^2 + 2*a + I)*log((
b*x + a + I)/b))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int e^{2i\arctan(a+bx)}x^2\,dx = -\frac{x^3}{3} - x\left(\frac{2ia}{b^2} - \frac{2}{b^2}\right) + \frac{ix^2}{b} + \frac{2i(a+i)^2\log(a+bx+i)}{b^3}$$

input
```
integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)*x**2,x)
```

output
```
-x**3/3 - x*(2*I*a/b**2 - 2/b**2) + I*x**2/b + 2*I*(a + I)**2*log(a + b*x
+ I)/b**3
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(42) = 84.

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.61

$$\int e^{2i\arctan(a+bx)}x^2\,dx = -\frac{b^2x^3 - 3i\,bx^2 + 6\,(i\,a-1)x}{3\,b^2} + \frac{2\,(a^2+2i\,a-1)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^3} + \frac{(i\,a^2-2\,a-i)\log(b^2x^2+2\,abx+a^2+1)}{b^3}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^2,x, algorithm="maxima")
```

output
```
-1/3*(b^2*x^3 - 3*I*b*x^2 + 6*(I*a - 1)*x)/b^2 + 2*(a^2 + 2*I*a - 1)*arcta
n((b^2*x + a*b)/b)/b^3 + (I*a^2 - 2*a - I)*log(b^2*x^2 + 2*a*b*x + a^2 + 1
)/b^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int e^{2i\arctan(a+bx)}x^2\,dx = -\frac{2\left(-i\,a^2+2\,a+i\right)\log\left(bx+a+i\right)}{b^3} - \frac{b^3x^3-3i\,b^2x^2+6i\,abx-6\,bx}{3\,b^3}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^2,x, algorithm="giac")
```

output
```
-2*(-I*a^2 + 2*a + I)*log(b*x + a + I)/b^3 - 1/3*(b^3*x^3 - 3*I*b^2*x^2 +
6*I*a*b*x - 6*b*x)/b^3
```

Mupad [B] (verification not implemented)

Time = 23.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.98

$$\int e^{2i\arctan(a+bx)}x^2\,dx = -\ln\left(x+\frac{a+1\mathrm{i}}{b}\right)\left(\frac{4\,a}{b^3}-\frac{\left(2\,a^2-2\right)\,1\mathrm{i}}{b^3}\right) - x^2\left(\frac{\left(-1+a\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,b}-\frac{\left(1+a\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,b}\right)-\frac{x^3}{3} - \frac{x\left(-1+a\,1\mathrm{i}\right)\left(\frac{\left(-1+a\,1\mathrm{i}\right)\,1\mathrm{i}}{b}-\frac{\left(1+a\,1\mathrm{i}\right)\,1\mathrm{i}}{b}\right)\,1\mathrm{i}}{b}$$

input
```
int((x^2*(a*1i + b*x*1i + 1)^2)/((a + b*x)^2 + 1),x)
```

output
```
- log(x + (a + 1i)/b)*((4*a)/b^3 - ((2*a^2 - 2)*1i)/b^3) - x^2*(((a*1i - 1
)*1i)/(2*b) - ((a*1i + 1)*1i)/(2*b)) - x^3/3 - (x*(a*1i - 1)*(((a*1i - 1)*
1i)/b - ((a*1i + 1)*1i)/b)*1i)/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.37

$$\int e^{2i\arctan(a+bx)}x^2\,dx$$
$$= \frac{6atan(bx+a)\,a^2+12atan(bx+a)\,ai-6atan(bx+a)+3\log(b^2x^2+2abx+a^2+1)\,a^2i-6\log(b^2x^2+}{3b^3}$$

input
```
int((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x^2,x)
```

output
```
(6*atan(a + b*x)*a**2 + 12*atan(a + b*x)*a*i - 6*atan(a + b*x) + 3*log(a**
2 + 2*a*b*x + b**2*x**2 + 1)*a**2*i - 6*log(a**2 + 2*a*b*x + b**2*x**2 + 1
)*a - 3*log(a**2 + 2*a*b*x + b**2*x**2 + 1)*i - 6*a*b*i*x - b**3*x**3 + 3*
b**2*i*x**2 + 6*b*x)/(3*b**3)
```

3.188 $\int e^{2i\arctan(a+bx)}x\,dx$

Optimal result

Integrand size = 14, antiderivative size = 37

$$\int e^{2i\arctan(a+bx)}x\,dx=\frac{2ix}{b}-\frac{x^2}{2}+\frac{2(1-ia)\log(i+a+bx)}{b^2}$$

output
```
2*I*x/b-1/2*x^2+2*(1-I*a)*ln(I+a+b*x)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{2i\arctan(a+bx)}x\,dx=\frac{2ix}{b}-\frac{x^2}{2}+\frac{2(1-ia)\log(i+a+bx)}{b^2}$$

input
```
Integrate[E^((2*I)*ArcTan[a + b*x])*x,x]
```

output
```
((2*I)*x)/b - x^2/2 + (2*(1 - I*a)*Log[I + a + b*x])/b^2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5618, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{2i\arctan(a+bx)}dx$$

$$\Big\downarrow \text{ 5618}$$

$$\int \frac{x(ia+ibx+1)}{-ia-ibx+1}dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(\frac{2(1-ia)}{b(a+bx+i)}+\frac{2i}{b}-x\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2(1-ia)\log(a+bx+i)}{b^2}+\frac{2ix}{b}-\frac{x^2}{2}$$

input
```
Int[E^((2*I)*ArcTan[a + b*x])*x,x]
```

output
```
((2*I)*x)/b - x^2/2 + (2*(1 - I*a)*Log[I + a + b*x])/b^2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

method	result	size
parallelrisch	$-\frac{b^{2} x^{2}-4 \ln \left(b x+a+i\right)+4 i \ln \left(b x+a+i\right) a-4 i b x}{2 b^{2}}$	41
risch	$-\frac{x^{2}}{2}+\frac{2 i x}{b}+\frac{\ln \left(b^{2} x^{2}+2 a b x+a^{2}+1\right)}{b^{2}}-\frac{2 i \arctan \left(b x+a\right)}{b^{2}}-\frac{i a \ln \left(b^{2} x^{2}+2 a b x+a^{2}+1\right)}{b^{2}}-\frac{2 a \arctan \left(b x+a\right)}{b^{2}}$	85
default	$\frac{-\frac{1}{2} b \,x^{2}+2 i x}{b}+\frac{\frac{\left(-2 i a b+2 b\right) \ln \left(b^{2} x^{2}+2 a b x+a^{2}+1\right)}{2 b^{2}}+\frac{\left(-2 i a^{2}-2 i-\frac{\left(-2 i a b+2 b\right) a}{b}\right) \arctan \left(\frac{2 b^{2} x+2 a b}{2 b}\right)}{b}}{b}$	99

input
```
int((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b^2*x^2-4*ln(I+a+b*x)+4*I*ln(I+a+b*x)*a-4*I*b*x)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int e^{2i \arctan(a+bx)} x \, dx = -\frac{b^2 x^2 - 4i \, b x + 4 \, (i \, a - 1) \log \left(\frac{b x + a + i}{b}\right)}{2 \, b^2}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x,x, algorithm="fricas")
```

output
```
-1/2*(b^2*x^2 - 4*I*b*x + 4*(I*a - 1)*log((b*x + a + I)/b))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int e^{2i\arctan(a+bx)}x\,dx = -\frac{x^2}{2} + \frac{2ix}{b} - \frac{2i(a+i)\log(a+bx+i)}{b^2}$$

input
```
integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)*x,x)
```

output
```
-x**2/2 + 2*I*x/b - 2*I*(a + I)*log(a + b*x + I)/b**2
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(29) = 58.

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.73

$$\int e^{2i\arctan(a+bx)}x\,dx = -\frac{bx^2-4i\,x}{2\,b} - \frac{2\,(a+i)\arctan\left(\frac{b^2x+ab}{b}\right)}{b^2} + \frac{(-i\,a+1)\log\left(b^2x^2+2\,abx+a^2+1\right)}{b^2}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x,x, algorithm="maxima")
```

output
```
-1/2*(b*x^2 - 4*I*x)/b - 2*(a + I)*arctan((b^2*x + a*b)/b)/b^2 + (-I*a + 1
)*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int e^{2i\arctan(a+bx)}x\,dx = -\frac{2\,(i\,a-1)\log\left(bx+a+i\right)}{b^2} - \frac{b^2x^2-4i\,bx}{2\,b^2}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x,x, algorithm="giac")
```

output
```
-2*(I*a - 1)*log(b*x + a + I)/b^2 - 1/2*(b^2*x^2 - 4*I*b*x)/b^2
```

Mupad [B] (verification not implemented)

Time = 23.44 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int e^{2i\arctan(a+bx)}x\,dx = -\ln\left(x+\frac{a+1\mathrm{i}}{b}\right)\left(-\frac{2}{b^2}+\frac{a\,2\mathrm{i}}{b^2}\right) - x\left(\frac{(-1+a\,1\mathrm{i})\,1\mathrm{i}}{b}-\frac{(1+a\,1\mathrm{i})\,1\mathrm{i}}{b}\right)-\frac{x^2}{2}$$

input
```
int((x*(a*1i + b*x*1i + 1)^2)/((a + b*x)^2 + 1),x)
```

output
```
- log(x + (a + 1i)/b)*((a*2i)/b^2 - 2/b^2) - x*(((a*1i - 1)*1i)/b - ((a*1i
 + 1)*1i)/b) - x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.14

$$\int e^{2i\arctan(a+bx)}x\,dx = \frac{-4atan(bx+a)\,a-4atan(bx+a)\,i-2\log(b^2x^2+2abx+a^2+1)\,ai+2\log(b^2x^2+2abx+a^2+1)-b}{2b^2}$$

input
```
int((1+I*(b*x+a))^2/(1+(b*x+a)^2)*x,x)
```

output
```
( - 4*atan(a + b*x)*a - 4*atan(a + b*x)*i - 2*log(a**2 + 2*a*b*x + b**2*x*
*2 + 1)*a*i + 2*log(a**2 + 2*a*b*x + b**2*x**2 + 1) - b**2*x**2 + 4*b*i*x)
/(2*b**2)
```

3.189 $\int e^{2i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 20

$$\int e^{2i\arctan(a+bx)}\,dx = -x + \frac{2i\log(i+a+bx)}{b}$$

output
```
-x+2*I*ln(I+a+b*x)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int e^{2i\arctan(a+bx)}\,dx = -x + \frac{2\arctan(a+bx)}{b} + \frac{i\log\left(1+(a+bx)^2\right)}{b}$$

input
```
Integrate[E^((2*I)*ArcTan[a + b*x]),x]
```

output
```
-x + (2*ArcTan[a + b*x])/b + (I*Log[1 + (a + b*x)^2])/b
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5616, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2i\arctan(a+bx)}dx$$

$$\Big\downarrow \ 5616$$

$$\int \frac{ia+ibx+1}{-ia-ibx+1}dx$$

$$\Big\downarrow \ 49$$

$$\int \left(-1+\frac{2i}{a+bx+i}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-x+\frac{2i\log(a+bx+i)}{b}$$

input
```
Int[E^((2*I)*ArcTan[a + b*x]),x]
```

output
```
-x + ((2*I)*Log[I + a + b*x])/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5616
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^(I*(n/2))/(1 + I*a*c + I*b*c*x)^(I*(n/2)), x] /; FreeQ[{a, b,
c, n}, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
parallelrisch	$\frac{2i\ln(bx+a+i)-bx}{b}$	21
risch	$-x+\frac{i\ln(b^2x^2+2abx+a^2+1)}{b}+\frac{2\arctan(bx+a)}{b}$	40
default	$-x+\frac{i\ln(b^2x^2+2abx+a^2+1)}{b}+\frac{2\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}$	51

input
```
int((1+I*(b*x+a))^2/(1+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
(2*I*ln(I+a+b*x)-b*x)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int e^{2i\arctan(a+bx)}\,dx=-\frac{bx-2i\log\left(\frac{bx+a+i}{b}\right)}{b}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2),x, algorithm="fricas")
```

output
```
-(b*x - 2*I*log((b*x + a + I)/b))/b
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int e^{2i\arctan(a+bx)}\,dx = -x + \frac{2i\log(a+bx+i)}{b}$$

input
```
integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2),x)
```

output
```
-x + 2*I*log(a + b*x + I)/b
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(16) = 32.

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.30

$$\int e^{2i\arctan(a+bx)}\,dx = -x + \frac{2\arctan\left(\frac{b^2x+ab}{b}\right)}{b} + \frac{i\log(b^2x^2+2abx+a^2+1)}{b}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2),x, algorithm="maxima")
```

output
```
-x + 2*arctan((b^2*x + a*b)/b)/b + I*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int e^{2i\arctan(a+bx)}\,dx = -x + \frac{2i\log(bx+a+i)}{b}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2),x, algorithm="giac")
```

output
```
-x + 2*I*log(b*x + a + I)/b
```

Mupad [B] (verification not implemented)

Time = 23.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int e^{2i\arctan(a+bx)}\,dx = -x + \frac{\ln\left(x + \frac{a+1\mathrm{i}}{b}\right)\,2\mathrm{i}}{b}$$

input
```
int((a*1i + b*x*1i + 1)^2/((a + b*x)^2 + 1),x)
```

output
```
(log(x + (a + 1i)/b)*2i)/b - x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int e^{2i\arctan(a+bx)}\,dx = \frac{2atan(bx+a) + \log(b^2x^2 + 2abx + a^2 + 1)\,i - bx}{b}$$

input
```
int((1+I*(b*x+a))^2/(1+(b*x+a)^2),x)
```

output
```
(2*atan(a + b*x) + log(a**2 + 2*a*b*x + b**2*x**2 + 1)*i - b*x)/b
```

3.190 $\int \frac{e^{2i\arctan(a+bx)}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 38

$$\int \frac{e^{2i\arctan(a+bx)}}{x}\,dx=\frac{(i-a)\log(x)}{i+a}-\frac{2\log(i+a+bx)}{1-ia}$$

output
```
(I-a)*ln(x)/(I+a)-2*ln(I+a+b*x)/(1-I*a)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{e^{2i\arctan(a+bx)}}{x}\,dx=-\frac{(-i+a)\log(x)+2i\log(i+a+bx)}{i+a}$$

input
```
Integrate[E^((2*I)*ArcTan[a + b*x])/x,x]
```

output
```
-(((-I + a)*Log[x] + (2*I)*Log[I + a + b*x])/(I + a))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5618, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2i\arctan(a+bx)}}{x}dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{ia+ibx+1}{x(-ia-ibx+1)}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{-a+i}{(a+i)x}-\frac{2ib}{(a+i)(a+bx+i)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(-a+i)\log(x)}{a+i}-\frac{2\log(a+bx+i)}{1-ia}$$

input
```
Int[E^((2*I)*ArcTan[a + b*x])/x,x]
```

output
```
((I - a)*Log[x])/(I + a) - (2*Log[I + a + b*x])/(1 - I*a)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

method	result	size
parallelrisch	$\frac{2ia\ln(x)-a^2\ln(x)-2\ln(bx+a+i)-2i\ln(bx+a+i)a+\ln(x)}{a^2+1}$	47
risch	$\frac{i\ln(-x)}{i+a}-\frac{\ln(-x)a}{i+a}-\frac{i\ln(b^2x^2+2abx+a^2+1)}{i+a}-\frac{2\arctan(bx+a)}{i+a}$	69
default	$-\frac{2b\left(\frac{(iab+b)\ln\left(b^2x^2+2abx+a^2+1\right)}{2b^2}+\frac{\left(ia^2-i+2a-\frac{(iab+b)a}{b}\right)\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}\right)}{a^2+1}+\frac{(-a^2+2ia+1)\ln(x)}{a^2+1}$	110

input
```
int((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x,x,method=_RETURNVERBOSE)
```

output
```
(2*I*a*ln(x)-a^2*ln(x)-2*ln(I+a+b*x)-2*I*ln(I+a+b*x)*a+ln(x))/(a^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{e^{2i\arctan(a+bx)}}{x}\,dx=-\frac{(a-i)\log(x)+2i\log\left(\frac{bx+a+i}{b}\right)}{a+i}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x,x, algorithm="fricas")
```

output
```
-((a - I)*log(x) + 2*I*log((b*x + a + I)/b))/(a + I)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(24) = 48$.

Time = 0.41 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.63

$$\int \frac{e^{2i\arctan(a+bx)}}{x}\,dx = -\frac{(a-i)\log\left(-\frac{a^2(a-i)}{a+i} + a^2 - \frac{2ia(a-i)}{a+i} + x(ab-3ib) + \frac{a-i}{a+i} + 1\right)}{a+i} - \frac{2i\log\left(a^2 - \frac{2ia^2}{a+i} + \frac{4a}{a+i} + x(ab-3ib) + 1 + \frac{2i}{a+i}\right)}{a+i}$$

input
```
integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)/x,x)
```

output
```
-(a - I)*log(-a**2*(a - I)/(a + I) + a**2 - 2*I*a*(a - I)/(a + I) + x*(a*b
 - 3*I*b) + (a - I)/(a + I) + 1)/(a + I) - 2*I*log(a**2 - 2*I*a**2/(a + I)
 + 4*a/(a + I) + x*(a*b - 3*I*b) + 1 + 2*I/(a + I))/(a + I)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(29) = 58$.

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.05

$$\int \frac{e^{2i\arctan(a+bx)}}{x}\,dx = -\frac{2\,(a-i)\arctan\left(\frac{b^2x+ab}{b}\right)}{a^2+1} - \frac{(i\,a+1)\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^2+1} - \frac{(a^2-2i\,a-1)\log\left(x\right)}{a^2+1}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x,x, algorithm="maxima")
```

output
```
-2*(a - I)*arctan((b^2*x + a*b)/b)/(a^2 + 1) - (I*a + 1)*log(b^2*x^2 + 2*a
*b*x + a^2 + 1)/(a^2 + 1) - (a^2 - 2*I*a - 1)*log(x)/(a^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{e^{2i\arctan(a+bx)}}{x}\,dx = -\frac{2i\,b\log(bx+a+i)}{ab+i\,b} - \frac{(a-i)\log(|x|)}{a+i}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x,x, algorithm="giac")
```

output
```
-2*I*b*log(b*x + a + I)/(a*b + I*b) - (a - I)*log(abs(x))/(a + I)
```

Mupad [B] (verification not implemented)

Time = 23.52 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{e^{2i\arctan(a+bx)}}{x}\,dx = \ln(x)\left(-1+\frac{2\mathrm{i}}{a+1\mathrm{i}}\right) - \frac{\ln(a+b\,x+1\mathrm{i})\,2\mathrm{i}}{a+1\mathrm{i}}$$

input
```
int((a*1i + b*x*1i + 1)^2/(x*((a + b*x)^2 + 1)),x)
```

output
```
log(x)*(2i/(a + 1i) - 1) - (log(a + b*x + 1i)*2i)/(a + 1i)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.21

$$\int \frac{e^{2i\arctan(a+bx)}}{x}\,dx = \frac{-2atan(bx+a)\,a+2atan(bx+a)\,i-\log(b^2x^2+2abx+a^2+1)\,ai-\log(b^2x^2+2abx+a^2+1)-\log(}{a^2+1}$$

input
```
int((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x,x)
```

output

```
( - 2*atan(a + b*x)*a + 2*atan(a + b*x)*i - log(a**2 + 2*a*b*x + b**2*x**2
 + 1)*a*i - log(a**2 + 2*a*b*x + b**2*x**2 + 1) - log(x)*a**2 + 2*log(x)*a
*i + log(x))/(a**2 + 1)
```

3.191 $\int \frac{e^{2i\arctan(a+bx)}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 55

$$\int \frac{e^{2i\arctan(a+bx)}}{x^2}\,dx = -\frac{i-a}{(i+a)x} - \frac{2ib\log(x)}{(i+a)^2} + \frac{2ib\log(i+a+bx)}{(i+a)^2}$$

output
```
-(I-a)/(I+a)/x-2*I*b*ln(x)/(I+a)^2+2*I*b*ln(I+a+b*x)/(I+a)^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.71

$$\int \frac{e^{2i\arctan(a+bx)}}{x^2}\,dx = \frac{1+a^2-2ibx\log(x)+2ibx\log(i+a+bx)}{(i+a)^2x}$$

input
```
Integrate[E^((2*I)*ArcTan[a + b*x])/x^2,x]
```

output
```
(1 + a^2 - (2*I)*b*x*Log[x] + (2*I)*b*x*Log[I + a + b*x])/((I + a)^2*x)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5618, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2i\arctan(a+bx)}}{x^2}dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{ia+ibx+1}{x^2(-ia-ibx+1)}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{2ib^2}{(a+i)^2(a+bx+i)}-\frac{2ib}{(a+i)^2x}+\frac{-a+i}{(a+i)x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2ib\log(x)}{(a+i)^2}+\frac{2ib\log(a+bx+i)}{(a+i)^2}-\frac{-a+i}{(a+i)x}$$

input
```
Int[E^((2*I)*ArcTan[a + b*x])/x^2,x]
```

output
```
-((I - a)/((I + a)*x)) - ((2*I)*b*Log[x])/(I + a)^2 + ((2*I)*b*Log[I + a +
 b*x])/(I + a)^2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.75

method	result
parallelrisch	$-\frac{2i\ln(x)x\,a^2b-2i\ln(bx+a+i)x\,a^2b+1-2ib\ln(x)x+2ib\ln(bx+a+i)x+2ia^3+4\ln(x)xab-4\ln(bx+a+i)xab-a^4+2ia}{(a^2+1)^2x}$
default	$\frac{2b^2\left(\frac{(ia^2b+2ab-ib)\ln(b^2x^2+2abx+a^2+1)}{2b^2}+\frac{\left(ia^3-3ia+3a^2-1-\frac{(ia^2b+2ab-ib)a}{b}\right)\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}\right)}{(a^2+1)^2}-\frac{-a^2+2ia+1}{(a^2+1)x}-2b$
risch	$-\frac{i}{(i+a)x}+\frac{a}{(i+a)x}+\frac{2b\ln((-2a^2b-2b)x)}{ia^2-2a-i}-\frac{b\ln(4a^4b^2x^2+8a^5bx+4a^6+8a^2b^2x^2+16a^3bx+12a^4+4b^2x^2+8abx+12a^2}{ia^2-2a-i}$

input
```
int((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(2*I*ln(x)*x*a^2*b-2*I*ln(I+a+b*x)*x*a^2*b+1-2*I*b*ln(x)*x+2*I*b*ln(I+a+b
*x)*x+2*I*a^3+4*ln(x)*x*a*b-4*ln(I+a+b*x)*x*a*b-a^4+2*I*a)/(a^2+1)^2/x
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int \frac{e^{2i\arctan(a+bx)}}{x^2}\,dx = \frac{-2i\,bx\log(x)+2i\,bx\log\left(\frac{bx+a+i}{b}\right)+a^2+1}{(a^2+2i\,a-1)x}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^2,x, algorithm="fricas")
```

output
```
(-2*I*b*x*log(x) + 2*I*b*x*log((b*x + a + I)/b) + a^2 + 1)/((a^2 + 2*I*a -
 1)*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(39) = 78$.

Time = 0.31 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.84

$$\int \frac{e^{2i\arctan(a+bx)}}{x^2}\,dx = -\frac{2ib\log\left(-\frac{2a^3b}{(a+i)^2} - \frac{6ia^2b}{(a+i)^2} + 2ab + \frac{6ab}{(a+i)^2} + 4b^2x + 2ib + \frac{2ib}{(a+i)^2}\right)}{(a+i)^2} + \frac{2ib\log\left(\frac{2a^3b}{(a+i)^2} + \frac{6ia^2b}{(a+i)^2} + 2ab - \frac{6ab}{(a+i)^2} + 4b^2x + 2ib - \frac{2ib}{(a+i)^2}\right)}{(a+i)^2} - \frac{-a+i}{x(a+i)}$$

input
```
integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)/x**2,x)
```

output
```
-2*I*b*log(-2*a**3*b/(a + I)**2 - 6*I*a**2*b/(a + I)**2 + 2*a*b + 6*a*b/(a
 + I)**2 + 4*b**2*x + 2*I*b + 2*I*b/(a + I)**2)/(a + I)**2 + 2*I*b*log(2*a
**3*b/(a + I)**2 + 6*I*a**2*b/(a + I)**2 + 2*a*b - 6*a*b/(a + I)**2 + 4*b*
*2*x + 2*I*b - 2*I*b/(a + I)**2)/(a + I)**2 - (-a + I)/(x*(a + I))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(38) = 76$.

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.29

$$\int \frac{e^{2i\arctan(a+bx)}}{x^2}\,dx = \frac{2(a^2 - 2ia - 1)b\arctan\left(\frac{b^2x+ab}{b}\right)}{a^4 + 2a^2 + 1} + \frac{(ia^2 + 2a - i)b\log(b^2x^2 + 2abx + a^2 + 1)}{a^4 + 2a^2 + 1} - \frac{2(ia^2 + 2a - i)b\log(x)}{a^4 + 2a^2 + 1} + \frac{a^2 - 2ia - 1}{(a^2 + 1)x}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^2,x, algorithm="maxima")
```

output
```
2*(a^2 - 2*I*a - 1)*b*arctan((b^2*x + a*b)/b)/(a^4 + 2*a^2 + 1) + (I*a^2 +
 2*a - I)*b*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/(a^4 + 2*a^2 + 1) - 2*(I*a^2
+ 2*a - I)*b*log(x)/(a^4 + 2*a^2 + 1) + (a^2 - 2*I*a - 1)/((a^2 + 1)*x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int \frac{e^{2i\arctan(a+bx)}}{x^2}\,dx = \frac{2\,b^2\log\left(bx+a+i\right)}{-i\,a^2b+2\,ab+i\,b}+\frac{2\,b\log\left(|x|\right)}{i\,a^2-2\,a-i}+\frac{a^2+1}{\left(a+i\right)^2x}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^2,x, algorithm="giac")
```

output
```
2*b^2*log(b*x + a + I)/(-I*a^2*b + 2*a*b + I*b) + 2*b*log(abs(x))/(I*a^2 -
 2*a - I) + (a^2 + 1)/((a + I)^2*x)
```

Mupad [B] (verification not implemented)

Time = 23.38 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.78

$$\int \frac{e^{2i\arctan(a+bx)}}{x^2}\,dx = \frac{a-\mathrm{i}}{x\,\left(a+1\mathrm{i}\right)}+\frac{b\,\mathrm{atanh}\left(\frac{a^2+a\,2\mathrm{i}-1}{{\left(a+1\mathrm{i}\right)}^2}-\frac{x\,\left(2\,a^4\,b^2+4\,a^2\,b^2+2\,b^2\right)}{{\left(a+1\mathrm{i}\right)}^2\,\left(-b\,a^3+1\mathrm{i}\,b\,a^2-b\,a+b\,1\mathrm{i}\right)}\right)\,4\mathrm{i}}{{\left(a+1\mathrm{i}\right)}^2}$$

input
```
int((a*1i + b*x*1i + 1)^2/(x^2*((a + b*x)^2 + 1)),x)
```

output
```
(a - 1i)/(x*(a + 1i)) + (b*atanh((a*2i + a^2 - 1)/(a + 1i)^2 - (x*(2*b^2 +
 4*a^2*b^2 + 2*a^4*b^2))/((a + 1i)^2*(b*1i - a*b + a^2*b*1i - a^3*b)))*4i)
/(a + 1i)^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.93

$$\int \frac{e^{2i\arctan(a+bx)}}{x^2}\,dx = \frac{2atan(bx+a)\,a^2bx - 4atan(bx+a)\,abix - 2atan(bx+a)\,bx + \log(b^2x^2+2abx+a^2+1)\,a^2bix + 2\log(}{}$$

input
```
int((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^2,x)
```

output
```
(2*atan(a + b*x)*a**2*b*x - 4*atan(a + b*x)*a*b*i*x - 2*atan(a + b*x)*b*x
+ log(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**2*b*i*x + 2*log(a**2 + 2*a*b*x +
b**2*x**2 + 1)*a*b*x - log(a**2 + 2*a*b*x + b**2*x**2 + 1)*b*i*x - 2*log(x
)*a**2*b*i*x - 4*log(x)*a*b*x + 2*log(x)*b*i*x + a**4 - 2*a**3*i - 2*a*i -
 1)/(x*(a**4 + 2*a**2 + 1))
```

3.192 $\int \frac{e^{2i\arctan(a+bx)}}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 76

$$\int \frac{e^{2i\arctan(a+bx)}}{x^3}\,dx = -\frac{i-a}{2(i+a)x^2} + \frac{2ib}{(i+a)^2x} - \frac{2b^2\log(x)}{(1-ia)^3} + \frac{2b^2\log(i+a+bx)}{(1-ia)^3}$$

output
```
-1/2*(I-a)/(I+a)/x^2+2*I*b/(I+a)^2/x-2*b^2*ln(x)/(1-I*a)^3+2*b^2*ln(I+a+b*
x)/(1-I*a)^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.83

$$\int \frac{e^{2i\arctan(a+bx)}}{x^3}\,dx = \frac{(i+a)\left(1+a^2+4ibx\right)+4ib^2x^2\log(x)-4ib^2x^2\log(i+a+bx)}{2(i+a)^3x^2}$$

input
```
Integrate[E^((2*I)*ArcTan[a + b*x])/x^3,x]
```

output
```
((I + a)*(1 + a^2 + (4*I)*b*x) + (4*I)*b^2*x^2*Log[x] - (4*I)*b^2*x^2*Log[
I + a + b*x])/(2*(I + a)^3*x^2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5618, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2i\arctan(a+bx)}}{x^3}dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{ia+ibx+1}{x^3(-ia-ibx+1)}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{2ib^3}{(a+i)^3(a+bx+i)}+\frac{2ib^2}{(a+i)^3x}-\frac{2ib}{(a+i)^2x^2}+\frac{-a+i}{(a+i)x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2b^2\log(x)}{(1-ia)^3}+\frac{2b^2\log(a+bx+i)}{(1-ia)^3}+\frac{2ib}{(a+i)^2x}-\frac{-a+i}{2(a+i)x^2}$$

input
```
Int[E^((2*I)*ArcTan[a + b*x])/x^3,x]
```

output
```
-1/2*(I - a)/((I + a)*x^2) + ((2*I)*b)/((I + a)^2*x) - (2*b^2*Log[x])/(1 -
 I*a)^3 + (2*b^2*Log[I + a + b*x])/(1 - I*a)^3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(67) = 134$.

Time = 0.27 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.64

method	result
parallelrisch	$\frac{-2ia^5+4ia^4bx-1-2ia-4ia^3-4ibx+12\ln(x)x^2a^2b^2-12\ln(bx+a+i)x^2a^2b^2-4i\ln(bx+a+i)x^2a^3b^2+a^6-4\ln(x)x^2b^2+4\ln(b}{2(a^4+2a^2+1)(a^2+1)x^2}$
default	$-\frac{2b^3\left(\frac{\left(ia^3b+3a^2b-3iab-b\right)\ln\left(b^2x^2+2abx+a^2+1\right)}{2b^2}+\frac{\left(ia^4-6ia^2+4a^3+i-4a-\frac{\left(ia^3b+3a^2b-3iab-b\right)a}{b}\right)\arctan\left(\frac{2b^2x+2ab}{2b}\right)}{b}\right)}{\left(a^2+1\right)^3}-$
risch	$\frac{\frac{2ibx}{a^2+2ia-1}+\frac{-i+a}{2i+2a}}{x^2}-\frac{2b^2\ln\left(\left(2a^4b+4a^2b+2b\right)x\right)}{ia^3-3a^2-3ia+1}+\frac{b^2\ln\left(4a^8b^2x^2+8a^9bx+4a^{10}+16a^6b^2x^2+32a^7bx+20a^8+24a^4b^2x^2+48\right.}{ia^3-3a^2-3}$

input
```
int((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-2*I*a^5+4*I*x*a^4*b-1-2*I*a-4*I*a^3-4*I*b*x+12*ln(x)*x^2*a^2*b^2-12*
ln(I+a+b*x)*x^2*a^2*b^2-4*I*ln(I+a+b*x)*x^2*a^3*b^2+a^6-4*ln(x)*x^2*b^2+4*
ln(I+a+b*x)*x^2*b^2+8*a^3*b*x+4*I*ln(x)*x^2*a^3*b^2+a^4+12*I*ln(I+a+b*x)*x
^2*a*b^2+8*a*b*x-12*I*ln(x)*x^2*a*b^2-a^2)/(a^4+2*a^2+1)/(a^2+1)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

$$\int \frac{e^{2i\arctan(a+bx)}}{x^3}\,dx = \frac{4i\,b^2x^2\log(x) - 4i\,b^2x^2\log\left(\frac{bx+a+i}{b}\right) + a^3 - 4(-i\,a+1)bx + i\,a^2 + a + i}{2\,(a^3 + 3i\,a^2 - 3\,a - i)x^2}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^3,x, algorithm="fricas")
```

output
```
1/2*(4*I*b^2*x^2*log(x) - 4*I*b^2*x^2*log((b*x + a + I)/b) + a^3 - 4*(-I*a
 + 1)*b*x + I*a^2 + a + I)/((a^3 + 3*I*a^2 - 3*a - I)*x^2)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(60) = 120.

Time = 0.44 (sec) , antiderivative size = 228, normalized size of antiderivative = 3.00

$$\int \frac{e^{2i\arctan(a+bx)}}{x^3}\,dx = \frac{2ib^2\log\left(-\frac{2a^4b^2}{(a+i)^3} - \frac{8ia^3b^2}{(a+i)^3} + \frac{12a^2b^2}{(a+i)^3} + 2ab^2 + \frac{8iab^2}{(a+i)^3} + 4b^3x + 2ib^2 - \frac{2b^2}{(a+i)^3}\right)}{(a+i)^3} - \frac{2ib^2\log\left(\frac{2a^4b^2}{(a+i)^3} + \frac{8ia^3b^2}{(a+i)^3} - \frac{12a^2b^2}{(a+i)^3} + 2ab^2 - \frac{8iab^2}{(a+i)^3} + 4b^3x + 2ib^2 + \frac{2b^2}{(a+i)^3}\right)}{(a+i)^3} - \frac{-a^2 - 4ibx - 1}{x^2\cdot(2a^2 + 4ia - 2)}$$

input
```
integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)/x**3,x)
```

output
```
2*I*b**2*log(-2*a**4*b**2/(a + I)**3 - 8*I*a**3*b**2/(a + I)**3 + 12*a**2*
b**2/(a + I)**3 + 2*a*b**2 + 8*I*a*b**2/(a + I)**3 + 4*b**3*x + 2*I*b**2 -
 2*b**2/(a + I)**3)/(a + I)**3 - 2*I*b**2*log(2*a**4*b**2/(a + I)**3 + 8*I
*a**3*b**2/(a + I)**3 - 12*a**2*b**2/(a + I)**3 + 2*a*b**2 - 8*I*a*b**2/(a
 + I)**3 + 4*b**3*x + 2*I*b**2 + 2*b**2/(a + I)**3)/(a + I)**3 - (-a**2 -
4*I*b*x - 1)/(x**2*(2*a**2 + 4*I*a - 2))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(58) = 116$.

Time = 0.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.47

$$\int \frac{e^{2i\arctan(a+bx)}}{x^3}\,dx = -\frac{2\,(a^3-3i\,a^2-3\,a+i)b^2\arctan\left(\frac{b^2x+ab}{b}\right)}{a^6+3\,a^4+3\,a^2+1} - \frac{(i\,a^3+3\,a^2-3i\,a-1)b^2\log\left(b^2x^2+2\,abx+a^2+1\right)}{a^6+3\,a^4+3\,a^2+1} - \frac{2\,(-i\,a^3-3\,a^2+3i\,a+1)b^2\log\left(x\right)}{a^6+3\,a^4+3\,a^2+1} + \frac{a^4-2i\,a^3-4\,(-i\,a^2-2\,a+i)bx-2i\,a-1}{2\,(a^4+2\,a^2+1)x^2}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^3,x, algorithm="maxima")
```

output
```
-2*(a^3 - 3*I*a^2 - 3*a + I)*b^2*arctan((b^2*x + a*b)/b)/(a^6 + 3*a^4 + 3*
a^2 + 1) - (I*a^3 + 3*a^2 - 3*I*a - 1)*b^2*log(b^2*x^2 + 2*a*b*x + a^2 + 1
)/(a^6 + 3*a^4 + 3*a^2 + 1) - 2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*b^2*log(x)/(a
^6 + 3*a^4 + 3*a^2 + 1) + 1/2*(a^4 - 2*I*a^3 - 4*(-I*a^2 - 2*a + I)*b*x -
2*I*a - 1)/((a^4 + 2*a^2 + 1)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.17

$$\int \frac{e^{2i\arctan(a+bx)}}{x^3}\,dx = \frac{2\,b^3\log(bx+a+i)}{i\,a^3b-3\,a^2b-3i\,ab+b} + \frac{2\,b^2\log(|x|)}{-i\,a^3+3\,a^2+3i\,a-1} + \frac{a^3+i\,a^2+4i\,(ab+i\,b)x+a+i}{2\,(a+i)^3x^2}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^3,x, algorithm="giac")
```

output
```
2*b^3*log(b*x + a + I)/(I*a^3*b - 3*a^2*b - 3*I*a*b + b) + 2*b^2*log(abs(x
))/(-I*a^3 + 3*a^2 + 3*I*a - 1) + 1/2*(a^3 + I*a^2 + 4*I*(a*b + I*b)*x + a
 + I)/((a + I)^3*x^2)
```

Mupad [B] (verification not implemented)

Time = 23.48 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.03

$$\int \frac{e^{2i\arctan(a+bx)}}{x^3}\,dx = \frac{\frac{a-\mathrm{i}}{2\,(a+1\mathrm{i})}+\frac{b\,x\,2\mathrm{i}}{(a+1\mathrm{i})^2}}{x^2} + \frac{b^2\,\mathrm{atanh}\left(\frac{-a^3-a^2\,3\mathrm{i}+3\,a+1\mathrm{i}}{(a+1\mathrm{i})^3}+\frac{x\,(2\,a^8\,b^2+8\,a^6\,b^2+12\,a^4\,b^2+8\,a^2\,b^2+2\,b^2)}{(a+1\mathrm{i})^3\,(-b\,a^6+2\mathrm{i}\,b\,a^5-b\,a^4+4\mathrm{i}\,b\,a^3+b\,a^2+2\mathrm{i}\,b\,a+b)}\right)\,4\mathrm{i}}{(a+1\mathrm{i})^3}$$

input
```
int((a*1i + b*x*1i + 1)^2/(x^3*((a + b*x)^2 + 1)),x)
```

output
```
((a - 1i)/(2*(a + 1i)) + (b*x*2i)/(a + 1i)^2)/x^2 + (b^2*atanh((3*a - a^2*
3i - a^3 + 1i)/(a + 1i)^3 + (x*(2*b^2 + 8*a^2*b^2 + 12*a^4*b^2 + 8*a^6*b^2
 + 2*a^8*b^2))/((a + 1i)^3*(b + a*b*2i + a^2*b + a^3*b*4i - a^4*b + a^5*b*
2i - a^6*b)))*4i)/(a + 1i)^3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.99

$$\int \frac{e^{2i\arctan(a+bx)}}{x^3}\,dx$$
$$= \frac{-4atan(bx+a)\,a^3b^2x^2 + 12atan(bx+a)\,a^2b^2i\,x^2 + 12atan(bx+a)\,a\,b^2x^2 - 4atan(bx+a)\,b^2i\,x^2 - 2\log}{}$$

input
```
int((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^3,x)
```

output
```
( - 4*atan(a + b*x)*a**3*b**2*x**2 + 12*atan(a + b*x)*a**2*b**2*i*x**2 + 1
2*atan(a + b*x)*a*b**2*x**2 - 4*atan(a + b*x)*b**2*i*x**2 - 2*log(a**2 + 2
*a*b*x + b**2*x**2 + 1)*a**3*b**2*i*x**2 - 6*log(a**2 + 2*a*b*x + b**2*x**
2 + 1)*a**2*b**2*x**2 + 6*log(a**2 + 2*a*b*x + b**2*x**2 + 1)*a*b**2*i*x**
2 + 2*log(a**2 + 2*a*b*x + b**2*x**2 + 1)*b**2*x**2 + 4*log(x)*a**3*b**2*i
*x**2 + 12*log(x)*a**2*b**2*x**2 - 12*log(x)*a*b**2*i*x**2 - 4*log(x)*b**2
*x**2 + a**6 - 2*a**5*i + 4*a**4*b*i*x + a**4 + 8*a**3*b*x - 4*a**3*i - a*
*2 + 8*a*b*x - 2*a*i - 4*b*i*x - 1)/(2*x**2*(a**6 + 3*a**4 + 3*a**2 + 1))
```

3.193 $\int \frac{e^{2i\arctan(a+bx)}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 93

$$\int \frac{e^{2i\arctan(a+bx)}}{x^4}\,dx=-\frac{i-a}{3(i+a)x^3}+\frac{ib}{(i+a)^2x^2}+\frac{2b^2}{(1-ia)^3x}-\frac{2ib^3\log(x)}{(i+a)^4}+\frac{2ib^3\log(i+a+bx)}{(i+a)^4}$$

output
```
-1/3*(I-a)/(I+a)/x^3+I*b/(I+a)^2/x^2+2*b^2/(1-I*a)^3/x-2*I*b^3*ln(x)/(I+a)
^4+2*I*b^3*ln(I+a+b*x)/(I+a)^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int \frac{e^{2i\arctan(a+bx)}}{x^4}\,dx=\frac{(i+a)\left(i+a+ia^2+a^3-3bx+3iabx-6ib^2x^2\right)-6ib^3x^3\log(x)+6ib^3x^3\log(i+a+bx)}{3(i+a)^4x^3}$$

input
```
Integrate[E^((2*I)*ArcTan[a + b*x])/x^4,x]
```

output
```
((I + a)*(I + a + I*a^2 + a^3 - 3*b*x + (3*I)*a*b*x - (6*I)*b^2*x^2) - (6*
I)*b^3*x^3*Log[x] + (6*I)*b^3*x^3*Log[I + a + b*x])/(3*(I + a)^4*x^3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5618, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2i\arctan(a+bx)}}{x^4}dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{ia+ibx+1}{x^4(-ia-ibx+1)}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{2ib^4}{(a+i)^4(a+bx+i)}-\frac{2ib^3}{(a+i)^4x}+\frac{2ib^2}{(a+i)^3x^2}-\frac{2ib}{(a+i)^2x^3}+\frac{-a+i}{(a+i)x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2ib^3\log(x)}{(a+i)^4}+\frac{2ib^3\log(a+bx+i)}{(a+i)^4}+\frac{2b^2}{(1-ia)^3x}+\frac{ib}{(a+i)^2x^2}-\frac{-a+i}{3(a+i)x^3}$$

input
```
Int[E^((2*I)*ArcTan[a + b*x])/x^4,x]
```

output
```
-1/3*(I - a)/((I + a)*x^3) + (I*b)/((I + a)^2*x^2) + (2*b^2)/((1 - I*a)^3*
x) - ((2*I)*b^3*Log[x])/(I + a)^4 + ((2*I)*b^3*Log[I + a + b*x])/(I + a)^4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(81) = 162$.

Time = 0.28 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.87

method	result
default	$\frac{2b^4\left(\frac{(ia^4b+4a^3b-6ia^2b-4ab+ib)\ln\left(b^2x^2+2abx+a^2+1\right)}{2b^2}+\frac{\left(ia^5-10ia^3+5a^4+5ia-10a^2+1-\frac{(ia^4b+4a^3b-6ia^2b-4ab+ib)a}{b}\right)\arctan\left(\right.}{b}\right.}{(a^2+1)^4}$
parallelrisch	$-\frac{1-6abx+2ia^7-18iab^2x^2+3ia^2bx-6b^2x^2+2ia+12a^2b^2x^2-12a^3bx+3ibx+6i\ln(x)x^3a^4b^3-6i\ln(bx+a+i)x^3a^4b^3-36i\ln(}{}$
risch	$\frac{-\frac{2ib^2x^2}{(a^2+2ia-1)(i+a)}+\frac{ibx}{a^2+2ia-1}+\frac{-i+a}{3i+3a}}{x^3}-\frac{b^3\ln\left(4a^{12}b^2x^2+8a^{13}bx+4a^{14}+24a^{10}b^2x^2+48a^{11}bx+28a^{12}+60a^8b^2x^2+120a^9b\right.}{}$

input
```
int((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
2*b^4/(a^2+1)^4*(1/2*(I*a^4*b-6*I*a^2*b+4*a^3*b+I*b-4*a*b)/b^2*ln(b^2*x^2+
2*a*b*x+a^2+1)+(I*a^5-10*I*a^3+5*a^4+5*I*a-10*a^2+1-(I*a^4*b-6*I*a^2*b+4*a
^3*b+I*b-4*a*b)*a/b)/b*arctan(1/2*(2*b^2*x+2*a*b)/b))-1/3*(2*I*a-a^2+1)/(a
^2+1)/x^3+b*(I*a^2-I+2*a)/(a^2+1)^2/x^2-2*b^2*(I*a^3-3*I*a+3*a^2-1)/(a^2+1
)^3/x-2*b^3*(I*a^4-6*I*a^2+4*a^3+I-4*a)/(a^2+1)^4*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01

$$\int \frac{e^{2i\arctan(a+bx)}}{x^4}\,dx = \frac{-6i\,b^3x^3\log(x)+6i\,b^3x^3\log\left(\frac{bx+a+i}{b}\right)-6\,(i\,a-1)b^2x^2+a^4+2i\,a^3-3\,(-i\,a^2+2\,a+i)bx+2i\,a-1}{3\,(a^4+4i\,a^3-6\,a^2-4i\,a+1)x^3}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^4,x, algorithm="fricas")
```

output
```
1/3*(-6*I*b^3*x^3*log(x) + 6*I*b^3*x^3*log((b*x + a + I)/b) - 6*(I*a - 1)*
b^2*x^2 + a^4 + 2*I*a^3 - 3*(-I*a^2 + 2*a + I)*b*x + 2*I*a - 1)/((a^4 + 4*
I*a^3 - 6*a^2 - 4*I*a + 1)*x^3)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(73) = 146.

Time = 0.56 (sec) , antiderivative size = 286, normalized size of antiderivative = 3.08

$$\int \frac{e^{2i\arctan(a+bx)}}{x^4}\,dx = -\frac{2ib^3\log\left(-\frac{2a^5b^3}{(a+i)^4}-\frac{10ia^4b^3}{(a+i)^4}+\frac{20a^3b^3}{(a+i)^4}+\frac{20ia^2b^3}{(a+i)^4}+2ab^3-\frac{10ab^3}{(a+i)^4}+4b^4x+2ib^3-\frac{2ib^3}{(a+i)^4}\right)}{(a+i)^4} + \frac{2ib^3\log\left(\frac{2a^5b^3}{(a+i)^4}+\frac{10ia^4b^3}{(a+i)^4}-\frac{20a^3b^3}{(a+i)^4}-\frac{20ia^2b^3}{(a+i)^4}+2ab^3+\frac{10ab^3}{(a+i)^4}+4b^4x+2ib^3+\frac{2ib^3}{(a+i)^4}\right)}{(a+i)^4} - \frac{-a^3-ia^2-a+6ib^2x^2+x(-3iab+3b)-i}{x^3\cdot(3a^3+9ia^2-9a-3i)}$$

input
```
integrate((1+I*(b*x+a))**2/(1+(b*x+a)**2)/x**4,x)
```

output
```
-2*I*b**3*log(-2*a**5*b**3/(a + I)**4 - 10*I*a**4*b**3/(a + I)**4 + 20*a**
3*b**3/(a + I)**4 + 20*I*a**2*b**3/(a + I)**4 + 2*a*b**3 - 10*a*b**3/(a +
I)**4 + 4*b**4*x + 2*I*b**3 - 2*I*b**3/(a + I)**4)/(a + I)**4 + 2*I*b**3*l
og(2*a**5*b**3/(a + I)**4 + 10*I*a**4*b**3/(a + I)**4 - 20*a**3*b**3/(a +
I)**4 - 20*I*a**2*b**3/(a + I)**4 + 2*a*b**3 + 10*a*b**3/(a + I)**4 + 4*b*
*4*x + 2*I*b**3 + 2*I*b**3/(a + I)**4)/(a + I)**4 - (-a**3 - I*a**2 - a +
6*I*b**2*x**2 + x*(-3*I*a*b + 3*b) - I)/(x**3*(3*a**3 + 9*I*a**2 - 9*a - 3
*I))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(69) = 138$.

Time = 0.12 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.83

$$\int \frac{e^{2i\arctan(a+bx)}}{x^4}\,dx = \frac{2\,(a^4 - 4i\,a^3 - 6\,a^2 + 4i\,a + 1)b^3\arctan\left(\frac{b^2x+ab}{b}\right)}{a^8 + 4\,a^6 + 6\,a^4 + 4\,a^2 + 1} + \frac{(i\,a^4 + 4\,a^3 - 6i\,a^2 - 4\,a + i)b^3\log\left(b^2x^2 + 2\,abx + a^2 + 1\right)}{a^8 + 4\,a^6 + 6\,a^4 + 4\,a^2 + 1} - \frac{2\,(i\,a^4 + 4\,a^3 - 6i\,a^2 - 4\,a + i)b^3\log\left(x\right)}{a^8 + 4\,a^6 + 6\,a^4 + 4\,a^2 + 1} + \frac{a^6 - 2i\,a^5 + 6\,(-i\,a^3 - 3\,a^2 + 3i\,a + 1)b^2x^2 + a^4 - 4i\,a^3 + 3\,(i\,a^4 + 2\,a^3 + 2\,a - i)bx - a^2 - 2i\,a - 1}{3\,(a^6 + 3\,a^4 + 3\,a^2 + 1)x^3}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^4,x, algorithm="maxima")
```

output
```
2*(a^4 - 4*I*a^3 - 6*a^2 + 4*I*a + 1)*b^3*arctan((b^2*x + a*b)/b)/(a^8 + 4
*a^6 + 6*a^4 + 4*a^2 + 1) + (I*a^4 + 4*a^3 - 6*I*a^2 - 4*a + I)*b^3*log(b^
2*x^2 + 2*a*b*x + a^2 + 1)/(a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1) - 2*(I*a^4 +
4*a^3 - 6*I*a^2 - 4*a + I)*b^3*log(x)/(a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1) +
1/3*(a^6 - 2*I*a^5 + 6*(-I*a^3 - 3*a^2 + 3*I*a + 1)*b^2*x^2 + a^4 - 4*I*a^
3 + 3*(I*a^4 + 2*a^3 + 2*a - I)*b*x - a^2 - 2*I*a - 1)/((a^6 + 3*a^4 + 3*a
^2 + 1)*x^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.35

$$\int \frac{e^{2i\arctan(a+bx)}}{x^4}\,dx = \frac{2\,b^4\log\left(bx+a+i\right)}{-i\,a^4b+4\,a^3b+6i\,a^2b-4\,ab-i\,b}+\frac{2\,b^3\log\left(|x|\right)}{i\,a^4-4\,a^3-6i\,a^2+4\,a+i} + \frac{a^4+2i\,a^3-6i\,(ab^2+i\,b^2)x^2+3i\,(a^2b+2i\,ab-b)x+2i\,a-1}{3\,(a+i)^4x^3}$$

input
```
integrate((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^4,x, algorithm="giac")
```

output
```
2*b^4*log(b*x + a + I)/(-I*a^4*b + 4*a^3*b + 6*I*a^2*b - 4*a*b - I*b) + 2*
b^3*log(abs(x))/(I*a^4 - 4*a^3 - 6*I*a^2 + 4*a + I) + 1/3*(a^4 + 2*I*a^3 -
 6*I*(a*b^2 + I*b^2)*x^2 + 3*I*(a^2*b + 2*I*a*b - b)*x + 2*I*a - 1)/((a +
I)^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 23.46 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.14

$$\int \frac{e^{2i\arctan(a+bx)}}{x^4}\,dx = \frac{\frac{a-\mathrm{i}}{3\,(a+1\mathrm{i})}-\frac{b^2\,x^2\,2\mathrm{i}}{(a+1\mathrm{i})^3}+\frac{b\,x\,1\mathrm{i}}{(a+1\mathrm{i})^2}}{x^3} + \frac{b^3\,\mathrm{atanh}\left(\frac{a^4+a^3\,4\mathrm{i}-6\,a^2-a\,4\mathrm{i}+1}{(a+1\mathrm{i})^4}-\frac{x\,\left(2\,a^{12}\,b^2+12\,a^{10}\,b^2+30\,a^8\,b^2+40\,a^6\,b^2+30\,a^4\,b^2+12\,a^2\,b^2+2\,b^2\right)}{(a+1\mathrm{i})^4\,\left(-b\,a^9+3\mathrm{i}\,b\,a^8+8\mathrm{i}\,b\,a^6+6\,b\,a^5+6\mathrm{i}\,b\,a^4+8\,b\,a^3+3\,b\,a-b\,1\mathrm{i}\right)}\right)\,4\mathrm{i}}{(a+1\mathrm{i})^4}$$

input
```
int((a*1i + b*x*1i + 1)^2/(x^4*((a + b*x)^2 + 1)),x)
```

output
```
((a - 1i)/(3*(a + 1i)) - (b^2*x^2*2i)/(a + 1i)^3 + (b*x*1i)/(a + 1i)^2)/x^
3 + (b^3*atanh((a^3*4i - 6*a^2 - a*4i + a^4 + 1)/(a + 1i)^4 - (x*(2*b^2 +
12*a^2*b^2 + 30*a^4*b^2 + 40*a^6*b^2 + 30*a^8*b^2 + 12*a^10*b^2 + 2*a^12*b
^2))/((a + 1i)^4*(3*a*b - b*1i + 8*a^3*b + a^4*b*6i + 6*a^5*b + a^6*b*8i +
 a^8*b*3i - a^9*b))))*4i)/(a + 1i)^4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 464, normalized size of antiderivative = 4.99

$$\int \frac{e^{2i\arctan(a+bx)}}{x^4}\,dx$$
$$= \frac{-1 + 2a^6 + 6atan(bx+a)\,b^3x^3 + 6a^5bx - 6a^5i - 18\log(b^2x^2 + 2abx + a^2 + 1)\,a^2b^3i\,x^3 - 6\log(x)\,a^4b^3i\,x}{}$$

input
```
int((1+I*(b*x+a))^2/(1+(b*x+a)^2)/x^4,x)
```

output
```
(6*atan(a + b*x)*a**4*b**3*x**3 - 24*atan(a + b*x)*a**3*b**3*i*x**3 - 36*a
tan(a + b*x)*a**2*b**3*x**3 + 24*atan(a + b*x)*a*b**3*i*x**3 + 6*atan(a +
b*x)*b**3*x**3 + 3*log(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**4*b**3*i*x**3 +
12*log(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**3*b**3*x**3 - 18*log(a**2 + 2*a*
b*x + b**2*x**2 + 1)*a**2*b**3*i*x**3 - 12*log(a**2 + 2*a*b*x + b**2*x**2
+ 1)*a*b**3*x**3 + 3*log(a**2 + 2*a*b*x + b**2*x**2 + 1)*b**3*i*x**3 - 6*l
og(x)*a**4*b**3*i*x**3 - 24*log(x)*a**3*b**3*x**3 + 36*log(x)*a**2*b**3*i*
x**3 + 24*log(x)*a*b**3*x**3 - 6*log(x)*b**3*i*x**3 + a**8 - 2*a**7*i + 3*
a**6*b*i*x + 2*a**6 - 6*a**5*b**2*i*x**2 + 6*a**5*b*x - 6*a**5*i - 18*a**4
*b**2*x**2 + 3*a**4*b*i*x + 12*a**3*b**2*i*x**2 + 12*a**3*b*x - 6*a**3*i -
 12*a**2*b**2*x**2 - 3*a**2*b*i*x - 2*a**2 + 18*a*b**2*i*x**2 + 6*a*b*x -
2*a*i + 6*b**2*x**2 - 3*b*i*x - 1)/(3*x**3*(a**8 + 4*a**6 + 6*a**4 + 4*a**
2 + 1))
```

3.194 $\int e^{3i\arctan(a+bx)}x^4\,dx$

Optimal result

Integrand size = 16, antiderivative size = 359

$$\begin{aligned}
&\int e^{3i\arctan(a+bx)}x^4\,dx \\
&= -\frac{3(19i+68a-88ia^2-48a^3+8ia^4)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} \\
&\quad-\frac{2ix^4(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} \\
&\quad-\frac{3(95i+272a-254ia^2-72a^3)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{40b^5} \\
&\quad+\frac{3(17i+16a)x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{20b^3} \\
&\quad-\frac{11x^3\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{5b^2} \\
&\quad+\frac{(61i+118a-52ia^2)(1-ia-ibx)^{3/2}(1+ia+ibx)^{3/2}}{20b^5} \\
&\quad-\frac{3(19-68ia-88a^2+48ia^3+8a^4)\operatorname{arcsinh}(a+bx)}{8b^5}
\end{aligned}$$

output
```
-3/8*(19*I+68*a-88*I*a^2-48*a^3+8*I*a^4)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)
^(1/2)/b^5-2*I*x^4*(1+I*a+I*b*x)^(3/2)/b/(1-I*a-I*b*x)^(1/2)-3/40*(95*I+27
2*a-254*I*a^2-72*a^3)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(3/2)/b^5+3/20*(17
*I+16*a)*x^2*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(3/2)/b^3-11/5*x^3*(1-I*a-I
*b*x)^(1/2)*(1+I*a+I*b*x)^(3/2)/b^2+1/20*(61*I+118*a-52*I*a^2)*(1-I*a-I*b*
x)^(3/2)*(1+I*a+I*b*x)^(3/2)/b^5-3/8*(19-68*I*a-88*a^2+48*I*a^3+8*a^4)*arc
sinh(b*x+a)/b^5
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.69

$$\int e^{3i\arctan(a+bx)}x^4\,dx =$$

$$-\frac{\sqrt{1+ia+ibx}(448i+418ia^4+8a^5+163bx+61ib^2x^2-34b^3x^3-22ib^4x^4+8b^5x^5+14ia^3(121i+8b}{40b^5\sqrt{-i(i+a+bx)}}$$

$$+\frac{3(-1)^{3/4}\left(19-68ia-88a^2+48ia^3+8a^4\right)\operatorname{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{4\sqrt{-ib}b^{9/2}}$$

input
```
Integrate[E^((3*I)*ArcTan[a + b*x])*x^4,x]
```

output
```
-1/40*(Sqrt[1 + I*a + I*b*x]*(448*I + (418*I)*a^4 + 8*a^5 + 163*b*x + (61*
I)*b^2*x^2 - 34*b^3*x^3 - (22*I)*b^4*x^4 + 8*b^5*x^5 + (14*I)*a^3*(121*I +
 8*b*x) - I*a^2*(2599 - (422*I)*b*x + 52*b^2*x^2) + a*(1763 - (458*I)*b*x
+ 118*b^2*x^2 + (32*I)*b^3*x^3)))/(b^5*Sqrt[(-I)*(I + a + b*x)]) + (3*(-1)
^(3/4)*(19 - (68*I)*a - 88*a^2 + (48*I)*a^3 + 8*a^4)*ArcSinh[((1/2 + I/2)*
Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/(4*Sqrt[(-I)*b]*b^(9/2))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.95, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {5618, 108, 27, 170, 27, 170, 25, 27, 164, 60, 62, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{3i\arctan(a+bx)}dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{x^4(ia+ibx+1)^{3/2}}{(-ia-ibx+1)^{3/2}}dx$$

$$\Big\downarrow \text{108}$$

$$\frac{2i\int \frac{x^3\sqrt{ia+ibx+1}(8(ia+1)+11ibx)}{2\sqrt{-ia-ibx+1}}dx}{b}-\frac{2ix^4(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{i\int \frac{x^3\sqrt{ia+ibx+1}(8(ia+1)+11ibx)}{\sqrt{-ia-ibx+1}}dx}{b}-\frac{2ix^4(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \text{170}$$

$$\frac{i\left(\frac{\int -\frac{3bx^2\sqrt{ia+ibx+1}(11(i-a)(1-ia)-(17-16ia)bx)}{\sqrt{-ia-ibx+1}}dx}{5b^2}+\frac{11ix^3\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{5b}\right)}{b}-\frac{2ix^4(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{i\left(\frac{11ix^3\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{5b}-\frac{3\int \frac{x^2\sqrt{ia+ibx+1}\left(11i\left(a^2+1\right)-(17-16ia)bx\right)}{\sqrt{-ia-ibx+1}}dx}{5b}\right)}{b}-\frac{2ix^4(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \text{170}$$

$$\frac{i\Bigg(\frac{11ix^3\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{5b}-\frac{3\Bigg(\frac{\int -\frac{bx\sqrt{ia+ibx+1}\left(2(i-a)(17-16ia)(a+i)-\left(-52ia^2+118a+61i\right)bx\right)}{\sqrt{-ia-ibx+1}}dx}{4b^2}-\frac{(17-16ia)x^2\sqrt{-ia-ibx+1}(ia+ibx\cdots}{4b}\cdots\Bigg)}{5b}\cdots\Bigg)}{b}$$

$$\frac{2ix^4(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

$\downarrow$ 25

$$\frac{i\Bigg(\frac{11ix^3\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{5b}-\frac{3\Bigg(-\frac{\int \frac{bx\sqrt{ia+ibx+1}\left(2(ia+1)(a+i)(16a+17i)-\left(-52ia^2+118a+61i\right)bx\right)}{\sqrt{-ia-ibx+1}}dx}{4b^2}-\frac{(17-16ia)x^2\sqrt{-ia-ibx+1}(ia+iba\cdots}{4b}\cdots\Bigg)}{5b}\cdots\Bigg)}{b}$$

$$\frac{2ix^4(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

$\downarrow$ 27

$$\frac{i\Bigg(\frac{11ix^3\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{5b}-\frac{3\Bigg(-\frac{\int \frac{x\sqrt{ia+ibx+1}\left(2(ia+1)(a+i)(16a+17i)-\left(-52ia^2+118a+61i\right)bx\right)}{\sqrt{-ia-ibx+1}}dx}{4b}-\frac{(17-16ia)x^2\sqrt{-ia-ibx+1}(ia+ibx\cdots}{4b}\cdots\Bigg)}{5b}\cdots\Bigg)}{b}$$

$$\frac{2ix^4(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

$\downarrow$ 164

$$\frac{i\Bigg(\frac{11ix^3\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{5b}-\frac{3\Bigg(-\frac{\frac{5\left(8ia^4-48a^3-88ia^2+68a+19i\right)\int\frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}dx}{2b}-\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}\left(112ia^3+2\left(-52ia^2+118\cdots\right)\right)}{6b^2}}{4b}\cdots\Bigg)}{5b}\cdots\Bigg)}{b}$$

$$\frac{2ix^4(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \ 60$$

$$i\left(\frac{11ix^3\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{5b}-\frac{3\left(-\frac{\frac{5\left(8ia^4-48a^3-88ia^2+68a+19i\right)\left(\int\frac{1}{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}-\frac{\sqrt{-ia-ibx}}{}}{4b}\right.}{}\right.$$

$$b$$

$$\frac{2ix^4(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \ 62$$

$$i\left(\frac{11ix^3\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{5b}-\frac{3\left(-\frac{\frac{5\left(8ia^4-48a^3-88ia^2+68a+19i\right)\left(\int\frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}-\frac{\sqrt{-ia}}{}}{4b}\right.}{}\right.$$

$$b$$

$$\frac{2ix^4(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \ 1090$$

$$\frac{i\left(\frac{11ix^3\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{5b}-\frac{3\left(-\frac{\frac{5\left(8ia^4-48a^3-88ia^2+68a+19i\right)\left(\frac{\int \frac{1}{\sqrt{\frac{\left(2xb^2+2ab\right)^2}{4b^2}+1}}d\left(2xb^2+2ab\right)}{2b^2}+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}}{4b}-\frac{\sqrt{-}\cdots}{\cdots}\right)}{\cdots}\right)}{b}$$

$$\frac{2ix^4(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

$\downarrow$ 222

$$\frac{i\left(\frac{11ix^3\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{5b}-\frac{3\left(-\frac{\frac{5\left(8ia^4-48a^3-88ia^2+68a+19i\right)\left(\frac{\operatorname{arcsinh}\left(\frac{2ab+2b^2x}{2b}\right)}{b}+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}}{4b}-\frac{\sqrt{-ia-ibx+1}(\cdots}{\cdots}\right)}{\cdots}\right)}{b}$$

$$\frac{2ix^4(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

input
```
Int[E^((3*I)*ArcTan[a + b*x])*x^4,x]
```

output
```
((-2*I)*x^4*(1 + I*a + I*b*x)^(3/2))/(b*Sqrt[1 - I*a - I*b*x]) + (I*((((11
*I)/5)*x^3*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/b - (3*(-1/4*((1
7 - (16*I)*a)*x^2*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/b - (-1/6
*(Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2)*(163 - (458*I)*a - 422*a^2
 + (112*I)*a^3 + 2*(61*I + 118*a - (52*I)*a^2)*b*x))/b^2 + (5*(19*I + 68*a
 - (88*I)*a^2 - 48*a^3 + (8*I)*a^4)*((I*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a
 + I*b*x])/b + ArcSinh[(2*a*b + 2*b^2*x)/(2*b)]/b))/(2*b))/(4*b)))/(5*b)))
/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.21

method	result
risch	$-\frac{i(8b^4x^4-8a\,b^3x^3-30ib^3x^3+8a^2b^2x^2+70ia\,b^2x^2-8a^3bx-130ia^2bx+8a^4+250ia^3-64b^2x^2+252abx+125ibx-804a^2-835ia+288}{40b^5}$
default	Expression too large to display

input
```
int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/40*I*(8*b^4*x^4-30*I*b^3*x^3-8*a*b^3*x^3+70*I*a*b^2*x^2+8*a^2*b^2*x^2-1
30*I*a^2*b*x-8*a^3*b*x+250*I*a^3+8*a^4-64*b^2*x^2+125*I*b*x+252*a*b*x-835*
I*a-804*a^2+288)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^5-1/8/b^4*(-204*I*a*ln((b
^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+144*I*a^3
*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-I*(
128*a^3-32*I*a^4-128*a+192*I*a^2-32*I)/b^2/(x+(I+a)/b)*((x+(I+a)/b)^2*b^2-
2*I*b*(x+(I+a)/b))^(1/2)+57*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^
2+1)^(1/2))/(b^2)^(1/2)-264*a^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*
x+a^2+1)^(1/2))/(b^2)^(1/2)+24*a^4*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a
*b*x+a^2+1)^(1/2))/(b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.74

$$\int e^{3i\arctan(a+bx)}x^4\,dx = \frac{-62i\,a^6+2687\,a^5+11575i\,a^4-20350\,a^3+(-62i\,a^5+2625\,a^4+8950i\,a^3-11400\,a^2-6340i\,a+1280}{}$$

input
```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^4,x, algorithm="fricas")
```

output
```
1/320*(-62*I*a^6 + 2687*a^5 + 11575*I*a^4 - 20350*a^3 + (-62*I*a^5 + 2625*
a^4 + 8950*I*a^3 - 11400*a^2 - 6340*I*a + 1280)*b*x - 17740*I*a^2 + 120*(8
*a^5 + 56*I*a^4 - 136*a^3 + (8*a^4 + 48*I*a^3 - 88*a^2 - 68*I*a + 19)*b*x
- 156*I*a^2 + 87*a + 19*I)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)) - 8*(8*I*b^5*x^5 + 22*b^4*x^4 - 2*(16*a + 17*I)*b^3*x^3 + 8*I*a^5 + (52
*a^2 + 118*I*a - 61)*b^2*x^2 - 418*a^4 - 1694*I*a^3 - (112*a^3 + 422*I*a^2
 - 458*a - 163*I)*b*x + 2599*a^2 + 1763*I*a - 448)*sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1) + 7620*a + 1280*I)/(b^6*x + (a + I)*b^5)
```

Sympy [F]

$$\int e^{3i\arctan(a+bx)}x^4\,dx = \text{Too large to display}$$

input
```
integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)*x**4,x)
```

output
```
-I*(Integral(I*x**4/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*s
qrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2
*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*a*x**
4/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(
a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a**3*x**4/(a**2*sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
+ b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1)), x) + Integral(-3*b*x**5/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
 + Integral(b**3*x**7/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*a
**2*x**4/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 +
2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
+ sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*b**2*x**6/(a**
2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1)), x) + Integral(3*a*b**2*x**6/(a**2*sqrt(a**2...
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3081 vs. $2(253) = 506$.

Time = 0.08 (sec) , antiderivative size = 3081, normalized size of antiderivative = 8.58

$$\int e^{3i\arctan(a+bx)}x^4\,dx = \text{Too large to display}$$

input
```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^4,x, algorithm="maxima")
```

output
```
-1/5*I*b*x^6/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 11/20*I*a*x^5/sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1) - 693/4*I*a^7*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)*b^2) - 33/20*I*a^2*x^4/(sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)*b) + 2415/8*I*(a^2 + 1)*a^5*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)*b^2) + 231/40*I*a^3*x^3/(sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)*b^2) - 2/5*(-I*a^2 - I)*x^4/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
*b) - 3/4*(I*a*b^2 + b^2)*x^5/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) - 23
1/8*I*(a^2 + 1)*a^6/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*b^3) + 945/4*(I*a*b^2 + b^2)*a^6*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4) - 105*(I*a^2*b + 2*a*b - I*b)*a^5*x/((a^
2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) - 2919/20*I*
(a^2 + 1)^2*a^3*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)*b^2) - 15*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^4*x/((a^2*b^2 - (a^2 + 1)*b^
2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) - 231/8*I*a^4*x^2/(sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)*b^3) - 111/40*I*(a^2 + 1)*a*x^3/(sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1)*b^2) + 9/4*(I*a*b^2 + b^2)*a*x^4/(sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)*b^3) - (I*a^2*b + 2*a*b - I*b)*x^4/(sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1)*b^2) + 189/5*I*(a^2 + 1)^2*a^4/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*b^3) - 2835/8*(I*a*b^2 + b^2)*(a^2 + 1)*a^4*x/((a^2*
b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4) + 265/2*(I*...
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.93

$$\int e^{3i\arctan(a+bx)}x^4\,dx =$$
$$-\frac{1}{40}\sqrt{(bx+a)^2+1}\left(\left(2\left(x\left(\frac{4i\,x}{b}-\frac{4i\,ab^{17}-15\,b^{17}}{b^{19}}\right)-\frac{-4i\,a^2b^{16}+35\,ab^{16}+32i\,b^{16}}{b^{19}}\right)x-\frac{8i\,a^3b^{15}-\right.\right.$$
$$+\frac{(8\,a^4+48i\,a^3-88\,a^2-68i\,a+19)\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab+a^3b+\left(x|b|-\sqrt{(bx+a)^2}\right.\right.}{}$$

input
```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^4,x, algorithm="giac")
```

output
```
-1/40*sqrt((b*x + a)^2 + 1)*((2*(x*(4*I*x/b - (4*I*a*b^17 - 15*b^17)/b^19)
 - (-4*I*a^2*b^16 + 35*a*b^16 + 32*I*b^16)/b^19)*x - (8*I*a^3*b^15 - 130*a
^2*b^15 - 252*I*a*b^15 + 125*b^15)/b^19)*x - (-8*I*a^4*b^14 + 250*a^3*b^14
 + 804*I*a^2*b^14 - 835*a*b^14 - 288*I*b^14)/b^19) + 1/8*(8*a^4 + 48*I*a^3
 - 88*a^2 - 68*I*a + 19)*log(3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b +
a^3*b + (x*abs(b) - sqrt((b*x + a)^2 + 1))^3*abs(b) + 3*(x*abs(b) - sqrt((
b*x + a)^2 + 1))*a^2*abs(b) + 2*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b +
 2*I*a^2*b + 4*(I*x*abs(b) - I*sqrt((b*x + a)^2 + 1))*a*abs(b) - a*b - (x*
abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^4*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{3i\arctan(a+bx)}x^4\,dx = \int \frac{x^4\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}{\left((a+b\,x)^2+1\right)^{3/2}}\,dx$$

input
```
int((x^4*(a*1i + b*x*1i + 1)^3)/((a + b*x)^2 + 1)^(3/2),x)
```

output
```
int((x^4*(a*1i + b*x*1i + 1)^3)/((a + b*x)^2 + 1)^(3/2), x)
```

Reduce [F]

$$\int e^{3i\arctan(a+bx)}x^4\,dx=\int\frac{(1+i(bx+a))^3x^4}{\left(1+(bx+a)^2\right)^{\frac{3}{2}}}dx$$

input
```
int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^4,x)
```

output
```
int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^4,x)
```

3.195 $\int e^{3i\arctan(a+bx)}x^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 282

$$\begin{aligned}\int e^{3i\arctan(a+bx)}x^3\,dx &= \frac{3(17-44ia-36a^2+8ia^3)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} \\ &\quad - \frac{2ix^3(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} \\ &\quad + \frac{3(17-32ia-14a^2)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{8b^4} \\ &\quad - \frac{9x^2\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{4b^2} \\ &\quad - \frac{(11-10ia)(1-ia-ibx)^{3/2}(1+ia+ibx)^{3/2}}{4b^4} \\ &\quad - \frac{3(17i+44a-36ia^2-8a^3)\operatorname{arcsinh}(a+bx)}{8b^4}\end{aligned}$$

output
```
3/8*(17-44*I*a-36*a^2+8*I*a^3)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^4
-2*I*x^3*(1+I*a+I*b*x)^(3/2)/b/(1-I*a-I*b*x)^(1/2)+3/8*(17-32*I*a-14*a^2)*
(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(3/2)/b^4-9/4*x^2*(1-I*a-I*b*x)^(1/2)*(1
+I*a+I*b*x)^(3/2)/b^2-1/4*(11-10*I*a)*(1-I*a-I*b*x)^(3/2)*(1+I*a+I*b*x)^(3
/2)/b^4-3/8*(17*I+44*a-36*I*a^2-8*a^3)*arcsinh(b*x+a)/b^4
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.71

$$\int e^{3i\arctan(a+bx)}x^3\,dx$$
$$= \frac{\sqrt{1+ia+ibx}(80+78ia^3+2a^4-29ibx+11b^2x^2+6ib^3x^3-2b^4x^4+a^2(-233+22ibx)-ia(237-54i}{8b^4\sqrt{-i(i+a+bx)}}$$
$$+\frac{3\sqrt[4]{-1}(-17i-44a+36ia^2+8a^3)\sqrt{-ib}\text{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{4b^{9/2}}$$

input
```
Integrate[E^((3*I)*ArcTan[a + b*x])*x^3,x]
```

output
```
(Sqrt[1 + I*a + I*b*x]*(80 + (78*I)*a^3 + 2*a^4 - (29*I)*b*x + 11*b^2*x^2
+ (6*I)*b^3*x^3 - 2*b^4*x^4 + a^2*(-233 + (22*I)*b*x) - I*a*(237 - (54*I)*
b*x + 10*b^2*x^2)))/(8*b^4*Sqrt[(-I)*(I + a + b*x)]) + (3*(-1)^(1/4)*(-17*
I - 44*a + (36*I)*a^2 + 8*a^3)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*S
qrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/(4*b^(9/2))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {5618, 108, 27, 170, 25, 27, 164, 60, 62, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3e^{3i\arctan(a+bx)}\,dx$$
$$\Big\downarrow \text{5618}$$
$$\int \frac{x^3(ia+ibx+1)^{3/2}}{(-ia-ibx+1)^{3/2}}dx$$
$$\Big\downarrow \text{108}$$

$$\frac{2i\int \frac{3x^2\sqrt{ia+ibx+1}(2(ia+1)+3ibx)}{2\sqrt{-ia-ibx+1}}dx}{b}-\frac{2ix^3(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

$\downarrow$ 27

$$\frac{3i\int \frac{x^2\sqrt{ia+ibx+1}(2(ia+1)+3ibx)}{\sqrt{-ia-ibx+1}}dx}{b}-\frac{2ix^3(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

$\downarrow$ 170

$$\frac{3i\left(\frac{\int -\frac{bx\sqrt{ia+ibx+1}\left(6i\left(a^2+1\right)-(11-10ia)bx\right)}{\sqrt{-ia-ibx+1}}dx}{4b^2}+\frac{3ix^2\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{4b}\right)}{b}-\frac{2ix^3(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

$\downarrow$ 25

$$\frac{3i\left(\frac{3ix^2\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{4b}-\frac{\int \frac{bx\sqrt{ia+ibx+1}\left(6i\left(a^2+1\right)-(11-10ia)bx\right)}{\sqrt{-ia-ibx+1}}dx}{4b^2}\right)}{b}-\frac{2ix^3(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

$\downarrow$ 27

$$\frac{3i\left(\frac{3ix^2\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{4b}-\frac{\int \frac{x\sqrt{ia+ibx+1}\left(6i\left(a^2+1\right)-(11-10ia)bx\right)}{\sqrt{-ia-ibx+1}}dx}{4b}\right)}{b}-\frac{2ix^3(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

$\downarrow$ 164

$$\frac{3i\left(\frac{3ix^2\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{4b}-\frac{\frac{\left(8ia^3-36a^2-44ia+17\right)\int \frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}dx}{2b}+\frac{\sqrt{-ia-ibx+1}\left(-22ia^2-2(11-10ia)bx+54a+29i\right)(ia+ibx+1)^{3/2}}{6b^2}}{4b}\right)}{b}$$

$$\frac{2ix^3(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

$\downarrow$ 60

$$\frac{3i\left(\frac{3ix^2\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{4b}-\frac{\frac{\left(8ia^3-36a^2-44ia+17\right)\left(\int \frac{1}{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}+\frac{\sqrt{-ia-ibx+1}\left(-22ia^2-\right.}{}}{4b}\right.}{b}$$

$$\frac{2ix^3(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

$\Big\downarrow$ 62

$$\frac{3i\left(\frac{3ix^2\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{4b}-\frac{\frac{\left(8ia^3-36a^2-44ia+17\right)\left(\int\frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}+\frac{\sqrt{-ia-ibx+1}\left(-22i\right.}{}}{4b}\right.}{b}$$

$$\frac{2ix^3(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

$\Big\downarrow$ 1090

$$\frac{3i\left(\frac{3ix^2\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{4b}-\frac{\frac{\left(8ia^3-36a^2-44ia+17\right)\left(\frac{\int\frac{1}{\sqrt{\frac{\left(2xb^2+2ab\right)^2}{4b^2}+1}}d\left(2xb^2+2ab\right)}{2b^2}+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}+\frac{\sqrt{-ia-ibx+1}\left(-\right.}{}}{4b}\right.}{b}$$

$$\frac{2ix^3(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

$\Big\downarrow$ 222

$$\frac{3i\left(\frac{3ix^2\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{4b}-\frac{\frac{\sqrt{-ia-ibx+1}\left(-22ia^2-2(11-10ia)bx+54a+29i\right)(ia+ibx+1)^{3/2}}{6b^2}+\frac{\left(8ia^3-36a^2-44ia+17\right)\left(\frac{\text{arcsinh}\left(\frac{2ab+2b}{2b}\right.}{b}\right.}{2b}}{4b}\right.}{b}$$

$$\frac{2ix^3(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

input

```
Int[E^((3*I)*ArcTan[a + b*x])*x^3,x]
```

output
```
((-2*I)*x^3*(1 + I*a + I*b*x)^(3/2))/(b*Sqrt[1 - I*a - I*b*x]) + ((3*I)*((
((3*I)/4)*x^2*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/b - ((Sqrt[1
- I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2)*(29*I + 54*a - (22*I)*a^2 - 2*(11 -
 (10*I)*a)*b*x))/(6*b^2) + ((17 - (44*I)*a - 36*a^2 + (8*I)*a^3)*((I*Sqrt[
1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b + ArcSinh[(2*a*b + 2*b^2*x)/(2*b
)]/b))/(2*b))/(4*b)))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.21

method	result
risch	$\frac{i(-2b^3x^3+2a\,b^2x^2+8ib^2x^2-2a^2bx-20iabx+2a^3+44ia^2+19bx-93a-48i)\sqrt{b^2x^2+2abx+a^2+1}}{8b^4}+\frac{-\frac{51i\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+}\right.}{\sqrt{b^2}}}{}$
default	Expression too large to display

input
```
int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*I*(-2*b^3*x^3+8*I*b^2*x^2+2*a*b^2*x^2-20*I*a*b*x-2*a^2*b*x+44*I*a^2+2*
a^3+19*b*x-48*I-93*a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^4+1/8/b^3*(-51*I*ln(
(b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+108*I*a
^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-I
*(96*a^2-32*I*a^3-32+96*I*a)/b^2/(x+(I+a)/b)*((x+(I+a)/b)^2*b^2-2*I*b*(x+(
I+a)/b))^(1/2)-132*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1
/2))/(b^2)^(1/2)+24*a^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)
^(1/2))/(b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.77

$$\int e^{3i\arctan(a+bx)}x^3\,dx$$
$$= \frac{15i\,a^5-495\,a^4-1664i\,a^3+(15i\,a^4-480\,a^3-1184i\,a^2+968\,a+256i)bx+2152\,a^2-24\,(8\,a^4+44i\,a^3}{}$$

input
```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^3,x, algorithm="fricas")
```

output
```
1/64*(15*I*a^5 - 495*a^4 - 1664*I*a^3 + (15*I*a^4 - 480*a^3 - 1184*I*a^2 +
 968*a + 256*I)*b*x + 2152*a^2 - 24*(8*a^4 + 44*I*a^3 + (8*a^3 + 36*I*a^2
- 44*a - 17*I)*b*x - 80*a^2 - 61*I*a + 17)*log(-b*x - a + sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1)) - 8*(2*I*b^4*x^4 + 6*b^3*x^3 - (10*a + 11*I)*b^2*x^2 -
2*I*a^4 + 78*a^3 + (22*a^2 + 54*I*a - 29)*b*x + 233*I*a^2 - 237*a - 80*I)*
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1224*I*a - 256)/(b^5*x + (a + I)*b^4)
```

Sympy [F]

$$\int e^{3i\arctan(a+bx)}x^3\,dx = \text{Too large to display}$$

input
```
integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)*x**3,x)
```

output
```
-I*(Integral(I*x**3/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*s
qrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2
*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*a*x**
3/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(
a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a**3*x**3/(a**2*sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
+ b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1)), x) + Integral(-3*b*x**4/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
 + Integral(b**3*x**6/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*a
**2*x**3/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 +
2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
+ sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*b**2*x**5/(a**
2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1)), x) + Integral(3*a*b**2*x**5/(a**2*sqrt(a**2...
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2295 vs. $2(198) = 396$.

Time = 0.06 (sec) , antiderivative size = 2295, normalized size of antiderivative = 8.14

$$\int e^{3i\arctan(a+bx)}x^3\,dx = \text{Too large to display}$$

input
```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^3,x, algorithm="maxima")
```

output
```
-1/4*I*b*x^5/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 315/4*I*a^6*x/((a^2*b^2 -
 (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) + 3/4*I*a*x^4/sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1) - 945/8*I*(a^2 + 1)*a^4*x/((a^2*b^2 - (a^2 + 1)
*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) - 21/8*I*a^2*x^3/(sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)*b) + 105/8*I*(a^2 + 1)*a^5/((a^2*b^2 - (a^2 + 1)*b^2)
*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) - 105*(I*a*b^2 + b^2)*a^5*x/((a^2*
b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) + 45*(I*a^2*b
+ 2*a*b - I*b)*a^4*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)*b^2) + 169/4*I*(a^2 + 1)^2*a^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1)*b) + 6*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^3*x/((a^
2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) + 105/8*I*a^3*
x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) - 5/8*(-I*a^2 - I)*x^3/(sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1)*b) - (I*a*b^2 + b^2)*x^4/(sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1)*b^2) - 14*I*(a^2 + 1)^2*a^3/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) + 265/2*(I*a*b^2 + b^2)*(a^2 + 1)*a^3*x/
((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^3) - 93/2*(
I*a^2*b + 2*a*b - I*b)*(a^2 + 1)*a^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)*b^2) - 15/8*I*(a^2 + 1)^3*x/((a^2*b^2 - (a^2 + 1
)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) - 5*(-I*a^3 - 3*a^2 + 3*I*a +
1)*(a^2 + 1)*a*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^...
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.01

$$\int e^{3i\arctan(a+bx)}x^3\,dx = -\frac{1}{8}\sqrt{(bx+a)^2+1}\left(\left(2\,x\left(\frac{i\,x}{b}-\frac{i\,ab^{11}-4\,b^{11}}{b^{13}}\right)-\frac{-2i\,a^2b^{10}+20\,ab^{10}+19i\,b^{10}}{b^{13}}\right)x-\frac{2i\,a^3b^9-44\,a^2b^9}{b}\right.$$
$$-\frac{(8\,a^3+36i\,a^2-44\,a-17i)\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab+a^3b+\left(x|b|-\sqrt{(bx+a)^2+1}\right)^3|b\right.}{}$$

input

```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^3,x, algorithm="giac")
```

output

```
-1/8*sqrt((b*x + a)^2 + 1)*((2*x*(I*x/b - (I*a*b^11 - 4*b^11)/b^13) - (-2*
I*a^2*b^10 + 20*a*b^10 + 19*I*b^10)/b^13)*x - (2*I*a^3*b^9 - 44*a^2*b^9 -
93*I*a*b^9 + 48*b^9)/b^13) - 1/8*(8*a^3 + 36*I*a^2 - 44*a - 17*I)*log(3*(x
*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b + (x*abs(b) - sqrt((b*x + a
)^2 + 1))^3*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*abs(b) + 2*I
*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b + 2*I*a^2*b + 4*(I*x*abs(b) - I*sq
rt((b*x + a)^2 + 1))*a*abs(b) - a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*a
bs(b))/(b^3*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{3i\arctan(a+bx)}x^3\,dx = \int \frac{x^3\,{(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})}^3}{{\left({(a+b\,x)}^2+1\right)}^{3/2}}\,dx$$

input

```
int((x^3*(a*1i + b*x*1i + 1)^3)/((a + b*x)^2 + 1)^(3/2),x)
```

output

```
int((x^3*(a*1i + b*x*1i + 1)^3)/((a + b*x)^2 + 1)^(3/2), x)
```

Reduce [F]

$$\int e^{3i\arctan(a+bx)}x^3\,dx = \int \frac{(1+i(bx+a))^3\,x^3}{\left(1+(bx+a)^2\right)^{\frac{3}{2}}}dx$$

input
```
int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^3,x)
```

output
```
int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^3,x)
```

3.196 $\int e^{3i\arctan(a+bx)}x^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 227

$$\int e^{3i\arctan(a+bx)}x^2\,dx = \frac{(11i+18a-6ia^2)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} + \frac{(11i+18a-6ia^2)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{6b^3} - \frac{i(i+a)^2(1+ia+ibx)^{5/2}}{b^3\sqrt{1-ia-ibx}} + \frac{i\sqrt{1-ia-ibx}(1+ia+ibx)^{5/2}}{3b^3} + \frac{(11-18ia-6a^2)\operatorname{arcsinh}(a+bx)}{2b^3}$$

output
```
1/2*(11*I+18*a-6*I*a^2)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^3+1/6*(1
1*I+18*a-6*I*a^2)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(3/2)/b^3-I*(I+a)^2*(1
+I*a+I*b*x)^(5/2)/b^3/(1-I*a-I*b*x)^(1/2)+1/3*I*(1-I*a-I*b*x)^(1/2)*(1+I*a
+I*b*x)^(5/2)/b^3+1/2*(11-18*I*a-6*a^2)*arcsinh(b*x+a)/b^3
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.70

$$\int e^{3i\arctan(a+bx)}x^2\,dx = \frac{\sqrt{1+ia+ibx}(52i-53ia^2-2a^3+19bx+7ib^2x^2-2b^3x^3+a(103-16ibx))}{6b^3\sqrt{-i(i+a+bx)}} + \frac{(-1)^{3/4}(-11+18ia+6a^2)\operatorname{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{\sqrt{-ib}b^{5/2}}$$

input
```
Integrate[E^((3*I)*ArcTan[a + b*x])*x^2,x]
```

output
```
(Sqrt[1 + I*a + I*b*x]*(52*I - (53*I)*a^2 - 2*a^3 + 19*b*x + (7*I)*b^2*x^2
 - 2*b^3*x^3 + a*(103 - (16*I)*b*x)))/(6*b^3*Sqrt[(-I)*(I + a + b*x)]) + (
(-1)^(3/4)*(-11 + (18*I)*a + 6*a^2)*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)
*(I + a + b*x)])/Sqrt[(-I)*b]])/(Sqrt[(-I)*b]*b^(5/2))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5618, 100, 27, 90, 60, 60, 62, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2e^{3i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{ 5618}$$

$$\int \frac{x^2(ia+ibx+1)^{3/2}}{(-ia-ibx+1)^{3/2}}dx$$

$$\Big\downarrow \text{ 100}$$

$$-\frac{i\int \frac{b((3-2ia)(a+i)-bx)(ia+ibx+1)^{3/2}}{\sqrt{-ia-ibx+1}}dx}{b^3}-\frac{i(a+i)^2(ia+ibx+1)^{5/2}}{b^3\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \ 27$$

$$-\frac{i\int \frac{((3-2ia)(a+i)-bx)(ia+ibx+1)^{3/2}}{\sqrt{-ia-ibx+1}}dx}{b^2}-\frac{i(a+i)^2(ia+ibx+1)^{5/2}}{b^3\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \ 90$$

$$-\frac{i\left(\frac{1}{3}\left(-6ia^2+18a+11i\right)\int \frac{(ia+ibx+1)^{3/2}}{\sqrt{-ia-ibx+1}}dx-\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{5/2}}{3b}\right)}{b^2}-\frac{i(a+i)^2(ia+ibx+1)^{5/2}}{b^3\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \ 60$$

$$-\frac{i\left(\frac{1}{3}\left(-6ia^2+18a+11i\right)\left(\frac{3}{2}\int \frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}dx+\frac{i\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2b}\right)-\frac{\sqrt{-ia-ibx+1}(ia+ibx+1)^{5/2}}{3b}\right)}{b^2}-\frac{i(a+i)^2(ia+ibx+1)^{5/2}}{b^3\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \ 60$$

$$-\frac{i\left(\frac{1}{3}\left(-6ia^2+18a+11i\right)\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)+\frac{i\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2b}\right)-\right.}{b^2}$$
$$\frac{i(a+i)^2(ia+ibx+1)^{5/2}}{b^3\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \ 62$$

$$-\frac{i\left(\frac{1}{3}\left(-6ia^2+18a+11i\right)\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)+\frac{i\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2b}\right)\right.}{b^2}$$
$$\frac{i(a+i)^2(ia+ibx+1)^{5/2}}{b^3\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \ 1090$$

$$-\frac{i\left(\frac{1}{3}\left(-6ia^2+18a+11i\right)\left(\frac{3}{2}\left(\frac{\int \frac{1}{\sqrt{\frac{\left(2xb^2+2ab\right)^2}{4b^2}+1}}d\left(2xb^2+2ab\right)}{2b^2}+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)+\frac{i\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2b}\right.\right.}{b^2}$$
$$\frac{i(a+i)^2(ia+ibx+1)^{5/2}}{b^3\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \text{222}$$

$$-\frac{i\left(\frac{1}{3}\left(-6ia^2+18a+11i\right)\left(\frac{3}{2}\left(\frac{\text{arcsinh}\left(\frac{2ab+2b^2x}{2b}\right)}{b}+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)+\frac{i\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2b}\right)-\frac{\sqrt{-ia\cdots}}{}\right)}{b^2}$$

$$\frac{i(a+i)^2(ia+ibx+1)^{5/2}}{b^3\sqrt{-ia-ibx+1}}$$

input
```
Int[E^((3*I)*ArcTan[a + b*x])*x^2,x]
```

output
```
((-I)*(I + a)^2*(1 + I*a + I*b*x)^(5/2))/(b^3*Sqrt[1 - I*a - I*b*x]) - (I*
(-1/3*(Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(5/2))/b + ((11*I + 18*a -
(6*I)*a^2)*(((I/2)*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/b + (3*(
(I*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b + ArcSinh[(2*a*b + 2*b^2
*x)/(2*b)]/b))/2))/3))/b^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.14

method	result
risch	$-\frac{i\left(2b^2x^2-2abx-9ibx+2a^2+27ia-28\right)\sqrt{b^2x^2+2abx+a^2+1}}{6b^3}-\frac{\frac{18ia\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}-\frac{i\left(-8ia^2+16a+8i\right)\sqrt{\left(x+\frac{i+a}{b}\right.}}{b^2\left(x+\frac{i+a}{b}\right.}}{}$
default	Expression too large to display

input
```
int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*I*(2*b^2*x^2-9*I*b*x-2*a*b*x+27*I*a+2*a^2-28)*(b^2*x^2+2*a*b*x+a^2+1)
^(1/2)/b^3-1/2/b^2*(18*I*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2
+1)^(1/2))/(b^2)^(1/2)-I*(16*a-8*I*a^2+8*I)/b^2/(x+(I+a)/b)*((x+(I+a)/b)^2
*b^2-2*I*b*(x+(I+a)/b))^(1/2)-11*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b
*x+a^2+1)^(1/2))/(b^2)^(1/2)+6*a^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a
*b*x+a^2+1)^(1/2))/(b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.77

$$\int e^{3i\arctan(a+bx)}x^2\,dx = \frac{-7i\,a^4 + 166\,a^3 + (-7i\,a^3 + 159\,a^2 + 249i\,a - 96)bx + 408i\,a^2 + 12\,(6\,a^3 + (6\,a^2 + 18i\,a - 11)bx + 24i\,a}{}$$

input
```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^2,x, algorithm="fricas")
```

output
```
1/24*(-7*I*a^4 + 166*a^3 + (-7*I*a^3 + 159*a^2 + 249*I*a - 96)*b*x + 408*I
*a^2 + 12*(6*a^3 + (6*a^2 + 18*I*a - 11)*b*x + 24*I*a^2 - 29*a - 11*I)*log
(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 4*(2*I*b^3*x^3 + 7*b^2*x^
2 + 2*I*a^3 - (16*a + 19*I)*b*x - 53*a^2 - 103*I*a + 52)*sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1) - 345*a - 96*I)/(b^4*x + (a + I)*b^3)
```

Sympy [F]

$$\int e^{3i\arctan(a+bx)}x^2\,dx = \text{Too large to display}$$

input
```
integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)*x**2,x)
```

output
```
-I*(Integral(I*x**2/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*s
qrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2
*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*a*x**
2/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(
a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a**3*x**2/(a**2*sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
+ b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1)), x) + Integral(-3*b*x**3/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
 + Integral(b**3*x**5/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*a
**2*x**2/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 +
2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
+ sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*b**2*x**4/(a**
2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1)), x) + Integral(3*a*b**2*x**4/(a**2*sqrt(a**2...
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1608 vs. $2(155) = 310$.

Time = 0.05 (sec) , antiderivative size = 1608, normalized size of antiderivative = 7.08

$$\int e^{3i\arctan(a+bx)}x^2\,dx = \text{Too large to display}$$

input
```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^2,x, algorithm="maxima")
```

output
```
-35*I*a^5*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))
- 1/3*I*b*x^4/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 265/6*I*(a^2 + 1)*a^3*x/
((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + 7/6*I*a*x^
3/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 35/6*I*(a^2 + 1)*a^4/((a^2*b^2 - (a^
2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) - 61/6*I*(a^2 + 1)^2*a*x/
((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 2*(-I*a^3
- 3*a^2 + 3*I*a + 1)*a^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1)) + 45*(I*a*b^2 + b^2)*a^4*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) - 18*(I*a^2*b + 2*a*b - I*b)*a^3*x/((a^2
*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) - 35/6*I*a^2*x^
2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) + 29/6*I*(a^2 + 1)^2*a^2/((a^2*b^2
 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) + (-I*a^3 - 3*a^2 +
 3*I*a + 1)*(a^2 + 1)*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)) - 93/2*(I*a*b^2 + b^2)*(a^2 + 1)*a^2*x/((a^2*b^2 - (a^2 + 1)*b
^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) + 15*(I*a^2*b + 2*a*b - I*b)*(a
^2 + 1)*a*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b
) - 4/3*(-I*a^2 - I)*x^2/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) - 3/2*(I*a*
b^2 + b^2)*x^3/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) + 35/2*I*a^3*arcsin
h(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 + 15/2*(I*a*b^2
+ b^2)*(a^2 + 1)*a^3/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x ...
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.07

$$\int e^{3i\arctan(a+bx)}x^2\,dx$$
$$= -\frac{1}{6}\sqrt{(bx+a)^2+1}\left(x\left(\frac{2i\,x}{b}-\frac{2i\,ab^6-9\,b^6}{b^8}\right)-\frac{-2i\,a^2b^5+27\,ab^5+28i\,b^5}{b^8}\right)$$
$$+\frac{(6\,a^2+18i\,a-11)\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab+a^3b+\left(x|b|-\sqrt{(bx+a)^2+1}\right)^3|b|+3\left(x\right.\right.}{}$$

input
```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^2,x, algorithm="giac")
```

output
```
-1/6*sqrt((b*x + a)^2 + 1)*(x*(2*I*x/b - (2*I*a*b^6 - 9*b^6)/b^8) - (-2*I*
a^2*b^5 + 27*a*b^5 + 28*I*b^5)/b^8) + 1/6*(6*a^2 + 18*I*a - 11)*log(3*(x*a
bs(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b + (x*abs(b) - sqrt((b*x + a)^
2 + 1))^3*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*abs(b) + 2*I*(
x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b + 2*I*a^2*b + 4*(I*x*abs(b) - I*sqrt
((b*x + a)^2 + 1))*a*abs(b) - a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs
(b))/(b^2*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{3i\arctan(a+bx)}x^2\,dx=\int \frac{x^2\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}{\left((a+b\,x)^2+1\right)^{3/2}}\,dx$$

input
```
int((x^2*(a*1i + b*x*1i + 1)^3)/((a + b*x)^2 + 1)^(3/2),x)
```

output
```
int((x^2*(a*1i + b*x*1i + 1)^3)/((a + b*x)^2 + 1)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 770, normalized size of antiderivative = 3.39

$$\int e^{3i\arctan(a+bx)}x^2\,dx$$
$$=\frac{-24+26\sqrt{b^2x^2+2abx+a^2+1}\,b^2i\,x^2+50\sqrt{b^2x^2+2abx+a^2+1}\,a^2i-33\sqrt{b^2x^2+2abx+a^2+1}\,bx-}{}$$

input
```
int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x^2,x)
```

output
```
( - 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**4*i - 2*sqrt(a**2 + 2*a*b*x
+ b**2*x**2 + 1)*a**3*b*i*x + 51*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**3
 + 69*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**2*b*x + 50*sqrt(a**2 + 2*a*b
*x + b**2*x**2 + 1)*a**2*i - 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a*b**3
*i*x**3 + 9*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a*b**2*x**2 + 106*sqrt(a*
*2 + 2*a*b*x + b**2*x**2 + 1)*a*b*i*x + 51*sqrt(a**2 + 2*a*b*x + b**2*x**2
 + 1)*a - 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*b**4*i*x**4 - 9*sqrt(a**2
 + 2*a*b*x + b**2*x**2 + 1)*b**3*x**3 + 26*sqrt(a**2 + 2*a*b*x + b**2*x**2
 + 1)*b**2*i*x**2 - 33*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*b*x + 52*sqrt(
a**2 + 2*a*b*x + b**2*x**2 + 1)*i - 18*log(sqrt(a**2 + 2*a*b*x + b**2*x**2
 + 1) + a + b*x)*a**4 - 36*log(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + a +
b*x)*a**3*b*x - 54*log(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + a + b*x)*a**
3*i - 18*log(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + a + b*x)*a**2*b**2*x**
2 - 108*log(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + a + b*x)*a**2*b*i*x + 1
5*log(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + a + b*x)*a**2 - 54*log(sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1) + a + b*x)*a*b**2*i*x**2 + 66*log(sqrt(a**2
 + 2*a*b*x + b**2*x**2 + 1) + a + b*x)*a*b*x - 54*log(sqrt(a**2 + 2*a*b*x
+ b**2*x**2 + 1) + a + b*x)*a*i + 33*log(sqrt(a**2 + 2*a*b*x + b**2*x**2 +
 1) + a + b*x)*b**2*x**2 + 33*log(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + a
 + b*x) + 24*a**4 + 48*a**3*b*x + 48*a**3*i + 24*a**2*b**2*x**2 + 96*a*...
```

3.197 $\int e^{3i\arctan(a+bx)}x\,dx$

Optimal result

Integrand size = 14, antiderivative size = 163

$$\int e^{3i\arctan(a+bx)}x\,dx=-\frac{3(3-2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2}-\frac{(3-2ia)\sqrt{1-ia-ibx}(1+ia+ibx)^{3/2}}{2b^2}-\frac{(1-ia)(1+ia+ibx)^{5/2}}{b^2\sqrt{1-ia-ibx}}+\frac{3(3i+2a)\operatorname{arcsinh}(a+bx)}{2b^2}$$

output
```
-3/2*(3-2*I*a)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^2-1/2*(3-2*I*a)*(
1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(3/2)/b^2-(1-I*a)*(1+I*a+I*b*x)^(5/2)/b^2
/(1-I*a-I*b*x)^(1/2)+3/2*(3*I+2*a)*arcsinh(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.81

$$\int e^{3i\arctan(a+bx)}x\,dx=\frac{\sqrt{1+ia+ibx}(-14+15ia+a^2+5ibx-b^2x^2)}{2b^2\sqrt{-i(i+a+bx)}}+\frac{3\sqrt[4]{-1}(3i+2a)\sqrt{-ib}\operatorname{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{b^{5/2}}$$

input
```
Integrate[E^((3*I)*ArcTan[a + b*x])*x,x]
```

output
```
(Sqrt[1 + I*a + I*b*x]*(-14 + (15*I)*a + a^2 + (5*I)*b*x - b^2*x^2))/(2*b^
2*Sqrt[(-I)*(I + a + b*x)]) + (3*(-1)^(1/4)*(3*I + 2*a)*Sqrt[(-I)*b]*ArcSi
nh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5618, 87, 60, 60, 62, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{3i\arctan(a+bx)}dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{x(ia+ibx+1)^{3/2}}{(-ia-ibx+1)^{3/2}}dx$$

$$\Big\downarrow \text{87}$$

$$\frac{(2a+3i)\int \frac{(ia+ibx+1)^{3/2}}{\sqrt{-ia-ibx+1}}dx}{b}-\frac{(1-ia)(ia+ibx+1)^{5/2}}{b^2\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \text{60}$$

$$\frac{(2a+3i)\left(\frac{3}{2}\int \frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}dx+\frac{i\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2b}\right)}{b}-\frac{(1-ia)(ia+ibx+1)^{5/2}}{b^2\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \text{60}$$

$$\frac{(2a+3i)\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)+\frac{i\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2b}\right)}{b}-\frac{(1-ia)(ia+ibx+1)^{5/2}}{b^2\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \text{62}$$

$$\frac{(2a+3i)\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)+\frac{i\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2b}\right)}{b}-\frac{(1-ia)(ia+ibx+1)^{5/2}}{b^2\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \; 1090$$

$$\frac{(2a+3i)\left(\frac{3}{2}\left(\frac{\int \frac{1}{\sqrt{\frac{(2xb^2+2ab)^2}{4b^2}+1}}d(2xb^2+2ab)}{2b^2}+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)+\frac{i\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2b}\right)}{b}-\frac{(1-ia)(ia+ibx+1)^{5/2}}{b^2\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \; 222$$

$$\frac{(2a+3i)\left(\frac{3}{2}\left(\frac{\operatorname{arcsinh}\left(\frac{2ab+2b^2x}{2b}\right)}{b}+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)+\frac{i\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}{2b}\right)}{b}-\frac{(1-ia)(ia+ibx+1)^{5/2}}{b^2\sqrt{-ia-ibx+1}}$$

input
```
Int[E^((3*I)*ArcTan[a + b*x])*x,x]
```

output
```
-(((1 - I*a)*(1 + I*a + I*b*x)^(5/2))/(b^2*Sqrt[1 - I*a - I*b*x])) + ((3*I
 + 2*a)*(((I/2)*Sqrt[1 - I*a - I*b*x]*(1 + I*a + I*b*x)^(3/2))/b + (3*((I*
Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b + ArcSinh[(2*a*b + 2*b^2*x)
/(2*b)]/b))/2))/b
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.14

method	result
risch	$\frac{i(-bx+a+6i)\sqrt{b^2x^2+2abx+a^2+1}}{2b^2}+\frac{\frac{9i\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}-\frac{i(-8ia+8)\sqrt{\left(x+\frac{i+a}{b}\right)^2b^2-2ib\left(x+\frac{i+a}{b}\right)}}{b^2\left(x+\frac{i+a}{b}\right)}+\frac{6a\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\right.}{}}{2b}$
default	Expression too large to display

input
```
int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*I*(-b*x+a+6*I)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^2+1/2/b*(9*I*ln((b^2*x+
a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-I*(8-8*I*a)/b^
2/(x+(I+a)/b)*((x+(I+a)/b)^2*b^2-2*I*b*(x+(I+a)/b))^(1/2)+6*a*ln((b^2*x+a*
b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.83

$$\int e^{3i\arctan(a+bx)}x\,dx$$
$$= \frac{3i\,a^3 + (3i\,a^2 - 44\,a - 32i)bx - 47\,a^2 - 12\,((2\,a + 3i)bx + 2\,a^2 + 5i\,a - 3)\log\left(-bx - a + \sqrt{b^2x^2 + 2\,ab}\right.}{8\,(b^3x + (a + i)b^2)}$$

input
```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x,x, algorithm="fricas")
```

output
```
1/8*(3*I*a^3 + (3*I*a^2 - 44*a - 32*I)*b*x - 47*a^2 - 12*((2*a + 3*I)*b*x
+ 2*a^2 + 5*I*a - 3)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 4
*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(I*b^2*x^2 - I*a^2 + 5*b*x + 15*a + 14*
I) - 76*I*a + 32)/(b^3*x + (a + I)*b^2)
```

Sympy [F]

$$\int e^{3i\arctan(a+bx)}x\,dx = \text{Too large to display}$$

input
```
integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)*x,x)
```

output
```
-I*(Integral(I*x/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt
(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x*
*2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*a*x/(a**
2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a**3*x/(a**2*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x*
*2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2
+ 1)), x) + Integral(-3*b*x**2/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
+ 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a
*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integr
al(b**3*x**4/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**
2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*a**2*x/(a*
*2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*b**2*x**3/(a**2*sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
+ b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1)), x) + Integral(3*a*b**2*x**3/(a**2*sqrt(a**2 + 2*a*b*x +...
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1108 vs. $2(113) = 226$.

Time = 0.04 (sec) , antiderivative size = 1108, normalized size of antiderivative = 6.80

$$\int e^{3i\arctan(a+bx)}x\,dx = \text{Too large to display}$$

input
```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x,x, algorithm="maxima")
```

output
```
15*I*a^4*b*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))
 - 31/2*I*(a^2 + 1)*a^2*b*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1)) - 1/2*I*b*x^3/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 5/2*I*(a
^2 + 1)*a^3/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))
+ 6*(I*a^2*b + 2*a*b - I*b)*a^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)) - 18*(I*a*b^2 + b^2)*a^3*x/((a^2*b^2 - (a^2 + 1)*b^2
)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) + 3/2*I*(a^2 + 1)^2*b*x/((a^2*b^2 -
 (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (-I*a^3 - 3*a^2 + 3*I
*a + 1)*a*b*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
) + 5/2*I*a*x^2/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 3/2*I*(a^2 + 1)^2*a/((
a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (-I*a^3 - 3*
a^2 + 3*I*a + 1)*a^2/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)) - 3*(I*a^2*b + 2*a*b - I*b)*(a^2 + 1)*x/((a^2*b^2 - (a^2 + 1)*b^2
)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + 15*(I*a*b^2 + b^2)*(a^2 + 1)*a*x/((
a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) - 15/2*I*a^2
*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^2 - 3*(I*a*
b^2 + b^2)*(a^2 + 1)*a^2/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)*b^2) + 3*(I*a^2*b + 2*a*b - I*b)*(a^2 + 1)*a/((a^2*b^2 - (a^2
+ 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) - 3*(I*a*b^2 + b^2)*x^2/(sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) - 3/2*(-I*a^2 - I)*arcsinh(2*(b^2*...
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.28

$$\int e^{3i\arctan(a+bx)}x\,dx = -\frac{1}{2}\sqrt{(bx+a)^2+1}\left(\frac{i\,x}{b}+\frac{-i\,ab^2+6\,b^2}{b^4}\right)$$
$$-\frac{(2\,a+3i)\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab+a^3b+\left(x|b|-\sqrt{(bx+a)^2+1}\right)^3|b|+3\left(x|b|-\sqrt{(b\ldots\right.\right.}{\ldots}$$

input
```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x,x, algorithm="giac")
```

output
```
-1/2*sqrt((b*x + a)^2 + 1)*(I*x/b + (-I*a*b^2 + 6*b^2)/b^4) - 1/2*(2*a + 3
*I)*log(3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b + (x*abs(b) - s
qrt((b*x + a)^2 + 1))^3*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*
abs(b) + 2*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b + 2*I*a^2*b + 4*(I*x*a
bs(b) - I*sqrt((b*x + a)^2 + 1))*a*abs(b) - a*b - (x*abs(b) - sqrt((b*x +
a)^2 + 1))*abs(b))/(b*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{3i\arctan(a+bx)}x\,dx = \int \frac{x\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}{\left((a+b\,x)^2+1\right)^{3/2}}\,dx$$

input
```
int((x*(a*1i + b*x*1i + 1)^3)/((a + b*x)^2 + 1)^(3/2),x)
```

output
```
int((x*(a*1i + b*x*1i + 1)^3)/((a + b*x)^2 + 1)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 555, normalized size of antiderivative = 3.40

$$\int e^{3i\arctan(a+bx)}x\,dx$$
$$= \frac{-33i - 32a - 33b^2 i\,x^2 - 80\sqrt{b^2x^2+2abx+a^2+1}\,abx - 4\sqrt{b^2x^2+2abx+a^2+1}\,b^3 i\,x^3 - 36\sqrt{b^2x^2+2}}{}$$

input
```
int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)*x,x)
```

output

```
(4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**3*i + 4*sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1)*a**2*b*i*x - 56*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**2 -
4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a*b**2*i*x**2 - 80*sqrt(a**2 + 2*a*
b*x + b**2*x**2 + 1)*a*b*x + 4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a*i -
4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*b**3*i*x**3 - 24*sqrt(a**2 + 2*a*b*
x + b**2*x**2 + 1)*b**2*x**2 - 36*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*b*i
*x - 56*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 24*log(sqrt(a**2 + 2*a*b*x
+ b**2*x**2 + 1) + a + b*x)*a**3 + 48*log(sqrt(a**2 + 2*a*b*x + b**2*x**2
+ 1) + a + b*x)*a**2*b*x + 36*log(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + a
 + b*x)*a**2*i + 24*log(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + a + b*x)*a*
b**2*x**2 + 72*log(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + a + b*x)*a*b*i*x
 + 24*log(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + a + b*x)*a + 36*log(sqrt(
a**2 + 2*a*b*x + b**2*x**2 + 1) + a + b*x)*b**2*i*x**2 + 36*log(sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1) + a + b*x)*i - 32*a**3 - 64*a**2*b*x - 33*a**2*
i - 32*a*b**2*x**2 - 66*a*b*i*x - 32*a - 33*b**2*i*x**2 - 33*i)/(8*b**2*(a
**2 + 2*a*b*x + b**2*x**2 + 1))
```

3.198 $\int e^{3i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 94

$$\int e^{3i\arctan(a+bx)}\,dx = -\frac{3i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b} - \frac{2i(1+ia+ibx)^{3/2}}{b\sqrt{1-ia-ibx}} - \frac{3\text{arcsinh}(a+bx)}{b}$$

output
```
-3*I*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b-2*I*(1+I*a+I*b*x)^(3/2)/b/(
1-I*a-I*b*x)^(1/2)-3*arcsinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.48

$$\int e^{3i\arctan(a+bx)}\,dx = \frac{\sqrt{1+(a+bx)^2}\left(-i+\frac{4}{i+a+bx}\right)}{b} - \frac{3\text{arcsinh}(a+bx)}{b}$$

input
```
Integrate[E^((3*I)*ArcTan[a + b*x]),x]
```

output
```
(Sqrt[1 + (a + b*x)^2]*(-I + 4/(I + a + b*x)))/b - (3*ArcSinh[a + b*x])/b
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5616, 57, 60, 62, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{3i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{ 5616}$$

$$\int \frac{(ia+ibx+1)^{3/2}}{(-ia-ibx+1)^{3/2}}dx$$

$$\Big\downarrow \text{ 57}$$

$$-3\int \frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}dx-\frac{2i(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \text{ 60}$$

$$-3\left(\int \frac{1}{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)-\frac{2i(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \text{ 62}$$

$$-3\left(\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)-\frac{2i(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \text{ 1090}$$

$$-3\left(\frac{\int \frac{1}{\sqrt{\frac{\left(2xb^2+2ab\right)^2}{4b^2}+1}}d\left(2xb^2+2ab\right)}{2b^2}+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)-\frac{2i(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \text{ 222}$$

$$-3\left(\frac{\operatorname{arcsinh}\left(\frac{2ab+2b^2x}{2b}\right)}{b}+\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)-\frac{2i(ia+ibx+1)^{3/2}}{b\sqrt{-ia-ibx+1}}$$

input
```
Int[E^((3*I)*ArcTan[a + b*x]),x]
```

output
```
((-2*I)*(1 + I*a + I*b*x)^(3/2))/(b*Sqrt[1 - I*a - I*b*x]) - 3*((I*Sqrt[1
- I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b + ArcSinh[(2*a*b + 2*b^2*x)/(2*b)]
/b)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5616
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^(I*(n/2))/(1 + I*a*c + I*b*c*x)^(I*(n/2)), x] /; FreeQ[{a, b,
c, n}, x]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.28

method	result
risch	$-\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{b}+\frac{4\sqrt{\left(x+\frac{i+a}{b}\right)^2b^2-2ib\left(x+\frac{i+a}{b}\right)}}{b^2\left(x+\frac{i+a}{b}\right)}-\frac{3\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}$
default	$\frac{2(ia+1)^3(2b^2x+2ab)}{(4b^2(a^2+1)-4a^2b^2)\sqrt{b^2x^2+2abx+a^2+1}}-ib^3\left(\frac{x^2}{b^2\sqrt{b^2x^2+2abx+a^2+1}}-\frac{3a\left(-\frac{x}{b^2\sqrt{b^2x^2+2abx+a^2+1}}-\frac{a\left(-\frac{1}{b^2\sqrt{b^2x^2+2abx+a}}\right.}{}\right.}{}\right.$

input
```
int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-I/b*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+4/b^2/(x+(I+a)/b)*((x+(I+a)/b)^2*b^2-2*
I*b*(x+(I+a)/b))^(1/2)-3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1
)^(1/2))/(b^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int e^{3i\arctan(a+bx)}\,dx$$
$$= \frac{(-i\,a+8)bx - i\,a^2 + 6\,(bx+a+i)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right) - 2\,\sqrt{b^2x^2+2\,abx+a^2+}}{2\,(b^2x+(a+i)b)}$$

input
```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((-I*a + 8)*b*x - I*a^2 + 6*(b*x + a + I)*log(-b*x - a + sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)) - 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(I*b*x + I*a -
 5) + 9*a + 8*I)/(b^2*x + (a + I)*b)
```

Sympy [F]

$$\int e^{3i\arctan(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2),x)
```

output
```
-I*(Integral(I/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2
 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*a/(a**2*sq
rt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x*
*2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a
*b*x + b**2*x**2 + 1)), x) + Integral(a**3/(a**2*sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqr
t(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)),
 x) + Integral(-3*I*a**2/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*
b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x +
 b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*
b*x/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b
*x + b**2*x**2 + 1) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqr
t(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(b**3*x**3/(a**2*sqrt(a**
2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1
) + b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x +
 b**2*x**2 + 1)), x) + Integral(-3*I*b**2*x**2/(a**2*sqrt(a**2 + 2*a*b*x +
 b**2*x**2 + 1) + 2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**2
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + sqrt(a**2 + 2*a*b*x + b**2*x**2 +
1)), x) + Integral(3*a*b**2*x**2/(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 ...
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 736 vs. 2(66) = 132.

Time = 0.04 (sec) , antiderivative size = 736, normalized size of antiderivative = 7.83

$$\int e^{3i\arctan(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

output
```
-6*I*a^3*b^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)) + 5*I*(a^2 + 1)*a*b^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1)) - I*(a^2 + 1)*a^2*b/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)) + 6*(I*a*b^2 + b^2)*a^2*x/((a^2*b^2 - (a^2 + 1)*b^2
)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 3*(I*a^2*b + 2*a*b - I*b)*a*b*x/((a
^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (I*a^3 + 3*a^
2 - 3*I*a - 1)*b^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)) - I*b*x^2/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 3*(I*a^2*b + 2*a*b
- I*b)*a^2/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) -
 (-I*a^3 - 3*a^2 + 3*I*a + 1)*a*b/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)) - 3*(I*a*b^2 + b^2)*(a^2 + 1)*x/((a^2*b^2 - (a^2 + 1
)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + 3*I*a*arcsinh(2*(b^2*x + a*b)/
sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b + 3*(I*a*b^2 + b^2)*(a^2 + 1)*a/((a^
2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) - 2*(I*a^2 + I
)/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b) - 3*(I*a*b^2 + b^2)*arcsinh(2*(b^2
*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 + 3*(I*a^2*b + 2*a*b - I
*b)/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(66) = 132$.

Time = 0.16 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.91

$$\int e^{3i\arctan(a+bx)}\,dx$$
$$= \frac{\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab+a^3b+\left(x|b|-\sqrt{(bx+a)^2+1}\right)^3|b|+3\left(x|b|-\sqrt{(bx+a)^2+1}\right)\right.}{}$$
$$-\frac{i\sqrt{(bx+a)^2+1}}{b}$$

input
```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2),x, algorithm="giac")
```

output
```
log(3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b + (x*abs(b) - sqrt(
(b*x + a)^2 + 1))^3*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*abs(
b) + 2*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b + 2*I*a^2*b + 4*(I*x*abs(b
) - I*sqrt((b*x + a)^2 + 1))*a*abs(b) - a*b - (x*abs(b) - sqrt((b*x + a)^2
 + 1))*abs(b))/abs(b) - I*sqrt((b*x + a)^2 + 1)/b
```

Mupad [F(-1)]

Timed out.

$$\int e^{3i\arctan(a+bx)}\,dx = \int \frac{(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}{\left((a+b\,x)^2+1\right)^{3/2}}\,dx$$

input
```
int((a*1i + b*x*1i + 1)^3/((a + b*x)^2 + 1)^(3/2),x)
```

output
```
int((a*1i + b*x*1i + 1)^3/((a + b*x)^2 + 1)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 298, normalized size of antiderivative = 3.17

$$\int e^{3i\arctan(a+bx)}\,dx$$
$$= \frac{-\sqrt{b^2x^2+2abx+a^2+1}\,a^2i-2\sqrt{b^2x^2+2abx+a^2+1}\,abix+4\sqrt{b^2x^2+2abx+a^2+1}\,a-\sqrt{b^2x^2+2a}}{}$$

input
```
int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2),x)
```

output
```
( - sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**2*i - 2*sqrt(a**2 + 2*a*b*x +
b**2*x**2 + 1)*a*b*i*x + 4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a - sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1)*b**2*i*x**2 + 4*sqrt(a**2 + 2*a*b*x + b**2*
x**2 + 1)*b*x - 5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i - 3*log(sqrt(a**2
 + 2*a*b*x + b**2*x**2 + 1) + a + b*x)*a**2 - 6*log(sqrt(a**2 + 2*a*b*x +
b**2*x**2 + 1) + a + b*x)*a*b*x - 3*log(sqrt(a**2 + 2*a*b*x + b**2*x**2 +
1) + a + b*x)*b**2*x**2 - 3*log(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + a +
 b*x) + 4*a**2 + 8*a*b*x + 4*b**2*x**2 + 4)/(b*(a**2 + 2*a*b*x + b**2*x**2
 + 1))
```

3.199 $\int \frac{e^{3i\arctan(a+bx)}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 134

$$\int \frac{e^{3i\arctan(a+bx)}}{x}\,dx = \frac{4\sqrt{1+ia+ibx}}{(1-ia)\sqrt{1-ia-ibx}} - i\text{arcsinh}(a+bx) - \frac{2(i-a)^{3/2}\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i+a)^{3/2}}$$

output
```
4*(1+I*a+I*b*x)^(1/2)/(1-I*a)/(1-I*a-I*b*x)^(1/2)-I*arcsinh(b*x+a)-2*(I-a)
^(3/2)*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-I*b*x)^(
1/2))/(I+a)^(3/2)
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.46

$$\int \frac{e^{3i\arctan(a+bx)}}{x}\,dx = \frac{2\left(\frac{2i\sqrt{1+ia+ibx}}{\sqrt{-i(i+a+bx)}} + \frac{\sqrt[4]{-1}(i+a)(-ib)^{3/2}\text{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{b^{3/2}} + \frac{\sqrt{-1-ia}(-i+a)\text{arctanh}\left(\frac{\sqrt{-1-ia}\sqrt{-i(i+a+bx)}}{\sqrt{-1+ia}\sqrt{1+ia+ibx}}\right)}{\sqrt{-1+ia}}\right)}{i+a}$$

input
```
Integrate[E^((3*I)*ArcTan[a + b*x])/x,x]
```

output
```
(2*(((2*I)*Sqrt[1 + I*a + I*b*x])/Sqrt[(-I)*(I + a + b*x)] + ((-1)^(1/4)*(
I + a)*((-I)*b)^(3/2)*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)
])/Sqrt[(-I)*b]])/b^(3/2) + (Sqrt[-1 - I*a]*(-I + a)*ArcTanh[(Sqrt[-1 - I*
a]*Sqrt[(-I)*(I + a + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/Sqrt
[-1 + I*a]))/(I + a)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5618, 109, 27, 175, 62, 104, 221, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3i\arctan(a+bx)}}{x}dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{(ia+ibx+1)^{3/2}}{x(-ia-ibx+1)^{3/2}}dx$$

$$\Big\downarrow \text{109}$$

$$\frac{4\sqrt{ia+ibx+1}}{(1-ia)\sqrt{-ia-ibx+1}}-\frac{2\int \frac{b(i(i-a)^2-(1-ia)bx)}{2x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{(a+i)b}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\sqrt{ia+ibx+1}}{(1-ia)\sqrt{-ia-ibx+1}}-\frac{\int \frac{i(i-a)^2-(1-ia)bx}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{a+i}$$

$$\Big\downarrow \text{175}$$

$$\frac{4\sqrt{ia+ibx+1}}{(1-ia)\sqrt{-ia-ibx+1}}-\frac{i(-a+i)^2\int \frac{1}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx-(1-ia)b\int \frac{1}{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{a+i}$$

$$\Big\downarrow \; 62$$

$$\frac{4\sqrt{ia+ibx+1}}{(1-ia)\sqrt{-ia-ibx+1}}-\frac{i(-a+i)^2\int \frac{1}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx-(1-ia)b\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx}{a+i}$$

$$\Big\downarrow \; 104$$

$$\frac{4\sqrt{ia+ibx+1}}{(1-ia)\sqrt{-ia-ibx+1}}-\frac{2i(-a+i)^2\int \frac{1}{-ia+\frac{(1-ia)(ia+ibx+1)}{-ia-ibx+1}-1}d\frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}-(1-ia)b\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx}{a+i}$$

$$\Big\downarrow \; 221$$

$$\frac{4\sqrt{ia+ibx+1}}{(1-ia)\sqrt{-ia-ibx+1}}-\frac{\frac{2(-a+i)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{a+i}}-(1-ia)b\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx}{a+i}$$

$$\Big\downarrow \; 1090$$

$$\frac{4\sqrt{ia+ibx+1}}{(1-ia)\sqrt{-ia-ibx+1}}-\frac{\frac{2(-a+i)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{a+i}}-\frac{(1-ia)\int \frac{1}{\sqrt{\frac{\left(2xb^2+2ab\right)^2}{4b^2}+1}}d(2xb^2+2ab)}{2b}}{a+i}$$

$$\Big\downarrow \; 222$$

$$\frac{4\sqrt{ia+ibx+1}}{(1-ia)\sqrt{-ia-ibx+1}}-\frac{\frac{2(-a+i)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{a+i}}-(1-ia)\text{arcsinh}\left(\frac{2ab+2b^2x}{2b}\right)}{a+i}$$

input
```
Int[E^((3*I)*ArcTan[a + b*x])/x,x]
```

output
```
(4*Sqrt[1 + I*a + I*b*x])/((1 - I*a)*Sqrt[1 - I*a - I*b*x]) - (-((1 - I*a)
*ArcSinh[(2*a*b + 2*b^2*x)/(2*b)]) + (2*(I - a)^(3/2)*ArcTanh[(Sqrt[I + a]
*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/Sqrt[I + a])
/(I + a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(104) = 208$.

Time = 0.52 (sec) , antiderivative size = 485, normalized size of antiderivative = 3.62

method	result
default	$(-ia^3 - 3a^2 + 3ia + 1)\left(\frac{1}{(a^2+1)\sqrt{b^2x^2+2abx+a^2+1}} - \frac{2ab(2b^2x+2ab)}{(a^2+1)(4b^2(a^2+1)-4a^2b^2)\sqrt{b^2x^2+2abx+a^2+1}} - \frac{\ln\left(2a^2+2\right.}{}\right.$

input
```
int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(-I*a^3-3*a^2+3*I*a+1)*(1/(a^2+1)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-2*a*b/(a^2
+1)*(2*b^2*x+2*a*b)/(4*b^2*(a^2+1)-4*a^2*b^2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2
)-1/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2
+1)^(1/2))/x))-I*b^3*(-x/b^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-a/b*(-1/b^2/(b^
2*x^2+2*a*b*x+a^2+1)^(1/2)-2*a/b*(2*b^2*x+2*a*b)/(4*b^2*(a^2+1)-4*a^2*b^2)
/(b^2*x^2+2*a*b*x+a^2+1)^(1/2))+1/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+
2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2))+6*I*(1+I*a)^2*b*(2*b^2*x+2*a*b)/(4*b^2*
(a^2+1)-4*a^2*b^2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3*(1+I*a)*b^2*(-1/b^2/(b^
2*x^2+2*a*b*x+a^2+1)^(1/2)-2*a/b*(2*b^2*x+2*a*b)/(4*b^2*(a^2+1)-4*a^2*b^2)
/(b^2*x^2+2*a*b*x+a^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(90) = 180$.

Time = 0.13 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.66

$$\int \frac{e^{3i\arctan(a+bx)}}{x}\,dx = -\frac{((a+i)bx+a^2+2i\,a-1)\sqrt{-\frac{a^3-3i\,a^2-3\,a+i}{a^3+3i\,a^2-3\,a-i}}\log\left(-\frac{(a-i)bx-\sqrt{b^2x^2+2\,abx+a^2+1}(a-i)-(i\,a^2-2\,a-i)\sqrt{-\frac{a^3-3i\,a^2-3\,a\cdots}{a^3+3i\,a^2-3\,a\cdots}}}{a-i}\right.}{}$$

input
```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x,x, algorithm="fricas")
```

output
```
-(((a + I)*b*x + a^2 + 2*I*a - 1)*sqrt(-(a^3 - 3*I*a^2 - 3*a + I)/(a^3 + 3
*I*a^2 - 3*a - I))*log(-((a - I)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(
a - I) - (I*a^2 - 2*a - I)*sqrt(-(a^3 - 3*I*a^2 - 3*a + I)/(a^3 + 3*I*a^2
- 3*a - I)))/(a - I)) - ((a + I)*b*x + a^2 + 2*I*a - 1)*sqrt(-(a^3 - 3*I*a
^2 - 3*a + I)/(a^3 + 3*I*a^2 - 3*a - I))*log(-((a - I)*b*x - sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)*(a - I) - (-I*a^2 + 2*a + I)*sqrt(-(a^3 - 3*I*a^2 - 3
*a + I)/(a^3 + 3*I*a^2 - 3*a - I)))/(a - I)) + 4*b*x + ((-I*a + 1)*b*x - I
*a^2 + 2*a + I)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + 4*a +
4*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 4*I)/((a + I)*b*x + a^2 + 2*I*a - 1)
```

Sympy [F]

$$\int \frac{e^{3i\arctan(a+bx)}}{x}\,dx = \text{Too large to display}$$

input
```
integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)/x,x)
```

output
```
-I*(Integral(I/(a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**2*s
qrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**3*sqrt(a**2 + 2*a*b*x + b**2
*x**2 + 1) + x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*a/(
a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**2*sqrt(a**2 + 2*a*b
*x + b**2*x**2 + 1) + b**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x*s
qrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a**3/(a**2*x*sqrt(a**2
 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
 1) + b**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x*sqrt(a**2 + 2*a*b
*x + b**2*x**2 + 1)), x) + Integral(-3*I*a**2/(a**2*x*sqrt(a**2 + 2*a*b*x
+ b**2*x**2 + 1) + 2*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*
x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x*sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1)), x) + Integral(-3*b*x/(a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
1) + 2*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**3*sqrt(a**2
 + 2*a*b*x + b**2*x**2 + 1) + x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
+ Integral(b**3*x**3/(a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*
x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**3*sqrt(a**2 + 2*a*b*x
+ b**2*x**2 + 1) + x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(
-3*I*b**2*x**2/(a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**2*s
qrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**3*sqrt(a**2 + 2*a*b*x + b**2
*x**2 + 1) + x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(3*a...
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 733 vs. $2(90) = 180$.

Time = 0.04 (sec) , antiderivative size = 733, normalized size of antiderivative = 5.47

$$\int \frac{e^{3i\arctan(a+bx)}}{x}\,dx = \text{Too large to display}$$

input
```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x,x, algorithm="maxima")
```

output
```
2*I*a^2*b^3*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
) + (-I*a^2 - I)*b^3*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)) + (-I*a^3 - 3*a^2 + 3*I*a + 1)*a*b^3*x/((a^2*b^2 - (a^2 + 1)*b^
2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) + I*(a^2 + 1)*a*b^2/((a^2*
b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (-I*a^3 - 3*a^2
+ 3*I*a + 1)*a^2*b^2/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)*(a^2 + 1)) - 3*(I*a*b^2 + b^2)*a*b*x/((a^2*b^2 - (a^2 + 1)*b^2)*sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + 3*(I*a^2*b + 2*a*b - I*b)*b^2*x/((a^2*b
^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 3*(I*a*b^2 + b^2)
*a^2/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + 3*(I*
a^2*b + 2*a*b - I*b)*a*b/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)) - (-I*a^3 - 3*a^2 + 3*I*a + 1)*arcsinh(2*a*b*x/(sqrt(-4*a^2*b
^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*
abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3/2) +
 (-I*a^3 - 3*a^2 + 3*I*a + 1)/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)
) + 3*(I*a*b^2 + b^2)/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2) - I*arcsinh(
2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(90) = 180$.

Time = 0.25 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.88

$$\int \frac{e^{3i\arctan(a+bx)}}{x}\,dx$$

$$= \frac{i\,b\log\left(-3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab-a^3b-\left(x|b|-\sqrt{(bx+a)^2+1}\right)^3|b|-3\left(x|b|-\sqrt{(bx+a)^2\cdots}\right.\right.}{\cdots}$$

$$-\frac{(i\,a^2+2\,a-i)\log\left(\frac{\left|-2\,x|b|+2\,\sqrt{(bx+a)^2+1}-2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\,\sqrt{(bx+a)^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{\sqrt{a^2+1}(a+i)}$$

input
```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x,x, algorithm="giac")
```

output
```
1/3*I*b*log(-3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b - a^3*b - (x*abs(b
) - sqrt((b*x + a)^2 + 1))^3*abs(b) - 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))
*a^2*abs(b) - 2*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b - 2*I*a^2*b - 4*(
I*x*abs(b) - I*sqrt((b*x + a)^2 + 1))*a*abs(b) + a*b + (x*abs(b) - sqrt((b
*x + a)^2 + 1))*abs(b))/abs(b) - (I*a^2 + 2*a - I)*log(abs(-2*x*abs(b) + 2
*sqrt((b*x + a)^2 + 1) - 2*sqrt(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt((b*x +
a)^2 + 1) + 2*sqrt(a^2 + 1)))/(sqrt(a^2 + 1)*(a + I))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3i\arctan(a+bx)}}{x}\,dx = \int \frac{(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}{x\left((a+b\,x)^2+1\right)^{3/2}}\,dx$$

input
```
int((a*1i + b*x*1i + 1)^3/(x*((a + b*x)^2 + 1)^(3/2)),x)
```

output
```
int((a*1i + b*x*1i + 1)^3/(x*((a + b*x)^2 + 1)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1336, normalized size of antiderivative = 9.97

$$\int \frac{e^{3i\arctan(a+bx)}}{x}\,dx = \text{Too large to display}$$

input
```
int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x,x)
```

output

```
(2*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/sq
rt(a**2 + 1))*a**5 + 4*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**
2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*a**4*b*x - 6*sqrt(a**2 + 1)*atan((sqrt(a
**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*a**4*i + 2*sqrt(
a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/sqrt(a**2
+ 1))*a**3*b**2*x**2 - 12*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*
x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*a**3*b*i*x - 4*sqrt(a**2 + 1)*atan((s
qrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*a**3 - 6*sq
rt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/sqrt(a*
*2 + 1))*a**2*b**2*i*x**2 - 12*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x +
b**2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*a**2*b*x - 4*sqrt(a**2 + 1)*atan
((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*a**2*i -
 6*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/sq
rt(a**2 + 1))*a*b**2*x**2 + 4*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*a*b*i*x - 6*sqrt(a**2 + 1)*atan((
sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*a + 2*sqrt
(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/sqrt(a**2
 + 1))*b**2*i*x**2 + 2*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**
2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*i - 4*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
1)*a**4 - 4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a**3*b*x + 8*sqrt(a**2...
```

3.200 $\int \frac{e^{3i\arctan(a+bx)}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 176

$$\int \frac{e^{3i\arctan(a+bx)}}{x^2}\,dx = -\frac{6ib\sqrt{1+ia+ibx}}{(i+a)^2\sqrt{1-ia-ibx}} - \frac{(1+ia+ibx)^{3/2}}{(1-ia)x\sqrt{1-ia-ibx}} + \frac{6i\sqrt{i-a}\,b\,\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i+a)^{5/2}}$$

output
```
-6*I*b*(1+I*a+I*b*x)^(1/2)/(I+a)^2/(1-I*a-I*b*x)^(1/2)-(1+I*a+I*b*x)^(3/2)
/(1-I*a)/x/(1-I*a-I*b*x)^(1/2)+6*I*(I-a)^(1/2)*b*arctanh((I+a)^(1/2)*(1+I*
a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-I*b*x)^(1/2))/(I+a)^(5/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.82

$$\int \frac{e^{3i\arctan(a+bx)}}{x^2}\,dx = \frac{\frac{\sqrt{1+ia+ibx}(1+a^2-5ibx+abx)}{x\sqrt{-i(i+a+bx)}} + \frac{6i\sqrt{-1-ia}\,b\,\text{arctanh}\left(\frac{\sqrt{-1-ia}\sqrt{-i(i+a+bx)}}{\sqrt{-1+ia}\sqrt{1+ia+ibx}}\right)}{\sqrt{-1+ia}}}{(i+a)^2}$$

input
```
Integrate[E^((3*I)*ArcTan[a + b*x])/x^2,x]
```

output
```
((Sqrt[1 + I*a + I*b*x]*(1 + a^2 - (5*I)*b*x + a*b*x))/(x*Sqrt[(-I)*(I + a
 + b*x)]) + ((6*I)*Sqrt[-1 - I*a]*b*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-I)*(I +
 a + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/Sqrt[-1 + I*a])/(I +
a)^2
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5618, 105, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3i\arctan(a+bx)}}{x^2}\,dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{(ia+ibx+1)^{3/2}}{x^2(-ia-ibx+1)^{3/2}}dx$$

$$\Big\downarrow \text{105}$$

$$-\frac{3b\int \frac{\sqrt{ia+ibx+1}}{x(-ia-ibx+1)^{3/2}}dx}{a+i}-\frac{(ia+ibx+1)^{3/2}}{(1-ia)x\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \text{105}$$

$$-\frac{3b\left(\frac{(-a+i)\int \frac{1}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{a+i}+\frac{2\sqrt{ia+ibx+1}}{(1-ia)\sqrt{-ia-ibx+1}}\right)}{a+i}-\frac{(ia+ibx+1)^{3/2}}{(1-ia)x\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{3b\left(\frac{2(-a+i)\int \frac{1}{-ia+\frac{(1-ia)(ia+ibx+1)}{-ia-ibx+1}-1}d\frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}}{a+i}+\frac{2\sqrt{ia+ibx+1}}{(1-ia)\sqrt{-ia-ibx+1}}\right)}{a+i}-\frac{(ia+ibx+1)^{3/2}}{(1-ia)x\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{3b\left(\frac{2\sqrt{ia+ibx+1}}{(1-ia)\sqrt{-ia-ibx+1}}-\frac{2i\sqrt{-a+i}\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(a+i)^{3/2}}\right)}{a+i}-\frac{(ia+ibx+1)^{3/2}}{(1-ia)x\sqrt{-ia-ibx+1}}$$

input
```
Int[E^((3*I)*ArcTan[a + b*x])/x^2,x]
```

output
```
-((1 + I*a + I*b*x)^(3/2)/((1 - I*a)*x*Sqrt[1 - I*a - I*b*x])) - (3*b*((2*
Sqrt[1 + I*a + I*b*x])/((1 - I*a)*Sqrt[1 - I*a - I*b*x]) - ((2*I)*Sqrt[I -
 a]*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a
- I*b*x])])/(I + a)^(3/2)))/(I + a)
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.99

method	result
risch	$\frac{i\sqrt{b^2x^2+2abx+a^2+1}\,(-i+a)}{(i+a)^2 x} + \frac{b\left(-\frac{3\sqrt{a^2+1}\,\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{x}\right)}{i+a} - \frac{4i(ia-1)\sqrt{\left(x+\frac{i+a}{b}\right)^2b^2-2ib\left(x+\frac{i+a}{b}\right)}}{b(i+a)\left(x+\frac{i+a}{b}\right)}\right.}{a^2+2ia-1}$
default	$(-ia^3-3a^2+3ia+1)\left(-\frac{1}{(a^2+1)x\sqrt{b^2x^2+2abx+a^2+1}} - \frac{3ab\left(\frac{1}{(a^2+1)\sqrt{b^2x^2+2abx+a^2+1}} - \frac{2ab(2b^2x}{(a^2+1)\left(4b^2(a^2+1)-4a^2b\right.}\right.}{}\right.$

input
```
int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
I*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*(-I+a)/(I+a)^2/x+1/(a^2-1+2*I*a)*b*(-3*(a^
2+1)^(1/2)/(I+a)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+
1)^(1/2))/x)-4*I*(I*a-1)/b/(I+a)/(x+(I+a)/b)*((x+(I+a)/b)^2*b^2-2*I*b*(x+(
I+a)/b))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 389 vs. 2(116) = 232.

Time = 0.14 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.21

$$\int \frac{e^{3i\arctan(a+bx)}}{x^2}\,dx =$$

$$-\frac{(-i\,a-5)b^2x^2+(-i\,a^2-4\,a-5i)bx-3\left((a^2+2i\,a-1)bx^2+(a^3+3i\,a^2-3\,a-i)x\right)\sqrt{\frac{(a-}{a^5+5i\,a^4-10\,a^3}}}{}$$

input
```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
-((-I*a - 5)*b^2*x^2 + (-I*a^2 - 4*a - 5*I)*b*x - 3*((a^2 + 2*I*a - 1)*b*x
^2 + (a^3 + 3*I*a^2 - 3*a - I)*x)*sqrt((a - I)*b^2/(a^5 + 5*I*a^4 - 10*a^3
 - 10*I*a^2 + 5*a + I))*log(-(b^2*x + (a^3 + 3*I*a^2 - 3*a - I)*sqrt((a -
I)*b^2/(a^5 + 5*I*a^4 - 10*a^3 - 10*I*a^2 + 5*a + I)) - sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1)*b)/b) + 3*((a^2 + 2*I*a - 1)*b*x^2 + (a^3 + 3*I*a^2 - 3*a
- I)*x)*sqrt((a - I)*b^2/(a^5 + 5*I*a^4 - 10*a^3 - 10*I*a^2 + 5*a + I))*lo
g(-(b^2*x - (a^3 + 3*I*a^2 - 3*a - I)*sqrt((a - I)*b^2/(a^5 + 5*I*a^4 - 10
*a^3 - 10*I*a^2 + 5*a + I)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b)/b) + sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1)*((-I*a - 5)*b*x - I*a^2 - I))/((a^2 + 2*I*
a - 1)*b*x^2 + (a^3 + 3*I*a^2 - 3*a - I)*x)
```

Sympy [F]

$$\int \frac{e^{3i\arctan(a+bx)}}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)/x**2,x)
```

output
```
-I*(Integral(I/(a**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**
3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**4*sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1) + x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(
-3*a/(a**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**3*sqrt(a**
2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
 1) + x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a**3/(a**2
*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**3*sqrt(a**2 + 2*a*b*
x + b**2*x**2 + 1) + b**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**2
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*a**2/(a**2*x**2
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**3*sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1) + b**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**2*sqrt
(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*b*x/(a**2*x**2*sqrt(a*
*2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2
 + 1) + b**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**2*sqrt(a**2 +
2*a*b*x + b**2*x**2 + 1)), x) + Integral(b**3*x**3/(a**2*x**2*sqrt(a**2 +
2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
 + b**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**2*sqrt(a**2 + 2*a*b
*x + b**2*x**2 + 1)), x) + Integral(-3*I*b**2*x**2/(a**2*x**2*sqrt(a**2 +
2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
 + b**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**2*sqrt(a**2 + 2*...
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 992 vs. 2(116) = 232.

Time = 0.04 (sec) , antiderivative size = 992, normalized size of antiderivative = 5.64

$$\int \frac{e^{3i\arctan(a+bx)}}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
-I*a*b^4*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) -
 3*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^2*b^4*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2) - I*a^2*b^3/((a^2*b^2 - (a^2 + 1
)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 3*(-I*a^3 - 3*a^2 + 3*I*a + 1)
*a^3*b^3/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2
 + 1)^2) - 3*(I*a^2*b + 2*a*b - I*b)*a*b^3*x/((a^2*b^2 - (a^2 + 1)*b^2)*sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) + 2*(-I*a^3 - 3*a^2 + 3*I*a + 1
)*b^4*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2
+ 1)) - 3*(I*a^2*b + 2*a*b - I*b)*a^2*b^2/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) + 2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a
*b^3/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1
)) + 3*(I*a*b^2 + b^2)*b^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1)) + 3*(I*a*b^2 + b^2)*a*b/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1)) + 3*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a*b*arcsinh(
2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b
^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(
x)))/(a^2 + 1)^(5/2) + I*b/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 3*(-I*a^3 -
 3*a^2 + 3*I*a + 1)*a*b/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2) +
3*(I*a^2*b + 2*a*b - I*b)*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b
^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sq...
```

Giac [F]

$$\int \frac{e^{3i\arctan(a+bx)}}{x^2}\,dx = \int \frac{(i\,bx + i\,a + 1)^3}{\left((bx+a)^2+1\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^2,x, algorithm="giac")
```

output
```
undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3i\arctan(a+bx)}}{x^2}\,dx = \int \frac{(1 + a\,1\mathrm{i} + b\,x\,1\mathrm{i})^3}{x^2\,\left((a+b\,x)^2+1\right)^{3/2}}\,dx$$

input
```
int((a*1i + b*x*1i + 1)^3/(x^2*((a + b*x)^2 + 1)^(3/2)),x)
```

output
```
int((a*1i + b*x*1i + 1)^3/(x^2*((a + b*x)^2 + 1)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1546, normalized size of antiderivative = 8.78

$$\int \frac{e^{3i\arctan(a+bx)}}{x^2}\,dx = \text{Too large to display}$$

input
```
int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^2,x)
```

output

```
(6*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/sq
rt(a**2 + 1))*a**6*b*i*x + 12*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*a**5*b**2*i*x**2 + 18*sqrt(a**2 +
 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*
a**5*b*x + 6*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i +
 b*i*x)/sqrt(a**2 + 1))*a**4*b**3*i*x**3 + 36*sqrt(a**2 + 1)*atan((sqrt(a*
*2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*a**4*b**2*x**2 -
12*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/sq
rt(a**2 + 1))*a**4*b*i*x + 18*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*a**3*b**3*x**3 - 36*sqrt(a**2 + 1
)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*a*
*3*b**2*i*x**2 + 12*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 +
 1)*i + b*i*x)/sqrt(a**2 + 1))*a**3*b*x - 18*sqrt(a**2 + 1)*atan((sqrt(a**
2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*a**2*b**3*i*x**3 -
 12*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/s
qrt(a**2 + 1))*a**2*b**2*x**2 - 18*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*
x + b**2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*a**2*b*i*x - 6*sqrt(a**2 + 1
)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*a*
b**3*x**3 - 6*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i
+ b*i*x)/sqrt(a**2 + 1))*a*b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*a...
```

3.201 $\int \frac{e^{3i\arctan(a+bx)}}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 264

$$\int \frac{e^{3i\arctan(a+bx)}}{x^3}\,dx = \frac{3(3i-2a)b^2\sqrt{1+ia+ibx}}{(1+ia)(i+a)^3\sqrt{1-ia-ibx}} + \frac{(3i-2a)b(1+ia+ibx)^{3/2}}{2(1+ia)(i+a)^2x\sqrt{1-ia-ibx}} - \frac{(1+ia+ibx)^{5/2}}{2(1+a^2)x^2\sqrt{1-ia-ibx}} + \frac{3(3+2ia)b^2\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{\sqrt{i-a}(i+a)^{7/2}}$$

output
```
3*(3*I-2*a)*b^2*(1+I*a+I*b*x)^(1/2)/(1+I*a)/(I+a)^3/(1-I*a-I*b*x)^(1/2)+1/
2*(3*I-2*a)*b*(1+I*a+I*b*x)^(3/2)/(1+I*a)/(I+a)^2/x/(1-I*a-I*b*x)^(1/2)-1/
2*(1+I*a+I*b*x)^(5/2)/(a^2+1)/x^2/(1-I*a-I*b*x)^(1/2)+3*(3+2*I*a)*b^2*arct
anh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-I*b*x)^(1/2))/(I-a)
^(1/2)/(I+a)^(7/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.73

$$\int \frac{e^{3i\arctan(a+bx)}}{x^3}\,dx = \frac{\frac{\sqrt{1+ia+ibx}(i+a+ia^2+a^3-5bx+5iabx+14ib^2x^2-ab^2x^2)}{x^2\sqrt{-i(i+a+bx)}}-\frac{6i\sqrt{-1-ia}(-3i+2a)b^2\operatorname{arctanh}\left(\frac{\sqrt{-1-ia}\sqrt{-i(i+a+bx)}}{\sqrt{-1+ia}\sqrt{1+ia+ibx}}\right)}{\sqrt{-1+ia}(-i+a)}}{2(i+a)^3}$$

input
```
Integrate[E^((3*I)*ArcTan[a + b*x])/x^3,x]
```

output
```
((Sqrt[1 + I*a + I*b*x]*(I + a + I*a^2 + a^3 - 5*b*x + (5*I)*a*b*x + (14*I
)*b^2*x^2 - a*b^2*x^2))/(x^2*Sqrt[(-I)*(I + a + b*x)]) - ((6*I)*Sqrt[-1 -
I*a]*(-3*I + 2*a)*b^2*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-I)*(I + a + b*x)])/(S
qrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/(Sqrt[-1 + I*a]*(-I + a)))/(2*(I +
a)^3)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5618, 107, 105, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3i\arctan(a+bx)}}{x^3}\,dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{(ia+ibx+1)^{3/2}}{x^3(-ia-ibx+1)^{3/2}}dx$$

$$\Big\downarrow \text{107}$$

$$\frac{(-2a+3i)b\int \frac{(ia+ibx+1)^{3/2}}{x^2(-ia-ibx+1)^{3/2}}dx}{2\left(a^2+1\right)}-\frac{(ia+ibx+1)^{5/2}}{2\left(a^2+1\right)x^2\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \; 105$$

$$\frac{(-2a+3i)b\left(-\frac{3b\int \frac{\sqrt{ia+ibx+1}}{x(-ia-ibx+1)^{3/2}}dx}{a+i}-\frac{(ia+ibx+1)^{3/2}}{(1-ia)x\sqrt{-ia-ibx+1}}\right)}{2\left(a^2+1\right)}-\frac{(ia+ibx+1)^{5/2}}{2\left(a^2+1\right)x^2\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \; 105$$

$$\frac{(-2a+3i)b\left(-\frac{3b\left(\frac{(-a+i)\int \frac{1}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{a+i}+\frac{2\sqrt{ia+ibx+1}}{(1-ia)\sqrt{-ia-ibx+1}}\right)}{a+i}-\frac{(ia+ibx+1)^{3/2}}{(1-ia)x\sqrt{-ia-ibx+1}}\right)}{2\left(a^2+1\right)}-\frac{(ia+ibx+1)^{5/2}}{2\left(a^2+1\right)x^2\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \; 104$$

$$\frac{(-2a+3i)b\left(-\frac{3b\left(\frac{2(-a+i)\int \frac{1}{-ia+\frac{(1-ia)(ia+ibx+1)}{-ia-ibx+1}-1}d\frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}}{a+i}+\frac{2\sqrt{ia+ibx+1}}{(1-ia)\sqrt{-ia-ibx+1}}\right)}{a+i}-\frac{(ia+ibx+1)^{3/2}}{(1-ia)x\sqrt{-ia-ibx+1}}\right)}{2\left(a^2+1\right)}-\frac{(ia+ibx+1)^{5/2}}{2\left(a^2+1\right)x^2\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \; 221$$

$$\frac{(-2a+3i)b\left(-\frac{3b\left(\frac{2\sqrt{ia+ibx+1}}{(1-ia)\sqrt{-ia-ibx+1}}-\frac{2i\sqrt{-a+i}\,\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(a+i)^{3/2}}\right)}{a+i}-\frac{(ia+ibx+1)^{3/2}}{(1-ia)x\sqrt{-ia-ibx+1}}\right)}{2\left(a^2+1\right)}-\frac{(ia+ibx+1)^{5/2}}{2\left(a^2+1\right)x^2\sqrt{-ia-ibx+1}}$$

input
```
Int[E^((3*I)*ArcTan[a + b*x])/x^3,x]
```

output
```
-1/2*(1 + I*a + I*b*x)^(5/2)/((1 + a^2)*x^2*Sqrt[1 - I*a - I*b*x]) + ((3*I
 - 2*a)*b*(-((1 + I*a + I*b*x)^(3/2)/((1 - I*a)*x*Sqrt[1 - I*a - I*b*x]))
- (3*b*((2*Sqrt[1 + I*a + I*b*x])/((1 - I*a)*Sqrt[1 - I*a - I*b*x]) - ((2*
I)*Sqrt[I - a]*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sq
rt[1 - I*a - I*b*x])])/(I + a)^(3/2)))/(I + a)))/(2*(1 + a^2))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.02

method	result
risch	$\frac{i(-a b^3x^3+6ib^3x^3-a^2b^2x^2+12ia\,b^2x^2+a^3bx+6ia^2bx+a^4+b^2x^2+abx+6ibx+2a^2+1)}{2x^2(i+a)^3\sqrt{b^2x^2+2abx+a^2+1}}+\frac{b^2\left(-\frac{3\left(-2a^2+ia-3\right)\ln\left(\frac{2a^2+2+2abx+2\sqrt{}}{}\right)}{(i+a)\sqrt{a^2}}\right.}{}$
default	$-\frac{2ib^3(2b^2x+2ab)}{(4b^2(a^2+1)-4a^2b^2)\sqrt{b^2x^2+2abx+a^2+1}}+(-ia^3-3a^2+3ia+1)\left(-\frac{1}{2(a^2+1)x^2\sqrt{b^2x^2+2abx+a^2+1}}-\frac{5ab\left(-\right.}{}\right.$

input
```
int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*I*(-a*b^3*x^3-a^2*b^2*x^2+a^3*b*x+6*I*b^3*x^3+a^4+b^2*x^2+12*I*a*b^2*x
^2+a*b*x+6*I*a^2*b*x+2*a^2+6*I*b*x+1)/x^2/(I+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^
(1/2)+1/2/(a^3-3*a+3*I*a^2-I)*b^2*(-3*(I*a-2*a^2-3)/(I+a)/(a^2+1)^(1/2)*ln
((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+8*I*(I
*a-1)/b/(I+a)/(x+(I+a)/b)*((x+(I+a)/b)^2*b^2-2*I*b*(x+(I+a)/b))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 574 vs. 2(182) = 364.

Time = 0.16 (sec) , antiderivative size = 574, normalized size of antiderivative = 2.17

$$\int \frac{e^{3i\arctan(a+bx)}}{x^3}\,dx$$

$$=\frac{(-i\,a-14)b^3x^3+(-i\,a^2-13\,a-14i)b^2x^2-3\left((a^3+3i\,a^2-3\,a-i)bx^3+(a^4+4i\,a^3-6\,a^2-4i\,a+1\right.}{}$$

input
```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
1/2*((-I*a - 14)*b^3*x^3 + (-I*a^2 - 13*a - 14*I)*b^2*x^2 - 3*((a^3 + 3*I*
a^2 - 3*a - I)*b*x^3 + (a^4 + 4*I*a^3 - 6*a^2 - 4*I*a + 1)*x^2)*sqrt((4*a^
2 - 12*I*a - 9)*b^4/(a^8 + 6*I*a^7 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2
 + 6*I*a - 1))*log(-((2*a - 3*I)*b^3*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
*(2*a - 3*I)*b^2 + (a^5 + 3*I*a^4 - 2*a^3 + 2*I*a^2 - 3*a - I)*sqrt((4*a^2
 - 12*I*a - 9)*b^4/(a^8 + 6*I*a^7 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2
+ 6*I*a - 1)))/((2*a - 3*I)*b^2)) + 3*((a^3 + 3*I*a^2 - 3*a - I)*b*x^3 + (
a^4 + 4*I*a^3 - 6*a^2 - 4*I*a + 1)*x^2)*sqrt((4*a^2 - 12*I*a - 9)*b^4/(a^8
 + 6*I*a^7 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*a - 1))*log(-((2*
a - 3*I)*b^3*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*a - 3*I)*b^2 - (a^5
+ 3*I*a^4 - 2*a^3 + 2*I*a^2 - 3*a - I)*sqrt((4*a^2 - 12*I*a - 9)*b^4/(a^8
+ 6*I*a^7 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*a - 1)))/((2*a - 3
*I)*b^2)) + ((-I*a - 14)*b^2*x^2 + I*a^3 - 5*(a + I)*b*x - a^2 + I*a - 1)*
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((a^3 + 3*I*a^2 - 3*a - I)*b*x^3 + (a^4
 + 4*I*a^3 - 6*a^2 - 4*I*a + 1)*x^2)
```

Sympy [F]

$$\int \frac{e^{3i\arctan(a+bx)}}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)/x**3,x)
```

output
```
-I*(Integral(I/(a**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**
4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**5*sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1) + x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(
-3*a/(a**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**4*sqrt(a**
2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
 1) + x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a**3/(a**2
*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**4*sqrt(a**2 + 2*a*b*
x + b**2*x**2 + 1) + b**2*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**3
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*a**2/(a**2*x**3
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**4*sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1) + b**2*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**3*sqrt
(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*b*x/(a**2*x**3*sqrt(a*
*2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2
 + 1) + b**2*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**3*sqrt(a**2 +
2*a*b*x + b**2*x**2 + 1)), x) + Integral(b**3*x**3/(a**2*x**3*sqrt(a**2 +
2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
 + b**2*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**3*sqrt(a**2 + 2*a*b
*x + b**2*x**2 + 1)), x) + Integral(-3*I*b**2*x**2/(a**2*x**3*sqrt(a**2 +
2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
 + b**2*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**3*sqrt(a**2 + 2*...
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1536 vs. $2(182) = 364$.

Time = 0.05 (sec) , antiderivative size = 1536, normalized size of antiderivative = 5.82

$$\int \frac{e^{3i\arctan(a+bx)}}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
15/2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^3*b^5*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^3) + 15/2*(-I*a^3 - 3*a^2 + 3*I*a
 + 1)*a^4*b^4/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
*(a^2 + 1)^3) + 9*(I*a^2*b + 2*a*b - I*b)*a^2*b^4*x/((a^2*b^2 - (a^2 + 1)*
b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2) + I*b^5*x/((a^2*b^2 -
(a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 13/2*(-I*a^3 - 3*a^2 +
 3*I*a + 1)*a*b^5*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*(a^2 + 1)^2) + 9*(I*a^2*b + 2*a*b - I*b)*a^3*b^3/((a^2*b^2 - (a^2 +
 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2) + I*a*b^4/((a^2*b^
2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 13/2*(-I*a^3 - 3*a
^2 + 3*I*a + 1)*a^2*b^4/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)*(a^2 + 1)^2) - 3*(I*a*b^2 + b^2)*a*b^3*x/((a^2*b^2 - (a^2 + 1)*
b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) - 6*(I*a^2*b + 2*a*b - I
*b)*b^4*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^
2 + 1)) - 3*(I*a*b^2 + b^2)*a^2*b^2/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) - 6*(I*a^2*b + 2*a*b - I*b)*a*b^3/((a^2*
b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) - 15/2*(
-I*a^3 - 3*a^2 + 3*I*a + 1)*a^2*b^2*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(
a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x))
+ 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(7/2) + 15/2...
```

Giac [F]

$$\int \frac{e^{3i\arctan(a+bx)}}{x^3}\,dx = \int \frac{(i\,bx + i\,a + 1)^3}{\left((bx + a)^2 + 1\right)^{\frac{3}{2}} x^3}\,dx$$

input
```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^3,x, algorithm="giac")
```

output
```
undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3i\arctan(a+bx)}}{x^3}\,dx=\int \frac{{\left(1+a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}\right)}^3}{x^3\,{\left({\left(a+b\,x\right)}^2+1\right)}^{3/2}}\,dx$$

input
```
int((a*1i + b*x*1i + 1)^3/(x^3*((a + b*x)^2 + 1)^(3/2)),x)
```

output
```
int((a*1i + b*x*1i + 1)^3/(x^3*((a + b*x)^2 + 1)^(3/2)), x)
```

Reduce [F]

$$\int \frac{e^{3i\arctan(a+bx)}}{x^3}\,dx=\int \frac{(1+i(bx+a))^3}{\left(1+(bx+a)^2\right)^{\frac{3}{2}} x^3}dx$$

input
```
int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^3,x)
```

output
```
int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^3,x)
```

3.202 $\int \frac{e^{3i\arctan(a+bx)}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 338

$$\int \frac{e^{3i\arctan(a+bx)}}{x^4}\,dx = \frac{(52+51ia-2a^2)\,b^3\sqrt{1+ia+ibx}}{6(i-a)(i+a)^4\sqrt{1-ia-ibx}} - \frac{(i-a)\sqrt{1+ia+ibx}}{3(i+a)x^3\sqrt{1-ia-ibx}} + \frac{7ib\sqrt{1+ia+ibx}}{6(i+a)^2x^2\sqrt{1-ia-ibx}} + \frac{(19+16ia)b^2\sqrt{1+ia+ibx}}{6(i-a)(i+a)^3x\sqrt{1-ia-ibx}} - \frac{(11i-18a-6ia^2)\,b^3\operatorname{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{3/2}(i+a)^{9/2}}$$

output
```
1/6*(52+51*I*a-2*a^2)*b^3*(1+I*a+I*b*x)^(1/2)/(I-a)/(I+a)^4/(1-I*a-I*b*x)^
(1/2)-1/3*(I-a)*(1+I*a+I*b*x)^(1/2)/(I+a)/x^3/(1-I*a-I*b*x)^(1/2)+7/6*I*b*
(1+I*a+I*b*x)^(1/2)/(I+a)^2/x^2/(1-I*a-I*b*x)^(1/2)+1/6*(19+16*I*a)*b^2*(1
+I*a+I*b*x)^(1/2)/(I-a)/(I+a)^3/x/(1-I*a-I*b*x)^(1/2)-(11*I-18*a-6*I*a^2)*
b^3*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-I*b*x)^(1/2
))/(I-a)^(3/2)/(I+a)^(9/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.83

$$\int \frac{e^{3i\arctan(a+bx)}}{x^4}\,dx =$$
$$-\frac{2(-1+ia)^{3/2}(1+ia)(i+a)^2(1+ia+ibx)^{5/2}+(3i-4a)(-1+ia)^{5/2}bx(1+ia+ibx)^{5/2}-i(-11-}{6(-1+}$$

input
```
Integrate[E^((3*I)*ArcTan[a + b*x])/x^4,x]
```

output
```
-1/6*(2*(-1 + I*a)^(3/2)*(1 + I*a)*(I + a)^2*(1 + I*a + I*b*x)^(5/2) + (3*
I - 4*a)*(-1 + I*a)^(5/2)*b*x*(1 + I*a + I*b*x)^(5/2) - I*(-11 - (18*I)*a
+ 6*a^2)*b^2*x^2*(I*Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x]*(1 + a^2 - (5*I)*
b*x + a*b*x) - 6*Sqrt[-1 - I*a]*b*x*Sqrt[(-I)*(I + a + b*x)]*ArcTanh[(Sqrt
[-1 - I*a]*Sqrt[(-I)*(I + a + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x]
)]))/((-1 + I*a)^(5/2)*(1 + a^2)^2*x^3*Sqrt[(-I)*(I + a + b*x)])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {5618, 109, 25, 27, 168, 27, 168, 25, 27, 169, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3i\arctan(a+bx)}}{x^4}dx$$
$$\Big\downarrow \ 5618$$
$$\int \frac{(ia+ibx+1)^{3/2}}{x^4(-ia-ibx+1)^{3/2}}dx$$
$$\Big\downarrow \ 109$$
$$-\frac{\int -\frac{b(7(i-a)-6bx)}{x^3(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}dx}{3(1-ia)}-\frac{(-a+i)\sqrt{ia+ibx+1}}{3(a+i)x^3\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b(7(i-a)-6bx)}{x^3(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}dx}{3(1-ia)}-\frac{(-a+i)\sqrt{ia+ibx+1}}{3(a+i)x^3\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{7(i-a)-6bx}{x^3(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}dx}{3(1-ia)}-\frac{(-a+i)\sqrt{ia+ibx+1}}{3(a+i)x^3\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \text{168}$$

$$\frac{b\left(\frac{7\sqrt{ia+ibx+1}}{2(a+i)x^2\sqrt{-ia-ibx+1}}-\frac{\int \frac{b\left(-16a^2+35ia+14(i-a)bx+19\right)}{x^2(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}dx}{2(a^2+1)}\right)}{3(1-ia)}-\frac{(-a+i)\sqrt{ia+ibx+1}}{3(a+i)x^3\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\left(\frac{7\sqrt{ia+ibx+1}}{2(a+i)x^2\sqrt{-ia-ibx+1}}-\frac{b\int \frac{-16a^2+35ia+14(i-a)bx+19}{x^2(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}dx}{2(a^2+1)}\right)}{3(1-ia)}-\frac{(-a+i)\sqrt{ia+ibx+1}}{3(a+i)x^3\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \text{168}$$

$$\frac{b\left(\frac{7\sqrt{ia+ibx+1}}{2(a+i)x^2\sqrt{-ia-ibx+1}}-\frac{b\left(-\frac{\int -\frac{b\left(3(i-a)\left(-6a^2+18ia+11\right)-\left(-16a^2+35ia+19\right)bx\right)}{x(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}dx}{a^2+1}-\frac{(-16a+19i)\sqrt{ia+ibx+1}}{(a+i)x\sqrt{-ia-ibx+1}}\right)}{2(a^2+1)}\right)}{3(1-ia)}-\frac{(-a+i)\sqrt{ia+ibx+1}}{3(a+i)x^3\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{7\sqrt{ia+ibx+1}}{2(a+i)x^2\sqrt{-ia-ibx+1}}-\frac{b\left(\frac{\int \frac{b\left(3(i-a)\left(-6a^2+18ia+11\right)-\left(-16a^2+35ia+19\right)bx\right)}{x(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}dx}{a^2+1}-\frac{(-16a+19i)\sqrt{ia+ibx+1}}{(a+i)x\sqrt{-ia-ibx+1}}\right)}{2(a^2+1)}\right)}{3(1-ia)}-\frac{(-a+i)\sqrt{ia+ibx+1}}{3(a+i)x^3\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{7\sqrt{ia+ibx+1}}{2(a+i)x^2\sqrt{-ia-ibx+1}}-\frac{b\left(\frac{b\int \frac{3(i-a)\left(-6a^2+18ia+11\right)-\left(-16a^2+35ia+19\right)bx}{x(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}dx}{a^2+1}-\frac{(-16a+19i)\sqrt{ia+ibx+1}}{(a+i)x\sqrt{-ia-ibx+1}}\right)}{2(a^2+1)}\right)}{3(1-ia)}-\frac{(-a+i)\sqrt{ia+ibx+1}}{3(a+i)x^3\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \; 169$$

$$\frac{b\left(\frac{7\sqrt{ia+ibx+1}}{2(a+i)x^2\sqrt{-ia-ibx+1}}-\frac{b\left(\frac{b\left(-\frac{\int \frac{3\left(-6ia^3-24a^2+29ia+11\right)b}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{(a+i)b}-\frac{\left(-2ia^3-53a^2+103ia+52\right)\sqrt{ia+ibx+1}}{(a+i)\sqrt{-ia-ibx+1}}\right)}{a^2+1}-\frac{(-16a+19i)\sqrt{ia+ibx+1}}{(a+i)x\sqrt{-ia-ibx+1}}\right)}{2(a^2+1)}\right)}{3(1-ia)}-\frac{(-a+i)\sqrt{ia+ibx+1}}{3(a+i)x^3\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{7\sqrt{ia+ibx+1}}{2(a+i)x^2\sqrt{-ia-ibx+1}}-\frac{b\left(\frac{b\left(-\frac{3\left(-6ia^3-24a^2+29ia+11\right)\int \frac{1}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{a+i}-\frac{\left(-2ia^3-53a^2+103ia+52\right)\sqrt{ia+ibx+1}}{(a+i)\sqrt{-ia-ibx+1}}\right)}{a^2+1}-\frac{(-16a+19i)\sqrt{ia+ibx+1}}{(a+i)x\sqrt{-ia-ibx+1}}\right)}{2(a^2+1)}\right)}{3(1-ia)}-\frac{(-a+i)\sqrt{ia+ibx+1}}{3(a+i)x^3\sqrt{-ia-ibx+1}}$$

$$\Big\downarrow \; 104$$

$$\frac{b\left(\frac{7\sqrt{ia+ibx+1}}{2(a+i)x^2\sqrt{-ia-ibx+1}}-\frac{b\left(\frac{b\left(-\frac{6\left(-6ia^3-24a^2+29ia+11\right)\int \frac{1}{-ia+\frac{(1-ia)(ia+ibx+1)}{-ia-ibx+1}-1}d\frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}}{a+i}-\frac{\left(-2ia^3-53a^2+103ia+52\right)\sqrt{ia+ibx+1}}{(a+i)\sqrt{-ia-ibx+1}}\right)}{a^2+1}\cdots}{2(a^2+1)}\cdots\right)}{3(1-ia)}$$

$$\frac{(-a+i)\sqrt{ia+ibx+1}}{3(a+i)x^3\sqrt{-ia-ibx+1}}$$

$\Big\downarrow$ 221

$$\frac{b\left(\frac{7\sqrt{ia+ibx+1}}{2(a+i)x^2\sqrt{-ia-ibx+1}}-\frac{b\left(\frac{b\left(\frac{6i\left(-6ia^3-24a^2+29ia+11\right)\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}(a+i)^{3/2}}-\frac{\left(-2ia^3-53a^2+103ia+52\right)\sqrt{ia+ibx+1}}{(a+i)\sqrt{-ia-ibx+1}}\right)}{a^2+1}-\frac{(-16a+1\cdots}{(a+i)x\cdots}}{2(a^2+1)}\cdots\right)}{3(1-ia)}$$

$$\frac{(-a+i)\sqrt{ia+ibx+1}}{3(a+i)x^3\sqrt{-ia-ibx+1}}$$

input
```
Int[E^((3*I)*ArcTan[a + b*x])/x^4,x]
```

output
```
-1/3*((I - a)*Sqrt[1 + I*a + I*b*x])/((I + a)*x^3*Sqrt[1 - I*a - I*b*x]) +
 (b*((7*Sqrt[1 + I*a + I*b*x])/(2*(I + a)*x^2*Sqrt[1 - I*a - I*b*x]) - (b*
(-(((19*I - 16*a)*Sqrt[1 + I*a + I*b*x])/((I + a)*x*Sqrt[1 - I*a - I*b*x])
) + (b*(-(((52 + (103*I)*a - 53*a^2 - (2*I)*a^3)*Sqrt[1 + I*a + I*b*x])/((
I + a)*Sqrt[1 - I*a - I*b*x])) + ((6*I)*(11 + (29*I)*a - 24*a^2 - (6*I)*a^
3)*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a -
 I*b*x])])/(Sqrt[I - a]*(I + a)^(3/2))))/(1 + a^2)))/(2*(1 + a^2))))/(3*(1
 - I*a))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.12

method	result
risch	$\frac{i\left(2a^2b^4x^4-27ia\,b^4x^4+2a^3b^3x^3-45ia^2b^3x^3-9ia^3b^2x^2-28b^4x^4+2a^5bx+9ia^4bx-58a\,b^3x^3+9ib^3x^3+2a^6-26a^2b^2x^2-9ia\,b^2x^2+4a^{3}\right.}{6x^3(-i+a)(i+a)^4\sqrt{b^2x^2+2abx+a^2+1}}$
default	Expression too large to display

input
```
int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*I*(9*I*a^4*b*x+2*a^2*b^4*x^4-9*I*a*b^2*x^2+2*a^3*b^3*x^3-45*I*a^2*b^3*
x^3-28*b^4*x^4-9*I*a^3*b^2*x^2+9*I*b^3*x^3+2*a^5*b*x-58*a*b^3*x^3-27*I*a*b
^4*x^4+2*a^6-26*a^2*b^2*x^2+9*I*b*x+4*a^3*b*x+6*a^4-26*b^2*x^2+18*I*a^2*b*
x+2*a*b*x+6*a^2+2)/x^3/(-I+a)/(I+a)^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/2/(-
I+a)/(a^4-6*a^2+4*I*a^3+1-4*I*a)*b^3*(-1/(I+a)*(12*I*a^2-6*a^3+11*I-7*a)/(
a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(
1/2))/x)-8*(a^2+1)/b/(I+a)/(x+(I+a)/b)*((x+(I+a)/b)^2*b^2-2*I*b*(x+(I+a)/b
))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 839 vs. 2(224) = 448.

Time = 0.13 (sec) , antiderivative size = 839, normalized size of antiderivative = 2.48

$$\int \frac{e^{3i\arctan(a+bx)}}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
1/6*((2*I*a^2 + 51*a - 52*I)*b^4*x^4 + (2*I*a^3 + 49*a^2 - I*a + 52)*b^3*x
^3 + 3*sqrt((36*a^4 - 216*I*a^3 - 456*a^2 + 396*I*a + 121)*b^6/(a^12 + 6*I
*a^11 - 12*a^10 - 2*I*a^9 - 27*a^8 - 36*I*a^7 - 36*I*a^5 + 27*a^4 - 2*I*a^
3 + 12*a^2 + 6*I*a - 1))*((a^5 + 3*I*a^4 - 2*a^3 + 2*I*a^2 - 3*a - I)*b*x^
4 + (a^6 + 4*I*a^5 - 5*a^4 - 5*a^2 - 4*I*a + 1)*x^3)*log(-((6*a^2 - 18*I*a
 - 11)*b^4*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(6*a^2 - 18*I*a - 11)*b^3
 + (a^7 + 3*I*a^6 - a^5 + 5*I*a^4 - 5*a^3 + I*a^2 - 3*a - I)*sqrt((36*a^4
- 216*I*a^3 - 456*a^2 + 396*I*a + 121)*b^6/(a^12 + 6*I*a^11 - 12*a^10 - 2*
I*a^9 - 27*a^8 - 36*I*a^7 - 36*I*a^5 + 27*a^4 - 2*I*a^3 + 12*a^2 + 6*I*a -
 1)))/((6*a^2 - 18*I*a - 11)*b^3)) - 3*sqrt((36*a^4 - 216*I*a^3 - 456*a^2
+ 396*I*a + 121)*b^6/(a^12 + 6*I*a^11 - 12*a^10 - 2*I*a^9 - 27*a^8 - 36*I*
a^7 - 36*I*a^5 + 27*a^4 - 2*I*a^3 + 12*a^2 + 6*I*a - 1))*((a^5 + 3*I*a^4 -
 2*a^3 + 2*I*a^2 - 3*a - I)*b*x^4 + (a^6 + 4*I*a^5 - 5*a^4 - 5*a^2 - 4*I*a
 + 1)*x^3)*log(-((6*a^2 - 18*I*a - 11)*b^4*x - sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*(6*a^2 - 18*I*a - 11)*b^3 - (a^7 + 3*I*a^6 - a^5 + 5*I*a^4 - 5*a^3
+ I*a^2 - 3*a - I)*sqrt((36*a^4 - 216*I*a^3 - 456*a^2 + 396*I*a + 121)*b^6
/(a^12 + 6*I*a^11 - 12*a^10 - 2*I*a^9 - 27*a^8 - 36*I*a^7 - 36*I*a^5 + 27*
a^4 - 2*I*a^3 + 12*a^2 + 6*I*a - 1)))/((6*a^2 - 18*I*a - 11)*b^3)) + ((2*I
*a^2 + 51*a - 52*I)*b^3*x^3 + 2*I*a^5 + (16*a^2 - 3*I*a + 19)*b^2*x^2 - 2*
a^4 + 4*I*a^3 - 7*(a^3 + I*a^2 + a + I)*b*x - 4*a^2 + 2*I*a - 2)*sqrt(b...
```

Sympy [F]

$$\int \frac{e^{3i\arctan(a+bx)}}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((1+I*(b*x+a))**3/(1+(b*x+a)**2)**(3/2)/x**4,x)
```

output
```
-I*(Integral(I/(a**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**
5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**6*sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1) + x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(
-3*a/(a**2*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**5*sqrt(a**
2 + 2*a*b*x + b**2*x**2 + 1) + b**2*x**6*sqrt(a**2 + 2*a*b*x + b**2*x**2 +
 1) + x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(a**3/(a**2
*x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**5*sqrt(a**2 + 2*a*b*
x + b**2*x**2 + 1) + b**2*x**6*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**4
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*I*a**2/(a**2*x**4
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**5*sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1) + b**2*x**6*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**4*sqrt
(a**2 + 2*a*b*x + b**2*x**2 + 1)), x) + Integral(-3*b*x/(a**2*x**4*sqrt(a*
*2 + 2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2
 + 1) + b**2*x**6*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**4*sqrt(a**2 +
2*a*b*x + b**2*x**2 + 1)), x) + Integral(b**3*x**3/(a**2*x**4*sqrt(a**2 +
2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
 + b**2*x**6*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**4*sqrt(a**2 + 2*a*b
*x + b**2*x**2 + 1)), x) + Integral(-3*I*b**2*x**2/(a**2*x**4*sqrt(a**2 +
2*a*b*x + b**2*x**2 + 1) + 2*a*b*x**5*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
 + b**2*x**6*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1) + x**4*sqrt(a**2 + 2*...
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2313 vs. $2(224) = 448$.

Time = 0.06 (sec) , antiderivative size = 2313, normalized size of antiderivative = 6.84

$$\int \frac{e^{3i\arctan(a+bx)}}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
-35/2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^4*b^6*x/((a^2*b^2 - (a^2 + 1)*b^2)*sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^4) - 35/2*(-I*a^3 - 3*a^2 + 3*I*
a + 1)*a^5*b^5/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)*(a^2 + 1)^4) - 45/2*(I*a^2*b + 2*a*b - I*b)*a^3*b^5*x/((a^2*b^2 - (a^2 +
 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^3) - I*a*b^6*x/((a^2*
b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)) + 115/6*
(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^2*b^6*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^3) - 45/2*(I*a^2*b + 2*a*b - I*b)*a^4*
b^4/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)
^3) - I*a^2*b^5/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1)*(a^2 + 1)) + 115/6*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a^3*b^5/((a^2*b^2 - (a^
2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^3) + 9*(I*a*b^2 +
b^2)*a^2*b^4*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)*(a^2 + 1)^2) + 39/2*(I*a^2*b + 2*a*b - I*b)*a*b^5*x/((a^2*b^2 - (a^2 + 1
)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2) - 8/3*(-I*a^3 - 3*a^
2 + 3*I*a + 1)*b^6*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)*(a^2 + 1)^2) + 9*(I*a*b^2 + b^2)*a^3*b^3/((a^2*b^2 - (a^2 + 1)*b^2
)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)^2) + 39/2*(I*a^2*b + 2*a*b -
 I*b)*a^2*b^4/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
*(a^2 + 1)^2) - 8/3*(-I*a^3 - 3*a^2 + 3*I*a + 1)*a*b^5/((a^2*b^2 - (a^2...
```

Giac [F]

$$\int \frac{e^{3i\arctan(a+bx)}}{x^4}\,dx = \int \frac{(i\,bx + i\,a + 1)^3}{\left((bx + a)^2 + 1\right)^{\frac{3}{2}} x^4}\,dx$$

input
```
integrate((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^4,x, algorithm="giac")
```

output
```
undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3i\arctan(a+bx)}}{x^4}\,dx = \int \frac{(1 + a\,1\mathrm{i} + b\,x\,1\mathrm{i})^3}{x^4\,\left((a + b\,x)^2 + 1\right)^{3/2}}\,dx$$

input
```
int((a*1i + b*x*1i + 1)^3/(x^4*((a + b*x)^2 + 1)^(3/2)),x)
```

output
```
int((a*1i + b*x*1i + 1)^3/(x^4*((a + b*x)^2 + 1)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 5.75 (sec) , antiderivative size = 2831, normalized size of antiderivative = 8.38

$$\int \frac{e^{3i\arctan(a+bx)}}{x^4}\,dx = \text{Too large to display}$$

input
```
int((1+I*(b*x+a))^3/(1+(b*x+a)^2)^(3/2)/x^4,x)
```

output

```
(36*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/s
qrt(a**2 + 1))*a**8*b**3*i*x**3 + 72*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*
b*x + b**2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*a**7*b**4*i*x**4 + 216*sqr
t(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/sqrt(a**
2 + 1))*a**7*b**3*x**3 + 36*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*a**6*b**5*i*x**5 + 432*sqrt(a**2 +
1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*a
**6*b**4*x**4 - 462*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 +
 1)*i + b*i*x)/sqrt(a**2 + 1))*a**6*b**3*i*x**3 + 216*sqrt(a**2 + 1)*atan(
(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*a**5*b**5
*x**5 - 996*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i +
b*i*x)/sqrt(a**2 + 1))*a**5*b**4*i*x**4 - 342*sqrt(a**2 + 1)*atan((sqrt(a*
*2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*a**5*b**3*x**3 -
498*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/s
qrt(a**2 + 1))*a**4*b**5*i*x**5 - 1116*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*
a*b*x + b**2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*a**4*b**4*x**4 - 192*sqr
t(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/sqrt(a**
2 + 1))*a**4*b**3*i*x**3 - 558*sqrt(a**2 + 1)*atan((sqrt(a**2 + 2*a*b*x +
b**2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1))*a**3*b**5*x**5 + 612*sqrt(a**2 +
 1)*atan((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*i + b*i*x)/sqrt(a**2 + 1...
```

3.203 $\int e^{-i\arctan(a+bx)}x^4\,dx$

Optimal result

Integrand size = 16, antiderivative size = 315

$$\begin{aligned}\int e^{-i\arctan(a+bx)}x^4\,dx = &-\frac{(3i-12a-24ia^2+16a^3+8ia^4)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5}\\&+\frac{7(i+16a+6ia^2-24a^3)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{120b^5}\\&+\frac{(i-8a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{20b^3}\\&+\frac{x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{5b^2}\\&-\frac{(13i-14a-36ia^2)(1-ia-ibx)^{5/2}\sqrt{1+ia+ibx}}{60b^5}\\&+\frac{(3+12ia-24a^2-16ia^3+8a^4)\operatorname{arcsinh}(a+bx)}{8b^5}\end{aligned}$$

output

```
-1/8*(3*I-12*a-24*I*a^2+16*a^3+8*I*a^4)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^
(1/2)/b^5+7/120*(I+16*a+6*I*a^2-24*a^3)*(1-I*a-I*b*x)^(3/2)*(1+I*a+I*b*x)^
(1/2)/b^5+1/20*(I-8*a)*x^2*(1-I*a-I*b*x)^(3/2)*(1+I*a+I*b*x)^(1/2)/b^3+1/5
*x^3*(1-I*a-I*b*x)^(3/2)*(1+I*a+I*b*x)^(1/2)/b^2-1/60*(13*I-14*a-36*I*a^2)
*(1-I*a-I*b*x)^(5/2)*(1+I*a+I*b*x)^(1/2)/b^5+1/8*(3+12*I*a-24*a^2-16*I*a^3
+8*a^4)*arcsinh(b*x+a)/b^5
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.79

$$\int e^{-i\arctan(a+bx)}x^4\,dx$$
$$= \frac{i\sqrt{1+ia+ibx}(-64+226a^4+24ia^5+109ibx+77b^2x^2-62ib^3x^3-54b^4x^4+24ib^5x^5+2a^3(-41i+72}{120b^5\sqrt{-i(i+a+bx)}}$$
$$+\frac{\sqrt[4]{-1}(-3i+12a+24ia^2-16a^3-8ia^4)\,\text{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{4\sqrt{-ib}b^{9/2}}$$

input
```
Integrate[x^4/E^(I*ArcTan[a + b*x]),x]
```

output
```
((I/120)*Sqrt[1 + I*a + I*b*x]*(-64 + 226*a^4 + (24*I)*a^5 + (109*I)*b*x +
 77*b^2*x^2 - (62*I)*b^3*x^3 - 54*b^4*x^4 + (24*I)*b^5*x^5 + 2*a^3*(-41*I
+ 72*b*x) + a^2*(57 - (346*I)*b*x - 84*b^2*x^2) + a*(-211*I - 346*b*x + (1
54*I)*b^2*x^2 + 64*b^3*x^3)))/(b^5*Sqrt[(-I)*(I + a + b*x)]) + ((-1)^(1/4)
*(-3*I + 12*a + (24*I)*a^2 - 16*a^3 - (8*I)*a^4)*ArcSinh[((1/2 + I/2)*Sqrt
[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/(4*Sqrt[(-I)*b]*b^(9/2))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.90, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {5618, 111, 25, 170, 25, 27, 164, 60, 62, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{-i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{x^4\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}dx$$

$$\Big\downarrow \text{111}$$

$$\frac{\int -\frac{x^2\sqrt{-ia-ibx+1}\left(3\left(a^2+1\right)-(i-8a)bx\right)}{\sqrt{ia+ibx+1}}dx}{5b^2}+\frac{x^3(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{5b^2}$$

$$\Big\downarrow \; 25$$

$$\frac{x^3(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{5b^2}-\frac{\int \frac{x^2\sqrt{-ia-ibx+1}\left(3\left(a^2+1\right)-(i-8a)bx\right)}{\sqrt{ia+ibx+1}}dx}{5b^2}$$

$$\Big\downarrow \; 170$$

$$\frac{x^3(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{5b^2}-\frac{\frac{\int -\frac{bx\sqrt{-ia-ibx+1}\left(2(i-8a)(i-a)(a+i)-\left(-36a^2+14ia+13\right)bx\right)}{\sqrt{ia+ibx+1}}dx}{4b^2}-\frac{(-8a+i)x^2(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{4b}}{5b^2}$$

$$\Big\downarrow \; 25$$

$$\frac{x^3(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{5b^2}-\frac{-\frac{\int \frac{bx\sqrt{-ia-ibx+1}\left(2(i-8a)(i-a)(a+i)-\left(-36a^2+14ia+13\right)bx\right)}{\sqrt{ia+ibx+1}}dx}{4b^2}-\frac{(-8a+i)x^2(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{4b}}{5b^2}$$

$$\Big\downarrow \; 27$$

$$\frac{x^3(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{5b^2}-\frac{-\frac{\int \frac{x\sqrt{-ia-ibx+1}\left(2(i-8a)(i-a)(a+i)-\left(-36a^2+14ia+13\right)bx\right)}{\sqrt{ia+ibx+1}}dx}{4b}-\frac{(-8a+i)x^2(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{4b}}{5b^2}$$

$$\Big\downarrow \; 164$$

$$\frac{x^3(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{5b^2}-\frac{-\frac{\frac{5\left(8a^4-16ia^3-24a^2+12ia+3\right)\int \frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}dx}{2b}-\frac{(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}\left(96a^3+2\left(-36a^2+14ia+13\right)bx-86ia^2-114a+19i\right)}{6b^2}}{4b}-\frac{(-8a+i)x^2(-ia-i\cdots}{}}{5b^2}$$

$$\Big\downarrow \; 60$$

$$\frac{x^3(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{5b^2}-\frac{-\frac{\frac{5\left(8a^4-16ia^3-24a^2+12ia+3\right)\left(\int \frac{1}{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}-\frac{(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}\left(96a^3+2\left(-36a^2+14ia+13\right)bx-8\cdots\right.}{6b^2}}{4b}\cdots}{5b^2}$$

$$\Big\downarrow \; 62$$

$$\frac{x^3(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{5b^2} - \frac{-\frac{\frac{5\left(8a^4-16ia^3-24a^2+12ia+3\right)\left(\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx - \frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b} - \frac{(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}\left(96a^3+2\left(-36a^2+14ia+13\right)bx \ldots}{6b^2}}{4b}}{5b^2}$$

$\big\downarrow$ 1090

$$\frac{x^3(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{5b^2} - \frac{-\frac{\frac{5\left(8a^4-16ia^3-24a^2+12ia+3\right)\left(\frac{\int \frac{1}{\sqrt{\frac{\left(2xb^2+2ab\right)^2}{4b^2}+1}}d\left(2xb^2+2ab\right)}{2b^2} - \frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b} - \frac{(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}\left(96a^3+2\left(-36a^2+14ia+13 \ldots}{6b^2}}{4b}}{5b^2}$$

$\big\downarrow$ 222

$$\frac{x^3(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{5b^2} - \frac{-\frac{\frac{5\left(8a^4-16ia^3-24a^2+12ia+3\right)\left(\frac{\text{arcsinh}\left(\frac{2ab+2b^2x}{2b}\right)}{b} - \frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b} - \frac{(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}\left(96a^3+2\left(-36a^2+14ia+13\right)bx-86ia^2 \ldots}{6b^2}}{4b}}{5b^2}$$

input
```
Int[x^4/E^(I*ArcTan[a + b*x]),x]
```

output
```
(x^3*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(5*b^2) - (-1/4*((I -
8*a)*x^2*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/b - (-1/6*((1 - I*
a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x]*(19*I - 114*a - (86*I)*a^2 + 96*a^3
 + 2*(13 + (14*I)*a - 36*a^2)*b*x))/b^2 + (5*(3 + (12*I)*a - 24*a^2 - (16*
I)*a^3 + 8*a^4)*(((-I)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b + Ar
cSinh[(2*a*b + 2*b^2*x)/(2*b)]/b))/(2*b))/(4*b))/(5*b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.63

method	result
risch	$-\frac{i(24b^4x^4-24a\,b^3x^3+30ib^3x^3+24a^2b^2x^2-70ia\,b^2x^2-24a^3bx+130ia^2bx+24a^4-250ia^3-32b^2x^2+116abx-45ibx-332a^2+275ia-}{120b^5}$
default	Expression too large to display

input
```
int(x^4/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/120*I*(24*b^4*x^4+30*I*b^3*x^3-24*a*b^3*x^3-70*I*a*b^2*x^2+24*a^2*b^2*x
^2+130*I*a^2*b*x-24*a^3*b*x-250*I*a^3+24*a^4-32*b^2*x^2-45*I*b*x+116*a*b*x
+275*I*a-332*a^2+64)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^5+1/8*(3+12*I*a-24*a^
2-16*I*a^3+8*a^4)/b^4*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(
1/2))/(b^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.56

$$\int e^{-i\arctan(a+bx)}x^4\,dx$$
$$= \frac{-186i\,a^5 - 1345\,a^4 + 1730i\,a^3 + 1320\,a^2 - 120\left(8\,a^4 - 16i\,a^3 - 24\,a^2 + 12i\,a + 3\right)\log\left(-bx - a + \sqrt{b^2x^2\right.}}{}$$

input
```
integrate(x^4/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/960*(-186*I*a^5 - 1345*a^4 + 1730*I*a^3 + 1320*a^2 - 120*(8*a^4 - 16*I*a
^3 - 24*a^2 + 12*I*a + 3)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
) - 8*(24*I*b^4*x^4 + 6*(-4*I*a - 5)*b^3*x^3 + 2*(12*I*a^2 + 35*a - 16*I)*
b^2*x^2 + 24*I*a^4 + 250*a^3 + (-24*I*a^3 - 130*a^2 + 116*I*a + 45)*b*x -
332*I*a^2 - 275*a + 64*I)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 300*I*a)/b^5
```

Sympy [F]

$$\int e^{-i\arctan(a+bx)}x^4\,dx = -i\int\frac{x^4\sqrt{a^2+2abx+b^2x^2+1}}{a+bx-i}\,dx$$

input
```
integrate(x**4/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2),x)
```

output
```
-I*Integral(x**4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a + b*x - I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 456 vs. 2(223) = 446.

Time = 0.14 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.45

$$\int e^{-i\arctan(a+bx)}x^4\,dx = \frac{2i\sqrt{b^2x^2+2\,abx+a^2+1}a^3x}{b^4} - \frac{i\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}x^2}{5\,b^3} + \frac{a^4\,\mathrm{arsinh}\,(bx+a)}{b^5} + \frac{i\sqrt{b^2x^2+2\,abx+a^2+1}a^4}{b^5} + \frac{3i\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}ax}{5\,b^4} + \frac{3\sqrt{b^2x^2+2\,abx+a^2+1}a^2x}{b^4} - \frac{2i\,a^3\,\mathrm{arsinh}\,(bx+a)}{b^5} - \frac{6i\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}a^2}{5\,b^5} - \frac{\sqrt{b^2x^2+2\,abx+a^2+1}a^3}{b^5} + \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}x}{4\,b^4} - \frac{5i\sqrt{b^2x^2+2\,abx+a^2+1}ax}{2\,b^4} - \frac{3\,a^2\,\mathrm{arsinh}\,(bx+a)}{b^5} - \frac{13\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}a}{12\,b^5} + \frac{7i\sqrt{b^2x^2+2\,abx+a^2+1}a^2}{2\,b^5} - \frac{5\sqrt{b^2x^2+2\,abx+a^2+1}x}{8\,b^4} + \frac{3i\,a\,\mathrm{arsinh}\,(bx+a)}{2\,b^5} + \frac{7i\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{15\,b^5} + \frac{27\sqrt{b^2x^2+2\,abx+a^2+1}a}{8\,b^5} + \frac{3\,\mathrm{arsinh}\,(bx+a)}{8\,b^5} - \frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{b^5}$$

input
```
integrate(x^4/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3*x/b^4 - 1/5*I*(b^2*x^2 + 2*a*b*x
 + a^2 + 1)^(3/2)*x^2/b^3 + a^4*arcsinh(b*x + a)/b^5 + I*sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)*a^4/b^5 + 3/5*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a*x/b
^4 + 3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*x/b^4 - 2*I*a^3*arcsinh(b*x +
 a)/b^5 - 6/5*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^2/b^5 - sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*a^3/b^5 + 1/4*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*x/
b^4 - 5/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*x/b^4 - 3*a^2*arcsinh(b*x
+ a)/b^5 - 13/12*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a/b^5 + 7/2*I*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)*a^2/b^5 - 5/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
*x/b^4 + 3/2*I*a*arcsinh(b*x + a)/b^5 + 7/15*I*(b^2*x^2 + 2*a*b*x + a^2 +
1)^(3/2)/b^5 + 27/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/b^5 + 3/8*arcsinh(
b*x + a)/b^5 - I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.65

$$\int e^{-i\arctan(a+bx)}x^4\,dx = -\frac{1}{120}\sqrt{(bx+a)^2+1}\left(\left(2\left(3x\left(\frac{4i\,x}{b}-\frac{4i\,ab^7+5\,b^7}{b^9}\right)-\frac{-12i\,a^2b^6-35\,ab^6+16i\,b^6}{b^9}\right)x-\frac{24i\,a^3b^5+1}{}\right.\right.$$
$$-\frac{(8\,a^4-16i\,a^3-24\,a^2+12i\,a+3)\log\left(\left|-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right|\right)}{8\,b^4|b|}$$

input
```
integrate(x^4/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/120*sqrt((b*x + a)^2 + 1)*((2*(3*x*(4*I*x/b - (4*I*a*b^7 + 5*b^7)/b^9)
- (-12*I*a^2*b^6 - 35*a*b^6 + 16*I*b^6)/b^9)*x - (24*I*a^3*b^5 + 130*a^2*b
^5 - 116*I*a*b^5 - 45*b^5)/b^9)*x - (-24*I*a^4*b^4 - 250*a^3*b^4 + 332*I*a
^2*b^4 + 275*a*b^4 - 64*I*b^4)/b^9) - 1/8*(8*a^4 - 16*I*a^3 - 24*a^2 + 12*
I*a + 3)*log(abs(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b)))/(b^4*a
bs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{-i\arctan(a+bx)}x^4\,dx = \int \frac{x^4\,\sqrt{(a+b\,x)^2+1}}{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,dx$$

input
```
int((x^4*((a + b*x)^2 + 1)^(1/2))/(a*1i + b*x*1i + 1),x)
```

output
```
int((x^4*((a + b*x)^2 + 1)^(1/2))/(a*1i + b*x*1i + 1), x)
```

Reduce [F]

$$\int e^{-i\arctan(a+bx)}x^4\,dx = \int \frac{\sqrt{b^2x^2+2abx+a^2+1}\,x^4}{bix+ai+1}dx$$

input
```
int(x^4/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x)
```

output
```
int((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*x**4)/(a*i + b*i*x + 1),x)
```

3.204 $\int e^{-i\arctan(a+bx)}x^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 233

$$\int e^{-i\arctan(a+bx)}x^3\,dx = -\frac{(3+12ia-12a^2-8ia^3)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4} - \frac{5(1-6a^2)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{24b^4} + \frac{x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2} - \frac{(1+6ia)(1-ia-ibx)^{5/2}\sqrt{1+ia+ibx}}{12b^4} - \frac{(3i-12a-12ia^2+8a^3)\,\text{arcsinh}(a+bx)}{8b^4}$$

output

```
-1/8*(3+12*I*a-12*a^2-8*I*a^3)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^4
-5/24*(-6*a^2+1)*(1-I*a-I*b*x)^(3/2)*(1+I*a+I*b*x)^(1/2)/b^4+1/4*x^2*(1-I*
a-I*b*x)^(3/2)*(1+I*a+I*b*x)^(1/2)/b^2-1/12*(1+6*I*a)*(1-I*a-I*b*x)^(5/2)*
(1+I*a+I*b*x)^(1/2)/b^4-1/8*(3*I-12*a-12*I*a^2+8*a^3)*arcsinh(b*x+a)/b^4
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.87

$$\int e^{-i\arctan(a+bx)}x^3\,dx$$
$$=\frac{\sqrt{1+ia+ibx}(-16-38ia^3+6a^4+25ibx+17b^2x^2-14ib^3x^3-6b^4x^4+5a^2(1-6ibx)+ia(-23+50i}{24b^4\sqrt{-i(i+a+bx)}}$$
$$+\frac{(-1)^{3/4}\left(-3-12ia+12a^2+8ia^3\right)\sqrt{-ib}\operatorname{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{4b^{9/2}}$$

input
```
Integrate[x^3/E^(I*ArcTan[a + b*x]),x]
```

output
```
(Sqrt[1 + I*a + I*b*x]*(-16 - (38*I)*a^3 + 6*a^4 + (25*I)*b*x + 17*b^2*x^2
 - (14*I)*b^3*x^3 - 6*b^4*x^4 + 5*a^2*(1 - (6*I)*b*x) + I*a*(-23 + (50*I)*
b*x + 18*b^2*x^2)))/(24*b^4*Sqrt[(-I)*(I + a + b*x)]) + ((-1)^(3/4)*(-3 -
(12*I)*a + 12*a^2 + (8*I)*a^3)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*S
qrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/(4*b^(9/2))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5618, 111, 25, 164, 60, 62, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3e^{-i\arctan(a+bx)}\,dx$$
$$\Big\downarrow \text{ 5618}$$
$$\int \frac{x^3\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}dx$$
$$\Big\downarrow \text{ 111}$$

$$\frac{\int -\frac{x\sqrt{-ia-ibx+1}\left(2\left(a^2+1\right)-(i-6a)bx\right)}{\sqrt{ia+ibx+1}}dx}{4b^2}+\frac{x^2(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{4b^2}$$

$\downarrow$ 25

$$\frac{x^2(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{4b^2}-\frac{\int \frac{x\sqrt{-ia-ibx+1}\left(2\left(a^2+1\right)-(i-6a)bx\right)}{\sqrt{ia+ibx+1}}dx}{4b^2}$$

$\downarrow$ 164

$$\frac{x^2(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{4b^2}-\frac{\frac{\left(8a^3-12ia^2-12a+3i\right)\int \frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}dx}{2b}+\frac{\sqrt{ia+ibx+1}\left(-18a^2-2(-6a+i)bx+10ia+7\right)(-ia-ibx+1)^{3/2}}{6b^2}}{4b^2}$$

$\downarrow$ 60

$$\frac{x^2(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{4b^2}-\frac{\frac{\left(8a^3-12ia^2-12a+3i\right)\left(\int \frac{1}{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}+\frac{\sqrt{ia+ibx+1}\left(-18a^2-2(-6a+i)bx+10ia+7\right)(-ia-ibx+1)^{3/2}}{6b^2}}{4b^2}$$

$\downarrow$ 62

$$\frac{x^2(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{4b^2}-\frac{\frac{\left(8a^3-12ia^2-12a+3i\right)\left(\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}+\frac{\sqrt{ia+ibx+1}\left(-18a^2-2(-6a+i)bx+10ia+7\right)(-ia-ibx+1)^{3/2}}{6b^2}}{4b^2}$$

$\downarrow$ 1090

$$\frac{x^2(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{4b^2}-\frac{\frac{\left(8a^3-12ia^2-12a+3i\right)\left(\frac{\int \frac{1}{\sqrt{\frac{\left(2xb^2+2ab\right)^2}{4b^2}+1}}d\left(2xb^2+2ab\right)}{2b^2}-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}+\frac{\sqrt{ia+ibx+1}\left(-18a^2-2(-6a+i)bx+10ia+7\right)(-ia-ibx+1)^{3/2}}{6b^2}}{4b^2}$$

$\downarrow$ 222

$$\frac{x^2(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{4b^2}-\frac{\frac{\sqrt{ia+ibx+1}\left(-18a^2-2(-6a+i)bx+10ia+7\right)(-ia-ibx+1)^{3/2}}{6b^2}+\frac{\left(8a^3-12ia^2-12a+3i\right)\left(\frac{\text{arcsinh}\left(\frac{2ab+2b^2x}{2b}\right)}{b}-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}}{4b^2}$$

input
```
Int[x^3/E^(I*ArcTan[a + b*x]),x]
```

output
```
(x^2*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(4*b^2) - (((1 - I*a -
 I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x]*(7 + (10*I)*a - 18*a^2 - 2*(I - 6*a)*b
*x))/(6*b^2) + ((3*I - 12*a - (12*I)*a^2 + 8*a^3)*(((-I)*Sqrt[1 - I*a - I*
b*x]*Sqrt[1 + I*a + I*b*x])/b + ArcSinh[(2*a*b + 2*b^2*x)/(2*b)]/b))/(2*b)
)/(4*b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.64

method	result
risch	$\frac{i(-6b^3x^3+6ab^2x^2-8ib^2x^2-6a^2bx+20iabx+6a^3-44ia^2+9bx-39a+16i)\sqrt{b^2x^2+2abx+a^2+1}}{24b^4}-\frac{(8a^3-12ia^2-12a+3i)\ln\left(\frac{b^2x+}{\sqrt{b^}}\right.}{8b^3\sqrt{b}}$
default	$-\frac{i\left(\frac{x(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}}{4b^2}-\frac{5a\left(\frac{(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}}{3b^2}-\frac{a\left(\frac{(2b^2x+2ab)\sqrt{b^2x^2+2abx+a^2+1}}{4b^2}+\frac{(4b^2(a^2+1)-4a^2b^2)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2}\right.}{8b^2\sqrt{b^2}}\right.}{b}\right.}{4b}\right.}{b}$

input
```
int(x^3/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*I*(-6*b^3*x^3-8*I*b^2*x^2+6*a*b^2*x^2+20*I*a*b*x-6*a^2*b*x-44*I*a^2+6
*a^3+9*b*x+16*I-39*a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^4-1/8*(3*I-12*a-12*I
*a^2+8*a^3)/b^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/
(b^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.60

$$\int e^{-i\arctan(a+bx)}x^3\,dx$$
$$=\frac{45i\,a^4+224\,a^3-192i\,a^2+24\,(8\,a^3-12i\,a^2-12\,a+3i)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-8\,(6}{}$$

input
```
integrate(x^3/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/192*(45*I*a^4 + 224*a^3 - 192*I*a^2 + 24*(8*a^3 - 12*I*a^2 - 12*a + 3*I)
*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 8*(6*I*b^3*x^3 + 2*(-
3*I*a - 4)*b^2*x^2 - 6*I*a^3 + (6*I*a^2 + 20*a - 9*I)*b*x - 44*a^2 + 39*I*
a + 16)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 72*a)/b^4
```

Sympy [F]

$$\int e^{-i\arctan(a+bx)}x^3\,dx = -i\int \frac{x^3\sqrt{a^2+2abx+b^2x^2+1}}{a+bx-i}\,dx$$

input
```
integrate(x**3/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2),x)
```

output
```
-I*Integral(x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a + b*x - I), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.32

$$\begin{aligned}\int e^{-i\arctan(a+bx)}x^3\,dx = &-\frac{3i\sqrt{b^2x^2+2\,abx+a^2+1}a^2x}{2\,b^3} - \frac{a^3\,\mathrm{arsinh}\,(bx+a)}{b^4} \\ &- \frac{i\sqrt{b^2x^2+2\,abx+a^2+1}a^3}{2\,b^4} - \frac{i\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}x}{4\,b^3} \\ &- \frac{3\sqrt{b^2x^2+2\,abx+a^2+1}ax}{2\,b^3} + \frac{3i\,a^2\,\mathrm{arsinh}\,(bx+a)}{2\,b^4} \\ &+ \frac{3i\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}a}{4\,b^4} + \frac{3\sqrt{b^2x^2+2\,abx+a^2+1}a^2}{2\,b^4} \\ &+ \frac{5i\sqrt{b^2x^2+2\,abx+a^2+1}x}{8\,b^3} + \frac{3\,a\,\mathrm{arsinh}\,(bx+a)}{2\,b^4} \\ &+ \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{3\,b^4} - \frac{19i\sqrt{b^2x^2+2\,abx+a^2+1}a}{8\,b^4} \\ &- \frac{3i\,\mathrm{arsinh}\,(bx+a)}{8\,b^4} - \frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{b^4}\end{aligned}$$

input
```
integrate(x^3/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-3/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*x/b^3 - a^3*arcsinh(b*x + a)/
b^4 - 1/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3/b^4 - 1/4*I*(b^2*x^2 + 2
*a*b*x + a^2 + 1)^(3/2)*x/b^3 - 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*x/
b^3 + 3/2*I*a^2*arcsinh(b*x + a)/b^4 + 3/4*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)
^(3/2)*a/b^4 + 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2/b^4 + 5/8*I*sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^3 + 3/2*a*arcsinh(b*x + a)/b^4 + 1/3*(b^2
*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/b^4 - 19/8*I*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)*a/b^4 - 3/8*I*arcsinh(b*x + a)/b^4 - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.67

$$\int e^{-i\arctan(a+bx)}x^3\,dx = -\frac{1}{24}\sqrt{(bx+a)^2+1}\left(\left(2x\left(\frac{3ix}{b}-\frac{3iab^5+4b^5}{b^7}\right)-\frac{-6ia^2b^4-20ab^4+9ib^4}{b^7}\right)x-\frac{6ia^3b^3+44a^2b^3-}{b^7}\right. + \frac{(8a^3-12ia^2-12a+3i)\log\left(\left|-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right|\right)}{8b^3|b|}$$

input
```
integrate(x^3/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/24*sqrt((b*x + a)^2 + 1)*((2*x*(3*I*x/b - (3*I*a*b^5 + 4*b^5)/b^7) - (-
6*I*a^2*b^4 - 20*a*b^4 + 9*I*b^4)/b^7)*x - (6*I*a^3*b^3 + 44*a^2*b^3 - 39*
I*a*b^3 - 16*b^3)/b^7) + 1/8*(8*a^3 - 12*I*a^2 - 12*a + 3*I)*log(abs(-a*b
- (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b)))/(b^3*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{-i\arctan(a+bx)}x^3\,dx = \int \frac{x^3\sqrt{(a+b\,x)^2+1}}{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,dx$$

input
```
int((x^3*((a + b*x)^2 + 1)^(1/2))/(a*1i + b*x*1i + 1),x)
```

output
```
int((x^3*((a + b*x)^2 + 1)^(1/2))/(a*1i + b*x*1i + 1), x)
```

Reduce [F]

$$\int e^{-i\arctan(a+bx)}x^3\,dx = \int \frac{\sqrt{b^2x^2+2abx+a^2+1}\,x^3}{bix+ai+1}dx$$

input
```
int(x^3/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x)
```

output
```
int((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*x**3)/(a*i + b*i*x + 1),x)
```

3.205 $\int e^{-i\arctan(a+bx)}x^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 171

$$\int e^{-i\arctan(a+bx)}x^2\,dx = \frac{(i-2a-2ia^2)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} + \frac{(i-4a)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{6b^3} + \frac{x(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{3b^2} - \frac{(1+2ia-2a^2)\operatorname{arcsinh}(a+bx)}{2b^3}$$

output
```
1/2*(I-2*a-2*I*a^2)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^3+1/6*(I-4*a
)*(1-I*a-I*b*x)^(3/2)*(1+I*a+I*b*x)^(1/2)/b^3+1/3*x*(1-I*a-I*b*x)^(3/2)*(1
+I*a+I*b*x)^(1/2)/b^2-1/2*(1+2*I*a-2*a^2)*arcsinh(b*x+a)/b^3
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.95

$$\int e^{-i\arctan(a+bx)}x^2\,dx = \frac{i\sqrt{1+ia+ibx}(4+7a^2+2ia^3-7ibx-5b^2x^2+2ib^3x^3+a(5i+8bx))}{6b^3\sqrt{-i(i+a+bx)}} + \frac{\sqrt[4]{-1}(-1-2ia+2a^2)\sqrt{-ib}\operatorname{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{b^{7/2}}$$

input
```
Integrate[x^2/E^(I*ArcTan[a + b*x]),x]
```

output
```
((I/6)*Sqrt[1 + I*a + I*b*x]*(4 + 7*a^2 + (2*I)*a^3 - (7*I)*b*x - 5*b^2*x^
2 + (2*I)*b^3*x^3 + a*(5*I + 8*b*x)))/(b^3*Sqrt[(-I)*(I + a + b*x)]) + ((-
1)^(1/4)*(-1 - (2*I)*a + 2*a^2)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*
Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/b^(7/2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5618, 101, 25, 90, 60, 62, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2e^{-i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{x^2\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}dx$$

$$\Big\downarrow \text{101}$$

$$\frac{\int -\frac{\sqrt{-ia-ibx+1}(a^2-(i-4a)bx+1)}{\sqrt{ia+ibx+1}}dx}{3b^2}+\frac{x\sqrt{ia+ibx+1}(-ia-ibx+1)^{3/2}}{3b^2}$$

$$\Big\downarrow \; 25$$

$$\frac{x(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{3b^2}-\frac{\int \frac{\sqrt{-ia-ibx+1}\left(a^2-(i-4a)bx+1\right)}{\sqrt{ia+ibx+1}}dx}{3b^2}$$

$$\Big\downarrow \; 90$$

$$\frac{x(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{3b^2}-\frac{\frac{3}{2}\left(-2a^2+2ia+1\right)\int \frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}dx-\frac{(-4a+i)(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{2b}}{3b^2}$$

$$\Big\downarrow \; 60$$

$$\frac{x(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{3b^2}-\frac{\frac{3}{2}\left(-2a^2+2ia+1\right)\left(\int \frac{1}{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)-\frac{(-4a+i)(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{2b}}{3b^2}$$

$$\Big\downarrow \; 62$$

$$\frac{x(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{3b^2}-\frac{\frac{3}{2}\left(-2a^2+2ia+1\right)\left(\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)-\frac{(-4a+i)(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{2b}}{3b^2}$$

$$\Big\downarrow \; 1090$$

$$\frac{x(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{3b^2}-\frac{\frac{3}{2}\left(-2a^2+2ia+1\right)\left(\frac{\int \frac{1}{\sqrt{\frac{\left(2xb^2+2ab\right)^2}{4b^2}+1}}d\left(2xb^2+2ab\right)}{2b^2}-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)-\frac{(-4a+i)(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{2b}}{3b^2}$$

$$\Big\downarrow \; 222$$

$$\frac{x(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{3b^2}-\frac{\frac{3}{2}\left(-2a^2+2ia+1\right)\left(\frac{\operatorname{arcsinh}\left(\frac{2ab+2b^2x}{2b}\right)}{b}-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)-\frac{(-4a+i)(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{2b}}{3b^2}$$

input
```
Int[x^2/E^(I*ArcTan[a + b*x]),x]
```

output
```
(x*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(3*b^2) - (-1/2*((I - 4*
a)*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/b + (3*(1 + (2*I)*a - 2*
a^2)*(((-I)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b + ArcSinh[(2*a*
b + 2*b^2*x)/(2*b)]/b))/2)/(3*b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{i\left(2b^2x^2-2abx+3ibx+2a^2-9ia-4\right)\sqrt{b^2x^2+2abx+a^2+1}}{6b^3}+\frac{\left(2a^2-2ia-1\right)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{2b^2\sqrt{b^2}}$
default	$-\frac{i\left(i\left(\frac{\left(2b^2x+2ab\right)\sqrt{b^2x^2+2abx+a^2+1}}{4b^2}+\frac{\left(4b^2\left(a^2+1\right)-4a^2b^2\right)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{8b^2\sqrt{b^2}}\right)+b\left(\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{3b^2}-\frac{a\left(\frac{(2b^2 \ldots}{\ldots}\right.}{\ldots}\right.\right.}{\ldots}$

input
```
int(x^2/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*I*(2*b^2*x^2+3*I*b*x-2*a*b*x-9*I*a+2*a^2-4)*(b^2*x^2+2*a*b*x+a^2+1)^(
1/2)/b^3+1/2*(-2*I*a+2*a^2-1)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*
b*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.62

$$\int e^{-i\arctan(a+bx)}x^2\,dx = \frac{-7i\,a^3 - 21\,a^2 - 12\,(2\,a^2 - 2i\,a - 1)\log\left(-bx - a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right) - 4\,\sqrt{b^2x^2 + 2\,abx + a^2 +}}{24\,b^3}$$

input
```
integrate(x^2/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/24*(-7*I*a^3 - 21*a^2 - 12*(2*a^2 - 2*I*a - 1)*log(-b*x - a + sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)) - 4*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*I*b^2*x^
2 + (-2*I*a - 3)*b*x + 2*I*a^2 + 9*a - 4*I) + 9*I*a)/b^3
```

Sympy [F]

$$\int e^{-i\arctan(a+bx)}x^2\,dx = -i\int \frac{x^2\sqrt{a^2 + 2abx + b^2x^2 + 1}}{a + bx - i}\,dx$$

input
```
integrate(x**2/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2),x)
```

output
```
-I*Integral(x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a + b*x - I), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.94

$$\begin{aligned}\int e^{-i\arctan(a+bx)}x^2\,dx &= \frac{i\,\sqrt{b^2x^2 + 2\,abx + a^2 + 1}ax}{b^2} + \frac{a^2\operatorname{arsinh}(bx + a)}{b^3} \\ &\quad + \frac{\sqrt{b^2x^2 + 2\,abx + a^2 + 1}x}{2\,b^2} - \frac{i\,a\operatorname{arsinh}(bx + a)}{b^3} \\ &\quad - \frac{i\,(b^2x^2 + 2\,abx + a^2 + 1)^{\frac{3}{2}}}{3\,b^3} - \frac{3\,\sqrt{b^2x^2 + 2\,abx + a^2 + 1}a}{2\,b^3} \\ &\quad - \frac{\operatorname{arsinh}(bx + a)}{2\,b^3} + \frac{i\,\sqrt{b^2x^2 + 2\,abx + a^2 + 1}}{b^3}\end{aligned}$$

input
```
integrate(x^2/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*x/b^2 + a^2*arcsinh(b*x + a)/b^3 + 1
/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^2 - I*a*arcsinh(b*x + a)/b^3 - 1/
3*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/b^3 - 3/2*sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)*a/b^3 - 1/2*arcsinh(b*x + a)/b^3 + I*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.67

$$\int e^{-i\arctan(a+bx)}x^2\,dx = -\frac{1}{6}\sqrt{(bx+a)^2+1}\left(x\left(\frac{2i\,x}{b}-\frac{2i\,ab^3+3\,b^3}{b^5}\right)-\frac{-2i\,a^2b^2-9\,ab^2+4i\,b^2}{b^5}\right) - \frac{(2\,a^2-2i\,a-1)\log\left(\left|-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right|\right)}{2\,b^2|b|}$$

input
```
integrate(x^2/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/6*sqrt((b*x + a)^2 + 1)*(x*(2*I*x/b - (2*I*a*b^3 + 3*b^3)/b^5) - (-2*I*
a^2*b^2 - 9*a*b^2 + 4*I*b^2)/b^5) - 1/2*(2*a^2 - 2*I*a - 1)*log(abs(-a*b -
 (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b)))/(b^2*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{-i\arctan(a+bx)}x^2\,dx = \int \frac{x^2\sqrt{(a+b\,x)^2+1}}{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,dx$$

input
```
int((x^2*((a + b*x)^2 + 1)^(1/2))/(a*1i + b*x*1i + 1),x)
```

output
```
int((x^2*((a + b*x)^2 + 1)^(1/2))/(a*1i + b*x*1i + 1), x)
```

Reduce [F]

$$\int e^{-i\arctan(a+bx)}x^2\,dx=\int\frac{\sqrt{b^2x^2+2abx+a^2+1}\,x^2}{bix+ai+1}dx$$

input
```
int(x^2/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x)
```

output
```
int((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*x**2)/(a*i + b*i*x + 1),x)
```

3.206 $\int e^{-i\arctan(a+bx)}x\,dx$

Optimal result

Integrand size = 14, antiderivative size = 110

$$\int e^{-i\arctan(a+bx)}x\,dx = \frac{(1+2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} + \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2b^2} + \frac{(i-2a)\operatorname{arcsinh}(a+bx)}{2b^2}$$

output
```
1/2*(1+2*I*a)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^2+1/2*(1-I*a-I*b*x
)^(3/2)*(1+I*a+I*b*x)^(1/2)/b^2+1/2*(I-2*a)*arcsinh(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.19

$$\int e^{-i\arctan(a+bx)}x\,dx = \frac{\sqrt{1+ia+ibx}(2-ia+a^2-3ibx-b^2x^2)}{2b^2\sqrt{-i(i+a+bx)}} + \frac{(-1)^{3/4}(1+2ia)\sqrt{-ib}\operatorname{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{b^{5/2}}$$

input
```
Integrate[x/E^(I*ArcTan[a + b*x]),x]
```

output
```
(Sqrt[1 + I*a + I*b*x]*(2 - I*a + a^2 - (3*I)*b*x - b^2*x^2))/(2*b^2*Sqrt[
(-I)*(I + a + b*x)]) + ((-1)^(3/4)*(1 + (2*I)*a)*Sqrt[(-I)*b]*ArcSinh[((1/
2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5618, 90, 60, 62, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{x\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}dx$$

$$\Big\downarrow \text{90}$$

$$\frac{(-2a+i)\int \frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}dx}{2b}+\frac{\sqrt{ia+ibx+1}(-ia-ibx+1)^{3/2}}{2b^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{(-2a+i)\left(\int \frac{1}{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}+\frac{\sqrt{ia+ibx+1}(-ia-ibx+1)^{3/2}}{2b^2}$$

$$\Big\downarrow \text{62}$$

$$\frac{(-2a+i)\left(\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}+\frac{\sqrt{ia+ibx+1}(-ia-ibx+1)^{3/2}}{2b^2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{(-2a+i)\left(\frac{\int \frac{1}{\sqrt{\frac{(2xb^2+2ab)^2}{4b^2}+1}}d(2xb^2+2ab)}{2b^2}-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}+\frac{\sqrt{ia+ibx+1}(-ia-ibx+1)^{3/2}}{2b^2}$$

$$\Big\downarrow \text{222}$$

$$\frac{(-2a+i)\left(\frac{\operatorname{arcsinh}\left(\frac{2ab+2b^2x}{2b}\right)}{b}-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}+\frac{\sqrt{ia+ibx+1}(-ia-ibx+1)^{3/2}}{2b^2}$$

input
```
Int[x/E^(I*ArcTan[a + b*x]),x]
```

output
```
((1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(2*b^2) + ((I - 2*a)*(((-I
)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b + ArcSinh[(2*a*b + 2*b^2*
x)/(2*b)]/b))/(2*b)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.79

method	result
risch	$\frac{i(-bx+a-2i)\sqrt{b^2x^2+2abx+a^2+1}}{2b^2}-\frac{(-i+2a)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{2b\sqrt{b^2}}$
default	$-\frac{i\left(\frac{\left(2b^2x+2ab\right)\sqrt{b^2x^2+2abx+a^2+1}}{4b^2}+\frac{\left(4b^2\left(a^2+1\right)-4a^2b^2\right)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{8b^2\sqrt{b^2}}\right)}{b}+\frac{(ia+1)\left(\sqrt{\left(x-\frac{i-a}{b}\right)^2b^2+2ib\left(x-\right.}\right.}{}$

input
```
int(x/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*(-b*x+a-2*I)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^2-1/2*(-I+2*a)/b*ln((b^
2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int e^{-i\arctan(a+bx)}x\,dx = \frac{3i\,a^2+4\,(2\,a-i)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-4\,\sqrt{b^2x^2+2\,abx+a^2+1}(i\,bx-i\,a-2)+}{8\,b^2}$$

input
```
integrate(x/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(3*I*a^2 + 4*(2*a - I)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)) - 4*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(I*b*x - I*a - 2) + 4*a)/b^2
```

Sympy [F]

$$\int e^{-i\arctan(a+bx)}x\,dx = -i\int\frac{x\sqrt{a^2+2abx+b^2x^2+1}}{a+bx-i}\,dx$$

input
```
integrate(x/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2),x)
```

output
```
-I*Integral(x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a + b*x - I), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.88

$$\int e^{-i\arctan(a+bx)}x\,dx = -\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}x}{2\,b}-\frac{a\,\mathrm{arsinh}\,(bx+a)}{b^2}+\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}a}{2\,b^2}+\frac{i\,\mathrm{arsinh}\,(bx+a)}{2\,b^2}+\frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{b^2}$$

input
```
integrate(x/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b - a*arcsinh(b*x + a)/b^2 + 1/
2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/b^2 + 1/2*I*arcsinh(b*x + a)/b^2 +
 sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.69

$$\int e^{-i\arctan(a+bx)}x\,dx = -\frac{1}{2}\sqrt{(bx+a)^2+1}\left(\frac{ix}{b}+\frac{-iab-2b}{b^3}\right) + \frac{(2a-i)\log\left(\left|-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right|\right)}{2b|b|}$$

input
```
integrate(x/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt((b*x + a)^2 + 1)*(I*x/b + (-I*a*b - 2*b)/b^3) + 1/2*(2*a - I)*lo
g(abs(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b)))/(b*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{-i\arctan(a+bx)}x\,dx = \int \frac{x\sqrt{(a+b\,x)^2+1}}{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,dx$$

input
```
int((x*((a + b*x)^2 + 1)^(1/2))/(a*1i + b*x*1i + 1),x)
```

output
```
int((x*((a + b*x)^2 + 1)^(1/2))/(a*1i + b*x*1i + 1), x)
```

Reduce [F]

$$\int e^{-i\arctan(a+bx)}x\,dx=\int\frac{\sqrt{b^2x^2+2abx+a^2+1}\,x}{bix+ai+1}dx$$

input
```
int(x/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x)
```

output
```
int((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*x)/(a*i + b*i*x + 1),x)
```

3.207 $\int e^{-i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 52

$$\int e^{-i\arctan(a+bx)}\,dx=-\frac{i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b}+\frac{\operatorname{arcsinh}(a+bx)}{b}$$

output
```
-I*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b+arcsinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.54

$$\int e^{-i\arctan(a+bx)}\,dx=\frac{-i\sqrt{1+(a+bx)^2}+\operatorname{arcsinh}(a+bx)}{b}$$

input
```
Integrate[E^((-I)*ArcTan[a + b*x]),x]
```

output
```
((-I)*Sqrt[1 + (a + b*x)^2] + ArcSinh[a + b*x])/b
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5616, 60, 62, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-i\arctan(a+bx)}dx$$

$$\Big\downarrow \text{5616}$$

$$\int \frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}dx$$

$$\Big\downarrow \text{60}$$

$$\int \frac{1}{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}$$

$$\Big\downarrow \text{62}$$

$$\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}$$

$$\Big\downarrow \text{1090}$$

$$\frac{\int \frac{1}{\sqrt{\frac{(2xb^2+2ab)^2}{4b^2}+1}}d(2xb^2+2ab)}{2b^2}-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}$$

$$\Big\downarrow \text{222}$$

$$\frac{\text{arcsinh}\left(\frac{2ab+2b^2x}{2b}\right)}{b}-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}$$

input
```
Int[E^((-I)*ArcTan[a + b*x]),x]
```

output
```
((-I)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b + ArcSinh[(2*a*b + 2*
b^2*x)/(2*b)]/b
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5616
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^(I*(n/2))/(1 + I*a*c + I*b*c*x)^(I*(n/2)), x] /; FreeQ[{a, b,
c, n}, x]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33

method	result	size
risch	$-\frac{i\sqrt{b^{2}x^{2}+2abx+a^{2}+1}}{b}+\frac{\ln\left(\frac{b^{2}x+ab}{\sqrt{b^{2}}}+\sqrt{b^{2}x^{2}+2abx+a^{2}+1}\right)}{\sqrt{b^{2}}}$	69
default	$-\frac{i\left(\sqrt{\left(x-\frac{i-a}{b}\right)^{2}b^{2}+2ib\left(x-\frac{i-a}{b}\right)}+\frac{ib\ln\left(\frac{ib+\left(x-\frac{i-a}{b}\right)b^{2}}{\sqrt{b^{2}}}+\sqrt{\left(x-\frac{i-a}{b}\right)^{2}b^{2}+2ib\left(x-\frac{i-a}{b}\right)}\right)}{\sqrt{b^{2}}}\right)}{b}$	125

input
```
int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I/b*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a
*b*x+a^2+1)^(1/2))/(b^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

$$\int e^{-i\arctan(a+bx)}\,dx = \frac{-i\,a-2i\,\sqrt{b^2x^2+2\,abx+a^2+1}-2\,\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}{2\,b}$$

input
```
integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(-I*a - 2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*log(-b*x - a + sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

Sympy [F]

$$\int e^{-i\arctan(a+bx)}\,dx = -i\int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{a+bx-i}\,dx$$

input
```
integrate(1/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2),x)
```

output
```
-I*Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a + b*x - I), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

$$\int e^{-i\arctan(a+bx)}\,dx = \frac{\operatorname{arsinh}(bx+a)}{b} - \frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{b}$$

input
```
integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(b*x + a)/b - I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int e^{-i\arctan(a+bx)}\,dx = -\frac{\log\left(\left|-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right|\right)}{|b|} - \frac{i\sqrt{(bx+a)^2+1}}{b}$$

input
```
integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-log(abs(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b)))/abs(b) - I*sqr
t((b*x + a)^2 + 1)/b
```

Mupad [F(-1)]

Timed out.

$$\int e^{-i\arctan(a+bx)}\,dx = \int \frac{\sqrt{(a+b\,x)^2+1}}{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,dx$$

input
```
int(((a + b*x)^2 + 1)^(1/2)/(a*1i + b*x*1i + 1),x)
```

output
```
int(((a + b*x)^2 + 1)^(1/2)/(a*1i + b*x*1i + 1), x)
```

Reduce [F]

$$\int e^{-i\arctan(a+bx)}\,dx = \int \frac{\sqrt{b^2x^2+2abx+a^2+1}}{bix+ai+1}dx$$

input
```
int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2),x)
```

output
```
int(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a*i + b*i*x + 1),x)
```

3.208 $\int \frac{e^{-i\arctan(a+bx)}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 89

$$\int \frac{e^{-i\arctan(a+bx)}}{x}\,dx = -i\text{arcsinh}(a+bx) - \frac{2\sqrt{i+a}\,\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{\sqrt{i-a}}$$

output
```
-I*arcsinh(b*x+a)-2*(I+a)^(1/2)*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I
-a)^(1/2)/(1-I*a-I*b*x)^(1/2))/(I-a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.60

$$\int \frac{e^{-i\arctan(a+bx)}}{x}\,dx = \frac{2\sqrt[4]{-1}(-ib)^{3/2}\text{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{b^{3/2}} - \frac{2\sqrt{-1+ia}\,\text{arctanh}\left(\frac{\sqrt{-1-ia}\sqrt{-i(i+a+bx)}}{\sqrt{-1+ia}\sqrt{1+ia+ibx}}\right)}{\sqrt{-1-ia}}$$

input
```
Integrate[1/(E^(I*ArcTan[a + b*x])*x),x]
```

output
```
(2*(-1)^(1/4)*((-I)*b)^(3/2)*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a
 + b*x)])/Sqrt[(-I)*b]])/b^(3/2) - (2*Sqrt[-1 + I*a]*ArcTanh[(Sqrt[-1 - I*
a]*Sqrt[(-I)*(I + a + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/Sqrt
[-1 - I*a]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.25, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5618, 140, 27, 62, 104, 221, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-i\arctan(a+bx)}}{x}dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{\sqrt{-ia-ibx+1}}{x\sqrt{ia+ibx+1}}dx$$

$$\Big\downarrow \text{140}$$

$$\int \frac{1-ia}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx-ib\int \frac{1}{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx$$

$$\Big\downarrow \text{27}$$

$$(1-ia)\int \frac{1}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx-ib\int \frac{1}{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx$$

$$\Big\downarrow \text{62}$$

$$(1-ia)\int \frac{1}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx-ib\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx$$

$$\Big\downarrow \text{104}$$

$$2(1-ia)\int \frac{1}{-ia+\frac{(1-ia)(ia+ibx+1)}{-ia-ibx+1}-1}d\frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}-ib\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx$$

$$\Big\downarrow \text{221}$$

$$-ib\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx-\frac{2i(1-ia)\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}\sqrt{a+i}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{i\int \frac{1}{\sqrt{\frac{(2xb^2+2ab)^2}{4b^2}+1}}d(2xb^2+2ab)}{2b}-\frac{2i(1-ia)\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}\sqrt{a+i}}$$

$$\Big\downarrow \text{222}$$

$$-i\text{arcsinh}\left(\frac{2ab+2b^2x}{2b}\right)-\frac{2i(1-ia)\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}\sqrt{a+i}}$$

input
```
Int[1/(E^(I*ArcTan[a + b*x])*x),x]
```

output
```
(-I)*ArcSinh[(2*a*b + 2*b^2*x)/(2*b)] - ((2*I)*(1 - I*a)*ArcTanh[(Sqrt[I +
 a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/(Sqrt[I -
 a]*Sqrt[I + a])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(68) = 136$.

Time = 0.31 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.92

method	result
default	$\frac{i\left(\sqrt{b^2x^2+2abx+a^2+1}+\frac{ab\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}-\sqrt{a^2+1}\,\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{x}\right)\right)}{i-a}-\frac{i\left(\sqrt{(\right.}\right.}{}$

input
```
int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output

```
I/(I-a)*((b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2
*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2
*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x))-I/(I-a)*(((x-(I-a)/b)^2*
b^2+2*I*b*(x-(I-a)/b))^(1/2)+I*b*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-
(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(59) = 118$.

Time = 0.15 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.62

$$\int \frac{e^{-i\arctan(a+bx)}}{x}\,dx = -\sqrt{-\frac{a+i}{a-i}}\log\left(-bx+(i\,a+1)\sqrt{-\frac{a+i}{a-i}}+\sqrt{b^2x^2+2\,abx+a^2+1}\right)+\sqrt{-\frac{a+i}{a-i}}\log\left(-bx+(-i\,a-1)\sqrt{-\frac{a+i}{a-i}}+\sqrt{b^2x^2+2\,abx+a^2+1}\right)+i\,\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)$$

input

```
integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x,x, algorithm="fricas")
```

output

```
-sqrt(-(a + I)/(a - I))*log(-b*x + (I*a + 1)*sqrt(-(a + I)/(a - I)) + sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)) + sqrt(-(a + I)/(a - I))*log(-b*x + (-I*a -
 1)*sqrt(-(a + I)/(a - I)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + I*log(-b
*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))
```

Sympy [F]

$$\int \frac{e^{-i\arctan(a+bx)}}{x}\,dx = -i\int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{ax+bx^2-ix}\,dx$$

input
```
integrate(1/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2)/x,x)
```

output
```
-I*Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a*x + b*x**2 - I*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e^{-i\arctan(a+bx)}}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a-1>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.27

$$\int \frac{e^{-i\arctan(a+bx)}}{x}\,dx = -\frac{(-i\,a+1)\log\left(\frac{\left|2\,x|b|-2\,\sqrt{(bx+a)^2+1}-2\,\sqrt{a^2+1}\right|}{\left|2\,x|b|-2\,\sqrt{(bx+a)^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{\sqrt{a^2+1}} + \frac{i\,b\log\left(\left|-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right|\right)}{|b|}$$

input
```
integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x,x, algorithm="giac")
```

output
```
-(-I*a + 1)*log(abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) - 2*sqrt(a^2 + 1)
)/abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) + 2*sqrt(a^2 + 1)))/sqrt(a^2 +
1) + I*b*log(abs(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b)))/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-i\arctan(a+bx)}}{x}\,dx = \int \frac{\sqrt{(a+b\,x)^2+1}}{x\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})}\,dx$$

input
```
int(((a + b*x)^2 + 1)^(1/2)/(x*(a*1i + b*x*1i + 1)),x)
```

output
```
int(((a + b*x)^2 + 1)^(1/2)/(x*(a*1i + b*x*1i + 1)), x)
```

Reduce [F]

$$\int \frac{e^{-i\arctan(a+bx)}}{x}\,dx = \int \frac{\sqrt{b^2x^2+2abx+a^2+1}}{bi\,x^2+aix+x}dx$$

input
```
int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x,x)
```

output
```
int(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a*i*x + b*i*x**2 + x),x)
```

3.209 $\int \frac{e^{-i\arctan(a+bx)}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 130

$$\int \frac{e^{-i\arctan(a+bx)}}{x^2}\,dx=-\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{(1+ia)x}-\frac{2ib\,\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{3/2}\sqrt{i+a}}$$

output
```
-(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/(1+I*a)/x-2*I*b*arctanh((I+a)^(1/
2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-I*b*x)^(1/2))/(I-a)^(3/2)/(I+a)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.92

$$\int \frac{e^{-i\arctan(a+bx)}}{x^2}\,dx=i\left(\frac{\sqrt{1+a^2+2abx+b^2x^2}}{(-i+a)x}+\frac{2b\,\text{arctanh}\left(\frac{\sqrt{-1-ia}\sqrt{-i(i+a+bx)}}{\sqrt{-1+ia}\sqrt{1+ia+ibx}}\right)}{(-1-ia)^{3/2}\sqrt{-1+ia}}\right)$$

input
```
Integrate[1/(E^(I*ArcTan[a + b*x])*x^2),x]
```

output
```
I*(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/((-I + a)*x) + (2*b*ArcTanh[(Sqrt[-1
- I*a]*Sqrt[(-I)*(I + a + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/
((-1 - I*a)^(3/2)*Sqrt[-1 + I*a]))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5618, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-i\arctan(a+bx)}}{x^2}dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{\sqrt{-ia-ibx+1}}{x^2\sqrt{ia+ibx+1}}dx$$

$$\Big\downarrow \text{105}$$

$$\frac{b\int \frac{1}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{-a+i} - \frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{(1+ia)x}$$

$$\Big\downarrow \text{104}$$

$$\frac{2b\int \frac{1}{-ia+\frac{(1-ia)(ia+ibx+1)}{-ia-ibx+1}-1}d\frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}}{-a+i} - \frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{(1+ia)x}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2ib\,\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{3/2}\sqrt{a+i}} - \frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{(1+ia)x}$$

input
```
Int[1/(E^(I*ArcTan[a + b*x])*x^2),x]
```

output
```
-((Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/((1 + I*a)*x)) - ((2*I)*b*
ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*
b*x])])/((I - a)^(3/2)*Sqrt[I + a])
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.72

method	result
risch	$\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{(-i+a)x}+\frac{b\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{x}\right)}{(-i+a)\sqrt{a^2+1}}$
default	$\frac{i\left(-\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{\left(a^2+1\right)x}+\frac{ab\left(\sqrt{b^2x^2+2abx+a^2+1}+\frac{ab\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}-\sqrt{a^2+1}\,\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2a}}{x}\right.\right.}{a^2+1}\right.}{i-a}$

input
```
int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
I*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/(-I+a)/x+1/(-I+a)*b/(a^2+1)^(1/2)*ln((2*a^
2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(86) = 172.

Time = 0.14 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.72

$$\int \frac{e^{-i\arctan(a+bx)}}{x^2}\,dx =$$

$$-\frac{(a-i)\sqrt{\frac{b^2}{a^4-2i\,a^3-2i\,a-1}}\,x\log\left(-\frac{b^2x-\sqrt{b^2x^2+2\,abx+a^2+1}\,b+(a^3-i\,a^2+a-i)\sqrt{\frac{b^2}{a^4-2i\,a^3-2i\,a-1}}}{b}\right)-(a-i)\sqrt{\frac{b^2}{a^4-2i\,a^3-2}}}{(a-i)}$$

input
```
integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-((a - I)*sqrt(b^2/(a^4 - 2*I*a^3 - 2*I*a - 1))*x*log(-(b^2*x - sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)*b + (a^3 - I*a^2 + a - I)*sqrt(b^2/(a^4 - 2*I*a^3
- 2*I*a - 1)))/b) - (a - I)*sqrt(b^2/(a^4 - 2*I*a^3 - 2*I*a - 1))*x*log(-(
b^2*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b - (a^3 - I*a^2 + a - I)*sqrt(b
^2/(a^4 - 2*I*a^3 - 2*I*a - 1)))/b) - I*b*x - I*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1))/((a - I)*x)
```

Sympy [F]

$$\int \frac{e^{-i\arctan(a+bx)}}{x^2}\,dx = -i\int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{ax^2+bx^3-ix^2}\,dx$$

input
```
integrate(1/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2)/x**2,x)
```

output
```
-I*Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a*x**2 + b*x**3 - I*x**2
), x)
```

Maxima [F]

$$\int \frac{e^{-i\arctan(a+bx)}}{x^2}\,dx = \int \frac{\sqrt{(bx+a)^2+1}}{(i\,bx+i\,a+1)x^2}\,dx$$

input
```
integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x + a)^2 + 1)/((I*b*x + I*a + 1)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12

$$\int \frac{e^{-i\arctan(a+bx)}}{x^2}\,dx = \frac{b\log\left(\frac{\left|2\,x|b|-2\,\sqrt{(bx+a)^2+1}-2\,\sqrt{a^2+1}\right|}{\left|2\,x|b|-2\,\sqrt{(bx+a)^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{\sqrt{a^2+1}(a-i)} - \frac{2\left(\left(x|b|-\sqrt{(bx+a)^2+1}\right)ab+a^2|b|+|b|\right)}{\left(\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2-a^2-1\right)(-i\,a-1)}$$

input
```
integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
b*log(abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) - 2*sqrt(a^2 + 1))/abs(2*x*
abs(b) - 2*sqrt((b*x + a)^2 + 1) + 2*sqrt(a^2 + 1)))/(sqrt(a^2 + 1)*(a - I
)) - 2*((x*abs(b) - sqrt((b*x + a)^2 + 1))*a*b + a^2*abs(b) + abs(b))/(((x
*abs(b) - sqrt((b*x + a)^2 + 1))^2 - a^2 - 1)*(-I*a - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-i\arctan(a+bx)}}{x^2}\,dx = \int \frac{\sqrt{(a+b\,x)^2+1}}{x^2\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})}\,dx$$

input
```
int(((a + b*x)^2 + 1)^(1/2)/(x^2*(a*1i + b*x*1i + 1)),x)
```

output
```
int(((a + b*x)^2 + 1)^(1/2)/(x^2*(a*1i + b*x*1i + 1)), x)
```

Reduce [F]

$$\int \frac{e^{-i\arctan(a+bx)}}{x^2}\,dx = \int \frac{\sqrt{b^2x^2+2abx+a^2+1}}{bi\,x^3+ai\,x^2+x^2}dx$$

input
```
int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^2,x)
```

output
```
int(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a*i*x**2 + b*i*x**3 + x**2),x)
```

3.210 $\int \frac{e^{-i\arctan(a+bx)}}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 201

$$\int \frac{e^{-i\arctan(a+bx)}}{x^3}\,dx = \frac{(1-2ia)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2(i-a)^2(i+a)x} - \frac{(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2(1+a^2)x^2} + \frac{(1-2ia)b^2\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{5/2}(i+a)^{3/2}}$$

output
```
1/2*(1-2*I*a)*b*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^2/(I+a)/x-1/
2*(1-I*a-I*b*x)^(3/2)*(1+I*a+I*b*x)^(1/2)/(a^2+1)/x^2+(1-2*I*a)*b^2*arctan
h((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-I*b*x)^(1/2))/(I-a)^(
5/2)/(I+a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.77

$$\int \frac{e^{-i\arctan(a+bx)}}{x^3}\,dx = \frac{\frac{i(1+a^2-2ibx-abx)\sqrt{1+a^2+2abx+b^2x^2}}{x^2}+\frac{2(i+2a)b^2\operatorname{arctanh}\left(\frac{\sqrt{-1-ia}\sqrt{-i(i+a+bx)}}{\sqrt{-1+ia}\sqrt{1+ia+ibx}}\right)}{\sqrt{-1-ia}\sqrt{-1+ia}}}{2(-i+a)^2(i+a)}$$

input

```
Integrate[1/(E^(I*ArcTan[a + b*x])*x^3),x]
```

output

```
((I*(1 + a^2 - (2*I)*b*x - a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/x^2 +
 (2*(I + 2*a)*b^2*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-I)*(I + a + b*x)])/(Sqrt[
-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/(Sqrt[-1 - I*a]*Sqrt[-1 + I*a]))/(2*(-I
 + a)^2*(I + a))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5618, 107, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-i\arctan(a+bx)}}{x^3}\,dx$$

$$\Big\downarrow \text{ 5618}$$

$$\int \frac{\sqrt{-ia-ibx+1}}{x^3\sqrt{ia+ibx+1}}dx$$

$$\Big\downarrow \text{ 107}$$

$$-\frac{(2a+i)b\int \frac{\sqrt{-ia-ibx+1}}{x^2\sqrt{ia+ibx+1}}dx}{2\left(a^2+1\right)}-\frac{\sqrt{ia+ibx+1}(-ia-ibx+1)^{3/2}}{2\left(a^2+1\right)x^2}$$

$$\Big\downarrow \text{ 105}$$

$$-\frac{(2a+i)b\left(\frac{b\int \frac{1}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{-a+i}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{(1+ia)x}\right)}{2\left(a^2+1\right)}-\frac{\sqrt{ia+ibx+1}(-ia-ibx+1)^{3/2}}{2\left(a^2+1\right)x^2}$$

$$\Big\downarrow \; 104$$

$$-\frac{(2a+i)b\left(\frac{2b\int \frac{1}{-ia+\frac{(1-ia)(ia+ibx+1)}{-ia-ibx+1}-1}d\frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}}{-a+i}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{(1+ia)x}\right)}{2\left(a^2+1\right)}-\frac{\sqrt{ia+ibx+1}(-ia-ibx+1)^{3/2}}{2\left(a^2+1\right)x^2}$$

$$\Big\downarrow \; 221$$

$$-\frac{(2a+i)b\left(-\frac{2ib\,\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{3/2}\sqrt{a+i}}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{(1+ia)x}\right)}{2\left(a^2+1\right)}-\frac{\sqrt{ia+ibx+1}(-ia-ibx+1)^{3/2}}{2\left(a^2+1\right)x^2}$$

input
```
Int[1/(E^(I*ArcTan[a + b*x])*x^3),x]
```

output
```
-1/2*((1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/((1 + a^2)*x^2) - ((I
 + 2*a)*b*(-((Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/((1 + I*a)*x))
- ((2*I)*b*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1
 - I*a - I*b*x])])/((I - a)^(3/2)*Sqrt[I + a])))/(2*(1 + a^2))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.93

method	result
risch	$\frac{i(-a b^3x^3-2ib^3x^3-a^2b^2x^2-4ia b^2x^2+a^3bx-2ia^2bx+a^4+b^2x^2+abx-2ibx+2a^2+1)}{2x^2(i+a)(-i+a)^2\sqrt{b^2x^2+2abx+a^2+1}} - \frac{b^2(i+2a)\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\sqrt{b^2x^2}}{x}\right)}{2(a^2+1)^{\frac{3}{2}}(-i+a)}$
default	$\frac{i\left(-\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{2\left(a^2+1\right)x^2}-\frac{ab\left(-\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{\left(a^2+1\right)x}+\frac{ab\left(\sqrt{b^2x^2+2abx+a^2+1}+\frac{ab\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}-\sqrt{a^2+1}\ln\left(2a\right.\right.}{a^2+1}\right.}{}\right.}{}$

input
```
int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*I*(-a*b^3*x^3-a^2*b^2*x^2+a^3*b*x-2*I*b^3*x^3+a^4+b^2*x^2-4*I*a*b^2*x^
2+a*b*x-2*I*a^2*b*x+2*a^2-2*I*b*x+1)/x^2/(I+a)/(-I+a)^2/(b^2*x^2+2*a*b*x+a
^2+1)^(1/2)-1/2*b^2*(I+2*a)/(a^2+1)^(3/2)/(-I+a)*ln((2*a^2+2+2*a*b*x+2*(a^
2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(135) = 270$.

Time = 0.15 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.25

$$\int \frac{e^{-i\arctan(a+bx)}}{x^3}\,dx$$

$$= \frac{(-ia+2)b^2x^2+\sqrt{\frac{(4a^2+4ia-1)b^4}{a^8-2ia^7+2a^6-6ia^5-6ia^3-2a^2-2ia-1}}(a^3-ia^2+a-i)x^2\log\left(-\frac{(2a+i)b^3x-\sqrt{b^2x^2+2abx+a^2+1}}{}\right.}{}$$

input
```
integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
1/2*((-I*a + 2)*b^2*x^2 + sqrt((4*a^2 + 4*I*a - 1)*b^4/(a^8 - 2*I*a^7 + 2*
a^6 - 6*I*a^5 - 6*I*a^3 - 2*a^2 - 2*I*a - 1))*(a^3 - I*a^2 + a - I)*x^2*lo
g(-((2*a + I)*b^3*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*a + I)*b^2 + (a
^5 - I*a^4 + 2*a^3 - 2*I*a^2 + a - I)*sqrt((4*a^2 + 4*I*a - 1)*b^4/(a^8 -
2*I*a^7 + 2*a^6 - 6*I*a^5 - 6*I*a^3 - 2*a^2 - 2*I*a - 1)))/((2*a + I)*b^2)
) - sqrt((4*a^2 + 4*I*a - 1)*b^4/(a^8 - 2*I*a^7 + 2*a^6 - 6*I*a^5 - 6*I*a^
3 - 2*a^2 - 2*I*a - 1))*(a^3 - I*a^2 + a - I)*x^2*log(-((2*a + I)*b^3*x -
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*a + I)*b^2 - (a^5 - I*a^4 + 2*a^3 - 2
*I*a^2 + a - I)*sqrt((4*a^2 + 4*I*a - 1)*b^4/(a^8 - 2*I*a^7 + 2*a^6 - 6*I*
a^5 - 6*I*a^3 - 2*a^2 - 2*I*a - 1)))/((2*a + I)*b^2)) + sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1)*((-I*a + 2)*b*x + I*a^2 + I))/((a^3 - I*a^2 + a - I)*x^2)
```

Sympy [F]

$$\int \frac{e^{-i\arctan(a+bx)}}{x^3}\,dx = -i\int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{ax^3+bx^4-ix^3}\,dx$$

input
```
integrate(1/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2)/x**3,x)
```

output
```
-I*Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a*x**3 + b*x**4 - I*x**3
), x)
```

Maxima [F]

$$\int \frac{e^{-i\arctan(a+bx)}}{x^3}\,dx = \int \frac{\sqrt{(bx+a)^2+1}}{(i\,bx+i\,a+1)x^3}\,dx$$

input
```
integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x + a)^2 + 1)/((I*b*x + I*a + 1)*x^3), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 471 vs. 2(135) = 270.

Time = 0.18 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.34

$$\int \frac{e^{-i\arctan(a+bx)}}{x^3}\,dx = -\frac{(2\,ab^2+i\,b^2)\log\left(\frac{\left|2\,x|b|-2\,\sqrt{(bx+a)^2+1}-2\,\sqrt{a^2+1}\right|}{\left|2\,x|b|-2\,\sqrt{(bx+a)^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{2\,(a^3-i\,a^2+a-i)\sqrt{a^2+1}} - \frac{4\left(i\,x|b|-i\,\sqrt{(bx+a)^2+1}\right)a^4b^2+2i\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2a^3b|b|+2i\,a^5b|b|+2\left(x|b|-\sqrt{(bx\right.}{}$$

input
```
integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

output
```
-1/2*(2*a*b^2 + I*b^2)*log(abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) - 2*sq
rt(a^2 + 1))/abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) + 2*sqrt(a^2 + 1)))/
((a^3 - I*a^2 + a - I)*sqrt(a^2 + 1)) - (4*(I*x*abs(b) - I*sqrt((b*x + a)^
2 + 1))*a^4*b^2 + 2*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a^3*b*abs(b) +
2*I*a^5*b*abs(b) + 2*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a*b^2 - 2*(x*abs
(b) - sqrt((b*x + a)^2 + 1))*a^3*b^2 + 2*(x*abs(b) - sqrt((b*x + a)^2 + 1)
)^2*a^2*b*abs(b) - 2*a^4*b*abs(b) + I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3
*b^2 + 5*(I*x*abs(b) - I*sqrt((b*x + a)^2 + 1))*a^2*b^2 + 2*I*(x*abs(b) -
sqrt((b*x + a)^2 + 1))^2*a*b*abs(b) + 4*I*a^3*b*abs(b) - 2*(x*abs(b) - sqr
t((b*x + a)^2 + 1))*a*b^2 + 2*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b*abs(b
) - 4*a^2*b*abs(b) - (-I*x*abs(b) + I*sqrt((b*x + a)^2 + 1))*b^2 + 2*I*a*b
*abs(b) - 2*b*abs(b))/((a^3 - I*a^2 + a - I)*((x*abs(b) - sqrt((b*x + a)^2
 + 1))^2 - a^2 - 1)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-i\arctan(a+bx)}}{x^3}\,dx = \int \frac{\sqrt{(a+b\,x)^2+1}}{x^3\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})}\,dx$$

input
```
int(((a + b*x)^2 + 1)^(1/2)/(x^3*(a*1i + b*x*1i + 1)),x)
```

output
```
int(((a + b*x)^2 + 1)^(1/2)/(x^3*(a*1i + b*x*1i + 1)), x)
```

Reduce [F]

$$\int \frac{e^{-i\arctan(a+bx)}}{x^3}\,dx = \int \frac{\sqrt{1+(bx+a)^2}}{(1+i\,(bx+a))\,x^3}dx$$

input
```
int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^3,x)
```

output
```
int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^3,x)
```

3.211 $\int \frac{e^{-i\arctan(a+bx)}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 283

$$\begin{aligned}\int \frac{e^{-i\arctan(a+bx)}}{x^4}\,dx = &-\frac{\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{3(1+ia)x^3}\\ &+\frac{(3-2ia)b\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(i-a)^2(i+a)x^2}\\ &+\frac{(4-9ia-2a^2)\,b^2\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{6(1+ia)\,(1+a^2)^2\,x}\\ &+\frac{(2a+i(1-2a^2))\,b^3\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{7/2}(i+a)^{5/2}}\end{aligned}$$

output
```
-1/3*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/(1+I*a)/x^3+1/6*(3-2*I*a)*b*(
1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^2/(I+a)/x^2+1/6*(4-9*I*a-2*a^
2)*b^2*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/(1+I*a)/(a^2+1)^2/x+(2*a+I*
(-2*a^2+1))*b^3*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a
-I*b*x)^(1/2))/(I-a)^(7/2)/(I+a)^(5/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.82

$$\int \frac{e^{-i\arctan(a+bx)}}{x^4}\,dx$$

$$= \frac{\frac{2(1+ia)(i+a)(i+a+bx)\sqrt{1+a^2+2abx+b^2x^2}}{x^3} + \frac{(1-4ia)b(i+a+bx)\sqrt{1+a^2+2abx+b^2x^2}}{x^2} - 3i(1-2ia-2a^2)\,b^2\left(\frac{\sqrt{1+a^2+2abx+b^2x^2}}{(-i+a)x}\right.}{6\left(1+a^2\right)^2}$$

input
```
Integrate[1/(E^(I*ArcTan[a + b*x])*x^4),x]
```

output
```
((2*(1 + I*a)*(I + a)*(I + a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/x^3
 + ((1 - (4*I)*a)*b*(I + a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/x^2 -
 (3*I)*(1 - (2*I)*a - 2*a^2)*b^2*(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/((-I +
 a)*x) + (2*b*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-I)*(I + a + b*x)])/(Sqrt[-1 +
 I*a]*Sqrt[1 + I*a + I*b*x])])/((-1 - I*a)^(3/2)*Sqrt[-1 + I*a])))/(6*(1 +
 a^2)^2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {5618, 110, 25, 27, 168, 25, 27, 168, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-i\arctan(a+bx)}}{x^4}\,dx$$

$$\Big\downarrow \; 5618$$

$$\int \frac{\sqrt{-ia-ibx+1}}{x^4\sqrt{ia+ibx+1}}dx$$

$$\Big\downarrow \; 110$$

$$\frac{\int -\frac{b(2a+2bx+3i)}{x^3\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{3(1+ia)}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{3(1+ia)x^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{b(2a+2bx+3i)}{x^3\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{3(1+ia)}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{3(1+ia)x^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{2a+2bx+3i}{x^3\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{3(1+ia)}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{3(1+ia)x^3}$$

$$\Big\downarrow \text{168}$$

$$-\frac{b\left(-\frac{\int -\frac{b\left(-2a^2-9ia-(2a+3i)bx+4\right)}{x^2\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{2(a^2+1)}-\frac{(2a+3i)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2(a^2+1)x^2}\right)}{3(1+ia)}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{3(1+ia)x^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\left(\frac{\int \frac{b\left(-2a^2-9ia-(2a+3i)bx+4\right)}{x^2\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{2(a^2+1)}-\frac{(2a+3i)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2(a^2+1)x^2}\right)}{3(1+ia)}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{3(1+ia)x^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\left(\frac{b\int \frac{-2a^2-9ia-(2a+3i)bx+4}{x^2\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{2(a^2+1)}-\frac{(2a+3i)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2(a^2+1)x^2}\right)}{3(1+ia)}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{3(1+ia)x^3}$$

$$\Big\downarrow \text{168}$$

$$-\frac{b\left(\frac{b\left(-\frac{\int \frac{3\left(-2ia^2+2a+i\right)b}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{a^2+1}-\frac{\left(-2a^2-9ia+4\right)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{\left(a^2+1\right)x}\right)}{2(a^2+1)}-\frac{(2a+3i)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2(a^2+1)x^2}\right)}{3(1+ia)}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{3(1+ia)x^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{b\left(\frac{b\left(-\frac{3\left(-2ia^2+2a+i\right)b\int \frac{1}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{a^2+1}-\frac{\left(-2a^2-9ia+4\right)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{\left(a^2+1\right)x}\right)}{2(a^2+1)}-\frac{(2a+3i)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2(a^2+1)x^2}\right)}{3(1+ia)}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{3(1+ia)x^3}$$

$$\Big\downarrow \ 104$$

$$-\frac{b\left(\frac{b\left(-\frac{6\left(-2ia^2+2a+i\right)b\int \frac{1}{-ia+\frac{(1-ia)(ia+ibx+1)}{-ia-ibx+1}-1}d\frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}}{a^2+1}-\frac{\left(-2a^2-9ia+4\right)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{\left(a^2+1\right)x}\right)}{2(a^2+1)}-\frac{(2a+3i)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2(a^2+1)x^2}\right)}{3(1+ia)}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{3(1+ia)x^3}$$

$$\Big\downarrow \ 221$$

$$-\frac{b\left(\frac{b\left(\frac{6i\left(-2ia^2+2a+i\right)b\,\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}\sqrt{a+i}\left(a^2+1\right)}-\frac{\left(-2a^2-9ia+4\right)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{\left(a^2+1\right)x}\right)}{2(a^2+1)}-\frac{(2a+3i)\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{2(a^2+1)x^2}\right)}{3(1+ia)}-\frac{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{3(1+ia)x^3}$$

input
```
Int[1/(E^(I*ArcTan[a + b*x])*x^4),x]
```

output
```
-1/3*(Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/((1 + I*a)*x^3) - (b*(-
1/2*((3*I + 2*a)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/((1 + a^2)*x
^2) + (b*(-(((4 - (9*I)*a - 2*a^2)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*
b*x])/((1 + a^2)*x)) + ((6*I)*(I + 2*a - (2*I)*a^2)*b*ArcTanh[(Sqrt[I + a]
*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/(Sqrt[I - a]
*Sqrt[I + a]*(1 + a^2))))/(2*(1 + a^2))))/(3*(1 + I*a))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.99

method	result
risch	$\frac{i\left(2a^2b^4x^4+9ia\,b^4x^4+2a^3b^3x^3+15ia^2b^3x^3+3ia^3b^2x^2-4b^4x^4+2a^5bx-3ia^4bx-10a\,b^3x^3-3ib^3x^3+2a^6-2a^2b^2x^2+3ia\,b^2x^2+4a^3bx\right.}{6x^3(i+a)^2(-i+a)^3\sqrt{b^2x^2+2abx+a^2+1}}$
default	Expression too large to display

input
```
int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*I*(-3*I*x*a^4*b+2*a^2*b^4*x^4+3*I*a*b^2*x^2+2*a^3*b^3*x^3+15*I*a^2*b^3
*x^3-4*b^4*x^4+3*I*a^3*b^2*x^2-3*I*b^3*x^3+2*a^5*b*x-10*a*b^3*x^3+9*I*a*b^
4*x^4+2*a^6-2*a^2*b^2*x^2-3*I*b*x+4*a^3*b*x+6*a^4-2*b^2*x^2-6*I*a^2*b*x+2*
a*b*x+6*a^2+2)/x^3/(I+a)^2/(-I+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/2*b^3*
(2*I*a+2*a^2-1)/(a^2+1)^(5/2)/(-I+a)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(
b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 690 vs. 2(194) = 388.

Time = 0.15 (sec) , antiderivative size = 690, normalized size of antiderivative = 2.44

$$\int \frac{e^{-i\arctan(a+bx)}}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
1/6*((2*I*a^2 - 9*a - 4*I)*b^3*x^3 - 3*sqrt((4*a^4 + 8*I*a^3 - 8*a^2 - 4*I
*a + 1)*b^6/(a^12 - 2*I*a^11 + 4*a^10 - 10*I*a^9 + 5*a^8 - 20*I*a^7 - 20*I
*a^5 - 5*a^4 - 10*I*a^3 - 4*a^2 - 2*I*a - 1))*(a^5 - I*a^4 + 2*a^3 - 2*I*a
^2 + a - I)*x^3*log(-((2*a^2 + 2*I*a - 1)*b^4*x - sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)*(2*a^2 + 2*I*a - 1)*b^3 + (a^7 - I*a^6 + 3*a^5 - 3*I*a^4 + 3*a^3
 - 3*I*a^2 + a - I)*sqrt((4*a^4 + 8*I*a^3 - 8*a^2 - 4*I*a + 1)*b^6/(a^12 -
 2*I*a^11 + 4*a^10 - 10*I*a^9 + 5*a^8 - 20*I*a^7 - 20*I*a^5 - 5*a^4 - 10*I
*a^3 - 4*a^2 - 2*I*a - 1)))/((2*a^2 + 2*I*a - 1)*b^3)) + 3*sqrt((4*a^4 + 8
*I*a^3 - 8*a^2 - 4*I*a + 1)*b^6/(a^12 - 2*I*a^11 + 4*a^10 - 10*I*a^9 + 5*a
^8 - 20*I*a^7 - 20*I*a^5 - 5*a^4 - 10*I*a^3 - 4*a^2 - 2*I*a - 1))*(a^5 - I
*a^4 + 2*a^3 - 2*I*a^2 + a - I)*x^3*log(-((2*a^2 + 2*I*a - 1)*b^4*x - sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*a^2 + 2*I*a - 1)*b^3 - (a^7 - I*a^6 + 3*a
^5 - 3*I*a^4 + 3*a^3 - 3*I*a^2 + a - I)*sqrt((4*a^4 + 8*I*a^3 - 8*a^2 - 4*
I*a + 1)*b^6/(a^12 - 2*I*a^11 + 4*a^10 - 10*I*a^9 + 5*a^8 - 20*I*a^7 - 20*
I*a^5 - 5*a^4 - 10*I*a^3 - 4*a^2 - 2*I*a - 1)))/((2*a^2 + 2*I*a - 1)*b^3))
 + ((2*I*a^2 - 9*a - 4*I)*b^2*x^2 + 2*I*a^4 + (-2*I*a^3 + 3*a^2 - 2*I*a +
3)*b*x + 4*I*a^2 + 2*I)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((a^5 - I*a^4 +
 2*a^3 - 2*I*a^2 + a - I)*x^3)
```

Sympy [F]

$$\int \frac{e^{-i\arctan(a+bx)}}{x^4}\,dx = -i\int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{ax^4+bx^5-ix^4}\,dx$$

input
```
integrate(1/(1+I*(b*x+a))*(1+(b*x+a)**2)**(1/2)/x**4,x)
```

output
```
-I*Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a*x**4 + b*x**5 - I*x**4
), x)
```

Maxima [F]

$$\int \frac{e^{-i\arctan(a+bx)}}{x^4}\,dx = \int \frac{\sqrt{(bx+a)^2+1}}{(i\,bx+i\,a+1)x^4}\,dx$$

input
```
integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x + a)^2 + 1)/((I*b*x + I*a + 1)*x^4), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 884 vs. 2(194) = 388.

Time = 0.17 (sec) , antiderivative size = 884, normalized size of antiderivative = 3.12

$$\int \frac{e^{-i\arctan(a+bx)}}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

output
```
1/2*(2*a^2*b^3 + 2*I*a*b^3 - b^3)*log(abs(2*x*abs(b) - 2*sqrt((b*x + a)^2
+ 1) - 2*sqrt(a^2 + 1))/abs(2*x*abs(b) - 2*sqrt((b*x + a)^2 + 1) + 2*sqrt(
a^2 + 1)))/((a^5 - I*a^4 + 2*a^3 - 2*I*a^2 + a - I)*sqrt(a^2 + 1)) + 1/3*(
-8*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a^5*b^3 + 24*(-I*x*abs(b) + I*sq
rt((b*x + a)^2 + 1))*a^7*b^3 - 24*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a
^6*b^2*abs(b) - 8*I*a^8*b^2*abs(b) + 6*(x*abs(b) - sqrt((b*x + a)^2 + 1))^
5*a^2*b^3 - 24*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a^4*b^3 + 18*(x*abs(b)
 - sqrt((b*x + a)^2 + 1))*a^6*b^3 - 12*(x*abs(b) - sqrt((b*x + a)^2 + 1))^
2*a^5*b^2*abs(b) + 12*a^7*b^2*abs(b) + 6*I*(x*abs(b) - sqrt((b*x + a)^2 +
1))^5*a*b^3 - 32*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a^3*b^3 + 54*(-I*x
*abs(b) + I*sqrt((b*x + a)^2 + 1))*a^5*b^3 - 60*I*(x*abs(b) - sqrt((b*x +
a)^2 + 1))^2*a^4*b^2*abs(b) - 20*I*a^6*b^2*abs(b) - 3*(x*abs(b) - sqrt((b*
x + a)^2 + 1))^5*b^3 - 24*(x*abs(b) - sqrt((b*x + a)^2 + 1))^3*a^2*b^3 + 3
9*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^4*b^3 - 24*(x*abs(b) - sqrt((b*x +
a)^2 + 1))^2*a^3*b^2*abs(b) + 36*a^5*b^2*abs(b) - 24*I*(x*abs(b) - sqrt((b
*x + a)^2 + 1))^3*a*b^3 + 36*(-I*x*abs(b) + I*sqrt((b*x + a)^2 + 1))*a^3*b
^3 - 48*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a^2*b^2*abs(b) - 12*I*a^4*b
^2*abs(b) + 24*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*b^3 - 12*(x*abs(b) -
 sqrt((b*x + a)^2 + 1))^2*a*b^2*abs(b) + 36*a^3*b^2*abs(b) + 6*(-I*x*abs(b
) + I*sqrt((b*x + a)^2 + 1))*a*b^3 - 12*I*(x*abs(b) - sqrt((b*x + a)^2 ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-i\arctan(a+bx)}}{x^4}\,dx = \int \frac{\sqrt{(a+b\,x)^2+1}}{x^4\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})}\,dx$$

input
```
int(((a + b*x)^2 + 1)^(1/2)/(x^4*(a*1i + b*x*1i + 1)),x)
```

output
```
int(((a + b*x)^2 + 1)^(1/2)/(x^4*(a*1i + b*x*1i + 1)), x)
```

Reduce [F]

$$\int \frac{e^{-i \arctan(a+bx)}}{x^4} \, dx = \int \frac{\sqrt{1+(bx+a)^2}}{(1+i(bx+a))\,x^4} dx$$

input
```
int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^4,x)
```

output
```
int(1/(1+I*(b*x+a))*(1+(b*x+a)^2)^(1/2)/x^4,x)
```

3.212 $\int e^{-2i\arctan(a+bx)}x^4\,dx$

Optimal result

Integrand size = 16, antiderivative size = 99

$$\int e^{-2i\arctan(a+bx)}x^4\,dx=-\frac{2(1+ia)^3x}{b^4}-\frac{i(i-a)^2x^2}{b^3}+\frac{2(1+ia)x^3}{3b^2}-\frac{ix^4}{2b}-\frac{x^5}{5}-\frac{2i(i-a)^4\log(i-a-bx)}{b^5}$$

output
```
-2*(1+I*a)^3*x/b^4-I*(I-a)^2*x^2/b^3+2/3*(1+I*a)*x^3/b^2-1/2*I*x^4/b-1/5*x
^5-2*I*(I-a)^4*ln(I-a-b*x)/b^5
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96

$$\int e^{-2i\arctan(a+bx)}x^4\,dx=-\frac{2(1+ia)^3x}{b^4}-\frac{i(-i+a)^2x^2}{b^3}+\frac{2(1+ia)x^3}{3b^2}-\frac{ix^4}{2b}-\frac{x^5}{5}-\frac{2i(-i+a)^4\log(i-a-bx)}{b^5}$$

input
```
Integrate[x^4/E^((2*I)*ArcTan[a + b*x]),x]
```

output
```
(-2*(1 + I*a)^3*x)/b^4 - (I*(-I + a)^2*x^2)/b^3 + (2*(1 + I*a)*x^3)/(3*b^2
) - ((I/2)*x^4)/b - x^5/5 - ((2*I)*(-I + a)^4*Log[I - a - b*x])/b^5
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5618, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{-2i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{x^4(-ia-ibx+1)}{ia+ibx+1}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{2i(a-i)^4}{b^4(a+bx-i)}+\frac{2(-1-ia)^3}{b^4}-\frac{2i(a-i)^2x}{b^3}+\frac{2(1+ia)x^2}{b^2}-\frac{2ix^3}{b}-x^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2i(-a+i)^4\log(-a-bx+i)}{b^5}-\frac{2(1+ia)^3x}{b^4}-\frac{i(-a+i)^2x^2}{b^3}+\frac{2(1+ia)x^3}{3b^2}-\frac{ix^4}{2b}-\frac{x^5}{5}$$

input
```
Int[x^4/E^((2*I)*ArcTan[a + b*x]),x]
```

output
```
(-2*(1 + I*a)^3*x)/b^4 - (I*(I - a)^2*x^2)/b^3 + (2*(1 + I*a)*x^3)/(3*b^2)
 - ((I/2)*x^4)/b - x^5/5 - ((2*I)*(I - a)^4*Log[I - a - b*x])/b^5
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.26

method	result
default	$-\frac{i\left(-\frac{1}{5} i b^{4} x^{5}+\frac{1}{2} b^{3} x^{4}+\frac{2}{3} i b^{2} x^{3}-\frac{2}{3} a\,b^{2} x^{3}-2 i a b\,x^{2}+a^{2} b\,x^{2}+6 i x\,a^{2}-2 a^{3} x-b\,x^{2}-2 i x+6 a x\right)}{b^{4}}+\frac{\left(-2 i a^{4}-8 a^{3}+12 i a^{2}+8 a-2 i\right)}{b^{5}}$
risch	$-\frac{x^{5}}{5}-\frac{6 i a x}{b^{4}}+\frac{2 x^{3}}{3 b^{2}}-\frac{i \ln\left(b^{2} x^{2}+2 a b x+a^{2}+1\right) a^{4}}{b^{5}}-\frac{2 a\,x^{2}}{b^{3}}+\frac{2 i a^{3} x}{b^{4}}+\frac{6 x\,a^{2}}{b^{4}}+\frac{6 i \ln\left(b^{2} x^{2}+2 a b x+a^{2}+1\right) a^{2}}{b^{5}}+\frac{i x^{2}}{b^{3}}$
parallelrisch	$\frac{60+90 i a\,b^{2} x^{2}+30 b^{2} x^{2}+300 i a+300 a^{4}-90 a^{2} b^{2} x^{2}+6 b^{6} x^{6}-10 i b^{3} x^{3}+20 a\,b^{3} x^{3}+60 i \ln\left(b x+a-i\right) x b+240 \ln\left(b x+a-i\right) x\,a^{3} b-240}{}$

input
```
int(x^4/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
-I/b^4*(-1/5*I*b^4*x^5+1/2*b^3*x^4+2/3*I*b^2*x^3-2/3*a*b^2*x^3-2*I*a*b*x^2
+a^2*b*x^2+6*I*x*a^2-2*a^3*x-b*x^2-2*I*x+6*a*x)+(-2*I*a^4+12*I*a^2-8*a^3-2
*I+8*a)/b^5*ln(I-a-b*x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06

$$\int e^{-2i\arctan(a+bx)}x^4\,dx = -\frac{6\,b^5x^5+15i\,b^4x^4+20\,(-i\,a-1)b^3x^3+30\,(i\,a^2+2\,a-i)b^2x^2+60\,(-i\,a^3-3\,a^2+3i\,a+1)bx+60\,(i}{30\,b^5}$$

input
```
integrate(x^4/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="fricas")
```

output
```
-1/30*(6*b^5*x^5 + 15*I*b^4*x^4 + 20*(-I*a - 1)*b^3*x^3 + 30*(I*a^2 + 2*a
- I)*b^2*x^2 + 60*(-I*a^3 - 3*a^2 + 3*I*a + 1)*b*x + 60*(I*a^4 + 4*a^3 - 6
*I*a^2 - 4*a + I)*log((b*x + a - I)/b))/b^5
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.15

$$\int e^{-2i\arctan(a+bx)}x^4\,dx = -\frac{x^5}{5}-x^3\left(-\frac{2ia}{3b^2}-\frac{2}{3b^2}\right)-x^2\left(\frac{ia^2}{b^3}+\frac{2a}{b^3}-\frac{i}{b^3}\right) - x\left(-\frac{2ia^3}{b^4}-\frac{6a^2}{b^4}+\frac{6ia}{b^4}+\frac{2}{b^4}\right) - \frac{ix^4}{2b}-\frac{2i(a-i)^4\log(a+bx-i)}{b^5}$$

input
```
integrate(x**4/(1+I*(b*x+a))**2*(1+(b*x+a)**2),x)
```

output
```
-x**5/5 - x**3*(-2*I*a/(3*b**2) - 2/(3*b**2)) - x**2*(I*a**2/b**3 + 2*a/b*
*3 - I/b**3) - x*(-2*I*a**3/b**4 - 6*a**2/b**4 + 6*I*a/b**4 + 2/b**4) - I*
x**4/(2*b) - 2*I*(a - I)**4*log(a + b*x - I)/b**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06

$$\int e^{-2i\arctan(a+bx)}x^4\,dx = -\frac{6\,b^4x^5+15i\,b^3x^4-20\,(i\,a+1)b^2x^3-30\,(-i\,a^2-2\,a+i)bx^2-60\,(i\,a^3+3\,a^2-3i\,a-1)x}{30\,b^4} - \frac{2\,(i\,a^4+4\,a^3-6i\,a^2-4\,a+i)\log\left(i\,bx+i\,a+1\right)}{b^5}$$

input
```
integrate(x^4/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="maxima")
```

output
```
-1/30*(6*b^4*x^5 + 15*I*b^3*x^4 - 20*(I*a + 1)*b^2*x^3 - 30*(-I*a^2 - 2*a
+ I)*b*x^2 - 60*(I*a^3 + 3*a^2 - 3*I*a - 1)*x)/b^4 - 2*(I*a^4 + 4*a^3 - 6*
I*a^2 - 4*a + I)*log(I*b*x + I*a + 1)/b^5
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(73) = 146.

Time = 0.14 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.17

$$\int e^{-2i\arctan(a+bx)}x^4\,dx = \frac{i\,(i\,bx+i\,a+1)^5\left(-\frac{15i\,(2\,ab-3i\,b)}{(i\,bx+i\,a+1)b}-\frac{20\,(3\,a^2b^2-10i\,ab^2-7\,b^2)}{(i\,bx+i\,a+1)^2b^2}+\frac{60i\,(a^3b^3-6i\,a^2b^3-9\,ab^3+4i\,b^3)}{(i\,bx+i\,a+1)^3b^3}+\frac{30\,(a^4b^4-12i\,a^3b^4-30\,a^2b^4-}{(i\,bx+i\,a+1)^4b}\right.}{30\,b^5} - \frac{2\,(-i\,a^4-4\,a^3+6i\,a^2+4\,a-i)\log\left(\frac{1}{\sqrt{(bx+a)^2+1}|b|}\right)}{b^5}$$

input
```
integrate(x^4/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="giac")
```

output
```
1/30*I*(I*b*x + I*a + 1)^5*(-15*I*(2*a*b - 3*I*b)/((I*b*x + I*a + 1)*b) -
20*(3*a^2*b^2 - 10*I*a*b^2 - 7*b^2)/((I*b*x + I*a + 1)^2*b^2) + 60*I*(a^3*
b^3 - 6*I*a^2*b^3 - 9*a*b^3 + 4*I*b^3)/((I*b*x + I*a + 1)^3*b^3) + 30*(a^4
*b^4 - 12*I*a^3*b^4 - 30*a^2*b^4 + 28*I*a*b^4 + 9*b^4)/((I*b*x + I*a + 1)^
4*b^4) + 6)/b^5 - 2*(-I*a^4 - 4*a^3 + 6*I*a^2 + 4*a - I)*log(1/(sqrt((b*x
+ a)^2 + 1)*abs(b)))/b^5
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.67

$$\int e^{-2i\arctan(a+bx)}x^4\,dx = \ln\left(x+\frac{a-\mathrm{i}}{b}\right)\left(\frac{8\,a-8\,a^3}{b^5}-\frac{(2\,a^4-12\,a^2+2)\,1\mathrm{i}}{b^5}\right) + x^4\left(\frac{a-\mathrm{i}}{4\,b}-\frac{a+1\mathrm{i}}{4\,b}\right)-\frac{x^5}{5}+\frac{x^2\left(\frac{a-\mathrm{i}}{b}-\frac{a+1\mathrm{i}}{b}\right)(a-\mathrm{i})^2}{2\,b^2} - \frac{x^3\left(\frac{a-\mathrm{i}}{b}-\frac{a+1\mathrm{i}}{b}\right)(a-\mathrm{i})}{3\,b}-\frac{x\left(\frac{a-\mathrm{i}}{b}-\frac{a+1\mathrm{i}}{b}\right)(a-\mathrm{i})^3}{b^3}$$

input
```
int((x^4*((a + b*x)^2 + 1))/(a*1i + b*x*1i + 1)^2,x)
```

output
```
log(x + (a - 1i)/b)*((8*a - 8*a^3)/b^5 - ((2*a^4 - 12*a^2 + 2)*1i)/b^5) +
x^4*((a - 1i)/(4*b) - (a + 1i)/(4*b)) - x^5/5 + (x^2*((a - 1i)/b - (a + 1i
)/b)*(a - 1i)^2)/(2*b^2) - (x^3*((a - 1i)/b - (a + 1i)/b)*(a - 1i))/(3*b)
- (x*((a - 1i)/b - (a + 1i)/b)*(a - 1i)^3)/b^3
```

Reduce [F]

$$\int e^{-2i\arctan(a+bx)}x^4\,dx = -\left(\int\frac{x^6}{b^2x^2+2abx-2bix+a^2-2ai-1}dx\right)b^2 - 2\left(\int\frac{x^5}{b^2x^2+2abx-2bix+a^2-2ai-1}dx\right)ab - \left(\int\frac{x^4}{b^2x^2+2abx-2bix+a^2-2ai-1}dx\right)a^2 - \left(\int\frac{x^4}{b^2x^2+2abx-2bix+a^2-2ai-1}dx\right)$$

input
```
int(x^4/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x)
```

output
```
- int(x**6/(a**2 + 2*a*b*x - 2*a*i + b**2*x**2 - 2*b*i*x - 1),x)*b**2 - 2
*int(x**5/(a**2 + 2*a*b*x - 2*a*i + b**2*x**2 - 2*b*i*x - 1),x)*a*b - int(
x**4/(a**2 + 2*a*b*x - 2*a*i + b**2*x**2 - 2*b*i*x - 1),x)*a**2 - int(x**4
/(a**2 + 2*a*b*x - 2*a*i + b**2*x**2 - 2*b*i*x - 1),x)
```

3.213 $\int e^{-2i\arctan(a+bx)}x^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int e^{-2i\arctan(a+bx)}x^3\,dx=-\frac{2i(i-a)^2x}{b^3}+\frac{(1+ia)x^2}{b^2}-\frac{2ix^3}{3b}-\frac{x^4}{4}-\frac{2(1+ia)^3\log(i-a-bx)}{b^4}$$

output
```
-2*I*(I-a)^2*x/b^3+(1+I*a)*x^2/b^2-2/3*I*x^3/b-1/4*x^4-2*(1+I*a)^3*ln(I-a-
b*x)/b^4
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int e^{-2i\arctan(a+bx)}x^3\,dx=-\frac{2i(i-a)^2x}{b^3}+\frac{(1+ia)x^2}{b^2}-\frac{2ix^3}{3b}-\frac{x^4}{4}-\frac{2(1+ia)^3\log(i-a-bx)}{b^4}$$

input
```
Integrate[x^3/E^((2*I)*ArcTan[a + b*x]),x]
```

output
```
((-2*I)*(I - a)^2*x)/b^3 + ((1 + I*a)*x^2)/b^2 - (((2*I)/3)*x^3)/b - x^4/4
 - (2*(1 + I*a)^3*Log[I - a - b*x])/b^4
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5618, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{-2i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{x^3(-ia-ibx+1)}{ia+ibx+1}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{2(-1-ia)^3}{b^3(a+bx-i)}-\frac{2i(a-i)^2}{b^3}+\frac{2(1+ia)x}{b^2}-\frac{2ix^2}{b}-x^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(1+ia)^3\log(-a-bx+i)}{b^4}-\frac{2i(-a+i)^2x}{b^3}+\frac{(1+ia)x^2}{b^2}-\frac{2ix^3}{3b}-\frac{x^4}{4}$$

input
```
Int[x^3/E^((2*I)*ArcTan[a + b*x]),x]
```

output
```
((-2*I)*(I - a)^2*x)/b^3 + ((1 + I*a)*x^2)/b^2 - (((2*I)/3)*x^3)/b - x^4/4
 - (2*(1 + I*a)^3*Log[I - a - b*x])/b^4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

method	result
default	$\frac{i\left(\frac{1}{4}ib^3x^4-\frac{2}{3}b^2x^3-ib\,x^2+ab\,x^2+4iax-2a^2x+2x\right)}{b^3}+\frac{\left(2ia^3+6a^2-6ia-2\right)\ln\left(-bx-a+i\right)}{b^4}$
risch	$-\frac{x^4}{4}-\frac{2ix^3}{3b}+\frac{x^2}{b^2}+\frac{ia\,x^2}{b^2}-\frac{4ax}{b^3}-\frac{2ia^2x}{b^3}+\frac{2ix}{b^3}+\frac{3\ln\left(b^2x^2+2abx+a^2+1\right)a^2}{b^4}+\frac{i\ln\left(b^2x^2+2abx+a^2+1\right)a^3}{b^4}-$
parallelrisch	$-\frac{-96a+96a^3+24i-72i\ln(bx+a-i)xab-24a\,b^2x^2+4b^3x^3-3a\,b^4x^4-3x^5b^5+12ib^2x^2-144i\ln(bx+a-i)a^2+24i\ln(bx+a-i)}{}$

input
```
int(x^3/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
I/b^3*(1/4*I*b^3*x^4-2/3*b^2*x^3-I*b*x^2+a*b*x^2+4*I*a*x-2*a^2*x+2*x)+(2*I
*a^3-6*I*a+6*a^2-2)/b^4*ln(I-a-b*x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int e^{-2i\arctan(a+bx)}x^3\,dx = -\frac{3\,b^4x^4+8i\,b^3x^3+12\,(-i\,a-1)b^2x^2+24\,(i\,a^2+2\,a-i)bx+24\,(-i\,a^3-3\,a^2+3i\,a+1)\log\left(\frac{bx+a-i}{b}\right)}{12\,b^4}$$

input
```
integrate(x^3/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="fricas")
```

output
```
-1/12*(3*b^4*x^4 + 8*I*b^3*x^3 + 12*(-I*a - 1)*b^2*x^2 + 24*(I*a^2 + 2*a -
 I)*b*x + 24*(-I*a^3 - 3*a^2 + 3*I*a + 1)*log((b*x + a - I)/b))/b^4
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int e^{-2i\arctan(a+bx)}x^3\,dx = -\frac{x^4}{4}-x^2\left(-\frac{ia}{b^2}-\frac{1}{b^2}\right)-x\left(\frac{2ia^2}{b^3}+\frac{4a}{b^3}-\frac{2i}{b^3}\right) - \frac{2ix^3}{3b}+\frac{2i(a-i)^3\log(a+bx-i)}{b^4}$$

input
```
integrate(x**3/(1+I*(b*x+a))**2*(1+(b*x+a)**2),x)
```

output
```
-x**4/4 - x**2*(-I*a/b**2 - 1/b**2) - x*(2*I*a**2/b**3 + 4*a/b**3 - 2*I/b*
*3) - 2*I*x**3/(3*b) + 2*I*(a - I)**3*log(a + b*x - I)/b**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int e^{-2i\arctan(a+bx)}x^3\,dx = -\frac{i\,(-3i\,b^3x^4+8\,b^2x^3-12\,(a-i)bx^2+24\,(a^2-2i\,a-1)x)}{12\,b^3} - \frac{2\,(-i\,a^3-3\,a^2+3i\,a+1)\log(i\,bx+i\,a+1)}{b^4}$$

input
```
integrate(x^3/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="maxima")
```

output
```
-1/12*I*(-3*I*b^3*x^4 + 8*b^2*x^3 - 12*(a - I)*b*x^2 + 24*(a^2 - 2*I*a - 1
)*x)/b^3 - 2*(-I*a^3 - 3*a^2 + 3*I*a + 1)*log(I*b*x + I*a + 1)/b^4
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(59) = 118.

Time = 0.12 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.05

$$\int e^{-2i\arctan(a+bx)}x^3\,dx = -\frac{(i\,bx+i\,a+1)^4\left(-\frac{4i\,(3\,ab-5i\,b)}{(i\,bx+i\,a+1)b}-\frac{18\,(a^2b^2-4i\,ab^2-3\,b^2)}{(i\,bx+i\,a+1)^2b^2}+\frac{12i\,(a^3b^3-9i\,a^2b^3-15\,ab^3+7i\,b^3)}{(i\,bx+i\,a+1)^3b^3}+3\right)}{12\,b^4} - \frac{2\,(i\,a^3+3\,a^2-3i\,a-1)\log\left(\frac{1}{\sqrt{(bx+a)^2+1}|b|}\right)}{b^4}$$

input
```
integrate(x^3/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="giac")
```

output
```
-1/12*(I*b*x + I*a + 1)^4*(-4*I*(3*a*b - 5*I*b)/((I*b*x + I*a + 1)*b) - 18
*(a^2*b^2 - 4*I*a*b^2 - 3*b^2)/((I*b*x + I*a + 1)^2*b^2) + 12*I*(a^3*b^3 -
 9*I*a^2*b^3 - 15*a*b^3 + 7*I*b^3)/((I*b*x + I*a + 1)^3*b^3) + 3)/b^4 - 2*
(I*a^3 + 3*a^2 - 3*I*a - 1)*log(1/(sqrt((b*x + a)^2 + 1)*abs(b)))/b^4
```

Mupad [B] (verification not implemented)

Time = 23.42 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.68

$$\int e^{-2i\arctan(a+bx)}x^3\,dx = x^3\left(\frac{a-\mathrm{i}}{3\,b}-\frac{a+1\mathrm{i}}{3\,b}\right)-\frac{x^4}{4} - \ln\left(x+\frac{a-\mathrm{i}}{b}\right)\left(-\frac{6\,a^2-2}{b^4}+\frac{(6\,a-2\,a^3)\,1\mathrm{i}}{b^4}\right) - \frac{x^2\left(\frac{a-\mathrm{i}}{b}-\frac{a+1\mathrm{i}}{b}\right)(a-\mathrm{i})}{2\,b}+\frac{x\left(\frac{a-\mathrm{i}}{b}-\frac{a+1\mathrm{i}}{b}\right)(a-\mathrm{i})^2}{b^2}$$

input
```
int((x^3*((a + b*x)^2 + 1))/(a*1i + b*x*1i + 1)^2,x)
```

output
```
x^3*((a - 1i)/(3*b) - (a + 1i)/(3*b)) - x^4/4 - log(x + (a - 1i)/b)*(((6*a
 - 2*a^3)*1i)/b^4 - (6*a^2 - 2)/b^4) - (x^2*((a - 1i)/b - (a + 1i)/b)*(a -
 1i))/(2*b) + (x*((a - 1i)/b - (a + 1i)/b)*(a - 1i)^2)/b^2
```

Reduce [F]

$$\int e^{-2i\arctan(a+bx)}x^3\,dx = -\left(\int \frac{x^5}{b^2x^2+2abx-2bix+a^2-2ai-1}dx\right)b^2 - 2\left(\int \frac{x^4}{b^2x^2+2abx-2bix+a^2-2ai-1}dx\right)ab - \left(\int \frac{x^3}{b^2x^2+2abx-2bix+a^2-2ai-1}dx\right)a^2 - \left(\int \frac{x^3}{b^2x^2+2abx-2bix+a^2-2ai-1}dx\right)$$

input
```
int(x^3/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x)
```

output
```
 - int(x**5/(a**2 + 2*a*b*x - 2*a*i + b**2*x**2 - 2*b*i*x - 1),x)*b**2 - 2
*int(x**4/(a**2 + 2*a*b*x - 2*a*i + b**2*x**2 - 2*b*i*x - 1),x)*a*b - int(
x**3/(a**2 + 2*a*b*x - 2*a*i + b**2*x**2 - 2*b*i*x - 1),x)*a**2 - int(x**3
/(a**2 + 2*a*b*x - 2*a*i + b**2*x**2 - 2*b*i*x - 1),x)
```

3.214 $\int e^{-2i\arctan(a+bx)}x^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 59

$$\int e^{-2i\arctan(a+bx)}x^2\,dx=\frac{2(1+ia)x}{b^2}-\frac{ix^2}{b}-\frac{x^3}{3}-\frac{2i(i-a)^2\log(i-a-bx)}{b^3}$$

output
```
2*(1+I*a)*x/b^2-I*x^2/b-1/3*x^3-2*I*(I-a)^2*ln(I-a-b*x)/b^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int e^{-2i\arctan(a+bx)}x^2\,dx=\frac{bx(6+6ia-3ibx-b^2x^2)-6i(-i+a)^2\log(i-a-bx)}{3b^3}$$

input
```
Integrate[x^2/E^((2*I)*ArcTan[a + b*x]),x]
```

output
```
(b*x*(6 + (6*I)*a - (3*I)*b*x - b^2*x^2) - (6*I)*(-I + a)^2*Log[I - a - b*
x])/(3*b^3)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5618, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{-2i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{x^2(-ia-ibx+1)}{ia+ibx+1}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{2i(a-i)^2}{b^2(a+bx-i)}+\frac{2i(a-i)}{b^2}-\frac{2ix}{b}-x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2i(-a+i)^2\log(-a-bx+i)}{b^3}+\frac{2(1+ia)x}{b^2}-\frac{ix^2}{b}-\frac{x^3}{3}$$

input
```
Int[x^2/E^((2*I)*ArcTan[a + b*x]),x]
```

output
```
(2*(1 + I*a)*x)/b^2 - (I*x^2)/b - x^3/3 - ((2*I)*(I - a)^2*Log[I - a - b*x
])/b^3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

method	result
default	$\frac{i\left(\frac{1}{3}ib^2x^3-b\,x^2-2ix+2ax\right)}{b^2}+\frac{\left(-2ia^2-4a+2i\right)\ln\left(-bx-a+i\right)}{b^3}$
risch	$-\frac{x^3}{3}-\frac{ix^2}{b}+\frac{2x}{b^2}+\frac{2iax}{b^2}-\frac{2\ln\left(b^2x^2+2abx+a^2+1\right)a}{b^3}-\frac{i\ln\left(b^2x^2+2abx+a^2+1\right)a^2}{b^3}+\frac{i\ln\left(b^2x^2+2abx+a^2+1\right)}{b^3}-$
parallelrisch	$\frac{b^4x^4+6i\ln\left(bx+a-i\right)x\,a^2b+a\,b^3x^3-3ia\,b^2x^2-18ia-6i\ln\left(bx+a-i\right)xb-6+6i\ln\left(bx+a-i\right)a^3-18i\ln\left(bx+a-i\right)a+2ib^3x^3+12\ln}{3b^3\left(-bx-a+i\right)}$

input
```
int(x^2/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
I/b^2*(1/3*I*b^2*x^3-b*x^2-2*I*x+2*a*x)+(-2*I*a^2+2*I-4*a)/b^3*ln(I-a-b*x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int e^{-2i\arctan(a+bx)}x^2\,dx = -\frac{b^3x^3+3i\,b^2x^2+6\,(-i\,a-1)bx+6\,(i\,a^2+2\,a-i)\log\left(\frac{bx+a-i}{b}\right)}{3\,b^3}$$

input
```
integrate(x^2/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="fricas")
```

output
```
-1/3*(b^3*x^3 + 3*I*b^2*x^2 + 6*(-I*a - 1)*b*x + 6*(I*a^2 + 2*a - I)*log((
b*x + a - I)/b))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int e^{-2i\arctan(a+bx)}x^2\,dx = -\frac{x^3}{3} - x\left(-\frac{2ia}{b^2} - \frac{2}{b^2}\right) - \frac{ix^2}{b} - \frac{2i(a-i)^2\log(a+bx-i)}{b^3}$$

input
```
integrate(x**2/(1+I*(b*x+a))**2*(1+(b*x+a)**2),x)
```

output
```
-x**3/3 - x*(-2*I*a/b**2 - 2/b**2) - I*x**2/b - 2*I*(a - I)**2*log(a + b*x
 - I)/b**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int e^{-2i\arctan(a+bx)}x^2\,dx = -\frac{b^2x^3 + 3i\,bx^2 + 6\,(-i\,a-1)x}{3\,b^2} - \frac{2\,(i\,a^2 + 2\,a - i)\log(i\,bx + i\,a + 1)}{b^3}$$

input
```
integrate(x^2/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="maxima")
```

output
```
-1/3*(b^2*x^3 + 3*I*b*x^2 + 6*(-I*a - 1)*x)/b^2 - 2*(I*a^2 + 2*a - I)*log(
I*b*x + I*a + 1)/b^3
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(45) = 90.

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.85

$$\int e^{-2i\arctan(a+bx)}x^2\,dx = -\frac{i\,(i\,bx + i\,a + 1)^3\left(-\frac{3i\,(ab-2i\,b)}{(i\,bx+i\,a+1)b} - \frac{3\,(a^2b^2-6i\,ab^2-5\,b^2)}{(i\,bx+i\,a+1)^2b^2} + 1\right)}{3\,b^3} - \frac{2\,(-i\,a^2 - 2\,a + i)\log\left(\frac{1}{\sqrt{(bx+a)^2+1}|b|}\right)}{b^3}$$

input
```
integrate(x^2/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="giac")
```

output
```
-1/3*I*(I*b*x + I*a + 1)^3*(-3*I*(a*b - 2*I*b)/((I*b*x + I*a + 1)*b) - 3*(
a^2*b^2 - 6*I*a*b^2 - 5*b^2)/((I*b*x + I*a + 1)^2*b^2) + 1)/b^3 - 2*(-I*a^
2 - 2*a + I)*log(1/(sqrt((b*x + a)^2 + 1)*abs(b)))/b^3
```

Mupad [B] (verification not implemented)

Time = 23.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.53

$$\int e^{-2i\arctan(a+bx)}x^2\,dx = -\ln\left(x+\frac{a-\mathrm{i}}{b}\right)\left(\frac{4\,a}{b^3}+\frac{(2\,a^2-2)\,1\mathrm{i}}{b^3}\right)+x^2\left(\frac{a-\mathrm{i}}{2\,b}-\frac{a+1\mathrm{i}}{2\,b}\right)-\frac{x^3}{3}-\frac{x\left(\frac{a-\mathrm{i}}{b}-\frac{a+1\mathrm{i}}{b}\right)(a-\mathrm{i})}{b}$$

input
```
int((x^2*((a + b*x)^2 + 1))/(a*1i + b*x*1i + 1)^2,x)
```

output
```
x^2*((a - 1i)/(2*b) - (a + 1i)/(2*b)) - log(x + (a - 1i)/b)*((4*a)/b^3 + (
(2*a^2 - 2)*1i)/b^3) - x^3/3 - (x*((a - 1i)/b - (a + 1i)/b)*(a - 1i))/b
```

Reduce [F]

$$\int e^{-2i\arctan(a+bx)}x^2\,dx = -\left(\int\frac{x^4}{b^2x^2+2abx-2bix+a^2-2ai-1}dx\right)b^2-2\left(\int\frac{x^3}{b^2x^2+2abx-2bix+a^2-2ai-1}dx\right)ab-\left(\int\frac{x^2}{b^2x^2+2abx-2bix+a^2-2ai-1}dx\right)a^2-\left(\int\frac{x^2}{b^2x^2+2abx-2bix+a^2-2ai-1}dx\right)$$

input
```
int(x^2/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x)
```

output
```
- int(x**4/(a**2 + 2*a*b*x - 2*a*i + b**2*x**2 - 2*b*i*x - 1),x)*b**2 - 2
*int(x**3/(a**2 + 2*a*b*x - 2*a*i + b**2*x**2 - 2*b*i*x - 1),x)*a*b - int(
x**2/(a**2 + 2*a*b*x - 2*a*i + b**2*x**2 - 2*b*i*x - 1),x)*a**2 - int(x**2
/(a**2 + 2*a*b*x - 2*a*i + b**2*x**2 - 2*b*i*x - 1),x)
```

3.215 $\int e^{-2i\arctan(a+bx)}x\,dx$

Optimal result

Integrand size = 14, antiderivative size = 40

$$\int e^{-2i\arctan(a+bx)}x\,dx=-\frac{2ix}{b}-\frac{x^2}{2}+\frac{2(1+ia)\log(i-a-bx)}{b^2}$$

output
```
-2*I*x/b-1/2*x^2+2*(1+I*a)*ln(I-a-b*x)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int e^{-2i\arctan(a+bx)}x\,dx=-\frac{2ix}{b}-\frac{x^2}{2}+\frac{2(1+ia)\log(i-a-bx)}{b^2}$$

input
```
Integrate[x/E^((2*I)*ArcTan[a + b*x]),x]
```

output
```
((-2*I)*x)/b - x^2/2 + (2*(1 + I*a)*Log[I - a - b*x])/b^2
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5618, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-2i\arctan(a+bx)}dx$$

$$\Big\downarrow \text{ 5618}$$

$$\int \frac{x(-ia-ibx+1)}{ia+ibx+1}dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(\frac{2(1+ia)}{b(a+bx-i)}-\frac{2i}{b}-x\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2(1+ia)\log(-a-bx+i)}{b^2}-\frac{2ix}{b}-\frac{x^2}{2}$$

input
```
Int[x/E^((2*I)*ArcTan[a + b*x]),x]
```

output
```
((-2*I)*x)/b - x^2/2 + (2*(1 + I*a)*Log[I - a - b*x])/b^2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

method	result
default	$-\frac{\frac{1}{2} b\,x^{2}+2 i x}{b}+\frac{\left(2 i a+2\right) \ln\left(-b x-a+i\right)}{b^{2}}$
risch	$-\frac{x^{2}}{2}-\frac{2 i x}{b}+\frac{\ln\left(b^{2} x^{2}+2 a b x+a^{2}+1\right)}{b^{2}}+\frac{2 i \arctan\left(b x+a\right)}{b^{2}}+\frac{i a \ln\left(b^{2} x^{2}+2 a b x+a^{2}+1\right)}{b^{2}}-\frac{2 a \arctan\left(b x+a\right)}{b^{2}}$
parallelrisch	$-\frac{-b^{3} x^{3}+4 i \ln\left(b x+a-i\right) x a b-3 i b^{2} x^{2}-a\,b^{2} x^{2}+4 i \ln\left(b x+a-i\right) a^{2}-4 i+4 i a^{2}+4 \ln\left(b x+a-i\right) x b-4 i \ln\left(b x+a-i\right)+8 \ln\left(b x+a-i\right) a}{2 b^{2}\left(-b x-a+i\right)}$

input
```
int(x/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/b*(1/2*b*x^2+2*I*x)+(2*I*a+2)/b^2*ln(I-a-b*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int e^{-2i\arctan(a+bx)}x\,dx=-\frac{b^{2}x^{2}+4i\,bx+4\left(-i\,a-1\right)\log\left(\frac{bx+a-i}{b}\right)}{2\,b^{2}}$$

input
```
integrate(x/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="fricas")
```

output
```
-1/2*(b^2*x^2 + 4*I*b*x + 4*(-I*a - 1)*log((b*x + a - I)/b))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int e^{-2i\arctan(a+bx)}x\,dx = -\frac{x^2}{2} - \frac{2ix}{b} + \frac{2i(a-i)\log(a+bx-i)}{b^2}$$

input
```
integrate(x/(1+I*(b*x+a))**2*(1+(b*x+a)**2),x)
```

output
```
-x**2/2 - 2*I*x/b + 2*I*(a - I)*log(a + b*x - I)/b**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int e^{-2i\arctan(a+bx)}x\,dx = \frac{i\,(i\,bx^2-4\,x)}{2\,b} - \frac{2\,(-i\,a-1)\log(i\,bx+i\,a+1)}{b^2}$$

input
```
integrate(x/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="maxima")
```

output
```
1/2*I*(I*b*x^2 - 4*x)/b - 2*(-I*a - 1)*log(I*b*x + I*a + 1)/b^2
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(32) = 64.

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.80

$$\int e^{-2i\arctan(a+bx)}x\,dx = -\frac{i\left(\frac{(i\,bx+i\,a+1)^2\left(-\frac{2i\,(i\,ab+3\,b)}{(i\,bx+i\,a+1)b}+i\right)}{b}+\frac{4\,(a-i)\log\left(\frac{1}{\sqrt{(bx+a)^2+1}|b|}\right)}{b}\right)}{2\,b}$$

input
```
integrate(x/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="giac")
```

output
```
-1/2*I*((I*b*x + I*a + 1)^2*(-2*I*(I*a*b + 3*b)/((I*b*x + I*a + 1)*b) + I)
/b + 4*(a - I)*log(1/(sqrt((b*x + a)^2 + 1)*abs(b)))/b)/b
```

Mupad [B] (verification not implemented)

Time = 23.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int e^{-2i\arctan(a+bx)}x\,dx=\ln\left(x+\frac{a-\mathrm{i}}{b}\right)\left(\frac{2}{b^2}+\frac{a\,2\mathrm{i}}{b^2}\right)-\frac{x^2}{2}+x\left(\frac{a-\mathrm{i}}{b}-\frac{a+1\mathrm{i}}{b}\right)$$

input
```
int((x*((a + b*x)^2 + 1))/(a*1i + b*x*1i + 1)^2,x)
```

output
```
log(x + (a - 1i)/b)*((a*2i)/b^2 + 2/b^2) - x^2/2 + x*((a - 1i)/b - (a + 1i
)/b)
```

Reduce [F]

$$\begin{aligned}\int e^{-2i\arctan(a+bx)}x\,dx=&-\left(\int\frac{x^3}{b^2x^2+2abx-2bix+a^2-2ai-1}dx\right)b^2\\&-2\left(\int\frac{x^2}{b^2x^2+2abx-2bix+a^2-2ai-1}dx\right)ab\\&-\left(\int\frac{x}{b^2x^2+2abx-2bix+a^2-2ai-1}dx\right)a^2\\&-\left(\int\frac{x}{b^2x^2+2abx-2bix+a^2-2ai-1}dx\right)\end{aligned}$$

input
```
int(x/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x)
```

output
```
 - int(x**3/(a**2 + 2*a*b*x - 2*a*i + b**2*x**2 - 2*b*i*x - 1),x)*b**2 - 2
*int(x**2/(a**2 + 2*a*b*x - 2*a*i + b**2*x**2 - 2*b*i*x - 1),x)*a*b - int(
x/(a**2 + 2*a*b*x - 2*a*i + b**2*x**2 - 2*b*i*x - 1),x)*a**2 - int(x/(a**2
 + 2*a*b*x - 2*a*i + b**2*x**2 - 2*b*i*x - 1),x)
```

3.216 $\int e^{-2i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 23

$$\int e^{-2i\arctan(a+bx)}\,dx = -x - \frac{2i\log(i-a-bx)}{b}$$

output
```
-x-2*I*ln(I-a-b*x)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int e^{-2i\arctan(a+bx)}\,dx = -x + \frac{2\arctan(a+bx)}{b} - \frac{i\log\left(1+(a+bx)^2\right)}{b}$$

input
```
Integrate[E^((-2*I)*ArcTan[a + b*x]),x]
```

output
```
-x + (2*ArcTan[a + b*x])/b - (I*Log[1 + (a + b*x)^2])/b
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5616, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2i\arctan(a+bx)}dx$$

$$\Big\downarrow \text{5616}$$

$$\int \frac{-ia-ibx+1}{ia+ibx+1}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-1-\frac{2i}{a+bx-i}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-x-\frac{2i\log(-a-bx+i)}{b}$$

input
```
Int[E^((-2*I)*ArcTan[a + b*x]),x]
```

output
```
-x - ((2*I)*Log[I - a - b*x])/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5616
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^(I*(n/2))/(1 + I*a*c + I*b*c*x)^(I*(n/2)), x] /; FreeQ[{a, b,
c, n}, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
default	$-x-\frac{2 i \ln \left(-b x-a+i\right)}{b}$	22
risch	$-x-\frac{i \ln \left(b^{2} x^{2}+2 a b x+a^{2}+1\right)}{b}+\frac{2 \arctan \left(b x+a\right)}{b}$	40
parallelrisch	$\frac{2 i \ln \left(b x+a-i\right) x b+b^{2} x^{2}+2 i \ln \left(b x+a-i\right) a+1+2 i a-a^{2}+2 \ln \left(b x+a-i\right)}{b \left(-b x-a+i\right)}$	70

input
```
int(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
-x-2*I*ln(I-a-b*x)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int e^{-2i\arctan(a+bx)}\,dx=-\frac{bx+2i\,\log\left(\frac{bx+a-i}{b}\right)}{b}$$

input
```
integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="fricas")
```

output
```
-(b*x + 2*I*log((b*x + a - I)/b))/b
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int e^{-2i\arctan(a+bx)}\,dx = -x - \frac{2i\log(a+bx-i)}{b}$$

input
```
integrate(1/(1+I*(b*x+a))**2*(1+(b*x+a)**2),x)
```

output
```
-x - 2*I*log(a + b*x - I)/b
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int e^{-2i\arctan(a+bx)}\,dx = -x - \frac{2i\log(i\,bx+i\,a+1)}{b}$$

input
```
integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="maxima")
```

output
```
-x - 2*I*log(I*b*x + I*a + 1)/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int e^{-2i\arctan(a+bx)}\,dx = \frac{i\,(i\,bx+i\,a+1)}{b} + \frac{2i\log\left(\frac{1}{\sqrt{(bx+a)^2+1}|b|}\right)}{b}$$

input
```
integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x, algorithm="giac")
```

output
```
I*(I*b*x + I*a + 1)/b + 2*I*log(1/(sqrt((b*x + a)^2 + 1)*abs(b)))/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int e^{-2i\arctan(a+bx)}\,dx=-x-\frac{\ln\left(x+\frac{a-\mathrm{i}}{b}\right)\,2\mathrm{i}}{b}$$

input
```
int(((a + b*x)^2 + 1)/(a*1i + b*x*1i + 1)^2,x)
```

output
```
- x - (log(x + (a - 1i)/b)*2i)/b
```

Reduce [F]

$$\int e^{-2i\arctan(a+bx)}\,dx=-\left(\int\frac{x^2}{b^2x^2+2abx-2bix+a^2-2ai-1}dx\right)b^2-2\left(\int\frac{x}{b^2x^2+2abx-2bix+a^2-2ai-1}dx\right)ab-\left(\int\frac{1}{b^2x^2+2abx-2bix+a^2-2ai-1}dx\right)a^2-\left(\int\frac{1}{b^2x^2+2abx-2bix+a^2-2ai-1}dx\right)$$

input
```
int(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2),x)
```

output
```
- int(x**2/(a**2 + 2*a*b*x - 2*a*i + b**2*x**2 - 2*b*i*x - 1),x)*b**2 - 2
*int(x/(a**2 + 2*a*b*x - 2*a*i + b**2*x**2 - 2*b*i*x - 1),x)*a*b - int(1/(
a**2 + 2*a*b*x - 2*a*i + b**2*x**2 - 2*b*i*x - 1),x)*a**2 - int(1/(a**2 +
2*a*b*x - 2*a*i + b**2*x**2 - 2*b*i*x - 1),x)
```

3.217 $\int \frac{e^{-2i\arctan(a+bx)}}{x} dx$

Optimal result

Integrand size = 16, antiderivative size = 41

$$\int \frac{e^{-2i\arctan(a+bx)}}{x} dx = \frac{(i+a)\log(x)}{i-a} - \frac{2\log(i-a-bx)}{1+ia}$$

output
```
(I+a)*ln(x)/(I-a)-2*ln(I-a-b*x)/(1+I*a)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{e^{-2i\arctan(a+bx)}}{x} dx = \frac{-((i+a)\log(x)) + 2i\log(i-a-bx)}{-i+a}$$

input
```
Integrate[1/(E^((2*I)*ArcTan[a + b*x])*x),x]
```

output
```
(-((I + a)*Log[x]) + (2*I)*Log[I - a - b*x])/(-I + a)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5618, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2i\arctan(a+bx)}}{x} dx$$
$$\Big\downarrow \text{5618}$$
$$\int \frac{-ia-ibx+1}{x(ia+ibx+1)}dx$$
$$\Big\downarrow \text{86}$$
$$\int \left(\frac{2ib}{(a-i)(a+bx-i)}+\frac{-a-i}{(a-i)x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{(a+i)\log(x)}{-a+i}-\frac{2\log(-a-bx+i)}{1+ia}$$

input
```
Int[1/(E^((2*I)*ArcTan[a + b*x])*x),x]
```

output
```
((I + a)*Log[x])/(I - a) - (2*Log[I - a - b*x])/(1 + I*a)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

method	result
default	$-\frac{2i\ln(-bx-a+i)}{i-a}+\frac{(-a^2-1)\ln(x)}{(i-a)^2}$
risch	$\frac{i\ln(x)}{i-a}+\frac{\ln(x)a}{i-a}-\frac{i\ln(b^2x^2+2abx+a^2+1)}{i-a}+\frac{2\arctan(bx+a)}{i-a}$
parallelrisch	$-\frac{20\ln(x)x\,a^7b-\ln(x)x\,a^9b-14\ln(x)x\,a^5b-28\ln(x)x\,a^3b+48i\ln(x)a^3+8i\ln(x)a^9+2i\ln(bx+a-i)a^9-48i\ln(x)a^7-72i\ln(b \ldots}{}$

input
```
int(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2*I/(I-a)*ln(I-a-b*x)+(-a^2-1)/(I-a)^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{e^{-2i\arctan(a+bx)}}{x}\,dx=-\frac{(a+i)\log(x)-2i\log\left(\frac{bx+a-i}{b}\right)}{a-i}$$

input
```
integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x,x, algorithm="fricas")
```

output
```
-((a + I)*log(x) - 2*I*log((b*x + a - I)/b))/(a - I)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(24) = 48.

Time = 0.42 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.41

$$\int \frac{e^{-2i\arctan(a+bx)}}{x}\,dx = -\frac{(a+i)\log\left(a^2-\frac{a^2(a+i)}{a-i}+\frac{2ia(a+i)}{a-i}+x(ab+3ib)+1+\frac{a+i}{a-i}\right)}{a-i} + \frac{2i\log\left(a^2+\frac{2ia^2}{a-i}+\frac{4a}{a-i}+x(ab+3ib)+1-\frac{2i}{a-i}\right)}{a-i}$$

input
```
integrate(1/(1+I*(b*x+a))**2*(1+(b*x+a)**2)/x,x)
```

output
```
-(a + I)*log(a**2 - a**2*(a + I)/(a - I) + 2*I*a*(a + I)/(a - I) + x*(a*b
+ 3*I*b) + 1 + (a + I)/(a - I))/(a - I) + 2*I*log(a**2 + 2*I*a**2/(a - I)
+ 4*a/(a - I) + x*(a*b + 3*I*b) + 1 - 2*I/(a - I))/(a - I)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

$$\int \frac{e^{-2i\arctan(a+bx)}}{x}\,dx = -\frac{2\,(-i\,a-1)\log\left(i\,bx+i\,a+1\right)}{a^2-2i\,a-1} - \frac{(a^2+1)\log\left(x\right)}{a^2-2i\,a-1}$$

input
```
integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x,x, algorithm="maxima")
```

output
```
-2*(-I*a - 1)*log(I*b*x + I*a + 1)/(a^2 - 2*I*a - 1) - (a^2 + 1)*log(x)/(a
^2 - 2*I*a - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(32) = 64.

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.66

$$\int \frac{e^{-2i\arctan(a+bx)}}{x}\,dx = ib\left(\frac{(a+i)\log\left(\frac{ia}{ibx+ia+1}+\frac{1}{ibx+ia+1}-1\right)}{-iab-b}-\frac{i\log\left(\frac{1}{\sqrt{(bx+a)^2+1}|b|}\right)}{b}\right)$$

input
```
integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x,x, algorithm="giac")
```

output
```
I*b*((a + I)*log(I*a/(I*b*x + I*a + 1) + 1/(I*b*x + I*a + 1) - 1)/(-I*a*b
- b) - I*log(1/(sqrt((b*x + a)^2 + 1)*abs(b)))/b)
```

Mupad [B] (verification not implemented)

Time = 23.36 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{e^{-2i\arctan(a+bx)}}{x}\,dx = -\frac{2\ln(a+bx-\mathrm{i})}{1+a\,1\mathrm{i}}+\ln(x)\left(\frac{2}{1+a\,1\mathrm{i}}-1\right)$$

input
```
int(((a + b*x)^2 + 1)/(x*(a*1i + b*x*1i + 1)^2),x)
```

output
```
log(x)*(2/(a*1i + 1) - 1) - (2*log(a + b*x - 1i))/(a*1i + 1)
```

Reduce [F]

$$\int \frac{e^{-2i\arctan(a+bx)}}{x}\,dx = -\left(\int \frac{x}{b^2x^2+2abx-2bix+a^2-2ai-1}dx\right)b^2 - 2\left(\int \frac{1}{b^2x^2+2abx-2bix+a^2-2ai-1}dx\right)ab - \left(\int \frac{1}{b^2x^3+2ab\,x^2-2bi\,x^2+a^2x-2aix-x}dx\right)a^2 - \left(\int \frac{1}{b^2x^3+2ab\,x^2-2bi\,x^2+a^2x-2aix-x}dx\right)$$

input
```
int(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x,x)
```

output
```
- int(x/(a**2 + 2*a*b*x - 2*a*i + b**2*x**2 - 2*b*i*x - 1),x)*b**2 - 2*in
t(1/(a**2 + 2*a*b*x - 2*a*i + b**2*x**2 - 2*b*i*x - 1),x)*a*b - int(1/(a**
2*x + 2*a*b*x**2 - 2*a*i*x + b**2*x**3 - 2*b*i*x**2 - x),x)*a**2 - int(1/(
a**2*x + 2*a*b*x**2 - 2*a*i*x + b**2*x**3 - 2*b*i*x**2 - x),x)
```

3.218 $\int \frac{e^{-2i\arctan(a+bx)}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 62

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^2}\,dx = -\frac{i+a}{(i-a)x} + \frac{2ib\log(x)}{(i-a)^2} - \frac{2ib\log(i-a-bx)}{(i-a)^2}$$

output
```
-(I+a)/(I-a)/x+2*I*b*ln(x)/(I-a)^2-2*I*b*ln(I-a-b*x)/(I-a)^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.68

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^2}\,dx = \frac{1+a^2+2ibx\log(x)-2ibx\log(i-a-bx)}{(-i+a)^2x}$$

input
```
Integrate[1/(E^((2*I)*ArcTan[a + b*x])*x^2),x]
```

output
```
(1 + a^2 + (2*I)*b*x*Log[x] - (2*I)*b*x*Log[I - a - b*x])/((-I + a)^2*x)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5618, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^2}dx$$

$$\Big\downarrow \text{ 5618}$$

$$\int \frac{-ia-ibx+1}{x^2(ia+ibx+1)}dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(-\frac{2ib^2}{(a-i)^2(a+bx-i)}+\frac{2ib}{(a-i)^2x}+\frac{-a-i}{(a-i)x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2ib\log(x)}{(-a+i)^2}-\frac{2ib\log(-a-bx+i)}{(-a+i)^2}-\frac{a+i}{(-a+i)x}$$

input
```
Int[1/(E^((2*I)*ArcTan[a + b*x])*x^2),x]
```

output
```
-((I + a)/((I - a)*x)) + ((2*I)*b*Log[x])/(I - a)^2 - ((2*I)*b*Log[I - a -
 b*x])/(I - a)^2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

method	result
default	$-\frac{-a^2-1}{x(i-a)^2}-\frac{2b(ia+1)\ln(x)}{(i-a)^3}+\frac{2b(ia+1)\ln(-bx-a+i)}{(i-a)^3}$
risch	$\frac{i}{(-i+a)x}+\frac{a}{(-i+a)x}-\frac{2b\ln((-2a^2b-2b)x)}{ia^2+2a-i}+\frac{b\ln(4a^4b^2x^2+8a^5bx+4a^6+8a^2b^2x^2+16a^3bx+12a^4+4b^2x^2+8abx+12a}{ia^2+2a-i}$
parallelrisch	$\frac{-3ab+a^5b+ib-2a^3b+2i\ln(x)x\,a^3b^2-2i\ln(bx+a-i)x\,a^3b^2-6i\ln(x)xa\,b^2+6i\ln(bx+a-i)xa\,b^2+2i\ln(x)x^2a^2b^3-2i\ln(bx+a-}{}$

input
```
int(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(-a^2-1)/x/(I-a)^2-2*b*(1+I*a)/(I-a)^3*ln(x)+2*b*(1+I*a)/(I-a)^3*ln(I-a-b
*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.65

$$\int\frac{e^{-2i\arctan(a+bx)}}{x^2}\,dx=\frac{2i\,bx\log(x)-2i\,bx\log\left(\frac{bx+a-i}{b}\right)+a^2+1}{(a^2-2i\,a-1)x}$$

input
```
integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^2,x, algorithm="fricas")
```

output
```
(2*I*b*x*log(x) - 2*I*b*x*log((b*x + a - I)/b) + a^2 + 1)/((a^2 - 2*I*a -
1)*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(41) = 82$.

Time = 0.30 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.55

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^2}\,dx = \frac{2ib\log\left(-\frac{2a^3b}{(a-i)^2}+\frac{6ia^2b}{(a-i)^2}+2ab+\frac{6ab}{(a-i)^2}+4b^2x-2ib-\frac{2ib}{(a-i)^2}\right)}{(a-i)^2} - \frac{2ib\log\left(\frac{2a^3b}{(a-i)^2}-\frac{6ia^2b}{(a-i)^2}+2ab-\frac{6ab}{(a-i)^2}+4b^2x-2ib+\frac{2ib}{(a-i)^2}\right)}{(a-i)^2} - \frac{-a-i}{x(a-i)}$$

input
```
integrate(1/(1+I*(b*x+a))**2*(1+(b*x+a)**2)/x**2,x)
```

output
```
2*I*b*log(-2*a**3*b/(a - I)**2 + 6*I*a**2*b/(a - I)**2 + 2*a*b + 6*a*b/(a
- I)**2 + 4*b**2*x - 2*I*b - 2*I*b/(a - I)**2)/(a - I)**2 - 2*I*b*log(2*a*
*3*b/(a - I)**2 - 6*I*a**2*b/(a - I)**2 + 2*a*b - 6*a*b/(a - I)**2 + 4*b**
2*x - 2*I*b + 2*I*b/(a - I)**2)/(a - I)**2 - (-a - I)/(x*(a - I))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(41) = 82$.

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.77

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^2}\,dx = -\frac{2(a-i)b\log(ibx+ia+1)}{-ia^3-3a^2+3ia+1} + \frac{2(a-i)b\log(x)}{-ia^3-3a^2+3ia+1} + \frac{a^3+(a^2+1)bx-ia^2+a-i}{(a^2-2ia-1)bx^2+(a^3-3ia^2-3a+i)x}$$

input
```
integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^2,x, algorithm="maxima")
```

output
```
-2*(a - I)*b*log(I*b*x + I*a + 1)/(-I*a^3 - 3*a^2 + 3*I*a + 1) + 2*(a - I)
*b*log(x)/(-I*a^3 - 3*a^2 + 3*I*a + 1) + (a^3 + (a^2 + 1)*b*x - I*a^2 + a
- I)/((a^2 - 2*I*a - 1)*b*x^2 + (a^3 - 3*I*a^2 - 3*a + I)*x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(41) = 82$.

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.53

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^2}\,dx = \frac{2\,b^2\log\left(-\frac{i\,a}{i\,bx+i\,a+1}-\frac{1}{i\,bx+i\,a+1}+1\right)}{-i\,a^2b-2\,ab+i\,b} - \frac{ab+i\,b}{(a-i)^2\left(\frac{i\,a}{i\,bx+i\,a+1}+\frac{1}{i\,bx+i\,a+1}-1\right)}$$

input
```
integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^2,x, algorithm="giac")
```

output
```
2*b^2*log(-I*a/(I*b*x + I*a + 1) - 1/(I*b*x + I*a + 1) + 1)/(-I*a^2*b - 2*
a*b + I*b) - (a*b + I*b)/((a - I)^2*(I*a/(I*b*x + I*a + 1) + 1/(I*b*x + I*
a + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 23.36 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.61

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^2}\,dx = \frac{-1+a\,1\mathrm{i}}{x\,(1+a\,1\mathrm{i})} - \frac{4\,b\,\mathrm{atan}\left(\frac{a^2\,1\mathrm{i}+2\,a-\mathrm{i}}{(a-\mathrm{i})^2}+\frac{x\,(2\,a^4\,b^2+4\,a^2\,b^2+2\,b^2)}{(a-\mathrm{i})^2\,(-1\mathrm{i}\,b\,a^3+b\,a^2-1\mathrm{i}\,b\,a+b)}\right)}{(a-\mathrm{i})^2}$$

input
```
int(((a + b*x)^2 + 1)/(x^2*(a*1i + b*x*1i + 1)^2),x)
```

output
```
(a*1i - 1)/(x*(a*1i + 1)) - (4*b*atan((2*a + a^2*1i - 1i)/(a - 1i)^2 + (x*
(2*b^2 + 4*a^2*b^2 + 2*a^4*b^2))/((a - 1i)^2*(b - a*b*1i + a^2*b - a^3*b*1
i))))/(a - 1i)^2
```

Reduce [F]

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^2}\,dx = -\left(\int \frac{1}{b^2x^2+2abx-2bix+a^2-2ai-1}dx\right)b^2 - \left(\int \frac{1}{b^2x^4+2ab\,x^3-2bi\,x^3+a^2x^2-2ai\,x^2-x^2}dx\right)a^2 - \left(\int \frac{1}{b^2x^4+2ab\,x^3-2bi\,x^3+a^2x^2-2ai\,x^2-x^2}dx\right) - 2\left(\int \frac{1}{b^2x^3+2ab\,x^2-2bi\,x^2+a^2x-2aix-x}dx\right)ab$$

input

```
int(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^2,x)
```

output

```
 - int(1/(a**2 + 2*a*b*x - 2*a*i + b**2*x**2 - 2*b*i*x - 1),x)*b**2 - int(
1/(a**2*x**2 + 2*a*b*x**3 - 2*a*i*x**2 + b**2*x**4 - 2*b*i*x**3 - x**2),x)
*a**2 - int(1/(a**2*x**2 + 2*a*b*x**3 - 2*a*i*x**2 + b**2*x**4 - 2*b*i*x**
3 - x**2),x) - 2*int(1/(a**2*x + 2*a*b*x**2 - 2*a*i*x + b**2*x**3 - 2*b*i*
x**2 - x),x)*a*b
```

3.219 $\int \frac{e^{-2i\arctan(a+bx)}}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 83

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^3}\,dx = \frac{-i-a}{2(i-a)x^2} - \frac{2ib}{(i-a)^2x} - \frac{2b^2\log(x)}{(1+ia)^3} + \frac{2b^2\log(i-a-bx)}{(1+ia)^3}$$

output
```
1/2*(-I-a)/(I-a)/x^2-2*I*b/(I-a)^2/x-2*b^2*ln(x)/(1+I*a)^3+2*b^2*ln(I-a-b*
x)/(1+I*a)^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^3}\,dx = \frac{(-i+a)\left(1+a^2-4ibx\right)-4ib^2x^2\log(x)+4ib^2x^2\log(i-a-bx)}{2(-i+a)^3x^2}$$

input
```
Integrate[1/(E^((2*I)*ArcTan[a + b*x])*x^3),x]
```

output
```
((-I + a)*(1 + a^2 - (4*I)*b*x) - (4*I)*b^2*x^2*Log[x] + (4*I)*b^2*x^2*Log
[I - a - b*x])/(2*(-I + a)^3*x^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5618, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^3}\,dx$$

$$\Big\downarrow \text{ 5618}$$

$$\int \frac{-ia-ibx+1}{x^3(ia+ibx+1)}dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(\frac{2ib^3}{(a-i)^3(a+bx-i)}-\frac{2ib^2}{(a-i)^3x}+\frac{2ib}{(a-i)^2x^2}+\frac{-a-i}{(a-i)x^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2b^2\log(x)}{(1+ia)^3}+\frac{2b^2\log(-a-bx+i)}{(1+ia)^3}-\frac{2ib}{(-a+i)^2x}-\frac{a+i}{2(-a+i)x^2}$$

input
```
Int[1/(E^((2*I)*ArcTan[a + b*x])*x^3),x]
```

output
```
-1/2*(I + a)/((I - a)*x^2) - ((2*I)*b)/((I - a)^2*x) - (2*b^2*Log[x])/(1 +
 I*a)^3 + (2*b^2*Log[I - a - b*x])/(1 + I*a)^3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.31

method	result
default	$-\frac{-a^4+2ia^3+2ia+1}{2(i-a)^4x^2}-\frac{2b(ia^2+2a-i)}{(i-a)^4x}-\frac{2b^2(ia+1)\ln(x)}{(i-a)^4}+\frac{2b^2(ia+1)\ln(-bx-a+i)}{(i-a)^4}$
risch	$\frac{-\frac{2ibx}{a^2-2ia-1}+\frac{i+a}{-2i+2a}}{x^2}+\frac{2b^2\ln((2a^4b+4a^2b+2b)x)}{ia^3+3a^2-3ia-1}-\frac{b^2\ln(4a^8b^2x^2+8a^9bx+4a^{10}+16a^6b^2x^2+32a^7bx+20a^8+24a^4b^2x^2}{ia^3+3a^2}$
parallelrisch	$\frac{-b+27a^2b+4i\ln(x)x^2a^7b^3-4b^3x^2+8ia^9b-4\ln(x)x^2b^3-25a\,b^2x-a^{10}b+27a^8b-4i\ln(bx+a-i)x^2a^7b^3-60i\ln(x)x^3a^4b^4+60i}{}$

input
```
int(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(I-a)^4*(-a^4+2*I*a^3+2*I*a+1)/x^2-2*b*(I*a^2-I+2*a)/(I-a)^4/x-2*b^2*
(1+I*a)/(I-a)^4*ln(x)+2*b^2*(1+I*a)/(I-a)^4*ln(I-a-b*x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^3}\,dx = \frac{-4i\,b^2x^2\log(x)+4i\,b^2x^2\log\left(\frac{bx+a-i}{b}\right)+a^3-4(i\,a+1)bx-i\,a^2+a-i}{2\,(a^3-3i\,a^2-3\,a+i)x^2}$$

input
```
integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^3,x, algorithm="fricas")
```

output
```
1/2*(-4*I*b^2*x^2*log(x) + 4*I*b^2*x^2*log((b*x + a - I)/b) + a^3 - 4*(I*a
 + 1)*b*x - I*a^2 + a - I)/((a^3 - 3*I*a^2 - 3*a + I)*x^2)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(61) = 122.

Time = 0.43 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.72

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^3}\,dx = -\frac{2ib^2\log\left(-\frac{2a^4b^2}{(a-i)^3}+\frac{8ia^3b^2}{(a-i)^3}+\frac{12a^2b^2}{(a-i)^3}+2ab^2-\frac{8iab^2}{(a-i)^3}+4b^3x-2ib^2-\frac{2b^2}{(a-i)^3}\right)}{(a-i)^3} + \frac{2ib^2\log\left(\frac{2a^4b^2}{(a-i)^3}-\frac{8ia^3b^2}{(a-i)^3}-\frac{12a^2b^2}{(a-i)^3}+2ab^2+\frac{8iab^2}{(a-i)^3}+4b^3x-2ib^2+\frac{2b^2}{(a-i)^3}\right)}{(a-i)^3} - \frac{-a^2+4ibx-1}{x^2\cdot(2a^2-4ia-2)}$$

input
```
integrate(1/(1+I*(b*x+a))**2*(1+(b*x+a)**2)/x**3,x)
```

output
```
-2*I*b**2*log(-2*a**4*b**2/(a - I)**3 + 8*I*a**3*b**2/(a - I)**3 + 12*a**2
*b**2/(a - I)**3 + 2*a*b**2 - 8*I*a*b**2/(a - I)**3 + 4*b**3*x - 2*I*b**2
- 2*b**2/(a - I)**3)/(a - I)**3 + 2*I*b**2*log(2*a**4*b**2/(a - I)**3 - 8*
I*a**3*b**2/(a - I)**3 - 12*a**2*b**2/(a - I)**3 + 2*a*b**2 + 8*I*a*b**2/(
a - I)**3 + 4*b**3*x - 2*I*b**2 + 2*b**2/(a - I)**3)/(a - I)**3 - (-a**2 +
 4*I*b*x - 1)/(x**2*(2*a**2 - 4*I*a - 2))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(61) = 122.

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.93

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^3}\,dx = -\frac{2(-ia-1)b^2\log(ibx+ia+1)}{a^4-4ia^3-6a^2+4ia+1} - \frac{2(ia+1)b^2\log(x)}{a^4-4ia^3-6a^2+4ia+1} + \frac{4(-ia-1)b^2x^2+a^4-2ia^3+(a^3-5ia^2-7a+3i)bx-2ia-1}{2((a^3-3ia^2-3a+i)bx^3+(a^4-4ia^3-6a^2+4ia+1)x^2)}$$

input
```
integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^3,x, algorithm="maxima")
```

output
```
-2*(-I*a - 1)*b^2*log(I*b*x + I*a + 1)/(a^4 - 4*I*a^3 - 6*a^2 + 4*I*a + 1)
 - 2*(I*a + 1)*b^2*log(x)/(a^4 - 4*I*a^3 - 6*a^2 + 4*I*a + 1) + 1/2*(4*(-I
*a - 1)*b^2*x^2 + a^4 - 2*I*a^3 + (a^3 - 5*I*a^2 - 7*a + 3*I)*b*x - 2*I*a
- 1)/((a^3 - 3*I*a^2 - 3*a + I)*b*x^3 + (a^4 - 4*I*a^3 - 6*a^2 + 4*I*a + 1
)*x^2)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(61) = 122.

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.71

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^3}\,dx = \frac{2b^3\log\left(-\frac{ia}{ibx+ia+1}-\frac{1}{ibx+ia+1}+1\right)}{ia^3b+3a^2b-3iab-b} + \frac{\frac{iab^2-5b^2}{-ia-1}+\frac{2i(ab^3+3ib^3)}{(ibx+ia+1)b}}{2(a-i)^2\left(\frac{ia}{ibx+ia+1}+\frac{1}{ibx+ia+1}-1\right)^2}$$

input
```
integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^3,x, algorithm="giac")
```

output
```
2*b^3*log(-I*a/(I*b*x + I*a + 1) - 1/(I*b*x + I*a + 1) + 1)/(I*a^3*b + 3*a
^2*b - 3*I*a*b - b) + 1/2*((I*a*b^2 - 5*b^2)/(-I*a - 1) + 2*I*(a*b^3 + 3*I
*b^3)/((I*b*x + I*a + 1)*b))/((a - I)^2*(I*a/(I*b*x + I*a + 1) + 1/(I*b*x
+ I*a + 1) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 23.35 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.88

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^3}\,dx = \frac{\frac{a+1\mathrm{i}}{2\,(a-\mathrm{i})}-\frac{b\,x\,2\mathrm{i}}{{(a-\mathrm{i})}^2}}{x^2}-\frac{b^2\,\mathrm{atanh}\left(\frac{-a^3+a^2\,3\mathrm{i}+3\,a-\mathrm{i}}{{(a-\mathrm{i})}^3}-\frac{x\,\left(2\,a^8\,b^2+8\,a^6\,b^2+12\,a^4\,b^2+8\,a^2\,b^2+2\,b^2\right)}{{(a-\mathrm{i})}^3\,\left(b\,a^6+2\mathrm{i}\,b\,a^5+b\,a^4+4\mathrm{i}\,b\,a^3-b\,a^2+2\mathrm{i}\,b\,a-b\right)}\right)\,4\mathrm{i}}{{(a-\mathrm{i})}^3}$$

input
```
int(((a + b*x)^2 + 1)/(x^3*(a*1i + b*x*1i + 1)^2),x)
```

output
```
((a + 1i)/(2*(a - 1i)) - (b*x*2i)/(a - 1i)^2)/x^2 - (b^2*atanh((3*a + a^2*
3i - a^3 - 1i)/(a - 1i)^3 - (x*(2*b^2 + 8*a^2*b^2 + 12*a^4*b^2 + 8*a^6*b^2
 + 2*a^8*b^2))/((a - 1i)^3*(a*b*2i - b - a^2*b + a^3*b*4i + a^4*b + a^5*b*
2i + a^6*b)))*4i)/(a - 1i)^3
```

Reduce [F]

$$\begin{aligned}\int \frac{e^{-2i\arctan(a+bx)}}{x^3}\,dx = &-\left(\int \frac{1}{b^2x^5+2ab\,x^4-2bi\,x^4+a^2x^3-2ai\,x^3-x^3}dx\right)a^2\\ &-\left(\int \frac{1}{b^2x^5+2ab\,x^4-2bi\,x^4+a^2x^3-2ai\,x^3-x^3}dx\right)\\ &-2\left(\int \frac{1}{b^2x^4+2ab\,x^3-2bi\,x^3+a^2x^2-2ai\,x^2-x^2}dx\right)ab\\ &-\left(\int \frac{1}{b^2x^3+2ab\,x^2-2bi\,x^2+a^2x-2aix-x}dx\right)b^2\end{aligned}$$

input
```
int(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^3,x)
```

output
```
- int(1/(a**2*x**3 + 2*a*b*x**4 - 2*a*i*x**3 + b**2*x**5 - 2*b*i*x**4 - x
**3),x)*a**2 - int(1/(a**2*x**3 + 2*a*b*x**4 - 2*a*i*x**3 + b**2*x**5 - 2*
b*i*x**4 - x**3),x) - 2*int(1/(a**2*x**2 + 2*a*b*x**3 - 2*a*i*x**2 + b**2*
x**4 - 2*b*i*x**3 - x**2),x)*a*b - int(1/(a**2*x + 2*a*b*x**2 - 2*a*i*x +
b**2*x**3 - 2*b*i*x**2 - x),x)*b**2
```

3.220 $\int \frac{e^{-2i\arctan(a+bx)}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 104

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^4}\,dx = \frac{-i-a}{3(i-a)x^3} - \frac{ib}{(i-a)^2x^2} + \frac{2b^2}{(1+ia)^3x} + \frac{2ib^3\log(x)}{(i-a)^4} - \frac{2ib^3\log(i-a-bx)}{(i-a)^4}$$

output
```
1/3*(-I-a)/(I-a)/x^3-I*b/(I-a)^2/x^2+2*b^2/(1+I*a)^3/x+2*I*b^3*ln(x)/(I-a)
^4-2*I*b^3*ln(I-a-b*x)/(I-a)^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^4}\,dx = \frac{(-i+a)\left(-i+a-ia^2+a^3-3bx-3iabx+6ib^2x^2\right)+6ib^3x^3\log(x)-6ib^3x^3\log(i-a-bx)}{3(-i+a)^4x^3}$$

input
```
Integrate[1/(E^((2*I)*ArcTan[a + b*x])*x^4),x]
```

output
```
((-I + a)*(-I + a - I*a^2 + a^3 - 3*b*x - (3*I)*a*b*x + (6*I)*b^2*x^2) + (
6*I)*b^3*x^3*Log[x] - (6*I)*b^3*x^3*Log[I - a - b*x])/(3*(-I + a)^4*x^3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5618, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^4}dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{-ia-ibx+1}{x^4(ia+ibx+1)}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{2ib^4}{(a-i)^4(a+bx-i)}+\frac{2ib^3}{(a-i)^4x}-\frac{2ib^2}{(a-i)^3x^2}+\frac{2ib}{(a-i)^2x^3}+\frac{-a-i}{(a-i)x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2ib^3\log(x)}{(-a+i)^4}-\frac{2ib^3\log(-a-bx+i)}{(-a+i)^4}+\frac{2b^2}{(1+ia)^3x}-\frac{ib}{(-a+i)^2x^2}-\frac{a+i}{3(-a+i)x^3}$$

input
```
Int[1/(E^((2*I)*ArcTan[a + b*x])*x^4),x]
```

output
```
-1/3*(I + a)/((I - a)*x^3) - (I*b)/((I - a)^2*x^2) + (2*b^2)/((1 + I*a)^3*
x) + ((2*I)*b^3*Log[x])/(I - a)^4 - ((2*I)*b^3*Log[I - a - b*x])/(I - a)^4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.44

method	result
default	$\frac{b\left(i a^{3}+3 a^{2}-3 i a-1\right)}{\left(i-a\right)^{5} x^{2}}-\frac{2 b^{2} \left(i a^{2}+2 a-i\right)}{\left(i-a\right)^{5} x}-\frac{2 b^{3} \left(i a+1\right) \ln \left(x\right)}{\left(i-a\right)^{5}}-\frac{a^{5}-3 i a^{4}-2 a^{3}-2 i a^{2}-3 a+i}{3 \left(i-a\right)^{5} x^{3}}+\frac{2 b^{3} \left(i a+1\right) \ln \left(-b x-a+i\right)}{\left(i-a\right)^{5}}$
risch	$\frac{\frac{2 i b^{2} x^{2}}{\left(a^{2}-2 i a-1\right) \left(-i+a\right)}-\frac{i b x}{a^{2}-2 i a-1}+\frac{i+a}{-3 i+3 a}}{x^{3}}-\frac{2 b^{3} \ln \left(\left(-2 a^{6} b-6 a^{4} b-6 a^{2} b-2 b\right) x\right)}{i a^{4}+4 a^{3}-6 i a^{2}-4 a+i}+\frac{b^{3} \ln \left(4 a^{12} b^{2} x^{2}+8 a^{13} b x+4 a^{14}+24 a^{10} b^{2}\right.}{}$
parallelrisch	$-\frac{204 i x\, a^{3} b^{2}+9 a b+210 i \ln \left(x \right) x^{3} a^{3} b^{4}+42 a^{5} b-24 a\, b^{3} x^{2}-42 i \ln \left(x \right) x^{3} a\, b^{4}+42 i \ln \left(b x+a-i\right) x^{3} a\, b^{4}+42 x^{3} a^{6} b^{4}-210 x^{3} a^{4} b^{4}+24}{}$

input
```
int(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
b*(I*a^3-3*I*a+3*a^2-1)/(I-a)^5/x^2-2*b^2*(I*a^2-I+2*a)/(I-a)^5/x-2*b^3*(1
+I*a)/(I-a)^5*ln(x)-1/3/(I-a)^5*(-3*I*a^4+a^5-2*I*a^2-2*a^3+I-3*a)/x^3+2*b
^3*(1+I*a)/(I-a)^5*ln(I-a-b*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^4}\,dx = \frac{6i\,b^3x^3\log(x) - 6i\,b^3x^3\log\left(\frac{bx+a-i}{b}\right) - 6(-i\,a-1)b^2x^2 + a^4 - 2i\,a^3 - 3(i\,a^2+2\,a-i)bx - 2i\,a - 1}{3(a^4 - 4i\,a^3 - 6\,a^2 + 4i\,a + 1)x^3}$$

input
```
integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^4,x, algorithm="fricas")
```

output
```
1/3*(6*I*b^3*x^3*log(x) - 6*I*b^3*x^3*log((b*x + a - I)/b) - 6*(-I*a - 1)*
b^2*x^2 + a^4 - 2*I*a^3 - 3*(I*a^2 + 2*a - I)*b*x - 2*I*a - 1)/((a^4 - 4*I
*a^3 - 6*a^2 + 4*I*a + 1)*x^3)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(75) = 150.

Time = 0.57 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.75

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^4}\,dx = \frac{2ib^3\log\left(-\frac{2a^5b^3}{(a-i)^4} + \frac{10ia^4b^3}{(a-i)^4} + \frac{20a^3b^3}{(a-i)^4} - \frac{20ia^2b^3}{(a-i)^4} + 2ab^3 - \frac{10ab^3}{(a-i)^4} + 4b^4x - 2ib^3 + \frac{2ib^3}{(a-i)^4}\right)}{(a-i)^4} - \frac{2ib^3\log\left(\frac{2a^5b^3}{(a-i)^4} - \frac{10ia^4b^3}{(a-i)^4} - \frac{20a^3b^3}{(a-i)^4} + \frac{20ia^2b^3}{(a-i)^4} + 2ab^3 + \frac{10ab^3}{(a-i)^4} + 4b^4x - 2ib^3 - \frac{2ib^3}{(a-i)^4}\right)}{(a-i)^4} - \frac{-a^3 + ia^2 - a - 6ib^2x^2 + x(3iab + 3b) + i}{x^3\cdot(3a^3 - 9ia^2 - 9a + 3i)}$$

input
```
integrate(1/(1+I*(b*x+a))**2*(1+(b*x+a)**2)/x**4,x)
```

output
```
2*I*b**3*log(-2*a**5*b**3/(a - I)**4 + 10*I*a**4*b**3/(a - I)**4 + 20*a**3
*b**3/(a - I)**4 - 20*I*a**2*b**3/(a - I)**4 + 2*a*b**3 - 10*a*b**3/(a - I
)**4 + 4*b**4*x - 2*I*b**3 + 2*I*b**3/(a - I)**4)/(a - I)**4 - 2*I*b**3*lo
g(2*a**5*b**3/(a - I)**4 - 10*I*a**4*b**3/(a - I)**4 - 20*a**3*b**3/(a - I
)**4 + 20*I*a**2*b**3/(a - I)**4 + 2*a*b**3 + 10*a*b**3/(a - I)**4 + 4*b**
4*x - 2*I*b**3 - 2*I*b**3/(a - I)**4)/(a - I)**4 - (-a**3 + I*a**2 - a - 6
*I*b**2*x**2 + x*(3*I*a*b + 3*b) + I)/(x**3*(3*a**3 - 9*I*a**2 - 9*a + 3*I
))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(72) = 144$.

Time = 0.04 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.10

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^4}\,dx$$
$$= \frac{2\,(a-i)b^3\log\left(i\,bx+i\,a+1\right)}{i\,a^5+5\,a^4-10i\,a^3-10\,a^2+5i\,a+1} - \frac{2\,(a-i)b^3\log\left(x\right)}{i\,a^5+5\,a^4-10i\,a^3-10\,a^2+5i\,a+1}$$
$$+ \frac{6\,(a-i)b^3x^3 - i\,a^5 + 3\,(a^2-2i\,a-1)b^2x^2 - 3\,a^4 + 2i\,a^3 - (i\,a^4+5\,a^3-9i\,a^2-7\,a+2i)bx - 2\,a^2 +}{3\,((-i\,a^4-4\,a^3+6i\,a^2+4\,a-i)bx^4 + (-i\,a^5-5\,a^4+10i\,a^3+10\,a^2-5i\,a-1)x^3)}$$

input
```
integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^4,x, algorithm="maxima")
```

output
```
2*(a - I)*b^3*log(I*b*x + I*a + 1)/(I*a^5 + 5*a^4 - 10*I*a^3 - 10*a^2 + 5*
I*a + 1) - 2*(a - I)*b^3*log(x)/(I*a^5 + 5*a^4 - 10*I*a^3 - 10*a^2 + 5*I*a
 + 1) + 1/3*(6*(a - I)*b^3*x^3 - I*a^5 + 3*(a^2 - 2*I*a - 1)*b^2*x^2 - 3*a
^4 + 2*I*a^3 - (I*a^4 + 5*a^3 - 9*I*a^2 - 7*a + 2*I)*b*x - 2*a^2 + 3*I*a +
 1)/((-I*a^4 - 4*a^3 + 6*I*a^2 + 4*a - I)*b*x^4 + (-I*a^5 - 5*a^4 + 10*I*a
^3 + 10*a^2 - 5*I*a - 1)*x^3)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(72) = 144$.

Time = 0.14 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.76

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^4}\,dx = \frac{2\,b^4\log\left(-\frac{i\,a}{i\,bx+i\,a+1}-\frac{1}{i\,bx+i\,a+1}+1\right)}{-i\,a^4b-4\,a^3b+6i\,a^2b+4\,ab-i\,b} + \frac{\frac{-i\,ab^3+10\,b^3}{i\,a+1}+\frac{3i\,(ab^4+8i\,b^4)}{(i\,bx+i\,a+1)b}+\frac{3\,(a^2b^5+4i\,ab^5+5\,b^5)}{(i\,bx+i\,a+1)^2b^2}}{3\,(a-i)^3\left(\frac{i\,a}{i\,bx+i\,a+1}+\frac{1}{i\,bx+i\,a+1}-1\right)^3}$$

input
```
integrate(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^4,x, algorithm="giac")
```

output
```
2*b^4*log(-I*a/(I*b*x + I*a + 1) - 1/(I*b*x + I*a + 1) + 1)/(-I*a^4*b - 4*
a^3*b + 6*I*a^2*b + 4*a*b - I*b) + 1/3*((-I*a*b^3 + 10*b^3)/(I*a + 1) + 3*
I*(a*b^4 + 8*I*b^4)/((I*b*x + I*a + 1)*b) + 3*(a^2*b^5 + 4*I*a*b^5 + 5*b^5
)/((I*b*x + I*a + 1)^2*b^2))/((a - I)^3*(I*a/(I*b*x + I*a + 1) + 1/(I*b*x
+ I*a + 1) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 23.43 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.91

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^4}\,dx = \frac{\frac{a+1\mathrm{i}}{3\,(a-\mathrm{i})}+\frac{b^2\,x^2\,2\mathrm{i}}{(a-\mathrm{i})^3}-\frac{b\,x\,1\mathrm{i}}{(a-\mathrm{i})^2}}{x^3} - \frac{4\,b^3\,\mathrm{atan}\left(\frac{(a^4-a^3\,4\mathrm{i}-6\,a^2+a\,4\mathrm{i}+1)\,1\mathrm{i}}{(a-\mathrm{i})^4}+\frac{x\,(2\,a^{12}\,b^2+12\,a^{10}\,b^2+30\,a^8\,b^2+40\,a^6\,b^2+30\,a^4\,b^2+12\,a^2\,b^2+2\,b^2)}{(a-\mathrm{i})^4\,(-1\mathrm{i}\,b\,a^9+3\,b\,a^8+8\,b\,a^6+6\mathrm{i}\,b\,a^5+6\,b\,a^4+8\mathrm{i}\,b\,a^3+3\mathrm{i}\,b\,a-b)}\right)}{(a-\mathrm{i})^4}$$

input
```
int(((a + b*x)^2 + 1)/(x^4*(a*1i + b*x*1i + 1)^2),x)
```

output
```
((a + 1i)/(3*(a - 1i)) + (b^2*x^2*2i)/(a - 1i)^3 - (b*x*1i)/(a - 1i)^2)/x^
3 - (4*b^3*atan(((a*4i - 6*a^2 - a^3*4i + a^4 + 1)*1i)/(a - 1i)^4 + (x*(2*
b^2 + 12*a^2*b^2 + 30*a^4*b^2 + 40*a^6*b^2 + 30*a^8*b^2 + 12*a^10*b^2 + 2*
a^12*b^2))/((a - 1i)^4*(a*b*3i - b + a^3*b*8i + 6*a^4*b + a^5*b*6i + 8*a^6
*b + 3*a^8*b - a^9*b*1i))))/(a - 1i)^4
```

Reduce [F]

$$\int \frac{e^{-2i\arctan(a+bx)}}{x^4}\,dx = -\left(\int \frac{1}{b^2x^6+2ab\,x^5-2bi\,x^5+a^2x^4-2ai\,x^4-x^4}dx\right)a^2 - \left(\int \frac{1}{b^2x^6+2ab\,x^5-2bi\,x^5+a^2x^4-2ai\,x^4-x^4}dx\right) - 2\left(\int \frac{1}{b^2x^5+2ab\,x^4-2bi\,x^4+a^2x^3-2ai\,x^3-x^3}dx\right)ab - \left(\int \frac{1}{b^2x^4+2ab\,x^3-2bi\,x^3+a^2x^2-2ai\,x^2-x^2}dx\right)b^2$$

input
```
int(1/(1+I*(b*x+a))^2*(1+(b*x+a)^2)/x^4,x)
```

output
```
 - int(1/(a**2*x**4 + 2*a*b*x**5 - 2*a*i*x**4 + b**2*x**6 - 2*b*i*x**5 - x
**4),x)*a**2 - int(1/(a**2*x**4 + 2*a*b*x**5 - 2*a*i*x**4 + b**2*x**6 - 2*
b*i*x**5 - x**4),x) - 2*int(1/(a**2*x**3 + 2*a*b*x**4 - 2*a*i*x**3 + b**2*
x**5 - 2*b*i*x**4 - x**3),x)*a*b - int(1/(a**2*x**2 + 2*a*b*x**3 - 2*a*i*x
**2 + b**2*x**4 - 2*b*i*x**3 - x**2),x)*b**2
```

3.221 $\int e^{-3i\arctan(a+bx)}x^4\,dx$

Optimal result

Integrand size = 16, antiderivative size = 360

$$\begin{aligned}\int e^{-3i\arctan(a+bx)}x^4\,dx &= \frac{2ix^4(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}} \\ &\quad+\frac{3(19i-68a-88ia^2+48a^3+8ia^4)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^5} \\ &\quad+\frac{(41i-344a-554ia^2+216a^3)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{40b^5} \\ &\quad-\frac{3(17i-16a)x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{20b^3} \\ &\quad-\frac{11x^3(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{5b^2} \\ &\quad-\frac{(118a-i(61-52a^2))(1-ia-ibx)^{5/2}\sqrt{1+ia+ibx}}{20b^5} \\ &\quad-\frac{3(19+68ia-88a^2-48ia^3+8a^4)\operatorname{arcsinh}(a+bx)}{8b^5}\end{aligned}$$

output
```
2*I*x^4*(1-I*a-I*b*x)^(3/2)/b/(1+I*a+I*b*x)^(1/2)+3/8*(19*I-68*a-88*I*a^2+
48*a^3+8*I*a^4)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^5+1/40*(41*I-344
*a-554*I*a^2+216*a^3)*(1-I*a-I*b*x)^(3/2)*(1+I*a+I*b*x)^(1/2)/b^5-3/20*(17
*I-16*a)*x^2*(1-I*a-I*b*x)^(3/2)*(1+I*a+I*b*x)^(1/2)/b^3-11/5*x^3*(1-I*a-I
*b*x)^(3/2)*(1+I*a+I*b*x)^(1/2)/b^2-1/20*(118*a-I*(-52*a^2+61))*(1-I*a-I*b
*x)^(5/2)*(1+I*a+I*b*x)^(1/2)/b^5-3/8*(19+68*I*a-88*a^2-48*I*a^3+8*a^4)*ar
csinh(b*x+a)/b^5
```

Mathematica [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.83

$$\int e^{-3i\arctan(a+bx)}x^4\,dx$$

$$=\frac{\frac{448i+8ia^6+285bx+224ib^2x^2+95b^3x^3-56ib^4x^4-30b^5x^5+8ib^6x^6+a^5(410+8ibx)+2a^4(-638i+265bx)+a^3(-905-2004ibx+60b^2x^2)-a^2(836i+}{\sqrt{1+a^2+2abx+b^2x^2}}}{}$$

input
```
Integrate[x^4/E^((3*I)*ArcTan[a + b*x]),x]
```

output
```
((448*I + (8*I)*a^6 + 285*b*x + (224*I)*b^2*x^2 + 95*b^3*x^3 - (56*I)*b^4*
x^4 - 30*b^5*x^5 + (8*I)*b^6*x^6 + a^5*(410 + (8*I)*b*x) + 2*a^4*(-638*I +
 265*b*x) + a^3*(-905 - (2004*I)*b*x + 60*b^2*x^2) - a^2*(836*I + 2635*b*x
 + (356*I)*b^2*x^2 + 20*b^3*x^3) + a*(-1315 + (1468*I)*b*x - 515*b^2*x^2 +
 (116*I)*b^3*x^3 + 10*b^4*x^4 + (8*I)*b^5*x^5))/Sqrt[1 + a^2 + 2*a*b*x + b
^2*x^2] + (30*(-1)^(1/4)*(19*I - 68*a - (88*I)*a^2 + 48*a^3 + (8*I)*a^4)*S
qrt[b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]
])/Sqrt[(-I)*b])/(40*b^5)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.95, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5618, 108, 27, 170, 27, 170, 27, 164, 60, 62, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 e^{-3i\arctan(a+bx)}dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{x^4(-ia-ibx+1)^{3/2}}{(ia+ibx+1)^{3/2}}dx$$

$$\Big\downarrow \text{108}$$

$$\frac{2ix^4(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}-\frac{2i\int \frac{x^3\sqrt{-ia-ibx+1}(8(1-ia)-11ibx)}{2\sqrt{ia+ibx+1}}dx}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2ix^4(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}-\frac{i\int \frac{x^3\sqrt{-ia-ibx+1}(8(1-ia)-11ibx)}{\sqrt{ia+ibx+1}}dx}{b}$$

$$\Big\downarrow \text{170}$$

$$\frac{2ix^4(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}-\frac{i\left(\frac{\int \frac{3bx^2\sqrt{-ia-ibx+1}(11(ia+1)(a+i)+(16ia+17)bx)}{\sqrt{ia+ibx+1}}dx}{5b^2}-\frac{11ix^3(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{5b}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2ix^4(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}-\frac{i\left(\frac{3\int \frac{x^2\sqrt{-ia-ibx+1}\left(11i\left(a^2+1\right)+(16ia+17)bx\right)}{\sqrt{ia+ibx+1}}dx}{5b}-\frac{11ix^3(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{5b}\right)}{b}$$

$$\Big\downarrow \text{170}$$

$$\frac{\frac{2ix^4(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}-i\left(\frac{3\left(\frac{\int \frac{bx\sqrt{-ia-ibx+1}\left(2(17i-16a)(i-a)(1-ia)+\left(-52ia^2-118a+61i\right)bx\right)}{\sqrt{ia+ibx+1}}dx}{4b^2}+\frac{(17+16ia)x^2(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{4b}\right)}{5b}-\frac{11ix^3(-ia-ibx+1)^{3/2}\sqrt{i}}{5b}\right.}{b}$$

$\downarrow$ 27

$$\frac{\frac{2ix^4(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}-i\left(\frac{3\left(\frac{\int \frac{x\sqrt{-ia-ibx+1}\left(2(17i-16a)(i-a)(1-ia)+\left(-52ia^2-118a+61i\right)bx\right)}{\sqrt{ia+ibx+1}}dx}{4b}+\frac{(17+16ia)x^2(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{4b}\right)}{5b}-\frac{11ix^3(-ia-ibx+1)^{3/2}\sqrt{ia}}{5b}\right.}{b}$$

$\downarrow$ 164

$$\frac{\frac{2ix^4(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}-i\left(\frac{3\left(\frac{-\frac{5\left(8ia^4+48a^3-88ia^2-68a+19i\right)\int \frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}dx}{2b}-\frac{\sqrt{ia+ibx+1}\left(-112ia^3-2\left(-52ia^2-118a+61i\right)bx-422a^2+458ia+163\right)(-ia-ibx+1)^{3/2}}{6b^2}}{4b}+\frac{(17+16}{}\right.}{5b}\right.}{b}$$

$\downarrow$ 60

$$\frac{\frac{2ix^4(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}-i\left(\frac{3\left(\frac{-\frac{5\left(8ia^4+48a^3-88ia^2-68a+19i\right)\left(\int \frac{1}{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}-\frac{\sqrt{ia+ibx+1}\left(-112ia^3-2\left(-52ia^2-118a+61i\right)bx-422a\right.}{6b^2}}{4b}\right.}{5b}\right.}{b}$$

$\downarrow$ 62

$$\frac{2ix^4(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}-\frac{i\left(\frac{3\left(-\frac{-\frac{5\left(8ia^4+48a^3-88ia^2-68a+19i\right)\left(\int\frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}-\frac{\sqrt{ia+ibx+1}\left(-112ia^3-2\left(-52ia^2-118a+61i\right)bx-\right.}{6b^2}}{4b}\right.}{5b}\right.}{b}$$

$\downarrow$ 1090

$$\frac{2ix^4(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}-\frac{i\left(\frac{3\left(-\frac{-\frac{5\left(8ia^4+48a^3-88ia^2-68a+19i\right)\left(\frac{\int\frac{1}{\sqrt{\frac{\left(2xb^2+2ab\right)^2}{4b^2}+1}}d\left(2xb^2+2ab\right)}{2b^2}-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}-\frac{\sqrt{ia+ibx+1}\left(-112ia^3-2\left(-52ia^2-118a+61i\right)b\right.}{6b^2}}{4b}\right.}{5b}\right.}{b}$$

$\downarrow$ 222

$$\frac{2ix^4(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}-$$

$$\frac{i\left(\frac{3\left(-\frac{-\frac{\sqrt{ia+ibx+1}\left(-112ia^3-2\left(-52ia^2-118a+61i\right)bx-422a^2+458ia+163\right)(-ia-ibx+1)^{3/2}}{6b^2}-\frac{5\left(8ia^4+48a^3-88ia^2-68a+19i\right)\left(\frac{\text{arcsinh}\left(\frac{2ab+2b^2x}{2b}\right)}{b}-\right.}{2b}}{4b}\right.}{5b}\right.}{b}$$

input
```
Int[x^4/E^((3*I)*ArcTan[a + b*x]),x]
```

output
```
((2*I)*x^4*(1 - I*a - I*b*x)^(3/2))/(b*Sqrt[1 + I*a + I*b*x]) - (I*((((-11
*I)/5)*x^3*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/b + (3*(((17 + (
16*I)*a)*x^2*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/(4*b) + (-1/6*
((1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x]*(163 + (458*I)*a - 422*a^2
- (112*I)*a^3 - 2*(61*I - 118*a - (52*I)*a^2)*b*x))/b^2 - (5*(19*I - 68*a
- (88*I)*a^2 + 48*a^3 + (8*I)*a^4)*(((-I)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I
*a + I*b*x])/b + ArcSinh[(2*a*b + 2*b^2*x)/(2*b)]/b))/(2*b))/(4*b)))/(5*b)
))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.24

method	result
risch	$\frac{i\left(8b^4x^4-8a\,b^3x^3+30ib^3x^3+8a^2b^2x^2-70ia\,b^2x^2-8a^3bx+130ia^2bx+8a^4-250ia^3-64b^2x^2+252abx-125ibx-804a^2+835ia+288\right)\sqrt{}}{40b^5}$
default	Expression too large to display

input
```
int(x^4/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/40*I*(8*b^4*x^4+30*I*b^3*x^3-8*a*b^3*x^3-70*I*a*b^2*x^2+8*a^2*b^2*x^2+13
0*I*a^2*b*x-8*a^3*b*x-250*I*a^3+8*a^4-64*b^2*x^2-125*I*b*x+252*a*b*x+835*I
*a-804*a^2+288)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^5-1/8/b^4*(-144*I*a^3*ln((
b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+204*I*a*
ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-I*(-
128*a^3-32*I*a^4+128*a+192*I*a^2-32*I)/b^2/(x-(I-a)/b)*((x-(I-a)/b)^2*b^2+
2*I*b*(x-(I-a)/b))^(1/2)+57*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^
2+1)^(1/2))/(b^2)^(1/2)-264*a^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*
x+a^2+1)^(1/2))/(b^2)^(1/2)+24*a^4*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a
*b*x+a^2+1)^(1/2))/(b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.73

$$\int e^{-3i\arctan(a+bx)}x^4\,dx = \frac{62i\,a^6 + 2687\,a^5 - 11575i\,a^4 - 20350\,a^3 + (62i\,a^5 + 2625\,a^4 - 8950i\,a^3 - 11400\,a^2 + 6340i\,a + 1280)bx}{}$$

input
```
integrate(x^4/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/320*(62*I*a^6 + 2687*a^5 - 11575*I*a^4 - 20350*a^3 + (62*I*a^5 + 2625*a^
4 - 8950*I*a^3 - 11400*a^2 + 6340*I*a + 1280)*b*x + 17740*I*a^2 + 120*(8*a
^5 - 56*I*a^4 - 136*a^3 + (8*a^4 - 48*I*a^3 - 88*a^2 + 68*I*a + 19)*b*x +
156*I*a^2 + 87*a - 19*I)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))
 - 8*(-8*I*b^5*x^5 + 22*b^4*x^4 - 2*(16*a - 17*I)*b^3*x^3 - 8*I*a^5 + (52*
a^2 - 118*I*a - 61)*b^2*x^2 - 418*a^4 + 1694*I*a^3 - (112*a^3 - 422*I*a^2
- 458*a + 163*I)*b*x + 2599*a^2 - 1763*I*a - 448)*sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1) + 7620*a - 1280*I)/(b^6*x + (a - I)*b^5)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3i\arctan(a+bx)}x^4\,dx = \text{Timed out}$$

input
```
integrate(x**4/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1368 vs. $2(253) = 506$.

Time = 0.15 (sec) , antiderivative size = 1368, normalized size of antiderivative = 3.80

$$\int e^{-3i\arctan(a+bx)}x^4\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

output
```
I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^4/(b^7*x^2 + 2*a*b^6*x + a^2*b^5 -
 2*I*b^6*x - 2*I*a*b^5 - b^5) + 4*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^3/
(b^7*x^2 + 2*a*b^6*x + a^2*b^5 - 2*I*b^6*x - 2*I*a*b^5 - b^5) + 4*(b^2*x^2
 + 2*a*b*x + a^2 + 1)^(3/2)*a^3/(2*I*b^6*x + 2*I*a*b^5 + 2*b^5) + 6*I*sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^4/(I*b^6*x + I*a*b^5 + b^5) - 6*I*(b^2*x^2
 + 2*a*b*x + a^2 + 1)^(3/2)*a^2/(b^7*x^2 + 2*a*b^6*x + a^2*b^5 - 2*I*b^6*x
 - 2*I*a*b^5 - b^5) - 12*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^2/(2*I*b^
6*x + 2*I*a*b^5 + 2*b^5) + 24*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3/(I*b^6
*x + I*a*b^5 + b^5) - 4*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a/(b^7*x^2 + 2
*a*b^6*x + a^2*b^5 - 2*I*b^6*x - 2*I*a*b^5 - b^5) - 12*(b^2*x^2 + 2*a*b*x
+ a^2 + 1)^(3/2)*a/(2*I*b^6*x + 2*I*a*b^5 + 2*b^5) - 36*I*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1)*a^2/(I*b^6*x + I*a*b^5 + b^5) + I*(b^2*x^2 + 2*a*b*x + a
^2 + 1)^(3/2)/(b^7*x^2 + 2*a*b^6*x + a^2*b^5 - 2*I*b^6*x - 2*I*a*b^5 - b^5
) + 4*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/(2*I*b^6*x + 2*I*a*b^5 + 2*b^5
) - 24*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/(I*b^6*x + I*a*b^5 + b^5) - 3*a
^4*arcsinh(b*x + a)/b^5 + 6*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(I*b^6*x +
 I*a*b^5 + b^5) - I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a*x/b^4 - 3*sqrt(-
b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)*a^2*x/b^4 + 18*I*a^3*arcsin
h(b*x + a)/b^5 + I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^2/b^5 + 6*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)*a^3/b^5 - 3*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + ...
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.93

$$\int e^{-3i\arctan(a+bx)}x^4\,dx =$$

$$-\frac{1}{40}\sqrt{(bx+a)^2+1}\left(\left(2\left(x\left(-\frac{4i\,x}{b}-\frac{-4i\,ab^{17}-15\,b^{17}}{b^{19}}\right)-\frac{4i\,a^2b^{16}+35\,ab^{16}-32i\,b^{16}}{b^{19}}\right)x-\frac{-8i\,a^3b^{1}}{}\right.\right.$$

$$+\frac{(8\,a^4-48i\,a^3-88\,a^2+68i\,a+19)\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab+a^3b+\left(x|b|-\sqrt{(bx+a)^2}\right.\right.}{}$$

input
```
integrate(x^4/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/40*sqrt((b*x + a)^2 + 1)*((2*(x*(-4*I*x/b - (-4*I*a*b^17 - 15*b^17)/b^1
9) - (4*I*a^2*b^16 + 35*a*b^16 - 32*I*b^16)/b^19)*x - (-8*I*a^3*b^15 - 130
*a^2*b^15 + 252*I*a*b^15 + 125*b^15)/b^19)*x - (8*I*a^4*b^14 + 250*a^3*b^1
4 - 804*I*a^2*b^14 - 835*a*b^14 + 288*I*b^14)/b^19) + 1/8*(8*a^4 - 48*I*a^
3 - 88*a^2 + 68*I*a + 19)*log(3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b +
 a^3*b + (x*abs(b) - sqrt((b*x + a)^2 + 1))^3*abs(b) + 3*(x*abs(b) - sqrt(
(b*x + a)^2 + 1))*a^2*abs(b) - 2*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b
- 2*I*a^2*b + 4*(-I*x*abs(b) + I*sqrt((b*x + a)^2 + 1))*a*abs(b) - a*b - (
x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^4*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3i\arctan(a+bx)}x^4\,dx=\int\frac{x^4\,{\left({\left(a+b\,x\right)}^2+1\right)}^{3/2}}{{\left(1+a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}\right)}^3}\,dx$$

input
```
int((x^4*((a + b*x)^2 + 1)^(3/2))/(a*1i + b*x*1i + 1)^3,x)
```

output
```
int((x^4*((a + b*x)^2 + 1)^(3/2))/(a*1i + b*x*1i + 1)^3, x)
```

Reduce [F]

$$\int e^{-3i\arctan(a+bx)}x^4\,dx$$
$$= -\left(\int \frac{\sqrt{b^2x^2+2abx+a^2+1}\,x^6}{b^3i\,x^3+3a\,b^2i\,x^2+3a^2bix+a^3i+3b^2x^2+6abx-3bix+3a^2-3ai-1}dx\right)b^2$$
$$-2\left(\int \frac{\sqrt{b^2x^2+2abx+a^2+1}\,x^5}{b^3i\,x^3+3a\,b^2i\,x^2+3a^2bix+a^3i+3b^2x^2+6abx-3bix+3a^2-3ai-1}dx\right)ab$$
$$-\left(\int \frac{\sqrt{b^2x^2+2abx+a^2+1}\,x^4}{b^3i\,x^3+3a\,b^2i\,x^2+3a^2bix+a^3i+3b^2x^2+6abx-3bix+3a^2-3ai-1}dx\right)a^2$$
$$-\left(\int \frac{\sqrt{b^2x^2+2abx+a^2+1}\,x^4}{b^3i\,x^3+3a\,b^2i\,x^2+3a^2bix+a^3i+3b^2x^2+6abx-3bix+3a^2-3ai-1}dx\right)$$

input
```
int(x^4/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x)
```

output
```
- int((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*x**6)/(a**3*i + 3*a**2*b*i*x
+ 3*a**2 + 3*a*b**2*i*x**2 + 6*a*b*x - 3*a*i + b**3*i*x**3 + 3*b**2*x**2 -
 3*b*i*x - 1),x)*b**2 - 2*int((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*x**5)/
(a**3*i + 3*a**2*b*i*x + 3*a**2 + 3*a*b**2*i*x**2 + 6*a*b*x - 3*a*i + b**3
*i*x**3 + 3*b**2*x**2 - 3*b*i*x - 1),x)*a*b - int((sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1)*x**4)/(a**3*i + 3*a**2*b*i*x + 3*a**2 + 3*a*b**2*i*x**2 + 6*
a*b*x - 3*a*i + b**3*i*x**3 + 3*b**2*x**2 - 3*b*i*x - 1),x)*a**2 - int((sq
rt(a**2 + 2*a*b*x + b**2*x**2 + 1)*x**4)/(a**3*i + 3*a**2*b*i*x + 3*a**2 +
 3*a*b**2*i*x**2 + 6*a*b*x - 3*a*i + b**3*i*x**3 + 3*b**2*x**2 - 3*b*i*x -
 1),x)
```

3.222 $\int e^{-3i\arctan(a+bx)}x^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 282

$$\begin{aligned}\int e^{-3i\arctan(a+bx)}x^3\,dx=&\frac{2ix^3(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}}\\&+\frac{3(17+44ia-36a^2-8ia^3)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{8b^4}\\&+\frac{7(1+8ia-6a^2)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{8b^4}\\&-\frac{9x^2(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{4b^2}\\&+\frac{(11+10ia)(1-ia-ibx)^{5/2}\sqrt{1+ia+ibx}}{4b^4}\\&+\frac{3(17i-44a-36ia^2+8a^3)\operatorname{arcsinh}(a+bx)}{8b^4}\end{aligned}$$

output
```
2*I*x^3*(1-I*a-I*b*x)^(3/2)/b/(1+I*a+I*b*x)^(1/2)+3/8*(17+44*I*a-36*a^2-8*
I*a^3)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^4+7/8*(1+8*I*a-6*a^2)*(1-
I*a-I*b*x)^(3/2)*(1+I*a+I*b*x)^(1/2)/b^4-9/4*x^2*(1-I*a-I*b*x)^(3/2)*(1+I*
a+I*b*x)^(1/2)/b^2+1/4*(11+10*I*a)*(1-I*a-I*b*x)^(5/2)*(1+I*a+I*b*x)^(1/2)
/b^4+3/8*(17*I-44*a-36*I*a^2+8*a^3)*arcsinh(b*x+a)/b^4
```

Mathematica [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.87

$$\int e^{-3i\arctan(a+bx)}x^3\,dx$$
$$= \frac{80-2ia^5-51ibx+40b^2x^2-17ib^3x^3-8b^4x^4+2ib^5x^5+a^4(-76-2ibx)-5a^3(-31i+20bx)+a^2(4+}{8b^4\sqrt{1+a^2+2abx+b^2x^2}}$$
$$+\frac{3\sqrt[4]{-1}(17i-44a-36ia^2+8a^3)\sqrt{-ib}\operatorname{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{4b^{9/2}}$$

input
```
Integrate[x^3/E^((3*I)*ArcTan[a + b*x]),x]
```

output
```
(80 - (2*I)*a^5 - (51*I)*b*x + 40*b^2*x^2 - (17*I)*b^3*x^3 - 8*b^4*x^4 + (
2*I)*b^5*x^5 + a^4*(-76 - (2*I)*b*x) - 5*a^3*(-31*I + 20*b*x) + a^2*(4 + (
265*I)*b*x - 12*b^2*x^2) + a*(157*I + 212*b*x + (53*I)*b^2*x^2 + 4*b^3*x^3
 + (2*I)*b^4*x^4))/(8*b^4*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]) + (3*(-1)^(1/
4)*(17*I - 44*a - (36*I)*a^2 + 8*a^3)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sq
rt[b]*Sqrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/(4*b^(9/2))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.94, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5618, 108, 27, 170, 27, 164, 60, 62, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{-3i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{x^3(-ia-ibx+1)^{3/2}}{(ia+ibx+1)^{3/2}}dx$$

$$\Big\downarrow \text{108}$$

$$\frac{2ix^3(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}-\frac{2i\int \frac{3x^2\sqrt{-ia-ibx+1}(2(1-ia)-3ibx)}{2\sqrt{ia+ibx+1}}dx}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{2ix^3(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}-\frac{3i\int \frac{x^2\sqrt{-ia-ibx+1}(2(1-ia)-3ibx)}{\sqrt{ia+ibx+1}}dx}{b}$$

$$\Big\downarrow \; 170$$

$$\frac{2ix^3(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}-\frac{3i\left(\frac{\int \frac{bx\sqrt{-ia-ibx+1}\left(6i\left(a^2+1\right)+(10ia+11)bx\right)}{\sqrt{ia+ibx+1}}dx}{4b^2}-\frac{3ix^2(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{4b}\right)}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{2ix^3(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}-\frac{3i\left(\frac{\int \frac{x\sqrt{-ia-ibx+1}\left(6i\left(a^2+1\right)+(10ia+11)bx\right)}{\sqrt{ia+ibx+1}}dx}{4b}-\frac{3ix^2(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{4b}\right)}{b}$$

$$\Big\downarrow \; 164$$

$$\frac{2ix^3(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}-\frac{3i\left(\frac{\frac{(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}\left(-22ia^2+2(11+10ia)bx-54a+29i\right)}{6b^2}-\frac{\left(-8ia^3-36a^2+44ia+17\right)\int \frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}dx}{2b}}{4b}-\frac{3ix^2(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{4b}\right.}{b}$$

$$\Big\downarrow \; 60$$

$$\frac{2ix^3(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}-\frac{3i\left(\frac{\frac{(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}\left(-22ia^2+2(11+10ia)bx-54a+29i\right)}{6b^2}-\frac{\left(-8ia^3-36a^2+44ia+17\right)\left(\int \frac{1}{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}}{4b}\right.}{b}$$

$$\Big\downarrow \; 62$$

$$\frac{2ix^3(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}-\frac{3i\left(\frac{\frac{(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}\left(-22ia^2+2(11+10ia)bx-54a+29i\right)}{6b^2}-\frac{\left(-8ia^3-36a^2+44ia+17\right)\left(\int\frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}}{4b}\right.}{b}$$

$$\Big\downarrow \; 1090$$

$$\frac{2ix^3(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}-\frac{3i\left(\frac{\frac{(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}\left(-22ia^2+2(11+10ia)bx-54a+29i\right)}{6b^2}-\frac{\left(-8ia^3-36a^2+44ia+17\right)\left(\frac{\int\frac{1}{\sqrt{\frac{\left(2xb^2+2ab\right)^2}{4b^2}+1}}d\left(2xb^2+2ab\right)}{2b^2}-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ib}}{b}\right.}{2b}}{4b}\right.}{b}$$

$$\Big\downarrow \; 222$$

$$\frac{2ix^3(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}-\frac{3i\left(\frac{\frac{(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}\left(-22ia^2+2(11+10ia)bx-54a+29i\right)}{6b^2}-\frac{\left(-8ia^3-36a^2+44ia+17\right)\left(\frac{\text{arcsinh}\left(\frac{2ab+2b^2x}{2b}\right)}{b}-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)}{2b}}{4b}-\right.}{b}$$

input
```
Int[x^3/E^((3*I)*ArcTan[a + b*x]),x]
```

output
```
((2*I)*x^3*(1 - I*a - I*b*x)^(3/2))/(b*Sqrt[1 + I*a + I*b*x]) - ((3*I)*(((
(-3*I)/4)*x^2*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/b + (((1 - I*
a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x]*(29*I - 54*a - (22*I)*a^2 + 2*(11 +
 (10*I)*a)*b*x))/(6*b^2) - ((17 + (44*I)*a - 36*a^2 - (8*I)*a^3)*(((-I)*Sq
rt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b + ArcSinh[(2*a*b + 2*b^2*x)/(
2*b)]/b))/(2*b))/(4*b)))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{i\left(-2b^{3}x^{3}+2a\,b^{2}x^{2}-8i b^{2}x^{2}-2a^{2}bx+20iabx+2a^{3}-44ia^{2}+19bx-93a+48i\right)\sqrt{b^{2}x^{2}+2abx+a^{2}+1}}{8b^{4}}+\frac{\frac{51i\ln\left(\frac{b^{2}x+ab}{\sqrt{b^{2}}}+\sqrt{b^{2}x^{2}+2abx}\right.}{\sqrt{b^{2}}}}{}$
default	$\frac{i\left(\frac{\left(2b^{2}x+2ab\right)\left(b^{2}x^{2}+2abx+a^{2}+1\right)^{\frac{3}{2}}}{8b^{2}}+\frac{3\left(4b^{2}\left(a^{2}+1\right)-4a^{2}b^{2}\right)\left(\frac{\left(2b^{2}x+2ab\right)\sqrt{b^{2}x^{2}+2abx+a^{2}+1}}{4b^{2}}+\frac{\left(4b^{2}\left(a^{2}+1\right)-4a^{2}b^{2}\right)\ln\left(\frac{b^{2}x+ab}{\sqrt{b^{2}}}+\sqrt{b^{2}x^{2}\right.}{8b^{2}\sqrt{b^{2}}}\right.}{16b^{2}}\right.}{b^{3}}$

input
```
int(x^3/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*I*(-2*b^3*x^3-8*I*b^2*x^2+2*a*b^2*x^2+20*I*a*b*x-2*a^2*b*x-44*I*a^2+2
*a^3+19*b*x+48*I-93*a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^4+1/8/b^3*(51*I*ln(
(b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-108*I*a
^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-I
*(-96*a^2-32*I*a^3+32+96*I*a)/b^2/(x-(I-a)/b)*((x-(I-a)/b)^2*b^2+2*I*b*(x-
(I-a)/b))^(1/2)-132*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(
1/2))/(b^2)^(1/2)+24*a^3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1
)^(1/2))/(b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.77

$$\int e^{-3i\arctan(a+bx)}x^3\,dx$$
$$= \frac{-15i\,a^5 - 495\,a^4 + 1664i\,a^3 + (-15i\,a^4 - 480\,a^3 + 1184i\,a^2 + 968\,a - 256i)bx + 2152\,a^2 - 24\,(8\,a^4 - 44}{}$$

input
```
integrate(x^3/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/64*(-15*I*a^5 - 495*a^4 + 1664*I*a^3 + (-15*I*a^4 - 480*a^3 + 1184*I*a^2
 + 968*a - 256*I)*b*x + 2152*a^2 - 24*(8*a^4 - 44*I*a^3 + (8*a^3 - 36*I*a^
2 - 44*a + 17*I)*b*x - 80*a^2 + 61*I*a + 17)*log(-b*x - a + sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)) - 8*(-2*I*b^4*x^4 + 6*b^3*x^3 - (10*a - 11*I)*b^2*x^2
 + 2*I*a^4 + 78*a^3 + (22*a^2 - 54*I*a - 29)*b*x - 233*I*a^2 - 237*a + 80*
I)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 1224*I*a - 256)/(b^5*x + (a - I)*b^
4)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3i\arctan(a+bx)}x^3\,dx = \text{Timed out}$$

input
```
integrate(x**3/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 979 vs. 2(198) = 396.

Time = 0.14 (sec) , antiderivative size = 979, normalized size of antiderivative = 3.47

$$\int e^{-3i\arctan(a+bx)}x^3\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

output
```
-I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^3/(b^6*x^2 + 2*a*b^5*x + a^2*b^4
- 2*I*b^5*x - 2*I*a*b^4 - b^4) - 3*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^2
/(b^6*x^2 + 2*a*b^5*x + a^2*b^4 - 2*I*b^5*x - 2*I*a*b^4 - b^4) - 3*(b^2*x^
2 + 2*a*b*x + a^2 + 1)^(3/2)*a^2/(2*I*b^5*x + 2*I*a*b^4 + 2*b^4) - 6*I*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3/(I*b^5*x + I*a*b^4 + b^4) + 3*I*(b^2*x^
2 + 2*a*b*x + a^2 + 1)^(3/2)*a/(b^6*x^2 + 2*a*b^5*x + a^2*b^4 - 2*I*b^5*x
- 2*I*a*b^4 - b^4) + 6*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a/(2*I*b^5*x
+ 2*I*a*b^4 + 2*b^4) - 18*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2/(I*b^5*x +
 I*a*b^4 + b^4) + (b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/(b^6*x^2 + 2*a*b^5*x
 + a^2*b^4 - 2*I*b^5*x - 2*I*a*b^4 - b^4) + 3*(b^2*x^2 + 2*a*b*x + a^2 + 1
)^(3/2)/(2*I*b^5*x + 2*I*a*b^4 + 2*b^4) + 18*I*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*a/(I*b^5*x + I*a*b^4 + b^4) + 3*a^3*arcsinh(b*x + a)/b^4 + 6*sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1)/(I*b^5*x + I*a*b^4 + b^4) + 1/4*I*(b^2*x^2 + 2
*a*b*x + a^2 + 1)^(3/2)*x/b^3 + 3/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*
x + 4*I*a + 3)*a*x/b^3 - 27/2*I*a^2*arcsinh(b*x + a)/b^4 - 3/4*I*(b^2*x^2
+ 2*a*b*x + a^2 + 1)^(3/2)*a/b^4 - 9/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a
^2/b^4 + 3/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)*a^2/b^4
+ 3/8*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^3 - 3/2*I*sqrt(-b^2*x^2 - 2*
a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)*x/b^3 - 3/2*a*arcsin(I*b*x + I*a + 2)/b
^4 - 18*a*arcsinh(b*x + a)/b^4 - (b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/b^...
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.01

$$\int e^{-3i\arctan(a+bx)}x^3\,dx = -\frac{1}{8}\sqrt{(bx+a)^2+1}\left(\left(2\,x\left(-\frac{i\,x}{b}-\frac{-i\,ab^{11}-4\,b^{11}}{b^{13}}\right)-\frac{2i\,a^2b^{10}+20\,ab^{10}-19i\,b^{10}}{b^{13}}\right)x-\frac{-2i\,a^3b^9-44\,a\right.\right.$$
$$-\frac{(8\,a^3-36i\,a^2-44\,a+17i)\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab+a^3b+\left(x|b|-\sqrt{(bx+a)^2+1}\right)^3|b\right.}{}$$

input
```
integrate(x^3/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/8*sqrt((b*x + a)^2 + 1)*((2*x*(-I*x/b - (-I*a*b^11 - 4*b^11)/b^13) - (2
*I*a^2*b^10 + 20*a*b^10 - 19*I*b^10)/b^13)*x - (-2*I*a^3*b^9 - 44*a^2*b^9
+ 93*I*a*b^9 + 48*b^9)/b^13) - 1/8*(8*a^3 - 36*I*a^2 - 44*a + 17*I)*log(3*
(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b + (x*abs(b) - sqrt((b*x +
 a)^2 + 1))^3*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*abs(b) - 2
*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b - 2*I*a^2*b + 4*(-I*x*abs(b) + I
*sqrt((b*x + a)^2 + 1))*a*abs(b) - a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1)
)*abs(b))/(b^3*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3i\arctan(a+bx)}x^3\,dx = \int \frac{x^3\,\left({\left(a+b\,x\right)}^2+1\right)^{3/2}}{{\left(1+a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}\right)}^3}\,dx$$

input
```
int((x^3*((a + b*x)^2 + 1)^(3/2))/(a*1i + b*x*1i + 1)^3,x)
```

output
```
int((x^3*((a + b*x)^2 + 1)^(3/2))/(a*1i + b*x*1i + 1)^3, x)
```

Reduce [F]

$$\int e^{-3i\arctan(a+bx)}x^3\,dx$$
$$= -\left(\int \frac{\sqrt{b^2x^2+2abx+a^2+1}\,x^5}{b^3i\,x^3+3a\,b^2i\,x^2+3a^2bix+a^3i+3b^2x^2+6abx-3bix+3a^2-3ai-1}dx\right)b^2$$
$$-2\left(\int \frac{\sqrt{b^2x^2+2abx+a^2+1}\,x^4}{b^3i\,x^3+3a\,b^2i\,x^2+3a^2bix+a^3i+3b^2x^2+6abx-3bix+3a^2-3ai-1}dx\right)ab$$
$$-\left(\int \frac{\sqrt{b^2x^2+2abx+a^2+1}\,x^3}{b^3i\,x^3+3a\,b^2i\,x^2+3a^2bix+a^3i+3b^2x^2+6abx-3bix+3a^2-3ai-1}dx\right)a^2$$
$$-\left(\int \frac{\sqrt{b^2x^2+2abx+a^2+1}\,x^3}{b^3i\,x^3+3a\,b^2i\,x^2+3a^2bix+a^3i+3b^2x^2+6abx-3bix+3a^2-3ai-1}dx\right)$$

input
```
int(x^3/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x)
```

output
```
- int((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*x**5)/(a**3*i + 3*a**2*b*i*x
+ 3*a**2 + 3*a*b**2*i*x**2 + 6*a*b*x - 3*a*i + b**3*i*x**3 + 3*b**2*x**2 -
 3*b*i*x - 1),x)*b**2 - 2*int((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*x**4)/
(a**3*i + 3*a**2*b*i*x + 3*a**2 + 3*a*b**2*i*x**2 + 6*a*b*x - 3*a*i + b**3
*i*x**3 + 3*b**2*x**2 - 3*b*i*x - 1),x)*a*b - int((sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1)*x**3)/(a**3*i + 3*a**2*b*i*x + 3*a**2 + 3*a*b**2*i*x**2 + 6*
a*b*x - 3*a*i + b**3*i*x**3 + 3*b**2*x**2 - 3*b*i*x - 1),x)*a**2 - int((sq
rt(a**2 + 2*a*b*x + b**2*x**2 + 1)*x**3)/(a**3*i + 3*a**2*b*i*x + 3*a**2 +
 3*a*b**2*i*x**2 + 6*a*b*x - 3*a*i + b**3*i*x**3 + 3*b**2*x**2 - 3*b*i*x -
 1),x)
```

3.223 $\int e^{-3i\arctan(a+bx)}x^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 229

$$\int e^{-3i\arctan(a+bx)}x^2\,dx = \frac{i(i-a)^2(1-ia-ibx)^{5/2}}{b^3\sqrt{1+ia+ibx}} - \frac{(11i-18a-6ia^2)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^3} - \frac{(11i-18a-6ia^2)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{6b^3} - \frac{i(1-ia-ibx)^{5/2}\sqrt{1+ia+ibx}}{3b^3} + \frac{(11+18ia-6a^2)\operatorname{arcsinh}(a+bx)}{2b^3}$$

output
```
I*(I-a)^2*(1-I*a-I*b*x)^(5/2)/b^3/(1+I*a+I*b*x)^(1/2)-1/2*(11*I-18*a-6*I*a
^2)*(1-I*a-I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^3-1/6*(11*I-18*a-6*I*a^2)*(1
-I*a-I*b*x)^(3/2)*(1+I*a+I*b*x)^(1/2)/b^3-1/3*I*(1-I*a-I*b*x)^(5/2)*(1+I*a
+I*b*x)^(1/2)/b^3+1/2*(11+18*I*a-6*a^2)*arcsinh(b*x+a)/b^3
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.86

$$\int e^{-3i\arctan(a+bx)}x^2\,dx$$
$$= \frac{2ia^4 + a^3(51 + 2ibx) + a^2(-50i + 69bx) + a(51 - 106ibx + 9b^2x^2 + 2ib^3x^3) + i(-52 + 33ibx - 26b^2x^2}{6b^3\sqrt{1 + a^2 + 2abx + b^2x^2}}$$
$$+ \frac{\sqrt[4]{-1}(11 + 18ia - 6a^2)\sqrt{-ib}\operatorname{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{b^{7/2}}$$

input
```
Integrate[x^2/E^((3*I)*ArcTan[a + b*x]),x]
```

output
```
((2*I)*a^4 + a^3*(51 + (2*I)*b*x) + a^2*(-50*I + 69*b*x) + a*(51 - (106*I)
*b*x + 9*b^2*x^2 + (2*I)*b^3*x^3) + I*(-52 + (33*I)*b*x - 26*b^2*x^2 + (9*
I)*b^3*x^3 + 2*b^4*x^4))/(6*b^3*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]) + ((-1)
^(1/4)*(11 + (18*I)*a - 6*a^2)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*S
qrt[(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/b^(7/2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5618, 100, 25, 27, 90, 60, 60, 62, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{-3i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{ 5618}$$

$$\int \frac{x^2(-ia - ibx + 1)^{3/2}}{(ia + ibx + 1)^{3/2}}dx$$

$$\Big\downarrow \text{ 100}$$

$$\frac{i\int -\frac{b(-ia-ibx+1)^{3/2}((i-a)(2ia+3)+bx)}{\sqrt{ia+ibx+1}}dx}{b^3}+\frac{i(-a+i)^2(-ia-ibx+1)^{5/2}}{b^3\sqrt{ia+ibx+1}}$$

$\Big\downarrow$ 25

$$\frac{i(-a+i)^2(-ia-ibx+1)^{5/2}}{b^3\sqrt{ia+ibx+1}}-\frac{i\int \frac{b(-ia-ibx+1)^{3/2}((i-a)(2ia+3)+bx)}{\sqrt{ia+ibx+1}}dx}{b^3}$$

$\Big\downarrow$ 27

$$\frac{i(-a+i)^2(-ia-ibx+1)^{5/2}}{b^3\sqrt{ia+ibx+1}}-\frac{i\int \frac{(-ia-ibx+1)^{3/2}((i-a)(2ia+3)+bx)}{\sqrt{ia+ibx+1}}dx}{b^2}$$

$\Big\downarrow$ 90

$$\frac{i(-a+i)^2(-ia-ibx+1)^{5/2}}{b^3\sqrt{ia+ibx+1}}-\frac{i\left(\frac{(-ia-ibx+1)^{5/2}\sqrt{ia+ibx+1}}{3b}-\frac{1}{3}\left(18a-i\left(11-6a^2\right)\right)\int \frac{(-ia-ibx+1)^{3/2}}{\sqrt{ia+ibx+1}}dx\right)}{b^2}$$

$\Big\downarrow$ 60

$$\frac{i(-a+i)^2(-ia-ibx+1)^{5/2}}{b^3\sqrt{ia+ibx+1}}-\frac{i\left(\frac{(-ia-ibx+1)^{5/2}\sqrt{ia+ibx+1}}{3b}-\frac{1}{3}\left(18a-i\left(11-6a^2\right)\right)\left(\frac{3}{2}\int \frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}dx-\frac{i(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{2b}\right)\right)}{b^2}$$

$\Big\downarrow$ 60

$$\frac{i(-a+i)^2(-ia-ibx+1)^{5/2}}{b^3\sqrt{ia+ibx+1}}-\frac{i\left(\frac{(-ia-ibx+1)^{5/2}\sqrt{ia+ibx+1}}{3b}-\frac{1}{3}\left(18a-i\left(11-6a^2\right)\right)\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)-\frac{i(-ia}{}\right.\right.}{b^2}$$

$\Big\downarrow$ 62

$$\frac{i(-a+i)^2(-ia-ibx+1)^{5/2}}{b^3\sqrt{ia+ibx+1}}-\frac{i\left(\frac{(-ia-ibx+1)^{5/2}\sqrt{ia+ibx+1}}{3b}-\frac{1}{3}\left(18a-i\left(11-6a^2\right)\right)\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)-\frac{i(}{}\right.\right.}{b^2}$$

$\Big\downarrow$ 1090

$$\frac{i(-a+i)^2(-ia-ibx+1)^{5/2}}{b^3\sqrt{ia+ibx+1}}-\frac{i\left(\frac{(-ia-ibx+1)^{5/2}\sqrt{ia+ibx+1}}{3b}-\frac{1}{3}\left(18a-i\left(11-6a^2\right)\right)\left(\frac{3}{2}\left(\frac{\int\frac{1}{\sqrt{\frac{\left(2xb^2+2ab\right)^2}{4b^2}+1}}d\left(2xb^2+2ab\right)}{2b^2}-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)-\right.\right.}{b^2}$$

$\downarrow$ 222

$$\frac{i(-a+i)^2(-ia-ibx+1)^{5/2}}{b^3\sqrt{ia+ibx+1}}-\frac{i\left(\frac{(-ia-ibx+1)^{5/2}\sqrt{ia+ibx+1}}{3b}-\frac{1}{3}\left(18a-i\left(11-6a^2\right)\right)\left(\frac{3}{2}\left(\frac{\text{arcsinh}\left(\frac{2ab+2b^2x}{2b}\right)}{b}-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)-\frac{i(-ia-ibx+\right.\right.}{b^2}$$

input
```
Int[x^2/E^((3*I)*ArcTan[a + b*x]),x]
```

output
```
(I*(I - a)^2*(1 - I*a - I*b*x)^(5/2))/(b^3*Sqrt[1 + I*a + I*b*x]) - (I*(((
1 - I*a - I*b*x)^(5/2)*Sqrt[1 + I*a + I*b*x])/(3*b) - ((18*a - I*(11 - 6*a
^2))*(((-1/2*I)*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/b + (3*(((-
I)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b + ArcSinh[(2*a*b + 2*b^2
*x)/(2*b)]/b))/2))/3))/b^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.17

method	result
risch	$\frac{i(2b^2x^2-2abx+9ibx+2a^2-27ia-28)\sqrt{b^2x^2+2abx+a^2+1}}{6b^3} - \frac{-\frac{18ia\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}} - \frac{i(-8ia^2-16a+8i)\sqrt{(x-\frac{i-a}{b}}}{b^2(x-\frac{i-a}{b}}}{}$
default	$\frac{i\left(\frac{\left(\left(x-\frac{i-a}{b}\right)^2b^2+2ib\left(x-\frac{i-a}{b}\right)\right)^{\frac{3}{2}}}{3}+ib\left(\frac{\left(2\left(x-\frac{i-a}{b}\right)b^2+2ib\right)\sqrt{\left(x-\frac{i-a}{b}\right)^2b^2+2ib\left(x-\frac{i-a}{b}\right)}}{4b^2}+\frac{\ln(\frac{ib+\left(x-\frac{i-a}{b}\right)b^2}{\sqrt{b^2}}+\sqrt{\left(x-\frac{i-a}{b}\right)^2b^2+2ib(x}}{2\sqrt{b^2}}\right.\right.}{b^3}$

input
```
int(x^2/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*I*(2*b^2*x^2+9*I*b*x-2*a*b*x-27*I*a+2*a^2-28)*(b^2*x^2+2*a*b*x+a^2+1)^
(1/2)/b^3-1/2/b^2*(-18*I*a*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2
+1)^(1/2))/(b^2)^(1/2)-I*(-16*a-8*I*a^2+8*I)/b^2/(x-(I-a)/b)*((x-(I-a)/b)^
2*b^2+2*I*b*(x-(I-a)/b))^(1/2)-11*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*
b*x+a^2+1)^(1/2))/(b^2)^(1/2)+6*a^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*
a*b*x+a^2+1)^(1/2))/(b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.76

$$\int e^{-3i\arctan(a+bx)}x^2\,dx$$
$$= \frac{7i\,a^4+166\,a^3+(7i\,a^3+159\,a^2-249i\,a-96)bx-408i\,a^2+12\,(6\,a^3+(6\,a^2-18i\,a-11)bx-24i\,a^2-}{}$$

input
```
integrate(x^2/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/24*(7*I*a^4 + 166*a^3 + (7*I*a^3 + 159*a^2 - 249*I*a - 96)*b*x - 408*I*a
^2 + 12*(6*a^3 + (6*a^2 - 18*I*a - 11)*b*x - 24*I*a^2 - 29*a + 11*I)*log(-
b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 4*(-2*I*b^3*x^3 + 7*b^2*x^2
 - 2*I*a^3 - (16*a - 19*I)*b*x - 53*a^2 + 103*I*a + 52)*sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1) - 345*a + 96*I)/(b^4*x + (a - I)*b^3)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-3i\arctan(a+bx)}x^2\,dx = \text{Timed out}$$

input
```
integrate(x**2/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(155) = 310$.

Time = 0.12 (sec) , antiderivative size = 624, normalized size of antiderivative = 2.72

$$\int e^{-3i\arctan(a+bx)}x^2\,dx = \frac{i\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}a^2}{b^5x^2+2\,ab^4x+a^2b^3-2i\,b^4x-2i\,ab^3-b^3} + \frac{2\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}a}{b^5x^2+2\,ab^4x+a^2b^3-2i\,b^4x-2i\,ab^3-b^3} + \frac{2\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}a}{2i\,b^4x+2i\,ab^3+2\,b^3} + \frac{6i\,\sqrt{b^2x^2+2\,abx+a^2+1}a^2}{i\,b^4x+i\,ab^3+b^3} - \frac{i\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{b^5x^2+2\,ab^4x+a^2b^3-2i\,b^4x-2i\,ab^3-b^3} - \frac{2i\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{2i\,b^4x+2i\,ab^3+2\,b^3} + \frac{12\,\sqrt{b^2x^2+2\,abx+a^2+1}a}{i\,b^4x+i\,ab^3+b^3} - \frac{3\,a^2\,\mathrm{arsinh}\,(bx+a)}{b^3} - \frac{6i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{i\,b^4x+i\,ab^3+b^3} - \frac{\sqrt{-b^2x^2-2\,abx-a^2+4i\,bx+4i\,a+3}x}{2\,b^2} + \frac{9i\,a\,\mathrm{arsinh}\,(bx+a)}{b^3} + \frac{i\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{3\,b^3} + \frac{3\,\sqrt{b^2x^2+2\,abx+a^2+1}a}{b^3} - \frac{\sqrt{-b^2x^2-2\,abx-a^2+4i\,bx+4i\,a+3}a}{2\,b^3} + \frac{\arcsin\,(i\,bx+i\,a+2)}{2\,b^3} + \frac{6\,\mathrm{arsinh}\,(bx+a)}{b^3} - \frac{3i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{b^3} + \frac{i\,\sqrt{-b^2x^2-2\,abx-a^2+4i\,bx+4i\,a+3}}{b^3}$$

input
```
integrate(x^2/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

output
```
I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a^2/(b^5*x^2 + 2*a*b^4*x + a^2*b^3 -
 2*I*b^4*x - 2*I*a*b^3 - b^3) + 2*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a/(b
^5*x^2 + 2*a*b^4*x + a^2*b^3 - 2*I*b^4*x - 2*I*a*b^3 - b^3) + 2*(b^2*x^2 +
 2*a*b*x + a^2 + 1)^(3/2)*a/(2*I*b^4*x + 2*I*a*b^3 + 2*b^3) + 6*I*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)*a^2/(I*b^4*x + I*a*b^3 + b^3) - I*(b^2*x^2 + 2*a
*b*x + a^2 + 1)^(3/2)/(b^5*x^2 + 2*a*b^4*x + a^2*b^3 - 2*I*b^4*x - 2*I*a*b
^3 - b^3) - 2*I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/(2*I*b^4*x + 2*I*a*b^3
 + 2*b^3) + 12*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/(I*b^4*x + I*a*b^3 + b^
3) - 3*a^2*arcsinh(b*x + a)/b^3 - 6*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(I
*b^4*x + I*a*b^3 + b^3) - 1/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x + 4*
I*a + 3)*x/b^2 + 9*I*a*arcsinh(b*x + a)/b^3 + 1/3*I*(b^2*x^2 + 2*a*b*x + a
^2 + 1)^(3/2)/b^3 + 3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/b^3 - 1/2*sqrt(-
b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)*a/b^3 + 1/2*arcsin(I*b*x +
I*a + 2)/b^3 + 6*arcsinh(b*x + a)/b^3 - 3*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1)/b^3 + I*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 4*I*b*x + 4*I*a + 3)/b^3
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.05

$$\int e^{-3i\arctan(a+bx)}x^2\,dx$$
$$= -\frac{1}{6}\sqrt{(bx+a)^2+1}\left(x\left(-\frac{2i\,x}{b}+\frac{2i\,ab^6+9\,b^6}{b^8}\right)+\frac{-2i\,a^2b^5-27\,ab^5+28i\,b^5}{b^8}\right)$$
$$+\frac{(6\,a^2-18i\,a-11)\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab+a^3b+\left(x|b|-\sqrt{(bx+a)^2+1}\right)^3|b|+3\left(x\right.\right.}{}$$

input
```
integrate(x^2/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/6*sqrt((b*x + a)^2 + 1)*(x*(-2*I*x/b + (2*I*a*b^6 + 9*b^6)/b^8) + (-2*I
*a^2*b^5 - 27*a*b^5 + 28*I*b^5)/b^8) + 1/6*(6*a^2 - 18*I*a - 11)*log(3*(x*
abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b + (x*abs(b) - sqrt((b*x + a)
^2 + 1))^3*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*abs(b) - 2*I*
(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b - 2*I*a^2*b + 4*(-I*x*abs(b) + I*sq
rt((b*x + a)^2 + 1))*a*abs(b) - a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*a
bs(b))/(b^2*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3i\arctan(a+bx)}x^2\,dx=\int \frac{x^2\left((a+b\,x)^2+1\right)^{3/2}}{(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((x^2*((a + b*x)^2 + 1)^(3/2))/(a*1i + b*x*1i + 1)^3,x)
```

output
```
int((x^2*((a + b*x)^2 + 1)^(3/2))/(a*1i + b*x*1i + 1)^3, x)
```

Reduce [F]

$$\begin{aligned}&\int e^{-3i\arctan(a+bx)}x^2\,dx\\&=-\left(\int \frac{\sqrt{b^2x^2+2abx+a^2+1}\,x^4}{b^3i\,x^3+3a\,b^2i\,x^2+3a^2bix+a^3i+3b^2x^2+6abx-3bix+3a^2-3ai-1}dx\right)b^2\\&\quad-2\left(\int \frac{\sqrt{b^2x^2+2abx+a^2+1}\,x^3}{b^3i\,x^3+3a\,b^2i\,x^2+3a^2bix+a^3i+3b^2x^2+6abx-3bix+3a^2-3ai-1}dx\right)ab\\&\quad-\left(\int \frac{\sqrt{b^2x^2+2abx+a^2+1}\,x^2}{b^3i\,x^3+3a\,b^2i\,x^2+3a^2bix+a^3i+3b^2x^2+6abx-3bix+3a^2-3ai-1}dx\right)a^2\\&\quad-\left(\int \frac{\sqrt{b^2x^2+2abx+a^2+1}\,x^2}{b^3i\,x^3+3a\,b^2i\,x^2+3a^2bix+a^3i+3b^2x^2+6abx-3bix+3a^2-3ai-1}dx\right)\end{aligned}$$

input
```
int(x^2/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x)
```

output

```
- int((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*x**4)/(a**3*i + 3*a**2*b*i*x
+ 3*a**2 + 3*a*b**2*i*x**2 + 6*a*b*x - 3*a*i + b**3*i*x**3 + 3*b**2*x**2 -
 3*b*i*x - 1),x)*b**2 - 2*int((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*x**3)/
(a**3*i + 3*a**2*b*i*x + 3*a**2 + 3*a*b**2*i*x**2 + 6*a*b*x - 3*a*i + b**3
*i*x**3 + 3*b**2*x**2 - 3*b*i*x - 1),x)*a*b - int((sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1)*x**2)/(a**3*i + 3*a**2*b*i*x + 3*a**2 + 3*a*b**2*i*x**2 + 6*
a*b*x - 3*a*i + b**3*i*x**3 + 3*b**2*x**2 - 3*b*i*x - 1),x)*a**2 - int((sq
rt(a**2 + 2*a*b*x + b**2*x**2 + 1)*x**2)/(a**3*i + 3*a**2*b*i*x + 3*a**2 +
 3*a*b**2*i*x**2 + 6*a*b*x - 3*a*i + b**3*i*x**3 + 3*b**2*x**2 - 3*b*i*x -
 1),x)
```

3.224 $\int e^{-3i\arctan(a+bx)}x\,dx$

Optimal result

Integrand size = 14, antiderivative size = 163

$$\int e^{-3i\arctan(a+bx)}x\,dx = -\frac{(1+ia)(1-ia-ibx)^{5/2}}{b^2\sqrt{1+ia+ibx}} - \frac{3(3+2ia)\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{2b^2} - \frac{(3+2ia)(1-ia-ibx)^{3/2}\sqrt{1+ia+ibx}}{2b^2} - \frac{3(3i-2a)\text{arcsinh}(a+bx)}{2b^2}$$

output
```
-(1+I*a)*(1-I*a-I*b*x)^(5/2)/b^2/(1+I*a+I*b*x)^(1/2)-3/2*(3+2*I*a)*(1-I*a-
I*b*x)^(1/2)*(1+I*a+I*b*x)^(1/2)/b^2-1/2*(3+2*I*a)*(1-I*a-I*b*x)^(3/2)*(1+
I*a+I*b*x)^(1/2)/b^2-3/2*(3*I-2*a)*arcsinh(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.96

$$\int e^{-3i\arctan(a+bx)}x\,dx = \frac{i(14i-a^3+9bx+6ib^2x^2+b^3x^3+a^2(14i-bx)+a(-1+20ibx+b^2x^2))}{2b^2\sqrt{1+a^2+2abx+b^2x^2}} + \frac{3\sqrt[4]{-1}(-3i+2a)\sqrt{-ib}\,\text{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{b^{5/2}}$$

input
```
Integrate[x/E^((3*I)*ArcTan[a + b*x]),x]
```

output
```
((I/2)*(14*I - a^3 + 9*b*x + (6*I)*b^2*x^2 + b^3*x^3 + a^2*(14*I - b*x) +
a*(-1 + (20*I)*b*x + b^2*x^2)))/(b^2*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]) +
(3*(-1)^(1/4)*(-3*I + 2*a)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[
(-I)*(I + a + b*x)])/Sqrt[(-I)*b]])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5618, 87, 60, 60, 62, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-3i\arctan(a+bx)}dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{x(-ia-ibx+1)^{3/2}}{(ia+ibx+1)^{3/2}}dx$$

$$\Big\downarrow \text{87}$$

$$-\frac{(-2a+3i)\int \frac{(-ia-ibx+1)^{3/2}}{\sqrt{ia+ibx+1}}dx}{b} - \frac{(1+ia)(-ia-ibx+1)^{5/2}}{b^2\sqrt{ia+ibx+1}}$$

$$\Big\downarrow \; 60$$

$$-\frac{(-2a+3i)\left(\frac{3}{2}\int \frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}dx-\frac{i(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{2b}\right)}{b}-\frac{(1+ia)(-ia-ibx+1)^{5/2}}{b^2\sqrt{ia+ibx+1}}$$

$$\Big\downarrow \; 60$$

$$-\frac{(-2a+3i)\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)-\frac{i(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{2b}\right)}{b}-\frac{(1+ia)(-ia-ibx+1)^{5/2}}{b^2\sqrt{ia+ibx+1}}$$

$$\Big\downarrow \; 62$$

$$-\frac{(-2a+3i)\left(\frac{3}{2}\left(\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)-\frac{i(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{2b}\right)}{b}-\frac{(1+ia)(-ia-ibx+1)^{5/2}}{b^2\sqrt{ia+ibx+1}}$$

$$\Big\downarrow \; 1090$$

$$-\frac{(-2a+3i)\left(\frac{3}{2}\left(\frac{\int \frac{1}{\sqrt{\frac{\left(2xb^2+2ab\right)^2}{4b^2}+1}}d\left(2xb^2+2ab\right)}{2b^2}-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)-\frac{i(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{2b}\right)}{b}-\frac{(1+ia)(-ia-ibx+1)^{5/2}}{b^2\sqrt{ia+ibx+1}}$$

$$\Big\downarrow \; 222$$

$$-\frac{(-2a+3i)\left(\frac{3}{2}\left(\frac{\text{arcsinh}\left(\frac{2ab+2b^2x}{2b}\right)}{b}-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)-\frac{i(-ia-ibx+1)^{3/2}\sqrt{ia+ibx+1}}{2b}\right)}{b}-\frac{(1+ia)(-ia-ibx+1)^{5/2}}{b^2\sqrt{ia+ibx+1}}$$

input

```
Int[x/E^((3*I)*ArcTan[a + b*x]),x]
```

output
```
-(((1 + I*a)*(1 - I*a - I*b*x)^(5/2))/(b^2*Sqrt[1 + I*a + I*b*x])) - ((3*I
 - 2*a)*(((-1/2*I)*(1 - I*a - I*b*x)^(3/2)*Sqrt[1 + I*a + I*b*x])/b + (3*(
((-I)*Sqrt[1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b + ArcSinh[(2*a*b + 2*
b^2*x)/(2*b)]/b))/2))/b
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.20

method	result
risch	$-\frac{i(-bx+a-6i)\sqrt{b^2x^2+2abx+a^2+1}}{2b^2}+\frac{-\frac{9i\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}-\frac{i(-8ia-8)\sqrt{\left(x-\frac{i-a}{b}\right)^2b^2+2ib\left(x-\frac{i-a}{b}\right)}}{b^2\left(x-\frac{i-a}{b}\right)}+\frac{6a\ln\left(\frac{b^2x+}{\sqrt{b}}\right.}{}}{2b}$
default	$\frac{i\left(-\frac{i\left(\left(x-\frac{i-a}{b}\right)^2b^2+2ib\left(x-\frac{i-a}{b}\right)\right)^{\frac{5}{2}}}{b\left(x-\frac{i-a}{b}\right)^2}+3ib\left(\frac{\left(\left(x-\frac{i-a}{b}\right)^2b^2+2ib\left(x-\frac{i-a}{b}\right)\right)^{\frac{3}{2}}}{3}+ib\left(\frac{\left(2\left(x-\frac{i-a}{b}\right)b^2+2ib\right)\sqrt{\left(x-\frac{i-a}{b}\right)^2b^2+2ib\left(x-\frac{i-a}{b}\right)}}{4b^2}+\right.\right.\right.}{b^3}$

input
```
int(x/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*(-b*x+a-6*I)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/b^2+1/2/b*(-9*I*ln((b^2*
x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-I*(-8-8*I*a)
/b^2/(x-(I-a)/b)*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)+6*a*ln((b^2*x
+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.83

$$\int e^{-3i\arctan(a+bx)}x\,dx$$
$$=\frac{-3i\,a^3+(-3i\,a^2-44\,a+32i)bx-47\,a^2-12\left((2\,a-3i)bx+2\,a^2-5i\,a-3\right)\log\left(-bx-a+\sqrt{b^2x^2+2}\right.}{8\left(b^3x+(a-i)\right.}$$

input
```
integrate(x/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/8*(-3*I*a^3 + (-3*I*a^2 - 44*a + 32*I)*b*x - 47*a^2 - 12*((2*a - 3*I)*b*
x + 2*a^2 - 5*I*a - 3)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) -
 4*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-I*b^2*x^2 + I*a^2 + 5*b*x + 15*a -
14*I) + 76*I*a + 32)/(b^3*x + (a - I)*b^2)
```

Sympy [F]

$$\int e^{-3i\arctan(a+bx)}x\,dx$$
$$= i\left(\int \frac{x\sqrt{a^2+2abx+b^2x^2+1}}{a^3+3a^2bx-3ia^2+3ab^2x^2-6iabx-3a+b^3x^3-3ib^2x^2-3bx+i}\,dx + \int \frac{a^2x\sqrt{a^2+2abx+b^2x^2+1}}{a^3+3a^2bx-3ia^2+3ab^2x^2-6iabx-3a+b^3x^3-3ib^2x^2-3bx+i}\,dx + \int \frac{b^2x^3\sqrt{a^2+2abx+b^2x^2+1}}{a^3+3a^2bx-3ia^2+3ab^2x^2-6iabx-3a+b^3x^3-3ib^2x^2-3bx+i}\,dx + \int \frac{2abx^2\sqrt{a^2+2abx+b^2x^2+1}}{a^3+3a^2bx-3ia^2+3ab^2x^2-6iabx-3a+b^3x^3-3ib^2x^2-3bx+i}\,dx\right)$$

input
```
integrate(x/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2),x)
```

output
```
I*(Integral(x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a**3 + 3*a**2*b*x - 3*
I*a**2 + 3*a*b**2*x**2 - 6*I*a*b*x - 3*a + b**3*x**3 - 3*I*b**2*x**2 - 3*b
*x + I), x) + Integral(a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a**3 +
 3*a**2*b*x - 3*I*a**2 + 3*a*b**2*x**2 - 6*I*a*b*x - 3*a + b**3*x**3 - 3*I
*b**2*x**2 - 3*b*x + I), x) + Integral(b**2*x**3*sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1)/(a**3 + 3*a**2*b*x - 3*I*a**2 + 3*a*b**2*x**2 - 6*I*a*b*x - 3*
a + b**3*x**3 - 3*I*b**2*x**2 - 3*b*x + I), x) + Integral(2*a*b*x**2*sqrt(
a**2 + 2*a*b*x + b**2*x**2 + 1)/(a**3 + 3*a**2*b*x - 3*I*a**2 + 3*a*b**2*x
**2 - 6*I*a*b*x - 3*a + b**3*x**3 - 3*I*b**2*x**2 - 3*b*x + I), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(112) = 224.

Time = 0.11 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.80

$$\int e^{-3i\arctan(a+bx)}x\,dx = -\frac{i\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}a}{b^4x^2+2\,ab^3x+a^2b^2-2i\,b^3x-2i\,ab^2-b^2} - \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{b^4x^2+2\,ab^3x+a^2b^2-2i\,b^3x-2i\,ab^2-b^2} - \frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{2i\,b^3x+2i\,ab^2+2\,b^2} - \frac{6i\,\sqrt{b^2x^2+2\,abx+a^2+1}a}{i\,b^3x+i\,ab^2+b^2} + \frac{3\,a\,\mathrm{arsinh}\,(bx+a)}{b^2} - \frac{6\,\sqrt{b^2x^2+2\,abx+a^2+1}}{i\,b^3x+i\,ab^2+b^2} - \frac{9i\,\mathrm{arsinh}\,(bx+a)}{2\,b^2} - \frac{3\,\sqrt{b^2x^2+2\,abx+a^2+1}}{2\,b^2}$$

input
```
integrate(x/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

output
```
-I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a/(b^4*x^2 + 2*a*b^3*x + a^2*b^2 -
2*I*b^3*x - 2*I*a*b^2 - b^2) - (b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/(b^4*x^
2 + 2*a*b^3*x + a^2*b^2 - 2*I*b^3*x - 2*I*a*b^2 - b^2) - (b^2*x^2 + 2*a*b*
x + a^2 + 1)^(3/2)/(2*I*b^3*x + 2*I*a*b^2 + 2*b^2) - 6*I*sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)*a/(I*b^3*x + I*a*b^2 + b^2) + 3*a*arcsinh(b*x + a)/b^2 -
6*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(I*b^3*x + I*a*b^2 + b^2) - 9/2*I*arcs
inh(b*x + a)/b^2 - 3/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^2
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.29

$$\int e^{-3i\arctan(a+bx)}x\,dx = -\frac{1}{2}\sqrt{(bx+a)^2+1}\left(-\frac{i\,x}{b} - \frac{-i\,ab^2-6\,b^2}{b^4}\right) - \frac{(2\,a-3i)\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab+a^3b+\left(x|b|-\sqrt{(bx+a)^2+1}\right)^3|b|+3\left(x|b|-\sqrt{(b\right.\right.}{}$$

input
```
integrate(x/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/2*sqrt((b*x + a)^2 + 1)*(-I*x/b - (-I*a*b^2 - 6*b^2)/b^4) - 1/2*(2*a -
3*I)*log(3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b + (x*abs(b) -
sqrt((b*x + a)^2 + 1))^3*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2
*abs(b) - 2*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b - 2*I*a^2*b + 4*(-I*x
*abs(b) + I*sqrt((b*x + a)^2 + 1))*a*abs(b) - a*b - (x*abs(b) - sqrt((b*x
+ a)^2 + 1))*abs(b))/(b*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3i\arctan(a+bx)}x\,dx = \int \frac{x\left((a+b\,x)^2+1\right)^{3/2}}{(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((x*((a + b*x)^2 + 1)^(3/2))/(a*1i + b*x*1i + 1)^3,x)
```

output
```
int((x*((a + b*x)^2 + 1)^(3/2))/(a*1i + b*x*1i + 1)^3, x)
```

Reduce [F]

$$\begin{aligned}
&\int e^{-3i\arctan(a+bx)}x\,dx \\
&= -\left(\int \frac{\sqrt{b^2x^2+2abx+a^2+1}\,x^3}{b^3i\,x^3+3a\,b^2i\,x^2+3a^2bix+a^3i+3b^2x^2+6abx-3bix+3a^2-3ai-1}dx\right)b^2 \\
&\quad -2\left(\int \frac{\sqrt{b^2x^2+2abx+a^2+1}\,x^2}{b^3i\,x^3+3a\,b^2i\,x^2+3a^2bix+a^3i+3b^2x^2+6abx-3bix+3a^2-3ai-1}dx\right)ab \\
&\quad -\left(\int \frac{\sqrt{b^2x^2+2abx+a^2+1}\,x}{b^3i\,x^3+3a\,b^2i\,x^2+3a^2bix+a^3i+3b^2x^2+6abx-3bix+3a^2-3ai-1}dx\right)a^2 \\
&\quad -\left(\int \frac{\sqrt{b^2x^2+2abx+a^2+1}\,x}{b^3i\,x^3+3a\,b^2i\,x^2+3a^2bix+a^3i+3b^2x^2+6abx-3bix+3a^2-3ai-1}dx\right)
\end{aligned}$$

input
```
int(x/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x)
```

output
```
- int((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*x**3)/(a**3*i + 3*a**2*b*i*x
+ 3*a**2 + 3*a*b**2*i*x**2 + 6*a*b*x - 3*a*i + b**3*i*x**3 + 3*b**2*x**2 -
 3*b*i*x - 1),x)*b**2 - 2*int((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*x**2)/
(a**3*i + 3*a**2*b*i*x + 3*a**2 + 3*a*b**2*i*x**2 + 6*a*b*x - 3*a*i + b**3
*i*x**3 + 3*b**2*x**2 - 3*b*i*x - 1),x)*a*b - int((sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1)*x)/(a**3*i + 3*a**2*b*i*x + 3*a**2 + 3*a*b**2*i*x**2 + 6*a*b
*x - 3*a*i + b**3*i*x**3 + 3*b**2*x**2 - 3*b*i*x - 1),x)*a**2 - int((sqrt(
a**2 + 2*a*b*x + b**2*x**2 + 1)*x)/(a**3*i + 3*a**2*b*i*x + 3*a**2 + 3*a*b
**2*i*x**2 + 6*a*b*x - 3*a*i + b**3*i*x**3 + 3*b**2*x**2 - 3*b*i*x - 1),x)
```

3.225 $\int e^{-3i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 94

$$\int e^{-3i\arctan(a+bx)}\,dx=\frac{2i(1-ia-ibx)^{3/2}}{b\sqrt{1+ia+ibx}}+\frac{3i\sqrt{1-ia-ibx}\sqrt{1+ia+ibx}}{b}-\frac{3\text{arcsinh}(a+bx)}{b}$$

output
```
2*I*(1-I*a-I*b*x)^(3/2)/b/(1+I*a+I*b*x)^(1/2)+3*I*(1-I*a-I*b*x)^(1/2)*(1+I
*a+I*b*x)^(1/2)/b-3*arcsinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.48

$$\int e^{-3i\arctan(a+bx)}\,dx=\frac{\sqrt{1+(a+bx)^2}\left(i+\frac{4}{-i+a+bx}\right)}{b}-\frac{3\text{arcsinh}(a+bx)}{b}$$

input
```
Integrate[E^((-3*I)*ArcTan[a + b*x]),x]
```

output
```
(Sqrt[1 + (a + b*x)^2]*(I + 4/(-I + a + b*x)))/b - (3*ArcSinh[a + b*x])/b
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5616, 57, 60, 62, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-3i\arctan(a+bx)}dx$$

$$\Big\downarrow \text{5616}$$

$$\int \frac{(-ia-ibx+1)^{3/2}}{(ia+ibx+1)^{3/2}}dx$$

$$\Big\downarrow \text{57}$$

$$\frac{2i(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}-3\int \frac{\sqrt{-ia-ibx+1}}{\sqrt{ia+ibx+1}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{2i(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}-3\left(\int \frac{1}{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)$$

$$\Big\downarrow \text{62}$$

$$\frac{2i(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}-3\left(\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)$$

$$\Big\downarrow \text{1090}$$

$$\frac{2i(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}-3\left(\frac{\int \frac{1}{\sqrt{\frac{\left(2xb^2+2ab\right)^2}{4b^2}+1}}d\left(2xb^2+2ab\right)}{2b^2}-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)$$

$$\Big\downarrow \text{222}$$

$$\frac{2i(-ia-ibx+1)^{3/2}}{b\sqrt{ia+ibx+1}}-3\left(\frac{\operatorname{arcsinh}\left(\frac{2ab+2b^2x}{2b}\right)}{b}-\frac{i\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}{b}\right)$$

input
```
Int[E^((-3*I)*ArcTan[a + b*x]),x]
```

output
```
((2*I)*(1 - I*a - I*b*x)^(3/2))/(b*Sqrt[1 + I*a + I*b*x]) - 3*(((-I)*Sqrt[
1 - I*a - I*b*x]*Sqrt[1 + I*a + I*b*x])/b + ArcSinh[(2*a*b + 2*b^2*x)/(2*b
)]/b)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5616
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^(I*(n/2))/(1 + I*a*c + I*b*c*x)^(I*(n/2)), x] /; FreeQ[{a, b,
c, n}, x]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.37

method	result
risch	$\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{b}+\frac{4\sqrt{\left(x-\frac{i-a}{b}\right)^2b^2+2ib\left(x-\frac{i-a}{b}\right)}}{b^2\left(x-\frac{i-a}{b}\right)}-\frac{3\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}$
default	$\frac{i\left(\frac{i\left(\left(x-\frac{i-a}{b}\right)^2b^2+2ib\left(x-\frac{i-a}{b}\right)\right)^{\frac{5}{2}}}{b\left(x-\frac{i-a}{b}\right)^3}-2ib\left(-\frac{i\left(\left(x-\frac{i-a}{b}\right)^2b^2+2ib\left(x-\frac{i-a}{b}\right)\right)^{\frac{5}{2}}}{b\left(x-\frac{i-a}{b}\right)^2}+3ib\left(\frac{\left(\left(x-\frac{i-a}{b}\right)^2b^2+2ib\left(x-\frac{i-a}{b}\right)\right)^{\frac{3}{2}}}{3}+ib\left(\frac{\left(2\left(x-\frac{i-a}{b}\right)\right.}{}\right.\right.\right.\right.}{b^3}$

input
```
int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
I/b*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+4/b^2/(x-(I-a)/b)*((x-(I-a)/b)^2*b^2+2*I
*b*(x-(I-a)/b))^(1/2)-3*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)
^(1/2))/(b^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int e^{-3i\arctan(a+bx)}\,dx$$
$$=\frac{(i\,a+8)bx+i\,a^2+6\,(bx+a-i)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-2\,\sqrt{b^2x^2+2\,abx+a^2+1}(}{2\,(b^2x+(a-i)b)}$$

input
```
integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((I*a + 8)*b*x + I*a^2 + 6*(b*x + a - I)*log(-b*x - a + sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)) - 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-I*b*x - I*a -
 5) + 9*a - 8*I)/(b^2*x + (a - I)*b)
```

Sympy [F]

$$\int e^{-3i\arctan(a+bx)}\,dx$$
$$= i\left(\int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{a^3+3a^2bx-3ia^2+3ab^2x^2-6iabx-3a+b^3x^3-3ib^2x^2-3bx+i}\,dx + \int \frac{a^2\sqrt{a^2+2abx+b^2x^2+1}}{a^3+3a^2bx-3ia^2+3ab^2x^2-6iabx-3a+b^3x^3-3ib^2x^2-3bx+i}\,dx + \int \frac{b^2x^2\sqrt{a^2+2abx+b^2x^2+1}}{a^3+3a^2bx-3ia^2+3ab^2x^2-6iabx-3a+b^3x^3-3ib^2x^2-3bx+i}\,dx + \int \frac{2abx\sqrt{a^2+2abx+b^2x^2+1}}{a^3+3a^2bx-3ia^2+3ab^2x^2-6iabx-3a+b^3x^3-3ib^2x^2-3bx+i}\,dx\right)$$

input
```
integrate(1/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2),x)
```

output
```
I*(Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a**3 + 3*a**2*b*x - 3*I*
a**2 + 3*a*b**2*x**2 - 6*I*a*b*x - 3*a + b**3*x**3 - 3*I*b**2*x**2 - 3*b*x
 + I), x) + Integral(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a**3 + 3*a
**2*b*x - 3*I*a**2 + 3*a*b**2*x**2 - 6*I*a*b*x - 3*a + b**3*x**3 - 3*I*b**
2*x**2 - 3*b*x + I), x) + Integral(b**2*x**2*sqrt(a**2 + 2*a*b*x + b**2*x*
*2 + 1)/(a**3 + 3*a**2*b*x - 3*I*a**2 + 3*a*b**2*x**2 - 6*I*a*b*x - 3*a +
b**3*x**3 - 3*I*b**2*x**2 - 3*b*x + I), x) + Integral(2*a*b*x*sqrt(a**2 +
2*a*b*x + b**2*x**2 + 1)/(a**3 + 3*a**2*b*x - 3*I*a**2 + 3*a*b**2*x**2 - 6
*I*a*b*x - 3*a + b**3*x**3 - 3*I*b**2*x**2 - 3*b*x + I), x))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.10

$$\int e^{-3i\arctan(a+bx)}\,dx = \frac{i\left(b^2x^2+2\,abx+a^2+1\right)^{\frac{3}{2}}}{b^3x^2+2\,ab^2x+a^2b-2i\,b^2x-2i\,ab-b} - \frac{3\,\mathrm{arsinh}\left(bx+a\right)}{b} + \frac{6i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{i\,b^2x+i\,ab+b}$$

input

```
integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

output

```
I*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/(b^3*x^2 + 2*a*b^2*x + a^2*b - 2*I*b
^2*x - 2*I*a*b - b) - 3*arcsinh(b*x + a)/b + 6*I*sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)/(I*b^2*x + I*a*b + b)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(66) = 132.

Time = 0.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.91

$$\int e^{-3i\arctan(a+bx)}\,dx = \frac{\log\left(3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab+a^3b+\left(x|b|-\sqrt{(bx+a)^2+1}\right)^3|b|+3\left(x|b|-\sqrt{(bx+a)^2+1}\right.\right.}{} + \frac{i\sqrt{(bx+a)^2+1}}{b}$$

input

```
integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x, algorithm="giac")
```

output

```
log(3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b + a^3*b + (x*abs(b) - sqrt(
(b*x + a)^2 + 1))^3*abs(b) + 3*(x*abs(b) - sqrt((b*x + a)^2 + 1))*a^2*abs(
b) - 2*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b - 2*I*a^2*b + 4*(-I*x*abs(
b) + I*sqrt((b*x + a)^2 + 1))*a*abs(b) - a*b - (x*abs(b) - sqrt((b*x + a)^
2 + 1))*abs(b))/abs(b) + I*sqrt((b*x + a)^2 + 1)/b
```

Mupad [F(-1)]

Timed out.

$$\int e^{-3i\arctan(a+bx)}\,dx=\int \frac{\left((a+b\,x)^2+1\right)^{3/2}}{(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}\,dx$$

input
```
int(((a + b*x)^2 + 1)^(3/2)/(a*1i + b*x*1i + 1)^3,x)
```

output
```
int(((a + b*x)^2 + 1)^(3/2)/(a*1i + b*x*1i + 1)^3, x)
```

Reduce [F]

$$\begin{aligned}&\int e^{-3i\arctan(a+bx)}\,dx\\&=-\left(\int \frac{\sqrt{b^2x^2+2abx+a^2+1}}{b^3i\,x^3+3a\,b^2i\,x^2+3a^2bix+a^3i+3b^2x^2+6abx-3bix+3a^2-3ai-1}dx\right)a^2\\&\quad-\left(\int \frac{\sqrt{b^2x^2+2abx+a^2+1}}{b^3i\,x^3+3a\,b^2i\,x^2+3a^2bix+a^3i+3b^2x^2+6abx-3bix+3a^2-3ai-1}dx\right)\\&\quad-\left(\int \frac{\sqrt{b^2x^2+2abx+a^2+1}\,x^2}{b^3i\,x^3+3a\,b^2i\,x^2+3a^2bix+a^3i+3b^2x^2+6abx-3bix+3a^2-3ai-1}dx\right)b^2\\&\quad-2\left(\int \frac{\sqrt{b^2x^2+2abx+a^2+1}\,x}{b^3i\,x^3+3a\,b^2i\,x^2+3a^2bix+a^3i+3b^2x^2+6abx-3bix+3a^2-3ai-1}dx\right)ab\end{aligned}$$

input
```
int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2),x)
```

output

```
 - int(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a**3*i + 3*a**2*b*i*x + 3*a**
2 + 3*a*b**2*i*x**2 + 6*a*b*x - 3*a*i + b**3*i*x**3 + 3*b**2*x**2 - 3*b*i*
x - 1),x)*a**2 - int(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a**3*i + 3*a**2
*b*i*x + 3*a**2 + 3*a*b**2*i*x**2 + 6*a*b*x - 3*a*i + b**3*i*x**3 + 3*b**2
*x**2 - 3*b*i*x - 1),x) - int((sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*x**2)/
(a**3*i + 3*a**2*b*i*x + 3*a**2 + 3*a*b**2*i*x**2 + 6*a*b*x - 3*a*i + b**3
*i*x**3 + 3*b**2*x**2 - 3*b*i*x - 1),x)*b**2 - 2*int((sqrt(a**2 + 2*a*b*x
+ b**2*x**2 + 1)*x)/(a**3*i + 3*a**2*b*i*x + 3*a**2 + 3*a*b**2*i*x**2 + 6*
a*b*x - 3*a*i + b**3*i*x**3 + 3*b**2*x**2 - 3*b*i*x - 1),x)*a*b
```

3.226 $\int \frac{e^{-3i\arctan(a+bx)}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 134

$$\int \frac{e^{-3i\arctan(a+bx)}}{x}\,dx = \frac{4\sqrt{1-ia-ibx}}{(1+ia)\sqrt{1+ia+ibx}} + i\operatorname{arcsinh}(a+bx) - \frac{2(i+a)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{3/2}}$$

output
```
4*(1-I*a-I*b*x)^(1/2)/(1+I*a)/(1+I*a+I*b*x)^(1/2)+I*arcsinh(b*x+a)-2*(I+a)
^(3/2)*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-I*b*x)^(
1/2))/(I-a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.41

$$\int \frac{e^{-3i\arctan(a+bx)}}{x}\,dx = \frac{2(-1)^{3/4}\sqrt{-ib}\operatorname{arcsinh}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{b}\sqrt{-i(i+a+bx)}}{\sqrt{-ib}}\right)}{\sqrt{b}} + \frac{2\left(-\frac{2\sqrt{1+a^2+2abx+b^2x^2}}{-i+a+bx} + \frac{\sqrt{-1+ia}(i+a)\operatorname{arctanh}\left(\frac{\sqrt{-1-ia}\sqrt{-i(i+a+bx)}}{\sqrt{-1+ia}\sqrt{1+ia+ibx}}\right)}{\sqrt{-1-ia}}\right)}{-i+a}$$

input
```
Integrate[1/(E^((3*I)*ArcTan[a + b*x])*x),x]
```

output
```
(2*(-1)^(3/4)*Sqrt[(-I)*b]*ArcSinh[((1/2 + I/2)*Sqrt[b]*Sqrt[(-I)*(I + a +
 b*x)])/Sqrt[(-I)*b]])/Sqrt[b] + (2*((-2*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]
)/(-I + a + b*x) + (Sqrt[-1 + I*a]*(I + a)*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-
I)*(I + a + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/Sqrt[-1 - I*a]
))/(-I + a)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.22, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5618, 109, 27, 175, 62, 104, 221, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3i\arctan(a+bx)}}{x}dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{(-ia-ibx+1)^{3/2}}{x(ia+ibx+1)^{3/2}}dx$$

$$\Big\downarrow \text{109}$$

$$\frac{2\int -\frac{b(i(a+i)^2+(ia+1)bx)}{2x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{(-a+i)b}+\frac{4\sqrt{-ia-ibx+1}}{(1+ia)\sqrt{ia+ibx+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\sqrt{-ia-ibx+1}}{(1+ia)\sqrt{ia+ibx+1}}-\frac{\int \frac{i(a+i)^2+(ia+1)bx}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{-a+i}$$

$$\Big\downarrow \text{175}$$

$$\frac{4\sqrt{-ia-ibx+1}}{(1+ia)\sqrt{ia+ibx+1}}-\frac{i(a+i)^2\int \frac{1}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx+(1+ia)b\int \frac{1}{\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{-a+i}$$

$$\Big\downarrow \; 62$$

$$\frac{4\sqrt{-ia-ibx+1}}{(1+ia)\sqrt{ia+ibx+1}}-\frac{(1+ia)b\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx+i(a+i)^2\int \frac{1}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{-a+i}$$

$$\Big\downarrow \; 104$$

$$\frac{4\sqrt{-ia-ibx+1}}{(1+ia)\sqrt{ia+ibx+1}}-\frac{(1+ia)b\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx+2i(a+i)^2\int \frac{1}{-ia+\frac{(1-ia)(ia+ibx+1)}{-ia-ibx+1}-1}d\frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}}{-a+i}$$

$$\Big\downarrow \; 221$$

$$\frac{4\sqrt{-ia-ibx+1}}{(1+ia)\sqrt{ia+ibx+1}}-\frac{(1+ia)b\int \frac{1}{\sqrt{b^2x^2+2abx+(1-ia)(ia+1)}}dx+\frac{2(a+i)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}}}{-a+i}$$

$$\Big\downarrow \; 1090$$

$$\frac{4\sqrt{-ia-ibx+1}}{(1+ia)\sqrt{ia+ibx+1}}-\frac{\frac{(1+ia)\int \frac{1}{\sqrt{\frac{\left(2xb^2+2ab\right)^2}{4b^2}+1}}d\left(2xb^2+2ab\right)}{2b}+\frac{2(a+i)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}}}{-a+i}$$

$$\Big\downarrow \; 222$$

$$\frac{4\sqrt{-ia-ibx+1}}{(1+ia)\sqrt{ia+ibx+1}}-\frac{(1+ia)\text{arcsinh}\left(\frac{2ab+2b^2x}{2b}\right)+\frac{2(a+i)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{\sqrt{-a+i}}}{-a+i}$$

input
```
Int[1/(E^((3*I)*ArcTan[a + b*x])*x),x]
```

output
```
(4*Sqrt[1 - I*a - I*b*x])/((1 + I*a)*Sqrt[1 + I*a + I*b*x]) - ((1 + I*a)*A
rcSinh[(2*a*b + 2*b^2*x)/(2*b)] + (2*(I + a)^(3/2)*ArcTanh[(Sqrt[I + a]*Sq
rt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a - I*b*x])])/Sqrt[I - a])/(I
 - a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1066 vs. $2(104) = 208$.

Time = 0.79 (sec) , antiderivative size = 1067, normalized size of antiderivative = 7.96

method	result
default	$-\frac{i\left(\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{3}+ab\left(\frac{\left(2b^2x+2ab\right)\sqrt{b^2x^2+2abx+a^2+1}}{4b^2}+\frac{\left(4b^2\left(a^2+1\right)-4a^2b^2\right)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{8b^2\sqrt{b^2}}\right)+\left(a^2+1\right)\left(\right.}{(i-a)^3}$

input
```
int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-I/(I-a)^3*(1/3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+a*b*(1/4*(2*b^2*x+2*a*b)/b^2
*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/8*(4*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x
+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2))+(a^2+1)*((b^
2*x^2+2*a*b*x+a^2+1)^(1/2)+a*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x
+a^2+1)^(1/2))/(b^2)^(1/2)-(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/
2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)))-I/(I-a)^2/b*(-I/b/(x-(I-a)/b)^2*((x
-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)+3*I*b*(1/3*((x-(I-a)/b)^2*b^2+2*I
*b*(x-(I-a)/b))^(3/2)+I*b*(1/4*(2*(x-(I-a)/b)*b^2+2*I*b)/b^2*((x-(I-a)/b)^
2*b^2+2*I*b*(x-(I-a)/b))^(1/2)+1/2*ln((I*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((
x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2))))+I/(I-a)^3*(1/3*(
(x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(3/2)+I*b*(1/4*(2*(x-(I-a)/b)*b^2+2*I
*b)/b^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)+1/2*ln((I*b+(x-(I-a)/b
)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))/(b^2)^(1/2
)))+I/(I-a)/b^2*(I/b/(x-(I-a)/b)^3*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(
5/2)-2*I*b*(-I/b/(x-(I-a)/b)^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(5/2)
+3*I*b*(1/3*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(3/2)+I*b*(1/4*(2*(x-(I-
a)/b)*b^2+2*I*b)/b^2*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2)+1/2*ln((I
*b+(x-(I-a)/b)*b^2)/(b^2)^(1/2)+((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2
))/(b^2)^(1/2)))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(90) = 180$.

Time = 0.15 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.66

$$\int \frac{e^{-3i\arctan(a+bx)}}{x}\,dx$$
$$= \frac{((a-i)bx+a^2-2i\,a-1)\sqrt{-\frac{a^3+3i\,a^2-3\,a-i}{a^3-3i\,a^2-3\,a+i}}\log\left(-\frac{(a+i)bx-\sqrt{b^2x^2+2\,abx+a^2+1}(a+i)-(i\,a^2+2\,a-i)\sqrt{-\frac{a^3+3i\,a^2-3\,a-i}{a^3-3i\,a^2-3\,a+i}}}{a+i}\right.}{}$$

input
```
integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x,x, algorithm="fricas")
```

output
```
(((a - I)*b*x + a^2 - 2*I*a - 1)*sqrt(-(a^3 + 3*I*a^2 - 3*a - I)/(a^3 - 3*
I*a^2 - 3*a + I))*log(-((a + I)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a
 + I) - (I*a^2 + 2*a - I)*sqrt(-(a^3 + 3*I*a^2 - 3*a - I)/(a^3 - 3*I*a^2 -
 3*a + I)))/(a + I)) - ((a - I)*b*x + a^2 - 2*I*a - 1)*sqrt(-(a^3 + 3*I*a^
2 - 3*a - I)/(a^3 - 3*I*a^2 - 3*a + I))*log(-((a + I)*b*x - sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)*(a + I) - (-I*a^2 - 2*a + I)*sqrt(-(a^3 + 3*I*a^2 - 3*
a - I)/(a^3 - 3*I*a^2 - 3*a + I)))/(a + I)) - 4*b*x - ((I*a + 1)*b*x + I*a
^2 + 2*a - I)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 4*a - 4*
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 4*I)/((a - I)*b*x + a^2 - 2*I*a - 1)
```

Sympy [F]

$$\int \frac{e^{-3i\arctan(a+bx)}}{x}\,dx$$
$$= i\left(\int \frac{\sqrt{a^2+2abx+b^2x^2+1}}{a^3x+3a^2bx^2-3ia^2x+3ab^2x^3-6iabx^2-3ax+b^3x^4-3ib^2x^3-3bx^2+ix}\,dx + \int \frac{a^2\sqrt{a^2+2abx+b^2x^2+1}}{a^3x+3a^2bx^2-3ia^2x+3ab^2x^3-6iabx^2-3ax+b^3x^4-3ib^2x^3-3bx^2+ix}\,dx + \int \frac{b^2x^2\sqrt{a^2+2abx+b^2x^2+1}}{a^3x+3a^2bx^2-3ia^2x+3ab^2x^3-6iabx^2-3ax+b^3x^4-3ib^2x^3-3bx^2+ix}\,dx + \int \frac{2abx\sqrt{a^2+2abx+b^2x^2+1}}{a^3x+3a^2bx^2-3ia^2x+3ab^2x^3-6iabx^2-3ax+b^3x^4-3ib^2x^3-3bx^2+ix}\,dx\right)$$

input
```
integrate(1/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2)/x,x)
```

output
```
I*(Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a**3*x + 3*a**2*b*x**2 -
 3*I*a**2*x + 3*a*b**2*x**3 - 6*I*a*b*x**2 - 3*a*x + b**3*x**4 - 3*I*b**2*
x**3 - 3*b*x**2 + I*x), x) + Integral(a**2*sqrt(a**2 + 2*a*b*x + b**2*x**2
 + 1)/(a**3*x + 3*a**2*b*x**2 - 3*I*a**2*x + 3*a*b**2*x**3 - 6*I*a*b*x**2
- 3*a*x + b**3*x**4 - 3*I*b**2*x**3 - 3*b*x**2 + I*x), x) + Integral(b**2*
x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(a**3*x + 3*a**2*b*x**2 - 3*I*a*
*2*x + 3*a*b**2*x**3 - 6*I*a*b*x**2 - 3*a*x + b**3*x**4 - 3*I*b**2*x**3 -
3*b*x**2 + I*x), x) + Integral(2*a*b*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1
)/(a**3*x + 3*a**2*b*x**2 - 3*I*a**2*x + 3*a*b**2*x**3 - 6*I*a*b*x**2 - 3*
a*x + b**3*x**4 - 3*I*b**2*x**3 - 3*b*x**2 + I*x), x))
```

Maxima [F]

$$\int \frac{e^{-3i\arctan(a+bx)}}{x}\,dx = \int \frac{\left((bx+a)^2+1\right)^{\frac{3}{2}}}{(i\,bx+i\,a+1)^3x}\,dx$$

input
```
integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(((b*x + a)^2 + 1)^(3/2)/((I*b*x + I*a + 1)^3*x), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(90) = 180$.

Time = 0.25 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.88

$$\int \frac{e^{-3i\arctan(a+bx)}}{x}\,dx =$$

$$-\frac{i\,b\log\left(-3\left(x|b|-\sqrt{(bx+a)^2+1}\right)^2ab-a^3b-\left(x|b|-\sqrt{(bx+a)^2+1}\right)^3|b|-3\left(x|b|-\sqrt{(bx+a\right.\right.}{}$$

$$-\frac{(-i\,a^2+2\,a+i)\log\left(\frac{\left|-2\,x|b|+2\,\sqrt{(bx+a)^2+1}-2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\,\sqrt{(bx+a)^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{\sqrt{a^2+1}(a-i)}$$

input
```
integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x,x, algorithm="giac")
```

output
```
-1/3*I*b*log(-3*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*a*b - a^3*b - (x*abs(
b) - sqrt((b*x + a)^2 + 1))^3*abs(b) - 3*(x*abs(b) - sqrt((b*x + a)^2 + 1)
)*a^2*abs(b) + 2*I*(x*abs(b) - sqrt((b*x + a)^2 + 1))^2*b + 2*I*a^2*b - 4*
(-I*x*abs(b) + I*sqrt((b*x + a)^2 + 1))*a*abs(b) + a*b + (x*abs(b) - sqrt(
(b*x + a)^2 + 1))*abs(b))/abs(b) - (-I*a^2 + 2*a + I)*log(abs(-2*x*abs(b)
+ 2*sqrt((b*x + a)^2 + 1) - 2*sqrt(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt((b*x
 + a)^2 + 1) + 2*sqrt(a^2 + 1)))/(sqrt(a^2 + 1)*(a - I))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3i\arctan(a+bx)}}{x}\,dx = \int \frac{\left((a+b\,x)^2+1\right)^{3/2}}{x\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}\,dx$$

input
```
int(((a + b*x)^2 + 1)^(3/2)/(x*(a*1i + b*x*1i + 1)^3),x)
```

output
```
int(((a + b*x)^2 + 1)^(3/2)/(x*(a*1i + b*x*1i + 1)^3), x)
```

Reduce [F]

$$\int \frac{e^{-3i\arctan(a+bx)}}{x}\,dx = \int \frac{\left(1+(bx+a)^2\right)^{\frac{3}{2}}}{(1+i(bx+a))^3\,x}dx$$

input
```
int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x,x)
```

output
```
int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x,x)
```

3.227 $\int \frac{e^{-3i\arctan(a+bx)}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 178

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^2}\,dx = \frac{6ib\sqrt{1-ia-ibx}}{(i-a)^2\sqrt{1+ia+ibx}} - \frac{(1-ia-ibx)^{3/2}}{(1+ia)x\sqrt{1+ia+ibx}} - \frac{6i\sqrt{i+a}\,b\,\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{5/2}}$$

output
```
6*I*b*(1-I*a-I*b*x)^(1/2)/(I-a)^2/(1+I*a+I*b*x)^(1/2)-(1-I*a-I*b*x)^(3/2)/
(1+I*a)/x/(1+I*a+I*b*x)^(1/2)-6*I*(I+a)^(1/2)*b*arctanh((I+a)^(1/2)*(1+I*a
+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-I*b*x)^(1/2))/(I-a)^(5/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.81

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^2}\,dx = \frac{\frac{\sqrt{-i(i+a+bx)}(1+a^2+5ibx+abx)}{x\sqrt{1+ia+ibx}} - \frac{6i\sqrt{-1+ia}\,b\,\text{arctanh}\left(\frac{\sqrt{-1-ia}\sqrt{-i(i+a+bx)}}{\sqrt{-1+ia}\sqrt{1+ia+ibx}}\right)}{\sqrt{-1-ia}}}{(-i+a)^2}$$

input
```
Integrate[1/(E^((3*I)*ArcTan[a + b*x])*x^2),x]
```

output
```
((Sqrt[(-I)*(I + a + b*x)]*(1 + a^2 + (5*I)*b*x + a*b*x))/(x*Sqrt[1 + I*a
+ I*b*x]) - ((6*I)*Sqrt[-1 + I*a]*b*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-I)*(I +
 a + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/Sqrt[-1 - I*a])/(-I +
 a)^2
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5618, 105, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^2}dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{(-ia-ibx+1)^{3/2}}{x^2(ia+ibx+1)^{3/2}}dx$$

$$\Big\downarrow \text{105}$$

$$\frac{3b\int \frac{\sqrt{-ia-ibx+1}}{x(ia+ibx+1)^{3/2}}dx}{-a+i}-\frac{(-ia-ibx+1)^{3/2}}{(1+ia)x\sqrt{ia+ibx+1}}$$

$$\Big\downarrow \text{105}$$

$$\frac{3b\left(\frac{(a+i)\int \frac{1}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{-a+i}+\frac{2\sqrt{-ia-ibx+1}}{(1+ia)\sqrt{ia+ibx+1}}\right)}{-a+i}-\frac{(-ia-ibx+1)^{3/2}}{(1+ia)x\sqrt{ia+ibx+1}}$$

$$\Big\downarrow \text{104}$$

$$\frac{3b\left(\frac{2(a+i)\int \frac{1}{-ia+\frac{(1-ia)(ia+ibx+1)}{-ia-ibx+1}-1}d\frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}}{-a+i}+\frac{2\sqrt{-ia-ibx+1}}{(1+ia)\sqrt{ia+ibx+1}}\right)}{-a+i}-\frac{(-ia-ibx+1)^{3/2}}{(1+ia)x\sqrt{ia+ibx+1}}$$

$$\Big\downarrow \text{221}$$

$$\frac{3b\left(\frac{2\sqrt{-ia-ibx+1}}{(1+ia)\sqrt{ia+ibx+1}}-\frac{2i\sqrt{a+i}\operatorname{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{3/2}}\right)}{-a+i}-\frac{(-ia-ibx+1)^{3/2}}{(1+ia)x\sqrt{ia+ibx+1}}$$

input
```
Int[1/(E^((3*I)*ArcTan[a + b*x])*x^2),x]
```

output
```
-((1 - I*a - I*b*x)^(3/2)/((1 + I*a)*x*Sqrt[1 + I*a + I*b*x])) + (3*b*((2*
Sqrt[1 - I*a - I*b*x])/((1 + I*a)*Sqrt[1 + I*a + I*b*x]) - ((2*I)*Sqrt[I +
 a]*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a
- I*b*x])])/(I - a)^(3/2)))/(I - a)
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.05

method	result
risch	$-\frac{i\sqrt{b^{2}x^{2}+2abx+a^{2}+1}\,(i+a)}{(-i+a)^{2}x}+\frac{b\left(\frac{3\sqrt{a^{2}+1}\,\ln\left(\frac{2a^{2}+2+2abx+2\sqrt{a^{2}+1}\,\sqrt{b^{2}x^{2}+2abx+a^{2}+1}}{x}\right)}{i-a}+\frac{4i(ia+1)\sqrt{\left(x-\frac{i-a}{b}\right)^{2}b^{2}+2ib\left(x-\frac{i-a}{b}\right)}}{b(i-a)\left(x-\frac{i-a}{b}\right)}\right.}{a^{2}-2ia-1}$
default	Expression too large to display

input
```
int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-I*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*(I+a)/(-I+a)^2/x+1/(-2*I*a+a^2-1)*b*(3*(a
^2+1)^(1/2)/(I-a)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2
+1)^(1/2))/x)+4*I*(1+I*a)/b/(I-a)/(x-(I-a)/b)*((x-(I-a)/b)^2*b^2+2*I*b*(x-
(I-a)/b))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 389 vs. 2(116) = 232.

Time = 0.14 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.19

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^2}\,dx =$$

$$-\frac{(i\,a-5)b^{2}x^{2}+(i\,a^{2}-4\,a+5i)bx-3\left((a^{2}-2i\,a-1)bx^{2}+(a^{3}-3i\,a^{2}-3\,a+i)x\right)\sqrt{\frac{(a+i)b^{2}}{a^{5}-5i\,a^{4}-10\,a^{3}+10i}}}{}$$

input
```
integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
-((I*a - 5)*b^2*x^2 + (I*a^2 - 4*a + 5*I)*b*x - 3*((a^2 - 2*I*a - 1)*b*x^2
 + (a^3 - 3*I*a^2 - 3*a + I)*x)*sqrt((a + I)*b^2/(a^5 - 5*I*a^4 - 10*a^3 +
 10*I*a^2 + 5*a - I))*log(-(b^2*x + (a^3 - 3*I*a^2 - 3*a + I)*sqrt((a + I)
*b^2/(a^5 - 5*I*a^4 - 10*a^3 + 10*I*a^2 + 5*a - I)) - sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1)*b)/b) + 3*((a^2 - 2*I*a - 1)*b*x^2 + (a^3 - 3*I*a^2 - 3*a +
I)*x)*sqrt((a + I)*b^2/(a^5 - 5*I*a^4 - 10*a^3 + 10*I*a^2 + 5*a - I))*log(
-(b^2*x - (a^3 - 3*I*a^2 - 3*a + I)*sqrt((a + I)*b^2/(a^5 - 5*I*a^4 - 10*a
^3 + 10*I*a^2 + 5*a - I)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b)/b) + sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)*((I*a - 5)*b*x + I*a^2 + I))/((a^2 - 2*I*a -
 1)*b*x^2 + (a^3 - 3*I*a^2 - 3*a + I)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2)/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^2}\,dx = \int \frac{\left((bx+a)^2+1\right)^{\frac{3}{2}}}{(i\,bx+i\,a+1)^3x^2}\,dx$$

input
```
integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(((b*x + a)^2 + 1)^(3/2)/((I*b*x + I*a + 1)^3*x^2), x)
```

Giac [F]

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^2}\,dx = \int \frac{\left((bx+a)^2+1\right)^{\frac{3}{2}}}{\left(i\,bx+i\,a+1\right)^3x^2}\,dx$$

input
```
integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^2,x, algorithm="giac")
```

output
```
undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^2}\,dx = \int \frac{\left((a+b\,x)^2+1\right)^{3/2}}{x^2\,\left(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}\right)^3}\,dx$$

input
```
int(((a + b*x)^2 + 1)^(3/2)/(x^2*(a*1i + b*x*1i + 1)^3),x)
```

output
```
int(((a + b*x)^2 + 1)^(3/2)/(x^2*(a*1i + b*x*1i + 1)^3), x)
```

Reduce [F]

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^2}\,dx = \int \frac{\left(1+(bx+a)^2\right)^{\frac{3}{2}}}{\left(1+i\,(bx+a)\right)^3x^2}dx$$

input
```
int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^2,x)
```

output
```
int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^2,x)
```

3.228 $\int \frac{e^{-3i\arctan(a+bx)}}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 264

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^3}\,dx = -\frac{3(3i+2a)b^2\sqrt{1-ia-ibx}}{(1+ia)^3(i+a)\sqrt{1+ia+ibx}} + \frac{(3-2ia)b(1-ia-ibx)^{3/2}}{2(i-a)^2(i+a)x\sqrt{1+ia+ibx}} - \frac{(1-ia-ibx)^{5/2}}{2(1+a^2)\,x^2\sqrt{1+ia+ibx}} + \frac{3(3-2ia)b^2\text{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{7/2}\sqrt{i+a}}$$

output
```
-3*(3*I+2*a)*b^2*(1-I*a-I*b*x)^(1/2)/(1+I*a)^3/(I+a)/(1+I*a+I*b*x)^(1/2)+1
/2*(3-2*I*a)*b*(1-I*a-I*b*x)^(3/2)/(I-a)^2/(I+a)/x/(1+I*a+I*b*x)^(1/2)-1/2
*(1-I*a-I*b*x)^(5/2)/(a^2+1)/x^2/(1+I*a+I*b*x)^(1/2)+3*(3-2*I*a)*b^2*arcta
nh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-I*b*x)^(1/2))/(I-a)^
(7/2)/(I+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.73

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^3}\,dx$$
$$= \frac{\frac{\sqrt{-i(i+a+bx)}(-i+a-ia^2+a^3-5bx-5iabx-14ib^2x^2-ab^2x^2)}{x^2\sqrt{1+ia+ibx}}+\frac{6i\sqrt{-1+ia}(3i+2a)b^2\operatorname{arctanh}\left(\frac{\sqrt{-1-ia}\sqrt{-i(i+a+bx)}}{\sqrt{-1+ia}\sqrt{1+ia+ibx}}\right)}{\sqrt{-1-ia}(i+a)}}{2(-i+a)^3}$$

input
```
Integrate[1/(E^((3*I)*ArcTan[a + b*x])*x^3),x]
```

output
```
((Sqrt[(-I)*(I + a + b*x)]*(-I + a - I*a^2 + a^3 - 5*b*x - (5*I)*a*b*x - (
14*I)*b^2*x^2 - a*b^2*x^2))/(x^2*Sqrt[1 + I*a + I*b*x]) + ((6*I)*Sqrt[-1 +
 I*a]*(3*I + 2*a)*b^2*ArcTanh[(Sqrt[-1 - I*a]*Sqrt[(-I)*(I + a + b*x)])/(S
qrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x])])/(Sqrt[-1 - I*a]*(I + a)))/(2*(-I +
a)^3)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5618, 107, 105, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^3}\,dx$$
$$\Big\downarrow \text{5618}$$
$$\int \frac{(-ia-ibx+1)^{3/2}}{x^3(ia+ibx+1)^{3/2}}dx$$
$$\Big\downarrow \text{107}$$
$$-\frac{(2a+3i)b\int \frac{(-ia-ibx+1)^{3/2}}{x^2(ia+ibx+1)^{3/2}}dx}{2\left(a^2+1\right)}-\frac{(-ia-ibx+1)^{5/2}}{2\left(a^2+1\right)x^2\sqrt{ia+ibx+1}}$$

$\Big\downarrow$ 105

$$-\frac{(2a+3i)b\left(\frac{3b\int \frac{\sqrt{-ia-ibx+1}}{x(ia+ibx+1)^{3/2}}dx}{-a+i}-\frac{(-ia-ibx+1)^{3/2}}{(1+ia)x\sqrt{ia+ibx+1}}\right)}{2\left(a^2+1\right)}-\frac{(-ia-ibx+1)^{5/2}}{2\left(a^2+1\right)x^2\sqrt{ia+ibx+1}}$$

$\Big\downarrow$ 105

$$-\frac{(2a+3i)b\left(\frac{3b\left(\frac{(a+i)\int \frac{1}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{-a+i}+\frac{2\sqrt{-ia-ibx+1}}{(1+ia)\sqrt{ia+ibx+1}}\right)}{-a+i}-\frac{(-ia-ibx+1)^{3/2}}{(1+ia)x\sqrt{ia+ibx+1}}\right)}{2\left(a^2+1\right)}-\frac{(-ia-ibx+1)^{5/2}}{2\left(a^2+1\right)x^2\sqrt{ia+ibx+1}}$$

$\Big\downarrow$ 104

$$-\frac{(2a+3i)b\left(\frac{3b\left(\frac{2(a+i)\int \frac{1}{-ia+\frac{(1-ia)(ia+ibx+1)}{-ia-ibx+1}-1}d\frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}}{-a+i}+\frac{2\sqrt{-ia-ibx+1}}{(1+ia)\sqrt{ia+ibx+1}}\right)}{-a+i}-\frac{(-ia-ibx+1)^{3/2}}{(1+ia)x\sqrt{ia+ibx+1}}\right)}{2\left(a^2+1\right)}-\frac{(-ia-ibx+1)^{5/2}}{2\left(a^2+1\right)x^2\sqrt{ia+ibx+1}}$$

$\Big\downarrow$ 221

$$-\frac{(2a+3i)b\left(\frac{3b\left(\frac{2\sqrt{-ia-ibx+1}}{(1+ia)\sqrt{ia+ibx+1}}-\frac{2i\sqrt{a+i}\,\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{3/2}}\right)}{-a+i}-\frac{(-ia-ibx+1)^{3/2}}{(1+ia)x\sqrt{ia+ibx+1}}\right)}{2\left(a^2+1\right)}-\frac{(-ia-ibx+1)^{5/2}}{2\left(a^2+1\right)x^2\sqrt{ia+ibx+1}}$$

input
```
Int[1/(E^((3*I)*ArcTan[a + b*x])*x^3),x]
```

output
```
-1/2*(1 - I*a - I*b*x)^(5/2)/((1 + a^2)*x^2*Sqrt[1 + I*a + I*b*x]) - ((3*I
 + 2*a)*b*(-((1 - I*a - I*b*x)^(3/2)/((1 + I*a)*x*Sqrt[1 + I*a + I*b*x]))
+ (3*b*((2*Sqrt[1 - I*a - I*b*x])/((1 + I*a)*Sqrt[1 + I*a + I*b*x]) - ((2*
I)*Sqrt[I + a]*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sq
rt[1 - I*a - I*b*x])])/(I - a)^(3/2)))/(I - a)))/(2*(1 + a^2))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{i\left(-a\,b^{3}x^{3}-6i b^{3}x^{3}-a^{2}b^{2}x^{2}-12ia\,b^{2}x^{2}+a^{3}bx-6ia^{2}bx+a^{4}+b^{2}x^{2}+abx-6ibx+2a^{2}+1\right)}{2x^{2}\left(-i+a\right)^{3}\sqrt{b^{2}x^{2}+2abx+a^{2}+1}}+\frac{b^{2}\left(-\frac{3\left(2a^{2}+ia+3\right)\ln\left(\frac{2a^{2}+2+2abx+2}{}\right.}{(i-a)\sqrt{a}}\right.}{}$
default	Expression too large to display

input
```
int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*(-a*b^3*x^3-a^2*b^2*x^2+a^3*b*x-6*I*b^3*x^3+a^4+b^2*x^2-12*I*a*b^2*
x^2+a*b*x-6*I*a^2*b*x+2*a^2-6*I*b*x+1)/x^2/(-I+a)^3/(b^2*x^2+2*a*b*x+a^2+1
)^(1/2)+1/2/(a^3-3*a-3*I*a^2+I)*b^2*(-3*(I*a+2*a^2+3)/(I-a)/(a^2+1)^(1/2)*
ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)-8*I*
(1+I*a)/b/(I-a)/(x-(I-a)/b)*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 574 vs. 2(180) = 360.

Time = 0.16 (sec) , antiderivative size = 574, normalized size of antiderivative = 2.17

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^3}\,dx$$

$$= \frac{(i\,a-14)b^{3}x^{3}+(i\,a^{2}-13\,a+14i)b^{2}x^{2}-3\left((a^{3}-3i\,a^{2}-3\,a+i)bx^{3}+(a^{4}-4i\,a^{3}-6\,a^{2}+4i\,a+1)x^{2}\right)}{}$$

input
```
integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
1/2*((I*a - 14)*b^3*x^3 + (I*a^2 - 13*a + 14*I)*b^2*x^2 - 3*((a^3 - 3*I*a^
2 - 3*a + I)*b*x^3 + (a^4 - 4*I*a^3 - 6*a^2 + 4*I*a + 1)*x^2)*sqrt((4*a^2
+ 12*I*a - 9)*b^4/(a^8 - 6*I*a^7 - 14*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 -
 6*I*a - 1))*log(-((2*a + 3*I)*b^3*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(
2*a + 3*I)*b^2 + (a^5 - 3*I*a^4 - 2*a^3 - 2*I*a^2 - 3*a + I)*sqrt((4*a^2 +
 12*I*a - 9)*b^4/(a^8 - 6*I*a^7 - 14*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 -
6*I*a - 1)))/((2*a + 3*I)*b^2)) + 3*((a^3 - 3*I*a^2 - 3*a + I)*b*x^3 + (a^
4 - 4*I*a^3 - 6*a^2 + 4*I*a + 1)*x^2)*sqrt((4*a^2 + 12*I*a - 9)*b^4/(a^8 -
 6*I*a^7 - 14*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 - 6*I*a - 1))*log(-((2*a
+ 3*I)*b^3*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*a + 3*I)*b^2 - (a^5 -
3*I*a^4 - 2*a^3 - 2*I*a^2 - 3*a + I)*sqrt((4*a^2 + 12*I*a - 9)*b^4/(a^8 -
6*I*a^7 - 14*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 - 6*I*a - 1)))/((2*a + 3*I
)*b^2)) + ((I*a - 14)*b^2*x^2 - I*a^3 - 5*(a - I)*b*x - a^2 - I*a - 1)*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1))/((a^3 - 3*I*a^2 - 3*a + I)*b*x^3 + (a^4 -
4*I*a^3 - 6*a^2 + 4*I*a + 1)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2)/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^3}\,dx = \int \frac{\left((bx+a)^2+1\right)^{\frac{3}{2}}}{(i\,bx+i\,a+1)^3x^3}\,dx$$

input
```
integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(((b*x + a)^2 + 1)^(3/2)/((I*b*x + I*a + 1)^3*x^3), x)
```

Giac [F]

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^3}\,dx = \int \frac{\left((bx+a)^2+1\right)^{\frac{3}{2}}}{(i\,bx+i\,a+1)^3x^3}\,dx$$

input
```
integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^3,x, algorithm="giac")
```

output
```
undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^3}\,dx = \int \frac{\left((a+b\,x)^2+1\right)^{3/2}}{x^3\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}\,dx$$

input
```
int(((a + b*x)^2 + 1)^(3/2)/(x^3*(a*1i + b*x*1i + 1)^3),x)
```

output
```
int(((a + b*x)^2 + 1)^(3/2)/(x^3*(a*1i + b*x*1i + 1)^3), x)
```

Reduce [F]

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^3}\,dx = \int \frac{\left(1+(bx+a)^2\right)^{\frac{3}{2}}}{(1+i\,(bx+a))^3\,x^3}dx$$

input
```
int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^3,x)
```

output
```
int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^3,x)
```

3.229 $\int \frac{e^{-3i\arctan(a+bx)}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 339

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^4}\,dx = -\frac{(52-51ia-2a^2)\,b^3\sqrt{1-ia-ibx}}{6(i-a)^4(i+a)\sqrt{1+ia+ibx}}-\frac{(i+a)\sqrt{1-ia-ibx}}{3(i-a)x^3\sqrt{1+ia+ibx}}-\frac{7ib\sqrt{1-ia-ibx}}{6(i-a)^2x^2\sqrt{1+ia+ibx}}+\frac{(19-16ia)b^2\sqrt{1-ia-ibx}}{6(i-a)^3(i+a)x\sqrt{1+ia+ibx}}+\frac{(11i+18a-6ia^2)\,b^3\operatorname{arctanh}\left(\frac{\sqrt{i+a}\sqrt{1+ia+ibx}}{\sqrt{i-a}\sqrt{1-ia-ibx}}\right)}{(i-a)^{9/2}(i+a)^{3/2}}$$

output
```
-1/6*(52-51*I*a-2*a^2)*b^3*(1-I*a-I*b*x)^(1/2)/(I-a)^4/(I+a)/(1+I*a+I*b*x)
^(1/2)-1/3*(I+a)*(1-I*a-I*b*x)^(1/2)/(I-a)/x^3/(1+I*a+I*b*x)^(1/2)-7/6*I*b
*(1-I*a-I*b*x)^(1/2)/(I-a)^2/x^2/(1+I*a+I*b*x)^(1/2)+1/6*(19-16*I*a)*b^2*(
1-I*a-I*b*x)^(1/2)/(I-a)^3/(I+a)/x/(1+I*a+I*b*x)^(1/2)+(11*I+18*a-6*I*a^2)
*b^3*arctanh((I+a)^(1/2)*(1+I*a+I*b*x)^(1/2)/(I-a)^(1/2)/(1-I*a-I*b*x)^(1/
2))/(I-a)^(9/2)/(I+a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.81

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^4}\,dx =$$
$$-\frac{-2(-1-ia)^{7/2}(1-ia)(-i(i+a+bx))^{5/2}-(-1-ia)^{5/2}(3i+4a)bx(-i(i+a+bx))^{5/2}+i(-11+\ldots}{6(-1\ldots}$$

input
```
Integrate[1/(E^((3*I)*ArcTan[a + b*x])*x^4),x]
```

output
```
-1/6*(-2*(-1 - I*a)^(7/2)*(1 - I*a)*((-I)*(I + a + b*x))^(5/2) - (-1 - I*a
)^(5/2)*(3*I + 4*a)*b*x*((-I)*(I + a + b*x))^(5/2) + I*(-11 + (18*I)*a + 6
*a^2)*b^2*x^2*((-I)*Sqrt[-1 - I*a]*Sqrt[(-I)*(I + a + b*x)]*(1 + a^2 + (5*
I)*b*x + a*b*x) - 6*Sqrt[-1 + I*a]*b*x*Sqrt[1 + I*a + I*b*x]*ArcTanh[(Sqrt
[-1 - I*a]*Sqrt[(-I)*(I + a + b*x)])/(Sqrt[-1 + I*a]*Sqrt[1 + I*a + I*b*x]
)]))/((-1 - I*a)^(5/2)*(1 + a^2)^2*x^3*Sqrt[1 + I*a + I*b*x])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5618, 109, 27, 168, 25, 27, 168, 27, 169, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^4}dx$$
$$\Big\downarrow \text{5618}$$
$$\int \frac{(-ia-ibx+1)^{3/2}}{x^4(ia+ibx+1)^{3/2}}dx$$
$$\Big\downarrow \text{109}$$
$$-\frac{\int \frac{b(7(a+i)+6bx)}{x^3\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}dx}{3(1+ia)}-\frac{(a+i)\sqrt{-ia-ibx+1}}{3(-a+i)x^3\sqrt{ia+ibx+1}}$$

$$\Big\downarrow \; 27$$

$$-\frac{b\int \frac{7(a+i)+6bx}{x^3\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}dx}{3(1+ia)}-\frac{(a+i)\sqrt{-ia-ibx+1}}{3(-a+i)x^3\sqrt{ia+ibx+1}}$$

$$\Big\downarrow \; 168$$

$$-\frac{b\left(\frac{7\sqrt{-ia-ibx+1}}{2(-a+i)x^2\sqrt{ia+ibx+1}}-\frac{\int -\frac{b\left(-16a^2-35ia-14(a+i)bx+19\right)}{x^2\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}dx}{2(a^2+1)}\right)}{3(1+ia)}-\frac{(a+i)\sqrt{-ia-ibx+1}}{3(-a+i)x^3\sqrt{ia+ibx+1}}$$

$$\Big\downarrow \; 25$$

$$-\frac{b\left(\frac{\int \frac{b\left(-16a^2-35ia-14(a+i)bx+19\right)}{x^2\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}dx}{2(a^2+1)}+\frac{7\sqrt{-ia-ibx+1}}{2(-a+i)x^2\sqrt{ia+ibx+1}}\right)}{3(1+ia)}-\frac{(a+i)\sqrt{-ia-ibx+1}}{3(-a+i)x^3\sqrt{ia+ibx+1}}$$

$$\Big\downarrow \; 27$$

$$-\frac{b\left(\frac{b\int \frac{-16a^2-35ia-14(a+i)bx+19}{x^2\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}dx}{2(a^2+1)}+\frac{7\sqrt{-ia-ibx+1}}{2(-a+i)x^2\sqrt{ia+ibx+1}}\right)}{3(1+ia)}-\frac{(a+i)\sqrt{-ia-ibx+1}}{3(-a+i)x^3\sqrt{ia+ibx+1}}$$

$$\Big\downarrow \; 168$$

$$-\frac{b\left(\frac{b\left(-\frac{\int \frac{b\left(3(a+i)\left(-6a^2-18ia+11\right)+\left(-16a^2-35ia+19\right)bx\right)}{x\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}dx}{a^2+1}-\frac{(16a+19i)\sqrt{-ia-ibx+1}}{(-a+i)x\sqrt{ia+ibx+1}}\right)}{2(a^2+1)}+\frac{7\sqrt{-ia-ibx+1}}{2(-a+i)x^2\sqrt{ia+ibx+1}}\right)}{3(1+ia)}-\frac{(a+i)\sqrt{-ia-ibx+1}}{3(-a+i)x^3\sqrt{ia+ibx+1}}$$

$$\Big\downarrow \; 27$$

$$-\frac{b\left(\frac{b\left(-\frac{b\int \frac{3(a+i)\left(-6a^2-18ia+11\right)+\left(-16a^2-35ia+19\right)bx}{x\sqrt{-ia-ibx+1}(ia+ibx+1)^{3/2}}dx}{a^2+1}-\frac{(16a+19i)\sqrt{-ia-ibx+1}}{(-a+i)x\sqrt{ia+ibx+1}}\right)}{2(a^2+1)}+\frac{7\sqrt{-ia-ibx+1}}{2(-a+i)x^2\sqrt{ia+ibx+1}}\right)}{3(1+ia)}-\frac{(a+i)\sqrt{-ia-ibx+1}}{3(-a+i)x^3\sqrt{ia+ibx+1}}$$

$$\Big\downarrow \ 169$$

$$-\frac{b\left(\frac{b\left(-\frac{b\left(\frac{\int -\frac{3\left(6ia^3-24a^2-29ia+11\right)b}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{(-a+i)b}-\frac{\left(2ia^3-53a^2-103ia+52\right)\sqrt{-ia-ibx+1}}{(-a+i)\sqrt{ia+ibx+1}}\right)}{a^2+1}-\frac{(16a+19i)\sqrt{-ia-ibx+1}}{(-a+i)x\sqrt{ia+ibx+1}}\right)}{2(a^2+1)}+\frac{7\sqrt{-ia-ibx+1}}{2(-a+i)x^2\sqrt{ia+ibx+1}}\right)}{3(1+ia)}-\frac{(a+i)\sqrt{-ia-ibx+1}}{3(-a+i)x^3\sqrt{ia+ibx+1}}$$

$$\Big\downarrow \ 27$$

$$-\frac{b\left(\frac{b\left(-\frac{b\left(-\frac{3\left(6ia^3-24a^2-29ia+11\right)\int \frac{1}{x\sqrt{-ia-ibx+1}\sqrt{ia+ibx+1}}dx}{-a+i}-\frac{\left(2ia^3-53a^2-103ia+52\right)\sqrt{-ia-ibx+1}}{(-a+i)\sqrt{ia+ibx+1}}\right)}{a^2+1}-\frac{(16a+19i)\sqrt{-ia-ibx+1}}{(-a+i)x\sqrt{ia+ibx+1}}\right)}{2(a^2+1)}+\frac{7\sqrt{\cdots}}{2(-a+i\cdots}}{3(1+ia)}$$

$$\frac{(a+i)\sqrt{-ia-ibx+1}}{3(-a+i)x^3\sqrt{ia+ibx+1}}$$

$$\Big\downarrow \ 104$$

$$-\frac{b\left(\frac{b\left(-\frac{b\left(-\frac{6\left(6ia^3-24a^2-29ia+11\right)\int \frac{1}{-ia+\frac{(1-ia)(ia+ibx+1)}{-ia-ibx+1}-1}d\frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}}{-a+i}-\frac{\left(2ia^3-53a^2-103ia+52\right)\sqrt{-ia-ibx+1}}{(-a+i)\sqrt{ia+ibx+1}}\right)}{a^2+1}-\frac{(16a+19i)\sqrt{-ia-ibx+1}}{(-a+i)x\sqrt{ia+ibx+1}}\right)}{2(a^2+1)}\right)}{3(1+ia)}$$

$$\frac{(a+i)\sqrt{-ia-ibx+1}}{3(-a+i)x^3\sqrt{ia+ibx+1}}$$

$$\Big\downarrow \; 221$$

$$-\frac{b\left(\frac{b\left(-\frac{b\left(\frac{6i\left(6ia^3-24a^2-29ia+11\right)\text{arctanh}\left(\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-a+i}\sqrt{-ia-ibx+1}}\right)}{(-a+i)^{3/2}\sqrt{a+i}}-\frac{\left(2ia^3-53a^2-103ia+52\right)\sqrt{-ia-ibx+1}}{(-a+i)\sqrt{ia+ibx+1}}\right)}{a^2+1}-\frac{(16a+19i)\sqrt{-ia-ibx+1}}{(-a+i)x\sqrt{ia+ibx+1}}\right)}{2(a^2+1)}+\frac{7\cdots}{2(-a\cdots}\right)}{3(1+ia)}$$

$$\frac{(a+i)\sqrt{-ia-ibx+1}}{3(-a+i)x^3\sqrt{ia+ibx+1}}$$

input
```
Int[1/(E^((3*I)*ArcTan[a + b*x])*x^4),x]
```

output
```
-1/3*((I + a)*Sqrt[1 - I*a - I*b*x])/((I - a)*x^3*Sqrt[1 + I*a + I*b*x]) -
 (b*((7*Sqrt[1 - I*a - I*b*x])/(2*(I - a)*x^2*Sqrt[1 + I*a + I*b*x]) + (b*
(-(((19*I + 16*a)*Sqrt[1 - I*a - I*b*x])/((I - a)*x*Sqrt[1 + I*a + I*b*x])
) - (b*(-(((52 - (103*I)*a - 53*a^2 + (2*I)*a^3)*Sqrt[1 - I*a - I*b*x])/((
I - a)*Sqrt[1 + I*a + I*b*x])) + ((6*I)*(11 - (29*I)*a - 24*a^2 + (6*I)*a^
3)*ArcTanh[(Sqrt[I + a]*Sqrt[1 + I*a + I*b*x])/(Sqrt[I - a]*Sqrt[1 - I*a -
 I*b*x])])/((I - a)^(3/2)*Sqrt[I + a])))/(1 + a^2)))/(2*(1 + a^2))))/(3*(1
 + I*a))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{i\left(2a^2b^4x^4+27ia\,b^4x^4+2a^3b^3x^3+45ia^2b^3x^3+9ia^3b^2x^2-28b^4x^4+2a^5bx-9ia^4bx-58a\,b^3x^3-9ib^3x^3+2a^6-26a^2b^2x^2+9ia\,b^2x^2+\right.}{6x^3(i+a)(-i+a)^4\sqrt{b^2x^2+2abx+a^2+1}}$
default	Expression too large to display

input
```
int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*I*(-9*I*a^4*b*x+2*a^2*b^4*x^4+9*I*a*b^2*x^2+2*a^3*b^3*x^3+45*I*a^2*b^
3*x^3-28*b^4*x^4+9*I*a^3*b^2*x^2-9*I*b^3*x^3+2*a^5*b*x-58*a*b^3*x^3+27*I*a
*b^4*x^4+2*a^6-26*a^2*b^2*x^2-9*I*b*x+4*a^3*b*x+6*a^4-26*b^2*x^2-18*I*a^2*
b*x+2*a*b*x+6*a^2+2)/x^3/(I+a)/(-I+a)^4/(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-1/2/
(I+a)/(a^4-6*a^2-4*I*a^3+1+4*I*a)*b^3*(-(12*I*a^2+6*a^3+11*I+7*a)/(I-a)/(a
^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1
/2))/x)+8*(a^2+1)/b/(I-a)/(x-(I-a)/b)*((x-(I-a)/b)^2*b^2+2*I*b*(x-(I-a)/b)
)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 839 vs. 2(223) = 446.

Time = 0.16 (sec) , antiderivative size = 839, normalized size of antiderivative = 2.47

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
1/6*((-2*I*a^2 + 51*a + 52*I)*b^4*x^4 + (-2*I*a^3 + 49*a^2 + I*a + 52)*b^3
*x^3 + 3*sqrt((36*a^4 + 216*I*a^3 - 456*a^2 - 396*I*a + 121)*b^6/(a^12 - 6
*I*a^11 - 12*a^10 + 2*I*a^9 - 27*a^8 + 36*I*a^7 + 36*I*a^5 + 27*a^4 + 2*I*
a^3 + 12*a^2 - 6*I*a - 1))*((a^5 - 3*I*a^4 - 2*a^3 - 2*I*a^2 - 3*a + I)*b*
x^4 + (a^6 - 4*I*a^5 - 5*a^4 - 5*a^2 + 4*I*a + 1)*x^3)*log(-((6*a^2 + 18*I
*a - 11)*b^4*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(6*a^2 + 18*I*a - 11)*b
^3 + (a^7 - 3*I*a^6 - a^5 - 5*I*a^4 - 5*a^3 - I*a^2 - 3*a + I)*sqrt((36*a^
4 + 216*I*a^3 - 456*a^2 - 396*I*a + 121)*b^6/(a^12 - 6*I*a^11 - 12*a^10 +
2*I*a^9 - 27*a^8 + 36*I*a^7 + 36*I*a^5 + 27*a^4 + 2*I*a^3 + 12*a^2 - 6*I*a
 - 1)))/((6*a^2 + 18*I*a - 11)*b^3)) - 3*sqrt((36*a^4 + 216*I*a^3 - 456*a^
2 - 396*I*a + 121)*b^6/(a^12 - 6*I*a^11 - 12*a^10 + 2*I*a^9 - 27*a^8 + 36*
I*a^7 + 36*I*a^5 + 27*a^4 + 2*I*a^3 + 12*a^2 - 6*I*a - 1))*((a^5 - 3*I*a^4
 - 2*a^3 - 2*I*a^2 - 3*a + I)*b*x^4 + (a^6 - 4*I*a^5 - 5*a^4 - 5*a^2 + 4*I
*a + 1)*x^3)*log(-((6*a^2 + 18*I*a - 11)*b^4*x - sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)*(6*a^2 + 18*I*a - 11)*b^3 - (a^7 - 3*I*a^6 - a^5 - 5*I*a^4 - 5*a^
3 - I*a^2 - 3*a + I)*sqrt((36*a^4 + 216*I*a^3 - 456*a^2 - 396*I*a + 121)*b
^6/(a^12 - 6*I*a^11 - 12*a^10 + 2*I*a^9 - 27*a^8 + 36*I*a^7 + 36*I*a^5 + 2
7*a^4 + 2*I*a^3 + 12*a^2 - 6*I*a - 1)))/((6*a^2 + 18*I*a - 11)*b^3)) + ((-
2*I*a^2 + 51*a + 52*I)*b^3*x^3 - 2*I*a^5 + (16*a^2 + 3*I*a + 19)*b^2*x^2 -
 2*a^4 - 4*I*a^3 - 7*(a^3 - I*a^2 + a - I)*b*x - 4*a^2 - 2*I*a - 2)*sqr...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3i \arctan(a+bx)}}{x^4} \, dx = \text{Timed out}$$

input
```
integrate(1/(1+I*(b*x+a))**3*(1+(b*x+a)**2)**(3/2)/x**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^4}\,dx = \int \frac{\left((bx+a)^2+1\right)^{\frac{3}{2}}}{(i\,bx+i\,a+1)^3x^4}\,dx$$

input
```
integrate(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^4,x, algorithm="giac")
```

output
```
undef
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^4}\,dx = \int \frac{\left((a+b\,x)^2+1\right)^{3/2}}{x^4\,(1+a\,1\mathrm{i}+b\,x\,1\mathrm{i})^3}\,dx$$

input
```
int(((a + b*x)^2 + 1)^(3/2)/(x^4*(a*1i + b*x*1i + 1)^3),x)
```

output
```
int(((a + b*x)^2 + 1)^(3/2)/(x^4*(a*1i + b*x*1i + 1)^3), x)
```

Reduce [F]

$$\int \frac{e^{-3i\arctan(a+bx)}}{x^4}\,dx = \int \frac{\left(1+(bx+a)^2\right)^{\frac{3}{2}}}{\left(1+i\,(bx+a)\right)^3 x^4}dx$$

input
```
int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^4,x)
```

output
```
int(1/(1+I*(b*x+a))^3*(1+(b*x+a)^2)^(3/2)/x^4,x)
```

3.230 $\int e^{\frac{1}{2}i\arctan(a+bx)}x^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 391

$$\begin{aligned}\int e^{\frac{1}{2}i\arctan(a+bx)}x^2\,dx &= -\frac{(3i+4a-8ia^2)\,(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{8b^3} \\ &\quad -\frac{(i+8a)(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{12b^3} \\ &\quad +\frac{x(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{3b^2} \\ &\quad +\frac{(3i+4a-8ia^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3} \\ &\quad -\frac{(3i+4a-8ia^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3} \\ &\quad +\frac{(3i+4a-8ia^2)\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\right)}\right)}{8\sqrt{2}b^3}\end{aligned}$$

output
```
-1/8*(3*I+4*a-8*I*a^2)*(1-I*a-I*b*x)^(3/4)*(1+I*a+I*b*x)^(1/4)/b^3-1/12*(I
+8*a)*(1-I*a-I*b*x)^(3/4)*(1+I*a+I*b*x)^(5/4)/b^3+1/3*x*(1-I*a-I*b*x)^(3/4
)*(1+I*a+I*b*x)^(5/4)/b^2+1/16*(3*I+4*a-8*I*a^2)*arctan(1-2^(1/2)*(1-I*a-I
*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4))*2^(1/2)/b^3-1/16*(3*I+4*a-8*I*a^2)*arctan
(1+2^(1/2)*(1-I*a-I*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4))*2^(1/2)/b^3+1/16*(3*I+
4*a-8*I*a^2)*arctanh(2^(1/2)*(1-I*a-I*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4)/(1+(1
-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2)))*2^(1/2)/b^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.31

$$\int e^{\frac{1}{2}i\arctan(a+bx)}x^2\,dx$$
$$= \frac{(-i(i+a+bx))^{3/4}\left(-i\sqrt[4]{1+ia+ibx}(1+8a^2+5ibx-4b^2x^2+a(-7i+4bx))+2i\sqrt[4]{2}(-3+4ia+8a^2\right.}{12b^3}$$

input
```
Integrate[E^((I/2)*ArcTan[a + b*x])*x^2,x]
```

output
```
(((-I)*(I + a + b*x))^(3/4)*((-I)*(1 + I*a + I*b*x)^(1/4)*(1 + 8*a^2 + (5*
I)*b*x - 4*b^2*x^2 + a*(-7*I + 4*b*x)) + (2*I)*2^(1/4)*(-3 + (4*I)*a + 8*a
^2)*Hypergeometric2F1[-1/4, 3/4, 7/4, (-1/2*I)*(I + a + b*x)]))/(12*b^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.89 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5618, 101, 27, 90, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{\frac{1}{2}i\arctan(a+bx)}\,dx$$

$\downarrow$ 5618

$$\int \frac{x^2 \sqrt[4]{ia+ibx+1}}{\sqrt[4]{-ia-ibx+1}}dx$$

$\downarrow$ 101

$$\frac{\int -\frac{\sqrt[4]{ia+ibx+1}(2a^2+(8a+i)bx+2)}{2\sqrt[4]{-ia-ibx+1}}dx}{3b^2}+\frac{x(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{3b^2}$$

$\downarrow$ 27

$$\frac{x(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{3b^2}-\frac{\int \frac{\sqrt[4]{ia+ibx+1}(2(a^2+1)+(8a+i)bx)}{\sqrt[4]{-ia-ibx+1}}dx}{6b^2}$$

$\downarrow$ 90

$$\frac{x(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{3b^2}-\frac{\frac{3}{4}\left(-8a^2-4ia+3\right)\int \frac{\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-ia-ibx+1}}dx+\frac{(8a+i)(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{2b}}{6b^2}$$

$\downarrow$ 60

$$\frac{x(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{3b^2}-\frac{\frac{3}{4}\left(-8a^2-4ia+3\right)\left(\frac{1}{2}\int \frac{1}{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}dx+\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}\right)+\frac{(8a+i)(-ia-ibx+1)^{3/4}(i}{2b}}{6b^2}$$

$\downarrow$ 73

$$\frac{x(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{3b^2}-\frac{\frac{3}{4}\left(-8a^2-4ia+3\right)\left(\frac{2i\int \frac{\sqrt{-ia-ibx+1}}{(ia+ibx+1)^{3/4}}d\sqrt[4]{-ia-ibx+1}}{b}+\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}\right)+\frac{(8a+i)(-ia-ibx+1)^{3/4}(ia+}{2b}}{6b^2}$$

$\downarrow$ 854

$$\frac{x(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{3b^2}-\frac{\frac{3}{4}\left(-8a^2-4ia+3\right)\left(\frac{2i\int \frac{\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{b}+\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}\right)+\frac{(8a+i)(-ia-ibx+1)^{3/4}(ia+}{2b}}{6b^2}$$

$$\Big\downarrow \; 826$$

$$\frac{x(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{3b^2} - \frac{\frac{3}{4}\left(-8a^2-4ia+3\right)\left(\frac{2i\left(\frac{1}{2}\int \frac{\sqrt{-ia-ibx+1}+1}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}-\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{b}+\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{\cdots}}{b}\cdots\right.}{6b^2}$$

$$\Big\downarrow \; 1476$$

$$\frac{x(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{3b^2} - \frac{\frac{3}{4}\left(-8a^2-4ia+3\right)\left(\frac{2i\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\int \frac{1}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx\cdots}}{\sqrt[4]{ia+ibx+\cdots}}}\cdots\right.\right.}{b}\cdots\right.}{\cdots}$$

$$\Big\downarrow \; 1082$$

$$\frac{x(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{3b^2} - \frac{\frac{3}{4}\left(-8a^2-4ia+3\right)\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\cdots\right)}{b}\cdots\right.}{6b^2}$$

$$\Big\downarrow \; 217$$

$$\frac{x(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{3b^2} - \frac{\frac{3}{4}\left(-8a^2-4ia+3\right)\left(\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia\cdots}}{\sqrt[4]{ia-\cdots}}\right)}{b}\cdots\right.}{6b^2}$$

$\downarrow$ 1479

$$\frac{x(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{3b^2}-$$

$$\frac{3}{4}\left(-8a^2-4ia+3\right)\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1 \ldots}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+ \ldots}}{\sqrt[4]{ia+ibx+1}}}}{2\sqrt{2}} \ldots\right.\right.}{b \ldots}\right.$$

$\downarrow$ 25

$$\frac{x(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{3b^2}-$$

$$\frac{3}{4}\left(-8a^2-4ia+3\right)\left(\frac{2i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right) \ldots}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+ \ldots}}{\sqrt[4]{ia+ibx+1}}}}{2\sqrt{2}} \ldots\right.\right.}{b \ldots}\right.$$

$\downarrow$ 27

$$\frac{x(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{3b^2}-\frac{\frac{3}{4}(-8a^2-4ia+3)\left(\frac{2i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\cdots}\cdots\right.\right.}{b}\cdots\right)}{\cdots}$$

$\downarrow$ 1103

$$\frac{x(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{3b^2}-\frac{\frac{3}{4}(-8a^2-4ia+3)\left(\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-\cdots}}{\sqrt[4]{i\cdots}}\cdots\right)}{2\sqrt{2}}\cdots\right)\right)}{b}\cdots\right)}{\cdots}$$

input
```
Int[E^((I/2)*ArcTan[a + b*x])*x^2,x]
```

output
```
(x*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(5/4))/(3*b^2) - (((I + 8*a)*
(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(5/4))/(2*b) + (3*(3 - (4*I)*a -
 8*a^2)*((I*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/b + ((2*I)*((
-(ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sq
rt[2]) + ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1
/4)]/Sqrt[2])/2 + (Log[1 + Sqrt[1 - I*a - I*b*x] - (Sqrt[2]*(1 - I*a - I*b
*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/(2*Sqrt[2]) - Log[1 + Sqrt[1 - I*a - I
*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/(2*Sqrt
[2]))/2))/b))/4)/(6*b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [F]

$$\int \sqrt{\frac{1+i\,(bx+a)}{\sqrt{1+(bx+a)^2}}}\,x^2dx$$

input
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x^2,x)
```

output
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 554 vs. 2(265) = 530.

Time = 0.15 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.42

$$\int e^{\frac{1}{2}i\arctan(a+bx)}x^2\,dx$$
$$= \frac{3\,b^3\sqrt{\frac{64i\,a^4-64\,a^3-64i\,a^2+24\,a+9i}{b^6}}\log\left(\frac{i\,b^3\sqrt{\frac{64i\,a^4-64\,a^3-64i\,a^2+24\,a+9i}{b^6}}+(8\,a^2+4i\,a-3)\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{8\,a^2+4i\,a-3}\right)-3\,b^3\sqrt{\frac{64i\,a^4\cdots}{}}}{}$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x^2,x, algorithm="fric
as")
```

output
```
1/48*(3*b^3*sqrt((64*I*a^4 - 64*a^3 - 64*I*a^2 + 24*a + 9*I)/b^6)*log((I*b
^3*sqrt((64*I*a^4 - 64*a^3 - 64*I*a^2 + 24*a + 9*I)/b^6) + (8*a^2 + 4*I*a
- 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*a^2 + 4*I
*a - 3)) - 3*b^3*sqrt((64*I*a^4 - 64*a^3 - 64*I*a^2 + 24*a + 9*I)/b^6)*log
((-I*b^3*sqrt((64*I*a^4 - 64*a^3 - 64*I*a^2 + 24*a + 9*I)/b^6) + (8*a^2 +
4*I*a - 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*a^2
 + 4*I*a - 3)) + 3*b^3*sqrt((-64*I*a^4 + 64*a^3 + 64*I*a^2 - 24*a - 9*I)/b
^6)*log((I*b^3*sqrt((-64*I*a^4 + 64*a^3 + 64*I*a^2 - 24*a - 9*I)/b^6) + (8
*a^2 + 4*I*a - 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))
/(8*a^2 + 4*I*a - 3)) - 3*b^3*sqrt((-64*I*a^4 + 64*a^3 + 64*I*a^2 - 24*a -
 9*I)/b^6)*log((-I*b^3*sqrt((-64*I*a^4 + 64*a^3 + 64*I*a^2 - 24*a - 9*I)/b
^6) + (8*a^2 + 4*I*a - 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x +
a + I)))/(8*a^2 + 4*I*a - 3)) + 2*(8*b^3*x^3 - 2*I*b^2*x^2 + 8*a^3 + (8*I*
a - 1)*b*x + 34*I*a^2 - 37*a - 11*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1)/(b*x + a + I)))/b^3
```

Sympy [F]

$$\int e^{\frac{1}{2}i\arctan(a+bx)}x^2\,dx=\int x^2\sqrt{\frac{i\,(a+bx-i)}{\sqrt{a^2+2abx+b^2x^2+1}}}\,dx$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2)*x**2,x)
```

output
```
Integral(x**2*sqrt(I*(a + b*x - I)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)),
x)
```

Maxima [F]

$$\int e^{\frac{1}{2}i\arctan(a+bx)}x^2\,dx=\int x^2\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}\,dx$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x^2,x, algorithm="maxi
ma")
```

output
```
integrate(x^2*sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{1}{2}i\arctan(a+bx)}x^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x^2,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{2}i\arctan(a+bx)}x^2\,dx = \int x^2\sqrt{\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}}\,dx$$

input
```
int(x^2*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2),x)
```

output
```
int(x^2*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int e^{\frac{1}{2}i\arctan(a+bx)}x^2\,dx = \int \sqrt{\frac{1+i\,(bx+a)}{\sqrt{1+(bx+a)^2}}}\,x^2dx$$

input
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x^2,x)
```

output
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x^2,x)
```

3.231 $\int e^{\frac{1}{2} i \arctan(a+bx)} x \, dx$

Optimal result

Integrand size = 16, antiderivative size = 314

$$\int e^{\frac{1}{2} i \arctan(a+bx)} x \, dx = \frac{(1-4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2} + \frac{(1-ia-ibx)^{3/4}(1+ia+ibx)^{5/4}}{2b^2} - \frac{(1-4ia)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2} + \frac{(1-4ia)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2} - \frac{(1-4ia)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\right)}\right)}{4\sqrt{2}b^2}$$

output
```
1/4*(1-4*I*a)*(1-I*a-I*b*x)^(3/4)*(1+I*a+I*b*x)^(1/4)/b^2+1/2*(1-I*a-I*b*x
)^(3/4)*(1+I*a+I*b*x)^(5/4)/b^2-1/8*(1-4*I*a)*arctan(1-2^(1/2)*(1-I*a-I*b*
x)^(1/4)/(1+I*a+I*b*x)^(1/4))*2^(1/2)/b^2+1/8*(1-4*I*a)*arctan(1+2^(1/2)*(
1-I*a-I*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4))*2^(1/2)/b^2-1/8*(1-4*I*a)*arctanh(
2^(1/2)*(1-I*a-I*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4)/(1+(1-I*a-I*b*x)^(1/2)/(1+
I*a+I*b*x)^(1/2)))*2^(1/2)/b^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.26

$$\int e^{\frac{1}{2}i\arctan(a+bx)}x\,dx = \frac{(-i(i+a+bx))^{3/4}\left(3(1+ia+ibx)^{5/4}+2\sqrt[4]{2}(1-4ia)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{1}{2}i(i+a+bx)\right)\right)}{6b^2}$$

input
```
Integrate[E^((I/2)*ArcTan[a + b*x])*x,x]
```

output
```
(((-I)*(I + a + b*x))^(3/4)*(3*(1 + I*a + I*b*x)^(5/4) + 2*2^(1/4)*(1 - (4
*I)*a)*Hypergeometric2F1[-1/4, 3/4, 7/4, (-1/2*I)*(I + a + b*x)]))/(6*b^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.77 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.13, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {5618, 90, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\frac{1}{2}i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{x\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-ia-ibx+1}}dx$$

$$\Big\downarrow \text{90}$$

$$\frac{(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{2b^2}-\frac{(4a+i)\int \frac{\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-ia-ibx+1}}dx}{4b}$$

$$\Big\downarrow \text{60}$$

$$\frac{(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{2b^2}-\frac{(4a+i)\left(\frac{1}{2}\int \frac{1}{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}dx+\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}\right)}{4b}$$

$$\Big\downarrow \; 73$$

$$\frac{(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{2b^2}-\frac{(4a+i)\left(\frac{2i\int \frac{\sqrt{-ia-ibx+1}}{(ia+ibx+1)^{3/4}}d\sqrt[4]{-ia-ibx+1}}{b}+\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}\right)}{4b}$$

$$\Big\downarrow \; 854$$

$$\frac{(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{2b^2}-\frac{(4a+i)\left(\frac{2i\int \frac{\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{b}+\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}\right)}{4b}$$

$$\Big\downarrow \; 826$$

$$\frac{(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{2b^2}-\frac{(4a+i)\left(\frac{2i\left(\frac{1}{2}\int \frac{\sqrt{-ia-ibx+1}+1}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}-\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{b}+\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+\cdots}}{b}\cdots\right)}{4b}$$

$$\Big\downarrow \; 1476$$

$$\frac{(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{2b^2}-\frac{(4a+i)\left(\frac{2i\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\int \frac{1}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-\cdots}}{\sqrt[4]{i\cdots}}\cdots\right.\right.}{b}\cdots\right)}{4b}$$

$$\Big\downarrow \; 1082$$

$$\frac{(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{2b^2}-\frac{(4a+i)\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\sqrt{-ia-ib}}{-ia-ibx+}}{b}\right.}{4b}$$

$\downarrow$ 217

$$\frac{(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{2b^2}-\frac{(4a+i)\left(\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{b}\right.}{4b}$$

$\downarrow$ 1479

$$\frac{(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{2b^2}-\frac{(4a+i)\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-i}}{\sqrt[4]{ia}}}{2\sqrt{2}}\right.\right.}{b}\right.}{4}$$

$\downarrow$ 25

$$\frac{(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{2b^2}-$$

$$\frac{(4a+i)\left(\frac{2i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia}}{\sqrt[4]{ia}}}{2\sqrt{2}}\right.\right.}{b}\right.}{4}$$

$\downarrow$ 27

$$\frac{(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{2b^2}-$$

$$\frac{(4a+i)\left(\frac{2i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-i}}{\sqrt[4]{ia}}\right.\right.}{b}\right.}{4}$$

$\downarrow$ 1103

$$\frac{(-ia-ibx+1)^{3/4}(ia+ibx+1)^{5/4}}{2b^2}-\frac{(4a+i)\left(\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+}}{\sqrt[4]{ia+ibx+1}}\right.}{2\sqrt{2}}\right.\right.}{b}\right.}{4b}$$

input
```
Int[E^((I/2)*ArcTan[a + b*x])*x,x]
```

output
```
((1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(5/4))/(2*b^2) - ((I + 4*a)*((I
*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/b + ((2*I)*((-(ArcTan[1
- (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2]) + Ar
cTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2
])/2 + (Log[1 + Sqrt[1 - I*a - I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/
(1 + I*a + I*b*x)^(1/4)]/(2*Sqrt[2]) - Log[1 + Sqrt[1 - I*a - I*b*x] + (Sq
rt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/(2*Sqrt[2]))/2))/b
))/(4*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [F]

$$\int \sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}\, x dx$$

input
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x,x)
```

output
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x,x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.32

$$\int e^{\frac{1}{2}i\arctan(a+bx)}x\,dx = -\frac{b^2\sqrt{\frac{16i\,a^2-8\,a-i}{b^4}}\log\left(\frac{i\,b^2\sqrt{\frac{16i\,a^2-8\,a-i}{b^4}}+(4\,a+i)\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{4\,a+i}\right)-b^2\sqrt{\frac{16i\,a^2-8\,a-i}{b^4}}\log\left(\frac{-i\,b^2\sqrt{\frac{16i\,a^2-8\,a-i}{b^4}}+(4\,a}{4}\right.}{}$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x,x, algorithm="fricas
")
```

output
```
-1/8*(b^2*sqrt((16*I*a^2 - 8*a - I)/b^4)*log((I*b^2*sqrt((16*I*a^2 - 8*a -
 I)/b^4) + (4*a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I
)))/(4*a + I)) - b^2*sqrt((16*I*a^2 - 8*a - I)/b^4)*log((-I*b^2*sqrt((16*I
*a^2 - 8*a - I)/b^4) + (4*a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/
(b*x + a + I)))/(4*a + I)) + b^2*sqrt((-16*I*a^2 + 8*a + I)/b^4)*log((I*b^
2*sqrt((-16*I*a^2 + 8*a + I)/b^4) + (4*a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)/(b*x + a + I)))/(4*a + I)) - b^2*sqrt((-16*I*a^2 + 8*a + I)/b
^4)*log((-I*b^2*sqrt((-16*I*a^2 + 8*a + I)/b^4) + (4*a + I)*sqrt(I*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(4*a + I)) - 2*(2*b^2*x^2 - 2*a
^2 - I*b*x - 5*I*a + 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a
+ I)))/b^2
```

Sympy [F]

$$\int e^{\frac{1}{2}i\arctan(a+bx)}x\,dx = \int x\sqrt{\frac{i\,(a+bx-i)}{\sqrt{a^2+2abx+b^2x^2+1}}}\,dx$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2)*x,x)
```

output
```
Integral(x*sqrt(I*(a + b*x - I)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
```

Maxima [F]

$$\int e^{\frac{1}{2}i\arctan(a+bx)}x\,dx = \int x\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}\,dx$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x,x, algorithm="maxima
")
```

output
```
integrate(x*sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{1}{2}i\arctan(a+bx)}x\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{2}i\arctan(a+bx)}x\,dx = \int x\sqrt{\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}}\,dx$$

input
```
int(x*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2),x)
```

output
```
int(x*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int e^{\frac{1}{2}i\arctan(a+bx)}x\,dx=\int\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}\,xdx$$

input
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x,x)
```

output
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)*x,x)
```

3.232 $\int e^{\frac{1}{2}i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 245

$$\int e^{\frac{1}{2}i\arctan(a+bx)}\,dx = \frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} - \frac{i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} + \frac{i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} - \frac{i\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\right)}\right)}{\sqrt{2}b}$$

output
```
I*(1-I*a-I*b*x)^(3/4)*(1+I*a+I*b*x)^(1/4)/b-1/2*I*arctan(1-2^(1/2)*(1-I*a-
I*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4))*2^(1/2)/b+1/2*I*arctan(1+2^(1/2)*(1-I*a-
I*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4))*2^(1/2)/b-1/2*I*arctanh(2^(1/2)*(1-I*a-I
*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4)/(1+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2)
))*2^(1/2)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.18

$$\int e^{\frac{1}{2}i\arctan(a+bx)}\,dx=-\frac{8ie^{\frac{5}{2}i\arctan(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{4},2,\frac{9}{4},-e^{2i\arctan(a+bx)}\right)}{5b}$$

input
```
Integrate[E^((I/2)*ArcTan[a + b*x]),x]
```

output
```
(((-8*I)/5)*E^(((5*I)/2)*ArcTan[a + b*x])*Hypergeometric2F1[5/4, 2, 9/4, -
E^((2*I)*ArcTan[a + b*x])])/b
```

Rubi [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.22, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {5616, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{1}{2}i\arctan(a+bx)}dx$$

$$\Big\downarrow \text{5616}$$

$$\int \frac{\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-ia-ibx+1}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\int \frac{1}{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}dx+\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}$$

$$\Big\downarrow \text{73}$$

$$\frac{2i\int \frac{\sqrt{-ia-ibx+1}}{(ia+ibx+1)^{3/4}}d\sqrt[4]{-ia-ibx+1}}{b}+\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}$$

$$\Big\downarrow \text{854}$$

$$\frac{2i\int \frac{\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{b}+\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}$$

$$\Big\downarrow \text{826}$$

$$\frac{2i\left(\frac{1}{2}\int \frac{\sqrt{-ia-ibx+1}+1}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}-\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{b}+\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2i\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\int \frac{1}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia}}{\sqrt[4]{ia+}}\right.\right.}{b}$$

$$\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+}}{-ia-ibx+2}\right.}{b}$$

$$\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{b}$$

$$\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}$$

$$\Big\downarrow \text{1479}$$

$$2i\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx}}{\sqrt[4]{ia+ibx}}}{2\sqrt{2}}\right.\right.$$

$$b$$

$$\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}$$

$$\downarrow 25$$

$$2i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx}}{\sqrt[4]{ia+ibx}}}{2\sqrt{2}}\right.\right.$$

$$b$$

$$\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}$$

$$\downarrow 27$$

$$2i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-}}{\sqrt[4]{ia+i}}\right.\right.$$

$$b$$

$$\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}$$

$$\downarrow 1103$$

$$\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+}}{\sqrt[4]{ia+ibx+1}}\right.}{2\sqrt{2}}\right.\right.}{b}$$

$$\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}$$

input
```
Int[E^((I/2)*ArcTan[a + b*x]),x]
```

output
```
(I*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/b + ((2*I)*((-(ArcTan[
1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2]) +
ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt
[2])/2 + (Log[1 + Sqrt[1 - I*a - I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4)
)/(1 + I*a + I*b*x)^(1/4)]/(2*Sqrt[2]) - Log[1 + Sqrt[1 - I*a - I*b*x] + (
Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/(2*Sqrt[2]))/2))
/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5616
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^(I*(n/2))/(1 + I*a*c + I*b*c*x)^(I*(n/2)), x] /; FreeQ[{a, b,
c, n}, x]
```

Maple [F]

$$\int \sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}\,dx$$

input
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

output
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.04

$$\int e^{\frac{1}{2}i\arctan(a+bx)}\,dx = \frac{b\sqrt{\frac{i}{b^2}}\log\left(ib\sqrt{\frac{i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{bx+a+i}}\right)-b\sqrt{\frac{i}{b^2}}\log\left(-ib\sqrt{\frac{i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{bx+a+i}}\right)+b\sqrt{-\frac{i}{b^2}}\log\left(\cdots\right.}{\cdots}$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(b*sqrt(I/b^2)*log(I*b*sqrt(I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)/(b*x + a + I))) - b*sqrt(I/b^2)*log(-I*b*sqrt(I/b^2) + sqrt(I*sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + b*sqrt(-I/b^2)*log(I*b*sqr
t(-I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - b*s
qrt(-I/b^2)*log(-I*b*sqrt(-I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1)/(b*x + a + I))) + 2*(b*x + a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1)/(b*x + a + I)))/b
```

Sympy [F]

$$\int e^{\frac{1}{2}i\arctan(a+bx)}\,dx=\int\sqrt{\frac{i\,(a+bx)+1}{\sqrt{(a+bx)^2+1}}}\,dx$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2),x)
```

output
```
Integral(sqrt((I*(a + b*x) + 1)/sqrt((a + b*x)**2 + 1)), x)
```

Maxima [F]

$$\int e^{\frac{1}{2}i\arctan(a+bx)}\,dx=\int\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}\,dx$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{1}{2}i\arctan(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{1}{2}i\arctan(a+bx)}\,dx = \int \sqrt{\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}}\,dx$$

input
```
int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2),x)
```

output
```
int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int e^{\frac{1}{2}i\arctan(a+bx)}\,dx = \int \sqrt{\frac{1+i\,(bx+a)}{\sqrt{1+(bx+a)^2}}}dx$$

input
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

output
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

3.233 $\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 319

$$\begin{aligned}\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = &-\frac{2\sqrt[4]{i-a}\arctan\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt[4]{i+a}}\\
&-\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right)\\
&+\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right)\\
&-\frac{2\sqrt[4]{i-a}\operatorname{arctanh}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt[4]{i+a}}\\
&+\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}\left(1+\frac{\sqrt{1+i(a+bx)}}{\sqrt{1-i(a+bx)}}\right)}\right)\end{aligned}$$

output
```
-2*(I-a)^(1/4)*arctan((I+a)^(1/4)*(1+I*(b*x+a))^(1/4)/(I-a)^(1/4)/(1-I*(b*
x+a))^(1/4))/(I+a)^(1/4)-2^(1/2)*arctan(1-2^(1/2)*(1+I*(b*x+a))^(1/4)/(1-I
*(b*x+a))^(1/4))+2^(1/2)*arctan(1+2^(1/2)*(1+I*(b*x+a))^(1/4)/(1-I*(b*x+a)
)^(1/4))-2*(I-a)^(1/4)*arctanh((I+a)^(1/4)*(1+I*(b*x+a))^(1/4)/(I-a)^(1/4)
/(1-I*(b*x+a))^(1/4))/(I+a)^(1/4)+2^(1/2)*arctanh(2^(1/2)*(1+I*(b*x+a))^(1
/4)/(1-I*(b*x+a))^(1/4)/(1+(1+I*(b*x+a))^(1/2)/(1-I*(b*x+a))^(1/2)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.39

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = \frac{2}{3}(-i(i+a+bx))^{3/4}\left(-\sqrt[4]{2}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},-\frac{1}{2}i(i+a+bx)\right)+\frac{2(-i+a)\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{1+a^2-ibx+abx}{1+a^2+ibx+abx}\right)}{(i+a)(1+ia+ibx)^{3/4}}\right)$$

input
```
Integrate[E^((I/2)*ArcTan[a + b*x])/x,x]
```

output
```
(2*((-I)*(I + a + b*x))^(3/4)*(-(2^(1/4)*Hypergeometric2F1[3/4, 3/4, 7/4,
(-1/2*I)*(I + a + b*x)]) + (2*(-I + a)*Hypergeometric2F1[3/4, 1, 7/4, (1 +
 a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)])/((I + a)*(1 + I*a + I*b*
x)^(3/4)))/3
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.37, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {5617, 947, 981, 755, 756, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x}\,dx$$

$$\Big\downarrow \text{5617}$$

$$8\int \frac{1-i(a+bx)}{(i(a+bx)+1)\left(\frac{1-i(a+bx)}{i(a+bx)+1}+1\right)\left(-ia-\frac{(ia+1)(1-i(a+bx))}{i(a+bx)+1}+1\right)}d\frac{\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}$$

$$\Big\downarrow \text{947}$$

$$8\int \frac{i(a+bx)+1}{(1-i(a+bx))\left(\frac{i(a+bx)+1}{1-i(a+bx)}+1\right)\left(-ia+\frac{(1-ia)(i(a+bx)+1)}{1-i(a+bx)}-1\right)}d\frac{\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}$$

$$\Big\downarrow \text{981}$$

$$8\left(\frac{1}{2}\int \frac{1}{\frac{i(a+bx)+1}{1-i(a+bx)}+1}d\frac{\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}+\frac{1}{2}(1+ia)\int \frac{1}{-ia+\frac{(1-ia)(i(a+bx)+1)}{1-i(a+bx)}-1}d\frac{\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}\right)$$

$$\Big\downarrow \text{755}$$

$$8\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1-\frac{\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}}{\frac{i(a+bx)+1}{1-i(a+bx)}+1}d\frac{\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}+\frac{1}{2}\int \frac{\frac{\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}+1}{\frac{i(a+bx)+1}{1-i(a+bx)}+1}d\frac{\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}\right)+\frac{1}{2}(1+ia)\int \frac{}{-ia+\frac{(1}{}}\right.$$

$$\Big\downarrow \text{756}$$

$$8\left(\frac{1}{2}(1+ia)\left(-\frac{i\int \frac{1}{\sqrt{i-a}-\frac{\sqrt{a+i}\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}}d\frac{\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}}{2\sqrt{-a+i}}-\frac{i\int \frac{1}{\sqrt{i-a}+\frac{\sqrt{a+i}\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}}d\frac{\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}}{2\sqrt{-a+i}}\right)+\frac{1}{2}\left(\frac{1}{2}\right.\right.$$

$$\Big\downarrow \text{218}$$

$$8\left(\frac{1}{2}(1+ia)\left(-\frac{i\int \frac{1}{\sqrt{i-a}-\frac{\sqrt{a+i}\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}}d\frac{\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}}{2\sqrt{-a+i}}-\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}\right)+\frac{1}{2}\left(\frac{1}{2}\int\right.\right.$$

$$\Big\downarrow \text{221}$$

$$8\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1-\frac{\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}}{\frac{i(a+bx)+1}{1-i(a+bx)}+1}d\frac{\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}+\frac{1}{2}\int\frac{\frac{\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}+1}{\frac{i(a+bx)+1}{1-i(a+bx)}+1}d\frac{\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}\right)+\frac{1}{2}(1+ia)\left(-\frac{i\arctan}{}\right.\right.$$

$$\Big\downarrow \text{1476}$$

$$8\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\frac{\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}-\frac{\sqrt{2}\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}+1}d\frac{\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}+\frac{1}{2}\int\frac{1}{\frac{\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}+\frac{\sqrt{2}\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}+}\right.\right.\right.$$

$$\Big\downarrow \text{1082}$$

$$8\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\frac{\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\frac{\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}-1}d\left(\frac{\sqrt{2}\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int\frac{1}{-}\right.\right.$$

$$\Big\downarrow \text{217}$$

$$8\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1-\frac{\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}}{\frac{i(a+bx)+1}{1-i(a+bx)}+1}d\frac{\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{1-i(a+bx)}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1+i(}}{\sqrt[4]{1-i(a}}\right.}{\sqrt{2}}\right.\right.\right.$$

$$\Big\downarrow \text{1479}$$

$$8\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}}{\frac{\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}-\frac{\sqrt{2}\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}+1}d\frac{\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}+1\right)}{\frac{\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}+\frac{\sqrt{2}\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}+1}d\frac{\sqrt[4]{i(a}}{\sqrt[4]{1-}}}{2\sqrt{2}}\right.\right.\right.$$

$$\Big\downarrow \ 25$$

$$8\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}}{\frac{\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}-\frac{\sqrt{2}\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}+1}d\frac{\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}+1\right)}{\frac{\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}+\frac{\sqrt{2}\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}+1}d\frac{\sqrt[4]{i(a+bx}}{\sqrt[4]{1-i(a}}}{2\sqrt{2}}\right.\right.\right.$$

$$\Big\downarrow \ 27$$

$$8\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}}{\frac{\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}-\frac{\sqrt{2}\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}+1}d\frac{\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}+1}{\frac{\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}+\frac{\sqrt{2}\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}+1}d\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.\right.\right.$$

$$\Big\downarrow \ 1103$$

$$8\left(\frac{1}{2}(1+ia)\left(-\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}-\frac{i\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}\right)+\frac{1}{2}\left(\frac{1}{2}\left(\text{arct}\right.\right.\right.$$

input
```
Int[E^((I/2)*ArcTan[a + b*x])/x,x]
```

output
```
8*(((1 + I*a)*(((-1/2*I)*ArcTan[((I + a)^(1/4)*(1 + I*(a + b*x))^(1/4))/((
I - a)^(1/4)*(1 - I*(a + b*x))^(1/4))])/((I - a)^(3/4)*(I + a)^(1/4)) - ((
I/2)*ArcTanh[((I + a)^(1/4)*(1 + I*(a + b*x))^(1/4))/((I - a)^(1/4)*(1 - I
*(a + b*x))^(1/4))])/((I - a)^(3/4)*(I + a)^(1/4))))/2 + ((-(ArcTan[1 - (S
qrt[2]*(1 + I*(a + b*x))^(1/4))/(1 - I*(a + b*x))^(1/4)]/Sqrt[2]) + ArcTan
[1 + (Sqrt[2]*(1 + I*(a + b*x))^(1/4))/(1 - I*(a + b*x))^(1/4)]/Sqrt[2])/2
 + (-1/2*Log[1 - (Sqrt[2]*(1 + I*(a + b*x))^(1/4))/(1 - I*(a + b*x))^(1/4)
 + Sqrt[1 + I*(a + b*x)]/Sqrt[1 - I*(a + b*x)]]/Sqrt[2] + Log[1 + (Sqrt[2]
*(1 + I*(a + b*x))^(1/4))/(1 - I*(a + b*x))^(1/4) + Sqrt[1 + I*(a + b*x)]/
Sqrt[1 - I*(a + b*x)]]/(2*Sqrt[2]))/2)/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 947
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

rule 981
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[(-a)*(e^n/(b*c - a*d))   Int[(e*x)^(m - n)/(a + b*x^n),
x], x] + Simp[c*(e^n/(b*c - a*d))   Int[(e*x)^(m - n)/(c + d*x^n), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
 m, 2*n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5617
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_))*(x_)^(m_), x_Symbol] :> Simp
[4/(I^m*n*b^(m + 1)*c^(m + 1))   Subst[Int[x^(2/(I*n))*((1 - I*a*c - (1 + I
*a*c)*x^(2/(I*n)))^m/(1 + x^(2/(I*n)))^(m + 2)), x], x, (1 - I*c*(a + b*x))
^(I*(n/2))/(1 + I*c*(a + b*x))^(I*(n/2))], x] /; FreeQ[{a, b, c}, x] && ILt
Q[m, 0] && LtQ[-1, I*n, 1]
```

Maple [F]

$$\int \frac{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}}{x}dx$$

input
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x)
```

output
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.30

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = \frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right) - \frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}\sqrt{4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right) + \frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}\sqrt{-4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right) - \frac{1}{2}\sqrt{-4i}\log\left(-\frac{1}{2}\sqrt{-4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right) - \left(-\frac{a-i}{a+i}\right)^{\frac{1}{4}}\log\left(\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}+\left(-\frac{a-i}{a+i}\right)^{\frac{1}{4}}\right) - i\left(-\frac{a-i}{a+i}\right)^{\frac{1}{4}}\log\left(\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}+i\left(-\frac{a-i}{a+i}\right)^{\frac{1}{4}}\right) + i\left(-\frac{a-i}{a+i}\right)^{\frac{1}{4}}\log\left(\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}-i\left(-\frac{a-i}{a+i}\right)^{\frac{1}{4}}\right) + \left(-\frac{a-i}{a+i}\right)^{\frac{1}{4}}\log\left(\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}-\left(-\frac{a-i}{a+i}\right)^{\frac{1}{4}}\right)$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x, algorithm="fricas
")
```

output
```
1/2*sqrt(4*I)*log(1/2*sqrt(4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
/(b*x + a + I))) - 1/2*sqrt(4*I)*log(-1/2*sqrt(4*I) + sqrt(I*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)/(b*x + a + I))) + 1/2*sqrt(-4*I)*log(1/2*sqrt(-4*I) +
 sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - 1/2*sqrt(-4*I)
*log(-1/2*sqrt(-4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a +
 I))) - (-(a - I)/(a + I))^(1/4)*log(sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1)/(b*x + a + I)) + (-(a - I)/(a + I))^(1/4)) - I*(-(a - I)/(a + I))^(1/4
)*log(sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) + I*(-(a - I
)/(a + I))^(1/4)) + I*(-(a - I)/(a + I))^(1/4)*log(sqrt(I*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1)/(b*x + a + I)) - I*(-(a - I)/(a + I))^(1/4)) + (-(a - I)
/(a + I))^(1/4)*log(sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)
) - (-(a - I)/(a + I))^(1/4))
```

Sympy [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = \int \frac{\sqrt{\frac{i(a+bx-i)}{\sqrt{a^2+2abx+b^2x^2+1}}}}{x}\,dx$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2)/x,x)
```

output
```
Integral(sqrt(I*(a + b*x - I)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x, x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = \int \frac{\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}{x}\,dx$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x, algorithm="maxima
")
```

output
```
integrate(sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = \int \frac{\sqrt{\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}}}{x}\,dx$$

input
```
int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2)/x,x)
```

output
```
int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2)/x, x)
```

Reduce [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = \int \frac{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}}{x}dx$$

input
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x)
```

output
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x)
```

3.234 $\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 205

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = -\frac{(i+a+bx)\sqrt[4]{1+i(a+bx)}}{(i+a)x\sqrt[4]{1-i(a+bx)}} + \frac{ib\arctan\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}\right)}{(i-a)^{3/4}(i+a)^{5/4}} + \frac{ib\operatorname{arctanh}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}\right)}{(i-a)^{3/4}(i+a)^{5/4}}$$

output

```
-(I+a+b*x)*(1+I*(b*x+a))^(1/4)/(I+a)/x/(1-I*(b*x+a))^(1/4)+I*b*arctan((I+a
)^(1/4)*(1+I*(b*x+a))^(1/4)/(I-a)^(1/4)/(1-I*(b*x+a))^(1/4))/(I-a)^(3/4)/(
I+a)^(5/4)+I*b*arctanh((I+a)^(1/4)*(1+I*(b*x+a))^(1/4)/(I-a)^(1/4)/(1-I*(b
*x+a))^(1/4))/(I-a)^(3/4)/(I+a)^(5/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.54

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = \frac{(-i(i+a+bx))^{3/4}\left(3(i+a)(-i+a+bx)+2ibx\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{1+a^2-ibx+abx}{1+a^2+ibx+abx}\right)\right)}{3(i+a)^2x(1+ia+ibx)^{3/4}}$$

input
```
Integrate[E^((I/2)*ArcTan[a + b*x])/x^2,x]
```

output
```
(((-I)*(I + a + b*x))^(3/4)*(3*(I + a)*(-I + a + b*x) + (2*I)*b*x*Hypergeo
metric2F1[3/4, 1, 7/4, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)
]))/(3*(I + a)^2*x*(1 + I*a + I*b*x)^(3/4))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.27, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5617, 795, 817, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx$$

$$\Big\downarrow \text{5617}$$

$$8ib\int \frac{1-i(a+bx)}{(i(a+bx)+1)\left(-ia-\frac{(ia+1)(1-i(a+bx))}{i(a+bx)+1}+1\right)^2}d\frac{\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}$$

$$\Big\downarrow \text{795}$$

$$8ib\int \frac{i(a+bx)+1}{(1-i(a+bx))\left(-ia+\frac{(1-ia)(i(a+bx)+1)}{1-i(a+bx)}-1\right)^2}d\frac{\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}$$

$$\Big\downarrow \ 817$$

$$8ib\left(\frac{\int \frac{1}{-ia+\frac{(1-ia)(i(a+bx)+1)}{1-i(a+bx)}-1}d\frac{\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}}{4(1-ia)}+\frac{\sqrt[4]{1+i(a+bx)}}{4(1-ia)\sqrt[4]{1-i(a+bx)}\left(-\frac{(1-ia)(1+i(a+bx))}{1-i(a+bx)}+ia+1\right)}\right)$$

$$\Big\downarrow \ 756$$

$$8ib\left(\frac{-\frac{i\int \frac{1}{\sqrt{i-a}-\frac{\sqrt{a+i}\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}}d\frac{\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}}{2\sqrt{-a+i}}-\frac{i\int \frac{1}{\sqrt{i-a}+\frac{\sqrt{a+i}\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}}d\frac{\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}}{2\sqrt{-a+i}}}{4(1-ia)}+\frac{}{4(1-ia)\sqrt[4]{1-i(a-}}\right.$$

$$\Big\downarrow \ 218$$

$$8ib\left(\frac{-\frac{i\int \frac{1}{\sqrt{i-a}-\frac{\sqrt{a+i}\sqrt{i(a+bx)+1}}{\sqrt{1-i(a+bx)}}}d\frac{\sqrt[4]{i(a+bx)+1}}{\sqrt[4]{1-i(a+bx)}}}{2\sqrt{-a+i}}-\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}}{4(1-ia)}+\frac{\sqrt[4]{1-}}{4(1-ia)\sqrt[4]{1-i(a+bx)}}\right.$$

$$\Big\downarrow \ 221$$

$$8ib\left(\frac{-\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}-\frac{i\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}}{4(1-ia)}+\frac{\sqrt[4]{1+}}{4(1-ia)\sqrt[4]{1-i(a+bx)}\,(}\right.$$

input
```
Int[E^((I/2)*ArcTan[a + b*x])/x^2,x]
```

output
```
(8*I)*b*((1 + I*(a + b*x))^(1/4)/(4*(1 - I*a)*(1 - I*(a + b*x))^(1/4)*(1 +
 I*a - ((1 - I*a)*(1 + I*(a + b*x)))/(1 - I*(a + b*x)))) + (((-1/2*I)*ArcT
an[((I + a)^(1/4)*(1 + I*(a + b*x))^(1/4))/((I - a)^(1/4)*(1 - I*(a + b*x)
)^(1/4))])/((I - a)^(3/4)*(I + a)^(1/4)) - ((I/2)*ArcTanh[((I + a)^(1/4)*(
1 + I*(a + b*x))^(1/4))/((I - a)^(1/4)*(1 - I*(a + b*x))^(1/4))])/((I - a)
^(3/4)*(I + a)^(1/4)))/(4*(1 - I*a)))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 5617
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_))*(x_)^(m_), x_Symbol] :> Simp
[4/(I^m*n*b^(m + 1)*c^(m + 1))   Subst[Int[x^(2/(I*n))*((1 - I*a*c - (1 + I
*a*c)*x^(2/(I*n)))^m/(1 + x^(2/(I*n)))^(m + 2)), x], x, (1 - I*c*(a + b*x))
^(I*(n/2))/(1 + I*c*(a + b*x))^(I*(n/2))], x] /; FreeQ[{a, b, c}, x] && ILt
Q[m, 0] && LtQ[-1, I*n, 1]
```

Maple [F]

$$\int \frac{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}}{x^2}dx$$

input
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x)
```

output
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 598 vs. 2(141) = 282.

Time = 0.16 (sec) , antiderivative size = 598, normalized size of antiderivative = 2.92

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = \frac{\left(-\frac{b^4}{a^8+2i\,a^7+2\,a^6+6i\,a^5+6i\,a^3-2\,a^2+2i\,a-1}\right)^{\frac{1}{4}}(-i\,a+1)x\log\left(\frac{b\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}+\left(-\frac{b^4}{a^8+2i\,a^7+2\,a^6+6i\,a^5+6i\,a^3-2\,a^2+2i\,a-}\right.}{b}\right.}{}$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x, algorithm="fric
as")
```

output
```
1/2*((-b^4/(a^8 + 2*I*a^7 + 2*a^6 + 6*I*a^5 + 6*I*a^3 - 2*a^2 + 2*I*a - 1)
)^(1/4)*(-I*a + 1)*x*log((b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x
+ a + I)) + (-b^4/(a^8 + 2*I*a^7 + 2*a^6 + 6*I*a^5 + 6*I*a^3 - 2*a^2 + 2*I
*a - 1))^(1/4)*(a^2 + 1))/b) + (-b^4/(a^8 + 2*I*a^7 + 2*a^6 + 6*I*a^5 + 6*
I*a^3 - 2*a^2 + 2*I*a - 1))^(1/4)*(I*a - 1)*x*log((b*sqrt(I*sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)/(b*x + a + I)) - (-b^4/(a^8 + 2*I*a^7 + 2*a^6 + 6*I*a^
5 + 6*I*a^3 - 2*a^2 + 2*I*a - 1))^(1/4)*(a^2 + 1))/b) - (-b^4/(a^8 + 2*I*a
^7 + 2*a^6 + 6*I*a^5 + 6*I*a^3 - 2*a^2 + 2*I*a - 1))^(1/4)*(a + I)*x*log((
b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) - (-b^4/(a^8 + 2
*I*a^7 + 2*a^6 + 6*I*a^5 + 6*I*a^3 - 2*a^2 + 2*I*a - 1))^(1/4)*(I*a^2 + I)
)/b) + (-b^4/(a^8 + 2*I*a^7 + 2*a^6 + 6*I*a^5 + 6*I*a^3 - 2*a^2 + 2*I*a -
1))^(1/4)*(a + I)*x*log((b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x +
 a + I)) - (-b^4/(a^8 + 2*I*a^7 + 2*a^6 + 6*I*a^5 + 6*I*a^3 - 2*a^2 + 2*I*
a - 1))^(1/4)*(-I*a^2 - I))/b) - 2*(b*x + a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1)/(b*x + a + I)))/((a + I)*x)
```

Sympy [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = \int \frac{\sqrt{\frac{i(a+bx-i)}{\sqrt{a^2+2abx+b^2x^2+1}}}}{x^2}\,dx$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(I*(a + b*x - I)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x**2,
x)
```

Maxima [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = \int \frac{\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}{x^2}\,dx$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x, algorithm="maxi
ma")
```

output
```
integrate(sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = \int \frac{\sqrt{\frac{1+a\,1i+b\,x\,1i}{\sqrt{(a+b\,x)^2+1}}}}{x^2}\,dx$$

input
```
int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2)/x^2,x)
```

output
```
int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2)/x^2, x)
```

Reduce [F]

$$\int \frac{e^{\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = \int \frac{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}}{x^2}dx$$

input
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x)
```

output
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x)
```

3.235 $\int e^{\frac{3}{2}i\arctan(a+bx)}x^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 391

$$\begin{aligned}\int e^{\frac{3}{2}i\arctan(a+bx)}x^2\,dx &= -\frac{(17i+36a-24ia^2)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{24b^3} \\ &\quad-\frac{(3i+8a)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{12b^3} \\ &\quad+\frac{x\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{3b^2} \\ &\quad+\frac{(17i+36a-24ia^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3} \\ &\quad-\frac{(17i+36a-24ia^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3} \\ &\quad-\frac{(17i+36a-24ia^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\right)}\right)}{8\sqrt{2}b^3}\end{aligned}$$

output
```
-1/24*(17*I+36*a-24*I*a^2)*(1-I*a-I*b*x)^(1/4)*(1+I*a+I*b*x)^(3/4)/b^3-1/1
2*(3*I+8*a)*(1-I*a-I*b*x)^(1/4)*(1+I*a+I*b*x)^(7/4)/b^3+1/3*x*(1-I*a-I*b*x
)^(1/4)*(1+I*a+I*b*x)^(7/4)/b^2+1/16*(17*I+36*a-24*I*a^2)*arctan(1-2^(1/2)
*(1-I*a-I*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4))*2^(1/2)/b^3-1/16*(17*I+36*a-24*I
*a^2)*arctan(1+2^(1/2)*(1-I*a-I*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4))*2^(1/2)/b^
3-1/16*(17*I+36*a-24*I*a^2)*arctanh(2^(1/2)*(1-I*a-I*b*x)^(1/4)/(1+I*a+I*b
*x)^(1/4)/(1+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2)))*2^(1/2)/b^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.31

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x^2\,dx$$
$$= \frac{\sqrt[4]{-i(i+a+bx)}\left(-i(1+ia+ibx)^{3/4}\left(3+8a^2+7ibx-4b^2x^2+a(-5i+4bx)\right)+2i2^{3/4}(-17+36ia+\cdots\right.}{12b^3}$$

input
```
Integrate[E^(((3*I)/2)*ArcTan[a + b*x])*x^2,x]
```

output
```
(((-I)*(I + a + b*x))^(1/4)*((-I)*(1 + I*a + I*b*x)^(3/4)*(3 + 8*a^2 + (7*
I)*b*x - 4*b^2*x^2 + a*(-5*I + 4*b*x)) + (2*I)*2^(3/4)*(-17 + (36*I)*a + 2
4*a^2)*Hypergeometric2F1[-3/4, 1/4, 5/4, (-1/2*I)*(I + a + b*x)]))/(12*b^3
)
```

Rubi [A] (warning: unable to verify)

Time = 0.89 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5618, 101, 27, 90, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{\frac{3}{2}i\arctan(a+bx)}dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{x^2(ia+ibx+1)^{3/4}}{(-ia-ibx+1)^{3/4}}dx$$

$$\Big\downarrow \text{101}$$

$$\frac{\int -\frac{(ia+ibx+1)^{3/4}(2a^2+(8a+3i)bx+2)}{2(-ia-ibx+1)^{3/4}}dx}{3b^2}+\frac{x\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{3b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{3b^2}-\frac{\int \frac{(ia+ibx+1)^{3/4}(2(a^2+1)+(8a+3i)bx)}{(-ia-ibx+1)^{3/4}}dx}{6b^2}$$

$$\Big\downarrow \text{90}$$

$$\frac{x\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{3b^2}-\frac{\frac{1}{4}\left(-24a^2-36ia+17\right)\int \frac{(ia+ibx+1)^{3/4}}{(-ia-ibx+1)^{3/4}}dx+\frac{(8a+3i)\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{2b}}{6b^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{x\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{3b^2}-\frac{\frac{1}{4}\left(-24a^2-36ia+17\right)\left(\frac{3}{2}\int \frac{1}{(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}dx+\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}\right)+\frac{(8a+3i)\sqrt[4]{-ia-i}}{}}{6b^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{x\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{3b^2}-\frac{\frac{1}{4}\left(-24a^2-36ia+17\right)\left(\frac{6i\int \frac{1}{\sqrt[4]{ia+ibx+1}}d\sqrt[4]{-ia-ibx+1}}{b}+\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}\right)+\frac{(8a+3i)\sqrt[4]{-ia-}}{}}{6b^2}$$

$$\Big\downarrow \text{770}$$

$$\frac{\frac{x\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{3b^2}-\frac{\frac{1}{4}\left(-24a^2-36ia+17\right)\left(\frac{6i\int\frac{1}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{b}+\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}\right)+\frac{(8a+3i)\sqrt[4]{-ia-ibx}}{2b}}{6b^2}}{}$$

$\downarrow$ 755

$$\frac{x\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{3b^2}-\frac{\frac{1}{4}\left(-24a^2-36ia+17\right)\left(\frac{6i\left(\frac{1}{2}\int\frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\int\frac{\sqrt{-ia-ibx+1}+1}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{b}+\frac{i\sqrt[4]{-ia-ib}}{}\right.}{6b^2}$$

$\downarrow$ 1476

$$\frac{x\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{3b^2}-\frac{\frac{1}{4}\left(-24a^2-36ia+17\right)\left(\frac{6i\left(\frac{1}{2}\int\frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-i}}{\sqrt[4]{ia+ib}}\right.\right.}{b}\right.}{}$$

$\downarrow$ 1082

$$\frac{x\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{3b^2}-\frac{\frac{1}{4}\left(-24a^2-36ia+17\right)\left(\frac{6i\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}\right.\right.}{b}\right.}{6b^2}$$

$\downarrow$ 217

$$\frac{\frac{1}{4}\left(-24a^2-36ia+17\right)\left(\frac{6i\left(\frac{1}{2}\int\frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia\ldots}}{\sqrt[4]{ia+\ldots}}\right)}{\sqrt{2}}\ldots\right.\right.}{b}\ldots\right.}{6b^2}\ldots\quad \frac{x\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{3b^2}-$$

$\downarrow$ 1479

$$\frac{\frac{1}{4}\left(-24a^2-36ia+17\right)\left(\frac{6i\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx\ldots}}{\sqrt[4]{ia+ibx+\ldots}}\ldots\right.}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-\ldots}}{\sqrt[4]{ia+ib\ldots}}\ldots}\ldots}{\ldots}\ldots\right.\right.}{\ldots}\ldots\right.}{\ldots}\quad \frac{x\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{3b^2}-$$

$\downarrow$ 25

$$\frac{x\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{3b^2}-\frac{\frac{1}{4}\left(-24a^2-36ia+17\right)}{\left(6i\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\right.}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx}}{\sqrt[4]{ia+ibx+}}}}{2\sqrt{2}}\right.\right.\right.}$$

$\downarrow$ 27

$$\frac{x\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{3b^2}-\frac{\frac{1}{4}\left(-24a^2-36ia+17\right)}{\left(6i\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ib}}{\sqrt[4]{ia+ibx-}}}\right.\right.\right.}$$

$\downarrow$ 1103

$$\frac{x\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{3b^2} - \frac{\frac{1}{4}\left(-24a^2-36ia+17\right)\left(\frac{6i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{-ia-ibx+1}+\frac{\sqrt{2}}{}\right.}{}\right.\right)}{b}\right.}{}$$

input
```
Int[E^(((3*I)/2)*ArcTan[a + b*x])*x^2,x]
```

output
```
(x*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(7/4))/(3*b^2) - (((3*I + 8*a
)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(7/4))/(2*b) + ((17 - (36*I)*a
 - 24*a^2)*((I*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/b + ((6*I)
*((-(ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]
/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)
^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 + Sqrt[1 - I*a - I*b*x] - (Sqrt[2]*(1 - I
*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - I*a
 - I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/(2*
Sqrt[2]))/2))/b))/4)/(6*b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [F]

$$\int \left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}} x^2 dx$$

input
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x^2,x)
```

output
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 561 vs. 2(265) = 530.

Time = 0.14 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.43

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x^2\,dx$$

$$= \frac{3\,b^3\sqrt{\frac{576i\,a^4-1728\,a^3-2112i\,a^2+1224\,a+289i}{b^6}}\log\left(\frac{b^3\sqrt{\frac{576i\,a^4-1728\,a^3-2112i\,a^2+1224\,a+289i}{b^6}}+(24\,a^2+36i\,a-17)\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{24\,a^2+36i\,a-17}\right.}{}$$

input

```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x^2,x, algorithm="fric
as")
```

output

```
1/48*(3*b^3*sqrt((576*I*a^4 - 1728*a^3 - 2112*I*a^2 + 1224*a + 289*I)/b^6)
*log((b^3*sqrt((576*I*a^4 - 1728*a^3 - 2112*I*a^2 + 1224*a + 289*I)/b^6) +
 (24*a^2 + 36*I*a - 17)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a
+ I)))/(24*a^2 + 36*I*a - 17)) - 3*b^3*sqrt((576*I*a^4 - 1728*a^3 - 2112*I
*a^2 + 1224*a + 289*I)/b^6)*log(-(b^3*sqrt((576*I*a^4 - 1728*a^3 - 2112*I*
a^2 + 1224*a + 289*I)/b^6) - (24*a^2 + 36*I*a - 17)*sqrt(I*sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)/(b*x + a + I)))/(24*a^2 + 36*I*a - 17)) - 3*b^3*sqrt((-
576*I*a^4 + 1728*a^3 + 2112*I*a^2 - 1224*a - 289*I)/b^6)*log((b^3*sqrt((-5
76*I*a^4 + 1728*a^3 + 2112*I*a^2 - 1224*a - 289*I)/b^6) + (24*a^2 + 36*I*a
 - 17)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(24*a^2 +
36*I*a - 17)) + 3*b^3*sqrt((-576*I*a^4 + 1728*a^3 + 2112*I*a^2 - 1224*a -
289*I)/b^6)*log(-(b^3*sqrt((-576*I*a^4 + 1728*a^3 + 2112*I*a^2 - 1224*a -
289*I)/b^6) - (24*a^2 + 36*I*a - 17)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1)/(b*x + a + I)))/(24*a^2 + 36*I*a - 17)) + 2*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)*(8*I*b^2*x^2 - 2*(4*I*a - 7)*b*x + 8*I*a^2 - 46*a - 23*I)*sqrt(I*s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/b^3
```

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x^2\,dx = \text{Timed out}$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2)*x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x^2\,dx = \int x^2\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x^2,x, algorithm="maxi
ma")
```

output
```
integrate(x^2*((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x^2,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x^2\,dx=\int x^2\left(\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}\right)^{3/2}\,dx$$

input
```
int(x^2*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2),x)
```

output
```
int(x^2*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2), x)
```

Reduce [F]

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x^2\,dx=\int\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}}x^2dx$$

input
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x^2,x)
```

output
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x^2,x)
```

3.236 $\int e^{\frac{3}{2}i\arctan(a+bx)}x\,dx$

Optimal result

Integrand size = 16, antiderivative size = 314

$$\begin{aligned}\int e^{\frac{3}{2}i\arctan(a+bx)}x\,dx &= \frac{(3-4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2}\\ &\quad+\frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{7/4}}{2b^2}\\ &\quad-\frac{3(3-4ia)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2}\\ &\quad+\frac{3(3-4ia)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2}\\ &\quad+\frac{3(3-4ia)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\right)}\right)}{4\sqrt{2}b^2}\end{aligned}$$

output
```
1/4*(3-4*I*a)*(1-I*a-I*b*x)^(1/4)*(1+I*a+I*b*x)^(3/4)/b^2+1/2*(1-I*a-I*b*x
)^(1/4)*(1+I*a+I*b*x)^(7/4)/b^2-3/8*(3-4*I*a)*arctan(1-2^(1/2)*(1-I*a-I*b*
x)^(1/4)/(1+I*a+I*b*x)^(1/4))*2^(1/2)/b^2+3/8*(3-4*I*a)*arctan(1+2^(1/2)*(
1-I*a-I*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4))*2^(1/2)/b^2+3/8*(3-4*I*a)*arctanh(
2^(1/2)*(1-I*a-I*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4)/(1+(1-I*a-I*b*x)^(1/2)/(1+
I*a+I*b*x)^(1/2)))*2^(1/2)/b^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.25

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x\,dx = \frac{\sqrt[4]{-i(i+a+bx)}\left((1+ia+ibx)^{7/4}+2\ 2^{3/4}(3-4ia)\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{4},\frac{5}{4},-\frac{1}{2}i(i+a+bx)\right)\right)}{2b^2}$$

input
```
Integrate[E^(((3*I)/2)*ArcTan[a + b*x])*x,x]
```

output
```
(((-I)*(I + a + b*x))^(1/4)*((1 + I*a + I*b*x)^(7/4) + 2*2^(3/4)*(3 - (4*I
)*a)*Hypergeometric2F1[-3/4, 1/4, 5/4, (-1/2*I)*(I + a + b*x)]))/(2*b^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.77 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.13, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {5618, 90, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\frac{3}{2}i\arctan(a+bx)}dx$$

$$\Big\downarrow \text{ 5618}$$

$$\int \frac{x(ia+ibx+1)^{3/4}}{(-ia-ibx+1)^{3/4}}dx$$

$$\Big\downarrow \text{ 90}$$

$$\frac{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{2b^2}-\frac{(4a+3i)\int \frac{(ia+ibx+1)^{3/4}}{(-ia-ibx+1)^{3/4}}dx}{4b}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{2b^2}-\frac{(4a+3i)\left(\frac{3}{2}\int \frac{1}{(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}dx+\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}\right)}{4b}$$

$\downarrow$ 73

$$\frac{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{2b^2}-\frac{(4a+3i)\left(\frac{6i\int \frac{1}{\sqrt[4]{ia+ibx+1}}d\sqrt[4]{-ia-ibx+1}}{b}+\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}\right)}{4b}$$

$\downarrow$ 770

$$\frac{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{2b^2}-\frac{(4a+3i)\left(\frac{6i\int \frac{1}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{b}+\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}\right)}{4b}$$

$\downarrow$ 755

$$\frac{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{2b^2}-\frac{(4a+3i)\left(\frac{6i\left(\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\int \frac{\sqrt{-ia-ibx+1}+1}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{b}+\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1}{b}\right.}{4b}$$

$\downarrow$ 1476

$$\frac{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{2b^2}-\frac{(4a+3i)\left(\frac{6i\left(\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\int \frac{}{\sqrt{-}}\right.\right.}{b}\right.}{4b}$$

$\downarrow$ 1082

$$\frac{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{2b^2}-\frac{(4a+3i)\left(\frac{6i\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int\frac{1-\sqrt{-ia-i\cdots}}{-ia-ibx\cdots}\cdots\right)}{b}\cdots\right)}{4b}$$

↓ 217

$$\frac{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{2b^2}-\frac{(4a+3i)\left(\frac{6i\left(\frac{1}{2}\int\frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}\right)\cdots\right)}{b}\cdots\right)}{4b}$$

↓ 1479

$$\frac{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{2b^2}-\frac{(4a+3i)\left(\frac{6i\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}}{\cdots}\cdots\right)\cdots\right)}{b}\cdots\right)}{\cdots}$$

↓ 25

$$\frac{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{2b^2}-$$

$$(4a+3i)\left(\frac{6i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia}}{\sqrt[4]{ia}}}{2\sqrt{2}}\right.\right.\right.}{b}\right.$$

4

$\downarrow$ 27

$$\frac{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{2b^2}-$$

$$(4a+3i)\left(\frac{6i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-i}}{\sqrt[4]{ia}}\right.\right.\right.}{b}\right.$$

$\downarrow$ 1103

$$\frac{\frac{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{7/4}}{2b^2}-}{(4a+3i)\left(\frac{6i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx}}{\sqrt[4]{ia+ibx+}}\right.}{2\sqrt{2}}\right.}{b}\right.}{4b}$$

input
```
Int[E^(((3*I)/2)*ArcTan[a + b*x])*x,x]
```

output
```
((1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(7/4))/(2*b^2) - ((3*I + 4*a)*(
(I*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/b + ((6*I)*((-(ArcTan[
1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2]) +
ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt
[2])/2 + (-1/2*Log[1 + Sqrt[1 - I*a - I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^
(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - I*a - I*b*x] +
(Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/(2*Sqrt[2]))/2)
)/b))/(4*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [F]

$$\int \left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}} x dx$$

input
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x,x)
```

output
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(212) = 424.

Time = 0.13 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.37

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x\,dx = \frac{3\,b^2\sqrt{-\frac{16i\,a^2-24\,a-9i}{b^4}}\log\left(\frac{b^2\sqrt{-\frac{16i\,a^2-24\,a-9i}{b^4}}+(4\,a+3i)\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{4\,a+3i}\right)-3\,b^2\sqrt{-\frac{16i\,a^2-24\,a-9i}{b^4}}\log\left(-\frac{b^2\sqrt{-1}}{}\right.}{}$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x,x, algorithm="fricas
")
```

output
```
1/8*(3*b^2*sqrt(-(16*I*a^2 - 24*a - 9*I)/b^4)*log((b^2*sqrt(-(16*I*a^2 - 2
4*a - 9*I)/b^4) + (4*a + 3*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*
x + a + I)))/(4*a + 3*I)) - 3*b^2*sqrt(-(16*I*a^2 - 24*a - 9*I)/b^4)*log(-
(b^2*sqrt(-(16*I*a^2 - 24*a - 9*I)/b^4) - (4*a + 3*I)*sqrt(I*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(4*a + 3*I)) - 3*b^2*sqrt(-(-16*I*a^2
 + 24*a + 9*I)/b^4)*log((b^2*sqrt(-(-16*I*a^2 + 24*a + 9*I)/b^4) + (4*a +
3*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(4*a + 3*I))
 + 3*b^2*sqrt(-(-16*I*a^2 + 24*a + 9*I)/b^4)*log(-(b^2*sqrt(-(-16*I*a^2 +
24*a + 9*I)/b^4) - (4*a + 3*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b
*x + a + I)))/(4*a + 3*I)) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(2*I*b*x
- 2*I*a + 5)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/b^2
```

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x\,dx = \text{Timed out}$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2)*x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x\,dx = \int x\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x,x, algorithm="maxima
")
```

output
```
integrate(x*((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x\,dx = \int x\left(\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}\right)^{3/2}\,dx$$

input
```
int(x*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2),x)
```

output
```
int(x*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2), x)
```

Reduce [F]

$$\int e^{\frac{3}{2}i\arctan(a+bx)}x\,dx = \int \left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}} xdx$$

input
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x,x)
```

output
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)*x,x)
```

3.237 $\int e^{\frac{3}{2}i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 245

$$\int e^{\frac{3}{2}i\arctan(a+bx)}\,dx=\frac{i\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{b}-\frac{3i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b}+\frac{3i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b}+\frac{3i\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\right)}\right)}{\sqrt{2}b}$$

output
```
I*(1-I*a-I*b*x)^(1/4)*(1+I*a+I*b*x)^(3/4)/b-3/2*I*arctan(1-2^(1/2)*(1-I*a-
I*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4))*2^(1/2)/b+3/2*I*arctan(1+2^(1/2)*(1-I*a-
I*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4))*2^(1/2)/b+3/2*I*arctanh(2^(1/2)*(1-I*a-I
*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4)/(1+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2)
))*2^(1/2)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.18

$$\int e^{\frac{3}{2}i\arctan(a+bx)}\,dx=-\frac{8ie^{\frac{7}{2}i\arctan(a+bx)}\text{Hypergeometric2F1}\left(\frac{7}{4},2,\frac{11}{4},-e^{2i\arctan(a+bx)}\right)}{7b}$$

input
```
Integrate[E^(((3*I)/2)*ArcTan[a + b*x]),x]
```

output
```
(((-8*I)/7)*E^(((7*I)/2)*ArcTan[a + b*x])*Hypergeometric2F1[7/4, 2, 11/4,
-E^((2*I)*ArcTan[a + b*x])])/b
```

Rubi [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.22, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.857$, Rules used = {5616, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{3}{2}i\arctan(a+bx)}dx$$

$$\Big\downarrow \text{5616}$$

$$\int \frac{(ia+ibx+1)^{3/4}}{(-ia-ibx+1)^{3/4}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{2}\int \frac{1}{(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}dx+\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}$$

$$\Big\downarrow \text{73}$$

$$\frac{6i\int \frac{1}{\sqrt[4]{ia+ibx+1}}d\sqrt[4]{-ia-ibx+1}}{b}+\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}$$

$$\Big\downarrow \text{770}$$

$$\frac{6i\int \frac{1}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{b}+\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}$$

$$\Big\downarrow \text{755}$$

$$\frac{6i\left(\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\int \frac{\sqrt{-ia-ibx+1}+1}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{b}+\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}$$

$$\Big\downarrow \text{1476}$$

$$\frac{6i\left(\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\int \frac{}{\sqrt{-i}}\right.\right.}{b}$$
$$\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}$$

$$\Big\downarrow \text{1082}$$

$$\frac{6i\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+}}{-ia-ibx+2}\right.}{b}$$
$$\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{6i\left(\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}\right)\right)}{b}$$
$$\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}$$

$$\Big\downarrow \text{1479}$$

$$6i\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-i}}{\sqrt[4]{ia+ib}}}{2\sqrt{2}}\right.\right.$$

$$\overline{\qquad b \qquad}$$

$$\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}$$

$\downarrow$ 25

$$6i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}\right.\right.$$

$$\overline{\qquad b \qquad}$$

$$\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}$$

$\downarrow$ 27

$$6i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ib}}{\sqrt[4]{ia+ibx}}\right.\right.$$

$$\overline{\qquad b \qquad}$$

$$\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}$$

$\downarrow$ 1103

$$\frac{6i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+}}{\sqrt[4]{ia+ibx+1}}\right.}{2\sqrt{2}}\right.\right.}{b}$$

$$\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}$$

input
```
Int[E^(((3*I)/2)*ArcTan[a + b*x]),x]
```

output
```
(I*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/b + ((6*I)*((-(ArcTan[
1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2]) +
ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt
[2])/2 + (-1/2*Log[1 + Sqrt[1 - I*a - I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^
(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - I*a - I*b*x] +
(Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/(2*Sqrt[2]))/2)
)/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5616
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^(I*(n/2))/(1 + I*a*c + I*b*c*x)^(I*(n/2)), x] /; FreeQ[{a, b,
c, n}, x]
```

Maple [F]

$$\int \left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}} dx$$

input
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

output
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.09

$$\int e^{\frac{3}{2}i\arctan(a+bx)}\,dx = \frac{b\sqrt{\frac{9i}{b^2}}\log\left(\frac{1}{3}b\sqrt{\frac{9i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)-b\sqrt{\frac{9i}{b^2}}\log\left(-\frac{1}{3}b\sqrt{\frac{9i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)-b\sqrt{-\frac{9i}{b^2}}\log\left(\right.}{}$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
1/2*(b*sqrt(9*I/b^2)*log(1/3*b*sqrt(9*I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1)/(b*x + a + I))) - b*sqrt(9*I/b^2)*log(-1/3*b*sqrt(9*I/b^2) +
 sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - b*sqrt(-9*I/b^
2)*log(1/3*b*sqrt(-9*I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*
x + a + I))) + b*sqrt(-9*I/b^2)*log(-1/3*b*sqrt(-9*I/b^2) + sqrt(I*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + 2*I*sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/b
```

Sympy [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}i\arctan(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{\frac{3}{2}i\arctan(a+bx)}\,dx = \int \left(\frac{i\,bx + i\,a + 1}{\sqrt{(bx+a)^2 + 1}}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
integrate(((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\frac{3}{2}i\arctan(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{3}{2}i\arctan(a+bx)}\,dx = \int \left(\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}\right)^{3/2}\,dx$$

input
```
int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2),x)
```

output
```
int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2), x)
```

Reduce [F]

$$\int e^{\frac{3}{2}i\arctan(a+bx)}\,dx = \int \left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}}\,dx$$

input
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

output
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

3.238 $\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 344

$$\begin{aligned}\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x}\,dx &= \frac{2(i-a)^{3/4}\arctan\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/4}} \\ &\quad+\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right) \\ &\quad-\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right) \\ &\quad-\frac{2(i-a)^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i+a)^{3/4}} \\ &\quad-\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\right)}\right)\end{aligned}$$

output
```
2*(I-a)^(3/4)*arctan((I+a)^(1/4)*(1+I*a+I*b*x)^(1/4)/(I-a)^(1/4)/(1-I*a-I*
b*x)^(1/4))/(I+a)^(3/4)+2^(1/2)*arctan(1-2^(1/2)*(1-I*a-I*b*x)^(1/4)/(1+I*
a+I*b*x)^(1/4))-2^(1/2)*arctan(1+2^(1/2)*(1-I*a-I*b*x)^(1/4)/(1+I*a+I*b*x)
^(1/4))-2*(I-a)^(3/4)*arctanh((I+a)^(1/4)*(1+I*a+I*b*x)^(1/4)/(I-a)^(1/4)/
(1-I*a-I*b*x)^(1/4))/(I+a)^(3/4)-2^(1/2)*arctanh(2^(1/2)*(1-I*a-I*b*x)^(1/
4)/(1+I*a+I*b*x)^(1/4)/(1+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.35

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x}\,dx = 2\sqrt[4]{-i(i+a+bx)}\left(-2^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},-\frac{1}{2}i(i+a+bx)\right) + \frac{2(-i+a)\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{1+a^2-ibx+abx}{1+a^2+ibx+abx}\right)}{(i+a)\sqrt[4]{1+ia+ibx}}\right)$$

input
```
Integrate[E^(((3*I)/2)*ArcTan[a + b*x])/x,x]
```

output
```
2*((-I)*(I + a + b*x))^(1/4)*(-(2^(3/4)*Hypergeometric2F1[1/4, 1/4, 5/4, (
-1/2*I)*(I + a + b*x)]) + (2*(-I + a)*Hypergeometric2F1[1/4, 1, 5/4, (1 +
a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)])/((I + a)*(1 + I*a + I*b*x
)^(1/4)))
```

Rubi [A] (warning: unable to verify)

Time = 0.96 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.24, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5618, 140, 27, 73, 104, 25, 770, 755, 827, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x}dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{(ia+ibx+1)^{3/4}}{x(-ia-ibx+1)^{3/4}}dx$$

$$\Big\downarrow \text{140}$$

$$ib\int \frac{1}{(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}dx+\int \frac{ia+1}{x(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}dx$$

$$\Big\downarrow \text{27}$$

$$ib\int \frac{1}{(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}dx+(1+ia)\int \frac{1}{x(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}dx$$

$$\Big\downarrow \text{73}$$

$$(1+ia)\int \frac{1}{x(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}dx-4\int \frac{1}{\sqrt[4]{ia+ibx+1}}d\sqrt[4]{-ia-ibx+1}$$

$$\Big\downarrow \text{104}$$

$$4(1+ia)\int -\frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}\left(ia-\frac{(1-ia)(ia+ibx+1)}{-ia-ibx+1}+1\right)}d\frac{\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-ia-ibx+1}}-4\int \frac{1}{\sqrt[4]{ia+ibx+1}}d\sqrt[4]{-ia-ibx+1}$$

$$\Big\downarrow \text{25}$$

$$-4\int \frac{1}{\sqrt[4]{ia+ibx+1}}d\sqrt[4]{-ia-ibx+1}-4(1+ia)\int \frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}\left(ia-\frac{(1-ia)(ia+ibx+1)}{-ia-ibx+1}+1\right)}d\frac{\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-ia-ibx+1}}$$

$$\Big\downarrow \text{770}$$

$$-4\int \frac{1}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}-4(1+ia)\int \frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}\left(ia-\frac{(1-ia)(ia+ibx+1)}{-ia-ibx+1}+1\right)}d\frac{\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-ia-ibx+1}}$$

$$\Big\downarrow \text{755}$$

$$-4\left(\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\int \frac{\sqrt{-ia-ibx+1}+1}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)-4(1+ia)\int \frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}\left(ia-\frac{(1-ia)(ia+ibx+1)}{-ia-ibx+1}+1\right)}d\frac{\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-ia-ibx+1}}$$

$$\Big\downarrow \text{827}$$

$$4(1+ia)\left(\frac{i\int \frac{1}{\sqrt{i-a}+\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}}d\frac{\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-ia-ibx+1}}}{2\sqrt{a+i}}-\frac{i\int \frac{1}{\sqrt{i-a}-\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}}d\frac{\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-ia-ibx+1}}}{2\sqrt{a+i}}\right)-4\left(\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\int \frac{\sqrt{-ia-ibx+1}+1}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)$$

$$\Big\downarrow \text{218}$$

$$4(1+ia)\left(\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2\sqrt[4]{-a+i}(a+i)^{3/4}}-\frac{i\int \frac{1}{\sqrt{i-a}-\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}}d\frac{\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-ia-ibx+1}}}{2\sqrt{a+i}}\right)-4\left(\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\int \frac{\sqrt{-ia-ibx+1}+1}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)$$

$$\Big\downarrow \text{221}$$

$$4(1+ia)\left(\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2\sqrt[4]{-a+i}(a+i)^{3/4}}-\frac{i\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2\sqrt[4]{-a+i}(a+i)^{3/4}}\right)-4\left(\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\int \frac{\sqrt{-ia-ibx+1}+1}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)$$

$$\Big\downarrow \text{1476}$$

$$4(1+ia)\left(\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2\sqrt[4]{-a+i}(a+i)^{3/4}}-\frac{i\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2\sqrt[4]{-a+i}(a+i)^{3/4}}\right)-$$
$$4\left(\frac{1}{2}\int\frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ib}}{\sqrt[4]{ia+ibx}}\right.\right.$$

$$\Big\downarrow \text{1082}$$

$$4(1+ia)\left(\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2\sqrt[4]{-a+i}(a+i)^{3/4}}-\frac{i\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2\sqrt[4]{-a+i}(a+i)^{3/4}}\right)-$$
$$4\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int\frac{1}{}\right.$$

$$\Big\downarrow \text{217}$$

$$4(1+ia)\left(\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2\sqrt[4]{-a+i}(a+i)^{3/4}}-\frac{i\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2\sqrt[4]{-a+i}(a+i)^{3/4}}\right)-$$
$$4\left(\frac{1}{2}\int\frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia}}{\sqrt[4]{ia+}}\right.}{\sqrt{2}}\right.\right.$$

$$\Big\downarrow \text{1479}$$

$$4(1+ia)\left(\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2\sqrt[4]{-a+i}(a+i)^{3/4}}-\frac{i\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2\sqrt[4]{-a+i}(a+i)^{3/4}}\right)-$$
$$4\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-i}}{\sqrt[4]{ia}}}{2\sqrt{2}}\right.\right.$$

$$\Big\downarrow \text{25}$$

$$4(1+ia)\left(\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2\sqrt[4]{-a+i}(a+i)^{3/4}}-\frac{i\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2\sqrt[4]{-a+i}(a+i)^{3/4}}\right)-$$
$$4\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ib}}{\sqrt[4]{ia+ibx}}}{2\sqrt{2}}\right.\right.$$

$\Big\downarrow$ 27

$$4(1+ia)\left(\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2\sqrt[4]{-a+i}(a+i)^{3/4}}-\frac{i\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2\sqrt[4]{-a+i}(a+i)^{3/4}}\right)-$$
$$4\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\right.\right.$$

$\Big\downarrow$ 1103

$$4(1+ia)\left(\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2\sqrt[4]{-a+i}(a+i)^{3/4}}-\frac{i\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2\sqrt[4]{-a+i}(a+i)^{3/4}}\right)-$$
$$4\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{-ia-ibx+1}+\right.}{2\sqrt{}}\right.\right.$$

input
```
Int[E^(((3*I)/2)*ArcTan[a + b*x])/x,x]
```

output
```
4*(1 + I*a)*(((I/2)*ArcTan[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a
)^(1/4)*(1 - I*a - I*b*x)^(1/4))])/((I - a)^(1/4)*(I + a)^(3/4)) - ((I/2)*
ArcTanh[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a -
I*b*x)^(1/4))])/((I - a)^(1/4)*(I + a)^(3/4))) - 4*((-(ArcTan[1 - (Sqrt[2]
*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (
Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2])/2 + (-1
/2*Log[1 + Sqrt[1 - I*a - I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 +
I*a + I*b*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - I*a - I*b*x] + (Sqrt[2]*(1
- I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [F]

$$\int \frac{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}}}{x}dx$$

input
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x)
```

output
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 690 vs. $2(228) = 456$.

Time = 0.10 (sec) , antiderivative size = 690, normalized size of antiderivative = 2.01

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x}\,dx = \text{Too large to display}$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x, algorithm="fricas
")
```

output
```
1/2*sqrt(4*I)*log(1/2*I*sqrt(4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1)/(b*x + a + I))) - 1/2*sqrt(4*I)*log(-1/2*I*sqrt(4*I) + sqrt(I*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - 1/2*sqrt(-4*I)*log(1/2*I*sqrt(-
4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + 1/2*sqrt
(-4*I)*log(-1/2*I*sqrt(-4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b
*x + a + I))) - (-(a^3 - 3*I*a^2 - 3*a + I)/(a^3 + 3*I*a^2 - 3*a - I))^(1/
4)*log(((a^2 + 2*I*a - 1)*(-(a^3 - 3*I*a^2 - 3*a + I)/(a^3 + 3*I*a^2 - 3*a
 - I))^(3/4) + (a^2 - 2*I*a - 1)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/
(b*x + a + I)))/(a^2 - 2*I*a - 1)) + (-(a^3 - 3*I*a^2 - 3*a + I)/(a^3 + 3*
I*a^2 - 3*a - I))^(1/4)*log(-((a^2 + 2*I*a - 1)*(-(a^3 - 3*I*a^2 - 3*a + I
)/(a^3 + 3*I*a^2 - 3*a - I))^(3/4) - (a^2 - 2*I*a - 1)*sqrt(I*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(a^2 - 2*I*a - 1)) + I*(-(a^3 - 3*I*
a^2 - 3*a + I)/(a^3 + 3*I*a^2 - 3*a - I))^(1/4)*log(((I*a^2 - 2*a - I)*(-(
a^3 - 3*I*a^2 - 3*a + I)/(a^3 + 3*I*a^2 - 3*a - I))^(3/4) + (a^2 - 2*I*a -
 1)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(a^2 - 2*I*a
- 1)) - I*(-(a^3 - 3*I*a^2 - 3*a + I)/(a^3 + 3*I*a^2 - 3*a - I))^(1/4)*log
(((-I*a^2 + 2*a + I)*(-(a^3 - 3*I*a^2 - 3*a + I)/(a^3 + 3*I*a^2 - 3*a - I)
)^(3/4) + (a^2 - 2*I*a - 1)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x
+ a + I)))/(a^2 - 2*I*a - 1))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x}\,dx = \text{Timed out}$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x}\,dx = \int \frac{\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x, algorithm="maxima
")
```

output
```
integrate(((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x}\,dx=\int \frac{\left(\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}\right)^{3/2}}{x}\,dx$$

input
```
int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2)/x,x)
```

output
```
int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2)/x, x)
```

Reduce [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x}\,dx=\int \frac{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}}}{x}dx$$

input
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x)
```

output
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x)
```

3.239 $\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 211

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = -\frac{\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{(1-ia)x} - \frac{3ib\arctan\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{\sqrt[4]{i-a}(i+a)^{7/4}} + \frac{3ib\operatorname{arctanh}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{\sqrt[4]{i-a}(i+a)^{7/4}}$$

output
```
-(1-I*a-I*b*x)^(1/4)*(1+I*a+I*b*x)^(3/4)/(1-I*a)/x-3*I*b*arctan((I+a)^(1/4
)*(1+I*a+I*b*x)^(1/4)/(I-a)^(1/4)/(1-I*a-I*b*x)^(1/4))/(I-a)^(1/4)/(I+a)^(
7/4)+3*I*b*arctanh((I+a)^(1/4)*(1+I*a+I*b*x)^(1/4)/(I-a)^(1/4)/(1-I*a-I*b*
x)^(1/4))/(I-a)^(1/4)/(I+a)^(7/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.50

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = \frac{\sqrt[4]{-i(i+a+bx)}\left(1+a^2+ibx+abx+6ibx\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{1+a^2-ibx+abx}{1+a^2+ibx+abx}\right)\right)}{(i+a)^2x\sqrt[4]{1+ia+ibx}}$$

input
```
Integrate[E^(((3*I)/2)*ArcTan[a + b*x])/x^2,x]
```

output
```
(((-I)*(I + a + b*x))^(1/4)*(1 + a^2 + I*b*x + a*b*x + (6*I)*b*x*Hypergeom
etric2F1[1/4, 1, 5/4, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)]
))/((I + a)^2*x*(1 + I*a + I*b*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5618, 105, 104, 25, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{(ia+ibx+1)^{3/4}}{x^2(-ia-ibx+1)^{3/4}}dx$$

$$\Big\downarrow \text{105}$$

$$-\frac{3b\int \frac{1}{x(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}dx}{2(a+i)}-\frac{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{(1-ia)x}$$

$$\Big\downarrow \; 104$$

$$-\frac{6b\int -\frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}\left(ia-\frac{(1-ia)(ia+ibx+1)}{-ia-ibx+1}+1\right)}d\frac{\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-ia-ibx+1}}}{a+i}-\frac{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{(1-ia)x}$$

$$\Big\downarrow \; 25$$

$$\frac{6b\int \frac{\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}\left(ia-\frac{(1-ia)(ia+ibx+1)}{-ia-ibx+1}+1\right)}d\frac{\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-ia-ibx+1}}}{a+i}-\frac{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{(1-ia)x}$$

$$\Big\downarrow \; 827$$

$$-\frac{6b\left(\frac{i\int \frac{1}{\sqrt{i-a}+\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}}d\frac{\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-ia-ibx+1}}}{2\sqrt{a+i}}-\frac{i\int \frac{1}{\sqrt{i-a}-\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}}d\frac{\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-ia-ibx+1}}}{2\sqrt{a+i}}\right)}{a+i}-\frac{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{(1-ia)x}$$

$$\Big\downarrow \; 218$$

$$-\frac{6b\left(\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2\sqrt[4]{-a+i}(a+i)^{3/4}}-\frac{i\int \frac{1}{\sqrt{i-a}-\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}}d\frac{\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-ia-ibx+1}}}{2\sqrt{a+i}}\right)}{a+i}-\frac{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{(1-ia)x}$$

$$\Big\downarrow \; 221$$

$$-\frac{6b\left(\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2\sqrt[4]{-a+i}(a+i)^{3/4}}-\frac{i\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2\sqrt[4]{-a+i}(a+i)^{3/4}}\right)}{a+i}-\frac{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{(1-ia)x}$$

input
```
Int[E^(((3*I)/2)*ArcTan[a + b*x])/x^2,x]
```

output
```
-(((1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/((1 - I*a)*x)) - (6*b*
(((I/2)*ArcTan[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 -
 I*a - I*b*x)^(1/4))]])/((I - a)^(1/4)*(I + a)^(3/4)) - ((I/2)*ArcTanh[((I
+ a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a - I*b*x)^(1/4)
)])/((I - a)^(1/4)*(I + a)^(3/4))))/(I + a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [F]

$$\int \frac{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}}}{x^2}dx$$

input
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x)
```

output
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 694 vs. $2(137) = 274$.

Time = 0.09 (sec) , antiderivative size = 694, normalized size of antiderivative = 3.29

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x, algorithm="fric
as")
```

output
```
1/2*(3*(-b^4/(a^8 + 6*I*a^7 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*
a - 1))^(1/4)*(-I*a + 1)*x*log((b^3*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1)/(b*x + a + I)) + (a^6 + 4*I*a^5 - 5*a^4 - 5*a^2 - 4*I*a + 1)*(-b^4/(a^8
 + 6*I*a^7 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*a - 1))^(3/4))/b^
3) + 3*(-b^4/(a^8 + 6*I*a^7 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*
a - 1))^(1/4)*(I*a - 1)*x*log((b^3*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)/(b*x + a + I)) - (a^6 + 4*I*a^5 - 5*a^4 - 5*a^2 - 4*I*a + 1)*(-b^4/(a^8
+ 6*I*a^7 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*a - 1))^(3/4))/b^3
) + 3*(-b^4/(a^8 + 6*I*a^7 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*a
 - 1))^(1/4)*(a + I)*x*log((b^3*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(
b*x + a + I)) - (I*a^6 - 4*a^5 - 5*I*a^4 - 5*I*a^2 + 4*a + I)*(-b^4/(a^8 +
 6*I*a^7 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*a - 1))^(3/4))/b^3)
 - 3*(-b^4/(a^8 + 6*I*a^7 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*a
- 1))^(1/4)*(a + I)*x*log((b^3*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b
*x + a + I)) - (-I*a^6 + 4*a^5 + 5*I*a^4 + 5*I*a^2 - 4*a - I)*(-b^4/(a^8 +
 6*I*a^7 - 14*a^6 - 14*I*a^5 - 14*I*a^3 + 14*a^2 + 6*I*a - 1))^(3/4))/b^3)
 - 2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)/(b*x + a + I)))/((a + I)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2)/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = \int \frac{\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x, algorithm="maxi
ma")
```

output
```
integrate(((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2)/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = \int \frac{\left(\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}\right)^{3/2}}{x^2}\,dx$$

input
```
int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2)/x^2,x)
```

output
```
int(((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2)/x^2, x)
```

Reduce [F]

$$\int \frac{e^{\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx=\int \frac{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}}}{x^2}dx$$

input
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x)
```

output
```
int(((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x)
```

3.240 $\int e^{-\frac{1}{2}i\arctan(a+bx)}x^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 391

$$\begin{aligned}\int e^{-\frac{1}{2}i\arctan(a+bx)}x^2\,dx &= \frac{(3i-4a-8ia^2)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{8b^3} \\ &+ \frac{(i-8a)(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{12b^3} \\ &+ \frac{x(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{3b^2} \\ &+ \frac{(3i-4a-8ia^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3} \\ &- \frac{(3i-4a-8ia^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3} \\ &- \frac{(3i-4a-8ia^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\right)}\right)}{8\sqrt{2}b^3}\end{aligned}$$

output
```
1/8*(3*I-4*a-8*I*a^2)*(1-I*a-I*b*x)^(1/4)*(1+I*a+I*b*x)^(3/4)/b^3+1/12*(I-
8*a)*(1-I*a-I*b*x)^(5/4)*(1+I*a+I*b*x)^(3/4)/b^3+1/3*x*(1-I*a-I*b*x)^(5/4)
*(1+I*a+I*b*x)^(3/4)/b^2+1/16*(3*I-4*a-8*I*a^2)*arctan(1-2^(1/2)*(1-I*a-I*
b*x)^(1/4)/(1+I*a+I*b*x)^(1/4))*2^(1/2)/b^3-1/16*(3*I-4*a-8*I*a^2)*arctan(
1+2^(1/2)*(1-I*a-I*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4))*2^(1/2)/b^3-1/16*(3*I-4
*a-8*I*a^2)*arctanh(2^(1/2)*(1-I*a-I*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4)/(1+(1-
I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2)))*2^(1/2)/b^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.25

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x^2\,dx$$
$$=\frac{(-i(i+a+bx))^{5/4}\left(5(1+ia+ibx)^{3/4}(i-8a+4bx)+3\,2^{3/4}(-3i+4a+8ia^2)\,\text{Hypergeometric2F1}\left(\frac{1}{4}\right.\right.}{60b^3}$$

input
```
Integrate[x^2/E^((I/2)*ArcTan[a + b*x]),x]
```

output
```
(((-I)*(I + a + b*x))^(5/4)*(5*(1 + I*a + I*b*x)^(3/4)*(I - 8*a + 4*b*x) +
 3*2^(3/4)*(-3*I + 4*a + (8*I)*a^2)*Hypergeometric2F1[1/4, 5/4, 9/4, (-1/2
*I)*(I + a + b*x)]))/(60*b^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.95 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5618, 101, 27, 90, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2e^{-\frac{1}{2}i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \; 5618$$

$$\int \frac{x^2 \sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}dx$$

$$\Big\downarrow \; 101$$

$$\frac{\int -\frac{\sqrt[4]{-ia-ibx+1}(2a^2-(i-8a)bx+2)}{2\sqrt[4]{ia+ibx+1}}dx}{3b^2}+\frac{x(ia+ibx+1)^{3/4}(-ia-ibx+1)^{5/4}}{3b^2}$$

$$\Big\downarrow \; 27$$

$$\frac{x(-ia-ibx+1)^{5/4}(ia+ibx+1)^{3/4}}{3b^2}-\frac{\int \frac{\sqrt[4]{-ia-ibx+1}(2(a^2+1)-(i-8a)bx)}{\sqrt[4]{ia+ibx+1}}dx}{6b^2}$$

$$\Big\downarrow \; 90$$

$$\frac{x(-ia-ibx+1)^{5/4}(ia+ibx+1)^{3/4}}{3b^2}-\frac{\frac{3}{4}\left(-8a^2+4ia+3\right)\int \frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}dx-\frac{(-8a+i)(-ia-ibx+1)^{5/4}(ia+ibx+1)^{3/4}}{2b}}{6b^2}$$

$$\Big\downarrow \; 60$$

$$\frac{x(-ia-ibx+1)^{5/4}(ia+ibx+1)^{3/4}}{3b^2}-\frac{\frac{3}{4}\left(-8a^2+4ia+3\right)\left(\frac{1}{2}\int \frac{1}{(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}dx-\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}\right)-\frac{(-8a+i)(-ia-ibx+1)^{5/4}}{2b}}{6b^2}$$

$$\Big\downarrow \; 73$$

$$\frac{x(-ia-ibx+1)^{5/4}(ia+ibx+1)^{3/4}}{3b^2}-\frac{\frac{3}{4}\left(-8a^2+4ia+3\right)\left(\frac{2i\int \frac{1}{\sqrt[4]{ia+ibx+1}}d\sqrt[4]{-ia-ibx+1}}{b}-\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}\right)-\frac{(-8a+i)(-ia-ibx+1)}{2b}}{6b^2}$$

$$\Big\downarrow \; 770$$

$$\frac{x(-ia-ibx+1)^{5/4}(ia+ibx+1)^{3/4}}{3b^2}-\frac{\frac{3}{4}\left(-8a^2+4ia+3\right)\left(\frac{2i\int \frac{1}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{b}-\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}\right)-\frac{(-8a+i)(-ia-ibx+1)^{5/4}(ia+}{2b}}{6b^2}$$

$$\Big\downarrow \text{755}$$

$$\frac{x(-ia-ibx+1)^{5/4}(ia+ibx+1)^{3/4}}{3b^2}-\frac{\frac{3}{4}\left(-8a^2+4ia+3\right)\left(\frac{2i\left(\frac{1}{2}\int\frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\int\frac{\sqrt{-ia-ibx+1}+1}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{b}-\frac{i\sqrt[4]{-ia-ibx+}}{b}\right.}{6b^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{x(-ia-ibx+1)^{5/4}(ia+ibx+1)^{3/4}}{3b^2}-\frac{\frac{3}{4}\left(-8a^2+4ia+3\right)\left(\frac{2i\left(\frac{1}{2}\int\frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx-}}{\sqrt[4]{ia+ibx+}}\right.\right.}{b}\right.}{}$$

$$\Big\downarrow \text{1082}$$

$$\frac{x(-ia-ibx+1)^{5/4}(ia+ibx+1)^{3/4}}{3b^2}-\frac{\frac{3}{4}\left(-8a^2+4ia+3\right)\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\right.}{b}\right.}{6b^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{x(-ia-ibx+1)^{5/4}(ia+ibx+1)^{3/4}}{3b^2}-\frac{\frac{3}{4}\left(-8a^2+4ia+3\right)\left(\frac{2i\left(\frac{1}{2}\int\frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-i}}{\sqrt[4]{ia+ibx}}\right.}{\sqrt{2}}\right.\right.}{b}\right.}{6b^2}$$

$\downarrow$ 1479

$$\frac{x(-ia-ibx+1)^{5/4}(ia+ibx+1)^{3/4}}{3b^2}-$$

$$\frac{3}{4}\left(-8a^2+4ia+3\right)\left(\frac{2i\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right.}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx}}{\sqrt[4]{ia+ibx+}}}}{2\sqrt{2}}\right.\right.}{}\right.$$

$\downarrow$ 25

$$\frac{x(-ia-ibx+1)^{5/4}(ia+ibx+1)^{3/4}}{3b^2}-$$

$$\frac{3}{4}\left(-8a^2+4ia+3\right)\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}}{2\sqrt{2}}\right.\right.}{b}\right.$$

$\downarrow$ 27

$$\frac{x(-ia-ibx+1)^{5/4}(ia+ibx+1)^{3/4}}{3b^2}-\frac{\frac{3}{4}(-8a^2+4ia+3)\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+}}{\sqrt[4]{ia+ibx+1}}}\right.\right.\right.}{b}}{}$$

$$\downarrow 1103$$

$$\frac{x(-ia-ibx+1)^{5/4}(ia+ibx+1)^{3/4}}{3b^2}-\frac{\frac{3}{4}(-8a^2+4ia+3)\left(\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-}}{\sqrt[4]{i}}\right.}{2\sqrt{2}}\right.\right.}{b}\right.}{}$$

input
```
Int[x^2/E^((I/2)*ArcTan[a + b*x]),x]
```

output
```
(x*(1 - I*a - I*b*x)^(5/4)*(1 + I*a + I*b*x)^(3/4))/(3*b^2) - (-1/2*((I -
8*a)*(1 - I*a - I*b*x)^(5/4)*(1 + I*a + I*b*x)^(3/4))/b + (3*(3 + (4*I)*a
- 8*a^2)*(((-I)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/b + ((2*I
)*((-(ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)
]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x
)^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 + Sqrt[1 - I*a - I*b*x] - (Sqrt[2]*(1 -
I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - I*
a - I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/(2
*Sqrt[2]))/2))/b))/4)/(6*b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}}dx$$

input
```
int(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

output
```
int(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 561 vs. 2(265) = 530.

Time = 0.10 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.43

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x^2\,dx$$
$$= \frac{3\,b^3\sqrt{\frac{64i\,a^4+64\,a^3-64i\,a^2-24\,a+9i}{b^6}}\log\left(\frac{b^3\sqrt{\frac{64i\,a^4+64\,a^3-64i\,a^2-24\,a+9i}{b^6}}+(8\,a^2-4i\,a-3)\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{8\,a^2-4i\,a-3}\right)-3\,b^3\sqrt{\frac{64i\,a^4+\cdots}{}}}{}$$

input
```
integrate(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="fric
as")
```

output
```
1/48*(3*b^3*sqrt((64*I*a^4 + 64*a^3 - 64*I*a^2 - 24*a + 9*I)/b^6)*log((b^3
*sqrt((64*I*a^4 + 64*a^3 - 64*I*a^2 - 24*a + 9*I)/b^6) + (8*a^2 - 4*I*a -
3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*a^2 - 4*I*a
 - 3)) - 3*b^3*sqrt((64*I*a^4 + 64*a^3 - 64*I*a^2 - 24*a + 9*I)/b^6)*log(-
(b^3*sqrt((64*I*a^4 + 64*a^3 - 64*I*a^2 - 24*a + 9*I)/b^6) - (8*a^2 - 4*I*
a - 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*a^2 - 4
*I*a - 3)) - 3*b^3*sqrt((-64*I*a^4 - 64*a^3 + 64*I*a^2 + 24*a - 9*I)/b^6)*
log((b^3*sqrt((-64*I*a^4 - 64*a^3 + 64*I*a^2 + 24*a - 9*I)/b^6) + (8*a^2 -
 4*I*a - 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(8*a^
2 - 4*I*a - 3)) + 3*b^3*sqrt((-64*I*a^4 - 64*a^3 + 64*I*a^2 + 24*a - 9*I)/
b^6)*log(-(b^3*sqrt((-64*I*a^4 - 64*a^3 + 64*I*a^2 + 24*a - 9*I)/b^6) - (8
*a^2 - 4*I*a - 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))
/(8*a^2 - 4*I*a - 3)) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(-8*I*b^2*x^2
- 2*(-4*I*a - 5)*b*x - 8*I*a^2 - 26*a + 11*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)/(b*x + a + I)))/b^3
```

Sympy [F]

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x^2\,dx = \int \frac{x^2}{\sqrt{\frac{i(a+bx-i)}{\sqrt{a^2+2abx+b^2x^2+1}}}}\,dx$$

input
```
integrate(x**2/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2),x)
```

output
```
Integral(x**2/sqrt(I*(a + b*x - I)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)),
x)
```

Maxima [F]

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x^2\,dx = \int \frac{x^2}{\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}\,dx$$

input
```
integrate(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(x^2/sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x^2\,dx = \int \frac{x^2}{\sqrt{\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}}}\,dx$$

input
```
int(x^2/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2),x)
```

output
```
int(x^2/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x^2\,dx=\int\frac{x^2}{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}}dx$$

input
```
int(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

output
```
int(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

3.241 $\int e^{-\frac{1}{2}i\arctan(a+bx)}x\,dx$

Optimal result

Integrand size = 16, antiderivative size = 314

$$\begin{aligned}\int e^{-\frac{1}{2}i\arctan(a+bx)}x\,dx &= \frac{(1+4ia)\sqrt[4]{1-ia-ibx}(1+ia+ibx)^{3/4}}{4b^2} \\ &+ \frac{(1-ia-ibx)^{5/4}(1+ia+ibx)^{3/4}}{2b^2} \\ &+ \frac{(1+4ia)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2} \\ &- \frac{(1+4ia)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2} \\ &- \frac{(1+4ia)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\right)}\right)}{4\sqrt{2}b^2}\end{aligned}$$

output
```
1/4*(1+4*I*a)*(1-I*a-I*b*x)^(1/4)*(1+I*a+I*b*x)^(3/4)/b^2+1/2*(1-I*a-I*b*x
)^(5/4)*(1+I*a+I*b*x)^(3/4)/b^2+1/8*(1+4*I*a)*arctan(1-2^(1/2)*(1-I*a-I*b*
x)^(1/4)/(1+I*a+I*b*x)^(1/4))*2^(1/2)/b^2-1/8*(1+4*I*a)*arctan(1+2^(1/2)*(
1-I*a-I*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4))*2^(1/2)/b^2-1/8*(1+4*I*a)*arctanh(
2^(1/2)*(1-I*a-I*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4)/(1+(1-I*a-I*b*x)^(1/2)/(1+
I*a+I*b*x)^(1/2)))*2^(1/2)/b^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.27

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x\,dx = -\frac{i(-i(i+a+bx))^{5/4}\left(5i(1+ia+ibx)^{3/4}+2^{3/4}(-i+4a)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{5}{4},\frac{9}{4},-\frac{1}{2}i(i+a+bx\right.\right.}{10b^2}$$

input
```
Integrate[x/E^((I/2)*ArcTan[a + b*x]),x]
```

output
```
((-1/10*I)*((-I)*(I + a + b*x))^(5/4)*((5*I)*(1 + I*a + I*b*x)^(3/4) + 2^(
3/4)*(-I + 4*a)*Hypergeometric2F1[1/4, 5/4, 9/4, (-1/2*I)*(I + a + b*x)]))
/b^2
```

Rubi [A] (warning: unable to verify)

Time = 0.80 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.13, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {5618, 90, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-\frac{1}{2}i\arctan(a+bx)}dx$$

$$\Big\downarrow \text{ 5618}$$

$$\int \frac{x\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}dx$$

$$\Big\downarrow \text{ 90}$$

$$\frac{(-4a+i)\int \frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}dx}{4b}+\frac{(ia+ibx+1)^{3/4}(-ia-ibx+1)^{5/4}}{2b^2}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{(-4a+i)\left(\frac{1}{2}\int\frac{1}{(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}dx-\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}\right)}{4b}+\frac{(ia+ibx+1)^{3/4}(-ia-ibx+1)^{5/4}}{2b^2}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{(-4a+i)\left(\frac{2i\int\frac{1}{\sqrt[4]{ia+ibx+1}}d\sqrt[4]{-ia-ibx+1}}{b}-\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}\right)}{4b}+\frac{(ia+ibx+1)^{3/4}(-ia-ibx+1)^{5/4}}{2b^2}$$

$$\Big\downarrow \text{ 770}$$

$$\frac{(-4a+i)\left(\frac{2i\int\frac{1}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{b}-\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}\right)}{4b}+\frac{(ia+ibx+1)^{3/4}(-ia-ibx+1)^{5/4}}{2b^2}$$

$$\Big\downarrow \text{ 755}$$

$$\frac{(-4a+i)\left(\frac{2i\left(\frac{1}{2}\int\frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\int\frac{\sqrt{-ia-ibx+1}+1}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{b}-\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+}{b}\right.}{4b}$$

$$\frac{(ia+ibx+1)^{3/4}(-ia-ibx+1)^{5/4}}{2b^2}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{(-4a+i)\left(\frac{2i\left(\frac{1}{2}\int\frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\int -\right.\right.}{b}\right.}{4b}$$

$$\frac{(ia+ibx+1)^{3/4}(-ia-ibx+1)^{5/4}}{2b^2}$$

$\Big\downarrow$ 1082

$$\frac{(-4a+i)\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-\sqrt{-ia-}}{-ia-ib}\cdots}{b}\cdots\right)}{4b}\cdots$$

$$\frac{(ia+ibx+1)^{3/4}(-ia-ibx+1)^{5/4}}{2b^2}$$

$\Big\downarrow$ 217

$$\frac{(-4a+i)\left(\frac{2i\left(\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}\right)\cdots}{b}\cdots\right)}{4b}\cdots$$

$$\frac{(ia+ibx+1)^{3/4}(-ia-ibx+1)^{5/4}}{2b^2}$$

$\Big\downarrow$ 1479

$$(-4a+i)\left(\frac{2i\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}}{2\sqrt{2}}\cdots\right.\right.}{b}\cdots\right)$$

$$\frac{(ia+ibx+1)^{3/4}(-ia-ibx+1)^{5/4}}{2b^2}$$

$\downarrow$ 25

$$(-4a+i)\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia\cdots}}{\sqrt[4]{ia\cdots}}}{2\sqrt{2}}\cdots\right.\right.}{b}\cdots\right)$$

$$4\cdots$$

$$\frac{(ia+ibx+1)^{3/4}(-ia-ibx+1)^{5/4}}{2b^2}$$

$\downarrow$ 27

$$\frac{(-4a+i)\left(\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-\ldots}}{\sqrt[4]{i\ldots}}\right.\right.\right.}{b}\right.}{\frac{(ia+ibx+1)^{3/4}(-ia-ibx+1)^{5/4}}{2b^2}}$$

$\downarrow$ 1103

$$\frac{(-4a+i)\left(\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx}\ldots}{\sqrt[4]{ia+\ldots}}\right.}{2\sqrt{2}}\right.\right.}{b}\right.}{4b}\ldots}{\frac{(ia+ibx+1)^{3/4}(-ia-ibx+1)^{5/4}}{2b^2}}$$

input
```
Int[x/E^((I/2)*ArcTan[a + b*x]),x]
```

output
```
((1 - I*a - I*b*x)^(5/4)*(1 + I*a + I*b*x)^(3/4))/(2*b^2) + ((I - 4*a)*(((
-I)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/b + ((2*I)*((-(ArcTan
[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2]) +
 ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqr
t[2])/2 + (-1/2*Log[1 + Sqrt[1 - I*a - I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)
^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - I*a - I*b*x] +
 (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/(2*Sqrt[2]))/2
))/b))/(4*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [F]

$$\int \frac{x}{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}}dx$$

input
```
int(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

output
```
int(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.34

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x\,dx = -\frac{b^2\sqrt{\frac{16i\,a^2+8\,a-i}{b^4}}\log\left(\frac{b^2\sqrt{\frac{16i\,a^2+8\,a-i}{b^4}}+(4\,a-i)\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{4\,a-i}\right)-b^2\sqrt{\frac{16i\,a^2+8\,a-i}{b^4}}\log\left(-\frac{b^2\sqrt{\frac{16i\,a^2+8\,a-i}{b^4}}-(4\,a-\ldots}{4\,\ldots}\right.}{\ldots}$$

input
```
integrate(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="fricas
")
```

output
```
-1/8*(b^2*sqrt((16*I*a^2 + 8*a - I)/b^4)*log((b^2*sqrt((16*I*a^2 + 8*a - I
)/b^4) + (4*a - I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))
)/(4*a - I)) - b^2*sqrt((16*I*a^2 + 8*a - I)/b^4)*log(-(b^2*sqrt((16*I*a^2
 + 8*a - I)/b^4) - (4*a - I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x
 + a + I)))/(4*a - I)) - b^2*sqrt((-16*I*a^2 - 8*a + I)/b^4)*log((b^2*sqrt
((-16*I*a^2 - 8*a + I)/b^4) + (4*a - I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)/(b*x + a + I)))/(4*a - I)) + b^2*sqrt((-16*I*a^2 - 8*a + I)/b^4)*lo
g(-(b^2*sqrt((-16*I*a^2 - 8*a + I)/b^4) - (4*a - I)*sqrt(I*sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)/(b*x + a + I)))/(4*a - I)) - 2*sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)*(-2*I*b*x + 2*I*a + 3)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/
(b*x + a + I)))/b^2
```

Sympy [F]

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x\,dx = \int \frac{x}{\sqrt{\frac{i(a+bx-i)}{\sqrt{a^2+2abx+b^2x^2+1}}}}\,dx$$

input
```
integrate(x/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2),x)
```

output
```
Integral(x/sqrt(I*(a + b*x - I)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
```

Maxima [F]

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x\,dx = \int \frac{x}{\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}\,dx$$

input
```
integrate(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(x/sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x\,dx = \int \frac{x}{\sqrt{\frac{1+a\,1i+b\,x\,1i}{\sqrt{(a+b\,x)^2+1}}}}\,dx$$

input
```
int(x/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2),x)
```

output
```
int(x/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}x\,dx = \int \frac{x}{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}}dx$$

input
```
int(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

output
```
int(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

3.242 $\int e^{-\frac{1}{2} i \arctan (a+b x)} dx$

Optimal result

Integrand size = 14, antiderivative size = 245

$$\int e^{-\frac{1}{2} i \arctan (a+b x)} dx = -\frac{i \sqrt[4]{1-i a-i b x}(1+i a+i b x)^{3/4}}{b} - \frac{i \arctan \left(1-\frac{\sqrt{2} \sqrt[4]{1-i a-i b x}}{\sqrt[4]{1+i a+i b x}}\right)}{\sqrt{2} b} + \frac{i \arctan \left(1+\frac{\sqrt{2} \sqrt[4]{1-i a-i b x}}{\sqrt[4]{1+i a+i b x}}\right)}{\sqrt{2} b} + \frac{i \text{arctanh}\left(\frac{\sqrt{2} \sqrt[4]{1-i a-i b x}}{\sqrt[4]{1+i a+i b x}\left(1+\frac{\sqrt{1-i a-i b x}}{\sqrt{1+i a+i b x}}\right)}\right)}{\sqrt{2} b}$$

output
```
-I*(1-I*a-I*b*x)^(1/4)*(1+I*a+I*b*x)^(3/4)/b-1/2*I*arctan(1-2^(1/2)*(1-I*a
-I*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4))*2^(1/2)/b+1/2*I*arctan(1+2^(1/2)*(1-I*a
-I*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4))*2^(1/2)/b+1/2*I*arctanh(2^(1/2)*(1-I*a-
I*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4)/(1+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2
)))*2^(1/2)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.18

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}\,dx=-\frac{8ie^{\frac{3}{2}i\arctan(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},-e^{2i\arctan(a+bx)}\right)}{3b}$$

input
```
Integrate[E^((-1/2*I)*ArcTan[a + b*x]),x]
```

output
```
(((-8*I)/3)*E^(((3*I)/2)*ArcTan[a + b*x])*Hypergeometric2F1[3/4, 2, 7/4, -
E^((2*I)*ArcTan[a + b*x])])/b
```

Rubi [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.22, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {5616, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}dx$$

$$\Big\downarrow \text{ 5616}$$

$$\int \frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}\int \frac{1}{(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}dx-\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2i\int \frac{1}{\sqrt[4]{ia+ibx+1}}d\sqrt[4]{-ia-ibx+1}}{b}-\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}$$

$$\Big\downarrow \text{770}$$

$$\frac{2i\int \frac{1}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{b}-\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}$$

$$\Big\downarrow \text{755}$$

$$\frac{2i\left(\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\int \frac{\sqrt{-ia-ibx+1}+1}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{b}-\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2i\left(\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\int \frac{}{\sqrt{-i}}\right.\right.}{b}$$

$$\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+}}{-ia-ibx+2}\right.}{b}$$

$$\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{2i\left(\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}\right)\right)}{b}$$

$$\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}$$

$$\Big\downarrow \text{1479}$$

$$\frac{2i\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-i\cdots}}{\sqrt[4]{ia+ib\cdots}}}{2\sqrt{2}}\cdots\right.\right.}{b}\cdots$$

$$\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}$$

$\downarrow$ 25

$$\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+\cdots}}{\sqrt[4]{ia+ibx+1\cdots}}}{2\sqrt{2}}\cdots\right.\right.}{b}\cdots$$

$$\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}$$

$\downarrow$ 27

$$\frac{2i\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ib\cdots}}{\sqrt[4]{ia+ib\cdots}}\cdots\right.\right.}{b}\cdots$$

$$\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}$$

$\downarrow$ 1103

$$\frac{2i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+}}{\sqrt[4]{ia+ibx+1}}\right.}{2\sqrt{2}}\right.\right.}{b}$$

$$\frac{i\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}{b}$$

input
```
Int[E^((-1/2*I)*ArcTan[a + b*x]),x]
```

output
```
((-I)*(1 - I*a - I*b*x)^(1/4)*(1 + I*a + I*b*x)^(3/4))/b + ((2*I)*((-(ArcT
an[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2])
 + ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/S
qrt[2])/2 + (-1/2*Log[1 + Sqrt[1 - I*a - I*b*x] - (Sqrt[2]*(1 - I*a - I*b*
x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - I*a - I*b*x]
 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/(2*Sqrt[2]))
/2))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5616
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^(I*(n/2))/(1 + I*a*c + I*b*c*x)^(I*(n/2)), x] /; FreeQ[{a, b,
c, n}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}}dx$$

input
```
int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

output
```
int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.09

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}\,dx = \frac{b\sqrt{\frac{i}{b^2}}\log\left(b\sqrt{\frac{i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{bx+a+i}}\right)-b\sqrt{\frac{i}{b^2}}\log\left(-b\sqrt{\frac{i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2abx+a^2+1}}{bx+a+i}}\right)-b\sqrt{-\frac{i}{b^2}}\log\left(b\sqrt{\right.}{}$$

input
```
integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="fricas
")
```

output
```
1/2*(b*sqrt(I/b^2)*log(b*sqrt(I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1)/(b*x + a + I))) - b*sqrt(I/b^2)*log(-b*sqrt(I/b^2) + sqrt(I*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - b*sqrt(-I/b^2)*log(b*sqrt(-I/b
^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + b*sqrt(-I
/b^2)*log(-b*sqrt(-I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x
+ a + I))) - 2*I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*sqrt(I*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1)/(b*x + a + I)))/b
```

Sympy [F]

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}\,dx=\int\frac{1}{\sqrt{\frac{i(a+bx)+1}{\sqrt{(a+bx)^2+1}}}}\,dx$$

input
```
integrate(1/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2),x)
```

output
```
Integral(1/sqrt((I*(a + b*x) + 1)/sqrt((a + b*x)**2 + 1)), x)
```

Maxima [F]

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}\,dx=\int\frac{1}{\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}\,dx$$

input
```
integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}\,dx = \int \frac{1}{\sqrt{\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}}}\,dx$$

input
```
int(1/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2),x)
```

output
```
int(1/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2), x)
```

Reduce [F]

$$\int e^{-\frac{1}{2}i\arctan(a+bx)}\,dx = \int \frac{1}{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}}dx$$

input
```
int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

output
```
int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2),x)
```

3.243 $\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 319

$$\begin{aligned}\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = &-\frac{2\sqrt[4]{i+a}\arctan\left(\frac{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt[4]{i-a}} \\ &-\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right) \\ &+\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right) \\ &-\frac{2\sqrt[4]{i+a}\,\mathrm{arctanh}\left(\frac{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt[4]{i-a}} \\ &+\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}\left(1+\frac{\sqrt{1-i(a+bx)}}{\sqrt{1+i(a+bx)}}\right)}\right)\end{aligned}$$

output
```
-2*(I+a)^(1/4)*arctan((I-a)^(1/4)*(1-I*(b*x+a))^(1/4)/(I+a)^(1/4)/(1+I*(b*
x+a))^(1/4))/(I-a)^(1/4)-2^(1/2)*arctan(1-2^(1/2)*(1-I*(b*x+a))^(1/4)/(1+I
*(b*x+a))^(1/4))+2^(1/2)*arctan(1+2^(1/2)*(1-I*(b*x+a))^(1/4)/(1+I*(b*x+a)
)^(1/4))-2*(I+a)^(1/4)*arctanh((I-a)^(1/4)*(1-I*(b*x+a))^(1/4)/(I+a)^(1/4)
/(1+I*(b*x+a))^(1/4))/(I-a)^(1/4)+2^(1/2)*arctanh(2^(1/2)*(1-I*(b*x+a))^(1
/4)/(1+I*(b*x+a))^(1/4)/(1+(1-I*(b*x+a))^(1/2)/(1+I*(b*x+a))^(1/2)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.39

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = \frac{2\sqrt[4]{-i(i+a+bx)}\left(2^{3/4}\sqrt[4]{1+ia+ibx}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},-\frac{1}{2}i(i+a+bx)\right)-2\,\text{Hypergeometr}\right.}{\sqrt[4]{1+ia+ibx}}$$

input
```
Integrate[1/(E^((I/2)*ArcTan[a + b*x])*x),x]
```

output
```
(2*((-I)*(I + a + b*x))^(1/4)*(2^(3/4)*(1 + I*a + I*b*x)^(1/4)*Hypergeomet
ric2F1[1/4, 1/4, 5/4, (-1/2*I)*(I + a + b*x)] - 2*Hypergeometric2F1[1/4, 1
, 5/4, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)]))/(1 + I*a + I
*b*x)^(1/4)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.37, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {5617, 981, 755, 756, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x}\,dx$$

$\big\downarrow$ 5617

$$-8\int \frac{1-i(a+bx)}{(i(a+bx)+1)\left(\frac{1-i(a+bx)}{i(a+bx)+1}+1\right)\left(-ia-\frac{(ia+1)(1-i(a+bx))}{i(a+bx)+1}+1\right)}d\frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}$$

$\big\downarrow$ 981

$$-8\left(\frac{1}{2}(1-ia)\int \frac{1}{-ia-\frac{(ia+1)(1-i(a+bx))}{i(a+bx)+1}+1}d\frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}-\frac{1}{2}\int \frac{1}{\frac{1-i(a+bx)}{i(a+bx)+1}+1}d\frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}\right)$$

$\big\downarrow$ 755

$$-8\left(\frac{1}{2}(1-ia)\int \frac{1}{-ia-\frac{(ia+1)(1-i(a+bx))}{i(a+bx)+1}+1}d\frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}+\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{\sqrt{1-i(a+bx)}}{\sqrt{i(a+bx)+1}}}{\frac{1-i(a+bx)}{i(a+bx)+1}+1}d\frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}-\frac{1}{2}\right.\right.$$

$\big\downarrow$ 756

$$-8\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{\sqrt{1-i(a+bx)}}{\sqrt{i(a+bx)+1}}}{\frac{1-i(a+bx)}{i(a+bx)+1}+1}d\frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}-\frac{1}{2}\int \frac{\frac{\sqrt{1-i(a+bx)}}{\sqrt{i(a+bx)+1}}+1}{\frac{1-i(a+bx)}{i(a+bx)+1}+1}d\frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}\right)+\frac{1}{2}(1-ia)\left(\frac{i\int \frac{}{\sqrt{a}}}{}\right.\right.$$

$\big\downarrow$ 218

$$-8\left(\frac{1}{2}(1-ia)\left(\frac{i\int \frac{1}{\sqrt{a+i}-\frac{\sqrt{i-a}\sqrt{1-i(a+bx)}}{\sqrt{i(a+bx)+1}}}d\frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}}{2\sqrt{a+i}}+\frac{i\arctan\left(\frac{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}\right)}{2\sqrt[4]{-a+i}(a+i)^{3/4}}\right)+\frac{1}{2}\left(-\frac{1}{2}\int\right.\right.$$

$\big\downarrow$ 221

$$-8\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-\frac{\sqrt{1-i(a+bx)}}{\sqrt{i(a+bx)+1}}}{\frac{1-i(a+bx)}{i(a+bx)+1}+1}d\frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}-\frac{1}{2}\int \frac{\frac{\sqrt{1-i(a+bx)}}{\sqrt{i(a+bx)+1}}+1}{\frac{1-i(a+bx)}{i(a+bx)+1}+1}d\frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}\right)+\frac{1}{2}(1-ia)\left(\frac{i\,\text{arcta}}{}\right.\right.$$

$\big\downarrow$ 1476

$$-8\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{\frac{\sqrt{1-i(a+bx)}}{\sqrt{i(a+bx)+1}}-\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}+1}d\frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}-\frac{1}{2}\int \frac{1}{\frac{\sqrt{1-i(a+bx)}}{\sqrt{i(a+bx)+1}}+\frac{\sqrt{2}\sqrt[4]{1-i(a+b}}{\sqrt[4]{i(a+bx)+}}\right.\right.\right.$$

$$\downarrow 1082$$

$$-8\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\frac{\sqrt{1-i(a+bx)}}{\sqrt{i(a+bx)+1}}-1}d\left(\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}+1\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\frac{\sqrt{1-i(a+bx)}}{\sqrt{i(a+bx)+1}}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\right.\right.$$

$$\downarrow 217$$

$$-8\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt{2}}-\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{1+i(a+bx)}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\frac{\sqrt{1-i(a+bx)}}{\sqrt{i(a+bx)+1}}}{\frac{1-i(a+bx)}{i(a+bx)+1}+1}d\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.\right.$$

$$\downarrow 1479$$

$$-8\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}}{\frac{\sqrt{1-i(a+bx)}}{\sqrt{i(a+bx)+1}}-\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}+1}d\frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}+1\right)}{\frac{\sqrt{1-i(a+bx)}}{\sqrt{i(a+bx)+1}}+\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}+1}d\frac{\sqrt[4]{1-}}{\sqrt[4]{i(}}}{2\sqrt{2}}\right.\right.\right.$$

$$\downarrow 25$$

$$-8\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}}{\frac{\sqrt{1-i(a+bx)}}{\sqrt{i(a+bx)+1}}-\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}+1}d\frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}+1\right)}{\frac{\sqrt{1-i(a+bx)}}{\sqrt{i(a+bx)+1}}+\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}+1}d\frac{\sqrt[4]{1-}}{\sqrt[4]{i(a-}}}{2\sqrt{2}}\right.\right.\right.$$

$$\Big\downarrow \text{27}$$

$$-8\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}}{\frac{\sqrt{1-i(a+bx)}}{\sqrt{i(a+bx)+1}}-\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}+1}d\frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}+1}{\frac{\sqrt{1-i(a+bx)}}{\sqrt{i(a+bx)+1}}+\frac{\sqrt{2}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}+1}d\right.\right.\right.$$

$$\Big\downarrow \text{1103}$$

$$-8\left(\frac{1}{2}(1-ia)\left(\frac{i\arctan\left(\frac{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}\right)}{2\sqrt[4]{-a+i}(a+i)^{3/4}}+\frac{i\text{arctanh}\left(\frac{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}\right)}{2\sqrt[4]{-a+i}(a+i)^{3/4}}\right)+\frac{1}{2}\left(\frac{1}{2}\left(\frac{\text{arct}}{\quad}\right.\right.\right.$$

input
```
Int[1/(E^((I/2)*ArcTan[a + b*x])*x),x]
```

output
```
-8*(((1 - I*a)*(((I/2)*ArcTan[((I - a)^(1/4)*(1 - I*(a + b*x))^(1/4))/((I
+ a)^(1/4)*(1 + I*(a + b*x))^(1/4))])/((I - a)^(1/4)*(I + a)^(3/4)) + ((I/
2)*ArcTanh[((I - a)^(1/4)*(1 - I*(a + b*x))^(1/4))/((I + a)^(1/4)*(1 + I*(
a + b*x))^(1/4))])/((I - a)^(1/4)*(I + a)^(3/4))))/2 + ((ArcTan[1 - (Sqrt[
2]*(1 - I*(a + b*x))^(1/4))/(1 + I*(a + b*x))^(1/4)]/Sqrt[2] - ArcTan[1 +
(Sqrt[2]*(1 - I*(a + b*x))^(1/4))/(1 + I*(a + b*x))^(1/4)]/Sqrt[2])/2 + (L
og[1 + Sqrt[1 - I*(a + b*x)]/Sqrt[1 + I*(a + b*x)] - (Sqrt[2]*(1 - I*(a +
b*x))^(1/4))/(1 + I*(a + b*x))^(1/4)]/(2*Sqrt[2]) - Log[1 + Sqrt[1 - I*(a
+ b*x)]/Sqrt[1 + I*(a + b*x)] + (Sqrt[2]*(1 - I*(a + b*x))^(1/4))/(1 + I*(
a + b*x))^(1/4)]/(2*Sqrt[2]))/2)/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 981
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[(-a)*(e^n/(b*c - a*d))   Int[(e*x)^(m - n)/(a + b*x^n),
x], x] + Simp[c*(e^n/(b*c - a*d))   Int[(e*x)^(m - n)/(c + d*x^n), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
 m, 2*n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5617
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_))*(x_)^(m_), x_Symbol] :> Simp
[4/(I^m*n*b^(m + 1)*c^(m + 1))   Subst[Int[x^(2/(I*n))*((1 - I*a*c - (1 + I
*a*c)*x^(2/(I*n)))^m/(1 + x^(2/(I*n)))^(m + 2)), x], x, (1 - I*c*(a + b*x))
^(I*(n/2))/(1 + I*c*(a + b*x))^(I*(n/2))], x] /; FreeQ[{a, b, c}, x] && ILt
Q[m, 0] && LtQ[-1, I*n, 1]
```

Maple [F]

$$\int \frac{1}{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}\, x}dx$$

input
```
int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x)
```

output
```
int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 470 vs. 2(227) = 454.

Time = 0.10 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.47

$$\begin{aligned}
&\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x}\,dx \\
&= -\frac{1}{2}\sqrt{4i}\log\left(\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right) \\
&+\frac{1}{2}\sqrt{4i}\log\left(-\frac{1}{2}i\sqrt{4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right) \\
&+\frac{1}{2}\sqrt{-4i}\log\left(\frac{1}{2}i\sqrt{-4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right) \\
&-\frac{1}{2}\sqrt{-4i}\log\left(-\frac{1}{2}i\sqrt{-4i}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right) \\
&+\left(-\frac{a+i}{a-i}\right)^{\frac{1}{4}}\log\left(\frac{(a-i)\left(-\frac{a+i}{a-i}\right)^{\frac{3}{4}}+(a+i)\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{a+i}\right) \\
&-\left(-\frac{a+i}{a-i}\right)^{\frac{1}{4}}\log\left(-\frac{(a-i)\left(-\frac{a+i}{a-i}\right)^{\frac{3}{4}}-(a+i)\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{a+i}\right) \\
&-i\left(-\frac{a+i}{a-i}\right)^{\frac{1}{4}}\log\left(\frac{(i\,a+1)\left(-\frac{a+i}{a-i}\right)^{\frac{3}{4}}+(a+i)\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{a+i}\right) \\
&+i\left(-\frac{a+i}{a-i}\right)^{\frac{1}{4}}\log\left(\frac{(-i\,a-1)\left(-\frac{a+i}{a-i}\right)^{\frac{3}{4}}+(a+i)\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{a+i}\right)
\end{aligned}$$

input
```
integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x, algorithm="fric
as")
```

output
```
-1/2*sqrt(4*I)*log(1/2*I*sqrt(4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1)/(b*x + a + I))) + 1/2*sqrt(4*I)*log(-1/2*I*sqrt(4*I) + sqrt(I*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + 1/2*sqrt(-4*I)*log(1/2*I*sqrt(
-4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - 1/2*sqr
t(-4*I)*log(-1/2*I*sqrt(-4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(
b*x + a + I))) + (-(a + I)/(a - I))^(1/4)*log(((a - I)*(-(a + I)/(a - I))^
(3/4) + (a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(
a + I)) - (-(a + I)/(a - I))^(1/4)*log(-((a - I)*(-(a + I)/(a - I))^(3/4)
- (a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(a + I)
) - I*(-(a + I)/(a - I))^(1/4)*log(((I*a + 1)*(-(a + I)/(a - I))^(3/4) + (
a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(a + I)) +
 I*(-(a + I)/(a - I))^(1/4)*log(((-I*a - 1)*(-(a + I)/(a - I))^(3/4) + (a
+ I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(a + I))
```

Sympy [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = \int \frac{1}{x\sqrt{\frac{i(a+bx-i)}{\sqrt{a^2+2abx+b^2x^2+1}}}}\,dx$$

input
```
integrate(1/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2)/x,x)
```

output
```
Integral(1/(x*sqrt(I*(a + b*x - I)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))),
 x)
```

Maxima [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = \int \frac{1}{x\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}\,dx$$

input
```
integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x, algorithm="maxi
ma")
```

output
```
integrate(1/(x*sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = \int \frac{1}{x\sqrt{\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}}}\,dx$$

input
```
int(1/(x*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2)),x)
```

output
```
int(1/(x*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x}\,dx = \int \frac{1}{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}\,x}dx$$

input
```
int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x)
```

output
```
int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x,x)
```

3.244 $\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 210

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = -\frac{(i-a-bx)\sqrt[4]{1-i(a+bx)}}{(i-a)x\sqrt[4]{1+i(a+bx)}} - \frac{ib\arctan\left(\frac{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}\right)}{(i-a)^{5/4}(i+a)^{3/4}} - \frac{ib\,\mathrm{arctanh}\left(\frac{\sqrt[4]{i-a}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i+a}\sqrt[4]{1+i(a+bx)}}\right)}{(i-a)^{5/4}(i+a)^{3/4}}$$

output

```
-(I-a-b*x)*(1-I*(b*x+a))^(1/4)/(I-a)/x/(1+I*(b*x+a))^(1/4)-I*b*arctan((I-a
)^(1/4)*(1-I*(b*x+a))^(1/4)/(I+a)^(1/4)/(1+I*(b*x+a))^(1/4))/(I-a)^(5/4)/(
I+a)^(3/4)-I*b*arctanh((I-a)^(1/4)*(1-I*(b*x+a))^(1/4)/(I+a)^(1/4)/(1+I*(b
*x+a))^(1/4))/(I-a)^(5/4)/(I+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.51

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = -\frac{\sqrt[4]{-i(i+a+bx)}\left(1+a^2+ibx+abx-2ibx\,\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},\frac{1+a^2-ibx+abx}{1+a^2+ibx+abx}\right)\right)}{(1+a^2)\,x\sqrt[4]{1+ia+ibx}}$$

input
```
Integrate[1/(E^((I/2)*ArcTan[a + b*x])*x^2),x]
```

output
```
-((((-I)*(I + a + b*x))^(1/4)*(1 + a^2 + I*b*x + a*b*x - (2*I)*b*x*Hyperge
ometric2F1[1/4, 1, 5/4, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x
)]))/((1 + a^2)*x*(1 + I*a + I*b*x)^(1/4)))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5617, 817, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx$$

$$\Big\downarrow \text{ 5617}$$

$$-8ib\int \frac{1-i(a+bx)}{(i(a+bx)+1)\left(-ia-\frac{(ia+1)(1-i(a+bx))}{i(a+bx)+1}+1\right)^2}d\frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}$$

$$\Big\downarrow \text{ 817}$$

$$-8ib\left(\frac{\sqrt[4]{1-i(a+bx)}}{4(1+ia)\sqrt[4]{1+i(a+bx)}\left(-\frac{(1+ia)(1-i(a+bx))}{1+i(a+bx)}-ia+1\right)}-\frac{\int\frac{1}{-ia-\frac{(ia+1)(1-i(a+bx))}{i(a+bx)+1}+1}d\frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}}{4(1+ia)}\right)$$

$\Big\downarrow$ 756

$$-8ib\left(\frac{\sqrt[4]{1-i(a+bx)}}{4(1+ia)\sqrt[4]{1+i(a+bx)}\left(-\frac{(1+ia)(1-i(a+bx))}{1+i(a+bx)}-ia+1\right)}-\frac{\frac{i\int\frac{1}{\sqrt{a+i}-\frac{\sqrt{i-a}\sqrt{1-i(a+bx)}}{\sqrt{i(a+bx)+1}}}d\frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}}{2\sqrt{a+i}}+\frac{i\int -}{}}{4(1+ia)}\right)$$

$\Big\downarrow$ 218

$$-8ib\left(\frac{\sqrt[4]{1-i(a+bx)}}{4(1+ia)\sqrt[4]{1+i(a+bx)}\left(-\frac{(1+ia)(1-i(a+bx))}{1+i(a+bx)}-ia+1\right)}-\frac{\frac{i\int\frac{1}{\sqrt{a+i}-\frac{\sqrt{i-a}\sqrt{1-i(a+bx)}}{\sqrt{i(a+bx)+1}}}d\frac{\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{i(a+bx)+1}}}{2\sqrt{a+i}}+\frac{i\,\mathrm{arcta}}{}}{4(1+ia)}\right)$$

$\Big\downarrow$ 221

$$-8ib\left(\frac{\sqrt[4]{1-i(a+bx)}}{4(1+ia)\sqrt[4]{1+i(a+bx)}\left(-\frac{(1+ia)(1-i(a+bx))}{1+i(a+bx)}-ia+1\right)}-\frac{\frac{i\arctan\left(\frac{\sqrt[4]{-a+i}\sqrt[4]{1-i(a+bx)}}{\sqrt[4]{a+i}\sqrt[4]{1+i(a+bx)}}\right)}{2\sqrt[4]{-a+i}(a+i)^{3/4}}+\frac{i\,\mathrm{arctanh}}{}}{4(1+ia)}\right)$$

input
```
Int[1/(E^((I/2)*ArcTan[a + b*x])*x^2),x]
```

output
```
(-8*I)*b*((1 - I*(a + b*x))^(1/4)/(4*(1 + I*a)*(1 + I*(a + b*x))^(1/4)*(1
- I*a - ((1 + I*a)*(1 - I*(a + b*x)))/(1 + I*(a + b*x)))) - (((I/2)*ArcTan
[((I - a)^(1/4)*(1 - I*(a + b*x))^(1/4))/((I + a)^(1/4)*(1 + I*(a + b*x))^
(1/4))])/((I - a)^(1/4)*(I + a)^(3/4)) + ((I/2)*ArcTanh[((I - a)^(1/4)*(1
- I*(a + b*x))^(1/4))/((I + a)^(1/4)*(1 + I*(a + b*x))^(1/4))])/((I - a)^(
1/4)*(I + a)^(3/4)))/(4*(1 + I*a)))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 5617
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_))*(x_)^(m_), x_Symbol] :> Simp
[4/(I^m*n*b^(m + 1)*c^(m + 1))   Subst[Int[x^(2/(I*n))*((1 - I*a*c - (1 + I
*a*c)*x^(2/(I*n)))^m/(1 + x^(2/(I*n)))^(m + 2)), x], x, (1 - I*c*(a + b*x))
^(I*(n/2))/(1 + I*c*(a + b*x))^(I*(n/2))], x] /; FreeQ[{a, b, c}, x] && ILt
Q[m, 0] && LtQ[-1, I*n, 1]
```

Maple [F]

$$\int \frac{1}{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}\,x^2}\,dx$$

input
```
int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x)
```

output
```
int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(141) = 282$.

Time = 0.09 (sec) , antiderivative size = 707, normalized size of antiderivative = 3.37

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x, algorithm="fr
icas")
```

output
```
1/2*((-b^4/(a^8 - 2*I*a^7 + 2*a^6 - 6*I*a^5 - 6*I*a^3 - 2*a^2 - 2*I*a - 1)
)^(1/4)*(-I*a - 1)*x*log((b^3*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*
x + a + I)) + (a^6 - 2*I*a^5 + a^4 - 4*I*a^3 - a^2 - 2*I*a - 1)*(-b^4/(a^8
 - 2*I*a^7 + 2*a^6 - 6*I*a^5 - 6*I*a^3 - 2*a^2 - 2*I*a - 1))^(3/4))/b^3) +
 (-b^4/(a^8 - 2*I*a^7 + 2*a^6 - 6*I*a^5 - 6*I*a^3 - 2*a^2 - 2*I*a - 1))^(1
/4)*(I*a + 1)*x*log((b^3*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a
 + I)) - (a^6 - 2*I*a^5 + a^4 - 4*I*a^3 - a^2 - 2*I*a - 1)*(-b^4/(a^8 - 2*
I*a^7 + 2*a^6 - 6*I*a^5 - 6*I*a^3 - 2*a^2 - 2*I*a - 1))^(3/4))/b^3) + (-b^
4/(a^8 - 2*I*a^7 + 2*a^6 - 6*I*a^5 - 6*I*a^3 - 2*a^2 - 2*I*a - 1))^(1/4)*(
a - I)*x*log((b^3*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))
- (I*a^6 + 2*a^5 + I*a^4 + 4*a^3 - I*a^2 + 2*a - I)*(-b^4/(a^8 - 2*I*a^7 +
 2*a^6 - 6*I*a^5 - 6*I*a^3 - 2*a^2 - 2*I*a - 1))^(3/4))/b^3) - (-b^4/(a^8
- 2*I*a^7 + 2*a^6 - 6*I*a^5 - 6*I*a^3 - 2*a^2 - 2*I*a - 1))^(1/4)*(a - I)*
x*log((b^3*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) - (-I*a
^6 - 2*a^5 - I*a^4 - 4*a^3 + I*a^2 - 2*a + I)*(-b^4/(a^8 - 2*I*a^7 + 2*a^6
 - 6*I*a^5 - 6*I*a^3 - 2*a^2 - 2*I*a - 1))^(3/4))/b^3) + 2*I*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I
)))/((a - I)*x)
```

Sympy [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = \int \frac{1}{x^2\sqrt{\frac{i(a+bx-i)}{\sqrt{a^2+2abx+b^2x^2+1}}}}\,dx$$

input
```
integrate(1/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(1/2)/x**2,x)
```

output
```
Integral(1/(x**2*sqrt(I*(a + b*x - I)/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)
)), x)
```

Maxima [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = \int \frac{1}{x^2\sqrt{\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}}}\,dx$$

input
```
integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x, algorithm="ma
xima")
```

output
```
integrate(1/(x^2*sqrt((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = \int \frac{1}{x^2\sqrt{\frac{1+a\,1i+b\,x\,1i}{\sqrt{(a+b\,x)^2+1}}}}\,dx$$

input
```
int(1/(x^2*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2)),x)
```

output
```
int(1/(x^2*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{e^{-\frac{1}{2}i\arctan(a+bx)}}{x^2}\,dx = \int \frac{1}{\sqrt{\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}}\,x^2}dx$$

input
```
int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x)
```

output
```
int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(1/2)/x^2,x)
```

3.245 $\int e^{-\frac{3}{2}i\arctan(a+bx)}x^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 391

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x^2\,dx = \frac{(17i-36a-24ia^2)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{24b^3} + \frac{(3i-8a)(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{12b^3} + \frac{x(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{3b^2} + \frac{(17i-36a-24ia^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3} - \frac{(17i-36a-24ia^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{8\sqrt{2}b^3} + \frac{(17i-36a-24ia^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\right)}\right)}{8\sqrt{2}b^3}$$

output
```
1/24*(17*I-36*a-24*I*a^2)*(1-I*a-I*b*x)^(3/4)*(1+I*a+I*b*x)^(1/4)/b^3+1/12
*(3*I-8*a)*(1-I*a-I*b*x)^(7/4)*(1+I*a+I*b*x)^(1/4)/b^3+1/3*x*(1-I*a-I*b*x)
^(7/4)*(1+I*a+I*b*x)^(1/4)/b^2+1/16*(17*I-36*a-24*I*a^2)*arctan(1-2^(1/2)*
(1-I*a-I*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4))*2^(1/2)/b^3-1/16*(17*I-36*a-24*I*
a^2)*arctan(1+2^(1/2)*(1-I*a-I*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4))*2^(1/2)/b^3
+1/16*(17*I-36*a-24*I*a^2)*arctanh(2^(1/2)*(1-I*a-I*b*x)^(1/4)/(1+I*a+I*b*
x)^(1/4)/(1+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2)))*2^(1/2)/b^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.25

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x^2\,dx$$
$$=\frac{(-i(i+a+bx))^{7/4}\left(7\sqrt[4]{1+ia+ibx}(3i-8a+4bx)+\sqrt[4]{2}(-17i+36a+24ia^2)\,\text{Hypergeometric2F1}\left(\frac{3}{4}\right.\right.}{84b^3}$$

input
```
Integrate[x^2/E^(((3*I)/2)*ArcTan[a + b*x]),x]
```

output
```
(((-I)*(I + a + b*x))^(7/4)*(7*(1 + I*a + I*b*x)^(1/4)*(3*I - 8*a + 4*b*x)
 + 2^(1/4)*(-17*I + 36*a + (24*I)*a^2)*Hypergeometric2F1[3/4, 7/4, 11/4, (
-1/2*I)*(I + a + b*x)]))/(84*b^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.98 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5618, 101, 27, 90, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2e^{-\frac{3}{2}i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{x^2(-ia-ibx+1)^{3/4}}{(ia+ibx+1)^{3/4}}dx$$

$$\Big\downarrow \text{101}$$

$$\frac{\int -\frac{(-ia-ibx+1)^{3/4}(2a^2-(3i-8a)bx+2)}{2(ia+ibx+1)^{3/4}}dx}{3b^2}+\frac{x\sqrt[4]{ia+ibx+1}(-ia-ibx+1)^{7/4}}{3b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x(-ia-ibx+1)^{7/4}\sqrt[4]{ia+ibx+1}}{3b^2}-\frac{\int \frac{(-ia-ibx+1)^{3/4}(2(a^2+1)-(3i-8a)bx)}{(ia+ibx+1)^{3/4}}dx}{6b^2}$$

$$\Big\downarrow \text{90}$$

$$\frac{x(-ia-ibx+1)^{7/4}\sqrt[4]{ia+ibx+1}}{3b^2}-\frac{\frac{1}{4}\left(-24a^2+36ia+17\right)\int \frac{(-ia-ibx+1)^{3/4}}{(ia+ibx+1)^{3/4}}dx-\frac{(-8a+3i)(-ia-ibx+1)^{7/4}\sqrt[4]{ia+ibx+1}}{2b}}{6b^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{x(-ia-ibx+1)^{7/4}\sqrt[4]{ia+ibx+1}}{3b^2}-\frac{\frac{1}{4}\left(-24a^2+36ia+17\right)\left(\frac{3}{2}\int \frac{1}{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}dx-\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}\right)-\frac{(-8a+3i)(-ia-ibx+}{}}{6b^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{x(-ia-ibx+1)^{7/4}\sqrt[4]{ia+ibx+1}}{3b^2}-\frac{\frac{1}{4}\left(-24a^2+36ia+17\right)\left(\frac{6i\int \frac{\sqrt{-ia-ibx+1}}{(ia+ibx+1)^{3/4}}d\sqrt[4]{-ia-ibx+1}}{b}-\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}\right)-\frac{(-8a+3i)(-ia-ibx+1)}{2b}}{6b^2}$$

$$\Big\downarrow \text{854}$$

$$\frac{x(-ia-ibx+1)^{7/4}\sqrt[4]{ia+ibx+1}}{3b^2}-\frac{\frac{1}{4}\left(-24a^2+36ia+17\right)\left(\frac{6i\int \frac{\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{b}-\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}\right)-\frac{(-8a+3i)(-ia-ibx+1)^{7}}{2b}}{6b^2}$$

$$\Big\downarrow \text{826}$$

$$\frac{\frac{x(-ia-ibx+1)^{7/4}\sqrt[4]{ia+ibx+1}}{3b^2}-\frac{\frac{1}{4}\left(-24a^2+36ia+17\right)\left(\frac{6i\left(\frac{1}{2}\int\frac{\sqrt{-ia-ibx+1}+1}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}-\frac{1}{2}\int\frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{b}-\frac{i(-ia-ibx+1)^{3}\cdots}{}\right.}{6b^2}}{}$$

$\downarrow$ 1476

$$\frac{x(-ia-ibx+1)^{7/4}\sqrt[4]{ia+ibx+1}}{3b^2}-\frac{\frac{1}{4}\left(-24a^2+36ia+17\right)\left(\frac{6i\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\int\frac{1}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-i\cdots}}{\sqrt[4]{ia+ibx\cdots}}}\cdots\right.\right.}{b}\right.}{}$$

$\downarrow$ 1082

$$\frac{x(-ia-ibx+1)^{7/4}\sqrt[4]{ia+ibx+1}}{3b^2}-\frac{\frac{1}{4}\left(-24a^2+36ia+17\right)\left(\frac{6i\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}\right)\cdots\right.}{b}\right.}{6b^2}$$

$\downarrow$ 217

$$\frac{x(-ia-ibx+1)^{7/4}\sqrt[4]{ia+ibx+1}}{3b^2}-\frac{\frac{1}{4}\left(-24a^2+36ia+17\right)\left(\frac{6i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-\cdots}}{\sqrt[4]{\cdots}}\cdots\right.}{b}\right.}{6b^2}$$

$\downarrow$ 1479

$$\frac{1}{4}\left(-24a^2+36ia+17\right)\left(\frac{x(-ia-ibx+1)^{7/4}\sqrt[4]{ia+ibx+1}}{3b^2}-\frac{6i\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+}}{\sqrt[4]{ia+ibx+1}}\right.}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ib}}{\sqrt[4]{ia+ibx}}}}{2\sqrt{}}\right.\right.}{}\right.$$

$\downarrow$ 25

$$\frac{1}{4}\left(-24a^2+36ia+17\right)\left(\frac{x(-ia-ibx+1)^{7/4}\sqrt[4]{ia+ibx+1}}{3b^2}-\frac{6i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right.}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx}}{\sqrt[4]{ia+ibx-}}}}{2\sqrt{}}\right.\right.}{}\right.$$

$\downarrow$ 27

$$\frac{x(-ia-ibx+1)^{7/4}\sqrt[4]{ia+ibx+1}}{3b^2}-$$

$$\frac{1}{4}(-24a^2+36ia+17)\left(\frac{6i\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt[4]{-ia-ibx+}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-i}}{\sqrt[4]{ia+ibx}}}\right.\right.\right.}{}\right.$$

↓ 1103

$$\frac{x(-ia-ibx+1)^{7/4}\sqrt[4]{ia+ibx+1}}{3b^2}-$$

$$\frac{1}{4}(-24a^2+36ia+17)\left(\frac{6i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{-ia-ibx+1}-\sqrt{2}\right.}{}\right.\right.}{b}\right.$$

input
```
Int[x^2/E^(((3*I)/2)*ArcTan[a + b*x]),x]
```

output
```
(x*(1 - I*a - I*b*x)^(7/4)*(1 + I*a + I*b*x)^(1/4))/(3*b^2) - (-1/2*((3*I
- 8*a)*(1 - I*a - I*b*x)^(7/4)*(1 + I*a + I*b*x)^(1/4))/b + ((17 + (36*I)*
a - 24*a^2)*(((-I)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/b + ((
6*I)*((-(ArcTan[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1
/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*
b*x)^(1/4)]/Sqrt[2])/2 + (Log[1 + Sqrt[1 - I*a - I*b*x] - (Sqrt[2]*(1 - I*
a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/(2*Sqrt[2]) - Log[1 + Sqrt[1 -
I*a - I*b*x] + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/
(2*Sqrt[2]))/2))/b))/4)/(6*b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [F]

$$\int \frac{x^2}{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

output
```
int(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 555 vs. 2(265) = 530.

Time = 0.13 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.42

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x^2\,dx$$

$$= \frac{3\,b^3\sqrt{\frac{576i\,a^4+1728\,a^3-2112i\,a^2-1224\,a+289i}{b^6}}\log\left(\frac{i\,b^3\sqrt{\frac{576i\,a^4+1728\,a^3-2112i\,a^2-1224\,a+289i}{b^6}}+(24\,a^2-36i\,a-17)\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+}}{bx+a+i}}\right.}{24\,a^2-36i\,a-17}}{}$$

input
```
integrate(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="fric
as")
```

output
```
1/48*(3*b^3*sqrt((576*I*a^4 + 1728*a^3 - 2112*I*a^2 - 1224*a + 289*I)/b^6)
*log((I*b^3*sqrt((576*I*a^4 + 1728*a^3 - 2112*I*a^2 - 1224*a + 289*I)/b^6)
 + (24*a^2 - 36*I*a - 17)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x +
a + I)))/(24*a^2 - 36*I*a - 17)) - 3*b^3*sqrt((576*I*a^4 + 1728*a^3 - 2112
*I*a^2 - 1224*a + 289*I)/b^6)*log((-I*b^3*sqrt((576*I*a^4 + 1728*a^3 - 211
2*I*a^2 - 1224*a + 289*I)/b^6) + (24*a^2 - 36*I*a - 17)*sqrt(I*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(24*a^2 - 36*I*a - 17)) + 3*b^3*sqr
t((-576*I*a^4 - 1728*a^3 + 2112*I*a^2 + 1224*a - 289*I)/b^6)*log((I*b^3*sq
rt((-576*I*a^4 - 1728*a^3 + 2112*I*a^2 + 1224*a - 289*I)/b^6) + (24*a^2 -
36*I*a - 17)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(24*
a^2 - 36*I*a - 17)) - 3*b^3*sqrt((-576*I*a^4 - 1728*a^3 + 2112*I*a^2 + 122
4*a - 289*I)/b^6)*log((-I*b^3*sqrt((-576*I*a^4 - 1728*a^3 + 2112*I*a^2 + 1
224*a - 289*I)/b^6) + (24*a^2 - 36*I*a - 17)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)/(b*x + a + I)))/(24*a^2 - 36*I*a - 17)) - 2*(8*b^3*x^3 + 22*I*
b^2*x^2 + 8*a^3 - (40*I*a + 37)*b*x - 38*I*a^2 + 23*a - 23*I)*sqrt(I*sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/b^3
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x^2\,dx = \text{Timed out}$$

input
```
integrate(x**2/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x^2\,dx = \int \frac{x^2}{\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate(x^2/((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x^2\,dx = \int \frac{x^2}{\left(\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}\right)^{3/2}}\,dx$$

input
```
int(x^2/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2),x)
```

output
```
int(x^2/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2), x)
```

Reduce [F]

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x^2\,dx = \int \frac{x^2}{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

output
```
int(x^2/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

3.246 $\int e^{-\frac{3}{2}i\arctan(a+bx)}x\,dx$

Optimal result

Integrand size = 16, antiderivative size = 314

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x\,dx=\frac{(3+4ia)(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{4b^2}+\frac{(1-ia-ibx)^{7/4}\sqrt[4]{1+ia+ibx}}{2b^2}+\frac{3(3+4ia)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2}-\frac{3(3+4ia)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{4\sqrt{2}b^2}+\frac{3(3+4ia)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\right)}\right)}{4\sqrt{2}b^2}$$

output
```
1/4*(3+4*I*a)*(1-I*a-I*b*x)^(3/4)*(1+I*a+I*b*x)^(1/4)/b^2+1/2*(1-I*a-I*b*x
)^(7/4)*(1+I*a+I*b*x)^(1/4)/b^2+3/8*(3+4*I*a)*arctan(1-2^(1/2)*(1-I*a-I*b*
x)^(1/4)/(1+I*a+I*b*x)^(1/4))*2^(1/2)/b^2-3/8*(3+4*I*a)*arctan(1+2^(1/2)*(
1-I*a-I*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4))*2^(1/2)/b^2+3/8*(3+4*I*a)*arctanh(
2^(1/2)*(1-I*a-I*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4)/(1+(1-I*a-I*b*x)^(1/2)/(1+
I*a+I*b*x)^(1/2)))*2^(1/2)/b^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.27

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x\,dx = -\frac{i(-i(i+a+bx))^{7/4}\left(7i\sqrt[4]{1+ia+ibx}+\sqrt[4]{2}(-3i+4a)\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{4},\frac{11}{4},-\frac{1}{2}i(i+a+bx)\right)\right)}{14b^2}$$

input
```
Integrate[x/E^(((3*I)/2)*ArcTan[a + b*x]),x]
```

output
```
((-1/14*I)*((-I)*(I + a + b*x))^(7/4)*((7*I)*(1 + I*a + I*b*x)^(1/4) + 2^(
1/4)*(-3*I + 4*a)*Hypergeometric2F1[3/4, 7/4, 11/4, (-1/2*I)*(I + a + b*x)
]))/b^2
```

Rubi [A] (warning: unable to verify)

Time = 0.80 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.13, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {5618, 90, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{-\frac{3}{2}i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{x(-ia-ibx+1)^{3/4}}{(ia+ibx+1)^{3/4}}dx$$

$$\Big\downarrow \text{90}$$

$$\frac{(-4a+3i)\int \frac{(-ia-ibx+1)^{3/4}}{(ia+ibx+1)^{3/4}}dx}{4b}+\frac{\sqrt[4]{ia+ibx+1}(-ia-ibx+1)^{7/4}}{2b^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{(-4a+3i)\left(\frac{3}{2}\int \frac{1}{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}dx-\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}\right)}{4b}+\frac{\sqrt[4]{ia+ibx+1}(-ia-ibx+1)^{7/4}}{2b^2}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{(-4a+3i)\left(\frac{6i\int \frac{\sqrt{-ia-ibx+1}}{(ia+ibx+1)^{3/4}}d\sqrt[4]{-ia-ibx+1}}{b}-\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}\right)}{4b}+\frac{\sqrt[4]{ia+ibx+1}(-ia-ibx+1)^{7/4}}{2b^2}$$

$$\Big\downarrow \text{ 854}$$

$$\frac{(-4a+3i)\left(\frac{6i\int \frac{\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{b}-\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}\right)}{4b}+\frac{\sqrt[4]{ia+ibx+1}(-ia-ibx+1)^{7/4}}{2b^2}$$

$$\Big\downarrow \text{ 826}$$

$$\frac{(-4a+3i)\left(\frac{6i\left(\frac{1}{2}\int \frac{\sqrt{-ia-ibx+1}+1}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}-\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{b}-\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}\right)}{4b}+\frac{\sqrt[4]{ia+ibx+1}(-ia-ibx+1)^{7/4}}{2b^2}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{(-4a+3i)\left(\frac{6i\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\int \frac{1}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{}}{}\right.\right.}{b}\right.}{4b}+\frac{\sqrt[4]{ia+ibx+1}(-ia-ibx+1)^{7/4}}{2b^2}$$

$$\Big\downarrow \text{ 1082}$$

$$(-4a+3i)\left(\frac{6i\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{-ia}}{-ia-}\right.}{b}\right.$$

$$4b$$

$$\frac{\sqrt[4]{ia+ibx+1}(-ia-ibx+1)^{7/4}}{2b^2}$$

$\big\downarrow$ 217

$$(-4a+3i)\left(\frac{6i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+}}{\sqrt[4]{ia+ibx+1}}\right.}{b}\right.$$

$$4b$$

$$\frac{\sqrt[4]{ia+ibx+1}(-ia-ibx+1)^{7/4}}{2b^2}$$

$\big\downarrow$ 1479

$$(-4a+3i)\left(\frac{6i\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{}}{\sqrt[4]{}}}{2\sqrt{2}}\right.\right.}{b}\right.$$

$$\frac{\sqrt[4]{ia+ibx+1}(-ia-ibx+1)^{7/4}}{2b^2}$$

$$\Big\downarrow \; 25$$

$$(-4a+3i)\left(\frac{6i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-}}{\sqrt[4]{}}}{2\sqrt{2}}\right)\right)}{b}\right)$$

$$\frac{\sqrt[4]{ia+ibx+1}(-ia-ibx+1)^{7/4}}{2b^2}$$

$$\Big\downarrow \; 27$$

$$(-4a+3i)\left(\frac{6i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{}}{}\right)\right)}{b}\right)$$

$$\frac{\sqrt[4]{ia+ibx+1}(-ia-ibx+1)^{7/4}}{2b^2}$$

$$\Big\downarrow \; 1103$$

$$(-4a+3i)\left(\frac{6i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx}}{\sqrt[4]{ia+ibx}}\right.}{2\sqrt{2}}\right.\right.}{b}\right.}{4b}$$

$$\frac{\sqrt[4]{ia+ibx+1}(-ia-ibx+1)^{7/4}}{2b^2}$$

input
```
Int[x/E^(((3*I)/2)*ArcTan[a + b*x]),x]
```

output
```
((1 - I*a - I*b*x)^(7/4)*(1 + I*a + I*b*x)^(1/4))/(2*b^2) + ((3*I - 4*a)*(
((-I)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/b + ((6*I)*((-(ArcT
an[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2])
 + ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/S
qrt[2])/2 + (Log[1 + Sqrt[1 - I*a - I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1
/4))/(1 + I*a + I*b*x)^(1/4)]/(2*Sqrt[2]) - Log[1 + Sqrt[1 - I*a - I*b*x]
+ (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/(2*Sqrt[2]))/
2))/b))/(4*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [F]

$$\int \frac{x}{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}}}dx$$

input
```
int(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

output
```
int(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(212) = 424$.

Time = 0.14 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.38

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x\,dx = \frac{3\,b^2\sqrt{-\frac{16i\,a^2+24\,a-9i}{b^4}}\log\left(-\frac{i\,b^2\sqrt{-\frac{16i\,a^2+24\,a-9i}{b^4}}-(4\,a-3i)\sqrt{\frac{i\,\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}}{4\,a-3i}\right)-3\,b^2\sqrt{-\frac{16i\,a^2+24\,a-9i}{b^4}}\log\left(-\frac{-i\,b\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="fricas
")
```

output
```
1/8*(3*b^2*sqrt(-(16*I*a^2 + 24*a - 9*I)/b^4)*log(-(I*b^2*sqrt(-(16*I*a^2
+ 24*a - 9*I)/b^4) - (4*a - 3*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/
(b*x + a + I)))/(4*a - 3*I)) - 3*b^2*sqrt(-(16*I*a^2 + 24*a - 9*I)/b^4)*lo
g(-(-I*b^2*sqrt(-(16*I*a^2 + 24*a - 9*I)/b^4) - (4*a - 3*I)*sqrt(I*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(4*a - 3*I)) + 3*b^2*sqrt(-(-16
*I*a^2 - 24*a + 9*I)/b^4)*log(-(I*b^2*sqrt(-(-16*I*a^2 - 24*a + 9*I)/b^4)
- (4*a - 3*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/(4*
a - 3*I)) - 3*b^2*sqrt(-(-16*I*a^2 - 24*a + 9*I)/b^4)*log(-(-I*b^2*sqrt(-(
-16*I*a^2 - 24*a + 9*I)/b^4) - (4*a - 3*I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)/(b*x + a + I)))/(4*a - 3*I)) - 2*(2*b^2*x^2 - 2*a^2 + 7*I*b*x +
3*I*a - 5)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/b^2
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x\,dx = \text{Timed out}$$

input
```
integrate(x/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x\,dx = \int \frac{x}{\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(x/((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x\,dx = \int \frac{x}{{\left(\frac{1+a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}}{\sqrt{{\left(a+b\,x\right)}^2+1}}\right)}^{3/2}}\,dx$$

input
```
int(x/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2),x)
```

output
```
int(x/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2), x)
```

Reduce [F]

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}x\,dx = \int \frac{x}{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}}}dx$$

input
```
int(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

output
```
int(x/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

3.247 $\int e^{-\frac{3}{2}i\arctan(a+bx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 245

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}\,dx = -\frac{i(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{b} - \frac{3i\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} + \frac{3i\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right)}{\sqrt{2}b} - \frac{3i\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\right)}\right)}{\sqrt{2}b}$$

output
```
-I*(1-I*a-I*b*x)^(3/4)*(1+I*a+I*b*x)^(1/4)/b-3/2*I*arctan(1-2^(1/2)*(1-I*a
-I*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4))*2^(1/2)/b+3/2*I*arctan(1+2^(1/2)*(1-I*a
-I*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4))*2^(1/2)/b-3/2*I*arctanh(2^(1/2)*(1-I*a-
I*b*x)^(1/4)/(1+I*a+I*b*x)^(1/4)/(1+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2
)))*2^(1/2)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.18

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}\,dx=-\frac{8ie^{\frac{1}{2}i\arctan(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},2,\frac{5}{4},-e^{2i\arctan(a+bx)}\right)}{b}$$

input
```
Integrate[E^(((-3*I)/2)*ArcTan[a + b*x]),x]
```

output
```
((-8*I)*E^((I/2)*ArcTan[a + b*x])*Hypergeometric2F1[1/4, 2, 5/4, -E^((2*I)
*ArcTan[a + b*x])])/b
```

Rubi [A] (warning: unable to verify)

Time = 0.67 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.22, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.857$, Rules used = {5616, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{5616}$$

$$\int \frac{(-ia-ibx+1)^{3/4}}{(ia+ibx+1)^{3/4}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{3}{2}\int \frac{1}{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}dx-\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}$$

$$\Big\downarrow \text{73}$$

$$\frac{6i\int \frac{\sqrt{-ia-ibx+1}}{(ia+ibx+1)^{3/4}}d\sqrt[4]{-ia-ibx+1}}{b}-\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}$$

$$\Big\downarrow \; 854$$

$$\frac{6i\int \frac{\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{b}-\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}$$

$$\Big\downarrow \; 826$$

$$\frac{6i\left(\frac{1}{2}\int \frac{\sqrt{-ia-ibx+1}+1}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}-\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{b}-\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}$$

$$\Big\downarrow \; 1476$$

$$\frac{6i\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\int \frac{1}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia}}{\sqrt[4]{ia+}}\right.\right.}{b}$$

$$\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}$$

$$\Big\downarrow \; 1082$$

$$\frac{6i\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+}}{-ia-ibx+2}\right.}{b}$$

$$\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}$$

$$\Big\downarrow \; 217$$

$$\frac{6i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{b}$$

$$\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}$$

$$\Big\downarrow \; 1479$$

$$6i\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx}}{\sqrt[4]{ia+ibx}}}{2\sqrt{2}}\right.\right.$$

$$b$$

$$\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}$$

$\big\downarrow$ 25

$$6i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx}}{\sqrt[4]{ia+ibx}}}{2\sqrt{2}}\right.\right.$$

$$b$$

$$\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}$$

$\big\downarrow$ 27

$$6i\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-}}{\sqrt[4]{ia+i}}\right.\right.$$

$$b$$

$$\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}$$

$\big\downarrow$ 1103

$$\frac{6i\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+}}{\sqrt[4]{ia+ibx+1}}\right.}{2\sqrt{2}}\right.}{b}$$

$$\frac{i(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{b}$$

input
```
Int[E^(((-3*I)/2)*ArcTan[a + b*x]),x]
```

output
```
((-I)*(1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/b + ((6*I)*((-(ArcT
an[1 - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2])
 + ArcTan[1 + (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/S
qrt[2])/2 + (Log[1 + Sqrt[1 - I*a - I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1
/4))/(1 + I*a + I*b*x)^(1/4)]/(2*Sqrt[2]) - Log[1 + Sqrt[1 - I*a - I*b*x]
+ (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/(2*Sqrt[2]))/
2))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5616
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^(I*(n/2))/(1 + I*a*c + I*b*c*x)^(I*(n/2)), x] /; FreeQ[{a, b,
c, n}, x]
```

Maple [F]

$$\int \frac{1}{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

output
```
int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.04

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}\,dx = \frac{b\sqrt{\frac{9i}{b^2}}\log\left(\frac{1}{3}i\,b\sqrt{\frac{9i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)-b\sqrt{\frac{9i}{b^2}}\log\left(-\frac{1}{3}i\,b\sqrt{\frac{9i}{b^2}}+\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}\right)+b\sqrt{-\frac{9i}{b^2}}\log$$

input
```
integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="fricas
")
```

output
```
1/2*(b*sqrt(9*I/b^2)*log(1/3*I*b*sqrt(9*I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1)/(b*x + a + I))) - b*sqrt(9*I/b^2)*log(-1/3*I*b*sqrt(9*I/b^
2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + b*sqrt(-9*
I/b^2)*log(1/3*I*b*sqrt(-9*I/b^2) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1)/(b*x + a + I))) - b*sqrt(-9*I/b^2)*log(-1/3*I*b*sqrt(-9*I/b^2) + sqrt(I
*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - 2*(b*x + a + I)*sqrt(
I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)))/b
```

Sympy [F]

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}\,dx=\int\frac{1}{\left(\frac{i(a+bx)+1}{\sqrt{(a+bx)^2+1}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2),x)
```

output
```
Integral(((I*(a + b*x) + 1)/sqrt((a + b*x)**2 + 1))**(-3/2), x)
```

Maxima [F]

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}\,dx=\int\frac{1}{\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(-3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}\,dx = \int \frac{1}{\left(\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}\right)^{3/2}}\,dx$$

input
```
int(1/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2),x)
```

output
```
int(1/((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2), x)
```

Reduce [F]

$$\int e^{-\frac{3}{2}i\arctan(a+bx)}\,dx = \int \frac{1}{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

output
```
int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2),x)
```

3.248 $\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 344

$$\begin{aligned}\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x}\,dx &= -\frac{2(i+a)^{3/4}\arctan\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/4}} \\ &\quad -\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right) \\ &\quad +\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}}\right) \\ &\quad -\frac{2(i+a)^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{3/4}} \\ &\quad -\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{1-ia-ibx}}{\sqrt[4]{1+ia+ibx}\left(1+\frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}}\right)}\right)\end{aligned}$$

output
```
-2*(I+a)^(3/4)*arctan((I+a)^(1/4)*(1+I*a+I*b*x)^(1/4)/(I-a)^(1/4)/(1-I*a-I
*b*x)^(1/4))/(I-a)^(3/4)-2^(1/2)*arctan(1-2^(1/2)*(1-I*a-I*b*x)^(1/4)/(1+I
*a+I*b*x)^(1/4))+2^(1/2)*arctan(1+2^(1/2)*(1-I*a-I*b*x)^(1/4)/(1+I*a+I*b*x
)^(1/4))-2*(I+a)^(3/4)*arctanh((I+a)^(1/4)*(1+I*a+I*b*x)^(1/4)/(I-a)^(1/4)
/(1-I*a-I*b*x)^(1/4))/(I-a)^(3/4)-2^(1/2)*arctanh(2^(1/2)*(1-I*a-I*b*x)^(1
/4)/(1+I*a+I*b*x)^(1/4)/(1+(1-I*a-I*b*x)^(1/2)/(1+I*a+I*b*x)^(1/2)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.37

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x}\,dx$$
$$= \frac{2(-i(i+a+bx))^{3/4}\left(\sqrt[4]{2}(1+ia+ibx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},-\frac{1}{2}i(i+a+bx)\right)-2\,\text{Hypergeor}\right.}{3(1+ia+ibx)^{3/4}}$$

input
```
Integrate[1/(E^(((3*I)/2)*ArcTan[a + b*x])*x),x]
```

output
```
(2*((-I)*(I + a + b*x))^(3/4)*(2^(1/4)*(1 + I*a + I*b*x)^(3/4)*Hypergeomet
ric2F1[3/4, 3/4, 7/4, (-1/2*I)*(I + a + b*x)] - 2*Hypergeometric2F1[3/4, 1
, 7/4, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)]))/(3*(1 + I*a
+ I*b*x)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.96 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.24, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.944$, Rules used = {5618, 140, 27, 73, 104, 756, 218, 221, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x}\,dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{(-ia-ibx+1)^{3/4}}{x(ia+ibx+1)^{3/4}}dx$$

$$\Big\downarrow \text{140}$$

$$\int \frac{1-ia}{x\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}dx-ib\int \frac{1}{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}dx$$

$$\Big\downarrow \text{27}$$

$$(1-ia)\int \frac{1}{x\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}dx-ib\int \frac{1}{\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}dx$$

$$\Big\downarrow \text{73}$$

$$(1-ia)\int \frac{1}{x\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}dx+4\int \frac{\sqrt{-ia-ibx+1}}{(ia+ibx+1)^{3/4}}d\sqrt[4]{-ia-ibx+1}$$

$$\Big\downarrow \text{104}$$

$$4\int \frac{\sqrt{-ia-ibx+1}}{(ia+ibx+1)^{3/4}}d\sqrt[4]{-ia-ibx+1}+4(1-ia)\int \frac{1}{-ia+\frac{(1-ia)(ia+ibx+1)}{-ia-ibx+1}-1}d\frac{\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-ia-ibx+1}}$$

$$\Big\downarrow \text{756}$$

$$4\int \frac{\sqrt{-ia-ibx+1}}{(ia+ibx+1)^{3/4}}d\sqrt[4]{-ia-ibx+1}+4(1-ia)\left(-\frac{i\int \frac{1}{\sqrt{i-a}-\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}}d\frac{\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-ia-ibx+1}}}{2\sqrt{-a+i}}-\frac{i\int \frac{1}{\sqrt{i-a}+\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}}d\frac{\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-ia-ibx+1}}}{2\sqrt{-a+i}}\right)$$

$$\Big\downarrow \text{218}$$

$$4(1-ia)\left(-\frac{i\int \frac{1}{\sqrt{i-a}-\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}}d\frac{\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-ia-ibx+1}}}{2\sqrt{-a+i}}-\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}\right)+4\int \frac{\sqrt{-ia-ibx+1}}{(ia+ibx+1)^{3/4}}d\sqrt[4]{-ia-ibx+1}$$

$$\Big\downarrow \; 221$$

$$4\int \frac{\sqrt{-ia-ibx+1}}{(ia+ibx+1)^{3/4}}d\sqrt[4]{-ia-ibx+1}+4(1-ia)\left(-\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}-\frac{i\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}\right)$$

$$\Big\downarrow \; 854$$

$$4\int \frac{\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+4(1-ia)\left(-\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}-\frac{i\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}\right)$$

$$\Big\downarrow \; 826$$

$$4\left(\frac{1}{2}\int \frac{\sqrt{-ia-ibx+1}+1}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}-\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)+4(1-ia)\left(-\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}-\frac{i\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}\right)$$

$$\Big\downarrow \; 1476$$

$$4\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+\frac{1}{2}\int \frac{1}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ib}}{\sqrt[4]{ia+ibx}}}\right.\right.$$
$$4(1-ia)\left(-\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}-\frac{i\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}\right)$$

$$\Big\downarrow \; 1082$$

$$4\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{-ia-ibx+1}-1}d\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int 1\right.$$

$$4(1-ia)\left(-\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}-\frac{i\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}\right)$$

$$\Big\downarrow \text{217}$$

$$4\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{-ia-ibx+1}}{-ia-ibx+2}d\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.$$

$$4(1-ia)\left(-\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}-\frac{i\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}\right)$$

$$\Big\downarrow \text{1479}$$

$$4\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia}}{\sqrt[4]{ia+}}}{2\sqrt{2}}\right.\right.$$

$$4(1-ia)\left(-\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}-\frac{i\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}\right)$$

$$\Big\downarrow \text{25}$$

$$4\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1\right)}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-}}{\sqrt[4]{ia+i}}}{2\sqrt{2}}\right.\right.$$

$$4(1-ia)\left(-\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}-\frac{i\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}\right)$$

$\downarrow$ 27

$$4\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{\sqrt{-ia-ibx+1}-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}d\frac{\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+1}{\sqrt{-ia-ibx+1}+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}+}\right.\right.$$

$$4(1-ia)\left(-\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}-\frac{i\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}\right)$$

$\downarrow$ 1103

$$4(1-ia)\left(-\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}-\frac{i\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}\right)+$$
$$4\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{-ia-ibx+1}}{\sqrt[4]{ia+ibx+1}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{-ia-ibx+1}-\right.}{2\sqrt{}}\right.\right.$$

input
```
Int[1/(E^(((3*I)/2)*ArcTan[a + b*x])*x),x]
```

output
```
4*(1 - I*a)*(((-1/2*I)*ArcTan[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I
 - a)^(1/4)*(1 - I*a - I*b*x)^(1/4))])/((I - a)^(3/4)*(I + a)^(1/4)) - ((I/
2)*ArcTanh[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a
 - I*b*x)^(1/4))])/((I - a)^(3/4)*(I + a)^(1/4))) + 4*((-(ArcTan[1 - (Sqrt
[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2]) + ArcTan[1
+ (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/Sqrt[2])/2 +
(Log[1 + Sqrt[1 - I*a - I*b*x] - (Sqrt[2]*(1 - I*a - I*b*x)^(1/4))/(1 + I*
a + I*b*x)^(1/4)]/(2*Sqrt[2]) - Log[1 + Sqrt[1 - I*a - I*b*x] + (Sqrt[2]*(
1 - I*a - I*b*x)^(1/4))/(1 + I*a + I*b*x)^(1/4)]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [F]

$$\int \frac{1}{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}} x}dx$$

input
```
int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x)
```

output
```
int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 629 vs. 2(228) = 456.

Time = 0.17 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.83

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x}\,dx = \text{Too large to display}$$

input
```
integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x, algorithm="fric
as")
```

output
```
-1/2*sqrt(4*I)*log(1/2*sqrt(4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)/(b*x + a + I))) + 1/2*sqrt(4*I)*log(-1/2*sqrt(4*I) + sqrt(I*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) - 1/2*sqrt(-4*I)*log(1/2*sqrt(-4*I)
+ sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I))) + 1/2*sqrt(-4*I
)*log(-1/2*sqrt(-4*I) + sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a
+ I))) + (-(a^3 + 3*I*a^2 - 3*a - I)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4)*log(
((a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) + (a - I)
*(-(a^3 + 3*I*a^2 - 3*a - I)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4))/(a + I)) -
(-(a^3 + 3*I*a^2 - 3*a - I)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4)*log(((a + I)*
sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) - (a - I)*(-(a^3 +
 3*I*a^2 - 3*a - I)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4))/(a + I)) + I*(-(a^3
+ 3*I*a^2 - 3*a - I)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4)*log(((a + I)*sqrt(I*
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) + (I*a + 1)*(-(a^3 + 3*I*
a^2 - 3*a - I)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4))/(a + I)) - I*(-(a^3 + 3*I
*a^2 - 3*a - I)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4)*log(((a + I)*sqrt(I*sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) + (-I*a - 1)*(-(a^3 + 3*I*a^2
- 3*a - I)/(a^3 - 3*I*a^2 - 3*a + I))^(1/4))/(a + I))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x}\,dx = \text{Timed out}$$

input
```
integrate(1/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x}\,dx = \int \frac{1}{x\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x, algorithm="maxi
ma")
```

output
```
integrate(1/(x*((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x}\,dx = \int \frac{1}{x\left(\frac{1+a\,1i+b\,x\,1i}{\sqrt{(a+b\,x)^2+1}}\right)^{3/2}}\,dx$$

input
```
int(1/(x*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2)),x)
```

output
```
int(1/(x*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2)), x)
```

Reduce [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x}\,dx = \int \frac{1}{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}} x}\,dx$$

input
```
int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x)
```

output
```
int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x,x)
```

3.249 $\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 211

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = -\frac{(1-ia-ibx)^{3/4}\sqrt[4]{1+ia+ibx}}{(1+ia)x} - \frac{3ib\arctan\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{7/4}\sqrt[4]{i+a}} - \frac{3ib\text{arctanh}\left(\frac{\sqrt[4]{i+a}\sqrt[4]{1+ia+ibx}}{\sqrt[4]{i-a}\sqrt[4]{1-ia-ibx}}\right)}{(i-a)^{7/4}\sqrt[4]{i+a}}$$

output
```
-(1-I*a-I*b*x)^(3/4)*(1+I*a+I*b*x)^(1/4)/(1+I*a)/x-3*I*b*arctan((I+a)^(1/4
)*(1+I*a+I*b*x)^(1/4)/(I-a)^(1/4)/(1-I*a-I*b*x)^(1/4))/(I-a)^(7/4)/(I+a)^(
1/4)-3*I*b*arctanh((I+a)^(1/4)*(1+I*a+I*b*x)^(1/4)/(I-a)^(1/4)/(1-I*a-I*b*
x)^(1/4))/(I-a)^(7/4)/(I+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.51

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = -\frac{(-i(i+a+bx))^{3/4}\left(1+a^2+ibx+abx-2ibx\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},\frac{1+a^2-ibx+abx}{1+a^2+ibx+abx}\right)\right)}{(1+a^2)\,x(1+ia+ibx)^{3/4}}$$

input
```
Integrate[1/(E^(((3*I)/2)*ArcTan[a + b*x])*x^2),x]
```

output
```
-((((-I)*(I + a + b*x))^(3/4)*(1 + a^2 + I*b*x + a*b*x - (2*I)*b*x*Hyperge
ometric2F1[3/4, 1, 7/4, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x
)]))/((1 + a^2)*x*(1 + I*a + I*b*x)^(3/4)))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5618, 105, 104, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x^2}dx$$

$$\Big\downarrow \text{ 5618}$$

$$\int \frac{(-ia-ibx+1)^{3/4}}{x^2(ia+ibx+1)^{3/4}}dx$$

$$\Big\downarrow \text{ 105}$$

$$\frac{3b\int \frac{1}{x\sqrt[4]{-ia-ibx+1}(ia+ibx+1)^{3/4}}dx}{2(-a+i)}-\frac{(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{(1+ia)x}$$

$$\Big\downarrow \text{104}$$

$$\frac{6b\int \frac{1}{-ia+\frac{(1-ia)(ia+ibx+1)}{-ia-ibx+1}-1}d\frac{\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-ia-ibx+1}}}{-a+i}-\frac{(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{(1+ia)x}$$

$$\Big\downarrow \text{756}$$

$$\frac{6b\left(-\frac{i\int \frac{1}{\sqrt{i-a}-\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}}d\frac{\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-ia-ibx+1}}}{2\sqrt{-a+i}}-\frac{i\int \frac{1}{\sqrt{i-a}+\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}}d\frac{\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-ia-ibx+1}}}{2\sqrt{-a+i}}\right)}{-a+i}-\frac{(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{(1+ia)x}$$

$$\Big\downarrow \text{218}$$

$$\frac{6b\left(-\frac{i\int \frac{1}{\sqrt{i-a}-\frac{\sqrt{a+i}\sqrt{ia+ibx+1}}{\sqrt{-ia-ibx+1}}}d\frac{\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-ia-ibx+1}}}{2\sqrt{-a+i}}-\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}\right)}{-a+i}-\frac{(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{(1+ia)x}$$

$$\Big\downarrow \text{221}$$

$$\frac{6b\left(-\frac{i\arctan\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}-\frac{i\text{arctanh}\left(\frac{\sqrt[4]{a+i}\sqrt[4]{ia+ibx+1}}{\sqrt[4]{-a+i}\sqrt[4]{-ia-ibx+1}}\right)}{2(-a+i)^{3/4}\sqrt[4]{a+i}}\right)}{-a+i}-\frac{(-ia-ibx+1)^{3/4}\sqrt[4]{ia+ibx+1}}{(1+ia)x}$$

input
```
Int[1/(E^(((3*I)/2)*ArcTan[a + b*x])*x^2),x]
```

output
```
-(((1 - I*a - I*b*x)^(3/4)*(1 + I*a + I*b*x)^(1/4))/((1 + I*a)*x)) + (6*b*
(((-1/2*I)*ArcTan[((I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(
1 - I*a - I*b*x)^(1/4))])/((I - a)^(3/4)*(I + a)^(1/4)) - ((I/2)*ArcTanh[(
(I + a)^(1/4)*(1 + I*a + I*b*x)^(1/4))/((I - a)^(1/4)*(1 - I*a - I*b*x)^(1
/4))])/((I - a)^(3/4)*(I + a)^(1/4))))/(I - a)
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [F]

$$\int \frac{1}{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}} x^2}\, dx$$

input
```
int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x)
```

output
```
int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(137) = 274$.

Time = 0.14 (sec) , antiderivative size = 613, normalized size of antiderivative = 2.91

$$\int \frac{e^{-\frac{3}{2} i \arctan(a+bx)}}{x^2}\, dx$$
$$= \frac{3\left(-\frac{b^4}{a^8-6i\,a^7-14\,a^6+14i\,a^5+14i\,a^3+14\,a^2-6i\,a-1}\right)^{\frac{1}{4}}(-i\,a-1)x\log\left(\frac{b\sqrt{\frac{i\sqrt{b^2x^2+2\,abx+a^2+1}}{bx+a+i}}+\left(-\frac{b^4}{a^8-6i\,a^7-14\,a^6+14i\,a^5+14i\,a^3+\cdots}\right.}{b}\right.}{\cdots}$$

input
```
integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x, algorithm="fr
icas")
```

output
```
1/2*(3*(-b^4/(a^8 - 6*I*a^7 - 14*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 - 6*I*
a - 1))^(1/4)*(-I*a - 1)*x*log((b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
/(b*x + a + I)) + (-b^4/(a^8 - 6*I*a^7 - 14*a^6 + 14*I*a^5 + 14*I*a^3 + 14
*a^2 - 6*I*a - 1))^(1/4)*(a^2 - 2*I*a - 1))/b) + 3*(-b^4/(a^8 - 6*I*a^7 -
14*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 - 6*I*a - 1))^(1/4)*(I*a + 1)*x*log(
(b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) - (-b^4/(a^8 -
6*I*a^7 - 14*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 - 6*I*a - 1))^(1/4)*(a^2 -
 2*I*a - 1))/b) - 3*(-b^4/(a^8 - 6*I*a^7 - 14*a^6 + 14*I*a^5 + 14*I*a^3 +
14*a^2 - 6*I*a - 1))^(1/4)*(a - I)*x*log((b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)/(b*x + a + I)) - (-b^4/(a^8 - 6*I*a^7 - 14*a^6 + 14*I*a^5 + 14*
I*a^3 + 14*a^2 - 6*I*a - 1))^(1/4)*(I*a^2 + 2*a - I))/b) + 3*(-b^4/(a^8 -
6*I*a^7 - 14*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 - 6*I*a - 1))^(1/4)*(a - I
)*x*log((b*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/(b*x + a + I)) - (-b^4
/(a^8 - 6*I*a^7 - 14*a^6 + 14*I*a^5 + 14*I*a^3 + 14*a^2 - 6*I*a - 1))^(1/4
)*(-I*a^2 - 2*a + I))/b) + 2*(b*x + a + I)*sqrt(I*sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)/(b*x + a + I)))/((a - I)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate(1/((1+I*(b*x+a))/(1+(b*x+a)**2)**(1/2))**(3/2)/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{i\,bx+i\,a+1}{\sqrt{(bx+a)^2+1}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x, algorithm="ma
xima")
```

output
```
integrate(1/(x^2*((I*b*x + I*a + 1)/sqrt((b*x + a)^2 + 1))^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = \int \frac{1}{x^2\left(\frac{1+a\,1\mathrm{i}+b\,x\,1\mathrm{i}}{\sqrt{(a+b\,x)^2+1}}\right)^{3/2}}\,dx$$

input
```
int(1/(x^2*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2)),x)
```

output
```
int(1/(x^2*((a*1i + b*x*1i + 1)/((a + b*x)^2 + 1)^(1/2))^(3/2)), x)
```

Reduce [F]

$$\int \frac{e^{-\frac{3}{2}i\arctan(a+bx)}}{x^2}\,dx = \int \frac{1}{\left(\frac{1+i(bx+a)}{\sqrt{1+(bx+a)^2}}\right)^{\frac{3}{2}} x^2}\,dx$$

input
```
int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x)
```

output
```
int(1/((1+I*(b*x+a))/(1+(b*x+a)^2)^(1/2))^(3/2)/x^2,x)
```

3.250 $\int e^{n\arctan(a+bx)}x^m\,dx$

Optimal result

Integrand size = 14, antiderivative size = 135

$$\int e^{n\arctan(a+bx)}x^m\,dx = -\frac{2^{1-\frac{in}{2}}x^m\left(-\frac{bx}{i+a}\right)^{-m}(1-ia-ibx)^{1+\frac{in}{2}}\operatorname{AppellF1}\left(1+\frac{in}{2},-m,\frac{in}{2},2+\frac{in}{2},\frac{1-ia-ibx}{1-ia},\frac{1}{2}(1-ia-ibx)\right)}{b(2i-n)}$$

output
```
-2^(1-1/2*I*n)*x^m*(1-I*a-I*b*x)^(1+1/2*I*n)*AppellF1(1+1/2*I*n,-m,1/2*I*n
,2+1/2*I*n,(1-I*a-I*b*x)/(1-I*a),1/2-1/2*I*a-1/2*I*b*x)/b/(2*I-n)/((-b*x/(
I+a))^m)
```

Mathematica [F]

$$\int e^{n\arctan(a+bx)}x^m\,dx = \int e^{n\arctan(a+bx)}x^m\,dx$$

input
```
Integrate[E^(n*ArcTan[a + b*x])*x^m,x]
```

output
```
Integrate[E^(n*ArcTan[a + b*x])*x^m, x]
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5618, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{n\arctan(a+bx)}dx$$

$$\Big\downarrow \text{ 5618}$$

$$\int x^m(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}dx$$

$$\Big\downarrow \text{ 152}$$

$$(-ia-ibx+1)^{\frac{in}{2}}\left(1+\frac{bx}{a+i}\right)^{-\frac{in}{2}}\int x^m(ia+ibx+1)^{-\frac{in}{2}}\left(\frac{bx}{a+i}+1\right)^{\frac{in}{2}}dx$$

$$\Big\downarrow \text{ 152}$$

$$(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}\left(1-\frac{bx}{-a+i}\right)^{\frac{in}{2}}\left(1+\frac{bx}{a+i}\right)^{-\frac{in}{2}}\int x^m\left(1-\frac{bx}{i-a}\right)^{-\frac{in}{2}}\left(\frac{bx}{a+i}+1\right)^{\frac{in}{2}}dx$$

$$\Big\downarrow \text{ 150}$$

$$\frac{x^{m+1}(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}\left(1-\frac{bx}{-a+i}\right)^{\frac{in}{2}}\left(1+\frac{bx}{a+i}\right)^{-\frac{in}{2}}\operatorname{AppellF1}\left(m+1,\frac{in}{2},-\frac{in}{2},m+2,\frac{bx}{i-a},-\frac{b}{a}\right.}{m+1}$$

input
```
Int[E^(n*ArcTan[a + b*x])*x^m,x]
```

output
```
(x^(1 + m)*(1 - I*a - I*b*x)^((I/2)*n)*(1 - (b*x)/(I - a))^((I/2)*n)*Appel
lF1[1 + m, (I/2)*n, (-1/2*I)*n, 2 + m, (b*x)/(I - a), -((b*x)/(I + a))])/(
(1 + m)*(1 + I*a + I*b*x)^((I/2)*n)*(1 + (b*x)/(I + a))^((I/2)*n))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [F]

$$\int e^{n \arctan(bx+a)} x^m dx$$

input
```
int(exp(n*arctan(b*x+a))*x^m,x)
```

output
```
int(exp(n*arctan(b*x+a))*x^m,x)
```

Fricas [F]

$$\int e^{n \arctan(a+bx)} x^m \, dx = \int x^m e^{(n \arctan(bx+a))} \, dx$$

input
```
integrate(exp(n*arctan(b*x+a))*x^m,x, algorithm="fricas")
```

output
```
integral(x^m*e^(n*arctan(b*x + a)), x)
```

Sympy [F]

$$\int e^{n\arctan(a+bx)}x^m\,dx = \int x^m e^{n\,\mathrm{atan}\,(a+bx)}\,dx$$

input
```
integrate(exp(n*atan(b*x+a))*x**m,x)
```

output
```
Integral(x**m*exp(n*atan(a + b*x)), x)
```

Maxima [F]

$$\int e^{n\arctan(a+bx)}x^m\,dx = \int x^m e^{(n\arctan(bx+a))}\,dx$$

input
```
integrate(exp(n*arctan(b*x+a))*x^m,x, algorithm="maxima")
```

output
```
integrate(x^m*e^(n*arctan(b*x + a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{n\arctan(a+bx)}x^m\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(exp(n*arctan(b*x+a))*x^m,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,1,2,0]%%%} / %%%{1,[0,0,0,0,2]%%%} Error: Bad Argum
ent Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(a+bx)}x^m\,dx = \int x^m\,\mathrm{e}^{n\,\mathrm{atan}(a+b\,x)}\,dx$$

input
```
int(x^m*exp(n*atan(a + b*x)),x)
```

output
```
int(x^m*exp(n*atan(a + b*x)), x)
```

Reduce [F]

$$\int e^{n\arctan(a+bx)}x^m\,dx = \int x^m e^{atan(bx+a)n}dx$$

input
```
int(exp(n*atan(b*x+a))*x^m,x)
```

output
```
int(x**m*e**(atan(a + b*x)*n),x)
```

3.251 $\int e^{n\arctan(a+bx)}x^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 320

$$\begin{aligned}
&\int e^{n\arctan(a+bx)}x^3\,dx\\
&= -\frac{(6-30a^2-2in-n^2-12a(i+n))\,(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{24b^4}\\
&\quad+\frac{x^2(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{4b^2}\\
&\quad-\frac{i(6a+n)(1-ia-ibx)^{2+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{12b^4}\\
&\quad+\frac{2^{-2-\frac{in}{2}}(24a^3+36a^2n-12a(2-n^2)-n(8-n^2))\,(1-ia-ibx)^{1+\frac{in}{2}}\operatorname{Hypergeometric2F1}\left(1+\frac{in}{2},\frac{in}{2},2\right.}{3b^4(2i-n)}
\end{aligned}$$

output

```
-1/24*(6-30*a^2-2*I*n-n^2-12*a*(I+n))*(1-I*a-I*b*x)^(1+1/2*I*n)*(1+I*a+I*b
*x)^(1-1/2*I*n)/b^4+1/4*x^2*(1-I*a-I*b*x)^(1+1/2*I*n)*(1+I*a+I*b*x)^(1-1/2
*I*n)/b^2-1/12*I*(6*a+n)*(1-I*a-I*b*x)^(2+1/2*I*n)*(1+I*a+I*b*x)^(1-1/2*I*
n)/b^4+1/3*2^(-2-1/2*I*n)*(24*a^3+36*a^2*n-12*a*(-n^2+2)-n*(-n^2+8))*(1-I*
a-I*b*x)^(1+1/2*I*n)*hypergeom([1/2*I*n, 1+1/2*I*n],[2+1/2*I*n],1/2-1/2*I*
a-1/2*I*b*x)/b^4/(2*I-n)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.85

$$\int e^{n\arctan(a+bx)}x^3\,dx = \frac{(-i(i+a+bx))^{1+\frac{in}{2}}\left(b^2(2i-n)x^2(1+ia+ibx)^{1-\frac{in}{2}}-2^{3-\frac{in}{2}}(6a+n)\,\text{Hypergeometric2F1}\left(-2+\frac{in}{2},1\right.\right.}{}$$

input
```
Integrate[E^(n*ArcTan[a + b*x])*x^3,x]
```

output
```
(((-I)*(I + a + b*x))^(1 + (I/2)*n)*(b^2*(2*I - n)*x^2*(1 + I*a + I*b*x)^(
1 - (I/2)*n) - 2^(3 - (I/2)*n)*(6*a + n)*Hypergeometric2F1[-2 + (I/2)*n, 1
 + (I/2)*n, 2 + (I/2)*n, (-1/2*I)*(I + a + b*x)] + 2^(3 - (I/2)*n)*(1 + I*
a)*(-I + 5*a + n)*Hypergeometric2F1[-1 + (I/2)*n, 1 + (I/2)*n, 2 + (I/2)*n
, (-1/2*I)*(I + a + b*x)] + 2^(1 - (I/2)*n)*(-I + a)^2*(-2*I + 4*a + n)*Hy
pergeometric2F1[1 + (I/2)*n, (I/2)*n, 2 + (I/2)*n, (-1/2*I)*(I + a + b*x)]
))/(4*b^4*(2*I - n))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5618, 111, 25, 164, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{n\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{5618}$$

$$\int x^3(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}\,dx$$

$$\Big\downarrow \text{111}$$

$$\frac{\int -x(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}\left(2\left(a^2+1\right)+b(6a+n)x\right)dx}{4b^2}+\frac{x^2(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{4b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^2(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{4b^2}-\frac{\int x(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}\left(2\left(a^2+1\right)+b(6a+n)x\right)dx}{4b^2}$$

$$\Big\downarrow \text{164}$$

$$\frac{x^2(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{4b^2}-\frac{\frac{(24a^3+36a^2n-12a(2-n^2)-n(8-n^2))\int(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}dx}{6b}+\frac{(-ia-ibx+1)^{1+\frac{in}{2}}(-18a^2+2bx(6a+n)-10an-n^2+6)(ia+ibx}{6b^2}}{4b^2}$$

$$\Big\downarrow \text{79}$$

$$\frac{x^2(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{4b^2}-\frac{\frac{(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}(-18a^2+2bx(6a+n)-10an-n^2+6)}{6b^2}-\frac{2^{-\frac{in}{2}}(24a^3+36a^2n-12a(2-n^2)-n(8-n^2))(-ia-ibx+1)^{1+\frac{in}{2}}\text{Hy}}{3b^2(-n+2i)}}{4b^2}$$

input
```
Int[E^(n*ArcTan[a + b*x])*x^3,x]
```

output
```
(x^2*(1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)*n))/(4*b
^2) - (((1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)*n)*(6
 - 18*a^2 - 10*a*n - n^2 + 2*b*(6*a + n)*x))/(6*b^2) - ((24*a^3 + 36*a^2*n
 - 12*a*(2 - n^2) - n*(8 - n^2))*(1 - I*a - I*b*x)^(1 + (I/2)*n)*Hypergeom
etric2F1[1 + (I/2)*n, (I/2)*n, 2 + (I/2)*n, (1 - I*a - I*b*x)/2])/(3*2^((I
/2)*n)*b^2*(2*I - n)))/(4*b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [F]

$$\int e^{n\arctan(bx+a)}x^3dx$$

input
```
int(exp(n*arctan(b*x+a))*x^3,x)
```

output
```
int(exp(n*arctan(b*x+a))*x^3,x)
```

Fricas [F]

$$\int e^{n\arctan(a+bx)}x^3\,dx = \int x^3e^{(n\arctan(bx+a))}\,dx$$

input
```
integrate(exp(n*arctan(b*x+a))*x^3,x, algorithm="fricas")
```

output
```
integral(x^3*e^(n*arctan(b*x + a)), x)
```

Sympy [F]

$$\int e^{n\arctan(a+bx)}x^3\,dx = \int x^3e^{n\,\mathrm{atan}\,(a+bx)}\,dx$$

input
```
integrate(exp(n*atan(b*x+a))*x**3,x)
```

output
```
Integral(x**3*exp(n*atan(a + b*x)), x)
```

Maxima [F]

$$\int e^{n\arctan(a+bx)}x^3\,dx = \int x^3e^{(n\arctan(bx+a))}\,dx$$

input
```
integrate(exp(n*arctan(b*x+a))*x^3,x, algorithm="maxima")
```

output
```
integrate(x^3*e^(n*arctan(b*x + a)), x)
```

Giac [F(-1)]

Timed out.

$$\int e^{n\arctan(a+bx)}x^3\,dx = \text{Timed out}$$

input
```
integrate(exp(n*arctan(b*x+a))*x^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(a+bx)}x^3\,dx = \int x^3\,\mathrm{e}^{n\,\mathrm{atan}(a+b\,x)}\,dx$$

input
```
int(x^3*exp(n*atan(a + b*x)),x)
```

output
```
int(x^3*exp(n*atan(a + b*x)), x)
```

Reduce [F]

$$\int e^{n\arctan(a+bx)}x^3\,dx=\int e^{atan(bx+a)n}x^3dx$$

input
```
int(exp(n*atan(b*x+a))*x^3,x)
```

output
```
int(e**(atan(a + b*x)*n)*x**3,x)
```

3.252 $\int e^{n\arctan(a+bx)}x^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 220

$$\int e^{n\arctan(a+bx)}x^2\,dx = -\frac{(4a+n)(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{6b^3} + \frac{x(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{3b^2} + \frac{2^{-\frac{in}{2}}(2-6a^2-6an-n^2)(1-ia-ibx)^{1+\frac{in}{2}}\,\text{Hypergeometric2F1}\left(1+\frac{in}{2},\frac{in}{2},2+\frac{in}{2},\frac{1}{2}(1-ia-ibx)\right)}{3b^3(2i-n)}$$

output
```
-1/6*(4*a+n)*(1-I*a-I*b*x)^(1+1/2*I*n)*(1+I*a+I*b*x)^(1-1/2*I*n)/b^3+1/3*x
*(1-I*a-I*b*x)^(1+1/2*I*n)*(1+I*a+I*b*x)^(1-1/2*I*n)/b^2+1/3*(-6*a^2-6*a*n
-n^2+2)*(1-I*a-I*b*x)^(1+1/2*I*n)*hypergeom([1/2*I*n, 1+1/2*I*n],[2+1/2*I*
n],1/2-1/2*I*a-1/2*I*b*x)/(2^(1/2*I*n))/b^3/(2*I-n)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.73

$$\int e^{n \arctan(a+bx)}x^2\,dx$$
$$= \frac{(-i(i+a+bx))^{1+\frac{in}{2}}\left(-\left((4a+n)(1+ia+ibx)^{1-\frac{in}{2}}\right)+2bx(1+ia+ibx)^{1-\frac{in}{2}}+\frac{2^{1-\frac{in}{2}}(-2+6a^2+6an+n^2)\,\mathrm{H}}{}\right.}{6b^3}$$

input
```
Integrate[E^(n*ArcTan[a + b*x])*x^2,x]
```

output
```
(((-I)*(I + a + b*x))^(1 + (I/2)*n)*(-((4*a + n)*(1 + I*a + I*b*x)^(1 - (I
/2)*n)) + 2*b*x*(1 + I*a + I*b*x)^(1 - (I/2)*n) + (2^(1 - (I/2)*n)*(-2 + 6
*a^2 + 6*a*n + n^2)*Hypergeometric2F1[1 + (I/2)*n, (I/2)*n, 2 + (I/2)*n, (
-1/2*I)*(I + a + b*x)])/(-2*I + n)))/(6*b^3)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5618, 101, 25, 90, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{n \arctan(a+bx)}\,dx$$
$$\Big\downarrow \text{5618}$$
$$\int x^2(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}\,dx$$
$$\Big\downarrow \text{101}$$
$$\frac{\int -(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}\left(a^2+b(4a+n)x+1\right)dx}{3b^2}+\frac{x(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{3b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{x(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{3b^2}-\frac{\int(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}\left(a^2+b(4a+n)x+1\right)dx}{3b^2}$$

$$\Big\downarrow \text{90}$$

$$\frac{x(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{3b^2}-\frac{\frac{1}{2}\left(-6a^2-6an-n^2+2\right)\int(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}dx+\frac{(4a+n)(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{2b}}{3b^2}$$

$$\Big\downarrow \text{79}$$

$$\frac{x(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{3b^2}-\frac{\frac{(4a+n)(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{2b}-\frac{2^{-\frac{in}{2}}\left(-6a^2-6an-n^2+2\right)(-ia-ibx+1)^{1+\frac{in}{2}}\text{Hypergeometric2F1}\left(\frac{in}{2}+1,\frac{in}{2},\frac{in}{2}+2,\frac{1}{2}(-ia-ibx+\right.}{b(-n+2i)}}{3b^2}$$

input
```
Int[E^(n*ArcTan[a + b*x])*x^2,x]
```

output
```
(x*(1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)*n))/(3*b^2
) - (((4*a + n)*(1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/
2)*n))/(2*b) - ((2 - 6*a^2 - 6*a*n - n^2)*(1 - I*a - I*b*x)^(1 + (I/2)*n)*
Hypergeometric2F1[1 + (I/2)*n, (I/2)*n, 2 + (I/2)*n, (1 - I*a - I*b*x)/2])
/(2^((I/2)*n)*b*(2*I - n)))/(3*b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [F]

$$\int e^{n \arctan(bx+a)} x^2 dx$$

input
```
int(exp(n*arctan(b*x+a))*x^2,x)
```

output
```
int(exp(n*arctan(b*x+a))*x^2,x)
```

Fricas [F]

$$\int e^{n \arctan(a+bx)} x^2 \, dx = \int x^2 e^{(n \arctan(bx+a))} \, dx$$

input
```
integrate(exp(n*arctan(b*x+a))*x^2,x, algorithm="fricas")
```

output
```
integral(x^2*e^(n*arctan(b*x + a)), x)
```

Sympy [F]

$$\int e^{n\arctan(a+bx)}x^2\,dx = \int x^2e^{n\,\mathrm{atan}\,(a+bx)}\,dx$$

input
```
integrate(exp(n*atan(b*x+a))*x**2,x)
```

output
```
Integral(x**2*exp(n*atan(a + b*x)), x)
```

Maxima [F]

$$\int e^{n\arctan(a+bx)}x^2\,dx = \int x^2e^{(n\arctan(bx+a))}\,dx$$

input
```
integrate(exp(n*arctan(b*x+a))*x^2,x, algorithm="maxima")
```

output
```
integrate(x^2*e^(n*arctan(b*x + a)), x)
```

Giac [F(-1)]

Timed out.

$$\int e^{n\arctan(a+bx)}x^2\,dx = \text{Timed out}$$

input
```
integrate(exp(n*arctan(b*x+a))*x^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(a+bx)}x^2\,dx=\int x^2\,\mathrm{e}^{n\,\mathrm{atan}(a+b\,x)}\,dx$$

input
```
int(x^2*exp(n*atan(a + b*x)),x)
```

output
```
int(x^2*exp(n*atan(a + b*x)), x)
```

Reduce [F]

$$\int e^{n\arctan(a+bx)}x^2\,dx=\int e^{atan(bx+a)n}x^2dx$$

input
```
int(exp(n*atan(b*x+a))*x^2,x)
```

output
```
int(e**(atan(a + b*x)*n)*x**2,x)
```

3.253 $\int e^{n\arctan(a+bx)}x\,dx$

Optimal result

Integrand size = 12, antiderivative size = 147

$$\int e^{n\arctan(a+bx)}x\,dx=\frac{(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{2b^2}+\frac{2^{-\frac{in}{2}}(2a+n)(1-ia-ibx)^{1+\frac{in}{2}}\text{Hypergeometric2F1}\left(1+\frac{in}{2},\frac{in}{2},2+\frac{in}{2},\frac{1}{2}(1-ia-ibx)\right)}{b^2(2i-n)}$$

output
```
1/2*(1-I*a-I*b*x)^(1+1/2*I*n)*(1+I*a+I*b*x)^(1-1/2*I*n)/b^2+(2*a+n)*(1-I*a
-I*b*x)^(1+1/2*I*n)*hypergeom([1/2*I*n, 1+1/2*I*n],[2+1/2*I*n],1/2-1/2*I*a
-1/2*I*b*x)/(2^(1/2*I*n))/b^2/(2*I-n)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.87

$$\int e^{n\arctan(a+bx)}x\,dx=\frac{i(-i(i+a+bx))^{1+\frac{in}{2}}\left((1+ia+ibx)^{-\frac{in}{2}}(-i+a+bx)+\frac{2^{1-\frac{in}{2}}(2a+n)\text{Hypergeometric2F1}\left(1+\frac{in}{2},\frac{in}{2},2+\frac{in}{2},-\frac{1}{2}i(i+a+\right.}{-2-in}\right.}{2b^2}$$

input
```
Integrate[E^(n*ArcTan[a + b*x])*x,x]
```

output
```
((I/2)*((-I)*(I + a + b*x))^(1 + (I/2)*n)*((-I + a + b*x)/(1 + I*a + I*b*x
)^((I/2)*n) + (2^(1 - (I/2)*n)*(2*a + n)*Hypergeometric2F1[1 + (I/2)*n, (I
/2)*n, 2 + (I/2)*n, (-1/2*I)*(I + a + b*x)])/(-2 - I*n)))/b^2
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5618, 90, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{n\arctan(a+bx)}\,dx$$

$$\Big\downarrow \text{5618}$$

$$\int x(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}dx$$

$$\Big\downarrow \text{90}$$

$$\frac{(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{2b^2}-\frac{(2a+n)\int(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}dx}{2b}$$

$$\Big\downarrow \text{79}$$

$$\frac{2^{-\frac{in}{2}}(2a+n)(-ia-ibx+1)^{1+\frac{in}{2}}\text{Hypergeometric2F1}\left(\frac{in}{2}+1,\frac{in}{2},\frac{in}{2}+2,\frac{1}{2}(-ia-ibx+1)\right)}{b^2(-n+2i)}+\frac{(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{2b^2}$$

input
```
Int[E^(n*ArcTan[a + b*x])*x,x]
```

output
```
((1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)*n))/(2*b^2)
+ ((2*a + n)*(1 - I*a - I*b*x)^(1 + (I/2)*n)*Hypergeometric2F1[1 + (I/2)*n
, (I/2)*n, 2 + (I/2)*n, (1 - I*a - I*b*x)/2])/(2^((I/2)*n)*b^2*(2*I - n))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [F]

$$\int \mathrm{e}^{n \arctan(bx+a)} x dx$$

input
```
int(exp(n*arctan(b*x+a))*x,x)
```

output
```
int(exp(n*arctan(b*x+a))*x,x)
```

Fricas [F]

$$\int e^{n \arctan(a+bx)} x \, dx = \int x e^{(n \arctan(bx+a))} \, dx$$

input
```
integrate(exp(n*arctan(b*x+a))*x,x, algorithm="fricas")
```

output
```
integral(x*e^(n*arctan(b*x + a)), x)
```

Sympy [F]

$$\int e^{n\arctan(a+bx)}x\,dx = \int xe^{n\,\mathrm{atan}\,(a+bx)}\,dx$$

input
```
integrate(exp(n*atan(b*x+a))*x,x)
```

output
```
Integral(x*exp(n*atan(a + b*x)), x)
```

Maxima [F]

$$\int e^{n\arctan(a+bx)}x\,dx = \int xe^{(n\arctan(bx+a))}\,dx$$

input
```
integrate(exp(n*arctan(b*x+a))*x,x, algorithm="maxima")
```

output
```
integrate(x*e^(n*arctan(b*x + a)), x)
```

Giac [F(-1)]

Timed out.

$$\int e^{n\arctan(a+bx)}x\,dx = \text{Timed out}$$

input
```
integrate(exp(n*arctan(b*x+a))*x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(a+bx)}x\,dx = \int x\,\mathrm{e}^{n\,\mathrm{atan}(a+b\,x)}\,dx$$

input
```
int(x*exp(n*atan(a + b*x)),x)
```

output
```
int(x*exp(n*atan(a + b*x)), x)
```

Reduce [F]

$$\int e^{n\arctan(a+bx)}x\,dx = \int e^{atan(bx+a)n}xdx$$

input
```
int(exp(n*atan(b*x+a))*x,x)
```

output
```
int(e**(atan(a + b*x)*n)*x,x)
```

3.254 $\int e^{n \arctan(a+bx)} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 91

$$\int e^{n \arctan(a+bx)} \, dx = -\frac{2^{1-\frac{in}{2}}(1-ia-ibx)^{1+\frac{in}{2}} \text{Hypergeometric2F1}\left(1+\frac{in}{2},\frac{in}{2},2+\frac{in}{2},\frac{1}{2}(1-ia-ibx)\right)}{b(2i-n)}$$

output
```
-2^(1-1/2*I*n)*(1-I*a-I*b*x)^(1+1/2*I*n)*hypergeom([1/2*I*n, 1+1/2*I*n],[2
+1/2*I*n],1/2-1/2*I*a-1/2*I*b*x)/b/(2*I-n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.66

$$\int e^{n \arctan(a+bx)} \, dx = \frac{4e^{(2i+n)\arctan(a+bx)} \text{Hypergeometric2F1}\left(2,1-\frac{in}{2},2-\frac{in}{2},-e^{2i\arctan(a+bx)}\right)}{b(2i+n)}$$

input
```
Integrate[E^(n*ArcTan[a + b*x]),x]
```

output
```
(4*E^((2*I + n)*ArcTan[a + b*x])*Hypergeometric2F1[2, 1 - (I/2)*n, 2 - (I/
2)*n, -E^((2*I)*ArcTan[a + b*x])])/(b*(2*I + n))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{5616, 79\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{n \arctan(a+bx)} \, dx$$

$$\Big\downarrow \text{ 5616}$$

$$\int (-ia - ibx + 1)^{\frac{in}{2}} (ia + ibx + 1)^{-\frac{in}{2}} dx$$

$$\Big\downarrow \text{ 79}$$

$$-\frac{2^{1-\frac{in}{2}}(-ia - ibx + 1)^{1+\frac{in}{2}} \text{Hypergeometric2F1}\left(\frac{in}{2}+1, \frac{in}{2}, \frac{in}{2}+2, \frac{1}{2}(-ia - ibx + 1)\right)}{b(-n+2i)}$$

input
```
Int[E^(n*ArcTan[a + b*x]),x]
```

output
```
-((2^(1 - (I/2)*n)*(1 - I*a - I*b*x)^(1 + (I/2)*n)*Hypergeometric2F1[1 + (
I/2)*n, (I/2)*n, 2 + (I/2)*n, (1 - I*a - I*b*x)/2])/(b*(2*I - n)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5616
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.)), x_Symbol] :> Int[(1 - I*a*
c - I*b*c*x)^(I*(n/2))/(1 + I*a*c + I*b*c*x)^(I*(n/2)), x] /; FreeQ[{a, b,
c, n}, x]
```

Maple [F]

$$\int e^{n\arctan(bx+a)}dx$$

input
```
int(exp(n*arctan(b*x+a)),x)
```

output
```
int(exp(n*arctan(b*x+a)),x)
```

Fricas [F]

$$\int e^{n\arctan(a+bx)}\,dx = \int e^{(n\arctan(bx+a))}\,dx$$

input
```
integrate(exp(n*arctan(b*x+a)),x, algorithm="fricas")
```

output
```
integral(e^(n*arctan(b*x + a)), x)
```

Sympy [F]

$$\int e^{n\arctan(a+bx)}\,dx = \int e^{n\,\mathrm{atan}\,(a+bx)}\,dx$$

input
```
integrate(exp(n*atan(b*x+a)),x)
```

output
```
Integral(exp(n*atan(a + b*x)), x)
```

Maxima [F]

$$\int e^{n\arctan(a+bx)}\,dx = \int e^{(n\arctan(bx+a))}\,dx$$

input
```
integrate(exp(n*arctan(b*x+a)),x, algorithm="maxima")
```

output
```
integrate(e^(n*arctan(b*x + a)), x)
```

Giac [F(-1)]

Timed out.

$$\int e^{n\arctan(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(exp(n*arctan(b*x+a)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(a+bx)}\,dx = \int \mathrm{e}^{n\,\mathrm{atan}(a+b\,x)}\,dx$$

input
```
int(exp(n*atan(a + b*x)),x)
```

output
```
int(exp(n*atan(a + b*x)), x)
```

Reduce [F]

$$\int e^{n\arctan(a+bx)}\,dx = \int e^{atan(bx+a)n}dx$$

input
```
int(exp(n*atan(b*x+a)),x)
```

output
```
int(e**(atan(a + b*x)*n),x)
```

3.255 $\int \frac{e^{n\arctan(a+bx)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 191

$$\int \frac{e^{n\arctan(a+bx)}}{x}\,dx = \frac{2i(1-ia-ibx)^{\frac{in}{2}}(1+ia+ibx)^{-\frac{in}{2}}\,\text{Hypergeometric2F1}\left(1,\frac{in}{2},1+\frac{in}{2},\frac{(i-a)(1-ia-ibx)}{(i+a)(1+ia+ibx)}\right)}{n} - \frac{i2^{1-\frac{in}{2}}(1-ia-ibx)^{\frac{in}{2}}\,\text{Hypergeometric2F1}\left(\frac{in}{2},\frac{in}{2},1+\frac{in}{2},\frac{1}{2}(1-ia-ibx)\right)}{n}$$

output

```
2*I*(1-I*a-I*b*x)^(1/2*I*n)*hypergeom([1, 1/2*I*n],[1+1/2*I*n],(I-a)*(1-I*
a-I*b*x)/(I+a)/(1+I*a+I*b*x))/n/((1+I*a+I*b*x)^(1/2*I*n))-I*2^(1-1/2*I*n)*
(1-I*a-I*b*x)^(1/2*I*n)*hypergeom([1/2*I*n, 1/2*I*n],[1+1/2*I*n],1/2-1/2*I
*a-1/2*I*b*x)/n
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.89

$$\int \frac{e^{n \arctan(a+bx)}}{x} \, dx = \frac{2i(1+ia+ibx)^{-\frac{in}{2}}(-i(i+a+bx))^{\frac{in}{2}} \left(\text{Hypergeometric2F1}\left(1, \frac{in}{2}, 1+\frac{in}{2}, \frac{1+a^2-ibx+abx}{1+a^2+ibx+abx}\right) - 2^{-\frac{in}{2}}(1+ia\right.}{n}$$

input
```
Integrate[E^(n*ArcTan[a + b*x])/x,x]
```

output
```
((2*I)*((-I)*(I + a + b*x))^((I/2)*n)*(Hypergeometric2F1[1, (I/2)*n, 1 + (
I/2)*n, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)] - ((1 + I*a +
 I*b*x)^((I/2)*n)*Hypergeometric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n, (-1/2*I
)*(I + a + b*x)])/2^((I/2)*n)))/(n*(1 + I*a + I*b*x)^((I/2)*n))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5618, 140, 27, 79, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n \arctan(a+bx)}}{x} \, dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}}{x} dx$$

$$\Big\downarrow \text{140}$$

$$\int \frac{(1-ia)(-ia-ibx+1)^{\frac{in}{2}-1}(ia+ibx+1)^{-\frac{in}{2}}}{x} dx - ib \int (-ia-ibx+1)^{\frac{in}{2}-1}(ia+ibx+1)^{-\frac{in}{2}} dx$$

$$\Big\downarrow \text{27}$$

$$(1-ia)\int \frac{(-ia-ibx+1)^{\frac{in}{2}-1}(ia+ibx+1)^{-\frac{in}{2}}}{x}dx-ib\int (-ia-ibx+1)^{\frac{in}{2}-1}(ia+ibx+1)^{-\frac{in}{2}}dx$$

$$\Big\downarrow \text{79}$$

$$(1-ia)\int \frac{(-ia-ibx+1)^{\frac{in}{2}-1}(ia+ibx+1)^{-\frac{in}{2}}}{x}dx-\frac{i2^{1-\frac{in}{2}}(-ia-ibx+1)^{\frac{in}{2}}\text{Hypergeometric2F1}\left(\frac{in}{2},\frac{in}{2},\frac{in}{2}+1,\frac{1}{2}(-ia-ibx+1)\right)}{n}$$

$$\Big\downarrow \text{141}$$

$$-\frac{2(1-ia)(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}\text{Hypergeometric2F1}\left(1,\frac{in}{2},\frac{in}{2}+1,\frac{(i-a)(-ia-ibx+1)}{(a+i)(ia+ibx+1)}\right)}{(a+i)n}-\frac{i2^{1-\frac{in}{2}}(-ia-ibx+1)^{\frac{in}{2}}\text{Hypergeometric2F1}\left(\frac{in}{2},\frac{in}{2},\frac{in}{2}+1,\frac{1}{2}(-ia-ibx+1)\right)}{n}$$

input
```
Int[E^(n*ArcTan[a + b*x])/x,x]
```

output
```
(-2*(1 - I*a)*(1 - I*a - I*b*x)^((I/2)*n)*Hypergeometric2F1[1, (I/2)*n, 1
+ (I/2)*n, ((I - a)*(1 - I*a - I*b*x))/((I + a)*(1 + I*a + I*b*x))])/((I +
 a)*n*(1 + I*a + I*b*x)^((I/2)*n)) - (I*2^(1 - (I/2)*n)*(1 - I*a - I*b*x)^
((I/2)*n)*Hypergeometric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n, (1 - I*a - I*b*
x)/2])/n
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [F]

$$\int \frac{e^{n \arctan(bx+a)}}{x} dx$$

input
```
int(exp(n*arctan(b*x+a))/x,x)
```

output
```
int(exp(n*arctan(b*x+a))/x,x)
```

Fricas [F]

$$\int \frac{e^{n\arctan(a+bx)}}{x}\,dx = \int \frac{e^{(n\arctan(bx+a))}}{x}\,dx$$

input
```
integrate(exp(n*arctan(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(e^(n*arctan(b*x + a))/x, x)
```

Sympy [F]

$$\int \frac{e^{n\arctan(a+bx)}}{x}\,dx = \int \frac{e^{n\,\mathrm{atan}\,(a+bx)}}{x}\,dx$$

input
```
integrate(exp(n*atan(b*x+a))/x,x)
```

output
```
Integral(exp(n*atan(a + b*x))/x, x)
```

Maxima [F]

$$\int \frac{e^{n\arctan(a+bx)}}{x}\,dx = \int \frac{e^{(n\arctan(bx+a))}}{x}\,dx$$

input
```
integrate(exp(n*arctan(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(e^(n*arctan(b*x + a))/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(a+bx)}}{x}\,dx = \text{Timed out}$$

input
```
integrate(exp(n*arctan(b*x+a))/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(a+bx)}}{x}\,dx = \int \frac{e^{n\,\mathrm{atan}(a+b\,x)}}{x}\,dx$$

input
```
int(exp(n*atan(a + b*x))/x,x)
```

output
```
int(exp(n*atan(a + b*x))/x, x)
```

Reduce [F]

$$\int \frac{e^{n\arctan(a+bx)}}{x}\,dx = \int \frac{e^{atan(bx+a)n}}{x}dx$$

input
```
int(exp(n*atan(b*x+a))/x,x)
```

output
```
int(e**(atan(a + b*x)*n)/x,x)
```

3.256 $\int \frac{e^{n \arctan(a+bx)}}{x^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 128

$$\int \frac{e^{n \arctan(a+bx)}}{x^2} \, dx = -\frac{4b(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{-1-\frac{in}{2}} \text{Hypergeometric2F1}\left(2,1+\frac{in}{2},2+\frac{in}{2},\frac{(i-a)(1-ia-ibx)}{(i+a)(1+ia+ibx)}\right)}{(i+a)^2(2i-n)}$$

output
```
-4*b*(1-I*a-I*b*x)^(1+1/2*I*n)*(1+I*a+I*b*x)^(-1-1/2*I*n)*hypergeom([2, 1+
1/2*I*n],[2+1/2*I*n],(I-a)*(1-I*a-I*b*x)/(I+a)/(1+I*a+I*b*x))/(I+a)^2/(2*I
-n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.98

$$\int \frac{e^{n \arctan(a+bx)}}{x^2} \, dx = -\frac{4ib(1+ia+ibx)^{-\frac{in}{2}}(-i(i+a+bx))^{1+\frac{in}{2}} \text{Hypergeometric2F1}\left(2,1+\frac{in}{2},2+\frac{in}{2},\frac{1+a^2-ibx+abx}{1+a^2+ibx+abx}\right)}{(i+a)^2(-2i+n)(-i+a+bx)}$$

input
```
Integrate[E^(n*ArcTan[a + b*x])/x^2,x]
```

output
```
((-4*I)*b*((-I)*(I + a + b*x))^(1 + (I/2)*n)*Hypergeometric2F1[2, 1 + (I/2
)*n, 2 + (I/2)*n, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)])/((
I + a)^2*(-2*I + n)*(1 + I*a + I*b*x)^((I/2)*n)*(-I + a + b*x))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5618, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\arctan(a+bx)}}{x^2}\,dx$$

$$\Big\downarrow \text{5618}$$

$$\int \frac{(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}}{x^2}dx$$

$$\Big\downarrow \text{141}$$

$$-\frac{4b(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{-1-\frac{in}{2}}\text{Hypergeometric2F1}\left(2,\frac{in}{2}+1,\frac{in}{2}+2,\frac{(i-a)(-ia-ibx+1)}{(a+i)(ia+ibx+1)}\right)}{(a+i)^2(-n+2i)}$$

input
```
Int[E^(n*ArcTan[a + b*x])/x^2,x]
```

output
```
(-4*b*(1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(-1 - (I/2)*n)*Hyp
ergeometric2F1[2, 1 + (I/2)*n, 2 + (I/2)*n, ((I - a)*(1 - I*a - I*b*x))/((
I + a)*(1 + I*a + I*b*x))])/((I + a)^2*(2*I - n))
```

Defintions of rubi rules used

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [F]

$$\int \frac{\mathrm{e}^{n\arctan(bx+a)}}{x^2}dx$$

input
```
int(exp(n*arctan(b*x+a))/x^2,x)
```

output
```
int(exp(n*arctan(b*x+a))/x^2,x)
```

Fricas [F]

$$\int \frac{e^{n\arctan(a+bx)}}{x^2}\,dx = \int \frac{e^{(n\arctan(bx+a))}}{x^2}\,dx$$

input
```
integrate(exp(n*arctan(b*x+a))/x^2,x, algorithm="fricas")
```

output
```
integral(e^(n*arctan(b*x + a))/x^2, x)
```

Sympy [F]

$$\int \frac{e^{n\arctan(a+bx)}}{x^2}\,dx = \int \frac{e^{n\,\mathrm{atan}\,(a+bx)}}{x^2}\,dx$$

input
```
integrate(exp(n*atan(b*x+a))/x**2,x)
```

output
```
Integral(exp(n*atan(a + b*x))/x**2, x)
```

Maxima [F]

$$\int \frac{e^{n\arctan(a+bx)}}{x^2}\,dx = \int \frac{e^{(n\arctan(bx+a))}}{x^2}\,dx$$

input
```
integrate(exp(n*arctan(b*x+a))/x^2,x, algorithm="maxima")
```

output
```
integrate(e^(n*arctan(b*x + a))/x^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(a+bx)}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate(exp(n*arctan(b*x+a))/x^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(a+bx)}}{x^2}\,dx = \int \frac{e^{n\,\mathrm{atan}(a+b\,x)}}{x^2}\,dx$$

input
```
int(exp(n*atan(a + b*x))/x^2,x)
```

output
```
int(exp(n*atan(a + b*x))/x^2, x)
```

Reduce [F]

$$\int \frac{e^{n\arctan(a+bx)}}{x^2}\,dx = \frac{-e^{atan(bx+a)n} + \left(\int \frac{e^{atan(bx+a)n}}{b^2x^3+2ab\,x^2+a^2x+x}dx\right)bnx}{x}$$

input
```
int(exp(n*atan(b*x+a))/x^2,x)
```

output
```
( - e**(atan(a + b*x)*n) + int(e**(atan(a + b*x)*n)/(a**2*x + 2*a*b*x**2 +
 b**2*x**3 + x),x)*b*n*x)/x
```

3.257 $\int \frac{e^{n\arctan(a+bx)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 207

$$\int \frac{e^{n\arctan(a+bx)}}{x^3}\,dx = -\frac{(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{1-\frac{in}{2}}}{2(1+a^2)x^2} - \frac{2b^2(2a-n)(1-ia-ibx)^{1+\frac{in}{2}}(1+ia+ibx)^{-1-\frac{in}{2}}\,\text{Hypergeometric2F1}\left(2,1+\frac{in}{2},2+\frac{in}{2},\frac{(i-a)(1-ia-ibx)}{(i+a)(1+ia+ibx)}\right)}{(i-a)(i+a)^3(2i-n)}$$

output
```
-1/2*(1-I*a-I*b*x)^(1+1/2*I*n)*(1+I*a+I*b*x)^(1-1/2*I*n)/(a^2+1)/x^2-2*b^2
*(2*a-n)*(1-I*a-I*b*x)^(1+1/2*I*n)*(1+I*a+I*b*x)^(-1-1/2*I*n)*hypergeom([2
, 1+1/2*I*n],[2+1/2*I*n],(I-a)*(1-I*a-I*b*x)/(I+a)/(1+I*a+I*b*x))/(I-a)/(I
+a)^3/(2*I-n)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.84

$$\int \frac{e^{n\arctan(a+bx)}}{x^3}\,dx = -\frac{i(1+ia+ibx)^{-\frac{in}{2}}(-i(i+a+bx))^{1+\frac{in}{2}}\left((i+a)^2(-2i+n)(-i+a+bx)^2+4b^2(-2a+n)x^2\,\text{Hyperge}\right.}{2(-i+a)(i+a)^3(-2i+n)x^2(-i+a+bx)}$$

input
```
Integrate[E^(n*ArcTan[a + b*x])/x^3,x]
```

output
```
((-1/2*I)*((-I)*(I + a + b*x))^(1 + (I/2)*n)*((I + a)^2*(-2*I + n)*(-I + a
 + b*x)^2 + 4*b^2*(-2*a + n)*x^2*Hypergeometric2F1[2, 1 + (I/2)*n, 2 + (I/
2)*n, (1 + a^2 - I*b*x + a*b*x)/(1 + a^2 + I*b*x + a*b*x)]))/((-I + a)*(I
+ a)^3*(-2*I + n)*x^2*(1 + I*a + I*b*x)^((I/2)*n)*(-I + a + b*x))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5618, 107, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\arctan(a+bx)}}{x^3}\,dx$$

$$\Big\downarrow \ 5618$$

$$\int \frac{(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}}{x^3}dx$$

$$\Big\downarrow \ 107$$

$$-\frac{b(2a-n)\int \frac{(-ia-ibx+1)^{\frac{in}{2}}(ia+ibx+1)^{-\frac{in}{2}}}{x^2}dx}{2\left(a^2+1\right)}-\frac{(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{2\left(a^2+1\right)x^2}$$

$$\Big\downarrow \ 141$$

$$\frac{2b^2(2a-n)(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{-1-\frac{in}{2}}\text{Hypergeometric2F1}\left(2,\frac{in}{2}+1,\frac{in}{2}+2,\frac{(i-a)(-ia-ibx+1)}{(a+i)(ia+ibx+1)}\right)}{(a+i)^2\left(a^2+1\right)(-n+2i)}-\frac{(-ia-ibx+1)^{1+\frac{in}{2}}(ia+ibx+1)^{1-\frac{in}{2}}}{2\left(a^2+1\right)x^2}$$

input
```
Int[E^(n*ArcTan[a + b*x])/x^3,x]
```

output
```
-1/2*((1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a + I*b*x)^(1 - (I/2)*n))/((1
 + a^2)*x^2) + (2*b^2*(2*a - n)*(1 - I*a - I*b*x)^(1 + (I/2)*n)*(1 + I*a +
 I*b*x)^(-1 - (I/2)*n)*Hypergeometric2F1[2, 1 + (I/2)*n, 2 + (I/2)*n, ((I
- a)*(1 - I*a - I*b*x))/((I + a)*(1 + I*a + I*b*x))])/((I + a)^2*(1 + a^2)
*(2*I - n))
```

Defintions of rubi rules used

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 5618
```
Int[E^(ArcTan[(c_.)*((a_) + (b_.)*(x_))]*(n_.))*((d_.) + (e_.)*(x_))^(m_.),
 x_Symbol] :> Int[(d + e*x)^m*((1 - I*a*c - I*b*c*x)^(I*(n/2))/(1 + I*a*c +
 I*b*c*x)^(I*(n/2))), x] /; FreeQ[{a, b, c, d, e, m, n}, x]
```

Maple [F]

$$\int \frac{e^{n \arctan(bx+a)}}{x^3} dx$$

input
```
int(exp(n*arctan(b*x+a))/x^3,x)
```

output
```
int(exp(n*arctan(b*x+a))/x^3,x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(a+bx)}}{x^3} \, dx = \int \frac{e^{(n \arctan(bx+a))}}{x^3} \, dx$$

input
```
integrate(exp(n*arctan(b*x+a))/x^3,x, algorithm="fricas")
```

output
```
integral(e^(n*arctan(b*x + a))/x^3, x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(a+bx)}}{x^3} \, dx = \int \frac{e^{n \operatorname{atan}(a+bx)}}{x^3} \, dx$$

input
```
integrate(exp(n*atan(b*x+a))/x**3,x)
```

output
```
Integral(exp(n*atan(a + b*x))/x**3, x)
```

Maxima [F]

$$\int \frac{e^{n \arctan(a+bx)}}{x^3} \, dx = \int \frac{e^{(n \arctan(bx+a))}}{x^3} \, dx$$

input
```
integrate(exp(n*arctan(b*x+a))/x^3,x, algorithm="maxima")
```

output
```
integrate(e^(n*arctan(b*x + a))/x^3, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(a+bx)}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate(exp(n*arctan(b*x+a))/x^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(a+bx)}}{x^3}\,dx = \int \frac{\mathrm{e}^{n\,\mathrm{atan}(a+b\,x)}}{x^3}\,dx$$

input
```
int(exp(n*atan(a + b*x))/x^3,x)
```

output
```
int(exp(n*atan(a + b*x))/x^3, x)
```

Reduce [F]

$$\int \frac{e^{n\arctan(a+bx)}}{x^3}\,dx$$
$$= \frac{-e^{atan(bx+a)n}a^2 - e^{atan(bx+a)n}b^2x^2 - e^{atan(bx+a)n}bnx - e^{atan(bx+a)n} - 2\left(\int \frac{e^{atan(bx+a)n}}{a^2b^2x^3+2a^3b\,x^2+a^4x+b^2x^3+2ab\,x^2+2a^2x}\right.}{}$$

input
```
int(exp(n*atan(b*x+a))/x^3,x)
```

output
```
( - e**(atan(a + b*x)*n)*a**2 - e**(atan(a + b*x)*n)*b**2*x**2 - e**(atan(
a + b*x)*n)*b*n*x - e**(atan(a + b*x)*n) - 2*int(e**(atan(a + b*x)*n)/(a**
4*x + 2*a**3*b*x**2 + a**2*b**2*x**3 + 2*a**2*x + 2*a*b*x**2 + b**2*x**3 +
 x),x)*a**3*b**2*n*x**2 + int(e**(atan(a + b*x)*n)/(a**4*x + 2*a**3*b*x**2
 + a**2*b**2*x**3 + 2*a**2*x + 2*a*b*x**2 + b**2*x**3 + x),x)*a**2*b**2*n*
*2*x**2 - 2*int(e**(atan(a + b*x)*n)/(a**4*x + 2*a**3*b*x**2 + a**2*b**2*x
**3 + 2*a**2*x + 2*a*b*x**2 + b**2*x**3 + x),x)*a*b**2*n*x**2 + int(e**(at
an(a + b*x)*n)/(a**4*x + 2*a**3*b*x**2 + a**2*b**2*x**3 + 2*a**2*x + 2*a*b
*x**2 + b**2*x**3 + x),x)*b**2*n**2*x**2)/(2*x**2*(a**2 + 1))
```

3.258 $\int e^{\arctan(ax)}(c+a^2cx^2)^p\,dx$

Optimal result

Integrand size = 19, antiderivative size = 102

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx = \frac{i2^{\left(1-\frac{i}{2}\right)+p}(1-iax)^{\left(1+\frac{i}{2}\right)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\text{Hypergeometric2F1}\left(\frac{i}{2}-p,\left(1+\frac{i}{2}\right)+p,\left(2+\frac{i}{2}\right)+\right.}{a((2+i)+2p)}$$

output
```
I*2^(1-1/2*I+p)*(1-I*a*x)^(1+1/2*I+p)*(a^2*c*x^2+c)^p*hypergeom([1/2*I-p,
1+1/2*I+p],[2+1/2*I+p],1/2-1/2*I*a*x)/a/(2+I+2*p)/((a^2*x^2+1)^p)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx = \frac{i2^{-\frac{i}{2}+p}(1-iax)^{\left(1+\frac{i}{2}\right)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\text{Hypergeometric2F1}\left(\frac{i}{2}-p,\left(1+\frac{i}{2}\right)+p,\left(2+\frac{i}{2}\right)+p,\right.}{a\left(\left(1+\frac{i}{2}\right)+p\right)}$$

input
```
Integrate[E^ArcTan[a*x]*(c + a^2*c*x^2)^p,x]
```

output
```
(I*2^(-1/2*I + p)*(1 - I*a*x)^((1 + I/2) + p)*(c + a^2*c*x^2)^p*Hypergeome
tric2F1[I/2 - p, (1 + I/2) + p, (2 + I/2) + p, (1 - I*a*x)/2])/(a*((1 + I/
2) + p)*(1 + a^2*x^2)^p)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5599, 5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\arctan(ax)}\left(a^2cx^2+c\right)^p\,dx$$

$$\Big\downarrow \text{5599}$$

$$\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\int e^{\arctan(ax)}\left(a^2x^2+1\right)^p\,dx$$

$$\Big\downarrow \text{5596}$$

$$\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\int (1-iax)^{p+\frac{i}{2}}(iax+1)^{p-\frac{i}{2}}dx$$

$$\Big\downarrow \text{79}$$

$$\frac{i2^{p+\left(1-\frac{i}{2}\right)}(1-iax)^{p+\left(1+\frac{i}{2}\right)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\text{Hypergeometric2F1}\left(\frac{i}{2}-p,p+\left(1+\frac{i}{2}\right),p+\left(2+\frac{i}{2}\right),\frac{1}{2}(1}{a(2p+(2+i))}$$

input
```
Int[E^ArcTan[a*x]*(c + a^2*c*x^2)^p,x]
```

output
```
(I*2^((1 - I/2) + p)*(1 - I*a*x)^((1 + I/2) + p)*(c + a^2*c*x^2)^p*Hyperge
ometric2F1[I/2 - p, (1 + I/2) + p, (2 + I/2) + p, (1 - I*a*x)/2])/(a*((2 +
 I) + 2*p)*(1 + a^2*x^2)^p)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \mathrm{e}^{\arctan(ax)} \left(a^{2} c\, x^{2}+c\right)^{p} dx$$

input
```
int(exp(arctan(a*x))*(a^2*c*x^2+c)^p,x)
```

output
```
int(exp(arctan(a*x))*(a^2*c*x^2+c)^p,x)
```

Fricas [F]

$$\int e^{\arctan(ax)} \left(c+a^2cx^2\right)^p \, dx = \int \left(a^{2} c x^{2} + c\right)^{p} e^{\left(\arctan\left(a x\right)\right)} \, dx$$

input
```
integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^p*e^(arctan(a*x)), x)
```

Sympy [F]

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int\left(c\left(a^2x^2+1\right)\right)^pe^{\operatorname{atan}(ax)}\,dx$$

input
```
integrate(exp(atan(a*x))*(a**2*c*x**2+c)**p,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**p*exp(atan(a*x)), x)
```

Maxima [F]

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int\left(a^2cx^2+c\right)^pe^{(\arctan(ax))}\,dx$$

input
```
integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^p*e^(arctan(a*x)), x)
```

Giac [F]

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int\left(a^2cx^2+c\right)^pe^{(\arctan(ax))}\,dx$$

input
```
integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^p*e^(arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int e^{\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^p\,dx$$

input
```
int(exp(atan(a*x))*(c + a^2*c*x^2)^p,x)
```

output
```
int(exp(atan(a*x))*(c + a^2*c*x^2)^p, x)
```

Reduce [F]

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int e^{atan(ax)}\left(a^2c\,x^2+c\right)^p dx$$

input
```
int(exp(atan(a*x))*(a^2*c*x^2+c)^p,x)
```

output
```
int(e**atan(a*x)*(a**2*c*x**2 + c)**p,x)
```

3.259 $\int e^{\arctan(ax)}(c+a^2cx^2)^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx = \frac{\left(\frac{1}{37}+\frac{6i}{37}\right)2^{3-\frac{i}{2}}c^2(1-iax)^{3+\frac{i}{2}}\,\text{Hypergeometric2F1}\left(-2+\frac{i}{2},3+\frac{i}{2},4+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

output
```
(1/37+6/37*I)*2^(3-1/2*I)*c^2*(1-I*a*x)^(3+1/2*I)*hypergeom([3+1/2*I, -2+1
/2*I],[4+1/2*I],1/2-1/2*I*a*x)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx = \frac{\left(\frac{1}{37}+\frac{6i}{37}\right)2^{3-\frac{i}{2}}c^2(1-iax)^{3+\frac{i}{2}}\,\text{Hypergeometric2F1}\left(-2+\frac{i}{2},3+\frac{i}{2},4+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

input
```
Integrate[E^ArcTan[a*x]*(c + a^2*c*x^2)^2,x]
```

output
```
((1/37 + (6*I)/37)*2^(3 - I/2)*c^2*(1 - I*a*x)^(3 + I/2)*Hypergeometric2F1
[-2 + I/2, 3 + I/2, 4 + I/2, (1 - I*a*x)/2])/a
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\arctan(ax)}\left(a^2cx^2+c\right)^2\,dx$$

$$\Big\downarrow \text{ 5596}$$

$$c^2\int (1-iax)^{2+\frac{i}{2}}(iax+1)^{2-\frac{i}{2}}dx$$

$$\Big\downarrow \text{ 79}$$

$$\frac{\left(\frac{1}{37}+\frac{6i}{37}\right)2^{3-\frac{i}{2}}c^2(1-iax)^{3+\frac{i}{2}}\text{Hypergeometric2F1}\left(-2+\frac{i}{2},3+\frac{i}{2},4+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

input
```
Int[E^ArcTan[a*x]*(c + a^2*c*x^2)^2,x]
```

output
```
((1/37 + (6*I)/37)*2^(3 - I/2)*c^2*(1 - I*a*x)^(3 + I/2)*Hypergeometric2F1
[-2 + I/2, 3 + I/2, 4 + I/2, (1 - I*a*x)/2])/a
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int e^{\arctan(ax)} \left(a^2 c\,x^2 + c\right)^2 dx$$

input
```
int(exp(arctan(a*x))*(a^2*c*x^2+c)^2,x)
```

output
```
int(exp(arctan(a*x))*(a^2*c*x^2+c)^2,x)
```

Fricas [F]

$$\int e^{\arctan(ax)} (c + a^2 cx^2)^2 \, dx = \int (a^2 cx^2 + c)^2 e^{(\arctan(ax))} \, dx$$

input
```
integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*e^(arctan(a*x)), x)
```

Sympy [F]

$$\int e^{\arctan(ax)} (c + a^2 cx^2)^2 \, dx = c^2 \left(\int 2a^2 x^2 e^{\operatorname{atan}(ax)} \, dx + \int a^4 x^4 e^{\operatorname{atan}(ax)} \, dx + \int e^{\operatorname{atan}(ax)} \, dx \right)$$

input
```
integrate(exp(atan(a*x))*(a**2*c*x**2+c)**2,x)
```

output
```
c**2*(Integral(2*a**2*x**2*exp(atan(a*x)), x) + Integral(a**4*x**4*exp(ata
n(a*x)), x) + Integral(exp(atan(a*x)), x))
```

Maxima [F]

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int \left(a^2cx^2+c\right)^2e^{(\arctan(ax))}\,dx$$

input
```
integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^2*e^(arctan(a*x)), x)
```

Giac [F]

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int \left(a^2cx^2+c\right)^2e^{(\arctan(ax))}\,dx$$

input
```
integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*e^(arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int e^{\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

input
```
int(exp(atan(a*x))*(c + a^2*c*x^2)^2,x)
```

output
```
int(exp(atan(a*x))*(c + a^2*c*x^2)^2, x)
```

Reduce [F]

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx = c^2\left(\int e^{atan(ax)}dx+\left(\int e^{atan(ax)}x^4dx\right)a^4+2\left(\int e^{atan(ax)}x^2dx\right)a^2\right)$$

input
```
int(exp(atan(a*x))*(a^2*c*x^2+c)^2,x)
```

output
```
c**2*(int(e**atan(a*x),x) + int(e**atan(a*x)*x**4,x)*a**4 + 2*int(e**atan(
a*x)*x**2,x)*a**2)
```

3.260 $\int e^{\arctan(ax)}(c+a^2cx^2)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int e^{\arctan(ax)}(c+a^2cx^2)\,dx = \frac{\left(\frac{1}{17}+\frac{4i}{17}\right)2^{2-\frac{i}{2}}c(1-iax)^{2+\frac{i}{2}}\,\text{Hypergeometric2F1}\left(-1+\frac{i}{2},2+\frac{i}{2},3+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

output
```
(1/17+4/17*I)*2^(2-1/2*I)*c*(1-I*a*x)^(2+1/2*I)*hypergeom([2+1/2*I, -1+1/2
*I],[3+1/2*I],1/2-1/2*I*a*x)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int e^{\arctan(ax)}(c+a^2cx^2)\,dx = \frac{\left(\frac{1}{17}+\frac{4i}{17}\right)2^{2-\frac{i}{2}}c(1-iax)^{2+\frac{i}{2}}\,\text{Hypergeometric2F1}\left(-1+\frac{i}{2},2+\frac{i}{2},3+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

input
```
Integrate[E^ArcTan[a*x]*(c + a^2*c*x^2),x]
```

output
```
((1/17 + (4*I)/17)*2^(2 - I/2)*c*(1 - I*a*x)^(2 + I/2)*Hypergeometric2F1[-
1 + I/2, 2 + I/2, 3 + I/2, (1 - I*a*x)/2])/a
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\arctan(ax)}\left(a^2cx^2+c\right)\,dx$$

$$\Big\downarrow \text{5596}$$

$$c\int (1-iax)^{1+\frac{i}{2}}(iax+1)^{1-\frac{i}{2}}dx$$

$$\Big\downarrow \text{79}$$

$$\frac{\left(\frac{1}{17}+\frac{4i}{17}\right)2^{2-\frac{i}{2}}c(1-iax)^{2+\frac{i}{2}}\text{Hypergeometric2F1}\left(-1+\frac{i}{2},2+\frac{i}{2},3+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

input
```
Int[E^ArcTan[a*x]*(c + a^2*c*x^2),x]
```

output

```
((1/17 + (4*I)/17)*2^(2 - I/2)*c*(1 - I*a*x)^(2 + I/2)*Hypergeometric2F1[-
1 + I/2, 2 + I/2, 3 + I/2, (1 - I*a*x)/2])/a
```


Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \mathrm{e}^{\arctan(ax)}\left(a^{2}c\,x^{2}+c\right)dx$$

input
```
int(exp(arctan(a*x))*(a^2*c*x^2+c),x)
```

output
```
int(exp(arctan(a*x))*(a^2*c*x^2+c),x)
```

Fricas [F]

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int\left(a^2cx^2+c\right)e^{(\arctan(ax))}\,dx$$

input
```
integrate(exp(arctan(a*x))*(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*e^(arctan(a*x)), x)
```

Sympy [F]

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)\,dx=c\left(\int a^2x^2e^{\operatorname{atan}(ax)}\,dx+\int e^{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate(exp(atan(a*x))*(a**2*c*x**2+c),x)
```

output
```
c*(Integral(a**2*x**2*exp(atan(a*x)), x) + Integral(exp(atan(a*x)), x))
```

Maxima [F]

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int \left(a^2cx^2+c\right)e^{(\arctan(ax))}\,dx$$

input
```
integrate(exp(arctan(a*x))*(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)*e^(arctan(a*x)), x)
```

Giac [F]

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int \left(a^2cx^2+c\right)e^{(\arctan(ax))}\,dx$$

input
```
integrate(exp(arctan(a*x))*(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*e^(arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int e^{\mathrm{atan}(a\,x)}\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(exp(atan(a*x))*(c + a^2*c*x^2),x)
```

output
```
int(exp(atan(a*x))*(c + a^2*c*x^2), x)
```

Reduce [F]

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)\,dx=c\left(\int e^{atan(ax)}dx+\left(\int e^{atan(ax)}x^2dx\right)a^2\right)$$

input
```
int(exp(atan(a*x))*(a^2*c*x^2+c),x)
```

output
```
c*(int(e**atan(a*x),x) + int(e**atan(a*x)*x**2,x)*a**2)
```

3.261 $\int e^{\arctan(ax)}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 60

$$\int e^{\arctan(ax)}\,dx = \frac{\left(\frac{1}{5}+\frac{2i}{5}\right)2^{1-\frac{i}{2}}(1-iax)^{1+\frac{i}{2}}\operatorname{Hypergeometric2F1}\left(\frac{i}{2},1+\frac{i}{2},2+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

output
```
(1/5+2/5*I)*2^(1-1/2*I)*(1-I*a*x)^(1+1/2*I)*hypergeom([1/2*I, 1+1/2*I],[2+
1/2*I],1/2-1/2*I*a*x)/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

$$\int e^{\arctan(ax)}\,dx = \frac{\left(\frac{4}{5}-\frac{8i}{5}\right)e^{(1+2i)\arctan(ax)}\operatorname{Hypergeometric2F1}\left(1-\frac{i}{2},2,2-\frac{i}{2},-e^{2i\arctan(ax)}\right)}{a}$$

input
```
Integrate[E^ArcTan[a*x],x]
```

output
```
((4/5 - (8*I)/5)*E^((1 + 2*I)*ArcTan[a*x])*Hypergeometric2F1[1 - I/2, 2, 2
 - I/2, -E^((2*I)*ArcTan[a*x])])/a
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5584, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\arctan(ax)}\,dx$$

$$\Big\downarrow \; 5584$$

$$\int (1-iax)^{\frac{i}{2}}(1+iax)^{-\frac{i}{2}}dx$$

$$\Big\downarrow \; 79$$

$$\frac{\left(\frac{1}{5}+\frac{2i}{5}\right)2^{1-\frac{i}{2}}(1-iax)^{1+\frac{i}{2}}\text{Hypergeometric2F1}\left(\frac{i}{2},1+\frac{i}{2},2+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

input
```
Int[E^ArcTan[a*x],x]
```

output
```
((1/5 + (2*I)/5)*2^(1 - I/2)*(1 - I*a*x)^(1 + I/2)*Hypergeometric2F1[I/2,
1 + I/2, 2 + I/2, (1 - I*a*x)/2])/a
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5584
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Maple [F]

$$\int e^{\arctan(ax)}dx$$

input
```
int(exp(arctan(a*x)),x)
```

output
```
int(exp(arctan(a*x)),x)
```

Fricas [F]

$$\int e^{\arctan(ax)}\,dx = \int e^{(\arctan(ax))}\,dx$$

input
```
integrate(exp(arctan(a*x)),x, algorithm="fricas")
```

output
```
integral(e^(arctan(a*x)), x)
```

Sympy [F]

$$\int e^{\arctan(ax)}\,dx = \int e^{\operatorname{atan}(ax)}\,dx$$

input
```
integrate(exp(atan(a*x)),x)
```

output
```
Integral(exp(atan(a*x)), x)
```

Maxima [F]

$$\int e^{\arctan(ax)}\,dx = \int e^{(\arctan(ax))}\,dx$$

input
```
integrate(exp(arctan(a*x)),x, algorithm="maxima")
```

output
```
integrate(e^(arctan(a*x)), x)
```

Giac [F]

$$\int e^{\arctan(ax)}\,dx = \int e^{(\arctan(ax))}\,dx$$

input
```
integrate(exp(arctan(a*x)),x, algorithm="giac")
```

output
```
integrate(e^(arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\arctan(ax)}\,dx = \int \mathrm{e}^{\mathrm{atan}(a\,x)}\,dx$$

input
```
int(exp(atan(a*x)),x)
```

output
```
int(exp(atan(a*x)), x)
```

Reduce [F]

$$\int e^{\arctan(ax)}\,dx = \int e^{atan(ax)}dx$$

input
```
int(exp(atan(a*x)),x)
```

output
```
int(e**atan(a*x),x)
```

3.262 $\int \frac{e^{\arctan(ax)}}{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 13

$$\int \frac{e^{\arctan(ax)}}{c+a^2cx^2}\,dx=\frac{e^{\arctan(ax)}}{ac}$$

output
```
exp(arctan(a*x))/a/c
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.69

$$\int \frac{e^{\arctan(ax)}}{c+a^2cx^2}\,dx=\frac{(1-iax)^{\frac{i}{2}}(1+iax)^{-\frac{i}{2}}}{ac}$$

input
```
Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2),x]
```

output
```
(1 - I*a*x)^(I/2)/(a*c*(1 + I*a*x)^(I/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5594}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\arctan(ax)}}{a^2cx^2+c}\,dx$$

$$\Big\downarrow \ 5594$$

$$\frac{e^{\arctan(ax)}}{ac}$$

input
```
Int[E^ArcTan[a*x]/(c + a^2*c*x^2),x]
```

output
```
E^ArcTan[a*x]/(a*c)
```

Defintions of rubi rules used

rule 5594
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{e^{\arctan(ax)}}{ac}$	13
parallelrisch	$\frac{e^{\arctan(ax)}}{ac}$	13
risch	$\frac{(-iax+1)^{\frac{i}{2}}(iax+1)^{-\frac{i}{2}}}{ac}$	28
orering	$\frac{(a^2x^2+1)e^{\arctan(ax)}}{a(a^2c\,x^2+c)}$	31

input
```
int(exp(arctan(a*x))/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
exp(arctan(a*x))/a/c
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{e^{\arctan(ax)}}{c + a^2cx^2} \, dx = \frac{e^{(\arctan(ax))}}{ac}$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
e^(arctan(a*x))/(a*c)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{e^{\arctan(ax)}}{c + a^2cx^2} \, dx = \begin{cases} \frac{e^{\operatorname{atan}(ax)}}{ac} & \text{for } a \neq 0 \\ \frac{x}{c} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(atan(a*x))/(a**2*c*x**2+c),x)
```

output
```
Piecewise((exp(atan(a*x))/(a*c), Ne(a, 0)), (x/c, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{e^{\arctan(ax)}}{c + a^2cx^2} \, dx = \frac{e^{(\arctan(ax))}}{ac}$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
e^(arctan(a*x))/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{e^{\arctan(ax)}}{c + a^2cx^2} \, dx = \frac{e^{(\arctan(ax))}}{ac}$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
e^(arctan(a*x))/(a*c)
```

Mupad [B] (verification not implemented)

Time = 23.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{e^{\arctan(ax)}}{c + a^2cx^2} \, dx = \frac{\mathrm{e}^{\mathrm{atan}(a\,x)}}{a\,c}$$

input
```
int(exp(atan(a*x))/(c + a^2*c*x^2),x)
```

output
```
exp(atan(a*x))/(a*c)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^{\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{e^{atan(ax)}}{ac}$$

input
```
int(exp(atan(a*x))/(a^2*c*x^2+c),x)
```

output
```
e**atan(a*x)/(a*c)
```

3.263 $\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{2e^{\arctan(ax)}}{5ac^2}+\frac{e^{\arctan(ax)}(1+2ax)}{5ac^2(1+a^2x^2)}$$

output
```
2/5*exp(arctan(a*x))/a/c^2+1/5*exp(arctan(a*x))*(2*a*x+1)/a/c^2/(a^2*x^2+1
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{(1-iax)^{\frac{i}{2}}(1+iax)^{-\frac{i}{2}}\,(3+2ax+2a^2x^2)}{5c^2\,(a+a^3x^2)}$$

input
```
Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^2,x]
```

output
```
((1 - I*a*x)^(I/2)*(3 + 2*a*x + 2*a^2*x^2))/(5*c^2*(1 + I*a*x)^(I/2)*(a +
a^3*x^2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {5593, 27, 5594}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\arctan(ax)}}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \ 5593$$

$$\frac{2\int \frac{e^{\arctan(ax)}}{c(a^2x^2+1)}dx}{5c}+\frac{(2ax+1)e^{\arctan(ax)}}{5ac^2(a^2x^2+1)}$$

$$\Big\downarrow \ 27$$

$$\frac{2\int \frac{e^{\arctan(ax)}}{a^2x^2+1}dx}{5c^2}+\frac{(2ax+1)e^{\arctan(ax)}}{5ac^2(a^2x^2+1)}$$

$$\Big\downarrow \ 5594$$

$$\frac{(2ax+1)e^{\arctan(ax)}}{5ac^2(a^2x^2+1)}+\frac{2e^{\arctan(ax)}}{5ac^2}$$

input
```
Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^2,x]
```

output
```
(2*E^ArcTan[a*x])/(5*a*c^2) + (E^ArcTan[a*x]*(1 + 2*a*x))/(5*a*c^2*(1 + a^
2*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5593
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2)))   I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
&& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
 0] && IntegerQ[2*p]
```

rule 5594
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{e^{\arctan(ax)}\left(2a^{2}x^{2}+2ax+3\right)}{5\left(a^{2}x^{2}+1\right)a\,c^{2}}$	39
orering	$\frac{\left(2a^{2}x^{2}+2ax+3\right)\left(a^{2}x^{2}+1\right)e^{\arctan(ax)}}{5a\left(a^{2}c\,x^{2}+c\right)^{2}}$	46
parallelrisch	$\frac{2a^{2}e^{\arctan(ax)}x^{2}+2\,e^{\arctan(ax)}ax+3\,e^{\arctan(ax)}}{5c^{2}\left(a^{2}x^{2}+1\right)a}$	50

input
```
int(exp(arctan(a*x))/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*exp(arctan(a*x))*(2*a^2*x^2+2*a*x+3)/(a^2*x^2+1)/a/c^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \frac{e^{\arctan(ax)}}{\left(c+a^2cx^2\right)^2}\,dx=\frac{\left(2\,a^{2}x^{2}+2\,ax+3\right)e^{\left(\arctan(ax)\right)}}{5\,\left(a^{3}c^{2}x^{2}+ac^{2}\right)}$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/5*(2*a^2*x^2 + 2*a*x + 3)*e^(arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(42) = 84$.

Time = 1.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.90

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \begin{cases} \frac{2a^2x^2e^{\operatorname{atan}(ax)}}{5a^3c^2x^2+5ac^2} + \frac{2axe^{\operatorname{atan}(ax)}}{5a^3c^2x^2+5ac^2} + \frac{3e^{\operatorname{atan}(ax)}}{5a^3c^2x^2+5ac^2} & \text{for } a \neq 0 \\ \frac{x}{c^2} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(atan(a*x))/(a**2*c*x**2+c)**2,x)
```

output
```
Piecewise((2*a**2*x**2*exp(atan(a*x))/(5*a**3*c**2*x**2 + 5*a*c**2) + 2*a*
x*exp(atan(a*x))/(5*a**3*c**2*x**2 + 5*a*c**2) + 3*exp(atan(a*x))/(5*a**3*
c**2*x**2 + 5*a*c**2), Ne(a, 0)), (x/c**2, True))
```

Maxima [F]

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{e^{(\arctan(ax))}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(44) = 88$.

Time = 0.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.18

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx$$
$$= \frac{3\,e^{(\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^4 - 4\,e^{(\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^3 + 2\,e^{(\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)}{5\left(ac^2\tan\left(\frac{1}{2}\arctan(ax)\right)^4 + 2\,ac^2\tan\left(\frac{1}{2}\arctan(ax)\right)^2 +\right.}$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
1/5*(3*e^(arctan(a*x))*tan(1/2*arctan(a*x))^4 - 4*e^(arctan(a*x))*tan(1/2*
arctan(a*x))^3 + 2*e^(arctan(a*x))*tan(1/2*arctan(a*x))^2 + 4*e^(arctan(a*
x))*tan(1/2*arctan(a*x)) + 3*e^(arctan(a*x)))/(a*c^2*tan(1/2*arctan(a*x))^
4 + 2*a*c^2*tan(1/2*arctan(a*x))^2 + a*c^2)
```

Mupad [B] (verification not implemented)

Time = 23.35 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{e^{\mathrm{atan}(a\,x)}\left(\frac{3}{5\,a^3\,c^2} + \frac{2\,x}{5\,a^2\,c^2} + \frac{2\,x^2}{5\,a\,c^2}\right)}{\frac{1}{a^2} + x^2}$$

input
```
int(exp(atan(a*x))/(c + a^2*c*x^2)^2,x)
```

output
```
(exp(atan(a*x))*(3/(5*a^3*c^2) + (2*x)/(5*a^2*c^2) + (2*x^2)/(5*a*c^2)))/(
1/a^2 + x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{e^{atan(ax)}(2a^2x^2+2ax+3)}{5a\,c^2\,(a^2x^2+1)}$$

input
```
int(exp(atan(a*x))/(a^2*c*x^2+c)^2,x)
```

output
```
(e**atan(a*x)*(2*a**2*x**2 + 2*a*x + 3))/(5*a*c**2*(a**2*x**2 + 1))
```

3.264 $\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 83

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{24e^{\arctan(ax)}}{85ac^3}+\frac{e^{\arctan(ax)}(1+4ax)}{17ac^3\left(1+a^2x^2\right)^2}+\frac{12e^{\arctan(ax)}(1+2ax)}{85ac^3\left(1+a^2x^2\right)}$$

output
```
24/85*exp(arctan(a*x))/a/c^3+1/17*exp(arctan(a*x))*(4*a*x+1)/a/c^3/(a^2*x^
2+1)^2+12/85*exp(arctan(a*x))*(2*a*x+1)/a/c^3/(a^2*x^2+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.07

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{5e^{\arctan(ax)}(1+4ax)+12(1-iax)^{\frac{i}{2}}(1+iax)^{-\frac{i}{2}}\left(1+a^2x^2\right)\left(3+2ax+2a^2x^2\right)}{85ac^3\left(1+a^2x^2\right)^2}$$

input
```
Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^3,x]
```

output
```
(5*E^ArcTan[a*x]*(1 + 4*a*x) + (12*(1 - I*a*x)^(I/2)*(1 + a^2*x^2)*(3 + 2*
a*x + 2*a^2*x^2))/(1 + I*a*x)^(I/2))/(85*a*c^3*(1 + a^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {5593, 27, 5593, 5594}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\arctan(ax)}}{(a^2cx^2+c)^3}dx$$

$$\Big\downarrow \; 5593$$

$$\frac{12\int \frac{e^{\arctan(ax)}}{c^2(a^2x^2+1)^2}dx}{17c}+\frac{(4ax+1)e^{\arctan(ax)}}{17ac^3(a^2x^2+1)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{12\int \frac{e^{\arctan(ax)}}{(a^2x^2+1)^2}dx}{17c^3}+\frac{(4ax+1)e^{\arctan(ax)}}{17ac^3(a^2x^2+1)^2}$$

$$\Big\downarrow \; 5593$$

$$\frac{12\left(\frac{2}{5}\int \frac{e^{\arctan(ax)}}{a^2x^2+1}dx+\frac{(2ax+1)e^{\arctan(ax)}}{5a(a^2x^2+1)}\right)}{17c^3}+\frac{(4ax+1)e^{\arctan(ax)}}{17ac^3(a^2x^2+1)^2}$$

$$\Big\downarrow \; 5594$$

$$\frac{(4ax+1)e^{\arctan(ax)}}{17ac^3(a^2x^2+1)^2}+\frac{12\left(\frac{(2ax+1)e^{\arctan(ax)}}{5a(a^2x^2+1)}+\frac{2e^{\arctan(ax)}}{5a}\right)}{17c^3}$$

input
```
Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^3,x]
```

output
```
(E^ArcTan[a*x]*(1 + 4*a*x))/(17*a*c^3*(1 + a^2*x^2)^2) + (12*((2*E^ArcTan[
a*x])/(5*a) + (E^ArcTan[a*x]*(1 + 2*a*x))/(5*a*(1 + a^2*x^2))))/(17*c^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5593
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2)))   I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
&& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
 0] && IntegerQ[2*p]
```

rule 5594
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Maple [A] (verified)

Time = 11.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.66

method	result	size
gosper	$\frac{e^{\arctan(ax)}\left(24a^4x^4+24a^3x^3+60a^2x^2+44ax+41\right)}{85\left(a^2x^2+1\right)^2 a\,c^3}$	55
orering	$\frac{\left(24a^4x^4+24a^3x^3+60a^2x^2+44ax+41\right)\left(a^2x^2+1\right)e^{\arctan(ax)}}{85a\left(a^2c\,x^2+c\right)^3}$	62
parallelrisch	$\frac{24a^4e^{\arctan(ax)}x^4+24a^3e^{\arctan(ax)}x^3+60a^2e^{\arctan(ax)}x^2+44\,e^{\arctan(ax)}ax+41\,e^{\arctan(ax)}}{85c^3\left(a^2x^2+1\right)^2 a}$	76

input
```
int(exp(arctan(a*x))/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/85*exp(arctan(a*x))*(24*a^4*x^4+24*a^3*x^3+60*a^2*x^2+44*a*x+41)/(a^2*x^
2+1)^2/a/c^3
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{(24\,a^4x^4+24\,a^3x^3+60\,a^2x^2+44\,ax+41)e^{(\arctan(ax))}}{85\,(a^5c^3x^4+2\,a^3c^3x^2+ac^3)}$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/85*(24*a^4*x^4 + 24*a^3*x^3 + 60*a^2*x^2 + 44*a*x + 41)*e^(arctan(a*x))/
(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(75) = 150.

Time = 3.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.69

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \begin{cases} \frac{24a^4x^4e^{\operatorname{atan}(ax)}}{85a^5c^3x^4+170a^3c^3x^2+85ac^3} + \frac{24a^3x^3e^{\operatorname{atan}(ax)}}{85a^5c^3x^4+170a^3c^3x^2+85ac^3} + \frac{60a^2x^2e^{\operatorname{atan}(ax)}}{85a^5c^3x^4+170a^3c^3x^2+85ac^3} + \frac{44axe^{\operatorname{atan}(ax)}}{85a^5c^3x^4+170a^3c^3x^2+85ac^3} + \frac{}{85a} \\ \frac{x}{c^3} \end{cases}$$

input
```
integrate(exp(atan(a*x))/(a**2*c*x**2+c)**3,x)
```

output
```
Piecewise((24*a**4*x**4*exp(atan(a*x))/(85*a**5*c**3*x**4 + 170*a**3*c**3*
x**2 + 85*a*c**3) + 24*a**3*x**3*exp(atan(a*x))/(85*a**5*c**3*x**4 + 170*a
**3*c**3*x**2 + 85*a*c**3) + 60*a**2*x**2*exp(atan(a*x))/(85*a**5*c**3*x**
4 + 170*a**3*c**3*x**2 + 85*a*c**3) + 44*a*x*exp(atan(a*x))/(85*a**5*c**3*
x**4 + 170*a**3*c**3*x**2 + 85*a*c**3) + 41*exp(atan(a*x))/(85*a**5*c**3*x
**4 + 170*a**3*c**3*x**2 + 85*a*c**3), Ne(a, 0)), (x/c**3, True))
```

Maxima [F]

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{e^{(\arctan(ax))}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(74) = 148.

Time = 0.14 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.45

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{41\,e^{(\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^8 - 88\,e^{(\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^7 + 76\,e^{(\arctan(ax))}\tan\left(\frac{1}{2}\arctan(a\right.}{85\left(ac^3\tan\right.}$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
1/85*(41*e^(arctan(a*x))*tan(1/2*arctan(a*x))^8 - 88*e^(arctan(a*x))*tan(1
/2*arctan(a*x))^7 + 76*e^(arctan(a*x))*tan(1/2*arctan(a*x))^6 + 72*e^(arct
an(a*x))*tan(1/2*arctan(a*x))^5 + 150*e^(arctan(a*x))*tan(1/2*arctan(a*x))
^4 - 72*e^(arctan(a*x))*tan(1/2*arctan(a*x))^3 + 76*e^(arctan(a*x))*tan(1/
2*arctan(a*x))^2 + 88*e^(arctan(a*x))*tan(1/2*arctan(a*x)) + 41*e^(arctan(
a*x)))/(a*c^3*tan(1/2*arctan(a*x))^8 + 4*a*c^3*tan(1/2*arctan(a*x))^6 + 6*
a*c^3*tan(1/2*arctan(a*x))^4 + 4*a*c^3*tan(1/2*arctan(a*x))^2 + a*c^3)
```

Mupad [B] (verification not implemented)

Time = 23.45 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{24\,\mathrm{e}^{\mathrm{atan}(a\,x)}}{85\,a\,c^3}+\frac{12\,\mathrm{e}^{\mathrm{atan}(a\,x)}\,(2\,a\,x+1)}{85\,a\,c^3\,\left(a^2\,x^2+1\right)}+\frac{\mathrm{e}^{\mathrm{atan}(a\,x)}\,(4\,a\,x+1)}{17\,a\,c^3\,{\left(a^2\,x^2+1\right)}^2}$$

input
```
int(exp(atan(a*x))/(c + a^2*c*x^2)^3,x)
```

output
```
(24*exp(atan(a*x)))/(85*a*c^3) + (12*exp(atan(a*x))*(2*a*x + 1))/(85*a*c^3
*(a^2*x^2 + 1)) + (exp(atan(a*x))*(4*a*x + 1))/(17*a*c^3*(a^2*x^2 + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.76

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{e^{atan(ax)}(24a^4x^4+24a^3x^3+60a^2x^2+44ax+41)}{85a\,c^3\,(a^4x^4+2a^2x^2+1)}$$

input
```
int(exp(atan(a*x))/(a^2*c*x^2+c)^3,x)
```

output
```
(e**atan(a*x)*(24*a**4*x**4 + 24*a**3*x**3 + 60*a**2*x**2 + 44*a*x + 41))/
(85*a*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

3.265 $\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^4} dx$

Optimal result

Integrand size = 19, antiderivative size = 116

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^4} dx = \frac{144e^{\arctan(ax)}}{629ac^4} + \frac{e^{\arctan(ax)}(1+6ax)}{37ac^4(1+a^2x^2)^3} + \frac{30e^{\arctan(ax)}(1+4ax)}{629ac^4(1+a^2x^2)^2} + \frac{72e^{\arctan(ax)}(1+2ax)}{629ac^4(1+a^2x^2)}$$

output
```
144/629*exp(arctan(a*x))/a/c^4+1/37*exp(arctan(a*x))*(6*a*x+1)/a/c^4/(a^2*
x^2+1)^3+30/629*exp(arctan(a*x))*(4*a*x+1)/a/c^4/(a^2*x^2+1)^2+72/629*exp(
arctan(a*x))*(2*a*x+1)/a/c^4/(a^2*x^2+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^4} dx = \frac{17ce^{\arctan(ax)}(1+6ax)+6(c+a^2cx^2)\left(5e^{\arctan(ax)}(1+4ax)+12(1-iax)^{\frac{i}{2}}(1+iax)^{-\frac{i}{2}}(-i+ax)(i+a\right.}{629ac^2(c+a^2cx^2)^3}$$

input
```
Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^4,x]
```

output
```
(17*c*E^ArcTan[a*x]*(1 + 6*a*x) + 6*(c + a^2*c*x^2)*(5*E^ArcTan[a*x]*(1 +
4*a*x) + (12*(1 - I*a*x)^(I/2)*(-I + a*x)*(I + a*x)*(3 + 2*a*x + 2*a^2*x^2
))/(1 + I*a*x)^(I/2)))/(629*a*c^2*(c + a^2*c*x^2)^3)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5593, 27, 5593, 5593, 5594}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\arctan(ax)}}{\left(a^2cx^2+c\right)^4}dx$$

$$\Big\downarrow \text{5593}$$

$$\frac{30\int \frac{e^{\arctan(ax)}}{c^3\left(a^2x^2+1\right)^3}dx}{37c}+\frac{(6ax+1)e^{\arctan(ax)}}{37ac^4\left(a^2x^2+1\right)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{30\int \frac{e^{\arctan(ax)}}{\left(a^2x^2+1\right)^3}dx}{37c^4}+\frac{(6ax+1)e^{\arctan(ax)}}{37ac^4\left(a^2x^2+1\right)^3}$$

$$\Big\downarrow \text{5593}$$

$$\frac{30\left(\frac{12}{17}\int \frac{e^{\arctan(ax)}}{\left(a^2x^2+1\right)^2}dx+\frac{(4ax+1)e^{\arctan(ax)}}{17a\left(a^2x^2+1\right)^2}\right)}{37c^4}+\frac{(6ax+1)e^{\arctan(ax)}}{37ac^4\left(a^2x^2+1\right)^3}$$

$$\Big\downarrow \text{5593}$$

$$\frac{30\left(\frac{12}{17}\left(\frac{2}{5}\int \frac{e^{\arctan(ax)}}{a^2x^2+1}dx+\frac{(2ax+1)e^{\arctan(ax)}}{5a\left(a^2x^2+1\right)}\right)+\frac{(4ax+1)e^{\arctan(ax)}}{17a\left(a^2x^2+1\right)^2}\right)}{37c^4}+\frac{(6ax+1)e^{\arctan(ax)}}{37ac^4\left(a^2x^2+1\right)^3}$$

$$\Big\downarrow \text{5594}$$

$$\frac{(6ax+1)e^{\arctan(ax)}}{37ac^4\left(a^2x^2+1\right)^3}+\frac{30\left(\frac{(4ax+1)e^{\arctan(ax)}}{17a(a^2x^2+1)^2}+\frac{12}{17}\left(\frac{(2ax+1)e^{\arctan(ax)}}{5a(a^2x^2+1)}+\frac{2e^{\arctan(ax)}}{5a}\right)\right)}{37c^4}$$

input
```
Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^4,x]
```

output
```
(E^ArcTan[a*x]*(1 + 6*a*x))/(37*a*c^4*(1 + a^2*x^2)^3) + (30*((E^ArcTan[a*
x]*(1 + 4*a*x))/(17*a*(1 + a^2*x^2)^2) + (12*((2*E^ArcTan[a*x])/(5*a) + (E
^ArcTan[a*x]*(1 + 2*a*x))/(5*a*(1 + a^2*x^2))))/17))/(37*c^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5593
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2)))   I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
&& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
 0] && IntegerQ[2*p]
```

rule 5594
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Maple [A] (verified)

Time = 36.44 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.61

method	result
gosper	$\frac{e^{\arctan(ax)}(144x^6a^6+144a^5x^5+504a^4x^4+408a^3x^3+606a^2x^2+366ax+263)}{629(a^2x^2+1)^3c^4a}$
orering	$\frac{(144x^6a^6+144a^5x^5+504a^4x^4+408a^3x^3+606a^2x^2+366ax+263)(a^2x^2+1)e^{\arctan(ax)}}{629a(a^2cx^2+c)^4}$
parallelrisch	$\frac{144a^6e^{\arctan(ax)}x^6+144a^5e^{\arctan(ax)}x^5+504a^4e^{\arctan(ax)}x^4+408a^3e^{\arctan(ax)}x^3+606a^2e^{\arctan(ax)}x^2+366\,e^{\arctan(ax)}ax+}{629c^4(a^2x^2+1)^3a}$

input
```
int(exp(arctan(a*x))/(a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/629*exp(arctan(a*x))*(144*a^6*x^6+144*a^5*x^5+504*a^4*x^4+408*a^3*x^3+60
6*a^2*x^2+366*a*x+263)/(a^2*x^2+1)^3/c^4/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.80

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{(144\,a^6x^6+144\,a^5x^5+504\,a^4x^4+408\,a^3x^3+606\,a^2x^2+366\,ax+263)e^{(\arctan(ax))}}{629\,(a^7c^4x^6+3\,a^5c^4x^4+3\,a^3c^4x^2+ac^4)}$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="fricas")
```

output
```
1/629*(144*a^6*x^6 + 144*a^5*x^5 + 504*a^4*x^4 + 408*a^3*x^3 + 606*a^2*x^2
 + 366*a*x + 263)*e^(arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3*c^4
*x^2 + a*c^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(107) = 214$.

Time = 8.31 (sec) , antiderivative size = 398, normalized size of antiderivative = 3.43

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^4}\,dx$$
$$= \begin{cases} \frac{144a^6x^6e^{\operatorname{atan}(ax)}}{629a^7c^4x^6+1887a^5c^4x^4+1887a^3c^4x^2+629ac^4} + \frac{144a^5x^5e^{\operatorname{atan}(ax)}}{629a^7c^4x^6+1887a^5c^4x^4+1887a^3c^4x^2+629ac^4} + \frac{504a^4x^4e^{\operatorname{atan}(ax)}}{629a^7c^4x^6+1887a^5c^4x^4+1887a^3c^4x^2} \\ \frac{x}{c^4} \end{cases}$$

input
```
integrate(exp(atan(a*x))/(a**2*c*x**2+c)**4,x)
```

output
```
Piecewise((144*a**6*x**6*exp(atan(a*x))/(629*a**7*c**4*x**6 + 1887*a**5*c*
*4*x**4 + 1887*a**3*c**4*x**2 + 629*a*c**4) + 144*a**5*x**5*exp(atan(a*x))
/(629*a**7*c**4*x**6 + 1887*a**5*c**4*x**4 + 1887*a**3*c**4*x**2 + 629*a*c
**4) + 504*a**4*x**4*exp(atan(a*x))/(629*a**7*c**4*x**6 + 1887*a**5*c**4*x
**4 + 1887*a**3*c**4*x**2 + 629*a*c**4) + 408*a**3*x**3*exp(atan(a*x))/(62
9*a**7*c**4*x**6 + 1887*a**5*c**4*x**4 + 1887*a**3*c**4*x**2 + 629*a*c**4)
 + 606*a**2*x**2*exp(atan(a*x))/(629*a**7*c**4*x**6 + 1887*a**5*c**4*x**4
+ 1887*a**3*c**4*x**2 + 629*a*c**4) + 366*a*x*exp(atan(a*x))/(629*a**7*c**
4*x**6 + 1887*a**5*c**4*x**4 + 1887*a**3*c**4*x**2 + 629*a*c**4) + 263*exp
(atan(a*x))/(629*a**7*c**4*x**6 + 1887*a**5*c**4*x**4 + 1887*a**3*c**4*x**
2 + 629*a*c**4), Ne(a, 0)), (x/c**4, True))
```

Maxima [F]

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \int \frac{e^{(\arctan(ax))}}{(a^2cx^2+c)^4}\,dx$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="maxima")
```

output
```
integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^4, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. 2(104) = 208.

Time = 0.14 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.56

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^4}\,dx$$
$$= \frac{263\,e^{(\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^{12} - 732\,e^{(\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^{11} + 846\,e^{(\arctan(ax))}\tan\left(\frac{1}{2}\operatorname{arct}\right.}{}$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="giac")
```

output
```
1/629*(263*e^(arctan(a*x))*tan(1/2*arctan(a*x))^12 - 732*e^(arctan(a*x))*t
an(1/2*arctan(a*x))^11 + 846*e^(arctan(a*x))*tan(1/2*arctan(a*x))^10 + 396
*e^(arctan(a*x))*tan(1/2*arctan(a*x))^9 + 2313*e^(arctan(a*x))*tan(1/2*arc
tan(a*x))^8 - 2136*e^(arctan(a*x))*tan(1/2*arctan(a*x))^7 + 2372*e^(arctan
(a*x))*tan(1/2*arctan(a*x))^6 + 2136*e^(arctan(a*x))*tan(1/2*arctan(a*x))^
5 + 2313*e^(arctan(a*x))*tan(1/2*arctan(a*x))^4 - 396*e^(arctan(a*x))*tan(
1/2*arctan(a*x))^3 + 846*e^(arctan(a*x))*tan(1/2*arctan(a*x))^2 + 732*e^(a
rctan(a*x))*tan(1/2*arctan(a*x)) + 263*e^(arctan(a*x)))/(a*c^4*tan(1/2*arc
tan(a*x))^12 + 6*a*c^4*tan(1/2*arctan(a*x))^10 + 15*a*c^4*tan(1/2*arctan(a
*x))^8 + 20*a*c^4*tan(1/2*arctan(a*x))^6 + 15*a*c^4*tan(1/2*arctan(a*x))^4
 + 6*a*c^4*tan(1/2*arctan(a*x))^2 + a*c^4)
```

Mupad [B] (verification not implemented)

Time = 23.34 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{144\,\mathrm{e}^{\mathrm{atan}(a\,x)}}{629\,a\,c^4} + \frac{72\,\mathrm{e}^{\mathrm{atan}(a\,x)}\,(2\,a\,x+1)}{629\,a\,c^4\,(a^2\,x^2+1)} + \frac{30\,\mathrm{e}^{\mathrm{atan}(a\,x)}\,(4\,a\,x+1)}{629\,a\,c^4\,(a^2\,x^2+1)^2} + \frac{\mathrm{e}^{\mathrm{atan}(a\,x)}\,(6\,a\,x+1)}{37\,a\,c^4\,(a^2\,x^2+1)^3}$$

input
```
int(exp(atan(a*x))/(c + a^2*c*x^2)^4,x)
```

output
```
(144*exp(atan(a*x)))/(629*a*c^4) + (72*exp(atan(a*x))*(2*a*x + 1))/(629*a*
c^4*(a^2*x^2 + 1)) + (30*exp(atan(a*x))*(4*a*x + 1))/(629*a*c^4*(a^2*x^2 +
 1)^2) + (exp(atan(a*x))*(6*a*x + 1))/(37*a*c^4*(a^2*x^2 + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.75

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{e^{atan(ax)}(144a^6x^6+144a^5x^5+504a^4x^4+408a^3x^3+606a^2x^2+366ax+263)}{629a\,c^4\,(a^6x^6+3a^4x^4+3a^2x^2+1)}$$

input
```
int(exp(atan(a*x))/(a^2*c*x^2+c)^4,x)
```

output
```
(e**atan(a*x)*(144*a**6*x**6 + 144*a**5*x**5 + 504*a**4*x**4 + 408*a**3*x*
*3 + 606*a**2*x**2 + 366*a*x + 263))/(629*a*c**4*(a**6*x**6 + 3*a**4*x**4
+ 3*a**2*x**2 + 1))
```

3.266 $\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^5} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 149

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^5} \, dx = \frac{8064e^{\arctan(ax)}}{40885ac^5} + \frac{e^{\arctan(ax)}(1+8ax)}{65ac^5(1+a^2x^2)^4} + \frac{56e^{\arctan(ax)}(1+6ax)}{2405ac^5(1+a^2x^2)^3} + \frac{336e^{\arctan(ax)}(1+4ax)}{8177ac^5(1+a^2x^2)^2} + \frac{4032e^{\arctan(ax)}(1+2ax)}{40885ac^5(1+a^2x^2)}$$

output
```
8064/40885*exp(arctan(a*x))/a/c^5+1/65*exp(arctan(a*x))*(8*a*x+1)/a/c^5/(a
^2*x^2+1)^4+56/2405*exp(arctan(a*x))*(6*a*x+1)/a/c^5/(a^2*x^2+1)^3+336/817
7*exp(arctan(a*x))*(4*a*x+1)/a/c^5/(a^2*x^2+1)^2+4032/40885*exp(arctan(a*x
))*(2*a*x+1)/a/c^5/(a^2*x^2+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^5} \, dx$$

$$= \frac{629e^{\arctan(ax)}(1+8ax) + \frac{56(c+a^2cx^2)\left(17ce^{\arctan(ax)}(1+6ax)+6(c+a^2cx^2)\left(5e^{\arctan(ax)}(1+4ax)+12(1-iax)^{\frac{i}{2}}(1+iax)^{-\frac{i}{2}}(-i+ax)\right.\right.}{c^2}}{40885ac(c+a^2cx^2)^4}$$

input
```
Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^5,x]
```

output
```
(629*E^ArcTan[a*x]*(1 + 8*a*x) + (56*(c + a^2*c*x^2)*(17*c*E^ArcTan[a*x]*(
1 + 6*a*x) + 6*(c + a^2*c*x^2)*(5*E^ArcTan[a*x]*(1 + 4*a*x) + (12*(1 - I*a
*x)^(I/2)*(-I + a*x)*(I + a*x)*(3 + 2*a*x + 2*a^2*x^2))/(1 + I*a*x)^(I/2))
))/c^2)/(40885*a*c*(c + a^2*c*x^2)^4)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5593, 27, 5593, 5593, 5593, 5594}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\arctan(ax)}}{(a^2cx^2+c)^5}dx$$

$$\Big\downarrow \text{5593}$$

$$\frac{56\int \frac{e^{\arctan(ax)}}{c^4(a^2x^2+1)^4}dx}{65c}+\frac{(8ax+1)e^{\arctan(ax)}}{65ac^5(a^2x^2+1)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{56\int \frac{e^{\arctan(ax)}}{(a^2x^2+1)^4}dx}{65c^5}+\frac{(8ax+1)e^{\arctan(ax)}}{65ac^5(a^2x^2+1)^4}$$

$$\Big\downarrow \text{5593}$$

$$\frac{56\left(\frac{30}{37}\int \frac{e^{\arctan(ax)}}{(a^2x^2+1)^3}dx+\frac{(6ax+1)e^{\arctan(ax)}}{37a(a^2x^2+1)^3}\right)}{65c^5}+\frac{(8ax+1)e^{\arctan(ax)}}{65ac^5(a^2x^2+1)^4}$$

$$\Big\downarrow \text{5593}$$

$$\frac{56\left(\frac{30}{37}\left(\frac{12}{17}\int \frac{e^{\arctan(ax)}}{(a^2x^2+1)^2}dx+\frac{(4ax+1)e^{\arctan(ax)}}{17a(a^2x^2+1)^2}\right)+\frac{(6ax+1)e^{\arctan(ax)}}{37a(a^2x^2+1)^3}\right)}{65c^5}+\frac{(8ax+1)e^{\arctan(ax)}}{65ac^5(a^2x^2+1)^4}$$

$$\Big\downarrow \text{5593}$$

$$\frac{56\left(\frac{30}{37}\left(\frac{12}{17}\left(\frac{2}{5}\int \frac{e^{\arctan(ax)}}{a^2x^2+1}dx+\frac{(2ax+1)e^{\arctan(ax)}}{5a(a^2x^2+1)}\right)+\frac{(4ax+1)e^{\arctan(ax)}}{17a(a^2x^2+1)^2}\right)+\frac{(6ax+1)e^{\arctan(ax)}}{37a(a^2x^2+1)^3}\right)}{65c^5}+\frac{(8ax+1)e^{\arctan(ax)}}{65ac^5\left(a^2x^2+1\right)^4}$$

$$\Big\downarrow \; 5594$$

$$\frac{(8ax+1)e^{\arctan(ax)}}{65ac^5\left(a^2x^2+1\right)^4}+\frac{56\left(\frac{(6ax+1)e^{\arctan(ax)}}{37a(a^2x^2+1)^3}+\frac{30}{37}\left(\frac{(4ax+1)e^{\arctan(ax)}}{17a(a^2x^2+1)^2}+\frac{12}{17}\left(\frac{(2ax+1)e^{\arctan(ax)}}{5a(a^2x^2+1)}+\frac{2e^{\arctan(ax)}}{5a}\right)\right)\right)}{65c^5}$$

input
```
Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^5,x]
```

output
```
(E^ArcTan[a*x]*(1 + 8*a*x))/(65*a*c^5*(1 + a^2*x^2)^4) + (56*((E^ArcTan[a*
x]*(1 + 6*a*x))/(37*a*(1 + a^2*x^2)^3) + (30*((E^ArcTan[a*x]*(1 + 4*a*x))/
(17*a*(1 + a^2*x^2)^2) + (12*((2*E^ArcTan[a*x])/(5*a) + (E^ArcTan[a*x]*(1
+ 2*a*x))/(5*a*(1 + a^2*x^2))))/17))/37))/(65*c^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5593
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2)))   I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
&& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
 0] && IntegerQ[2*p]
```

rule 5594
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Maple [A] (verified)

Time = 100.00 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.58

method	result
gosper	$\frac{e^{\arctan(ax)}(8064a^8x^8+8064a^7x^7+36288x^6a^6+30912a^5x^5+62160a^4x^4+43344a^3x^3+48664a^2x^2+25528ax+15357)}{40885(a^2x^2+1)^4c^5a}$
orering	$\frac{(8064a^8x^8+8064a^7x^7+36288x^6a^6+30912a^5x^5+62160a^4x^4+43344a^3x^3+48664a^2x^2+25528ax+15357)(a^2x^2+1)e^{\arctan(ax}}{40885a(a^2cx^2+c)^5}$
parallelrisch	$\frac{8064a^8e^{\arctan(ax)}x^8+8064a^7e^{\arctan(ax)}x^7+36288a^6e^{\arctan(ax)}x^6+30912a^5e^{\arctan(ax)}x^5+62160a^4e^{\arctan(ax)}x^4+43344a^3e}{40885c^5(a^2x^2+1)^4a}$

input
```
int(exp(arctan(a*x))/(a^2*c*x^2+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/40885*exp(arctan(a*x))*(8064*a^8*x^8+8064*a^7*x^7+36288*a^6*x^6+30912*a^
5*x^5+62160*a^4*x^4+43344*a^3*x^3+48664*a^2*x^2+25528*a*x+15357)/(a^2*x^2+
1)^4/c^5/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^5}\,dx = \frac{(8064\,a^8x^8+8064\,a^7x^7+36288\,a^6x^6+30912\,a^5x^5+62160\,a^4x^4+43344\,a^3x^3+48664\,a^2x^2+25528\,ax}{40885\,(a^9c^5x^8+4\,a^7c^5x^6+6\,a^5c^5x^4+4\,a^3c^5x^2+ac^5)}$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^5,x, algorithm="fricas")
```

output
```
1/40885*(8064*a^8*x^8 + 8064*a^7*x^7 + 36288*a^6*x^6 + 30912*a^5*x^5 + 621
60*a^4*x^4 + 43344*a^3*x^3 + 48664*a^2*x^2 + 25528*a*x + 15357)*e^(arctan(
a*x))/(a^9*c^5*x^8 + 4*a^7*c^5*x^6 + 6*a^5*c^5*x^4 + 4*a^3*c^5*x^2 + a*c^5
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(139) = 278$.

Time = 23.45 (sec) , antiderivative size = 620, normalized size of antiderivative = 4.16

$$\int \frac{e^{\arctan(ax)}}{(c + a^2cx^2)^5}\,dx = \begin{cases} \frac{8064a^8x^8e^{\operatorname{atan}(ax)}}{40885a^9c^5x^8+163540a^7c^5x^6+245310a^5c^5x^4+163540a^3c^5x^2+40885ac^5} + \frac{8064a^7x^7e^{\operatorname{atan}(ax)}}{40885a^9c^5x^8+163540a^7c^5x^6+245310a^5c^5x^4+163540a^3c^5x^2+4} \\ \frac{x}{c^5} \end{cases}$$

input
```
integrate(exp(atan(a*x))/(a**2*c*x**2+c)**5,x)
```

output
```
Piecewise((8064*a**8*x**8*exp(atan(a*x))/(40885*a**9*c**5*x**8 + 163540*a*
*7*c**5*x**6 + 245310*a**5*c**5*x**4 + 163540*a**3*c**5*x**2 + 40885*a*c**
5) + 8064*a**7*x**7*exp(atan(a*x))/(40885*a**9*c**5*x**8 + 163540*a**7*c**
5*x**6 + 245310*a**5*c**5*x**4 + 163540*a**3*c**5*x**2 + 40885*a*c**5) + 3
6288*a**6*x**6*exp(atan(a*x))/(40885*a**9*c**5*x**8 + 163540*a**7*c**5*x**
6 + 245310*a**5*c**5*x**4 + 163540*a**3*c**5*x**2 + 40885*a*c**5) + 30912*
a**5*x**5*exp(atan(a*x))/(40885*a**9*c**5*x**8 + 163540*a**7*c**5*x**6 + 2
45310*a**5*c**5*x**4 + 163540*a**3*c**5*x**2 + 40885*a*c**5) + 62160*a**4*
x**4*exp(atan(a*x))/(40885*a**9*c**5*x**8 + 163540*a**7*c**5*x**6 + 245310
*a**5*c**5*x**4 + 163540*a**3*c**5*x**2 + 40885*a*c**5) + 43344*a**3*x**3*
exp(atan(a*x))/(40885*a**9*c**5*x**8 + 163540*a**7*c**5*x**6 + 245310*a**5
*c**5*x**4 + 163540*a**3*c**5*x**2 + 40885*a*c**5) + 48664*a**2*x**2*exp(a
tan(a*x))/(40885*a**9*c**5*x**8 + 163540*a**7*c**5*x**6 + 245310*a**5*c**5
*x**4 + 163540*a**3*c**5*x**2 + 40885*a*c**5) + 25528*a*x*exp(atan(a*x))/(
40885*a**9*c**5*x**8 + 163540*a**7*c**5*x**6 + 245310*a**5*c**5*x**4 + 163
540*a**3*c**5*x**2 + 40885*a*c**5) + 15357*exp(atan(a*x))/(40885*a**9*c**5
*x**8 + 163540*a**7*c**5*x**6 + 245310*a**5*c**5*x**4 + 163540*a**3*c**5*x
**2 + 40885*a*c**5), Ne(a, 0)), (x/c**5, True))
```

Maxima [F]

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^5}\,dx = \int \frac{e^{(\arctan(ax))}}{(a^2cx^2+c)^5}\,dx$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^5,x, algorithm="maxima")
```

output
```
integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^5, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(134) = 268.

Time = 0.21 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.62

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^5}\,dx = \text{Too large to display}$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^5,x, algorithm="giac")
```

output
```
1/40885*(15357*e^(arctan(a*x))*tan(1/2*arctan(a*x))^16 - 51056*e^(arctan(a
*x))*tan(1/2*arctan(a*x))^15 + 71800*e^(arctan(a*x))*tan(1/2*arctan(a*x))^
14 + 10640*e^(arctan(a*x))*tan(1/2*arctan(a*x))^13 + 256620*e^(arctan(a*x)
)*tan(1/2*arctan(a*x))^12 - 327600*e^(arctan(a*x))*tan(1/2*arctan(a*x))^11
 + 404040*e^(arctan(a*x))*tan(1/2*arctan(a*x))^10 + 254800*e^(arctan(a*x))
*tan(1/2*arctan(a*x))^9 + 568750*e^(arctan(a*x))*tan(1/2*arctan(a*x))^8 -
254800*e^(arctan(a*x))*tan(1/2*arctan(a*x))^7 + 404040*e^(arctan(a*x))*tan
(1/2*arctan(a*x))^6 + 327600*e^(arctan(a*x))*tan(1/2*arctan(a*x))^5 + 2566
20*e^(arctan(a*x))*tan(1/2*arctan(a*x))^4 - 10640*e^(arctan(a*x))*tan(1/2*
arctan(a*x))^3 + 71800*e^(arctan(a*x))*tan(1/2*arctan(a*x))^2 + 51056*e^(a
rctan(a*x))*tan(1/2*arctan(a*x)) + 15357*e^(arctan(a*x)))/(a*c^5*tan(1/2*a
rctan(a*x))^16 + 8*a*c^5*tan(1/2*arctan(a*x))^14 + 28*a*c^5*tan(1/2*arctan
(a*x))^12 + 56*a*c^5*tan(1/2*arctan(a*x))^10 + 70*a*c^5*tan(1/2*arctan(a*x
))^8 + 56*a*c^5*tan(1/2*arctan(a*x))^6 + 28*a*c^5*tan(1/2*arctan(a*x))^4 +
 8*a*c^5*tan(1/2*arctan(a*x))^2 + a*c^5)
```

Mupad [B] (verification not implemented)

Time = 23.50 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.90

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^5}\,dx$$
$$= \frac{e^{\operatorname{atan}(a\,x)}\left(\frac{15357}{40885\,a^9\,c^5}+\frac{25528\,x}{40885\,a^8\,c^5}+\frac{8064\,x^8}{40885\,a\,c^5}+\frac{8064\,x^7}{40885\,a^2\,c^5}+\frac{36288\,x^6}{40885\,a^3\,c^5}+\frac{30912\,x^5}{40885\,a^4\,c^5}+\frac{336\,x^4}{221\,a^5\,c^5}+\frac{43344\,x^3}{40885\,a^6\,c^5}+\frac{486}{4088}\right.}{\frac{1}{a^8}+x^8+\frac{4\,x^6}{a^2}+\frac{6\,x^4}{a^4}+\frac{4\,x^2}{a^6}}$$

input
```
int(exp(atan(a*x))/(c + a^2*c*x^2)^5,x)
```

output
```
(exp(atan(a*x))*(15357/(40885*a^9*c^5) + (25528*x)/(40885*a^8*c^5) + (8064
*x^8)/(40885*a*c^5) + (8064*x^7)/(40885*a^2*c^5) + (36288*x^6)/(40885*a^3*
c^5) + (30912*x^5)/(40885*a^4*c^5) + (336*x^4)/(221*a^5*c^5) + (43344*x^3)
/(40885*a^6*c^5) + (48664*x^2)/(40885*a^7*c^5)))/(1/a^8 + x^8 + (4*x^6)/a^
2 + (6*x^4)/a^4 + (4*x^2)/a^6)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.74

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^5}\,dx$$
$$= \frac{e^{atan(ax)}(8064a^8x^8+8064a^7x^7+36288a^6x^6+30912a^5x^5+62160a^4x^4+43344a^3x^3+48664a^2x^2+2552}{40885a\,c^5\,(a^8x^8+4a^6x^6+6a^4x^4+4a^2x^2+1)}$$

input
```
int(exp(atan(a*x))/(a^2*c*x^2+c)^5,x)
```

output
```
(e**atan(a*x)*(8064*a**8*x**8 + 8064*a**7*x**7 + 36288*a**6*x**6 + 30912*a
**5*x**5 + 62160*a**4*x**4 + 43344*a**3*x**3 + 48664*a**2*x**2 + 25528*a*x
 + 15357))/(40885*a*c**5*(a**8*x**8 + 4*a**6*x**6 + 6*a**4*x**4 + 4*a**2*x
**2 + 1))
```

3.267 $\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\frac{\left(\frac{1}{13}+\frac{5i}{13}\right)2^{\frac{3}{2}-\frac{i}{2}}c(1-iax)^{\frac{5}{2}+\frac{i}{2}}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}+\frac{i}{2},\frac{5}{2}+\frac{i}{2},\frac{7}{2}+\frac{i}{2},\frac{1}{2}(1-\ldots\right)}{a\sqrt{1+a^2x^2}}$$

output
```
(1/13+5/13*I)*2^(3/2-1/2*I)*c*(1-I*a*x)^(5/2+1/2*I)*(a^2*c*x^2+c)^(1/2)*hy
pergeom([5/2+1/2*I, -3/2+1/2*I],[7/2+1/2*I],1/2-1/2*I*a*x)/a/(a^2*x^2+1)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\frac{\left(\frac{1}{13}+\frac{5i}{13}\right)2^{\frac{3}{2}-\frac{i}{2}}c(1-iax)^{\frac{5}{2}+\frac{i}{2}}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}+\frac{i}{2},\frac{5}{2}+\frac{i}{2},\frac{7}{2}+\frac{i}{2},\frac{1}{2}(1-\ldots\right)}{a\sqrt{1+a^2x^2}}$$

input
```
Integrate[E^ArcTan[a*x]*(c + a^2*c*x^2)^(3/2),x]
```

output
```
((1/13 + (5*I)/13)*2^(3/2 - I/2)*c*(1 - I*a*x)^(5/2 + I/2)*Sqrt[c + a^2*c*
x^2]*Hypergeometric2F1[-3/2 + I/2, 5/2 + I/2, 7/2 + I/2, (1 - I*a*x)/2])/(
a*Sqrt[1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5599, 5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}\,dx$$

$$\Big\downarrow \text{5599}$$

$$\frac{c\sqrt{a^2cx^2+c}\int e^{\arctan(ax)}\left(a^2x^2+1\right)^{3/2}dx}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{5596}$$

$$\frac{c\sqrt{a^2cx^2+c}\int (1-iax)^{\frac{3}{2}+\frac{i}{2}}(iax+1)^{\frac{3}{2}-\frac{i}{2}}dx}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{79}$$

$$\frac{\left(\frac{1}{13}+\frac{5i}{13}\right)2^{\frac{3}{2}-\frac{i}{2}}c(1-iax)^{\frac{5}{2}+\frac{i}{2}}\sqrt{a^2cx^2+c}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}+\frac{i}{2},\frac{5}{2}+\frac{i}{2},\frac{7}{2}+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

input
```
Int[E^ArcTan[a*x]*(c + a^2*c*x^2)^(3/2),x]
```

output

```
((1/13 + (5*I)/13)*2^(3/2 - I/2)*c*(1 - I*a*x)^(5/2 + I/2)*Sqrt[c + a^2*c*
x^2]*Hypergeometric2F1[-3/2 + I/2, 5/2 + I/2, 7/2 + I/2, (1 - I*a*x)/2])/(
a*Sqrt[1 + a^2*x^2])
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int e^{\arctan(ax)} \left(a^2 c\, x^2 + c\right)^{\frac{3}{2}} dx$$

input
```
int(exp(arctan(a*x))*(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(exp(arctan(a*x))*(a^2*c*x^2+c)^(3/2),x)
```

Fricas [F]

$$\int e^{\arctan(ax)} \left(c + a^2 c x^2\right)^{3/2} dx = \int \left(a^2 c x^2 + c\right)^{\frac{3}{2}} e^{\left(\arctan(ax)\right)} dx$$

input
```
integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*e^(arctan(a*x)), x)
```

Sympy [F]

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}e^{\operatorname{atan}(ax)}\,dx$$

input
```
integrate(exp(atan(a*x))*(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*exp(atan(a*x)), x)
```

Maxima [F]

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int\left(a^2cx^2+c\right)^{\frac{3}{2}}e^{(\arctan(ax))}\,dx$$

input
```
integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*e^(arctan(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int e^{\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(exp(atan(a*x))*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(exp(atan(a*x))*(c + a^2*c*x^2)^(3/2), x)
```

Reduce [F]

$$\int e^{\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\sqrt{c}\,c\left(\left(\int e^{atan(ax)}\sqrt{a^2x^2+1}\,x^2dx\right)a^2+\int e^{atan(ax)}\sqrt{a^2x^2+1}dx\right)$$

input
```
int(exp(atan(a*x))*(a^2*c*x^2+c)^(3/2),x)
```

output
```
sqrt(c)*c*(int(e**atan(a*x)*sqrt(a**2*x**2 + 1)*x**2,x)*a**2 + int(e**atan
(a*x)*sqrt(a**2*x**2 + 1),x))
```

3.268 $\int e^{\arctan(ax)}\sqrt{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int e^{\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{3}{2}+\frac{i}{2}}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}+\frac{i}{2},\frac{3}{2}+\frac{i}{2},\frac{5}{2}+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

output
```
(1/5+3/5*I)*2^(1/2-1/2*I)*(1-I*a*x)^(3/2+1/2*I)*(a^2*c*x^2+c)^(1/2)*hyperg
eom([3/2+1/2*I, -1/2+1/2*I],[5/2+1/2*I],1/2-1/2*I*a*x)/a/(a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int e^{\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{3}{2}+\frac{i}{2}}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}+\frac{i}{2},\frac{3}{2}+\frac{i}{2},\frac{5}{2}+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

input
```
Integrate[E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2],x]
```

output
```
((1/5 + (3*I)/5)*2^(1/2 - I/2)*(1 - I*a*x)^(3/2 + I/2)*Sqrt[c + a^2*c*x^2]
*Hypergeometric2F1[-1/2 + I/2, 3/2 + I/2, 5/2 + I/2, (1 - I*a*x)/2])/(a*Sq
rt[1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5599, 5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\arctan(ax)}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5599}$$

$$\frac{\sqrt{a^2cx^2+c}\int e^{\arctan(ax)}\sqrt{a^2x^2+1}dx}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{5596}$$

$$\frac{\sqrt{a^2cx^2+c}\int (1-iax)^{\frac{1}{2}+\frac{i}{2}}(iax+1)^{\frac{1}{2}-\frac{i}{2}}dx}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{79}$$

$$\frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{3}{2}+\frac{i}{2}}\sqrt{a^2cx^2+c}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}+\frac{i}{2},\frac{3}{2}+\frac{i}{2},\frac{5}{2}+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

input
```
Int[E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2],x]
```

output
```
((1/5 + (3*I)/5)*2^(1/2 - I/2)*(1 - I*a*x)^(3/2 + I/2)*Sqrt[c + a^2*c*x^2]
*Hypergeometric2F1[-1/2 + I/2, 3/2 + I/2, 5/2 + I/2, (1 - I*a*x)/2])/(a*Sq
rt[1 + a^2*x^2])
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \mathrm{e}^{\arctan(ax)}\sqrt{a^2c\,x^2+c}dx$$

input
```
int(exp(arctan(a*x))*(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(exp(arctan(a*x))*(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int e^{\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \int \sqrt{a^2cx^2+c}e^{(\arctan(ax))}\,dx$$

input
```
integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*e^(arctan(a*x)), x)
```

Sympy [F]

$$\int e^{\arctan(ax)}\sqrt{c+a^2cx^2}\,dx=\int\sqrt{c\,(a^2x^2+1)}e^{\operatorname{atan}(ax)}\,dx$$

input
```
integrate(exp(atan(a*x))*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*exp(atan(a*x)), x)
```

Maxima [F]

$$\int e^{\arctan(ax)}\sqrt{c+a^2cx^2}\,dx=\int\sqrt{a^2cx^2+c}e^{(\arctan(ax))}\,dx$$

input
```
integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*e^(arctan(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{\arctan(ax)}\sqrt{c+a^2cx^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(exp(arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{\arctan(ax)}\sqrt{c+a^2cx^2}\,dx=\int e^{\mathrm{atan}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(exp(atan(a*x))*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(exp(atan(a*x))*(c + a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int e^{\arctan(ax)}\sqrt{c+a^2cx^2}\,dx=\sqrt{c}\left(\int e^{atan(ax)}\sqrt{a^2x^2+1}dx\right)$$

input
```
int(exp(atan(a*x))*(a^2*c*x^2+c)^(1/2),x)
```

output
```
sqrt(c)*int(e**atan(a*x)*sqrt(a**2*x**2 + 1),x)
```

3.269 $\int \frac{e^{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int \frac{e^{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{(1+i)2^{-\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{1}{2}+\frac{i}{2}}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}+\frac{i}{2},\frac{1}{2}+\frac{i}{2},\frac{3}{2}+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}$$

output
```
(1+I)*(1-I*a*x)^(1/2+1/2*I)*(a^2*x^2+1)^(1/2)*hypergeom([1/2+1/2*I, 1/2+1/
2*I],[3/2+1/2*I],1/2-1/2*I*a*x)/(2^(1/2+1/2*I))/a/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\int \frac{e^{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{(1+i)2^{-\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{1}{2}+\frac{i}{2}}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}+\frac{i}{2},\frac{1}{2}+\frac{i}{2},\frac{3}{2}+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}$$

input
```
Integrate[E^ArcTan[a*x]/Sqrt[c + a^2*c*x^2],x]
```

output
```
((1 + I)*(1 - I*a*x)^(1/2 + I/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[1/2 +
 I/2, 1/2 + I/2, 3/2 + I/2, (1 - I*a*x)/2])/(2^(1/2 + I/2)*a*Sqrt[c + a^2*
c*x^2])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5599, 5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5599}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{\arctan(ax)}}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5596}$$

$$\frac{\sqrt{a^2x^2+1}\int (1-iax)^{-\frac{1}{2}+\frac{i}{2}}(iax+1)^{-\frac{1}{2}-\frac{i}{2}}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{79}$$

$$\frac{(1+i)2^{-\frac{1}{2}-\frac{i}{2}}(1-iax)^{\frac{1}{2}+\frac{i}{2}}\sqrt{a^2x^2+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2}+\frac{i}{2},\frac{1}{2}+\frac{i}{2},\frac{3}{2}+\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

input
```
Int[E^ArcTan[a*x]/Sqrt[c + a^2*c*x^2],x]
```

output
```
((1 + I)*(1 - I*a*x)^(1/2 + I/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[1/2 +
 I/2, 1/2 + I/2, 3/2 + I/2, (1 - I*a*x)/2])/(2^(1/2 + I/2)*a*Sqrt[c + a^2*
c*x^2])
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{\mathrm{e}^{\arctan(ax)}}{\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(e^(arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{e^{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{\operatorname{atan}(ax)}}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(exp(atan(a*x))/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(exp(atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{e^{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(e^(arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{e^{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(e^(arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{e}^{\mathrm{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(exp(atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(exp(atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{e^{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{e^{atan(ax)}}{\sqrt{a^2x^2+1}}dx}{\sqrt{c}}$$

input
```
int(exp(atan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(e**atan(a*x)/sqrt(a**2*x**2 + 1),x)/sqrt(c)
```

3.270 $\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 35

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{e^{\arctan(ax)}(1+ax)}{2ac\sqrt{c+a^2cx^2}}$$

output
```
1/2*exp(arctan(a*x))*(a*x+1)/a/c/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{e^{\arctan(ax)}(1+ax)}{2ac\sqrt{c+a^2cx^2}}$$

input
```
Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(E^ArcTan[a*x]*(1 + a*x))/(2*a*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{5592\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\arctan(ax)}}{(a^2cx^2+c)^{3/2}} \, dx$$

$$\Big\downarrow \; 5592$$

$$\frac{(ax+1)e^{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

input
```
Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(E^ArcTan[a*x]*(1 + a*x))/(2*a*c*Sqrt[c + a^2*c*x^2])
```

Defintions of rubi rules used

rule 5592
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{(a^2x^2+1)(ax+1)e^{\arctan(ax)}}{2a(a^2cx^2+c)^{\frac{3}{2}}}$	37
orering	$\frac{(a^2x^2+1)(ax+1)e^{\arctan(ax)}}{2a(a^2cx^2+c)^{\frac{3}{2}}}$	37

input
```
int(exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a^2*x^2+1)*(a*x+1)*exp(arctan(a*x))/a/(a^2*c*x^2+c)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{a^2cx^2+c}(ax+1)e^{(\arctan(ax))}}{2\,(a^3c^2x^2+ac^2)}$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(a^2*c*x^2 + c)*(a*x + 1)*e^(arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

Sympy [F]

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{e^{\operatorname{atan}(ax)}}{(c\,(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(atan(a*x))/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(exp(atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{e^{(\arctan(ax))}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(30) = 60$.

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.83

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{e^{(\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^2 - 2\,e^{(\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right) - e^{(\arctan(ax))}}{2\left(\sqrt{c}\tan\left(\frac{1}{2}\arctan(ax)\right)^2+\sqrt{c}\right)ac}$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(e^(arctan(a*x))*tan(1/2*arctan(a*x))^2 - 2*e^(arctan(a*x))*tan(1/2*a
rctan(a*x)) - e^(arctan(a*x)))/((sqrt(c)*tan(1/2*arctan(a*x))^2 + sqrt(c))
*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\mathrm{e}^{\mathrm{atan}(a\,x)}\,\left(\frac{x}{2\,c}+\frac{1}{2\,a\,c}\right)}{\sqrt{c\,a^2\,x^2+c}}$$

input
```
int(exp(atan(a*x))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
(exp(atan(a*x))*(x/(2*c) + 1/(2*a*c)))/(c + a^2*c*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{e^{atan(ax)}\sqrt{c}\,\sqrt{a^2x^2+1}\,(ax+1)}{2a\,c^2\,(a^2x^2+1)}$$

input
```
int(exp(atan(a*x))/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(e**atan(a*x)*sqrt(c)*sqrt(a**2*x**2 + 1)*(a*x + 1))/(2*a*c**2*(a**2*x**2
+ 1))
```

3.271 $\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{e^{\arctan(ax)}(1+3ax)}{10ac\,(c+a^2cx^2)^{3/2}} + \frac{3e^{\arctan(ax)}(1+ax)}{10ac^2\sqrt{c+a^2cx^2}}$$

output
```
1/10*exp(arctan(a*x))*(3*a*x+1)/a/c/(a^2*c*x^2+c)^(3/2)+3/10*exp(arctan(a*
x))*(a*x+1)/a/c^2/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{e^{\arctan(ax)}(4+6ax+3a^2x^2+3a^3x^3)}{10c^2\,(a+a^3x^2)\sqrt{c+a^2cx^2}}$$

input
```
Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(E^ArcTan[a*x]*(4 + 6*a*x + 3*a^2*x^2 + 3*a^3*x^3))/(10*c^2*(a + a^3*x^2)*
Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5593, 5592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5593}$$

$$\frac{3\int \frac{e^{\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx}{5c}+\frac{(3ax+1)e^{\arctan(ax)}}{10ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5592}$$

$$\frac{3(ax+1)e^{\arctan(ax)}}{10ac^2\sqrt{a^2cx^2+c}}+\frac{(3ax+1)e^{\arctan(ax)}}{10ac\,(a^2cx^2+c)^{3/2}}$$

input
```
Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(E^ArcTan[a*x]*(1 + 3*a*x))/(10*a*c*(c + a^2*c*x^2)^(3/2)) + (3*E^ArcTan[a
*x]*(1 + a*x))/(10*a*c^2*Sqrt[c + a^2*c*x^2])
```

Defintions of rubi rules used

rule 5592
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

rule 5593
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2)))   I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
&& EqQ[d, a^2*c] && LtQ[p, -1] && !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.75

method	result	size
gosper	$\frac{(a^2x^2+1)(3a^3x^3+3a^2x^2+6ax+4)\,\mathrm{e}^{\arctan(ax)}}{10a(a^2c\,x^2+c)^{\frac{5}{2}}}$	54
orering	$\frac{(a^2x^2+1)(3a^3x^3+3a^2x^2+6ax+4)\,\mathrm{e}^{\arctan(ax)}}{10a(a^2c\,x^2+c)^{\frac{5}{2}}}$	54

input
```
int(exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/10*(a^2*x^2+1)*(3*a^3*x^3+3*a^2*x^2+6*a*x+4)*exp(arctan(a*x))/a/(a^2*c*x
^2+c)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{(3\,a^3x^3+3\,a^2x^2+6\,ax+4)\sqrt{a^2cx^2+c}\,e^{(\arctan(ax))}}{10\,(a^5c^3x^4+2\,a^3c^3x^2+ac^3)}$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/10*(3*a^3*x^3 + 3*a^2*x^2 + 6*a*x + 4)*sqrt(a^2*c*x^2 + c)*e^(arctan(a*x
))/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F]

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{e^{\operatorname{atan}(ax)}}{(c\,(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(atan(a*x))/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(exp(atan(a*x))/(c*(a**2*x**2 + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{e^{(\arctan(ax))}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.49

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx =$$
$$-\frac{2\left(e^{(\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^6-3\,e^{(\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^5-3\,e^{(\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right.\right.}{5\left(ac^{\frac{5}{2}}\tan\left(\frac{1}{2}\arctan(ax)\right)^6+3\,ac^{\frac{5}{2}}\tan\left(\frac{1}{2}\arctan(ax)\right)^4+3\,ac^{\frac{5}{2}}\tan\left(\frac{1}{2}\arctan(ax)\right)^2+ac\right.}$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
-2/5*(e^(arctan(a*x))*tan(1/2*arctan(a*x))^6 - 3*e^(arctan(a*x))*tan(1/2*a
rctan(a*x))^5 - 3*e^(arctan(a*x))*tan(1/2*arctan(a*x)) - e^(arctan(a*x)))/
(a*c^(5/2)*tan(1/2*arctan(a*x))^6 + 3*a*c^(5/2)*tan(1/2*arctan(a*x))^4 + 3
*a*c^(5/2)*tan(1/2*arctan(a*x))^2 + a*c^(5/2))
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{e^{\operatorname{atan}(a\,x)}\left(\frac{2}{5\,a^3\,c^2}+\frac{3\,x^3}{10\,c^2}+\frac{3\,x}{5\,a^2\,c^2}+\frac{3\,x^2}{10\,a\,c^2}\right)}{\frac{\sqrt{c\,a^2\,x^2+c}}{a^2}+x^2\,\sqrt{c\,a^2\,x^2+c}}$$

input
```
int(exp(atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
(exp(atan(a*x))*(2/(5*a^3*c^2) + (3*x^3)/(10*c^2) + (3*x)/(5*a^2*c^2) + (3
*x^2)/(10*a*c^2)))/((c + a^2*c*x^2)^(1/2)/a^2 + x^2*(c + a^2*c*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{e^{atan(ax)}\sqrt{c}\,\sqrt{a^2x^2+1}\,(3a^3x^3+3a^2x^2+6ax+4)}{10a\,c^3\,(a^4x^4+2a^2x^2+1)}$$

input
```
int(exp(atan(a*x))/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(e**atan(a*x)*sqrt(c)*sqrt(a**2*x**2 + 1)*(3*a**3*x**3 + 3*a**2*x**2 + 6*a
*x + 4))/(10*a*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

3.272 $\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \frac{e^{\arctan(ax)}(1+5ax)}{26ac\,(c+a^2cx^2)^{5/2}} + \frac{e^{\arctan(ax)}(1+3ax)}{13ac^2\,(c+a^2cx^2)^{3/2}} + \frac{3e^{\arctan(ax)}(1+ax)}{13ac^3\sqrt{c+a^2cx^2}}$$

output
```
1/26*exp(arctan(a*x))*(5*a*x+1)/a/c/(a^2*c*x^2+c)^(5/2)+1/13*exp(arctan(a*
x))*(3*a*x+1)/a/c^2/(a^2*c*x^2+c)^(3/2)+3/13*exp(arctan(a*x))*(a*x+1)/a/c^
3/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.73

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \frac{e^{\arctan(ax)}(9+17ax+14a^2x^2+18a^3x^3+6a^4x^4+6a^5x^5)}{26ac^3\,(1+a^2x^2)^2\sqrt{c+a^2cx^2}}$$

input
```
Integrate[E^ArcTan[a*x]/(c + a^2*c*x^2)^(7/2),x]
```

output
```
(E^ArcTan[a*x]*(9 + 17*a*x + 14*a^2*x^2 + 18*a^3*x^3 + 6*a^4*x^4 + 6*a^5*x
^5))/(26*a*c^3*(1 + a^2*x^2)^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5593, 5593, 5592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\arctan(ax)}}{(a^2cx^2+c)^{7/2}}dx$$

$$\Big\downarrow \text{5593}$$

$$\frac{10\int \frac{e^{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx}{13c}+\frac{(5ax+1)e^{\arctan(ax)}}{26ac\,(a^2cx^2+c)^{5/2}}$$

$$\Big\downarrow \text{5593}$$

$$\frac{10\left(\frac{3\int \frac{e^{\arctan(ax)}}{\left(a^2cx^2+c\right)^{3/2}}dx}{5c}+\frac{(3ax+1)e^{\arctan(ax)}}{10ac(a^2cx^2+c)^{3/2}}\right)}{13c}+\frac{(5ax+1)e^{\arctan(ax)}}{26ac\,(a^2cx^2+c)^{5/2}}$$

$$\Big\downarrow \text{5592}$$

$$\frac{10\left(\frac{3(ax+1)e^{\arctan(ax)}}{10ac^2\sqrt{a^2cx^2+c}}+\frac{(3ax+1)e^{\arctan(ax)}}{10ac(a^2cx^2+c)^{3/2}}\right)}{13c}+\frac{(5ax+1)e^{\arctan(ax)}}{26ac\,(a^2cx^2+c)^{5/2}}$$

input
```
Int[E^ArcTan[a*x]/(c + a^2*c*x^2)^(7/2),x]
```

output
```
(E^ArcTan[a*x]*(1 + 5*a*x))/(26*a*c*(c + a^2*c*x^2)^(5/2)) + (10*((E^ArcTa
n[a*x]*(1 + 3*a*x))/(10*a*c*(c + a^2*c*x^2)^(3/2)) + (3*E^ArcTan[a*x]*(1 +
 a*x))/(10*a*c^2*Sqrt[c + a^2*c*x^2])))/(13*c)
```

Defintions of rubi rules used

rule 5592
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

rule 5593
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2)))   I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
&& EqQ[d, a^2*c] && LtQ[p, -1] && !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{\left(a^{2} x^{2}+1\right) \left(6 a^{5} x^{5}+6 a^{4} x^{4}+18 a^{3} x^{3}+14 a^{2} x^{2}+17 a x +9\right) {\mathrm e}^{\arctan \left(a x \right)}}{26 a \left(a^{2} c \,x^{2}+c \right)^{\frac{7}{2}}}$	70
orering	$\frac{\left(a^{2} x^{2}+1\right) \left(6 a^{5} x^{5}+6 a^{4} x^{4}+18 a^{3} x^{3}+14 a^{2} x^{2}+17 a x +9\right) {\mathrm e}^{\arctan \left(a x \right)}}{26 a \left(a^{2} c \,x^{2}+c \right)^{\frac{7}{2}}}$	70

input
```
int(exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/26*(a^2*x^2+1)*(6*a^5*x^5+6*a^4*x^4+18*a^3*x^3+14*a^2*x^2+17*a*x+9)*exp(
arctan(a*x))/a/(a^2*c*x^2+c)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \frac{(6\,a^5x^5+6\,a^4x^4+18\,a^3x^3+14\,a^2x^2+17\,ax+9)\sqrt{a^2cx^2+c}e^{(\arctan(ax))}}{26\,(a^7c^4x^6+3\,a^5c^4x^4+3\,a^3c^4x^2+ac^4)}$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
1/26*(6*a^5*x^5 + 6*a^4*x^4 + 18*a^3*x^3 + 14*a^2*x^2 + 17*a*x + 9)*sqrt(a
^2*c*x^2 + c)*e^(arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3*c^4*x^2
 + a*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(exp(atan(a*x))/(a**2*c*x**2+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{e^{(\arctan(ax))}}{(a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(e^(arctan(a*x))/(a^2*c*x^2 + c)^(7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(93) = 186$.

Time = 0.19 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.31

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = -\frac{9\,e^{(\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^{10} - 34\,e^{(\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^{9} + 11\,e^{(\arctan(ax))}\tan\left(\frac{1}{2}\arctan(a\right.}{}$$

input
```
integrate(exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
-1/26*(9*e^(arctan(a*x))*tan(1/2*arctan(a*x))^10 - 34*e^(arctan(a*x))*tan(
1/2*arctan(a*x))^9 + 11*e^(arctan(a*x))*tan(1/2*arctan(a*x))^8 - 8*e^(arct
an(a*x))*tan(1/2*arctan(a*x))^7 + 18*e^(arctan(a*x))*tan(1/2*arctan(a*x))^
6 - 108*e^(arctan(a*x))*tan(1/2*arctan(a*x))^5 - 18*e^(arctan(a*x))*tan(1/
2*arctan(a*x))^4 - 8*e^(arctan(a*x))*tan(1/2*arctan(a*x))^3 - 11*e^(arctan
(a*x))*tan(1/2*arctan(a*x))^2 - 34*e^(arctan(a*x))*tan(1/2*arctan(a*x)) -
9*e^(arctan(a*x)))/(a*c^(7/2)*tan(1/2*arctan(a*x))^10 + 5*a*c^(7/2)*tan(1/
2*arctan(a*x))^8 + 10*a*c^(7/2)*tan(1/2*arctan(a*x))^6 + 10*a*c^(7/2)*tan(
1/2*arctan(a*x))^4 + 5*a*c^(7/2)*tan(1/2*arctan(a*x))^2 + a*c^(7/2))
```

Mupad [B] (verification not implemented)

Time = 23.55 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \frac{e^{\mathrm{atan}(a\,x)}\left(\frac{9}{26\,a^5\,c^3}+\frac{3\,x^5}{13\,c^3}+\frac{17\,x}{26\,a^4\,c^3}+\frac{3\,x^4}{13\,a\,c^3}+\frac{9\,x^3}{13\,a^2\,c^3}+\frac{7\,x^2}{13\,a^3\,c^3}\right)}{\frac{\sqrt{c\,a^2\,x^2+c}}{a^4}+x^4\,\sqrt{c\,a^2\,x^2+c}+\frac{2\,x^2\,\sqrt{c\,a^2\,x^2+c}}{a^2}}$$

input
```
int(exp(atan(a*x))/(c + a^2*c*x^2)^(7/2),x)
```

output
```
(exp(atan(a*x))*(9/(26*a^5*c^3) + (3*x^5)/(13*c^3) + (17*x)/(26*a^4*c^3) +
 (3*x^4)/(13*a*c^3) + (9*x^3)/(13*a^2*c^3) + (7*x^2)/(13*a^3*c^3)))/((c +
a^2*c*x^2)^(1/2)/a^4 + x^4*(c + a^2*c*x^2)^(1/2) + (2*x^2*(c + a^2*c*x^2)^
(1/2))/a^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.84

$$\int \frac{e^{\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \frac{e^{atan(ax)}\sqrt{c}\,\sqrt{a^2x^2+1}\,(6a^5x^5+6a^4x^4+18a^3x^3+14a^2x^2+17ax+9)}{26a\,c^4\,(a^6x^6+3a^4x^4+3a^2x^2+1)}$$

input
```
int(exp(atan(a*x))/(a^2*c*x^2+c)^(7/2),x)
```

output
```
(e**atan(a*x)*sqrt(c)*sqrt(a**2*x**2 + 1)*(6*a**5*x**5 + 6*a**4*x**4 + 18*
a**3*x**3 + 14*a**2*x**2 + 17*a*x + 9))/(26*a*c**4*(a**6*x**6 + 3*a**4*x**
4 + 3*a**2*x**2 + 1))
```

3.273 $\int e^{2\arctan(ax)}(c+a^2cx^2)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 90

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx = \frac{i2^{-i+p}(1-iax)^{(1+i)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\text{Hypergeometric2F1}\left(i-p,(1+i)+p,(2+i)+p,\frac{1}{2}(1\right.}{a((1+i)+p)}$$

output
```
I*2^(-I+p)*(1-I*a*x)^(1+I+p)*(a^2*c*x^2+c)^p*hypergeom([I-p, 1+I+p],[2+I+p
],1/2-1/2*I*a*x)/a/(1+I+p)/((a^2*x^2+1)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx = \frac{i2^{-i+p}(1-iax)^{(1+i)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\text{Hypergeometric2F1}\left(i-p,(1+i)+p,(2+i)+p,\frac{1}{2}(1\right.}{a((1+i)+p)}$$

input
```
Integrate[E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^p,x]
```

output
```
(I*2^(-I + p)*(1 - I*a*x)^((1 + I) + p)*(c + a^2*c*x^2)^p*Hypergeometric2F
1[I - p, (1 + I) + p, (2 + I) + p, (1 - I*a*x)/2])/(a*((1 + I) + p)*(1 + a
^2*x^2)^p)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5599, 5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2\arctan(ax)}\left(a^2cx^2+c\right)^p\,dx$$

$$\Big\downarrow \text{5599}$$

$$\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\int e^{2\arctan(ax)}\left(a^2x^2+1\right)^p dx$$

$$\Big\downarrow \text{5596}$$

$$\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\int (1-iax)^{p+i}(iax+1)^{p-i}dx$$

$$\Big\downarrow \text{79}$$

$$\frac{i2^{p-i}(1-iax)^{p+(1+i)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\text{Hypergeometric2F1}\left(i-p,p+(1+i),p+(2+i),\frac{1}{2}(1-iax)\right)}{a(p+(1+i))}$$

input
```
Int[E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^p,x]
```

output
```
(I*2^(-I + p)*(1 - I*a*x)^((1 + I) + p)*(c + a^2*c*x^2)^p*Hypergeometric2F
1[I - p, (1 + I) + p, (2 + I) + p, (1 - I*a*x)/2])/(a*((1 + I) + p)*(1 + a
^2*x^2)^p)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \mathrm{e}^{2 \arctan(ax)} \left(a^2 c\, x^2 + c\right)^p dx$$

input
```
int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^p,x)
```

output
```
int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^p,x)
```

Fricas [F]

$$\int e^{2 \arctan(ax)} \left(c + a^2 c x^2\right)^p \, dx = \int \left(a^2 c x^2 + c\right)^p e^{\left(2 \arctan\left(ax\right)\right)} \, dx$$

input
```
integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^p*e^(2*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int \left(c\left(a^2x^2+1\right)\right)^p e^{2\operatorname{atan}(ax)}\,dx$$

input
```
integrate(exp(2*atan(a*x))*(a**2*c*x**2+c)**p,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**p*exp(2*atan(a*x)), x)
```

Maxima [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int \left(a^2cx^2+c\right)^p e^{(2\arctan(ax))}\,dx$$

input
```
integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^p*e^(2*arctan(a*x)), x)
```

Giac [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int \left(a^2cx^2+c\right)^p e^{(2\arctan(ax))}\,dx$$

input
```
integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^p*e^(2*arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int e^{2\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^p\,dx$$

input
```
int(exp(2*atan(a*x))*(c + a^2*c*x^2)^p,x)
```

output
```
int(exp(2*atan(a*x))*(c + a^2*c*x^2)^p, x)
```

Reduce [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int e^{2atan(ax)}\left(a^2c\,x^2+c\right)^p dx$$

input
```
int(exp(2*atan(a*x))*(a^2*c*x^2+c)^p,x)
```

output
```
int(e**(2*atan(a*x))*(a**2*c*x**2 + c)**p,x)
```

3.274 $\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx = \frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{1-i}c^2(1-iax)^{3+i}\,\text{Hypergeometric2F1}\left(-2+i,3+i,4+i,\frac{1}{2}(1-iax)\right)}{a}$$

output
```
(1/5+3/5*I)*2^(1-I)*c^2*(1-I*a*x)^(3+I)*hypergeom([3+I, -2+I],[4+I],1/2-1/
2*I*a*x)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx = \frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{1-i}c^2(1-iax)^{3+i}\,\text{Hypergeometric2F1}\left(-2+i,3+i,4+i,\frac{1}{2}(1-iax)\right)}{a}$$

input
```
Integrate[E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^2,x]
```

output
```
((1/5 + (3*I)/5)*2^(1 - I)*c^2*(1 - I*a*x)^(3 + I)*Hypergeometric2F1[-2 +
I, 3 + I, 4 + I, (1 - I*a*x)/2])/a
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2\arctan(ax)}\left(a^2cx^2+c\right)^2\,dx$$

$$\Big\downarrow \text{ 5596}$$

$$c^2\int (1-iax)^{2+i}(iax+1)^{2-i}dx$$

$$\Big\downarrow \text{ 79}$$

$$\frac{\left(\frac{1}{5}+\frac{3i}{5}\right)2^{1-i}c^2(1-iax)^{3+i}\,\text{Hypergeometric2F1}\left(-2+i,3+i,4+i,\frac{1}{2}(1-iax)\right)}{a}$$

input
```
Int[E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^2,x]
```

output

```
((1/5 + (3*I)/5)*2^(1 - I)*c^2*(1 - I*a*x)^(3 + I)*Hypergeometric2F1[-2 +
I, 3 + I, 4 + I, (1 - I*a*x)/2])/a
```


Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int e^{2\arctan(ax)}\left(a^{2}c\,x^{2}+c\right)^{2}dx$$

input
```
int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^2,x)
```

output
```
int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^2,x)
```

Fricas [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int\left(a^2cx^2+c\right)^2e^{\left(2\arctan(ax)\right)}\,dx$$

input
```
integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*e^(2*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=c^2\left(\int 2a^2x^2e^{2\,\mathrm{atan}\,(ax)}\,dx+\int a^4x^4e^{2\,\mathrm{atan}\,(ax)}\,dx+\int e^{2\,\mathrm{atan}\,(ax)}\,dx\right)$$

input
```
integrate(exp(2*atan(a*x))*(a**2*c*x**2+c)**2,x)
```

output
```
c**2*(Integral(2*a**2*x**2*exp(2*atan(a*x)), x) + Integral(a**4*x**4*exp(2
*atan(a*x)), x) + Integral(exp(2*atan(a*x)), x))
```

Maxima [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int \left(a^2cx^2+c\right)^2e^{(2\arctan(ax))}\,dx$$

input
```
integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^2*e^(2*arctan(a*x)), x)
```

Giac [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int \left(a^2cx^2+c\right)^2e^{(2\arctan(ax))}\,dx$$

input
```
integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*e^(2*arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int e^{2\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

input
```
int(exp(2*atan(a*x))*(c + a^2*c*x^2)^2,x)
```

output
```
int(exp(2*atan(a*x))*(c + a^2*c*x^2)^2, x)
```

Reduce [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx = c^2\left(\int e^{2atan(ax)}dx+\left(\int e^{2atan(ax)}x^4dx\right)a^4+2\left(\int e^{2atan(ax)}x^2dx\right)a^2\right)$$

input

```
int(exp(2*atan(a*x))*(a^2*c*x^2+c)^2,x)
```

output

```
c**2*(int(e**(2*atan(a*x)),x) + int(e**(2*atan(a*x))*x**4,x)*a**4 + 2*int(
e**(2*atan(a*x))*x**2,x)*a**2)
```

3.275 $\int e^{2\arctan(ax)}(c+a^2cx^2)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int e^{2\arctan(ax)}(c+a^2cx^2)\,dx = \frac{\left(\frac{1}{5}+\frac{2i}{5}\right)2^{1-i}c(1-iax)^{2+i}\,\text{Hypergeometric2F1}\left(-1+i,2+i,3+i,\frac{1}{2}(1-iax)\right)}{a}$$

output
```
(1/5+2/5*I)*2^(1-I)*c*(1-I*a*x)^(2+I)*hypergeom([-1+I, 2+I],[3+I],1/2-1/2*
I*a*x)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int e^{2\arctan(ax)}(c+a^2cx^2)\,dx = \frac{\left(\frac{1}{5}+\frac{2i}{5}\right)2^{1-i}c(1-iax)^{2+i}\,\text{Hypergeometric2F1}\left(-1+i,2+i,3+i,\frac{1}{2}(1-iax)\right)}{a}$$

input
```
Integrate[E^(2*ArcTan[a*x])*(c + a^2*c*x^2),x]
```

output
```
((1/5 + (2*I)/5)*2^(1 - I)*c*(1 - I*a*x)^(2 + I)*Hypergeometric2F1[-1 + I,
 2 + I, 3 + I, (1 - I*a*x)/2])/a
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2\arctan(ax)}\left(a^2cx^2+c\right)\,dx$$

$$\Big\downarrow \text{5596}$$

$$c\int (1-iax)^{1+i}(iax+1)^{1-i}dx$$

$$\Big\downarrow \text{79}$$

$$\frac{\left(\frac{1}{5}+\frac{2i}{5}\right)2^{1-i}c(1-iax)^{2+i}\,\text{Hypergeometric2F1}\left(-1+i,2+i,3+i,\frac{1}{2}(1-iax)\right)}{a}$$

input
```
Int[E^(2*ArcTan[a*x])*(c + a^2*c*x^2),x]
```

output
```
((1/5 + (2*I)/5)*2^(1 - I)*c*(1 - I*a*x)^(2 + I)*Hypergeometric2F1[-1 + I,
 2 + I, 3 + I, (1 - I*a*x)/2])/a
```


Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \mathrm{e}^{2 \arctan(ax)} \left(a^{2} c\, x^{2}+c\right) dx$$

input
```
int(exp(2*arctan(a*x))*(a^2*c*x^2+c),x)
```

output
```
int(exp(2*arctan(a*x))*(a^2*c*x^2+c),x)
```

Fricas [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)\, dx = \int \left(a^{2} c x^{2} + c\right) e^{\left(2 \, \arctan\left(a x\right)\right)} \,d x$$

input
```
integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*e^(2*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)\, dx = c \left(\int a^{2} x^{2} e^{2 \operatorname{atan}{\left(a x \right)}}\, dx + \int e^{2 \operatorname{atan}{\left(a x \right)}}\, dx\right)$$

input
```
integrate(exp(2*atan(a*x))*(a**2*c*x**2+c),x)
```

output
```
c*(Integral(a**2*x**2*exp(2*atan(a*x)), x) + Integral(exp(2*atan(a*x)), x)
)
```

Maxima [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int \left(a^2cx^2+c\right)e^{\left(2\arctan(ax)\right)}\,dx$$

input
```
integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)*e^(2*arctan(a*x)), x)
```

Giac [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int \left(a^2cx^2+c\right)e^{\left(2\arctan(ax)\right)}\,dx$$

input
```
integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*e^(2*arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int \mathrm{e}^{2\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(exp(2*atan(a*x))*(c + a^2*c*x^2),x)
```

output
```
int(exp(2*atan(a*x))*(c + a^2*c*x^2), x)
```

Reduce [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)\,dx = c\left(\int e^{2atan(ax)}dx+\left(\int e^{2atan(ax)}x^2dx\right)a^2\right)$$

input
```
int(exp(2*atan(a*x))*(a^2*c*x^2+c),x)
```

output
```
c*(int(e**(2*atan(a*x)),x) + int(e**(2*atan(a*x))*x**2,x)*a**2)
```

3.276 $\int e^{2\arctan(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 46

$$\int e^{2\arctan(ax)}\,dx = \frac{(1+i)2^{-1-i}(1-iax)^{1+i}\,\text{Hypergeometric2F1}\left(i,1+i,2+i,\frac{1}{2}(1-iax)\right)}{a}$$

output
```
(1+I)*(1-I*a*x)^(1+I)*hypergeom([I, 1+I],[2+I],1/2-1/2*I*a*x)/(2^(1+I))/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int e^{2\arctan(ax)}\,dx = \frac{(1-i)e^{(2+2i)\arctan(ax)}\,\text{Hypergeometric2F1}\left(1-i,2,2-i,-e^{2i\arctan(ax)}\right)}{a}$$

input
```
Integrate[E^(2*ArcTan[a*x]),x]
```

output
```
((1 - I)*E^((2 + 2*I)*ArcTan[a*x])*Hypergeometric2F1[1 - I, 2, 2 - I, -E^(
(2*I)*ArcTan[a*x])])/a
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5584, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2\arctan(ax)}dx$$

$$\Big\downarrow \text{5584}$$

$$\int (1-iax)^i(1+iax)^{-i}dx$$

$$\Big\downarrow \text{79}$$

$$\frac{(1+i)2^{-1-i}(1-iax)^{1+i}\text{Hypergeometric2F1}\left(i,1+i,2+i,\frac{1}{2}(1-iax)\right)}{a}$$

input
```
Int[E^(2*ArcTan[a*x]),x]
```

output
```
((1 + I)*(1 - I*a*x)^(1 + I)*Hypergeometric2F1[I, 1 + I, 2 + I, (1 - I*a*x
)/2])/(2^(1 + I)*a)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5584
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Maple [F]

$$\int e^{2\arctan(ax)}dx$$

input
```
int(exp(2*arctan(a*x)),x)
```

output
```
int(exp(2*arctan(a*x)),x)
```

Fricas [F]

$$\int e^{2\arctan(ax)}\,dx = \int e^{(2\arctan(ax))}\,dx$$

input
```
integrate(exp(2*arctan(a*x)),x, algorithm="fricas")
```

output
```
integral(e^(2*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{2\arctan(ax)}\,dx = \int e^{2\operatorname{atan}(ax)}\,dx$$

input
```
integrate(exp(2*atan(a*x)),x)
```

output
```
Integral(exp(2*atan(a*x)), x)
```

Maxima [F]

$$\int e^{2\arctan(ax)}\,dx = \int e^{(2\arctan(ax))}\,dx$$

input
```
integrate(exp(2*arctan(a*x)),x, algorithm="maxima")
```

output
```
integrate(e^(2*arctan(a*x)), x)
```

Giac [F]

$$\int e^{2\arctan(ax)}\,dx = \int e^{(2\arctan(ax))}\,dx$$

input
```
integrate(exp(2*arctan(a*x)),x, algorithm="giac")
```

output
```
integrate(e^(2*arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\arctan(ax)}\,dx = \int e^{2\,\mathrm{atan}(a\,x)}\,dx$$

input
```
int(exp(2*atan(a*x)),x)
```

output
```
int(exp(2*atan(a*x)), x)
```

Reduce [F]

$$\int e^{2\arctan(ax)}\,dx = \int e^{2atan(ax)}dx$$

input
```
int(exp(2*atan(a*x)),x)
```

output
```
int(e**(2*atan(a*x)),x)
```

3.277 $\int \frac{e^{2\arctan(ax)}}{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \frac{e^{2\arctan(ax)}}{c+a^2cx^2}\,dx=\frac{e^{2\arctan(ax)}}{2ac}$$

output
```
1/2*exp(2*arctan(a*x))/a/c
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{e^{2\arctan(ax)}}{c+a^2cx^2}\,dx=\frac{(1-iax)^i(1+iax)^{-i}}{2ac}$$

input
```
Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

output
```
(1 - I*a*x)^I/(2*a*c*(1 + I*a*x)^I)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {5594}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\arctan(ax)}}{a^2cx^2+c}\,dx$$
$$\Big\downarrow \text{ 5594}$$
$$\frac{e^{2\arctan(ax)}}{2ac}$$

input
```
Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

output
```
E^(2*ArcTan[a*x])/(2*a*c)
```

Defintions of rubi rules used

rule 5594
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result	size
gosper	$\frac{e^{2\arctan(ax)}}{2ac}$	16
parallelrisch	$\frac{e^{2\arctan(ax)}}{2ac}$	16
risch	$\frac{(-iax+1)^{i}(iax+1)^{-i}}{2ac}$	29
orering	$\frac{(a^2x^2+1)e^{2\arctan(ax)}}{2a(a^2c\,x^2+c)}$	34

input
```
int(exp(2*arctan(a*x))/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(2*arctan(a*x))/a/c
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^{2\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{e^{(2\arctan(ax))}}{2\,ac}$$

input
```
integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
1/2*e^(2*arctan(a*x))/(a*c)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^{2\arctan(ax)}}{c+a^2cx^2}\,dx = \begin{cases} \frac{e^{2\operatorname{atan}(ax)}}{2ac} & \text{for}\ a \neq 0 \\ \frac{x}{c} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(2*atan(a*x))/(a**2*c*x**2+c),x)
```

output
```
Piecewise((exp(2*atan(a*x))/(2*a*c), Ne(a, 0)), (x/c, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^{2\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{e^{(2\arctan(ax))}}{2\,ac}$$

input
```
integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/2*e^(2*arctan(a*x))/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^{2\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{e^{(2\arctan(ax))}}{2\,ac}$$

input
```
integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
1/2*e^(2*arctan(a*x))/(a*c)
```

Mupad [B] (verification not implemented)

Time = 23.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^{2\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{e^{2\,\mathrm{atan}(a\,x)}}{2\,a\,c}$$

input
```
int(exp(2*atan(a*x))/(c + a^2*c*x^2),x)
```

output
```
exp(2*atan(a*x))/(2*a*c)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{e^{2\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{e^{2atan(ax)}}{2ac}$$

input
```
int(exp(2*atan(a*x))/(a^2*c*x^2+c),x)
```

output
```
e**(2*atan(a*x))/(2*a*c)
```

3.278 $\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{e^{2\arctan(ax)}}{8ac^2} + \frac{e^{2\arctan(ax)}(1+ax)}{4ac^2(1+a^2x^2)}$$

output
```
1/8*exp(2*arctan(a*x))/a/c^2+1/4*exp(2*arctan(a*x))*(a*x+1)/a/c^2/(a^2*x^2
+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{(1-iax)^i(1+iax)^{-i}\,(3+2ax+a^2x^2)}{8c^2\,(a+a^3x^2)}$$

input
```
Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

output
```
((1 - I*a*x)^I*(3 + 2*a*x + a^2*x^2))/(8*c^2*(1 + I*a*x)^I*(a + a^3*x^2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5593, 27, 5594}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\arctan(ax)}}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \ 5593$$

$$\frac{\int \frac{e^{2\arctan(ax)}}{c(a^2x^2+1)}dx}{4c}+\frac{(ax+1)e^{2\arctan(ax)}}{4ac^2(a^2x^2+1)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{e^{2\arctan(ax)}}{a^2x^2+1}dx}{4c^2}+\frac{(ax+1)e^{2\arctan(ax)}}{4ac^2(a^2x^2+1)}$$

$$\Big\downarrow \ 5594$$

$$\frac{(ax+1)e^{2\arctan(ax)}}{4ac^2(a^2x^2+1)}+\frac{e^{2\arctan(ax)}}{8ac^2}$$

input
```
Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

output
```
E^(2*ArcTan[a*x])/(8*a*c^2) + (E^(2*ArcTan[a*x])*(1 + a*x))/(4*a*c^2*(1 +
a^2*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5593
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2)))   I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
&& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
 0] && IntegerQ[2*p]
```

rule 5594
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

method	result	size
gosper	$\frac{\mathrm{e}^{2 \arctan \left(a x \right)} \left(a^{2} x^{2}+2 a x +3\right)}{8 \left(a^{2} x^{2}+1\right) a \,c^{2}}$	40
orering	$\frac{\left(a^{2} x^{2}+2 a x +3\right) \left(a^{2} x^{2}+1\right) \mathrm{e}^{2 \arctan \left(a x \right)}}{8 a \left(a^{2} c \,x^{2}+c \right)^{2}}$	47
parallelrisch	$\frac{a^{2} \mathrm{e}^{2 \arctan \left(a x \right)} x^{2}+2 \,\mathrm{e}^{2 \arctan \left(a x \right)} a x +3 \,\mathrm{e}^{2 \arctan \left(a x \right)}}{8 c^{2} \left(a^{2} x^{2}+1\right) a}$	55

input
```
int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*exp(2*arctan(a*x))*(a^2*x^2+2*a*x+3)/(a^2*x^2+1)/a/c^2
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

$$\int \frac{e^{2 \arctan(ax)}}{(c+a^2cx^2)^2} \, dx = \frac{(a^2x^2 + 2\,ax + 3)e^{(2\,\arctan(ax))}}{8\,(a^3c^2x^2 + ac^2)}$$

input
```
integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/8*(a^2*x^2 + 2*a*x + 3)*e^(2*arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(42) = 84$.

Time = 1.18 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.87

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \begin{cases} \frac{a^2x^2e^{2\operatorname{atan}(ax)}}{8a^3c^2x^2+8ac^2} + \frac{2axe^{2\operatorname{atan}(ax)}}{8a^3c^2x^2+8ac^2} + \frac{3e^{2\operatorname{atan}(ax)}}{8a^3c^2x^2+8ac^2} & \text{for } a \neq 0 \\ \frac{x}{c^2} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**2,x)
```

output
```
Piecewise((a**2*x**2*exp(2*atan(a*x))/(8*a**3*c**2*x**2 + 8*a*c**2) + 2*a*
x*exp(2*atan(a*x))/(8*a**3*c**2*x**2 + 8*a*c**2) + 3*exp(2*atan(a*x))/(8*a
**3*c**2*x**2 + 8*a*c**2), Ne(a, 0)), (x/c**2, True))
```

Maxima [F]

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{e^{(2\arctan(ax))}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(47) = 94$.

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.25

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{3\,e^{(2\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^4 - 4\,e^{(2\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^3 - 2\,e^{(2\arctan(ax))}\tan\left(\frac{1}{2}\arctan(\right.}{8\left(ac^2\tan\left(\frac{1}{2}\arctan(ax)\right)^4 + 2\,ac^2\tan\left(\frac{1}{2}\arctan(ax)\right)\right.}$$

input
```
integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
1/8*(3*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^4 - 4*e^(2*arctan(a*x))*tan(
1/2*arctan(a*x))^3 - 2*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^2 + 4*e^(2*a
rctan(a*x))*tan(1/2*arctan(a*x)) + 3*e^(2*arctan(a*x)))/(a*c^2*tan(1/2*arc
tan(a*x))^4 + 2*a*c^2*tan(1/2*arctan(a*x))^2 + a*c^2)
```

Mupad [B] (verification not implemented)

Time = 23.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{e^{2\,\mathrm{atan}(a\,x)}\left(\frac{3}{8\,a^3\,c^2} + \frac{x}{4\,a^2\,c^2} + \frac{x^2}{8\,a\,c^2}\right)}{\frac{1}{a^2} + x^2}$$

input
```
int(exp(2*atan(a*x))/(c + a^2*c*x^2)^2,x)
```

output
```
(exp(2*atan(a*x))*(3/(8*a^3*c^2) + x/(4*a^2*c^2) + x^2/(8*a*c^2)))/(1/a^2
+ x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{e^{2atan(ax)}(a^2x^2+2ax+3)}{8a\,c^2\,(a^2x^2+1)}$$

input
```
int(exp(2*atan(a*x))/(a^2*c*x^2+c)^2,x)
```

output
```
(e**(2*atan(a*x))*(a**2*x**2 + 2*a*x + 3))/(8*a*c**2*(a**2*x**2 + 1))
```

3.279 $\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 88

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{3e^{2\arctan(ax)}}{40ac^3} + \frac{e^{2\arctan(ax)}(1+2ax)}{10ac^3(1+a^2x^2)^2} + \frac{3e^{2\arctan(ax)}(1+ax)}{20ac^3(1+a^2x^2)}$$

output
```
3/40*exp(2*arctan(a*x))/a/c^3+1/10*exp(2*arctan(a*x))*(2*a*x+1)/a/c^3/(a^2
*x^2+1)^2+3/20*exp(2*arctan(a*x))*(a*x+1)/a/c^3/(a^2*x^2+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.98

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{4e^{2\arctan(ax)}(1+2ax)+3(1-iax)^i(1+iax)^{-i}(1+a^2x^2)(3+2ax+a^2x^2)}{40ac^3(1+a^2x^2)^2}$$

input
```
Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

output
```
(4*E^(2*ArcTan[a*x])*(1 + 2*a*x) + (3*(1 - I*a*x)^I*(1 + a^2*x^2)*(3 + 2*a
*x + a^2*x^2))/(1 + I*a*x)^I)/(40*a*c^3*(1 + a^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5593, 27, 5593, 5594}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\arctan(ax)}}{(a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{ 5593}$$

$$\frac{3\int \frac{e^{2\arctan(ax)}}{c^2(a^2x^2+1)^2}dx}{5c}+\frac{(2ax+1)e^{2\arctan(ax)}}{10ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3\int \frac{e^{2\arctan(ax)}}{(a^2x^2+1)^2}dx}{5c^3}+\frac{(2ax+1)e^{2\arctan(ax)}}{10ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{ 5593}$$

$$\frac{3\left(\frac{1}{4}\int \frac{e^{2\arctan(ax)}}{a^2x^2+1}dx+\frac{(ax+1)e^{2\arctan(ax)}}{4a(a^2x^2+1)}\right)}{5c^3}+\frac{(2ax+1)e^{2\arctan(ax)}}{10ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{ 5594}$$

$$\frac{(2ax+1)e^{2\arctan(ax)}}{10ac^3\left(a^2x^2+1\right)^2}+\frac{3\left(\frac{(ax+1)e^{2\arctan(ax)}}{4a(a^2x^2+1)}+\frac{e^{2\arctan(ax)}}{8a}\right)}{5c^3}$$

input
```
Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

output
```
(E^(2*ArcTan[a*x])*(1 + 2*a*x))/(10*a*c^3*(1 + a^2*x^2)^2) + (3*(E^(2*ArcT
an[a*x])/(8*a) + (E^(2*ArcTan[a*x])*(1 + a*x))/(4*a*(1 + a^2*x^2))))/(5*c^
3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5593
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2)))   I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
&& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
 0] && IntegerQ[2*p]
```

rule 5594
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Maple [A] (verified)

Time = 11.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{e^{2 \arctan (a x)} (3 a^{4} x^{4}+6 a^{3} x^{3}+12 a^{2} x^{2}+14 a x+13)}{40 (a^{2} x^{2}+1)^{2} a \,c^{3}}$	57
orering	$\frac{(3 a^{4} x^{4}+6 a^{3} x^{3}+12 a^{2} x^{2}+14 a x+13) (a^{2} x^{2}+1) e^{2 \arctan (a x)}}{40 a (a^{2} c \,x^{2}+c)^{3}}$	64
parallelrisch	$\frac{3 a^{4} e^{2 \arctan (a x)} x^{4}+6 a^{3} e^{2 \arctan (a x)} x^{3}+12 a^{2} e^{2 \arctan (a x)} x^{2}+14 \,e^{2 \arctan (a x)} a x +13 \,e^{2 \arctan (a x)}}{40 c^{3} (a^{2} x^{2}+1)^{2} a}$	86

input
```
int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/40*exp(2*arctan(a*x))*(3*a^4*x^4+6*a^3*x^3+12*a^2*x^2+14*a*x+13)/(a^2*x^
2+1)^2/a/c^3
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{(3\,a^4x^4+6\,a^3x^3+12\,a^2x^2+14\,ax+13)e^{(2\,\arctan(ax))}}{40\,(a^5c^3x^4+2\,a^3c^3x^2+ac^3)}$$

input
```
integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/40*(3*a^4*x^4 + 6*a^3*x^3 + 12*a^2*x^2 + 14*a*x + 13)*e^(2*arctan(a*x))/
(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(78) = 156.

Time = 3.24 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.62

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$$
$$= \begin{cases} \frac{3a^4x^4e^{2\operatorname{atan}(ax)}}{40a^5c^3x^4+80a^3c^3x^2+40ac^3} + \frac{6a^3x^3e^{2\operatorname{atan}(ax)}}{40a^5c^3x^4+80a^3c^3x^2+40ac^3} + \frac{12a^2x^2e^{2\operatorname{atan}(ax)}}{40a^5c^3x^4+80a^3c^3x^2+40ac^3} + \frac{14axe^{2\operatorname{atan}(ax)}}{40a^5c^3x^4+80a^3c^3x^2+40ac^3} + \frac{1}{40a^5c^3x} \\ \frac{x}{c^3} \end{cases}$$

input
```
integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**3,x)
```

output
```
Piecewise((3*a**4*x**4*exp(2*atan(a*x))/(40*a**5*c**3*x**4 + 80*a**3*c**3*
x**2 + 40*a*c**3) + 6*a**3*x**3*exp(2*atan(a*x))/(40*a**5*c**3*x**4 + 80*a
**3*c**3*x**2 + 40*a*c**3) + 12*a**2*x**2*exp(2*atan(a*x))/(40*a**5*c**3*x
**4 + 80*a**3*c**3*x**2 + 40*a*c**3) + 14*a*x*exp(2*atan(a*x))/(40*a**5*c*
*3*x**4 + 80*a**3*c**3*x**2 + 40*a*c**3) + 13*exp(2*atan(a*x))/(40*a**5*c*
*3*x**4 + 80*a**3*c**3*x**2 + 40*a*c**3), Ne(a, 0)), (x/c**3, True))
```

Maxima [F]

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{e^{(2\arctan(ax))}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(79) = 158.

Time = 0.15 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.51

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{13\,e^{(2\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^8 - 28\,e^{(2\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^7 - 4\,e^{(2\arctan(ax))}\tan\left(\frac{1}{2}\arctan\right.}{40\left(\right.}$$

input
```
integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
1/40*(13*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^8 - 28*e^(2*arctan(a*x))*t
an(1/2*arctan(a*x))^7 - 4*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^6 + 36*e^
(2*arctan(a*x))*tan(1/2*arctan(a*x))^5 + 30*e^(2*arctan(a*x))*tan(1/2*arct
an(a*x))^4 - 36*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^3 - 4*e^(2*arctan(a
*x))*tan(1/2*arctan(a*x))^2 + 28*e^(2*arctan(a*x))*tan(1/2*arctan(a*x)) +
13*e^(2*arctan(a*x)))/(a*c^3*tan(1/2*arctan(a*x))^8 + 4*a*c^3*tan(1/2*arct
an(a*x))^6 + 6*a*c^3*tan(1/2*arctan(a*x))^4 + 4*a*c^3*tan(1/2*arctan(a*x))
^2 + a*c^3)
```

Mupad [B] (verification not implemented)

Time = 23.41 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.90

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{3\,\mathrm{e}^{2\,\mathrm{atan}(a\,x)}}{40\,a\,c^3} + \frac{3\,\mathrm{e}^{2\,\mathrm{atan}(a\,x)}\,(a\,x+1)}{20\,a\,c^3\,\left(a^2\,x^2+1\right)} + \frac{\mathrm{e}^{2\,\mathrm{atan}(a\,x)}\,(2\,a\,x+1)}{10\,a\,c^3\,\left(a^2\,x^2+1\right)^2}$$

input
```
int(exp(2*atan(a*x))/(c + a^2*c*x^2)^3,x)
```

output
```
(3*exp(2*atan(a*x)))/(40*a*c^3) + (3*exp(2*atan(a*x))*(a*x + 1))/(20*a*c^3
*(a^2*x^2 + 1)) + (exp(2*atan(a*x))*(2*a*x + 1))/(10*a*c^3*(a^2*x^2 + 1)^2
)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.74

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{e^{2atan(ax)}(3a^4x^4+6a^3x^3+12a^2x^2+14ax+13)}{40a\,c^3\,(a^4x^4+2a^2x^2+1)}$$

input
```
int(exp(2*atan(a*x))/(a^2*c*x^2+c)^3,x)
```

output
```
(e**(2*atan(a*x))*(3*a**4*x**4 + 6*a**3*x**3 + 12*a**2*x**2 + 14*a*x + 13)
)/(40*a*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

3.280 $\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{9e^{2\arctan(ax)}}{160ac^4}+\frac{e^{2\arctan(ax)}(1+3ax)}{20ac^4(1+a^2x^2)^3}+\frac{3e^{2\arctan(ax)}(1+2ax)}{40ac^4(1+a^2x^2)^2}+\frac{9e^{2\arctan(ax)}(1+ax)}{80ac^4(1+a^2x^2)}$$

output
```
9/160*exp(2*arctan(a*x))/a/c^4+1/20*exp(2*arctan(a*x))*(3*a*x+1)/a/c^4/(a^
2*x^2+1)^3+3/40*exp(2*arctan(a*x))*(2*a*x+1)/a/c^4/(a^2*x^2+1)^2+9/80*exp(
2*arctan(a*x))*(a*x+1)/a/c^4/(a^2*x^2+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.99

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{8ce^{2\arctan(ax)}(1+3ax)+3(c+a^2cx^2)\left(4e^{2\arctan(ax)}(1+2ax)+3(1-iax)^{i}(1+iax)^{-i}(-i+ax)(i+ax)\right.}{160ac^2(c+a^2cx^2)^3}$$

input
```
Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^4,x]
```

output
```
(8*c*E^(2*ArcTan[a*x])*(1 + 3*a*x) + 3*(c + a^2*c*x^2)*(4*E^(2*ArcTan[a*x]
)*(1 + 2*a*x) + (3*(1 - I*a*x)^I*(-I + a*x)*(I + a*x)*(3 + 2*a*x + a^2*x^2
))/(1 + I*a*x)^I))/(160*a*c^2*(c + a^2*c*x^2)^3)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5593, 27, 5593, 5593, 5594}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\arctan(ax)}}{(a^2cx^2+c)^4}dx$$

$$\Big\downarrow \text{5593}$$

$$\frac{3\int \frac{e^{2\arctan(ax)}}{c^3(a^2x^2+1)^3}dx}{4c}+\frac{(3ax+1)e^{2\arctan(ax)}}{20ac^4(a^2x^2+1)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{e^{2\arctan(ax)}}{(a^2x^2+1)^3}dx}{4c^4}+\frac{(3ax+1)e^{2\arctan(ax)}}{20ac^4(a^2x^2+1)^3}$$

$$\Big\downarrow \text{5593}$$

$$\frac{3\left(\frac{3}{5}\int \frac{e^{2\arctan(ax)}}{(a^2x^2+1)^2}dx+\frac{(2ax+1)e^{2\arctan(ax)}}{10a(a^2x^2+1)^2}\right)}{4c^4}+\frac{(3ax+1)e^{2\arctan(ax)}}{20ac^4(a^2x^2+1)^3}$$

$$\Big\downarrow \text{5593}$$

$$\frac{3\left(\frac{3}{5}\left(\frac{1}{4}\int \frac{e^{2\arctan(ax)}}{a^2x^2+1}dx+\frac{(ax+1)e^{2\arctan(ax)}}{4a(a^2x^2+1)}\right)+\frac{(2ax+1)e^{2\arctan(ax)}}{10a(a^2x^2+1)^2}\right)}{4c^4}+\frac{(3ax+1)e^{2\arctan(ax)}}{20ac^4(a^2x^2+1)^3}$$

$$\Big\downarrow \text{5594}$$

$$\frac{(3ax+1)e^{2\arctan(ax)}}{20ac^4\left(a^2x^2+1\right)^3}+\frac{3\left(\frac{(2ax+1)e^{2\arctan(ax)}}{10a(a^2x^2+1)^2}+\frac{3}{5}\left(\frac{(ax+1)e^{2\arctan(ax)}}{4a(a^2x^2+1)}+\frac{e^{2\arctan(ax)}}{8a}\right)\right)}{4c^4}$$

input
```
Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^4,x]
```

output
```
(E^(2*ArcTan[a*x])*(1 + 3*a*x))/(20*a*c^4*(1 + a^2*x^2)^3) + (3*((E^(2*Arc
Tan[a*x])*(1 + 2*a*x))/(10*a*(1 + a^2*x^2)^2) + (3*(E^(2*ArcTan[a*x])/(8*a
) + (E^(2*ArcTan[a*x])*(1 + a*x))/(4*a*(1 + a^2*x^2))))/5))/(4*c^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5593
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2)))   I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
&& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
 0] && IntegerQ[2*p]
```

rule 5594
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Maple [A] (verified)

Time = 36.80 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.59

method	result
gosper	$\frac{e^{2\arctan(ax)}\left(9x^{6}a^{6}+18a^{5}x^{5}+45a^{4}x^{4}+60a^{3}x^{3}+75a^{2}x^{2}+66ax+47\right)}{160\left(a^{2}x^{2}+1\right)^{3}c^{4}a}$
orering	$\frac{\left(9x^{6}a^{6}+18a^{5}x^{5}+45a^{4}x^{4}+60a^{3}x^{3}+75a^{2}x^{2}+66ax+47\right)\left(a^{2}x^{2}+1\right)e^{2\arctan(ax)}}{160a\left(a^{2}c\,x^{2}+c\right)^{4}}$
parallelrisch	$\frac{9a^{6}e^{2\arctan(ax)}x^{6}+18a^{5}e^{2\arctan(ax)}x^{5}+45a^{4}e^{2\arctan(ax)}x^{4}+60a^{3}e^{2\arctan(ax)}x^{3}+75a^{2}e^{2\arctan(ax)}x^{2}+66\,e^{2\arctan(ax)}ax+}{160c^{4}\left(a^{2}x^{2}+1\right)^{3}a}$

input
```
int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/160*exp(2*arctan(a*x))*(9*a^6*x^6+18*a^5*x^5+45*a^4*x^4+60*a^3*x^3+75*a^
2*x^2+66*a*x+47)/(a^2*x^2+1)^3/c^4/a
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.77

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{(9\,a^6x^6+18\,a^5x^5+45\,a^4x^4+60\,a^3x^3+75\,a^2x^2+66\,ax+47)e^{(2\,\arctan(ax))}}{160\,(a^7c^4x^6+3\,a^5c^4x^4+3\,a^3c^4x^2+ac^4)}$$

input
```
integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="fricas")
```

output
```
1/160*(9*a^6*x^6 + 18*a^5*x^5 + 45*a^4*x^4 + 60*a^3*x^3 + 75*a^2*x^2 + 66*
a*x + 47)*e^(2*arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3*c^4*x^2 +
 a*c^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(112) = 224$.

Time = 8.58 (sec) , antiderivative size = 410, normalized size of antiderivative = 3.33

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx$$
$$= \begin{cases} \frac{9a^6x^6e^{2\,\mathrm{atan}\,(ax)}}{160a^7c^4x^6+480a^5c^4x^4+480a^3c^4x^2+160ac^4} + \frac{18a^5x^5e^{2\,\mathrm{atan}\,(ax)}}{160a^7c^4x^6+480a^5c^4x^4+480a^3c^4x^2+160ac^4} + \frac{45a^4x^4e^{2\,\mathrm{atan}\,(ax)}}{160a^7c^4x^6+480a^5c^4x^4+480a^3c^4x^2+160a} \\ \frac{x}{c^4} \end{cases}$$

input
```
integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**4,x)
```

output
```
Piecewise((9*a**6*x**6*exp(2*atan(a*x))/(160*a**7*c**4*x**6 + 480*a**5*c**
4*x**4 + 480*a**3*c**4*x**2 + 160*a*c**4) + 18*a**5*x**5*exp(2*atan(a*x))/
(160*a**7*c**4*x**6 + 480*a**5*c**4*x**4 + 480*a**3*c**4*x**2 + 160*a*c**4
) + 45*a**4*x**4*exp(2*atan(a*x))/(160*a**7*c**4*x**6 + 480*a**5*c**4*x**4
 + 480*a**3*c**4*x**2 + 160*a*c**4) + 60*a**3*x**3*exp(2*atan(a*x))/(160*a
**7*c**4*x**6 + 480*a**5*c**4*x**4 + 480*a**3*c**4*x**2 + 160*a*c**4) + 75
*a**2*x**2*exp(2*atan(a*x))/(160*a**7*c**4*x**6 + 480*a**5*c**4*x**4 + 480
*a**3*c**4*x**2 + 160*a*c**4) + 66*a*x*exp(2*atan(a*x))/(160*a**7*c**4*x**
6 + 480*a**5*c**4*x**4 + 480*a**3*c**4*x**2 + 160*a*c**4) + 47*exp(2*atan(
a*x))/(160*a**7*c**4*x**6 + 480*a**5*c**4*x**4 + 480*a**3*c**4*x**2 + 160*
a*c**4), Ne(a, 0)), (x/c**4, True))
```

Maxima [F]

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \int \frac{e^{(2\arctan(ax))}}{(a^2cx^2+c)^4}\,dx$$

input
```
integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="maxima")
```

output
```
integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^4, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(111) = 222$.

Time = 0.16 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.63

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx$$
$$= \frac{47\,e^{(2\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^{12} - 132\,e^{(2\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^{11} + 18\,e^{(2\arctan(ax))}\tan\left(\frac{1}{2}\ \mathrm{ar}\right.}{}$$

input
```
integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="giac")
```

output
```
1/160*(47*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^12 - 132*e^(2*arctan(a*x)
)*tan(1/2*arctan(a*x))^11 + 18*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^10 +
 180*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^9 + 225*e^(2*arctan(a*x))*tan(
1/2*arctan(a*x))^8 - 456*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^7 - 4*e^(2
*arctan(a*x))*tan(1/2*arctan(a*x))^6 + 456*e^(2*arctan(a*x))*tan(1/2*arcta
n(a*x))^5 + 225*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^4 - 180*e^(2*arctan
(a*x))*tan(1/2*arctan(a*x))^3 + 18*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^
2 + 132*e^(2*arctan(a*x))*tan(1/2*arctan(a*x)) + 47*e^(2*arctan(a*x)))/(a*
c^4*tan(1/2*arctan(a*x))^12 + 6*a*c^4*tan(1/2*arctan(a*x))^10 + 15*a*c^4*t
an(1/2*arctan(a*x))^8 + 20*a*c^4*tan(1/2*arctan(a*x))^6 + 15*a*c^4*tan(1/2
*arctan(a*x))^4 + 6*a*c^4*tan(1/2*arctan(a*x))^2 + a*c^4)
```

Mupad [B] (verification not implemented)

Time = 23.63 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{9\,e^{2\,\mathrm{atan}(a\,x)}}{160\,a\,c^4} + \frac{9\,e^{2\,\mathrm{atan}(a\,x)}\,(a\,x+1)}{80\,a\,c^4\,(a^2\,x^2+1)} + \frac{3\,e^{2\,\mathrm{atan}(a\,x)}\,(2\,a\,x+1)}{40\,a\,c^4\,(a^2\,x^2+1)^2} + \frac{e^{2\,\mathrm{atan}(a\,x)}\,(3\,a\,x+1)}{20\,a\,c^4\,(a^2\,x^2+1)^3}$$

input
```
int(exp(2*atan(a*x))/(c + a^2*c*x^2)^4,x)
```

output
```
(9*exp(2*atan(a*x)))/(160*a*c^4) + (9*exp(2*atan(a*x))*(a*x + 1))/(80*a*c^
4*(a^2*x^2 + 1)) + (3*exp(2*atan(a*x))*(2*a*x + 1))/(40*a*c^4*(a^2*x^2 + 1
)^2) + (exp(2*atan(a*x))*(3*a*x + 1))/(20*a*c^4*(a^2*x^2 + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.72

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{e^{2atan(ax)}(9a^6x^6+18a^5x^5+45a^4x^4+60a^3x^3+75a^2x^2+66ax+47)}{160a\,c^4\,(a^6x^6+3a^4x^4+3a^2x^2+1)}$$

input
```
int(exp(2*atan(a*x))/(a^2*c*x^2+c)^4,x)
```

output
```
(e**(2*atan(a*x))*(9*a**6*x**6 + 18*a**5*x**5 + 45*a**4*x**4 + 60*a**3*x**
3 + 75*a**2*x**2 + 66*a*x + 47))/(160*a*c**4*(a**6*x**6 + 3*a**4*x**4 + 3*
a**2*x**2 + 1))
```

3.281 $\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx = \frac{\left(\frac{2}{29}+\frac{5i}{29}\right)2^{\frac{5}{2}-i}c(1-iax)^{\frac{5}{2}+i}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}+i,\frac{5}{2}+i,\frac{7}{2}+i,\frac{1}{2}(1-i\right.}{a\sqrt{1+a^2x^2}}$$

output
```
(2/29+5/29*I)*2^(5/2-I)*c*(1-I*a*x)^(5/2+I)*(a^2*c*x^2+c)^(1/2)*hypergeom(
[5/2+I, -3/2+I],[7/2+I],1/2-1/2*I*a*x)/a/(a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx = \frac{\left(\frac{2}{29}+\frac{5i}{29}\right)2^{\frac{5}{2}-i}c(1-iax)^{\frac{5}{2}+i}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}+i,\frac{5}{2}+i,\frac{7}{2}+i,\frac{1}{2}(1-i\right.}{a\sqrt{1+a^2x^2}}$$

input
```
Integrate[E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2),x]
```

output
```
((2/29 + (5*I)/29)*2^(5/2 - I)*c*(1 - I*a*x)^(5/2 + I)*Sqrt[c + a^2*c*x^2]
*Hypergeometric2F1[-3/2 + I, 5/2 + I, 7/2 + I, (1 - I*a*x)/2])/(a*Sqrt[1 +
 a^2*x^2])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5599, 5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}dx$$

$$\Big\downarrow \text{5599}$$

$$\frac{c\sqrt{a^2cx^2+c}\int e^{2\arctan(ax)}\left(a^2x^2+1\right)^{3/2}dx}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{5596}$$

$$\frac{c\sqrt{a^2cx^2+c}\int (1-iax)^{\frac{3}{2}+i}(iax+1)^{\frac{3}{2}-i}dx}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{79}$$

$$\frac{\left(\frac{2}{29}+\frac{5i}{29}\right)2^{\frac{5}{2}-i}c(1-iax)^{\frac{5}{2}+i}\sqrt{a^2cx^2+c}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}+i,\frac{5}{2}+i,\frac{7}{2}+i,\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

input
```
Int[E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2),x]
```

output

```
((2/29 + (5*I)/29)*2^(5/2 - I)*c*(1 - I*a*x)^(5/2 + I)*Sqrt[c + a^2*c*x^2]
*Hypergeometric2F1[-3/2 + I, 5/2 + I, 7/2 + I, (1 - I*a*x)/2])/(a*Sqrt[1 +
 a^2*x^2])
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int e^{2\arctan(ax)}\left(a^2c\,x^2+c\right)^{\frac{3}{2}}dx$$

input
```
int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x)
```

Fricas [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int\left(a^2cx^2+c\right)^{\frac{3}{2}}e^{(2\arctan(ax))}\,dx$$

input
```
integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*e^(2*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}e^{2\operatorname{atan}(ax)}\,dx$$

input
```
integrate(exp(2*atan(a*x))*(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*exp(2*atan(a*x)), x)
```

Maxima [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int\left(a^2cx^2+c\right)^{\frac{3}{2}}e^{(2\arctan(ax))}\,dx$$

input
```
integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*e^(2*arctan(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int e^{2\operatorname{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(exp(2*atan(a*x))*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(exp(2*atan(a*x))*(c + a^2*c*x^2)^(3/2), x)
```

Reduce [F]

$$\int e^{2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\sqrt{c}\,c\left(\left(\int e^{2atan(ax)}\sqrt{a^2x^2+1}\,x^2dx\right)a^2+\int e^{2atan(ax)}\sqrt{a^2x^2+1}dx\right)$$

input
```
int(exp(2*atan(a*x))*(a^2*c*x^2+c)^(3/2),x)
```

output
```
sqrt(c)*c*(int(e**(2*atan(a*x))*sqrt(a**2*x**2 + 1)*x**2,x)*a**2 + int(e**
(2*atan(a*x))*sqrt(a**2*x**2 + 1),x))
```

3.282 $\int e^{2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int e^{2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \frac{\left(\frac{2}{13}+\frac{3i}{13}\right)2^{\frac{3}{2}-i}(1-iax)^{\frac{3}{2}+i}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}+i,\frac{3}{2}+i,\frac{5}{2}+i,\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

output
```
(2/13+3/13*I)*2^(3/2-I)*(1-I*a*x)^(3/2+I)*(a^2*c*x^2+c)^(1/2)*hypergeom([-
1/2+I, 3/2+I],[5/2+I],1/2-1/2*I*a*x)/a/(a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int e^{2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \frac{\left(\frac{2}{13}+\frac{3i}{13}\right)2^{\frac{3}{2}-i}(1-iax)^{\frac{3}{2}+i}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}+i,\frac{3}{2}+i,\frac{5}{2}+i,\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

input
```
Integrate[E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2],x]
```

output
```
((2/13 + (3*I)/13)*2^(3/2 - I)*(1 - I*a*x)^(3/2 + I)*Sqrt[c + a^2*c*x^2]*H
ypergeometric2F1[-1/2 + I, 3/2 + I, 5/2 + I, (1 - I*a*x)/2])/(a*Sqrt[1 + a
^2*x^2])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5599, 5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2\arctan(ax)}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5599}$$

$$\frac{\sqrt{a^2cx^2+c}\int e^{2\arctan(ax)}\sqrt{a^2x^2+1}dx}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{5596}$$

$$\frac{\sqrt{a^2cx^2+c}\int (1-iax)^{\frac{1}{2}+i}(iax+1)^{\frac{1}{2}-i}dx}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{79}$$

$$\frac{\left(\frac{2}{13}+\frac{3i}{13}\right)2^{\frac{3}{2}-i}(1-iax)^{\frac{3}{2}+i}\sqrt{a^2cx^2+c}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}+i,\frac{3}{2}+i,\frac{5}{2}+i,\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

input
```
Int[E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2],x]
```

output

```
((2/13 + (3*I)/13)*2^(3/2 - I)*(1 - I*a*x)^(3/2 + I)*Sqrt[c + a^2*c*x^2]*H
ypergeometric2F1[-1/2 + I, 3/2 + I, 5/2 + I, (1 - I*a*x)/2])/(a*Sqrt[1 + a
^2*x^2])
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \mathrm{e}^{2\arctan(ax)}\sqrt{a^{2}c\,x^{2}+c}dx$$

input
```
int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int e^{2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx=\int \sqrt{a^2cx^2+c}e^{(2\arctan(ax))}\,dx$$

input
```
integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*e^(2*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \int \sqrt{c\left(a^2x^2+1\right)}e^{2\operatorname{atan}(ax)}\,dx$$

input
```
integrate(exp(2*atan(a*x))*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*exp(2*atan(a*x)), x)
```

Maxima [F]

$$\int e^{2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \int \sqrt{a^2cx^2+c}e^{(2\arctan(ax))}\,dx$$

input
```
integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*e^(2*arctan(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(exp(2*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \int e^{2\,\mathrm{atan}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(exp(2*atan(a*x))*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(exp(2*atan(a*x))*(c + a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int e^{2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \sqrt{c}\left(\int e^{2atan(ax)}\sqrt{a^2x^2+1}dx\right)$$

input
```
int(exp(2*atan(a*x))*(a^2*c*x^2+c)^(1/2),x)
```

output
```
sqrt(c)*int(e**(2*atan(a*x))*sqrt(a**2*x**2 + 1),x)
```

3.283 $\int \frac{e^{2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{e^{2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\left(\frac{2}{5}+\frac{i}{5}\right)2^{\frac{1}{2}-i}(1-iax)^{\frac{1}{2}+i}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}+i,\frac{1}{2}+i,\frac{3}{2}+i,\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}$$

output
```
(2/5+1/5*I)*2^(1/2-I)*(1-I*a*x)^(1/2+I)*(a^2*x^2+1)^(1/2)*hypergeom([1/2+I
, 1/2+I],[3/2+I],1/2-1/2*I*a*x)/a/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\left(\frac{2}{5}+\frac{i}{5}\right)2^{\frac{1}{2}-i}(1-iax)^{\frac{1}{2}+i}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}+i,\frac{1}{2}+i,\frac{3}{2}+i,\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}$$

input
```
Integrate[E^(2*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
((2/5 + I/5)*2^(1/2 - I)*(1 - I*a*x)^(1/2 + I)*Sqrt[1 + a^2*x^2]*Hypergeom
etric2F1[1/2 + I, 1/2 + I, 3/2 + I, (1 - I*a*x)/2])/(a*Sqrt[c + a^2*c*x^2]
)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5599, 5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \ 5599$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{2\arctan(ax)}}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \ 5596$$

$$\frac{\sqrt{a^2x^2+1}\int (1-iax)^{-\frac{1}{2}+i}(iax+1)^{-\frac{1}{2}-i}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \ 79$$

$$\frac{\left(\frac{2}{5}+\frac{i}{5}\right)2^{\frac{1}{2}-i}(1-iax)^{\frac{1}{2}+i}\sqrt{a^2x^2+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2}+i,\frac{1}{2}+i,\frac{3}{2}+i,\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

input
```
Int[E^(2*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output

```
((2/5 + I/5)*2^(1/2 - I)*(1 - I*a*x)^(1/2 + I)*Sqrt[1 + a^2*x^2]*Hypergeom
etric2F1[1/2 + I, 1/2 + I, 3/2 + I, (1 - I*a*x)/2])/(a*Sqrt[c + a^2*c*x^2]
)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{\mathrm{e}^{2 \arctan(ax)}}{\sqrt{a^2 c\, x^2 + c}} dx$$

input
```
int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{2 \arctan(ax)}}{\sqrt{c + a^2 c x^2}} dx = \int \frac{e^{(2 \arctan(ax))}}{\sqrt{a^2 c x^2 + c}} dx$$

input
```
integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(e^(2*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{e^{2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{2\operatorname{atan}(ax)}}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(exp(2*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{e^{2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(2\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(e^(2*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{e^{2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(2\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(e^(2*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{2\,\mathrm{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(exp(2*atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(exp(2*atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{e^{2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{e^{2atan(ax)}}{\sqrt{a^2x^2+1}}dx}{\sqrt{c}}$$

input
```
int(exp(2*atan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(e**(2*atan(a*x))/sqrt(a**2*x**2 + 1),x)/sqrt(c)
```

3.284 $\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 37

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx=\frac{e^{2\arctan(ax)}(2+ax)}{5ac\sqrt{c+a^2cx^2}}$$

output
```
1/5*exp(2*arctan(a*x))*(a*x+2)/a/c/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx=\frac{e^{2\arctan(ax)}(2+ax)}{5ac\sqrt{c+a^2cx^2}}$$

input
```
Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(E^(2*ArcTan[a*x])*(2 + a*x))/(5*a*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {5592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5592}$$

$$\frac{(ax+2)e^{2\arctan(ax)}}{5ac\sqrt{a^2cx^2+c}}$$

input
```
Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(E^(2*ArcTan[a*x])*(2 + a*x))/(5*a*c*Sqrt[c + a^2*c*x^2])
```

Defintions of rubi rules used

rule 5592
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{(a^2x^2+1)(ax+2)e^{2\arctan(ax)}}{5a(a^2cx^2+c)^{\frac{3}{2}}}$	39
orering	$\frac{(a^2x^2+1)(ax+2)e^{2\arctan(ax)}}{5a(a^2cx^2+c)^{\frac{3}{2}}}$	39

input
```
int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*(a^2*x^2+1)*(a*x+2)*exp(2*arctan(a*x))/a/(a^2*c*x^2+c)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{a^2cx^2+c}(ax+2)e^{(2\arctan(ax))}}{5\,(a^3c^2x^2+ac^2)}$$

input
```
integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
1/5*sqrt(a^2*c*x^2 + c)*(a*x + 2)*e^(2*arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

Sympy [F]

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{e^{2\operatorname{atan}(ax)}}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(exp(2*atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{e^{(2\arctan(ax))}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(32) = 64$.

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.89

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{2\left(e^{(2\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^2 - e^{(2\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right) - e^{(2\arctan(ax))}\right)}{5\left(\sqrt{c}\tan\left(\frac{1}{2}\arctan(ax)\right)^2+\sqrt{c}\right)ac}$$

input
```
integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
-2/5*(e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^2 - e^(2*arctan(a*x))*tan(1/2
*arctan(a*x)) - e^(2*arctan(a*x)))/((sqrt(c)*tan(1/2*arctan(a*x))^2 + sqrt
(c))*a*c)
```

Mupad [B] (verification not implemented)

Time = 23.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\mathrm{e}^{2\,\mathrm{atan}(a\,x)}\,\left(\frac{x}{5\,c}+\frac{2}{5\,a\,c}\right)}{\sqrt{c\,a^2\,x^2+c}}$$

input
```
int(exp(2*atan(a*x))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
(exp(2*atan(a*x))*(x/(5*c) + 2/(5*a*c)))/(c + a^2*c*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{e^{2atan(ax)}\sqrt{c}\,\sqrt{a^2x^2+1}\,(ax+2)}{5a\,c^2\,(a^2x^2+1)}$$

input
```
int(exp(2*atan(a*x))/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(e**(2*atan(a*x))*sqrt(c)*sqrt(a**2*x**2 + 1)*(a*x + 2))/(5*a*c**2*(a**2*x
**2 + 1))
```

3.285 $\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx=\frac{e^{2\arctan(ax)}(2+3ax)}{13ac\,(c+a^2cx^2)^{3/2}}+\frac{6e^{2\arctan(ax)}(2+ax)}{65ac^2\sqrt{c+a^2cx^2}}$$

output
```
1/13*exp(2*arctan(a*x))*(3*a*x+2)/a/c/(a^2*c*x^2+c)^(3/2)+6/65*exp(2*arcta
n(a*x))*(a*x+2)/a/c^2/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx=\frac{e^{2\arctan(ax)}(22+21ax+12a^2x^2+6a^3x^3)}{65c^2\,(a+a^3x^2)\,\sqrt{c+a^2cx^2}}$$

input
```
Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(E^(2*ArcTan[a*x])*(22 + 21*a*x + 12*a^2*x^2 + 6*a^3*x^3))/(65*c^2*(a + a^
3*x^2)*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5593, 5592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5593}$$

$$\frac{6\int \frac{e^{2\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx}{13c}+\frac{(3ax+2)e^{2\arctan(ax)}}{13ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5592}$$

$$\frac{6(ax+2)e^{2\arctan(ax)}}{65ac^2\sqrt{a^2cx^2+c}}+\frac{(3ax+2)e^{2\arctan(ax)}}{13ac\,(a^2cx^2+c)^{3/2}}$$

input
```
Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(E^(2*ArcTan[a*x])*(2 + 3*a*x))/(13*a*c*(c + a^2*c*x^2)^(3/2)) + (6*E^(2*A
rcTan[a*x])*(2 + a*x))/(65*a*c^2*Sqrt[c + a^2*c*x^2])
```

Defintions of rubi rules used

rule 5592
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

rule 5593
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2)))   I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
&& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

method	result	size
gosper	$\frac{(a^2x^2+1)(6a^3x^3+12a^2x^2+21ax+22)\mathrm{e}^{2\arctan(ax)}}{65a(a^2c\,x^2+c)^{\frac{5}{2}}}$	56
orering	$\frac{(a^2x^2+1)(6a^3x^3+12a^2x^2+21ax+22)\mathrm{e}^{2\arctan(ax)}}{65a(a^2c\,x^2+c)^{\frac{5}{2}}}$	56

input
```
int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/65*(a^2*x^2+1)*(6*a^3*x^3+12*a^2*x^2+21*a*x+22)*exp(2*arctan(a*x))/a/(a^
2*c*x^2+c)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{(6\,a^3x^3+12\,a^2x^2+21\,ax+22)\sqrt{a^2cx^2+c}e^{(2\,\arctan(ax))}}{65\,(a^5c^3x^4+2\,a^3c^3x^2+ac^3)}$$

input
```
integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/65*(6*a^3*x^3 + 12*a^2*x^2 + 21*a*x + 22)*sqrt(a^2*c*x^2 + c)*e^(2*arcta
n(a*x))/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F]

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{e^{2\,\mathrm{atan}\,(ax)}}{(c\,(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(exp(2*atan(a*x))/(c*(a**2*x**2 + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{e^{(2\,\arctan(ax))}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(66) = 132.

Time = 0.14 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.24

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx =$$

$$-\frac{2\left(11\,e^{(2\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^6 - 21\,e^{(2\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^5 - 9\,e^{(2\arctan(ax))}\tan\left(\frac{1}{2}\mathrm{arc}\right.}{65\left(ac^{\frac{5}{2}}\tan\left(\frac{1}{2}\arctan(ax)\right)\right.}$$

input
```
integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
-2/65*(11*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^6 - 21*e^(2*arctan(a*x))*
tan(1/2*arctan(a*x))^5 - 9*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^4 + 18*e
^(2*arctan(a*x))*tan(1/2*arctan(a*x))^3 + 9*e^(2*arctan(a*x))*tan(1/2*arct
an(a*x))^2 - 21*e^(2*arctan(a*x))*tan(1/2*arctan(a*x)) - 11*e^(2*arctan(a*
x)))/(a*c^(5/2)*tan(1/2*arctan(a*x))^6 + 3*a*c^(5/2)*tan(1/2*arctan(a*x))^
4 + 3*a*c^(5/2)*tan(1/2*arctan(a*x))^2 + a*c^(5/2))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{e^{2\,\mathrm{atan}(a\,x)}\left(\frac{22}{65\,a^3\,c^2}+\frac{6\,x^3}{65\,c^2}+\frac{21\,x}{65\,a^2\,c^2}+\frac{12\,x^2}{65\,a\,c^2}\right)}{\frac{\sqrt{c\,a^2\,x^2+c}}{a^2}+x^2\,\sqrt{c\,a^2\,x^2+c}}$$

input
```
int(exp(2*atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
(exp(2*atan(a*x))*(22/(65*a^3*c^2) + (6*x^3)/(65*c^2) + (21*x)/(65*a^2*c^2
) + (12*x^2)/(65*a*c^2)))/((c + a^2*c*x^2)^(1/2)/a^2 + x^2*(c + a^2*c*x^2)
^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{e^{2atan(ax)}\sqrt{c}\,\sqrt{a^2x^2+1}\,(6a^3x^3+12a^2x^2+21ax+22)}{65a\,c^3\,(a^4x^4+2a^2x^2+1)}$$

input
```
int(exp(2*atan(a*x))/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(e**(2*atan(a*x))*sqrt(c)*sqrt(a**2*x**2 + 1)*(6*a**3*x**3 + 12*a**2*x**2
+ 21*a*x + 22))/(65*a*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

3.286 $\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 114

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \frac{e^{2\arctan(ax)}(2+5ax)}{29ac\,(c+a^2cx^2)^{5/2}} + \frac{20e^{2\arctan(ax)}(2+3ax)}{377ac^2\,(c+a^2cx^2)^{3/2}} + \frac{24e^{2\arctan(ax)}(2+ax)}{377ac^3\sqrt{c+a^2cx^2}}$$

output
```
1/29*exp(2*arctan(a*x))*(5*a*x+2)/a/c/(a^2*c*x^2+c)^(5/2)+20/377*exp(2*arc
tan(a*x))*(3*a*x+2)/a/c^2/(a^2*c*x^2+c)^(3/2)+24/377*exp(2*arctan(a*x))*(a
*x+2)/a/c^3/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.71

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \frac{e^{2\arctan(ax)}(114+149ax+136a^2x^2+108a^3x^3+48a^4x^4+24a^5x^5)}{377ac^3\,(1+a^2x^2)^2\,\sqrt{c+a^2cx^2}}$$

input
```
Integrate[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^(7/2),x]
```

output
```
(E^(2*ArcTan[a*x])*(114 + 149*a*x + 136*a^2*x^2 + 108*a^3*x^3 + 48*a^4*x^4
 + 24*a^5*x^5))/(377*a*c^3*(1 + a^2*x^2)^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5593, 5593, 5592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2\arctan(ax)}}{(a^2cx^2+c)^{7/2}}dx$$

$$\Big\downarrow \text{5593}$$

$$\frac{20\int \frac{e^{2\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx}{29c}+\frac{(5ax+2)e^{2\arctan(ax)}}{29ac\,(a^2cx^2+c)^{5/2}}$$

$$\Big\downarrow \text{5593}$$

$$\frac{20\left(\frac{6\int \frac{e^{2\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx}{13c}+\frac{(3ax+2)e^{2\arctan(ax)}}{13ac(a^2cx^2+c)^{3/2}}\right)}{29c}+\frac{(5ax+2)e^{2\arctan(ax)}}{29ac\,(a^2cx^2+c)^{5/2}}$$

$$\Big\downarrow \text{5592}$$

$$\frac{20\left(\frac{6(ax+2)e^{2\arctan(ax)}}{65ac^2\sqrt{a^2cx^2+c}}+\frac{(3ax+2)e^{2\arctan(ax)}}{13ac(a^2cx^2+c)^{3/2}}\right)}{29c}+\frac{(5ax+2)e^{2\arctan(ax)}}{29ac\,(a^2cx^2+c)^{5/2}}$$

input
```
Int[E^(2*ArcTan[a*x])/(c + a^2*c*x^2)^(7/2),x]
```

output
```
(E^(2*ArcTan[a*x])*(2 + 5*a*x))/(29*a*c*(c + a^2*c*x^2)^(5/2)) + (20*((E^(
2*ArcTan[a*x])*(2 + 3*a*x))/(13*a*c*(c + a^2*c*x^2)^(3/2)) + (6*E^(2*ArcTa
n[a*x])*(2 + a*x))/(65*a*c^2*Sqrt[c + a^2*c*x^2])))/(29*c)
```

Defintions of rubi rules used

rule 5592
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

rule 5593
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2)))   I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
&& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.63

method	result	size
gosper	$\frac{\left(a^{2} x^{2}+1\right) \left(24 a^{5} x^{5}+48 a^{4} x^{4}+108 a^{3} x^{3}+136 a^{2} x^{2}+149 a x +114\right) {\mathrm e}^{2 \arctan \left(a x \right)}}{377 a \left(a^{2} c \,x^{2}+c \right)^{\frac{7}{2}}}$	72
orering	$\frac{\left(a^{2} x^{2}+1\right) \left(24 a^{5} x^{5}+48 a^{4} x^{4}+108 a^{3} x^{3}+136 a^{2} x^{2}+149 a x +114\right) {\mathrm e}^{2 \arctan \left(a x \right)}}{377 a \left(a^{2} c \,x^{2}+c \right)^{\frac{7}{2}}}$	72

input
```
int(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/377*(a^2*x^2+1)*(24*a^5*x^5+48*a^4*x^4+108*a^3*x^3+136*a^2*x^2+149*a*x+1
14)*exp(2*arctan(a*x))/a/(a^2*c*x^2+c)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.87

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \frac{(24\,a^5x^5+48\,a^4x^4+108\,a^3x^3+136\,a^2x^2+149\,ax+114)\sqrt{a^2cx^2+c}\,e^{(2\arctan(ax))}}{377\,(a^7c^4x^6+3\,a^5c^4x^4+3\,a^3c^4x^2+ac^4)}$$

input
```
integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
1/377*(24*a^5*x^5 + 48*a^4*x^4 + 108*a^3*x^3 + 136*a^2*x^2 + 149*a*x + 114
)*sqrt(a^2*c*x^2 + c)*e^(2*arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a
^3*c^4*x^2 + a*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(exp(2*atan(a*x))/(a**2*c*x**2+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{e^{(2\arctan(ax))}}{(a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(e^(2*arctan(a*x))/(a^2*c*x^2 + c)^(7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(99) = 198$.

Time = 0.16 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.39

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx =$$
$$-\frac{2\left(57\,e^{(2\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^{10}-149\,e^{(2\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^{9}-13\,e^{(2\arctan(ax))}\tan\left(\frac{1}{2}\right.\right.}{}$$

input
```
integrate(exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
-2/377*(57*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^10 - 149*e^(2*arctan(a*x
))*tan(1/2*arctan(a*x))^9 - 13*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^8 +
164*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^7 + 138*e^(2*arctan(a*x))*tan(1
/2*arctan(a*x))^6 - 414*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^5 - 138*e^(
2*arctan(a*x))*tan(1/2*arctan(a*x))^4 + 164*e^(2*arctan(a*x))*tan(1/2*arct
an(a*x))^3 + 13*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^2 - 149*e^(2*arctan
(a*x))*tan(1/2*arctan(a*x)) - 57*e^(2*arctan(a*x)))/(a*c^(7/2)*tan(1/2*arc
tan(a*x))^10 + 5*a*c^(7/2)*tan(1/2*arctan(a*x))^8 + 10*a*c^(7/2)*tan(1/2*a
rctan(a*x))^6 + 10*a*c^(7/2)*tan(1/2*arctan(a*x))^4 + 5*a*c^(7/2)*tan(1/2*
arctan(a*x))^2 + a*c^(7/2))
```

Mupad [B] (verification not implemented)

Time = 22.95 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.07

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \frac{e^{2\,\mathrm{atan}(a\,x)}\left(\frac{114}{377\,a^5\,c^3}+\frac{24\,x^5}{377\,c^3}+\frac{149\,x}{377\,a^4\,c^3}+\frac{48\,x^4}{377\,a\,c^3}+\frac{108\,x^3}{377\,a^2\,c^3}+\frac{136\,x^2}{377\,a^3\,c^3}\right)}{\frac{\sqrt{c\,a^2\,x^2+c}}{a^4}+x^4\,\sqrt{c\,a^2\,x^2+c}+\frac{2\,x^2\,\sqrt{c\,a^2\,x^2+c}}{a^2}}$$

input
```
int(exp(2*atan(a*x))/(c + a^2*c*x^2)^(7/2),x)
```

output
```
(exp(2*atan(a*x))*(114/(377*a^5*c^3) + (24*x^5)/(377*c^3) + (149*x)/(377*a
^4*c^3) + (48*x^4)/(377*a*c^3) + (108*x^3)/(377*a^2*c^3) + (136*x^2)/(377*
a^3*c^3)))/((c + a^2*c*x^2)^(1/2)/a^4 + x^4*(c + a^2*c*x^2)^(1/2) + (2*x^2
*(c + a^2*c*x^2)^(1/2))/a^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.82

$$\int \frac{e^{2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \frac{e^{2atan(ax)}\sqrt{c}\,\sqrt{a^2x^2+1}\,(24a^5x^5+48a^4x^4+108a^3x^3+136a^2x^2+149ax+114)}{377a\,c^4\,(a^6x^6+3a^4x^4+3a^2x^2+1)}$$

input
```
int(exp(2*atan(a*x))/(a^2*c*x^2+c)^(7/2),x)
```

output
```
(e**(2*atan(a*x))*sqrt(c)*sqrt(a**2*x**2 + 1)*(24*a**5*x**5 + 48*a**4*x**4
 + 108*a**3*x**3 + 136*a**2*x**2 + 149*a*x + 114))/(377*a*c**4*(a**6*x**6
+ 3*a**4*x**4 + 3*a**2*x**2 + 1))
```

3.287 $\int e^{-\arctan(ax)}(c+a^2cx^2)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 101

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx = \frac{2^{\left(1+\frac{i}{2}\right)+p}(1-iax)^{\left(1-\frac{i}{2}\right)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p \text{Hypergeometric2F1}\left(-\frac{i}{2}-p,\left(1-\frac{i}{2}\right)+p,\left(2-\frac{i}{2}\right)+\right.}{a((-1-2i)-2ip)}$$

output
```
2^(1+1/2*I+p)*(1-I*a*x)^(1-1/2*I+p)*(a^2*c*x^2+c)^p*hypergeom([-1/2*I-p, 1
-1/2*I+p],[2-1/2*I+p],1/2-1/2*I*a*x)/a/(-1-2*I-2*I*p)/((a^2*x^2+1)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx = \frac{i2^{\frac{i}{2}+p}(1-iax)^{\left(1-\frac{i}{2}\right)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p \text{Hypergeometric2F1}\left(-\frac{i}{2}-p,\left(1-\frac{i}{2}\right)+p,\left(2-\frac{i}{2}\right)+p,\right.}{a\left(\left(1-\frac{i}{2}\right)+p\right)}$$

input
```
Integrate[(c + a^2*c*x^2)^p/E^ArcTan[a*x],x]
```

output
```
(I*2^(I/2 + p)*(1 - I*a*x)^((1 - I/2) + p)*(c + a^2*c*x^2)^p*Hypergeometri
c2F1[-1/2*I - p, (1 - I/2) + p, (2 - I/2) + p, (1 - I*a*x)/2])/(a*((1 - I/
2) + p)*(1 + a^2*x^2)^p)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5599, 5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\arctan(ax)}\left(a^2cx^2+c\right)^p\,dx$$

$$\Big\downarrow \text{5599}$$

$$\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\int e^{-\arctan(ax)}\left(a^2x^2+1\right)^p dx$$

$$\Big\downarrow \text{5596}$$

$$\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\int (1-iax)^{p-\frac{i}{2}}(iax+1)^{p+\frac{i}{2}}dx$$

$$\Big\downarrow \text{79}$$

$$\frac{2^{p+\left(1+\frac{i}{2}\right)}(1-iax)^{p+\left(1-\frac{i}{2}\right)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\text{Hypergeometric2F1}\left(-p-\frac{i}{2},p+\left(1-\frac{i}{2}\right),p+\left(2-\frac{i}{2}\right),\frac{1}{2}(\right.}{a(-2ip-(1+2i))}$$

input
```
Int[(c + a^2*c*x^2)^p/E^ArcTan[a*x],x]
```

output
```
(2^((1 + I/2) + p)*(1 - I*a*x)^((1 - I/2) + p)*(c + a^2*c*x^2)^p*Hypergeom
etric2F1[-1/2*I - p, (1 - I/2) + p, (2 - I/2) + p, (1 - I*a*x)/2])/(a*((-1
 - 2*I) - (2*I)*p)*(1 + a^2*x^2)^p)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \left(a^{2} c\,x^{2}+c \right)^{p} {\mathrm e}^{-\arctan \left(a x \right)}d x$$

input
```
int((a^2*c*x^2+c)^p/exp(arctan(a*x)),x)
```

output
```
int((a^2*c*x^2+c)^p/exp(arctan(a*x)),x)
```

Fricas [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int \left(a^{2} c x^{2} + c\right)^{p} e^{\left(-\arctan\left(a x\right)\right)}\,{d x}$$

input
```
integrate((a^2*c*x^2+c)^p/exp(arctan(a*x)),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^p*e^(-arctan(a*x)), x)
```

Sympy [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int\left(c\left(a^2x^2+1\right)\right)^pe^{-\operatorname{atan}(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**p/exp(atan(a*x)),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**p*exp(-atan(a*x)), x)
```

Maxima [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int\left(a^2cx^2+c\right)^pe^{(-\arctan(ax))}\,dx$$

input
```
integrate((a^2*c*x^2+c)^p/exp(arctan(a*x)),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^p*e^(-arctan(a*x)), x)
```

Giac [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int\left(a^2cx^2+c\right)^pe^{(-\arctan(ax))}\,dx$$

input
```
integrate((a^2*c*x^2+c)^p/exp(arctan(a*x)),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^p*e^(-arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int \mathrm{e}^{-\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^p\,dx$$

input
```
int(exp(-atan(a*x))*(c + a^2*c*x^2)^p,x)
```

output
```
int(exp(-atan(a*x))*(c + a^2*c*x^2)^p, x)
```

Reduce [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int \frac{(a^2c\,x^2+c)^p}{e^{atan(ax)}}dx$$

input
```
int((a^2*c*x^2+c)^p/exp(atan(a*x)),x)
```

output
```
int((a**2*c*x**2 + c)**p/e**atan(a*x),x)
```

3.288 $\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx = -\frac{\left(\frac{1}{37}-\frac{6i}{37}\right)2^{3+\frac{i}{2}}c^2(1-iax)^{3-\frac{i}{2}}\,\text{Hypergeometric2F1}\left(-2-\frac{i}{2},3-\frac{i}{2},4-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

output
```
(-1/37+6/37*I)*2^(3+1/2*I)*c^2*(1-I*a*x)^(3-1/2*I)*hypergeom([-2-1/2*I, 3-
1/2*I],[4-1/2*I],1/2-1/2*I*a*x)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx = -\frac{\left(\frac{1}{37}-\frac{6i}{37}\right)2^{3+\frac{i}{2}}c^2(1-iax)^{3-\frac{i}{2}}\,\text{Hypergeometric2F1}\left(-2-\frac{i}{2},3-\frac{i}{2},4-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

input
```
Integrate[(c + a^2*c*x^2)^2/E^ArcTan[a*x],x]
```

output
```
((-1/37 + (6*I)/37)*2^(3 + I/2)*c^2*(1 - I*a*x)^(3 - I/2)*Hypergeometric2F
1[-2 - I/2, 3 - I/2, 4 - I/2, (1 - I*a*x)/2])/a
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\arctan(ax)}\left(a^2cx^2+c\right)^2\,dx$$

$$\Big\downarrow \text{5596}$$

$$c^2\int (1-iax)^{2-\frac{i}{2}}(iax+1)^{2+\frac{i}{2}}dx$$

$$\Big\downarrow \text{79}$$

$$-\frac{\left(\frac{1}{37}-\frac{6i}{37}\right)2^{3+\frac{i}{2}}c^2(1-iax)^{3-\frac{i}{2}}\text{Hypergeometric2F1}\left(-2-\frac{i}{2},3-\frac{i}{2},4-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

input
```
Int[(c + a^2*c*x^2)^2/E^ArcTan[a*x],x]
```

output

```
((-1/37 + (6*I)/37)*2^(3 + I/2)*c^2*(1 - I*a*x)^(3 - I/2)*Hypergeometric2F
1[-2 - I/2, 3 - I/2, 4 - I/2, (1 - I*a*x)/2])/a
```


Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \left(a^{2} c \,x^{2}+c \right)^{2} {\mathrm e}^{-\arctan \left(a x \right)}d x$$

input
```
int((a^2*c*x^2+c)^2/exp(arctan(a*x)),x)
```

output
```
int((a^2*c*x^2+c)^2/exp(arctan(a*x)),x)
```

Fricas [F]

$$\int e^{-\arctan(ax)} \left(c+a^2cx^2\right)^2 \, dx = \int \left(a^{2} c x^{2} + c\right)^{2} e^{\left(-\arctan\left(a x\right)\right)} \,d x$$

input
```
integrate((a^2*c*x^2+c)^2/exp(arctan(a*x)),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*e^(-arctan(a*x)), x)
```

Sympy [F]

$$\int e^{-\arctan(ax)} \left(c+a^2cx^2\right)^2 \, dx = c^{2} \left(\int 2 a^{2} x^{2} e^{- \operatorname{atan}{\left(a x \right)}}\, dx + \int a^{4} x^{4} e^{- \operatorname{atan}{\left(a x \right)}}\, dx + \int e^{- \operatorname{atan}{\left(a x \right)}}\, dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/exp(atan(a*x)),x)
```

output
```
c**2*(Integral(2*a**2*x**2*exp(-atan(a*x)), x) + Integral(a**4*x**4*exp(-a
tan(a*x)), x) + Integral(exp(-atan(a*x)), x))
```

Maxima [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int\left(a^2cx^2+c\right)^2e^{(-\arctan(ax))}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/exp(arctan(a*x)),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^2*e^(-arctan(a*x)), x)
```

Giac [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int\left(a^2cx^2+c\right)^2e^{(-\arctan(ax))}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/exp(arctan(a*x)),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*e^(-arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int e^{-\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

input
```
int(exp(-atan(a*x))*(c + a^2*c*x^2)^2,x)
```

output
```
int(exp(-atan(a*x))*(c + a^2*c*x^2)^2, x)
```

Reduce [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx = c^2\left(\left(\int \frac{x^4}{e^{atan(ax)}}dx\right)a^4+2\left(\int \frac{x^2}{e^{atan(ax)}}dx\right)a^2+\int \frac{1}{e^{atan(ax)}}dx\right)$$

input
```
int((a^2*c*x^2+c)^2/exp(atan(a*x)),x)
```

output
```
c**2*(int(x**4/e**atan(a*x),x)*a**4 + 2*int(x**2/e**atan(a*x),x)*a**2 + in
t(1/e**atan(a*x),x))
```

3.289 $\int e^{-\arctan(ax)}(c+a^2cx^2)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 61

$$\int e^{-\arctan(ax)}(c+a^2cx^2)\,dx = -\frac{\left(\frac{1}{17}-\frac{4i}{17}\right)2^{2+\frac{i}{2}}c(1-iax)^{2-\frac{i}{2}}\,\text{Hypergeometric2F1}\left(-1-\frac{i}{2},2-\frac{i}{2},3-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

output
```
(-1/17+4/17*I)*2^(2+1/2*I)*c*(1-I*a*x)^(2-1/2*I)*hypergeom([-1-1/2*I, 2-1/
2*I],[3-1/2*I],1/2-1/2*I*a*x)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int e^{-\arctan(ax)}(c+a^2cx^2)\,dx = -\frac{\left(\frac{1}{17}-\frac{4i}{17}\right)2^{2+\frac{i}{2}}c(1-iax)^{2-\frac{i}{2}}\,\text{Hypergeometric2F1}\left(-1-\frac{i}{2},2-\frac{i}{2},3-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

input
```
Integrate[(c + a^2*c*x^2)/E^ArcTan[a*x],x]
```

output
```
((-1/17 + (4*I)/17)*2^(2 + I/2)*c*(1 - I*a*x)^(2 - I/2)*Hypergeometric2F1[
-1 - I/2, 2 - I/2, 3 - I/2, (1 - I*a*x)/2])/a
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{5596, 79\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\arctan(ax)}\left(a^2cx^2+c\right)\,dx$$

$$\Big\downarrow \text{5596}$$

$$c\int (1-iax)^{1-\frac{i}{2}}(iax+1)^{1+\frac{i}{2}}dx$$

$$\Big\downarrow \text{79}$$

$$-\frac{\left(\frac{1}{17}-\frac{4i}{17}\right)2^{2+\frac{i}{2}}c(1-iax)^{2-\frac{i}{2}}\text{Hypergeometric2F1}\left(-1-\frac{i}{2},2-\frac{i}{2},3-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

input
```
Int[(c + a^2*c*x^2)/E^ArcTan[a*x],x]
```

output

```
((-1/17 + (4*I)/17)*2^(2 + I/2)*c*(1 - I*a*x)^(2 - I/2)*Hypergeometric2F1[
-1 - I/2, 2 - I/2, 3 - I/2, (1 - I*a*x)/2])/a
```


Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \left(a^{2} c \,x^{2}+c \right) {\mathrm e}^{-\arctan \left(a x \right)}d x$$

input
```
int((a^2*c*x^2+c)/exp(arctan(a*x)),x)
```

output
```
int((a^2*c*x^2+c)/exp(arctan(a*x)),x)
```

Fricas [F]

$$\int e^{-\arctan (a x)} \left(c+a^2 c x^2\right) \, dx=\int { {\left(a^{2} c x^{2} + c\right)} e^{\left(-\arctan \left(a x\right)\right)} \,d x }$$

input
```
integrate((a^2*c*x^2+c)/exp(arctan(a*x)),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*e^(-arctan(a*x)), x)
```

Sympy [F]

$$\int e^{-\arctan (a x)} \left(c+a^2 c x^2\right) \, dx=c \left(\int a^{2} x^{2} e^{- \operatorname{atan}{\left(a x \right)}}\, dx + \int e^{- \operatorname{atan}{\left(a x \right)}}\, dx\right)$$

input
```
integrate((a**2*c*x**2+c)/exp(atan(a*x)),x)
```

output
```
c*(Integral(a**2*x**2*exp(-atan(a*x)), x) + Integral(exp(-atan(a*x)), x))
```

Maxima [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int \left(a^2cx^2+c\right)e^{(-\arctan(ax))}\,dx$$

input
```
integrate((a^2*c*x^2+c)/exp(arctan(a*x)),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)*e^(-arctan(a*x)), x)
```

Giac [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int \left(a^2cx^2+c\right)e^{(-\arctan(ax))}\,dx$$

input
```
integrate((a^2*c*x^2+c)/exp(arctan(a*x)),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*e^(-arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int \mathrm{e}^{-\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(exp(-atan(a*x))*(c + a^2*c*x^2),x)
```

output
```
int(exp(-atan(a*x))*(c + a^2*c*x^2), x)
```

Reduce [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)\,dx = c\left(\left(\int \frac{x^2}{e^{atan(ax)}}dx\right)a^2+\int \frac{1}{e^{atan(ax)}}dx\right)$$

input
```
int((a^2*c*x^2+c)/exp(atan(a*x)),x)
```

output
```
c*(int(x**2/e**atan(a*x),x)*a**2 + int(1/e**atan(a*x),x))
```

3.290 $\int e^{-\arctan(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 60

$$\int e^{-\arctan(ax)}\,dx = -\frac{\left(\frac{1}{5}-\frac{2i}{5}\right)2^{1+\frac{i}{2}}(1-iax)^{1-\frac{i}{2}}\,\text{Hypergeometric2F1}\left(-\frac{i}{2},1-\frac{i}{2},2-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

output
```
(-1/5+2/5*I)*2^(1+1/2*I)*(1-I*a*x)^(1-1/2*I)*hypergeom([-1/2*I, 1-1/2*I],[
2-1/2*I],1/2-1/2*I*a*x)/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

$$\int e^{-\arctan(ax)}\,dx = -\frac{\left(\frac{4}{5}+\frac{8i}{5}\right)e^{(-1+2i)\arctan(ax)}\,\text{Hypergeometric2F1}\left(1+\frac{i}{2},2,2+\frac{i}{2},-e^{2i\arctan(ax)}\right)}{a}$$

input
```
Integrate[E^(-ArcTan[a*x]),x]
```

output
```
((-4/5 - (8*I)/5)*Hypergeometric2F1[1 + I/2, 2, 2 + I/2, -E^((2*I)*ArcTan[
a*x])])/(a*E^((1 - 2*I)*ArcTan[a*x]))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5584, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\arctan(ax)}dx$$

$$\Big\downarrow \text{ 5584}$$

$$\int (1-iax)^{-\frac{i}{2}}(1+iax)^{\frac{i}{2}}dx$$

$$\Big\downarrow \text{ 79}$$

$$-\frac{\left(\frac{1}{5}-\frac{2i}{5}\right)2^{1+\frac{i}{2}}(1-iax)^{1-\frac{i}{2}}\text{Hypergeometric2F1}\left(-\frac{i}{2},1-\frac{i}{2},2-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a}$$

input
```
Int[E^(-ArcTan[a*x]),x]
```

output
```
((-1/5 + (2*I)/5)*2^(1 + I/2)*(1 - I*a*x)^(1 - I/2)*Hypergeometric2F1[-1/2
*I, 1 - I/2, 2 - I/2, (1 - I*a*x)/2])/a
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5584
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Maple [F]

$$\int \mathrm{e}^{-\arctan(ax)}dx$$

input
```
int(exp(-arctan(a*x)),x)
```

output
```
int(exp(-arctan(a*x)),x)
```

Fricas [F]

$$\int e^{-\arctan(ax)}\,dx = \int e^{(-\arctan(ax))}\,dx$$

input
```
integrate(exp(-arctan(a*x)),x, algorithm="fricas")
```

output
```
integral(e^(-arctan(a*x)), x)
```

Sympy [F]

$$\int e^{-\arctan(ax)}\,dx = \int e^{-\operatorname{atan}(ax)}\,dx$$

input
```
integrate(exp(-atan(a*x)),x)
```

output
```
Integral(exp(-atan(a*x)), x)
```

Maxima [F]

$$\int e^{-\arctan(ax)}\,dx = \int e^{(-\arctan(ax))}\,dx$$

input
```
integrate(exp(-arctan(a*x)),x, algorithm="maxima")
```

output
```
integrate(e^(-arctan(a*x)), x)
```

Giac [F]

$$\int e^{-\arctan(ax)}\,dx = \int e^{(-\arctan(ax))}\,dx$$

input
```
integrate(exp(-arctan(a*x)),x, algorithm="giac")
```

output
```
integrate(e^(-arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\arctan(ax)}\,dx = \int \mathrm{e}^{-\mathrm{atan}(a\,x)}\,dx$$

input
```
int(exp(-atan(a*x)),x)
```

output
```
int(exp(-atan(a*x)), x)
```

Reduce [F]

$$\int e^{-\arctan(ax)}\,dx = \int \frac{1}{e^{atan(ax)}}dx$$

input
```
int(exp(-atan(a*x)),x)
```

output
```
int(1/e**atan(a*x),x)
```

3.291 $\int \frac{e^{-\arctan(ax)}}{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 16

$$\int \frac{e^{-\arctan(ax)}}{c+a^2cx^2}\,dx=-\frac{e^{-\arctan(ax)}}{ac}$$

output
```
-1/a/c/exp(arctan(a*x))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \frac{e^{-\arctan(ax)}}{c+a^2cx^2}\,dx=-\frac{(1-iax)^{-\frac{i}{2}}(1+iax)^{\frac{i}{2}}}{ac}$$

input
```
Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)),x]
```

output
```
-((1 + I*a*x)^(I/2)/(a*c*(1 - I*a*x)^(I/2)))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5594}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\arctan(ax)}}{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5594}$$

$$-\frac{e^{-\arctan(ax)}}{ac}$$

input
```
Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)),x]
```

output
```
-(1/(a*c*E^ArcTan[a*x]))
```

Defintions of rubi rules used

rule 5594
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{\mathrm{e}^{-\arctan(ax)}}{ac}$	16
parallelrisch	$-\frac{\mathrm{e}^{-\arctan(ax)}}{ac}$	16
risch	$-\frac{(-iax+1)^{-\frac{i}{2}}(iax+1)^{\frac{i}{2}}}{ac}$	33
orering	$-\frac{(a^2x^2+1)\mathrm{e}^{-\arctan(ax)}}{a(a^2c\,x^2+c)}$	34

input
```
int(1/exp(arctan(a*x))/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/a/c/exp(arctan(a*x))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{e^{-\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{e^{(-\arctan(ax))}}{ac}$$

input
```
integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
-e^(-arctan(a*x))/(a*c)
```

Sympy [A] (verification not implemented)

Time = 3.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{e^{-\arctan(ax)}}{c+a^2cx^2}\,dx = \begin{cases} -\frac{e^{-\operatorname{atan}(ax)}}{ac} & \text{for } a \neq 0 \\ \frac{x}{c} & \text{otherwise} \end{cases}$$

input
```
integrate(1/exp(atan(a*x))/(a**2*c*x**2+c),x)
```

output
```
Piecewise((-exp(-atan(a*x))/(a*c), Ne(a, 0)), (x/c, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{e^{-\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{2\,e^{(-\arctan(ax))}}{a^3cx^2+ac}$$

input
```
integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
-2*e^(-arctan(a*x))/(a^3*c*x^2 + a*c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{e^{-\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{e^{(-\arctan(ax))}}{ac}$$

input
```
integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
-e^(-arctan(a*x))/(a*c)
```

Mupad [B] (verification not implemented)

Time = 22.88 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{e^{-\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{\mathrm{e}^{-\mathrm{atan}(a\,x)}}{a\,c}$$

input
```
int(exp(-atan(a*x))/(c + a^2*c*x^2),x)
```

output
```
-exp(-atan(a*x))/(a*c)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{e^{-\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{1}{e^{atan(ax)}ac}$$

input
```
int(1/exp(atan(a*x))/(a^2*c*x^2+c),x)
```

output
```
( - 1)/(e**atan(a*x)*a*c)
```

3.292 $\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = -\frac{2e^{-\arctan(ax)}}{5ac^2} - \frac{e^{-\arctan(ax)}(1-2ax)}{5ac^2(1+a^2x^2)}$$

output
```
-2/5/a/c^2/exp(arctan(a*x))-1/5*(-2*a*x+1)/a/c^2/exp(arctan(a*x))/(a^2*x^2
+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = -\frac{(1-iax)^{-\frac{i}{2}}(1+iax)^{\frac{i}{2}}(3-2ax+2a^2x^2)}{5c^2(a+a^3x^2)}$$

input
```
Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^2),x]
```

output
```
-1/5*((1 + I*a*x)^(I/2)*(3 - 2*a*x + 2*a^2*x^2))/(c^2*(1 - I*a*x)^(I/2)*(a
 + a^3*x^2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5593, 27, 5594}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\arctan(ax)}}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5593}$$

$$\frac{2\int \frac{e^{-\arctan(ax)}}{c(a^2x^2+1)}dx}{5c}-\frac{(1-2ax)e^{-\arctan(ax)}}{5ac^2(a^2x^2+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{e^{-\arctan(ax)}}{a^2x^2+1}dx}{5c^2}-\frac{(1-2ax)e^{-\arctan(ax)}}{5ac^2(a^2x^2+1)}$$

$$\Big\downarrow \text{5594}$$

$$-\frac{(1-2ax)e^{-\arctan(ax)}}{5ac^2(a^2x^2+1)}-\frac{2e^{-\arctan(ax)}}{5ac^2}$$

input
```
Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^2),x]
```

output
```
-2/(5*a*c^2*E^ArcTan[a*x]) - (1 - 2*a*x)/(5*a*c^2*E^ArcTan[a*x]*(1 + a^2*x
^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5593
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2)))   I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
&& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
 0] && IntegerQ[2*p]
```

rule 5594
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Maple [A] (verified)

Time = 3.54 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

method	result	size
gosper	$-\frac{\left(2 a^{2} x^{2}-2 a x +3\right) {\mathrm e}^{-\arctan \left(a x \right)}}{5 \left(a^{2} x^{2}+1\right) c^{2} a}$	41
parallelrisch	$\frac{\left(-2 a^{2} x^{2}+2 a x -3\right) {\mathrm e}^{-\arctan \left(a x \right)}}{5 c^{2} \left(a^{2} x^{2}+1\right) a}$	41
orering	$-\frac{\left(2 a^{2} x^{2}-2 a x +3\right) \left(a^{2} x^{2}+1\right) {\mathrm e}^{-\arctan \left(a x \right)}}{5 a \left(a^{2} c \,x^{2}+c \right)^{2}}$	48

input
```
int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(2*a^2*x^2-2*a*x+3)/(a^2*x^2+1)/c^2/exp(arctan(a*x))/a
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = -\frac{(2\,a^2x^2-2\,ax+3)e^{(-\arctan(ax))}}{5\,(a^3c^2x^2+ac^2)}$$

input
```
integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/5*(2*a^2*x^2 - 2*a*x + 3)*e^(-arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(46) = 92.

Time = 19.75 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.15

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \begin{cases} -\frac{2a^2x^2}{5a^3c^2x^2e^{\operatorname{atan}(ax)}+5ac^2e^{\operatorname{atan}(ax)}} + \frac{2ax}{5a^3c^2x^2e^{\operatorname{atan}(ax)}+5ac^2e^{\operatorname{atan}(ax)}} - \frac{3}{5a^3c^2x^2e^{\operatorname{atan}(ax)}+5ac^2e^{\operatorname{atan}(ax)}} & \text{for } a \neq 0 \\ \frac{x}{c^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**2,x)
```

output
```
Piecewise((-2*a**2*x**2/(5*a**3*c**2*x**2*exp(atan(a*x)) + 5*a*c**2*exp(at
an(a*x))) + 2*a*x/(5*a**3*c**2*x**2*exp(atan(a*x)) + 5*a*c**2*exp(atan(a*x
))) - 3/(5*a**3*c**2*x**2*exp(atan(a*x)) + 5*a*c**2*exp(atan(a*x))), Ne(a,
 0)), (x/c**2, True))
```

Maxima [F]

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{e^{(-\arctan(ax))}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(48) = 96$.

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.81

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = -\frac{3\tan\left(\frac{1}{2}\arctan(ax)\right)^4 + 4\tan\left(\frac{1}{2}\arctan(ax)\right)^3 + 2\tan\left(\frac{1}{2}\arctan(ax)\right)^2 - 4\tan\left(\frac{1}{2}\arctan(ax)\right) + 3}{5\left(ac^2e^{(\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^4 + 2\,ac^2e^{(\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^2 + ac^2e^{(\arctan(ax))}\right)}$$

input
```
integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
-1/5*(3*tan(1/2*arctan(a*x))^4 + 4*tan(1/2*arctan(a*x))^3 + 2*tan(1/2*arct
an(a*x))^2 - 4*tan(1/2*arctan(a*x)) + 3)/(a*c^2*e^(arctan(a*x))*tan(1/2*ar
ctan(a*x))^4 + 2*a*c^2*e^(arctan(a*x))*tan(1/2*arctan(a*x))^2 + a*c^2*e^(a
rctan(a*x)))
```

Mupad [B] (verification not implemented)

Time = 23.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = -\frac{\mathrm{e}^{-\mathrm{atan}(a\,x)}\left(\frac{3}{5\,a^3\,c^2} - \frac{2\,x}{5\,a^2\,c^2} + \frac{2\,x^2}{5\,a\,c^2}\right)}{\frac{1}{a^2} + x^2}$$

input
```
int(exp(-atan(a*x))/(c + a^2*c*x^2)^2,x)
```

output
```
-(exp(-atan(a*x))*(3/(5*a^3*c^2) - (2*x)/(5*a^2*c^2) + (2*x^2)/(5*a*c^2)))
/(1/a^2 + x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{-2a^2x^2+2ax-3}{5e^{atan(ax)}a\,c^2\,(a^2x^2+1)}$$

input
```
int(1/exp(atan(a*x))/(a^2*c*x^2+c)^2,x)
```

output
```
( - 2*a**2*x**2 + 2*a*x - 3)/(5*e**atan(a*x)*a*c**2*(a**2*x**2 + 1))
```

3.293 $\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^3} dx = -\frac{24e^{-\arctan(ax)}}{85ac^3} - \frac{e^{-\arctan(ax)}(1-4ax)}{17ac^3(1+a^2x^2)^2} - \frac{12e^{-\arctan(ax)}(1-2ax)}{85ac^3(1+a^2x^2)}$$

output
```
-24/85/a/c^3/exp(arctan(a*x))-1/17*(-4*a*x+1)/a/c^3/exp(arctan(a*x))/(a^2*
x^2+1)^2-12/85*(-2*a*x+1)/a/c^3/exp(arctan(a*x))/(a^2*x^2+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^3} dx = \frac{5e^{-\arctan(ax)}(-1+4ax) - 12(1-iax)^{-\frac{i}{2}}(1+iax)^{\frac{i}{2}}(1+a^2x^2)(3-2ax+2a^2x^2)}{85ac^3(1+a^2x^2)^2}$$

input
```
Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^3),x]
```

output
```
((5*(-1 + 4*a*x))/E^ArcTan[a*x] - (12*(1 + I*a*x)^(I/2)*(1 + a^2*x^2)*(3 -
 2*a*x + 2*a^2*x^2))/(1 - I*a*x)^(I/2))/(85*a*c^3*(1 + a^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5593, 27, 5593, 5594}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\arctan(ax)}}{(a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5593}$$

$$\frac{12\int \frac{e^{-\arctan(ax)}}{c^2(a^2x^2+1)^2}dx}{17c}-\frac{(1-4ax)e^{-\arctan(ax)}}{17ac^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{12\int \frac{e^{-\arctan(ax)}}{(a^2x^2+1)^2}dx}{17c^3}-\frac{(1-4ax)e^{-\arctan(ax)}}{17ac^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{5593}$$

$$\frac{12\left(\frac{2}{5}\int \frac{e^{-\arctan(ax)}}{a^2x^2+1}dx-\frac{(1-2ax)e^{-\arctan(ax)}}{5a(a^2x^2+1)}\right)}{17c^3}-\frac{(1-4ax)e^{-\arctan(ax)}}{17ac^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{5594}$$

$$\frac{12\left(-\frac{(1-2ax)e^{-\arctan(ax)}}{5a(a^2x^2+1)}-\frac{2e^{-\arctan(ax)}}{5a}\right)}{17c^3}-\frac{(1-4ax)e^{-\arctan(ax)}}{17ac^3(a^2x^2+1)^2}$$

input
```
Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^3),x]
```

output
```
-1/17*(1 - 4*a*x)/(a*c^3*E^ArcTan[a*x]*(1 + a^2*x^2)^2) + (12*(-2/(5*a*E^A
rcTan[a*x]) - (1 - 2*a*x)/(5*a*E^ArcTan[a*x]*(1 + a^2*x^2))))/(17*c^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5593
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2)))   I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
&& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
 0] && IntegerQ[2*p]
```

rule 5594
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Maple [A] (verified)

Time = 14.63 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{\left(24a^{4}x^{4}-24a^{3}x^{3}+60a^{2}x^{2}-44ax+41\right)\mathrm{e}^{-\arctan\left(ax\right)}}{85\left(a^{2}x^{2}+1\right)^{2}c^{3}a}$	57
parallelrisch	$\frac{\left(-24a^{4}x^{4}+24a^{3}x^{3}-60a^{2}x^{2}+44ax-41\right)\mathrm{e}^{-\arctan\left(ax\right)}}{85c^{3}\left(a^{2}x^{2}+1\right)^{2}a}$	57
orering	$-\frac{\left(24a^{4}x^{4}-24a^{3}x^{3}+60a^{2}x^{2}-44ax+41\right)\left(a^{2}x^{2}+1\right)\mathrm{e}^{-\arctan\left(ax\right)}}{85a\left(a^{2}c\,x^{2}+c\right)^{3}}$	64

input
```
int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/85*(24*a^4*x^4-24*a^3*x^3+60*a^2*x^2-44*a*x+41)/(a^2*x^2+1)^2/c^3/exp(a
rctan(a*x))/a
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = -\frac{(24\,a^4x^4 - 24\,a^3x^3 + 60\,a^2x^2 - 44\,ax + 41)e^{(-\arctan(ax))}}{85\,(a^5c^3x^4 + 2\,a^3c^3x^2 + ac^3)}$$

input
```
integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/85*(24*a^4*x^4 - 24*a^3*x^3 + 60*a^2*x^2 - 44*a*x + 41)*e^(-arctan(a*x)
)/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(78) = 156.

Time = 90.23 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.27

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$$
$$= \begin{cases} -\frac{24a^4x^4}{85a^5c^3x^4e^{\operatorname{atan}(ax)}+170a^3c^3x^2e^{\operatorname{atan}(ax)}+85ac^3e^{\operatorname{atan}(ax)}} + \frac{24a^3x^3}{85a^5c^3x^4e^{\operatorname{atan}(ax)}+170a^3c^3x^2e^{\operatorname{atan}(ax)}+85ac^3e^{\operatorname{atan}(ax)}} - \frac{}{85a^5c^3x^4e^{\operatorname{at}}} \\ \frac{x}{c^3} \end{cases}$$

input
```
integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**3,x)
```

output
```
Piecewise((-24*a**4*x**4/(85*a**5*c**3*x**4*exp(atan(a*x)) + 170*a**3*c**3
*x**2*exp(atan(a*x)) + 85*a*c**3*exp(atan(a*x))) + 24*a**3*x**3/(85*a**5*c
**3*x**4*exp(atan(a*x)) + 170*a**3*c**3*x**2*exp(atan(a*x)) + 85*a*c**3*ex
p(atan(a*x))) - 60*a**2*x**2/(85*a**5*c**3*x**4*exp(atan(a*x)) + 170*a**3*
c**3*x**2*exp(atan(a*x)) + 85*a*c**3*exp(atan(a*x))) + 44*a*x/(85*a**5*c**
3*x**4*exp(atan(a*x)) + 170*a**3*c**3*x**2*exp(atan(a*x)) + 85*a*c**3*exp(
atan(a*x))) - 41/(85*a**5*c**3*x**4*exp(atan(a*x)) + 170*a**3*c**3*x**2*ex
p(atan(a*x)) + 85*a*c**3*exp(atan(a*x))), Ne(a, 0)), (x/c**3, True))
```

Maxima [F]

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{e^{(-\arctan(ax))}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(80) = 160.

Time = 0.14 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.04

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = -\frac{41\,\tan\left(\frac{1}{2}\arctan(ax)\right)^8 + 88\,\tan\left(\frac{1}{2}\arctan(ax)\right)^7 + 76\,\tan\left(\frac{1}{2}\arctan(ax)\right)^6 - 72\,\tan\left(\frac{1}{2}\arctan(ax\right)}{85\left(ac^3e^{(\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^8 + 4\,ac^3e^{(\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^6 + \right.}$$

input
```
integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
-1/85*(41*tan(1/2*arctan(a*x))^8 + 88*tan(1/2*arctan(a*x))^7 + 76*tan(1/2*
arctan(a*x))^6 - 72*tan(1/2*arctan(a*x))^5 + 150*tan(1/2*arctan(a*x))^4 +
72*tan(1/2*arctan(a*x))^3 + 76*tan(1/2*arctan(a*x))^2 - 88*tan(1/2*arctan(
a*x)) + 41)/(a*c^3*e^(arctan(a*x))*tan(1/2*arctan(a*x))^8 + 4*a*c^3*e^(arc
tan(a*x))*tan(1/2*arctan(a*x))^6 + 6*a*c^3*e^(arctan(a*x))*tan(1/2*arctan(
a*x))^4 + 4*a*c^3*e^(arctan(a*x))*tan(1/2*arctan(a*x))^2 + a*c^3*e^(arctan
(a*x)))
```

Mupad [B] (verification not implemented)

Time = 23.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{12\,\mathrm{e}^{-\mathrm{atan}(a\,x)}\,(2\,a\,x-1)}{85\,a\,c^3\,(a^2\,x^2+1)} - \frac{24\,\mathrm{e}^{-\mathrm{atan}(a\,x)}}{85\,a\,c^3} + \frac{\mathrm{e}^{-\mathrm{atan}(a\,x)}\,(4\,a\,x-1)}{17\,a\,c^3\,(a^2\,x^2+1)^2}$$

input
```
int(exp(-atan(a*x))/(c + a^2*c*x^2)^3,x)
```

output
```
(12*exp(-atan(a*x))*(2*a*x - 1))/(85*a*c^3*(a^2*x^2 + 1)) - (24*exp(-atan(
a*x)))/(85*a*c^3) + (exp(-atan(a*x))*(4*a*x - 1))/(17*a*c^3*(a^2*x^2 + 1)^
2)
```

Reduce [F]

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{-4e^{atan(ax)}\left(\int \frac{x}{e^{atan(ax)}a^6x^6+3e^{atan(ax)}a^4x^4+3e^{atan(ax)}a^2x^2+e^{atan(ax)}}dx\right)a^6x^4 - 8e^{atan(ax)}\left(\int \frac{x}{e^{atan(ax)}a^6x^6+3e^{atan(ax)}a^4x^4}\right.}{e^{atan(ax)}a\,c^3\,(a^4x^4}$$

input
```
int(1/exp(atan(a*x))/(a^2*c*x^2+c)^3,x)
```

output
```
( - 4*e**atan(a*x)*int(x/(e**atan(a*x)*a**6*x**6 + 3*e**atan(a*x)*a**4*x**
4 + 3*e**atan(a*x)*a**2*x**2 + e**atan(a*x)),x)*a**6*x**4 - 8*e**atan(a*x)
*int(x/(e**atan(a*x)*a**6*x**6 + 3*e**atan(a*x)*a**4*x**4 + 3*e**atan(a*x)
*a**2*x**2 + e**atan(a*x)),x)*a**4*x**2 - 4*e**atan(a*x)*int(x/(e**atan(a*
x)*a**6*x**6 + 3*e**atan(a*x)*a**4*x**4 + 3*e**atan(a*x)*a**2*x**2 + e**at
an(a*x)),x)*a**2 - 1)/(e**atan(a*x)*a*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

3.294 $\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 124

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = -\frac{144e^{-\arctan(ax)}}{629ac^4} - \frac{e^{-\arctan(ax)}(1-6ax)}{37ac^4(1+a^2x^2)^3} - \frac{30e^{-\arctan(ax)}(1-4ax)}{629ac^4(1+a^2x^2)^2} - \frac{72e^{-\arctan(ax)}(1-2ax)}{629ac^4(1+a^2x^2)}$$

output
```
-144/629/a/c^4/exp(arctan(a*x))-1/37*(-6*a*x+1)/a/c^4/exp(arctan(a*x))/(a^
2*x^2+1)^3-30/629*(-4*a*x+1)/a/c^4/exp(arctan(a*x))/(a^2*x^2+1)^2-72/629*(
-2*a*x+1)/a/c^4/exp(arctan(a*x))/(a^2*x^2+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{17ce^{-\arctan(ax)}(-1+6ax) - 6(c+a^2cx^2)\left(5e^{-\arctan(ax)}(1-4ax) + 12(1-iax)^{-\frac{i}{2}}(1+iax)^{\frac{i}{2}}(-i+ax)(\right.}{629ac^2(c+a^2cx^2)^3}$$

input
```
Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^4),x]
```

output
```
((17*c*(-1 + 6*a*x))/E^ArcTan[a*x] - 6*(c + a^2*c*x^2)*((5*(1 - 4*a*x))/E^
ArcTan[a*x] + (12*(1 + I*a*x)^(I/2)*(-I + a*x)*(I + a*x)*(3 - 2*a*x + 2*a^
2*x^2))/(1 - I*a*x)^(I/2)))/(629*a*c^2*(c + a^2*c*x^2)^3)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5593, 27, 5593, 5593, 5594}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\arctan(ax)}}{(a^2cx^2+c)^4}dx$$

$$\Big\downarrow \text{5593}$$

$$\frac{30\int \frac{e^{-\arctan(ax)}}{c^3(a^2x^2+1)^3}dx}{37c}-\frac{(1-6ax)e^{-\arctan(ax)}}{37ac^4(a^2x^2+1)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{30\int \frac{e^{-\arctan(ax)}}{(a^2x^2+1)^3}dx}{37c^4}-\frac{(1-6ax)e^{-\arctan(ax)}}{37ac^4(a^2x^2+1)^3}$$

$$\Big\downarrow \text{5593}$$

$$\frac{30\left(\frac{12}{17}\int \frac{e^{-\arctan(ax)}}{(a^2x^2+1)^2}dx-\frac{(1-4ax)e^{-\arctan(ax)}}{17a(a^2x^2+1)^2}\right)}{37c^4}-\frac{(1-6ax)e^{-\arctan(ax)}}{37ac^4(a^2x^2+1)^3}$$

$$\Big\downarrow \text{5593}$$

$$\frac{30\left(\frac{12}{17}\left(\frac{2}{5}\int \frac{e^{-\arctan(ax)}}{a^2x^2+1}dx-\frac{(1-2ax)e^{-\arctan(ax)}}{5a(a^2x^2+1)}\right)-\frac{(1-4ax)e^{-\arctan(ax)}}{17a(a^2x^2+1)^2}\right)}{37c^4}-\frac{(1-6ax)e^{-\arctan(ax)}}{37ac^4(a^2x^2+1)^3}$$

$$\Big\downarrow \text{5594}$$

$$\frac{30\left(\frac{12}{17}\left(-\frac{(1-2ax)e^{-\arctan(ax)}}{5a(a^2x^2+1)}-\frac{2e^{-\arctan(ax)}}{5a}\right)-\frac{(1-4ax)e^{-\arctan(ax)}}{17a(a^2x^2+1)^2}\right)}{37c^4}-\frac{(1-6ax)e^{-\arctan(ax)}}{37ac^4\left(a^2x^2+1\right)^3}$$

input
```
Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^4),x]
```

output
```
-1/37*(1 - 6*a*x)/(a*c^4*E^ArcTan[a*x]*(1 + a^2*x^2)^3) + (30*(-1/17*(1 -
4*a*x)/(a*E^ArcTan[a*x]*(1 + a^2*x^2)^2) + (12*(-2/(5*a*E^ArcTan[a*x]) - (
1 - 2*a*x)/(5*a*E^ArcTan[a*x]*(1 + a^2*x^2))))/17))/(37*c^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5593
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2)))   I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
&& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
 0] && IntegerQ[2*p]
```

rule 5594
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Maple [A] (verified)

Time = 46.46 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{(144x^6a^6-144a^5x^5+504a^4x^4-408a^3x^3+606a^2x^2-366ax+263)\,\mathrm{e}^{-\arctan(ax)}}{629(a^2x^2+1)^3c^4a}$	73
parallelrisch	$\frac{(-144x^6a^6+144a^5x^5-504a^4x^4+408a^3x^3-606a^2x^2+366ax-263)\,\mathrm{e}^{-\arctan(ax)}}{629c^4(a^2x^2+1)^3a}$	73
orering	$-\frac{(144x^6a^6-144a^5x^5+504a^4x^4-408a^3x^3+606a^2x^2-366ax+263)\,(a^2x^2+1)\,\mathrm{e}^{-\arctan(ax)}}{629a(a^2c\,x^2+c)^4}$	80

input
```
int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/629*(144*a^6*x^6-144*a^5*x^5+504*a^4*x^4-408*a^3*x^3+606*a^2*x^2-366*a*
x+263)/(a^2*x^2+1)^3/c^4/exp(arctan(a*x))/a
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.77

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = -\frac{(144\,a^6x^6-144\,a^5x^5+504\,a^4x^4-408\,a^3x^3+606\,a^2x^2-366\,ax+263)e^{(-\arctan(ax))}}{629\,(a^7c^4x^6+3\,a^5c^4x^4+3\,a^3c^4x^2+ac^4)}$$

input
```
integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="fricas")
```

output
```
-1/629*(144*a^6*x^6 - 144*a^5*x^5 + 504*a^4*x^4 - 408*a^3*x^3 + 606*a^2*x^
2 - 366*a*x + 263)*e^(-arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3*c
^4*x^2 + a*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \text{Timed out}$$

input
```
integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \int \frac{e^{(-\arctan(ax))}}{(a^2cx^2+c)^4}\,dx$$

input
```
integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="maxima")
```

output
```
integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^4, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(112) = 224$.

Time = 0.15 (sec) , antiderivative size = 266, normalized size of antiderivative = 2.15

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = -\frac{263\tan\left(\frac{1}{2}\arctan(ax)\right)^{12}+732\tan\left(\frac{1}{2}\arctan(ax)\right)^{11}+846\tan\left(\frac{1}{2}\arctan(ax)\right)^{10}-396\tan\left(\frac{1}{2}\operatorname{arct}\right.}{629\left(ac^4e^{(\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^{12}+6\,ac^4e^{(\arctan(ax)}\right.}$$

input
```
integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="giac")
```

output
```
-1/629*(263*tan(1/2*arctan(a*x))^12 + 732*tan(1/2*arctan(a*x))^11 + 846*ta
n(1/2*arctan(a*x))^10 - 396*tan(1/2*arctan(a*x))^9 + 2313*tan(1/2*arctan(a
*x))^8 + 2136*tan(1/2*arctan(a*x))^7 + 2372*tan(1/2*arctan(a*x))^6 - 2136*
tan(1/2*arctan(a*x))^5 + 2313*tan(1/2*arctan(a*x))^4 + 396*tan(1/2*arctan(
a*x))^3 + 846*tan(1/2*arctan(a*x))^2 - 732*tan(1/2*arctan(a*x)) + 263)/(a*
c^4*e^(arctan(a*x))*tan(1/2*arctan(a*x))^12 + 6*a*c^4*e^(arctan(a*x))*tan(
1/2*arctan(a*x))^10 + 15*a*c^4*e^(arctan(a*x))*tan(1/2*arctan(a*x))^8 + 20
*a*c^4*e^(arctan(a*x))*tan(1/2*arctan(a*x))^6 + 15*a*c^4*e^(arctan(a*x))*t
an(1/2*arctan(a*x))^4 + 6*a*c^4*e^(arctan(a*x))*tan(1/2*arctan(a*x))^2 + a
*c^4*e^(arctan(a*x)))
```

Mupad [B] (verification not implemented)

Time = 23.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{72\,\mathrm{e}^{-\mathrm{atan}(a\,x)}\,(2\,a\,x-1)}{629\,a\,c^4\,\left(a^2\,x^2+1\right)} - \frac{144\,\mathrm{e}^{-\mathrm{atan}(a\,x)}}{629\,a\,c^4} + \frac{30\,\mathrm{e}^{-\mathrm{atan}(a\,x)}\,(4\,a\,x-1)}{629\,a\,c^4\,{\left(a^2\,x^2+1\right)}^2} + \frac{\mathrm{e}^{-\mathrm{atan}(a\,x)}\,(6\,a\,x-1)}{37\,a\,c^4\,{\left(a^2\,x^2+1\right)}^3}$$

input
```
int(exp(-atan(a*x))/(c + a^2*c*x^2)^4,x)
```

output
```
(72*exp(-atan(a*x))*(2*a*x - 1))/(629*a*c^4*(a^2*x^2 + 1)) - (144*exp(-ata
n(a*x)))/(629*a*c^4) + (30*exp(-atan(a*x))*(4*a*x - 1))/(629*a*c^4*(a^2*x^
2 + 1)^2) + (exp(-atan(a*x))*(6*a*x - 1))/(37*a*c^4*(a^2*x^2 + 1)^3)
```

Reduce [F]

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{-6e^{atan(ax)}\left(\int \frac{x}{e^{atan(ax)}a^8x^8+4e^{atan(ax)}a^6x^6+6e^{atan(ax)}a^4x^4+4e^{atan(ax)}a^2x^2+e^{atan(ax)}}dx\right)a^8x^6 - 18e^{atan(ax)}\left(\int \frac{}{e^{atan(ax)}a^8}\right.}{}$$

input
```
int(1/exp(atan(a*x))/(a^2*c*x^2+c)^4,x)
```

output
```
( - 6*e**atan(a*x)*int(x/(e**atan(a*x)*a**8*x**8 + 4*e**atan(a*x)*a**6*x**
6 + 6*e**atan(a*x)*a**4*x**4 + 4*e**atan(a*x)*a**2*x**2 + e**atan(a*x)),x)
*a**8*x**6 - 18*e**atan(a*x)*int(x/(e**atan(a*x)*a**8*x**8 + 4*e**atan(a*x
)*a**6*x**6 + 6*e**atan(a*x)*a**4*x**4 + 4*e**atan(a*x)*a**2*x**2 + e**ata
n(a*x)),x)*a**6*x**4 - 18*e**atan(a*x)*int(x/(e**atan(a*x)*a**8*x**8 + 4*e
**atan(a*x)*a**6*x**6 + 6*e**atan(a*x)*a**4*x**4 + 4*e**atan(a*x)*a**2*x**
2 + e**atan(a*x)),x)*a**4*x**2 - 6*e**atan(a*x)*int(x/(e**atan(a*x)*a**8*x
**8 + 4*e**atan(a*x)*a**6*x**6 + 6*e**atan(a*x)*a**4*x**4 + 4*e**atan(a*x)
*a**2*x**2 + e**atan(a*x)),x)*a**2 - 1)/(e**atan(a*x)*a*c**4*(a**6*x**6 +
3*a**4*x**4 + 3*a**2*x**2 + 1))
```

3.295 $\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 98

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx = -\frac{\left(\frac{1}{13}-\frac{5i}{13}\right)2^{\frac{3}{2}+\frac{i}{2}}c(1-iax)^{\frac{5}{2}-\frac{i}{2}}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}-\frac{i}{2},\frac{5}{2}-\frac{i}{2},\frac{7}{2}-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

output
```
(-1/13+5/13*I)*2^(3/2+1/2*I)*c*(1-I*a*x)^(5/2-1/2*I)*(a^2*c*x^2+c)^(1/2)*h
ypergeom([5/2-1/2*I, -3/2-1/2*I],[7/2-1/2*I],1/2-1/2*I*a*x)/a/(a^2*x^2+1)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx = -\frac{\left(\frac{1}{13}-\frac{5i}{13}\right)2^{\frac{3}{2}+\frac{i}{2}}c(1-iax)^{\frac{5}{2}-\frac{i}{2}}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}-\frac{i}{2},\frac{5}{2}-\frac{i}{2},\frac{7}{2}-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/E^ArcTan[a*x],x]
```

output
```
((-1/13 + (5*I)/13)*2^(3/2 + I/2)*c*(1 - I*a*x)^(5/2 - I/2)*Sqrt[c + a^2*c
*x^2]*Hypergeometric2F1[-3/2 - I/2, 5/2 - I/2, 7/2 - I/2, (1 - I*a*x)/2])/
(a*Sqrt[1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5599, 5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}dx$$

$$\Big\downarrow \text{5599}$$

$$\frac{c\sqrt{a^2cx^2+c}\int e^{-\arctan(ax)}\left(a^2x^2+1\right)^{3/2}dx}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{5596}$$

$$\frac{c\sqrt{a^2cx^2+c}\int (1-iax)^{\frac{3}{2}-\frac{i}{2}}(iax+1)^{\frac{3}{2}+\frac{i}{2}}dx}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{\left(\frac{1}{13}-\frac{5i}{13}\right)2^{\frac{3}{2}+\frac{i}{2}}c(1-iax)^{\frac{5}{2}-\frac{i}{2}}\sqrt{a^2cx^2+c}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}-\frac{i}{2},\frac{5}{2}-\frac{i}{2},\frac{7}{2}-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/E^ArcTan[a*x],x]
```

output
```
((-1/13 + (5*I)/13)*2^(3/2 + I/2)*c*(1 - I*a*x)^(5/2 - I/2)*Sqrt[c + a^2*c
*x^2]*Hypergeometric2F1[-3/2 - I/2, 5/2 - I/2, 7/2 - I/2, (1 - I*a*x)/2])/
(a*Sqrt[1 + a^2*x^2])
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} {\mathrm e}^{-\arctan \left(a x \right)}d x$$

input
```
int((a^2*c*x^2+c)^(3/2)/exp(arctan(a*x)),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/exp(arctan(a*x)),x)
```

Fricas [F]

$$\int e^{-\arctan(ax)} \left(c+a^2 c x^2\right)^{3/2} \, dx = \int { {\left(a^{2} c x^{2} + c\right)}^{\frac{3}{2}} e^{\left(-\arctan\left(a x\right)\right)} \,d x }$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/exp(arctan(a*x)),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*e^(-arctan(a*x)), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/exp(atan(a*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int\left(a^2cx^2+c\right)^{\frac{3}{2}}e^{\left(-\arctan(ax)\right)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/exp(arctan(a*x)),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*e^(-arctan(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/exp(arctan(a*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int \mathrm{e}^{-\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(exp(-atan(a*x))*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(exp(-atan(a*x))*(c + a^2*c*x^2)^(3/2), x)
```

Reduce [F]

$$\int e^{-\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\sqrt{c}\,c\left(\int\frac{\sqrt{a^2x^2+1}}{e^{atan(ax)}}dx+\left(\int\frac{\sqrt{a^2x^2+1}\,x^2}{e^{atan(ax)}}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)/exp(atan(a*x)),x)
```

output
```
sqrt(c)*c*(int(sqrt(a**2*x**2 + 1)/e**atan(a*x),x) + int((sqrt(a**2*x**2 +
 1)*x**2)/e**atan(a*x),x)*a**2)
```

3.296 $\int e^{-\arctan(ax)}\sqrt{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 97

$$\int e^{-\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = -\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{3}{2}-\frac{i}{2}}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}-\frac{i}{2},\frac{3}{2}-\frac{i}{2},\frac{5}{2}-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

output
```
(-1/5+3/5*I)*2^(1/2+1/2*I)*(1-I*a*x)^(3/2-1/2*I)*(a^2*c*x^2+c)^(1/2)*hyper
geom([-1/2-1/2*I, 3/2-1/2*I],[5/2-1/2*I],1/2-1/2*I*a*x)/a/(a^2*x^2+1)^(1/2
)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int e^{-\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = -\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{3}{2}-\frac{i}{2}}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}-\frac{i}{2},\frac{3}{2}-\frac{i}{2},\frac{5}{2}-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/E^ArcTan[a*x],x]
```

output
```
((-1/5 + (3*I)/5)*2^(1/2 + I/2)*(1 - I*a*x)^(3/2 - I/2)*Sqrt[c + a^2*c*x^2
]*Hypergeometric2F1[-1/2 - I/2, 3/2 - I/2, 5/2 - I/2, (1 - I*a*x)/2])/(a*S
qrt[1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5599, 5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\arctan(ax)}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{ 5599}$$

$$\frac{\sqrt{a^2cx^2+c}\int e^{-\arctan(ax)}\sqrt{a^2x^2+1}dx}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{ 5596}$$

$$\frac{\sqrt{a^2cx^2+c}\int (1-iax)^{\frac{1}{2}-\frac{i}{2}}(iax+1)^{\frac{1}{2}+\frac{i}{2}}dx}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{ 79}$$

$$-\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{3}{2}-\frac{i}{2}}\sqrt{a^2cx^2+c}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}-\frac{i}{2},\frac{3}{2}-\frac{i}{2},\frac{5}{2}-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

input
```
Int[Sqrt[c + a^2*c*x^2]/E^ArcTan[a*x],x]
```

output
```
((-1/5 + (3*I)/5)*2^(1/2 + I/2)*(1 - I*a*x)^(3/2 - I/2)*Sqrt[c + a^2*c*x^2
]*Hypergeometric2F1[-1/2 - I/2, 3/2 - I/2, 5/2 - I/2, (1 - I*a*x)/2])/(a*S
qrt[1 + a^2*x^2])
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \sqrt{a^{2} c \,x^{2}+c}\, {\mathrm e}^{-\arctan \left(a x \right)}d x$$

input
```
int((a^2*c*x^2+c)^(1/2)/exp(arctan(a*x)),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/exp(arctan(a*x)),x)
```

Fricas [F]

$$\int e^{-\arctan (a x)} \sqrt{c+a^2 c x^2} \, dx=\int { \sqrt{a^{2} c x^{2} + c} e^{\left (-\arctan \left (a x\right)\right)} \,d x }$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/exp(arctan(a*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*e^(-arctan(a*x)), x)
```

Sympy [F]

$$\int e^{-\arctan(ax)}\sqrt{c+a^2cx^2}\,dx=\int\sqrt{c\left(a^2x^2+1\right)}e^{-\operatorname{atan}(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/exp(atan(a*x)),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*exp(-atan(a*x)), x)
```

Maxima [F]

$$\int e^{-\arctan(ax)}\sqrt{c+a^2cx^2}\,dx=\int\sqrt{a^2cx^2+c}e^{(-\arctan(ax))}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/exp(arctan(a*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*e^(-arctan(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-\arctan(ax)}\sqrt{c+a^2cx^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/exp(arctan(a*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \int e^{-\mathrm{atan}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(exp(-atan(a*x))*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(exp(-atan(a*x))*(c + a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int e^{-\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}}{e^{atan(ax)}}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)/exp(atan(a*x)),x)
```

output
```
sqrt(c)*int(sqrt(a**2*x**2 + 1)/e**atan(a*x),x)
```

3.297 $\int \frac{e^{-\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 93

$$\int \frac{e^{-\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{(1-i)2^{-\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{1}{2}-\frac{i}{2}}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2},\frac{1}{2}-\frac{i}{2},\frac{3}{2}-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}$$

output
```
(-1+I)*(1-I*a*x)^(1/2-1/2*I)*(a^2*x^2+1)^(1/2)*hypergeom([1/2-1/2*I, 1/2-1
/2*I],[3/2-1/2*I],1/2-1/2*I*a*x)/(2^(1/2-1/2*I))/a/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\int \frac{e^{-\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{(1-i)2^{-\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{1}{2}-\frac{i}{2}}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2},\frac{1}{2}-\frac{i}{2},\frac{3}{2}-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}$$

input
```
Integrate[1/(E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2]),x]
```

output
```
((-1 + I)*(1 - I*a*x)^(1/2 - I/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[1/2
- I/2, 1/2 - I/2, 3/2 - I/2, (1 - I*a*x)/2])/(2^(1/2 - I/2)*a*Sqrt[c + a^2
*c*x^2])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5599, 5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5599}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{-\arctan(ax)}}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5596}$$

$$\frac{\sqrt{a^2x^2+1}\int (1-iax)^{-\frac{1}{2}-\frac{i}{2}}(iax+1)^{-\frac{1}{2}+\frac{i}{2}}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{(1-i)2^{-\frac{1}{2}+\frac{i}{2}}(1-iax)^{\frac{1}{2}-\frac{i}{2}}\sqrt{a^2x^2+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2},\frac{1}{2}-\frac{i}{2},\frac{3}{2}-\frac{i}{2},\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

input
```
Int[1/(E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2]),x]
```

output

```
((-1 + I)*(1 - I*a*x)^(1/2 - I/2)*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[1/2
- I/2, 1/2 - I/2, 3/2 - I/2, (1 - I*a*x)/2])/(2^(1/2 - I/2)*a*Sqrt[c + a^2
*c*x^2])
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{\mathrm{e}^{-\arctan(ax)}}{\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{-\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(-\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(e^(-arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{e^{-\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{-\operatorname{atan}(ax)}}{\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(exp(-atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(-\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(e^(-arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{e^{-\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(-\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(e^(-arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{-\mathrm{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(exp(-atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(exp(-atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{e^{-\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{1}{e^{atan(ax)}\sqrt{a^2x^2+1}}dx}{\sqrt{c}}$$

input
```
int(1/exp(atan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(1/(e**atan(a*x)*sqrt(a**2*x**2 + 1)),x)/sqrt(c)
```

3.298 $\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 38

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{e^{-\arctan(ax)}(1-ax)}{2ac\sqrt{c+a^2cx^2}}$$

output
```
-1/2*(-a*x+1)/a/c/exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{e^{-\arctan(ax)}(-1+ax)}{2ac\sqrt{c+a^2cx^2}}$$

input
```
Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
(-1 + a*x)/(2*a*c*E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {5592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx$$
$$\Big\downarrow \ 5592$$
$$-\frac{(1-ax)e^{-\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

input
```
Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
-1/2*(1 - a*x)/(a*c*E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2])
```

Defintions of rubi rules used

rule 5592
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

method	result	size
gosper	$\frac{(a^2x^2+1)(ax-1)e^{-\arctan(ax)}}{2a(a^2cx^2+c)^{\frac{3}{2}}}$	39
orering	$\frac{(a^2x^2+1)(ax-1)e^{-\arctan(ax)}}{2a(a^2cx^2+c)^{\frac{3}{2}}}$	39

input
```
int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a^2*x^2+1)*(a*x-1)/a/exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{a^2cx^2+c}(ax-1)e^{(-\arctan(ax))}}{2\,(a^3c^2x^2+ac^2)}$$

input
```
integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(a^2*c*x^2 + c)*(a*x - 1)*e^(-arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

Sympy [F]

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{e^{-\operatorname{atan}(ax)}}{(c\,(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(exp(-atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{e^{(-\arctan(ax))}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(32) = 64$.

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.47

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx =$$
$$-\frac{\cos\left(\frac{1}{2}\arctan(ax)\right)^4 e^{(-\arctan(ax))} - 2\cos\left(\frac{1}{2}\arctan(ax)\right)^3 e^{(-\arctan(ax))}\sin\left(\frac{1}{2}\arctan(ax)\right) - 2\cos\left(\frac{1}{2}\arc}{2\,ac^{\frac{3}{2}}}$$

input
```
integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(cos(1/2*arctan(a*x))^4*e^(-arctan(a*x)) - 2*cos(1/2*arctan(a*x))^3*e
^(-arctan(a*x))*sin(1/2*arctan(a*x)) - 2*cos(1/2*arctan(a*x))*e^(-arctan(a
*x))*sin(1/2*arctan(a*x))^3 - e^(-arctan(a*x))*sin(1/2*arctan(a*x))^4)/(a*
c^(3/2))
```

Mupad [B] (verification not implemented)

Time = 23.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{e^{-\arctan(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{\mathrm{e}^{-\mathrm{atan}(a\,x)}\,\left(\frac{x}{2\,c}-\frac{1}{2\,a\,c}\right)}{\sqrt{c\,a^2\,x^2+c}}$$

input

```
int(exp(-atan(a*x))/(c + a^2*c*x^2)^(3/2),x)
```

output

```
(exp(-atan(a*x))*(x/(2*c) - 1/(2*a*c)))/(c + a^2*c*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{e^{-\arctan(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a^2x^2+1}\,(ax-1)}{2e^{atan(ax)}a\,c^2\,(a^2x^2+1)}$$

input

```
int(1/exp(atan(a*x))/(a^2*c*x^2+c)^(3/2),x)
```

output

```
(sqrt(c)*sqrt(a**2*x**2 + 1)*(a*x - 1))/(2*e**atan(a*x)*a*c**2*(a**2*x**2
+ 1))
```

3.299 $\int \frac{e^{-\arctan(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{e^{-\arctan(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx=-\frac{e^{-\arctan(ax)}(1-3ax)}{10ac\left(c+a^2cx^2\right)^{3/2}}-\frac{3e^{-\arctan(ax)}(1-ax)}{10ac^2\sqrt{c+a^2cx^2}}$$

output
```
-1/10*(-3*a*x+1)/a/c/exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2)-3/10*(-a*x+1)/a/
c^2/exp(arctan(a*x))/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int \frac{e^{-\arctan(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx=\frac{e^{-\arctan(ax)}\left(-4+6ax-3a^2x^2+3a^3x^3\right)}{10c^2\left(a+a^3x^2\right)\sqrt{c+a^2cx^2}}$$

input
```
Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
(-4 + 6*a*x - 3*a^2*x^2 + 3*a^3*x^3)/(10*c^2*E^ArcTan[a*x]*(a + a^3*x^2)*S
qrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5593, 5592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5593}$$

$$\frac{3\int \frac{e^{-\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx}{5c} - \frac{(1-3ax)e^{-\arctan(ax)}}{10ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5592}$$

$$-\frac{3(1-ax)e^{-\arctan(ax)}}{10ac^2\sqrt{a^2cx^2+c}} - \frac{(1-3ax)e^{-\arctan(ax)}}{10ac\,(a^2cx^2+c)^{3/2}}$$

input
```
Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
-1/10*(1 - 3*a*x)/(a*c*E^ArcTan[a*x]*(c + a^2*c*x^2)^(3/2)) - (3*(1 - a*x)
)/(10*a*c^2*E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2])
```

Defintions of rubi rules used

rule 5592
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

rule 5593
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2)))   I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
&& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.73

method	result	size
gosper	$\frac{(a^2x^2+1)(3a^3x^3-3a^2x^2+6ax-4)e^{-\arctan(ax)}}{10a(a^2c\,x^2+c)^{\frac{5}{2}}}$	56
orering	$\frac{(a^2x^2+1)(3a^3x^3-3a^2x^2+6ax-4)e^{-\arctan(ax)}}{10a(a^2c\,x^2+c)^{\frac{5}{2}}}$	56

input
```
int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/10*(a^2*x^2+1)*(3*a^3*x^3-3*a^2*x^2+6*a*x-4)/a/exp(arctan(a*x))/(a^2*c*x
^2+c)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{(3\,a^3x^3-3\,a^2x^2+6\,ax-4)\sqrt{a^2cx^2+c}e^{(-\arctan(ax))}}{10\,(a^5c^3x^4+2\,a^3c^3x^2+ac^3)}$$

input
```
integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/10*(3*a^3*x^3 - 3*a^2*x^2 + 6*a*x - 4)*sqrt(a^2*c*x^2 + c)*e^(-arctan(a*
x))/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{e^{(-\arctan(ax))}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.36

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{2\left(\tan\left(\frac{1}{2}\arctan(ax)\right)^6+3\tan\left(\frac{1}{2}\arctan(ax)\right)^5+3\,\mathrm{t}\right.}{5\left(ac^{\frac{5}{2}}e^{(\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^6+3\,ac^{\frac{5}{2}}e^{(\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^4+3\,a\right.}$$

input
```
integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
2/5*(tan(1/2*arctan(a*x))^6 + 3*tan(1/2*arctan(a*x))^5 + 3*tan(1/2*arctan(
a*x)) - 1)/(a*c^(5/2)*e^(arctan(a*x))*tan(1/2*arctan(a*x))^6 + 3*a*c^(5/2)
*e^(arctan(a*x))*tan(1/2*arctan(a*x))^4 + 3*a*c^(5/2)*e^(arctan(a*x))*tan(
1/2*arctan(a*x))^2 + a*c^(5/2)*e^(arctan(a*x)))
```

Mupad [B] (verification not implemented)

Time = 23.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{\mathrm{e}^{-\mathrm{atan}(a\,x)}\left(\frac{2}{5\,a^3\,c^2}-\frac{3\,x^3}{10\,c^2}-\frac{3\,x}{5\,a^2\,c^2}+\frac{3\,x^2}{10\,a\,c^2}\right)}{\frac{\sqrt{c\,a^2\,x^2+c}}{a^2}+x^2\,\sqrt{c\,a^2\,x^2+c}}$$

input
```
int(exp(-atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
-(exp(-atan(a*x))*(2/(5*a^3*c^2) - (3*x^3)/(10*c^2) - (3*x)/(5*a^2*c^2) +
(3*x^2)/(10*a*c^2)))/((c + a^2*c*x^2)^(1/2)/a^2 + x^2*(c + a^2*c*x^2)^(1/2
))
```

Reduce [F]

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\int \frac{1}{e^{atan(ax)}\sqrt{a^2x^2+1}\,a^4x^4+2e^{atan(ax)}\sqrt{a^2x^2+1}\,a^2x^2+e^{atan(ax)}\sqrt{a^2x^2+1}}dx}{\sqrt{c}\,c^2}$$

input
```
int(1/exp(atan(a*x))/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(1/(e**atan(a*x)*sqrt(a**2*x**2 + 1)*a**4*x**4 + 2*e**atan(a*x)*sqrt(a*
*2*x**2 + 1)*a**2*x**2 + e**atan(a*x)*sqrt(a**2*x**2 + 1)),x)/(sqrt(c)*c**
2)
```

3.300 $\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 115

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx=-\frac{e^{-\arctan(ax)}(1-5ax)}{26ac\,(c+a^2cx^2)^{5/2}}-\frac{e^{-\arctan(ax)}(1-3ax)}{13ac^2\,(c+a^2cx^2)^{3/2}}-\frac{3e^{-\arctan(ax)}(1-ax)}{13ac^3\sqrt{c+a^2cx^2}}$$

output
```
-1/26*(-5*a*x+1)/a/c/exp(arctan(a*x))/(a^2*c*x^2+c)^(5/2)-1/13*(-3*a*x+1)/
a/c^2/exp(arctan(a*x))/(a^2*c*x^2+c)^(3/2)-3/13*(-a*x+1)/a/c^3/exp(arctan(
a*x))/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx=\frac{e^{-\arctan(ax)}(-9+17ax-14a^2x^2+18a^3x^3-6a^4x^4+6a^5x^5)}{26ac^3\,(1+a^2x^2)^2\sqrt{c+a^2cx^2}}$$

input
```
Integrate[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^(7/2)),x]
```

output
```
(-9 + 17*a*x - 14*a^2*x^2 + 18*a^3*x^3 - 6*a^4*x^4 + 6*a^5*x^5)/(26*a*c^3*
E^ArcTan[a*x]*(1 + a^2*x^2)^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5593, 5593, 5592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-\arctan(ax)}}{(a^2cx^2+c)^{7/2}}dx$$

$$\Big\downarrow \text{5593}$$

$$\frac{10\int \frac{e^{-\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx}{13c}-\frac{(1-5ax)e^{-\arctan(ax)}}{26ac\left(a^2cx^2+c\right)^{5/2}}$$

$$\Big\downarrow \text{5593}$$

$$\frac{10\left(\frac{3\int \frac{e^{-\arctan(ax)}}{\left(a^2cx^2+c\right)^{3/2}}dx}{5c}-\frac{(1-3ax)e^{-\arctan(ax)}}{10ac(a^2cx^2+c)^{3/2}}\right)}{13c}-\frac{(1-5ax)e^{-\arctan(ax)}}{26ac\left(a^2cx^2+c\right)^{5/2}}$$

$$\Big\downarrow \text{5592}$$

$$\frac{10\left(-\frac{3(1-ax)e^{-\arctan(ax)}}{10ac^2\sqrt{a^2cx^2+c}}-\frac{(1-3ax)e^{-\arctan(ax)}}{10ac(a^2cx^2+c)^{3/2}}\right)}{13c}-\frac{(1-5ax)e^{-\arctan(ax)}}{26ac\left(a^2cx^2+c\right)^{5/2}}$$

input
```
Int[1/(E^ArcTan[a*x]*(c + a^2*c*x^2)^(7/2)),x]
```

output
```
-1/26*(1 - 5*a*x)/(a*c*E^ArcTan[a*x]*(c + a^2*c*x^2)^(5/2)) + (10*(-1/10*(
1 - 3*a*x)/(a*c*E^ArcTan[a*x]*(c + a^2*c*x^2)^(3/2)) - (3*(1 - a*x))/(10*a
*c^2*E^ArcTan[a*x]*Sqrt[c + a^2*c*x^2])))/(13*c)
```

Defintions of rubi rules used

rule 5592
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

rule 5593
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2)))   I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
&& EqQ[d, a^2*c] && LtQ[p, -1] && !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.63

method	result	size
gosper	$\frac{\left(a^{2} x^{2}+1\right) \left(6 a^{5} x^{5}-6 a^{4} x^{4}+18 a^{3} x^{3}-14 a^{2} x^{2}+17 a x -9\right) {\mathrm e}^{-\arctan \left(a x \right)}}{26 a \left(a^{2} c \,x^{2}+c \right)^{\frac{7}{2}}}$	72
orering	$\frac{\left(a^{2} x^{2}+1\right) \left(6 a^{5} x^{5}-6 a^{4} x^{4}+18 a^{3} x^{3}-14 a^{2} x^{2}+17 a x -9\right) {\mathrm e}^{-\arctan \left(a x \right)}}{26 a \left(a^{2} c \,x^{2}+c \right)^{\frac{7}{2}}}$	72

input
```
int(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/26*(a^2*x^2+1)*(6*a^5*x^5-6*a^4*x^4+18*a^3*x^3-14*a^2*x^2+17*a*x-9)/a/ex
p(arctan(a*x))/(a^2*c*x^2+c)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.86

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \frac{(6\,a^5x^5 - 6\,a^4x^4 + 18\,a^3x^3 - 14\,a^2x^2 + 17\,ax - 9)\sqrt{a^2cx^2 + c}e^{(-\arctan(ax))}}{26\,(a^7c^4x^6 + 3\,a^5c^4x^4 + 3\,a^3c^4x^2 + ac^4)}$$

input
```
integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
1/26*(6*a^5*x^5 - 6*a^4*x^4 + 18*a^3*x^3 - 14*a^2*x^2 + 17*a*x - 9)*sqrt(a
^2*c*x^2 + c)*e^(-arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3*c^4*x^
2 + a*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/exp(atan(a*x))/(a**2*c*x**2+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{e^{(-\arctan(ax))}}{(a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(e^(-arctan(a*x))/(a^2*c*x^2 + c)^(7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(99) = 198$.

Time = 0.18 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.95

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \frac{9\tan\left(\frac{1}{2}\arctan(ax)\right)^{10} + 34\tan\left(\frac{1}{2}\arctan(ax)\right)^{9} + 11\tan\left(\frac{1}{2}\arctan(ax)\right)^{8} + 8\,\mathrm{ta}}{26\left(ac^{\frac{7}{2}}e^{(\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^{10} + 5\,ac^{\frac{7}{2}}e^{(\arctan(ax))}\,\mathrm{t}\right.}$$

input
```
integrate(1/exp(arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
1/26*(9*tan(1/2*arctan(a*x))^10 + 34*tan(1/2*arctan(a*x))^9 + 11*tan(1/2*a
rctan(a*x))^8 + 8*tan(1/2*arctan(a*x))^7 + 18*tan(1/2*arctan(a*x))^6 + 108
*tan(1/2*arctan(a*x))^5 - 18*tan(1/2*arctan(a*x))^4 + 8*tan(1/2*arctan(a*x
))^3 - 11*tan(1/2*arctan(a*x))^2 + 34*tan(1/2*arctan(a*x)) - 9)/(a*c^(7/2)
*e^(arctan(a*x))*tan(1/2*arctan(a*x))^10 + 5*a*c^(7/2)*e^(arctan(a*x))*tan
(1/2*arctan(a*x))^8 + 10*a*c^(7/2)*e^(arctan(a*x))*tan(1/2*arctan(a*x))^6
+ 10*a*c^(7/2)*e^(arctan(a*x))*tan(1/2*arctan(a*x))^4 + 5*a*c^(7/2)*e^(arc
tan(a*x))*tan(1/2*arctan(a*x))^2 + a*c^(7/2)*e^(arctan(a*x)))
```

Mupad [B] (verification not implemented)

Time = 22.97 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = -\frac{\mathrm{e}^{-\mathrm{atan}(a\,x)}\left(\frac{9}{26\,a^5\,c^3} - \frac{3\,x^5}{13\,c^3} - \frac{17\,x}{26\,a^4\,c^3} + \frac{3\,x^4}{13\,a\,c^3} - \frac{9\,x^3}{13\,a^2\,c^3} + \frac{7\,x^2}{13\,a^3\,c^3}\right)}{\frac{\sqrt{c\,a^2\,x^2+c}}{a^4} + x^4\sqrt{c\,a^2\,x^2+c} + \frac{2\,x^2\sqrt{c\,a^2\,x^2+c}}{a^2}}$$

input
```
int(exp(-atan(a*x))/(c + a^2*c*x^2)^(7/2),x)
```

output
```
-(exp(-atan(a*x))*(9/(26*a^5*c^3) - (3*x^5)/(13*c^3) - (17*x)/(26*a^4*c^3)
 + (3*x^4)/(13*a*c^3) - (9*x^3)/(13*a^2*c^3) + (7*x^2)/(13*a^3*c^3)))/((c
+ a^2*c*x^2)^(1/2)/a^4 + x^4*(c + a^2*c*x^2)^(1/2) + (2*x^2*(c + a^2*c*x^2
)^(1/2))/a^2)
```

Reduce [F]

$$\int \frac{e^{-\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \frac{\int \frac{1}{e^{atan(ax)}\sqrt{a^2x^2+1}\,a^6x^6+3e^{atan(ax)}\sqrt{a^2x^2+1}\,a^4x^4+3e^{atan(ax)}\sqrt{a^2x^2+1}\,a^2x^2+e^{atan(ax)}\sqrt{a^2x^2+1}}dx}{\sqrt{c}\,c^3}$$

input
```
int(1/exp(atan(a*x))/(a^2*c*x^2+c)^(7/2),x)
```

output
```
int(1/(e**atan(a*x)*sqrt(a**2*x**2 + 1)*a**6*x**6 + 3*e**atan(a*x)*sqrt(a*
*2*x**2 + 1)*a**4*x**4 + 3*e**atan(a*x)*sqrt(a**2*x**2 + 1)*a**2*x**2 + e*
*atan(a*x)*sqrt(a**2*x**2 + 1)),x)/(sqrt(c)*c**3)
```

3.301 $\int e^{-2\arctan(ax)}(c+a^2cx^2)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 90

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx = \frac{i2^{i+p}(1-iax)^{(1-i)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p \text{Hypergeometric2F1}\left(-i-p,(1-i)+p,(2-i)+p,\frac{1}{2}(1\right.}{a((1-i)+p)}$$

output
```
I*2^(I+p)*(1-I*a*x)^(1-I+p)*(a^2*c*x^2+c)^p*hypergeom([-I-p, 1-I+p],[2-I+p
],1/2-1/2*I*a*x)/a/(1-I+p)/((a^2*x^2+1)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx = \frac{i2^{i+p}(1-iax)^{(1-i)+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p \text{Hypergeometric2F1}\left(-i-p,(1-i)+p,(2-i)+p,\frac{1}{2}(1\right.}{a((1-i)+p)}$$

input
```
Integrate[(c + a^2*c*x^2)^p/E^(2*ArcTan[a*x]),x]
```

output
```
(I*2^(I + p)*(1 - I*a*x)^((1 - I) + p)*(c + a^2*c*x^2)^p*Hypergeometric2F1
[-I - p, (1 - I) + p, (2 - I) + p, (1 - I*a*x)/2])/(a*((1 - I) + p)*(1 + a
^2*x^2)^p)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5599, 5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2\arctan(ax)}\left(a^2cx^2+c\right)^p\,dx$$

$$\Big\downarrow \text{5599}$$

$$\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\int e^{-2\arctan(ax)}\left(a^2x^2+1\right)^p dx$$

$$\Big\downarrow \text{5596}$$

$$\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\int (1-iax)^{p-i}(iax+1)^{p+i}dx$$

$$\Big\downarrow \text{79}$$

$$\frac{i2^{p+i}(1-iax)^{p+(1-i)}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p \text{Hypergeometric2F1}\left(-p-i,p+(1-i),p+(2-i),\frac{1}{2}(1-iax)\right)}{a(p+(1-i))}$$

input
```
Int[(c + a^2*c*x^2)^p/E^(2*ArcTan[a*x]),x]
```

output
```
(I*2^(I + p)*(1 - I*a*x)^((1 - I) + p)*(c + a^2*c*x^2)^p*Hypergeometric2F1
[-I - p, (1 - I) + p, (2 - I) + p, (1 - I*a*x)/2])/(a*((1 - I) + p)*(1 + a
^2*x^2)^p)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \left(a^{2} c \,x^{2}+c \right)^{p} {\mathrm e}^{-2 \arctan \left(a x \right)}d x$$

input
```
int((a^2*c*x^2+c)^p/exp(2*arctan(a*x)),x)
```

output
```
int((a^2*c*x^2+c)^p/exp(2*arctan(a*x)),x)
```

Fricas [F]

$$\int e^{-2 \arctan (a x)} \left(c+a^2 c x^2\right)^p \, dx=\int { {\left(a^{2} c x^{2} + c\right)}^{p} e^{\left(-2 \, \arctan \left(a x\right)\right)} \,d x }$$

input
```
integrate((a^2*c*x^2+c)^p/exp(2*arctan(a*x)),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^p*e^(-2*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int\left(c\left(a^2x^2+1\right)\right)^pe^{-2\operatorname{atan}(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**p/exp(2*atan(a*x)),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**p*exp(-2*atan(a*x)), x)
```

Maxima [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int\left(a^2cx^2+c\right)^pe^{(-2\arctan(ax))}\,dx$$

input
```
integrate((a^2*c*x^2+c)^p/exp(2*arctan(a*x)),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^p*e^(-2*arctan(a*x)), x)
```

Giac [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int\left(a^2cx^2+c\right)^pe^{(-2\arctan(ax))}\,dx$$

input
```
integrate((a^2*c*x^2+c)^p/exp(2*arctan(a*x)),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^p*e^(-2*arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int \mathrm{e}^{-2\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^p\,dx$$

input
```
int(exp(-2*atan(a*x))*(c + a^2*c*x^2)^p,x)
```

output
```
int(exp(-2*atan(a*x))*(c + a^2*c*x^2)^p, x)
```

Reduce [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int \frac{(a^2c\,x^2+c)^p}{e^{2atan(ax)}}dx$$

input
```
int((a^2*c*x^2+c)^p/exp(2*atan(a*x)),x)
```

output
```
int((a**2*c*x**2 + c)**p/e**(2*atan(a*x)),x)
```

3.302 $\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx = -\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{1+i}c^2(1-iax)^{3-i}\text{Hypergeometric2F1}\left(-2-i,3-i,4-i,\frac{1}{2}(1-iax)\right)}{a}$$

output
```
(-1/5+3/5*I)*2^(1+I)*c^2*(1-I*a*x)^(3-I)*hypergeom([3-I, -2-I],[4-I],1/2-1
/2*I*a*x)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx = -\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{1+i}c^2(1-iax)^{3-i}\text{Hypergeometric2F1}\left(-2-i,3-i,4-i,\frac{1}{2}(1-iax)\right)}{a}$$

input
```
Integrate[(c + a^2*c*x^2)^2/E^(2*ArcTan[a*x]),x]
```

output
```
((-1/5 + (3*I)/5)*2^(1 + I)*c^2*(1 - I*a*x)^(3 - I)*Hypergeometric2F1[-2 -
 I, 3 - I, 4 - I, (1 - I*a*x)/2])/a
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2\arctan(ax)}\left(a^2cx^2+c\right)^2\,dx$$

$$\Big\downarrow \ 5596$$

$$c^2\int (1-iax)^{2-i}(iax+1)^{2+i}dx$$

$$\Big\downarrow \ 79$$

$$-\frac{\left(\frac{1}{5}-\frac{3i}{5}\right)2^{1+i}c^2(1-iax)^{3-i}\,\text{Hypergeometric2F1}\left(-2-i,3-i,4-i,\frac{1}{2}(1-iax)\right)}{a}$$

input
```
Int[(c + a^2*c*x^2)^2/E^(2*ArcTan[a*x]),x]
```

output
```
((-1/5 + (3*I)/5)*2^(1 + I)*c^2*(1 - I*a*x)^(3 - I)*Hypergeometric2F1[-2 -
 I, 3 - I, 4 - I, (1 - I*a*x)/2])/a
```


Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \left(a^{2} c\,x^{2}+c \right)^{2} {\mathrm e}^{-2 \arctan \left(a x \right)}d x$$

input
```
int((a^2*c*x^2+c)^2/exp(2*arctan(a*x)),x)
```

output
```
int((a^2*c*x^2+c)^2/exp(2*arctan(a*x)),x)
```

Fricas [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int \left(a^{2} c x^{2} + c\right)^{2} e^{\left(-2 \, \arctan\left(a x\right)\right)}\,{d x}$$

input
```
integrate((a^2*c*x^2+c)^2/exp(2*arctan(a*x)),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*e^(-2*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=c^{2} \left(\int 2 a^{2} x^{2} e^{- 2 \operatorname{atan}{\left(a x \right)}}\, dx + \int a^{4} x^{4} e^{- 2 \operatorname{atan}{\left(a x \right)}}\, dx + \int e^{- 2 \operatorname{atan}{\left(a x \right)}}\, dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/exp(2*atan(a*x)),x)
```

output
```
c**2*(Integral(2*a**2*x**2*exp(-2*atan(a*x)), x) + Integral(a**4*x**4*exp(
-2*atan(a*x)), x) + Integral(exp(-2*atan(a*x)), x))
```

Maxima [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int\left(a^2cx^2+c\right)^2e^{(-2\arctan(ax))}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/exp(2*arctan(a*x)),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^2*e^(-2*arctan(a*x)), x)
```

Giac [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int\left(a^2cx^2+c\right)^2e^{(-2\arctan(ax))}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/exp(2*arctan(a*x)),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*e^(-2*arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int e^{-2\,\mathrm{atan}(a\,x)}\left(c\,a^2\,x^2+c\right)^2\,dx$$

input
```
int(exp(-2*atan(a*x))*(c + a^2*c*x^2)^2,x)
```

output
```
int(exp(-2*atan(a*x))*(c + a^2*c*x^2)^2, x)
```

Reduce [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=c^2\left(\left(\int\frac{x^4}{e^{2atan(ax)}}dx\right)a^4+2\left(\int\frac{x^2}{e^{2atan(ax)}}dx\right)a^2+\int\frac{1}{e^{2atan(ax)}}dx\right)$$

input
```
int((a^2*c*x^2+c)^2/exp(2*atan(a*x)),x)
```

output
```
c**2*(int(x**4/e**(2*atan(a*x)),x)*a**4 + 2*int(x**2/e**(2*atan(a*x)),x)*a
**2 + int(1/e**(2*atan(a*x)),x))
```

3.303 $\int e^{-2\arctan(ax)}(c+a^2cx^2)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int e^{-2\arctan(ax)}(c+a^2cx^2)\,dx = -\frac{\left(\frac{1}{5}-\frac{2i}{5}\right)2^{1+i}c(1-iax)^{2-i}\,\text{Hypergeometric2F1}\left(-1-i,2-i,3-i,\frac{1}{2}(1-iax)\right)}{a}$$

output
```
(-1/5+2/5*I)*2^(1+I)*c*(1-I*a*x)^(2-I)*hypergeom([-1-I, 2-I],[3-I],1/2-1/2
*I*a*x)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int e^{-2\arctan(ax)}(c+a^2cx^2)\,dx = -\frac{\left(\frac{1}{5}-\frac{2i}{5}\right)2^{1+i}c(1-iax)^{2-i}\,\text{Hypergeometric2F1}\left(-1-i,2-i,3-i,\frac{1}{2}(1-iax)\right)}{a}$$

input
```
Integrate[(c + a^2*c*x^2)/E^(2*ArcTan[a*x]),x]
```

output
```
((-1/5 + (2*I)/5)*2^(1 + I)*c*(1 - I*a*x)^(2 - I)*Hypergeometric2F1[-1 - I
, 2 - I, 3 - I, (1 - I*a*x)/2])/a
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2\arctan(ax)}\left(a^2cx^2+c\right)\,dx$$

$$\Big\downarrow \text{5596}$$

$$c\int (1-iax)^{1-i}(iax+1)^{1+i}dx$$

$$\Big\downarrow \text{79}$$

$$-\frac{\left(\frac{1}{5}-\frac{2i}{5}\right)2^{1+i}c(1-iax)^{2-i}\text{Hypergeometric2F1}\left(-1-i,2-i,3-i,\frac{1}{2}(1-iax)\right)}{a}$$

input
```
Int[(c + a^2*c*x^2)/E^(2*ArcTan[a*x]),x]
```

output
```
((-1/5 + (2*I)/5)*2^(1 + I)*c*(1 - I*a*x)^(2 - I)*Hypergeometric2F1[-1 - I
, 2 - I, 3 - I, (1 - I*a*x)/2])/a
```


Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \left(a^{2} c \,x^{2}+c \right) {\mathrm e}^{-2 \arctan \left(a x \right)}d x$$

input
```
int((a^2*c*x^2+c)/exp(2*arctan(a*x)),x)
```

output
```
int((a^2*c*x^2+c)/exp(2*arctan(a*x)),x)
```

Fricas [F]

$$\int e^{-2 \arctan (a x)} \left(c+a^2 c x^2\right) \, dx=\int { {\left(a^{2} c x^{2} + c\right)} e^{\left(-2 \, \arctan \left(a x\right)\right)} \,d x }$$

input
```
integrate((a^2*c*x^2+c)/exp(2*arctan(a*x)),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*e^(-2*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{-2 \arctan (a x)} \left(c+a^2 c x^2\right) \, dx=c \left(\int a^{2} x^{2} e^{- 2 \operatorname{atan}{\left(a x \right)}}\, dx + \int e^{- 2 \operatorname{atan}{\left(a x \right)}}\, dx\right)$$

input
```
integrate((a**2*c*x**2+c)/exp(2*atan(a*x)),x)
```

output
```
c*(Integral(a**2*x**2*exp(-2*atan(a*x)), x) + Integral(exp(-2*atan(a*x)),
x))
```

Maxima [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int\left(a^2cx^2+c\right)e^{(-2\arctan(ax))}\,dx$$

input
```
integrate((a^2*c*x^2+c)/exp(2*arctan(a*x)),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)*e^(-2*arctan(a*x)), x)
```

Giac [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int\left(a^2cx^2+c\right)e^{(-2\arctan(ax))}\,dx$$

input
```
integrate((a^2*c*x^2+c)/exp(2*arctan(a*x)),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*e^(-2*arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int e^{-2\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(exp(-2*atan(a*x))*(c + a^2*c*x^2),x)
```

output
```
int(exp(-2*atan(a*x))*(c + a^2*c*x^2), x)
```

Reduce [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)\,dx=c\left(\left(\int\frac{x^2}{e^{2atan(ax)}}dx\right)a^2+\int\frac{1}{e^{2atan(ax)}}dx\right)$$

input
```
int((a^2*c*x^2+c)/exp(2*atan(a*x)),x)
```

output
```
c*(int(x**2/e**(2*atan(a*x)),x)*a**2 + int(1/e**(2*atan(a*x)),x))
```

3.304 $\int e^{-2\arctan(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 46

$$\int e^{-2\arctan(ax)}\,dx = -\frac{(1-i)2^{-1+i}(1-iax)^{1-i}\,\text{Hypergeometric2F1}\left(-i,1-i,2-i,\frac{1}{2}(1-iax)\right)}{a}$$

output
```
(-1+I)*(1-I*a*x)^(1-I)*hypergeom([-I, 1-I],[2-I],1/2-1/2*I*a*x)/(2^(1-I))/
a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int e^{-2\arctan(ax)}\,dx = -\frac{(1+i)e^{(-2+2i)\arctan(ax)}\,\text{Hypergeometric2F1}\left(1+i,2,2+i,-e^{2i\arctan(ax)}\right)}{a}$$

input
```
Integrate[E^(-2*ArcTan[a*x]),x]
```

output
```
((-1 - I)*Hypergeometric2F1[1 + I, 2, 2 + I, -E^((2*I)*ArcTan[a*x])])/(a*E
^((2 - 2*I)*ArcTan[a*x]))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5584, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2\arctan(ax)}dx$$

$$\Big\downarrow \text{5584}$$

$$\int (1-iax)^{-i}(1+iax)^{i}dx$$

$$\Big\downarrow \text{79}$$

$$-\frac{(1-i)2^{-1+i}(1-iax)^{1-i}\text{Hypergeometric2F1}\left(-i,1-i,2-i,\frac{1}{2}(1-iax)\right)}{a}$$

input
```
Int[E^(-2*ArcTan[a*x]),x]
```

output
```
((-1 + I)*(1 - I*a*x)^(1 - I)*Hypergeometric2F1[-I, 1 - I, 2 - I, (1 - I*a
*x)/2])/(2^(1 - I)*a)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5584
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Maple [F]

$$\int e^{-2\arctan(ax)}dx$$

input
```
int(exp(-2*arctan(a*x)),x)
```

output
```
int(exp(-2*arctan(a*x)),x)
```

Fricas [F]

$$\int e^{-2\arctan(ax)}\,dx = \int e^{\left(-2\,\arctan\left(ax\right)\right)}\,dx$$

input
```
integrate(exp(-2*arctan(a*x)),x, algorithm="fricas")
```

output
```
integral(e^(-2*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{-2\arctan(ax)}\,dx = \int e^{-2\,\mathrm{atan}\,(ax)}\,dx$$

input
```
integrate(exp(-2*atan(a*x)),x)
```

output
```
Integral(exp(-2*atan(a*x)), x)
```

Maxima [F]

$$\int e^{-2\arctan(ax)}\,dx = \int e^{(-2\arctan(ax))}\,dx$$

input
```
integrate(exp(-2*arctan(a*x)),x, algorithm="maxima")
```

output
```
integrate(e^(-2*arctan(a*x)), x)
```

Giac [F]

$$\int e^{-2\arctan(ax)}\,dx = \int e^{(-2\arctan(ax))}\,dx$$

input
```
integrate(exp(-2*arctan(a*x)),x, algorithm="giac")
```

output
```
integrate(e^(-2*arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\arctan(ax)}\,dx = \int \mathrm{e}^{-2\,\mathrm{atan}(a\,x)}\,dx$$

input
```
int(exp(-2*atan(a*x)),x)
```

output
```
int(exp(-2*atan(a*x)), x)
```

Reduce [F]

$$\int e^{-2\arctan(ax)}\,dx = \int \frac{1}{e^{2atan(ax)}}dx$$

input
```
int(exp(-2*atan(a*x)),x)
```

output
```
int(1/e**(2*atan(a*x)),x)
```

3.305 $\int \frac{e^{-2\arctan(ax)}}{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \frac{e^{-2\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{e^{-2\arctan(ax)}}{2ac}$$

output
```
-1/2/a/c/exp(2*arctan(a*x))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{e^{-2\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{(1-iax)^{-i}(1+iax)^i}{2ac}$$

input
```
Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)),x]
```

output
```
-1/2*(1 + I*a*x)^I/(a*c*(1 - I*a*x)^I)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5594}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\arctan(ax)}}{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5594}$$

$$-\frac{e^{-2\arctan(ax)}}{2ac}$$

input
```
Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)),x]
```

output
```
-1/2*1/(a*c*E^(2*ArcTan[a*x]))
```

Defintions of rubi rules used

rule 5594
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{e^{-2\arctan(ax)}}{2ac}$	18
parallelrisch	$-\frac{e^{-2\arctan(ax)}}{2ac}$	18
risch	$-\frac{(-iax+1)^{-i}(iax+1)^{i}}{2ac}$	33
orering	$-\frac{(a^2x^2+1)e^{-2\arctan(ax)}}{2a(a^2c\,x^2+c)}$	36

input
```
int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/c/exp(2*arctan(a*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^{-2\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{e^{(-2\arctan(ax))}}{2\,ac}$$

input
```
integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
-1/2*e^(-2*arctan(a*x))/(a*c)
```

Sympy [A] (verification not implemented)

Time = 6.66 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{e^{-2\arctan(ax)}}{c+a^2cx^2}\,dx = \begin{cases} -\frac{e^{-2\operatorname{atan}(ax)}}{2ac} & \text{for}\ a \neq 0 \\ \frac{x}{c} & \text{otherwise} \end{cases}$$

input
```
integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c),x)
```

output
```
Piecewise((-exp(-2*atan(a*x))/(2*a*c), Ne(a, 0)), (x/c, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{e^{-2\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{e^{(-2\arctan(ax))}}{a^3cx^2+ac}$$

input
```
integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
-e^(-2*arctan(a*x))/(a^3*c*x^2 + a*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^{-2\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{e^{(-2\arctan(ax))}}{2\,ac}$$

input
```
integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
-1/2*e^(-2*arctan(a*x))/(a*c)
```

Mupad [B] (verification not implemented)

Time = 23.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^{-2\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{e^{-2\,\mathrm{atan}(a\,x)}}{2\,a\,c}$$

input
```
int(exp(-2*atan(a*x))/(c + a^2*c*x^2),x)
```

output
```
-exp(-2*atan(a*x))/(2*a*c)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{1}{2e^{2atan(ax)}ac}$$

input
```
int(1/exp(2*atan(a*x))/(a^2*c*x^2+c),x)
```

output
```
( - 1)/(2*e**(2*atan(a*x))*a*c)
```

3.306 $\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = -\frac{e^{-2\arctan(ax)}}{8ac^2} - \frac{e^{-2\arctan(ax)}(1-ax)}{4ac^2(1+a^2x^2)}$$

output
```
-1/8/a/c^2/exp(2*arctan(a*x))-1/4*(-a*x+1)/a/c^2/exp(2*arctan(a*x))/(a^2*x
^2+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = -\frac{(1-iax)^{-i}(1+iax)^{i}(3-2ax+a^2x^2)}{8c^2(a+a^3x^2)}$$

input
```
Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^2),x]
```

output
```
-1/8*((1 + I*a*x)^I*(3 - 2*a*x + a^2*x^2))/(c^2*(1 - I*a*x)^I*(a + a^3*x^2
))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5593, 27, 5594}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\arctan(ax)}}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \; 5593$$

$$\frac{\int \frac{e^{-2\arctan(ax)}}{c(a^2x^2+1)}dx}{4c}-\frac{(1-ax)e^{-2\arctan(ax)}}{4ac^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{e^{-2\arctan(ax)}}{a^2x^2+1}dx}{4c^2}-\frac{(1-ax)e^{-2\arctan(ax)}}{4ac^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \; 5594$$

$$-\frac{(1-ax)e^{-2\arctan(ax)}}{4ac^2\left(a^2x^2+1\right)}-\frac{e^{-2\arctan(ax)}}{8ac^2}$$

input
```
Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^2),x]
```

output
```
-1/8*1/(a*c^2*E^(2*ArcTan[a*x])) - (1 - a*x)/(4*a*c^2*E^(2*ArcTan[a*x])*(1
 + a^2*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5593
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2)))   I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
&& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
 0] && IntegerQ[2*p]
```

rule 5594
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Maple [A] (verified)

Time = 3.66 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

method	result	size
gosper	$-\frac{(a^{2} x^{2}-2 a x +3) {\mathrm e}^{-2 \arctan \left(a x \right)}}{8 (a^{2} x^{2}+1) c^{2} a}$	42
parallelrisch	$\frac{(-a^{2} x^{2}+2 a x -3) {\mathrm e}^{-2 \arctan \left(a x \right)}}{8 c^{2} (a^{2} x^{2}+1) a}$	43
orering	$-\frac{(a^{2} x^{2}-2 a x +3) (a^{2} x^{2}+1) {\mathrm e}^{-2 \arctan \left(a x \right)}}{8 a (a^{2} c \,x^{2}+c)^{2}}$	49

input
```
int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(a^2*x^2-2*a*x+3)/(a^2*x^2+1)/c^2/exp(2*arctan(a*x))/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.74

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = -\frac{(a^2x^2 - 2\,ax + 3)e^{(-2\,\arctan(ax))}}{8\,(a^3c^2x^2 + ac^2)}$$

input
```
integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/8*(a^2*x^2 - 2*a*x + 3)*e^(-2*arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(48) = 96.

Time = 38.44 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.30

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \begin{cases} -\frac{a^2x^2}{8a^3c^2x^2e^{2\,\mathrm{atan}\,(ax)}+8ac^2e^{2\,\mathrm{atan}\,(ax)}} + \frac{2ax}{8a^3c^2x^2e^{2\,\mathrm{atan}\,(ax)}+8ac^2e^{2\,\mathrm{atan}\,(ax)}} - \frac{3}{8a^3c^2x^2e^{2\,\mathrm{atan}\,(ax)}+8ac^2e^{2\,\mathrm{atan}\,(ax)}} & \text{for } a \neq 0 \\ \frac{x}{c^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**2,x)
```

output
```
Piecewise((-a**2*x**2/(8*a**3*c**2*x**2*exp(2*atan(a*x)) + 8*a*c**2*exp(2*
atan(a*x))) + 2*a*x/(8*a**3*c**2*x**2*exp(2*atan(a*x)) + 8*a*c**2*exp(2*at
an(a*x))) - 3/(8*a**3*c**2*x**2*exp(2*atan(a*x)) + 8*a*c**2*exp(2*atan(a*x
))), Ne(a, 0)), (x/c**2, True))
```

Maxima [F]

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{e^{(-2\,\arctan(ax))}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(47) = 94$.

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.93

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = -\frac{3\tan\left(\frac{1}{2}\arctan(ax)\right)^4 + 4\tan\left(\frac{1}{2}\arctan(ax)\right)^3 - 2\tan\left(\frac{1}{2}\arctan(ax)\right)^2 - 4\tan\left(\frac{1}{2}\arctan(ax)\right) + 3}{8\left(ac^2e^{(2\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^4 + 2ac^2e^{(2\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^2 + ac^2e^{(2\arctan(ax))}\right)}$$

input
```
integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
-1/8*(3*tan(1/2*arctan(a*x))^4 + 4*tan(1/2*arctan(a*x))^3 - 2*tan(1/2*arct
an(a*x))^2 - 4*tan(1/2*arctan(a*x)) + 3)/(a*c^2*e^(2*arctan(a*x))*tan(1/2*
arctan(a*x))^4 + 2*a*c^2*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^2 + a*c^2*
e^(2*arctan(a*x)))
```

Mupad [B] (verification not implemented)

Time = 24.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = -\frac{e^{-2\,\mathrm{atan}(a\,x)}\left(\frac{3}{8\,a^3\,c^2} - \frac{x}{4\,a^2\,c^2} + \frac{x^2}{8\,a\,c^2}\right)}{\frac{1}{a^2} + x^2}$$

input
```
int(exp(-2*atan(a*x))/(c + a^2*c*x^2)^2,x)
```

output
```
-(exp(-2*atan(a*x))*(3/(8*a^3*c^2) - x/(4*a^2*c^2) + x^2/(8*a*c^2)))/(1/a^
2 + x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{-a^2x^2+2ax-3}{8e^{2atan(ax)}a\,c^2\left(a^2x^2+1\right)}$$

input
```
int(1/exp(2*atan(a*x))/(a^2*c*x^2+c)^2,x)
```

output
```
( - a**2*x**2 + 2*a*x - 3)/(8*e**(2*atan(a*x))*a*c**2*(a**2*x**2 + 1))
```

3.307 $\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = -\frac{3e^{-2\arctan(ax)}}{40ac^3} - \frac{e^{-2\arctan(ax)}(1-2ax)}{10ac^3(1+a^2x^2)^2} - \frac{3e^{-2\arctan(ax)}(1-ax)}{20ac^3(1+a^2x^2)}$$

output
```
-3/40/a/c^3/exp(2*arctan(a*x))-1/10*(-2*a*x+1)/a/c^3/exp(2*arctan(a*x))/(a
^2*x^2+1)^2-3/20*(-a*x+1)/a/c^3/exp(2*arctan(a*x))/(a^2*x^2+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{e^{-2\arctan(ax)}(-4+8ax) - 3(1-iax)^{-i}(1+iax)^i(1+a^2x^2)(3-2ax+a^2x^2)}{40ac^3(1+a^2x^2)^2}$$

input
```
Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^3),x]
```

output
```
((-4 + 8*a*x)/E^(2*ArcTan[a*x]) - (3*(1 + I*a*x)^I*(1 + a^2*x^2)*(3 - 2*a*
x + a^2*x^2))/(1 - I*a*x)^I)/(40*a*c^3*(1 + a^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5593, 27, 5593, 5594}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\arctan(ax)}}{(a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{ 5593}$$

$$\frac{3\int \frac{e^{-2\arctan(ax)}}{c^2(a^2x^2+1)^2}dx}{5c}-\frac{(1-2ax)e^{-2\arctan(ax)}}{10ac^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3\int \frac{e^{-2\arctan(ax)}}{(a^2x^2+1)^2}dx}{5c^3}-\frac{(1-2ax)e^{-2\arctan(ax)}}{10ac^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{ 5593}$$

$$\frac{3\left(\frac{1}{4}\int \frac{e^{-2\arctan(ax)}}{a^2x^2+1}dx-\frac{(1-ax)e^{-2\arctan(ax)}}{4a(a^2x^2+1)}\right)}{5c^3}-\frac{(1-2ax)e^{-2\arctan(ax)}}{10ac^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{ 5594}$$

$$\frac{3\left(-\frac{(1-ax)e^{-2\arctan(ax)}}{4a(a^2x^2+1)}-\frac{e^{-2\arctan(ax)}}{8a}\right)}{5c^3}-\frac{(1-2ax)e^{-2\arctan(ax)}}{10ac^3(a^2x^2+1)^2}$$

input
```
Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^3),x]
```

output
```
-1/10*(1 - 2*a*x)/(a*c^3*E^(2*ArcTan[a*x])*(1 + a^2*x^2)^2) + (3*(-1/8*1/(
a*E^(2*ArcTan[a*x])) - (1 - a*x)/(4*a*E^(2*ArcTan[a*x])*(1 + a^2*x^2))))/(
5*c^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5593
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2)))   I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
&& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
 0] && IntegerQ[2*p]
```

rule 5594
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Maple [A] (verified)

Time = 14.87 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{\left(3 a^{4} x^{4}-6 a^{3} x^{3}+12 a^{2} x^{2}-14 a x +13\right) {\mathrm e}^{-2 \arctan \left(a x \right)}}{40 \left(a^{2} x^{2}+1\right)^{2} c^{3} a}$	59
parallelrisch	$\frac{\left(-3 a^{4} x^{4}+6 a^{3} x^{3}-12 a^{2} x^{2}+14 a x -13\right) {\mathrm e}^{-2 \arctan \left(a x \right)}}{40 c^{3} \left(a^{2} x^{2}+1\right)^{2} a}$	59
orering	$-\frac{\left(3 a^{4} x^{4}-6 a^{3} x^{3}+12 a^{2} x^{2}-14 a x +13\right) \left(a^{2} x^{2}+1\right) {\mathrm e}^{-2 \arctan \left(a x \right)}}{40 a \left(a^{2} c \,x^{2}+c \right)^{3}}$	66

input
```
int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/40*(3*a^4*x^4-6*a^3*x^3+12*a^2*x^2-14*a*x+13)/(a^2*x^2+1)^2/c^3/exp(2*a
rctan(a*x))/a
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = -\frac{(3\,a^4x^4-6\,a^3x^3+12\,a^2x^2-14\,ax+13)e^{(-2\,\arctan(ax))}}{40\,(a^5c^3x^4+2\,a^3c^3x^2+ac^3)}$$

input
```
integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/40*(3*a^4*x^4 - 6*a^3*x^3 + 12*a^2*x^2 - 14*a*x + 13)*e^(-2*arctan(a*x)
)/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(82) = 164.

Time = 173.99 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.55

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \begin{cases} -\frac{3a^4x^4}{40a^5c^3x^4e^{2\operatorname{atan}(ax)}+80a^3c^3x^2e^{2\operatorname{atan}(ax)}+40ac^3e^{2\operatorname{atan}(ax)}} + \frac{6a^3x^3}{40a^5c^3x^4e^{2\operatorname{atan}(ax)}+80a^3c^3x^2e^{2\operatorname{atan}(ax)}+40ac^3e^{2\operatorname{atan}(ax)}} - \frac{}{40a^5c^3} \\ \frac{x}{c^3} \end{cases}$$

input
```
integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**3,x)
```

output
```
Piecewise((-3*a**4*x**4/(40*a**5*c**3*x**4*exp(2*atan(a*x)) + 80*a**3*c**3
*x**2*exp(2*atan(a*x)) + 40*a*c**3*exp(2*atan(a*x))) + 6*a**3*x**3/(40*a**
5*c**3*x**4*exp(2*atan(a*x)) + 80*a**3*c**3*x**2*exp(2*atan(a*x)) + 40*a*c
**3*exp(2*atan(a*x))) - 12*a**2*x**2/(40*a**5*c**3*x**4*exp(2*atan(a*x)) +
 80*a**3*c**3*x**2*exp(2*atan(a*x)) + 40*a*c**3*exp(2*atan(a*x))) + 14*a*x
/(40*a**5*c**3*x**4*exp(2*atan(a*x)) + 80*a**3*c**3*x**2*exp(2*atan(a*x))
+ 40*a*c**3*exp(2*atan(a*x))) - 13/(40*a**5*c**3*x**4*exp(2*atan(a*x)) + 8
0*a**3*c**3*x**2*exp(2*atan(a*x)) + 40*a*c**3*exp(2*atan(a*x))), Ne(a, 0))
, (x/c**3, True))
```

Maxima [F]

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{e^{(-2\arctan(ax))}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(79) = 158$.

Time = 0.13 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.16

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = -\frac{13\tan\left(\frac{1}{2}\arctan(ax)\right)^8 + 28\tan\left(\frac{1}{2}\arctan(ax)\right)^7 - 4\tan\left(\frac{1}{2}\arctan(ax)\right)^6 - 36\tan\left(\frac{1}{2}\arctan(ax)\right)}{40\left(ac^3e^{(2\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^8 + 4\,ac^3e^{(2\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^6 + 6\,a\right.}$$

input
```
integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
-1/40*(13*tan(1/2*arctan(a*x))^8 + 28*tan(1/2*arctan(a*x))^7 - 4*tan(1/2*a
rctan(a*x))^6 - 36*tan(1/2*arctan(a*x))^5 + 30*tan(1/2*arctan(a*x))^4 + 36
*tan(1/2*arctan(a*x))^3 - 4*tan(1/2*arctan(a*x))^2 - 28*tan(1/2*arctan(a*x
)) + 13)/(a*c^3*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^8 + 4*a*c^3*e^(2*ar
ctan(a*x))*tan(1/2*arctan(a*x))^6 + 6*a*c^3*e^(2*arctan(a*x))*tan(1/2*arct
an(a*x))^4 + 4*a*c^3*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^2 + a*c^3*e^(2
*arctan(a*x)))
```

Mupad [B] (verification not implemented)

Time = 23.90 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{3\,\mathrm{e}^{-2\,\mathrm{atan}(a\,x)}\,(a\,x-1)}{20\,a\,c^3\,\left(a^2\,x^2+1\right)} - \frac{3\,\mathrm{e}^{-2\,\mathrm{atan}(a\,x)}}{40\,a\,c^3} + \frac{\mathrm{e}^{-2\,\mathrm{atan}(a\,x)}\,(2\,a\,x-1)}{10\,a\,c^3\,{\left(a^2\,x^2+1\right)}^2}$$

input
```
int(exp(-2*atan(a*x))/(c + a^2*c*x^2)^3,x)
```

output
```
(3*exp(-2*atan(a*x))*(a*x - 1))/(20*a*c^3*(a^2*x^2 + 1)) - (3*exp(-2*atan(
a*x)))/(40*a*c^3) + (exp(-2*atan(a*x))*(2*a*x - 1))/(10*a*c^3*(a^2*x^2 + 1
)^2)
```

Reduce [F]

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{-4e^{2atan(ax)}\left(\int \frac{x}{e^{2atan(ax)}a^6x^6+3e^{2atan(ax)}a^4x^4+3e^{2atan(ax)}a^2x^2+e^{2atan(ax)}}dx\right)a^6x^4 - 8e^{2atan(ax)}\left(\int \frac{}{e^{2atan(ax)}a^6x^6+3e^{2atan}}\right.}{2e^{2atan(ax)}a\,c^3}$$

input
```
int(1/exp(2*atan(a*x))/(a^2*c*x^2+c)^3,x)
```

output
```
( - 4*e**(2*atan(a*x))*int(x/(e**(2*atan(a*x))*a**6*x**6 + 3*e**(2*atan(a*
x))*a**4*x**4 + 3*e**(2*atan(a*x))*a**2*x**2 + e**(2*atan(a*x))),x)*a**6*x
**4 - 8*e**(2*atan(a*x))*int(x/(e**(2*atan(a*x))*a**6*x**6 + 3*e**(2*atan(
a*x))*a**4*x**4 + 3*e**(2*atan(a*x))*a**2*x**2 + e**(2*atan(a*x))),x)*a**4
*x**2 - 4*e**(2*atan(a*x))*int(x/(e**(2*atan(a*x))*a**6*x**6 + 3*e**(2*ata
n(a*x))*a**4*x**4 + 3*e**(2*atan(a*x))*a**2*x**2 + e**(2*atan(a*x))),x)*a*
*2 - 1)/(2*e**(2*atan(a*x))*a*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

3.308 $\int \frac{e^{-2\arctan(ax)}}{\left(c+a^2cx^2\right)^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 124

$$\int \frac{e^{-2\arctan(ax)}}{\left(c+a^2cx^2\right)^4}\,dx = -\frac{9e^{-2\arctan(ax)}}{160ac^4} - \frac{e^{-2\arctan(ax)}(1-3ax)}{20ac^4\left(1+a^2x^2\right)^3} - \frac{3e^{-2\arctan(ax)}(1-2ax)}{40ac^4\left(1+a^2x^2\right)^2} - \frac{9e^{-2\arctan(ax)}(1-ax)}{80ac^4\left(1+a^2x^2\right)}$$

output
```
-9/160/a/c^4/exp(2*arctan(a*x))-1/20*(-3*a*x+1)/a/c^4/exp(2*arctan(a*x))/(
a^2*x^2+1)^3-3/40*(-2*a*x+1)/a/c^4/exp(2*arctan(a*x))/(a^2*x^2+1)^2-9/80*(
-a*x+1)/a/c^4/exp(2*arctan(a*x))/(a^2*x^2+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int \frac{e^{-2\arctan(ax)}}{\left(c+a^2cx^2\right)^4}\,dx = \frac{8ce^{-2\arctan(ax)}(-1+3ax) - 3(c+a^2cx^2)\left(e^{-2\arctan(ax)}(4-8ax) + 3(1-iax)^{-i}(1+iax)^{i}(-i+ax)(i+\right.}{160ac^2\left(c+a^2cx^2\right)^3}$$

input
```
Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^4),x]
```

output
```
((8*c*(-1 + 3*a*x))/E^(2*ArcTan[a*x]) - 3*(c + a^2*c*x^2)*((4 - 8*a*x)/E^(
2*ArcTan[a*x]) + (3*(1 + I*a*x)^I*(-I + a*x)*(I + a*x)*(3 - 2*a*x + a^2*x^
2))/(1 - I*a*x)^I))/(160*a*c^2*(c + a^2*c*x^2)^3)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5593, 27, 5593, 5593, 5594}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\arctan(ax)}}{(a^2cx^2+c)^4}dx$$

$$\Big\downarrow \text{ 5593}$$

$$\frac{3\int \frac{e^{-2\arctan(ax)}}{c^3(a^2x^2+1)^3}dx}{4c}-\frac{(1-3ax)e^{-2\arctan(ax)}}{20ac^4(a^2x^2+1)^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3\int \frac{e^{-2\arctan(ax)}}{(a^2x^2+1)^3}dx}{4c^4}-\frac{(1-3ax)e^{-2\arctan(ax)}}{20ac^4(a^2x^2+1)^3}$$

$$\Big\downarrow \text{ 5593}$$

$$\frac{3\left(\frac{3}{5}\int \frac{e^{-2\arctan(ax)}}{(a^2x^2+1)^2}dx-\frac{(1-2ax)e^{-2\arctan(ax)}}{10a(a^2x^2+1)^2}\right)}{4c^4}-\frac{(1-3ax)e^{-2\arctan(ax)}}{20ac^4(a^2x^2+1)^3}$$

$$\Big\downarrow \text{ 5593}$$

$$\frac{3\left(\frac{3}{5}\left(\frac{1}{4}\int \frac{e^{-2\arctan(ax)}}{a^2x^2+1}dx-\frac{(1-ax)e^{-2\arctan(ax)}}{4a(a^2x^2+1)}\right)-\frac{(1-2ax)e^{-2\arctan(ax)}}{10a(a^2x^2+1)^2}\right)}{4c^4}-\frac{(1-3ax)e^{-2\arctan(ax)}}{20ac^4(a^2x^2+1)^3}$$

$$\Big\downarrow \text{ 5594}$$

$$\frac{3\left(\frac{3}{5}\left(-\frac{(1-ax)e^{-2\arctan(ax)}}{4a(a^2x^2+1)}-\frac{e^{-2\arctan(ax)}}{8a}\right)-\frac{(1-2ax)e^{-2\arctan(ax)}}{10a(a^2x^2+1)^2}\right)}{4c^4}-\frac{(1-3ax)e^{-2\arctan(ax)}}{20ac^4\left(a^2x^2+1\right)^3}$$

input
```
Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^4),x]
```

output
```
-1/20*(1 - 3*a*x)/(a*c^4*E^(2*ArcTan[a*x])*(1 + a^2*x^2)^3) + (3*(-1/10*(1
 - 2*a*x)/(a*E^(2*ArcTan[a*x])*(1 + a^2*x^2)^2) + (3*(-1/8*1/(a*E^(2*ArcTa
n[a*x])) - (1 - a*x)/(4*a*E^(2*ArcTan[a*x])*(1 + a^2*x^2))))/5))/(4*c^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5593
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2)))   I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
&& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
 0] && IntegerQ[2*p]
```

rule 5594
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Maple [A] (verified)

Time = 47.54 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{\left(9x^{6}a^{6}-18a^{5}x^{5}+45a^{4}x^{4}-60a^{3}x^{3}+75a^{2}x^{2}-66ax+47\right) {\mathrm e}^{-2 \arctan \left(a x \right)}}{160 \left(a^{2} x^{2}+1\right)^{3} c^{4} a}$	75
parallelrisch	$\frac{\left(-9x^{6}a^{6}+18a^{5}x^{5}-45a^{4}x^{4}+60a^{3}x^{3}-75a^{2}x^{2}+66ax-47\right) {\mathrm e}^{-2 \arctan \left(a x \right)}}{160 c^{4} \left(a^{2} x^{2}+1\right)^{3} a}$	75
orering	$-\frac{\left(9x^{6}a^{6}-18a^{5}x^{5}+45a^{4}x^{4}-60a^{3}x^{3}+75a^{2}x^{2}-66ax+47\right) \left(a^{2} x^{2}+1\right) {\mathrm e}^{-2 \arctan \left(a x \right)}}{160 a \left(a^{2} c \,x^{2}+c \right)^{4}}$	82

input
```
int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/160*(9*a^6*x^6-18*a^5*x^5+45*a^4*x^4-60*a^3*x^3+75*a^2*x^2-66*a*x+47)/(
a^2*x^2+1)^3/c^4/exp(2*arctan(a*x))/a
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.77

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = -\frac{(9\,a^6x^6-18\,a^5x^5+45\,a^4x^4-60\,a^3x^3+75\,a^2x^2-66\,ax+47)e^{(-2\arctan(ax))}}{160\,(a^7c^4x^6+3\,a^5c^4x^4+3\,a^3c^4x^2+ac^4)}$$

input
```
integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="fricas")
```

output
```
-1/160*(9*a^6*x^6 - 18*a^5*x^5 + 45*a^4*x^4 - 60*a^3*x^3 + 75*a^2*x^2 - 66
*a*x + 47)*e^(-2*arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*a^3*c^4*x^2
 + a*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \text{Timed out}$$

input
```
integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \int \frac{e^{(-2\arctan(ax))}}{(a^2cx^2+c)^4}\,dx$$

input
```
integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="maxima")
```

output
```
integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^4, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(111) = 222$.

Time = 0.15 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.26

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = -\frac{47\tan\left(\frac{1}{2}\arctan(ax)\right)^{12} + 132\tan\left(\frac{1}{2}\arctan(ax)\right)^{11} + 18\tan\left(\frac{1}{2}\arctan(ax)\right)^{10} - 180\tan\left(\frac{1}{2}\arctan\right.}{160\left(ac^4e^{(2\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^{12} + 6\,ac^4e^{(2\arctan(ax))}\tan\left(\frac{1}{2}\right.\right.}$$

input
```
integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="giac")
```

output
```
-1/160*(47*tan(1/2*arctan(a*x))^12 + 132*tan(1/2*arctan(a*x))^11 + 18*tan(
1/2*arctan(a*x))^10 - 180*tan(1/2*arctan(a*x))^9 + 225*tan(1/2*arctan(a*x)
)^8 + 456*tan(1/2*arctan(a*x))^7 - 4*tan(1/2*arctan(a*x))^6 - 456*tan(1/2*
arctan(a*x))^5 + 225*tan(1/2*arctan(a*x))^4 + 180*tan(1/2*arctan(a*x))^3 +
 18*tan(1/2*arctan(a*x))^2 - 132*tan(1/2*arctan(a*x)) + 47)/(a*c^4*e^(2*ar
ctan(a*x))*tan(1/2*arctan(a*x))^12 + 6*a*c^4*e^(2*arctan(a*x))*tan(1/2*arc
tan(a*x))^10 + 15*a*c^4*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^8 + 20*a*c^
4*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^6 + 15*a*c^4*e^(2*arctan(a*x))*ta
n(1/2*arctan(a*x))^4 + 6*a*c^4*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^2 +
a*c^4*e^(2*arctan(a*x)))
```

Mupad [B] (verification not implemented)

Time = 23.41 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{9\,\mathrm{e}^{-2\,\mathrm{atan}(a\,x)}\,(a\,x-1)}{80\,a\,c^4\,(a^2\,x^2+1)} - \frac{9\,\mathrm{e}^{-2\,\mathrm{atan}(a\,x)}}{160\,a\,c^4} + \frac{3\,\mathrm{e}^{-2\,\mathrm{atan}(a\,x)}\,(2\,a\,x-1)}{40\,a\,c^4\,(a^2\,x^2+1)^2} + \frac{\mathrm{e}^{-2\,\mathrm{atan}(a\,x)}\,(3\,a\,x-1)}{20\,a\,c^4\,(a^2\,x^2+1)^3}$$

input
```
int(exp(-2*atan(a*x))/(c + a^2*c*x^2)^4,x)
```

output
```
(9*exp(-2*atan(a*x))*(a*x - 1))/(80*a*c^4*(a^2*x^2 + 1)) - (9*exp(-2*atan(
a*x)))/(160*a*c^4) + (3*exp(-2*atan(a*x))*(2*a*x - 1))/(40*a*c^4*(a^2*x^2
+ 1)^2) + (exp(-2*atan(a*x))*(3*a*x - 1))/(20*a*c^4*(a^2*x^2 + 1)^3)
```

Reduce [F]

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{-6e^{2atan(ax)}\left(\int \frac{x}{e^{2atan(ax)}a^8x^8+4e^{2atan(ax)}a^6x^6+6e^{2atan(ax)}a^4x^4+4e^{2atan(ax)}a^2x^2+e^{2atan(ax)}}dx\right)a^8x^6-18e^{2atan(ax)}\left(\int \frac{}{e^{2ata}}\right.}{}$$

input
```
int(1/exp(2*atan(a*x))/(a^2*c*x^2+c)^4,x)
```

output

```
( - 6*e**(2*atan(a*x))*int(x/(e**(2*atan(a*x))*a**8*x**8 + 4*e**(2*atan(a*
x))*a**6*x**6 + 6*e**(2*atan(a*x))*a**4*x**4 + 4*e**(2*atan(a*x))*a**2*x**
2 + e**(2*atan(a*x))),x)*a**8*x**6 - 18*e**(2*atan(a*x))*int(x/(e**(2*atan
(a*x))*a**8*x**8 + 4*e**(2*atan(a*x))*a**6*x**6 + 6*e**(2*atan(a*x))*a**4*
x**4 + 4*e**(2*atan(a*x))*a**2*x**2 + e**(2*atan(a*x))),x)*a**6*x**4 - 18*
e**(2*atan(a*x))*int(x/(e**(2*atan(a*x))*a**8*x**8 + 4*e**(2*atan(a*x))*a*
*6*x**6 + 6*e**(2*atan(a*x))*a**4*x**4 + 4*e**(2*atan(a*x))*a**2*x**2 + e*
*(2*atan(a*x))),x)*a**4*x**2 - 6*e**(2*atan(a*x))*int(x/(e**(2*atan(a*x))*
a**8*x**8 + 4*e**(2*atan(a*x))*a**6*x**6 + 6*e**(2*atan(a*x))*a**4*x**4 +
4*e**(2*atan(a*x))*a**2*x**2 + e**(2*atan(a*x))),x)*a**2 - 1)/(2*e**(2*ata
n(a*x))*a*c**4*(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1))
```

3.309 $\int e^{-2\arctan(ax)}(c+a^2cx^2)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx = -\frac{\left(\frac{2}{29}-\frac{5i}{29}\right)2^{\frac{5}{2}+i}c(1-iax)^{\frac{5}{2}-i}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}-i,\frac{5}{2}-i,\frac{7}{2}-i,\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

output
```
(-2/29+5/29*I)*2^(5/2+I)*c*(1-I*a*x)^(5/2-I)*(a^2*c*x^2+c)^(1/2)*hypergeom
([-3/2-I, 5/2-I],[7/2-I],1/2-1/2*I*a*x)/a/(a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx = -\frac{\left(\frac{2}{29}-\frac{5i}{29}\right)2^{\frac{5}{2}+i}c(1-iax)^{\frac{5}{2}-i}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}-i,\frac{5}{2}-i,\frac{7}{2}-i,\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/E^(2*ArcTan[a*x]),x]
```

output
```
((-2/29 + (5*I)/29)*2^(5/2 + I)*c*(1 - I*a*x)^(5/2 - I)*Sqrt[c + a^2*c*x^2
]*Hypergeometric2F1[-3/2 - I, 5/2 - I, 7/2 - I, (1 - I*a*x)/2])/(a*Sqrt[1
+ a^2*x^2])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5599, 5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}dx$$

$$\Big\downarrow \ 5599$$

$$\frac{c\sqrt{a^2cx^2+c}\int e^{-2\arctan(ax)}\left(a^2x^2+1\right)^{3/2}dx}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \ 5596$$

$$\frac{c\sqrt{a^2cx^2+c}\int (1-iax)^{\frac{3}{2}-i}(iax+1)^{\frac{3}{2}+i}dx}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \ 79$$

$$-\frac{\left(\frac{2}{29}-\frac{5i}{29}\right)2^{\frac{5}{2}+i}c(1-iax)^{\frac{5}{2}-i}\sqrt{a^2cx^2+c}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}-i,\frac{5}{2}-i,\frac{7}{2}-i,\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/E^(2*ArcTan[a*x]),x]
```

output
```
((-2/29 + (5*I)/29)*2^(5/2 + I)*c*(1 - I*a*x)^(5/2 - I)*Sqrt[c + a^2*c*x^2
]*Hypergeometric2F1[-3/2 - I, 5/2 - I, 7/2 - I, (1 - I*a*x)/2])/(a*Sqrt[1
+ a^2*x^2])
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} {\mathrm e}^{-2 \arctan \left(a x \right)}d x$$

input
```
int((a^2*c*x^2+c)^(3/2)/exp(2*arctan(a*x)),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/exp(2*arctan(a*x)),x)
```

Fricas [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int \left(a^{2} c x^{2} + c\right)^{\frac{3}{2}} e^{\left(-2 \, \arctan\left(a x\right)\right)}\,{d x}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/exp(2*arctan(a*x)),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*e^(-2*arctan(a*x)), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/exp(2*atan(a*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int\left(a^2cx^2+c\right)^{\frac{3}{2}}e^{(-2\arctan(ax))}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/exp(2*arctan(a*x)),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*e^(-2*arctan(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/exp(2*arctan(a*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int \mathrm{e}^{-2\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(exp(-2*atan(a*x))*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(exp(-2*atan(a*x))*(c + a^2*c*x^2)^(3/2), x)
```

Reduce [F]

$$\int e^{-2\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\sqrt{c}\,c\left(\int\frac{\sqrt{a^2x^2+1}}{e^{2atan(ax)}}dx+\left(\int\frac{\sqrt{a^2x^2+1}\,x^2}{e^{2atan(ax)}}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)/exp(2*atan(a*x)),x)
```

output
```
sqrt(c)*c*(int(sqrt(a**2*x**2 + 1)/e**(2*atan(a*x)),x) + int((sqrt(a**2*x*
*2 + 1)*x**2)/e**(2*atan(a*x)),x)*a**2)
```

3.310 $\int e^{-2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int e^{-2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = -\frac{\left(\frac{2}{13}-\frac{3i}{13}\right)2^{\frac{3}{2}+i}(1-iax)^{\frac{3}{2}-i}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}-i,\frac{3}{2}-i,\frac{5}{2}-i,\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

output
```
(-2/13+3/13*I)*2^(3/2+I)*(1-I*a*x)^(3/2-I)*(a^2*c*x^2+c)^(1/2)*hypergeom([
-1/2-I, 3/2-I],[5/2-I],1/2-1/2*I*a*x)/a/(a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int e^{-2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = -\frac{\left(\frac{2}{13}-\frac{3i}{13}\right)2^{\frac{3}{2}+i}(1-iax)^{\frac{3}{2}-i}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}-i,\frac{3}{2}-i,\frac{5}{2}-i,\frac{1}{2}(1-iax)\right)}{a\sqrt{1+a^2x^2}}$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/E^(2*ArcTan[a*x]),x]
```

output
```
((-2/13 + (3*I)/13)*2^(3/2 + I)*(1 - I*a*x)^(3/2 - I)*Sqrt[c + a^2*c*x^2]*
Hypergeometric2F1[-1/2 - I, 3/2 - I, 5/2 - I, (1 - I*a*x)/2])/(a*Sqrt[1 +
a^2*x^2])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5599, 5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2\arctan(ax)}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5599}$$

$$\frac{\sqrt{a^2cx^2+c}\int e^{-2\arctan(ax)}\sqrt{a^2x^2+1}dx}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{5596}$$

$$\frac{\sqrt{a^2cx^2+c}\int (1-iax)^{\frac{1}{2}-i}(iax+1)^{\frac{1}{2}+i}dx}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{\left(\frac{2}{13}-\frac{3i}{13}\right)2^{\frac{3}{2}+i}(1-iax)^{\frac{3}{2}-i}\sqrt{a^2cx^2+c}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}-i,\frac{3}{2}-i,\frac{5}{2}-i,\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2x^2+1}}$$

input
```
Int[Sqrt[c + a^2*c*x^2]/E^(2*ArcTan[a*x]),x]
```

output
```
((-2/13 + (3*I)/13)*2^(3/2 + I)*(1 - I*a*x)^(3/2 - I)*Sqrt[c + a^2*c*x^2]*
Hypergeometric2F1[-1/2 - I, 3/2 - I, 5/2 - I, (1 - I*a*x)/2])/(a*Sqrt[1 +
a^2*x^2])
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \sqrt{a^{2} c \,x^{2}+c}\, \mathrm{e}^{-2 \arctan \left(a x \right)}d x$$

input
```
int((a^2*c*x^2+c)^(1/2)/exp(2*arctan(a*x)),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/exp(2*arctan(a*x)),x)
```

Fricas [F]

$$\int e^{-2 \arctan (a x)} \sqrt{c+a^2 c x^2} \, dx = \int \sqrt{a^{2} c x^{2} + c} e^{\left(-2 \, \arctan \left(a x\right)\right)} \,d x$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/exp(2*arctan(a*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*e^(-2*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{-2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \int \sqrt{c\,(a^2x^2+1)}e^{-2\operatorname{atan}(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/exp(2*atan(a*x)),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*exp(-2*atan(a*x)), x)
```

Maxima [F]

$$\int e^{-2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \int \sqrt{a^2cx^2+c}e^{(-2\arctan(ax))}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/exp(2*arctan(a*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*e^(-2*arctan(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{-2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/exp(2*arctan(a*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{-2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \int e^{-2\,\mathrm{atan}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(exp(-2*atan(a*x))*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(exp(-2*atan(a*x))*(c + a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int e^{-2\arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}}{e^{2atan(ax)}}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)/exp(2*atan(a*x)),x)
```

output
```
sqrt(c)*int(sqrt(a**2*x**2 + 1)/e**(2*atan(a*x)),x)
```

3.311 $\int \frac{e^{-2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{e^{-2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{\left(\frac{2}{5}-\frac{i}{5}\right)2^{\frac{1}{2}+i}(1-iax)^{\frac{1}{2}-i}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-i,\frac{1}{2}-i,\frac{3}{2}-i,\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}$$

output
```
(-2/5+1/5*I)*2^(1/2+I)*(1-I*a*x)^(1/2-I)*(a^2*x^2+1)^(1/2)*hypergeom([1/2-
I, 1/2-I],[3/2-I],1/2-1/2*I*a*x)/a/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{e^{-2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{\left(\frac{2}{5}-\frac{i}{5}\right)2^{\frac{1}{2}+i}(1-iax)^{\frac{1}{2}-i}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-i,\frac{1}{2}-i,\frac{3}{2}-i,\frac{1}{2}(1-iax)\right)}{a\sqrt{c+a^2cx^2}}$$

input
```
Integrate[1/(E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

output
```
((-2/5 + I/5)*2^(1/2 + I)*(1 - I*a*x)^(1/2 - I)*Sqrt[1 + a^2*x^2]*Hypergeo
metric2F1[1/2 - I, 1/2 - I, 3/2 - I, (1 - I*a*x)/2])/(a*Sqrt[c + a^2*c*x^2
])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5599, 5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5599}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{-2\arctan(ax)}}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5596}$$

$$\frac{\sqrt{a^2x^2+1}\int (1-iax)^{-\frac{1}{2}-i}(iax+1)^{-\frac{1}{2}+i}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{\left(\frac{2}{5}-\frac{i}{5}\right)2^{\frac{1}{2}+i}(1-iax)^{\frac{1}{2}-i}\sqrt{a^2x^2+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-i,\frac{1}{2}-i,\frac{3}{2}-i,\frac{1}{2}(1-iax)\right)}{a\sqrt{a^2cx^2+c}}$$

input
```
Int[1/(E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

output

```
((-2/5 + I/5)*2^(1/2 + I)*(1 - I*a*x)^(1/2 - I)*Sqrt[1 + a^2*x^2]*Hypergeo
metric2F1[1/2 - I, 1/2 - I, 3/2 - I, (1 - I*a*x)/2])/(a*Sqrt[c + a^2*c*x^2
])
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{\mathrm{e}^{-2\arctan(ax)}}{\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{-2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(-2\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(e^(-2*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{e^{-2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{e^{-2\,\mathrm{atan}\,(ax)}}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(exp(-2*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{e^{-2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{e^{(-2\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(e^(-2*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{e^{-2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{e^{(-2\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(e^(-2*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{-2\,\mathrm{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(exp(-2*atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(exp(-2*atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{e^{-2\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{1}{e^{2atan(ax)}\sqrt{a^2x^2+1}}dx}{\sqrt{c}}$$

input
```
int(1/exp(2*atan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(1/(e**(2*atan(a*x))*sqrt(a**2*x**2 + 1)),x)/sqrt(c)
```

3.312 $\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 38

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{e^{-2\arctan(ax)}(2-ax)}{5ac\sqrt{c+a^2cx^2}}$$

output
```
-1/5*(-a*x+2)/a/c/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{e^{-2\arctan(ax)}(-2+ax)}{5ac\sqrt{c+a^2cx^2}}$$

input
```
Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
(-2 + a*x)/(5*a*c*E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {5592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\arctan(ax)}}{(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 5592}$$

$$-\frac{(2-ax)e^{-2\arctan(ax)}}{5ac\sqrt{a^2cx^2+c}}$$

input
```
Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
-1/5*(2 - a*x)/(a*c*E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2])
```

Defintions of rubi rules used

rule 5592
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

method	result	size
gosper	$\frac{(a^2x^2+1)(ax-2)\mathrm{e}^{-2\arctan(ax)}}{5a(a^2cx^2+c)^{\frac{3}{2}}}$	41
orering	$\frac{(a^2x^2+1)(ax-2)\mathrm{e}^{-2\arctan(ax)}}{5a(a^2cx^2+c)^{\frac{3}{2}}}$	41

input
```
int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*(a^2*x^2+1)*(a*x-2)/a/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{a^2cx^2+c}(ax-2)e^{(-2\arctan(ax))}}{5\left(a^3c^2x^2+ac^2\right)}$$

input
```
integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
1/5*sqrt(a^2*c*x^2 + c)*(a*x - 2)*e^(-2*arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

Sympy [F]

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{e^{-2\,\mathrm{atan}\,(ax)}}{(c\,(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(exp(-2*atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{e^{(-2\arctan(ax))}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(32) = 64$.

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.47

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{2\left(\cos\left(\frac{1}{2}\arctan(ax)\right)^4 e^{(-2\arctan(ax))} - \cos\left(\frac{1}{2}\arctan(ax)\right)^3 e^{(-2\arctan(ax))}\sin\left(\frac{1}{2}\arctan(ax)\right) - \cos\left(\frac{1}{2}\text{ a}\right.\right.}{5\,ac^{\frac{3}{2}}}$$

input
```
integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
-2/5*(cos(1/2*arctan(a*x))^4*e^(-2*arctan(a*x)) - cos(1/2*arctan(a*x))^3*e
^(-2*arctan(a*x))*sin(1/2*arctan(a*x)) - cos(1/2*arctan(a*x))*e^(-2*arctan
(a*x))*sin(1/2*arctan(a*x))^3 - e^(-2*arctan(a*x))*sin(1/2*arctan(a*x))^4)
/(a*c^(3/2))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\mathrm{e}^{-2\,\mathrm{atan}(a\,x)}\,\left(\frac{x}{5\,c}-\frac{2}{5\,a\,c}\right)}{\sqrt{c\,a^2\,x^2+c}}$$

input
```
int(exp(-2*atan(a*x))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
(exp(-2*atan(a*x))*(x/(5*c) - 2/(5*a*c)))/(c + a^2*c*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a^2x^2+1}\,(ax-2)}{5e^{2atan(ax)}a\,c^2\,(a^2x^2+1)}$$

input
```
int(1/exp(2*atan(a*x))/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a**2*x**2 + 1)*(a*x - 2))/(5*e**(2*atan(a*x))*a*c**2*(a**2*x
**2 + 1))
```

3.313 $\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{e^{-2\arctan(ax)}(2-3ax)}{13ac\,(c+a^2cx^2)^{3/2}} - \frac{6e^{-2\arctan(ax)}(2-ax)}{65ac^2\sqrt{c+a^2cx^2}}$$

output
```
-1/13*(-3*a*x+2)/a/c/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2)-6/65*(-a*x+2)/
a/c^2/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{e^{-2\arctan(ax)}(-22+21ax-12a^2x^2+6a^3x^3)}{65c^2\,(a+a^3x^2)\,\sqrt{c+a^2cx^2}}$$

input
```
Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
(-22 + 21*a*x - 12*a^2*x^2 + 6*a^3*x^3)/(65*c^2*E^(2*ArcTan[a*x])*(a + a^3
*x^2)*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5593, 5592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\arctan(ax)}}{(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \text{5593}$$

$$\frac{6\int \frac{e^{-2\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx}{13c}-\frac{(2-3ax)e^{-2\arctan(ax)}}{13ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5592}$$

$$-\frac{6(2-ax)e^{-2\arctan(ax)}}{65ac^2\sqrt{a^2cx^2+c}}-\frac{(2-3ax)e^{-2\arctan(ax)}}{13ac\,(a^2cx^2+c)^{3/2}}$$

input
```
Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
-1/13*(2 - 3*a*x)/(a*c*E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)) - (6*(2 -
a*x))/(65*a*c^2*E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2])
```

Defintions of rubi rules used

rule 5592
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

rule 5593
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2)))   I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
&& EqQ[d, a^2*c] && LtQ[p, -1] && !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

method	result	size
gosper	$\frac{\left(a^{2} x^{2}+1\right) \left(6 a^{3} x^{3}-12 a^{2} x^{2}+21 a x -22\right) {\mathrm e}^{-2 \arctan \left(a x \right)}}{65 a \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}$	58
orering	$\frac{\left(a^{2} x^{2}+1\right) \left(6 a^{3} x^{3}-12 a^{2} x^{2}+21 a x -22\right) {\mathrm e}^{-2 \arctan \left(a x \right)}}{65 a \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}$	58

input
```
int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/65*(a^2*x^2+1)*(6*a^3*x^3-12*a^2*x^2+21*a*x-22)/a/exp(2*arctan(a*x))/(a^
2*c*x^2+c)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{(6\,a^3x^3-12\,a^2x^2+21\,ax-22)\sqrt{a^2cx^2+c}e^{(-2\arctan(ax))}}{65\,(a^5c^3x^4+2\,a^3c^3x^2+ac^3)}$$

input
```
integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/65*(6*a^3*x^3 - 12*a^2*x^2 + 21*a*x - 22)*sqrt(a^2*c*x^2 + c)*e^(-2*arct
an(a*x))/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{e^{(-2\arctan(ax))}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(66) = 132$.

Time = 0.16 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.92

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{2\left(11\tan\left(\frac{1}{2}\arctan(ax)\right)^6 + 21\tan\left(\frac{1}{2}\arctan(ax)\right)^5 - 9\tan\left(\frac{1}{2}\arctan(ax)\right)^4 - 1\right.}{65\left(ac^{\frac{5}{2}}e^{(2\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^6 + 3ac^{\frac{5}{2}}e^{(2\arctan(ax))}\tan\left(\frac{1}{2}\operatorname{arcta}\right.\right.}$$

input
```
integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
2/65*(11*tan(1/2*arctan(a*x))^6 + 21*tan(1/2*arctan(a*x))^5 - 9*tan(1/2*ar
ctan(a*x))^4 - 18*tan(1/2*arctan(a*x))^3 + 9*tan(1/2*arctan(a*x))^2 + 21*t
an(1/2*arctan(a*x)) - 11)/(a*c^(5/2)*e^(2*arctan(a*x))*tan(1/2*arctan(a*x)
)^6 + 3*a*c^(5/2)*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^4 + 3*a*c^(5/2)*e
^(2*arctan(a*x))*tan(1/2*arctan(a*x))^2 + a*c^(5/2)*e^(2*arctan(a*x)))
```

Mupad [B] (verification not implemented)

Time = 23.40 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{e^{-2\operatorname{atan}(a\,x)}\left(\frac{22}{65\,a^3\,c^2} - \frac{6\,x^3}{65\,c^2} - \frac{21\,x}{65\,a^2\,c^2} + \frac{12\,x^2}{65\,a\,c^2}\right)}{\frac{\sqrt{c\,a^2\,x^2+c}}{a^2} + x^2\,\sqrt{c\,a^2\,x^2+c}}$$

input
```
int(exp(-2*atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
-(exp(-2*atan(a*x))*(22/(65*a^3*c^2) - (6*x^3)/(65*c^2) - (21*x)/(65*a^2*c
^2) + (12*x^2)/(65*a*c^2)))/((c + a^2*c*x^2)^(1/2)/a^2 + x^2*(c + a^2*c*x^
2)^(1/2))
```

Reduce [F]

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\int \frac{1}{e^{2atan(ax)}\sqrt{a^2x^2+1}\,a^4x^4+2e^{2atan(ax)}\sqrt{a^2x^2+1}\,a^2x^2+e^{2atan(ax)}\sqrt{a^2x^2+1}}dx}{\sqrt{c}\,c^2}$$

input
```
int(1/exp(2*atan(a*x))/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(1/(e**(2*atan(a*x))*sqrt(a**2*x**2 + 1)*a**4*x**4 + 2*e**(2*atan(a*x))
*sqrt(a**2*x**2 + 1)*a**2*x**2 + e**(2*atan(a*x))*sqrt(a**2*x**2 + 1)),x)/
(sqrt(c)*c**2)
```

3.314 $\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 115

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = -\frac{e^{-2\arctan(ax)}(2-5ax)}{29ac\,(c+a^2cx^2)^{5/2}} - \frac{20e^{-2\arctan(ax)}(2-3ax)}{377ac^2\,(c+a^2cx^2)^{3/2}} - \frac{24e^{-2\arctan(ax)}(2-ax)}{377ac^3\sqrt{c+a^2cx^2}}$$

output
```
-1/29*(-5*a*x+2)/a/c/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(5/2)-20/377*(-3*a*x
+2)/a/c^2/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(3/2)-24/377*(-a*x+2)/a/c^3/exp
(2*arctan(a*x))/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \frac{e^{-2\arctan(ax)}(-114+149ax-136a^2x^2+108a^3x^3-48a^4x^4+24a^5x^5)}{377ac^3\,(1+a^2x^2)^2\,\sqrt{c+a^2cx^2}}$$

input
```
Integrate[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(7/2)),x]
```

output
```
(-114 + 149*a*x - 136*a^2*x^2 + 108*a^3*x^3 - 48*a^4*x^4 + 24*a^5*x^5)/(37
7*a*c^3*E^(2*ArcTan[a*x])*(1 + a^2*x^2)^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5593, 5593, 5592}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2\arctan(ax)}}{\left(a^2cx^2+c\right)^{7/2}}dx$$

$$\Big\downarrow \text{5593}$$

$$\frac{20\int \frac{e^{-2\arctan(ax)}}{\left(a^2cx^2+c\right)^{5/2}}dx}{29c}-\frac{(2-5ax)e^{-2\arctan(ax)}}{29ac\left(a^2cx^2+c\right)^{5/2}}$$

$$\Big\downarrow \text{5593}$$

$$\frac{20\left(\frac{6\int \frac{e^{-2\arctan(ax)}}{\left(a^2cx^2+c\right)^{3/2}}dx}{13c}-\frac{(2-3ax)e^{-2\arctan(ax)}}{13ac\left(a^2cx^2+c\right)^{3/2}}\right)}{29c}-\frac{(2-5ax)e^{-2\arctan(ax)}}{29ac\left(a^2cx^2+c\right)^{5/2}}$$

$$\Big\downarrow \text{5592}$$

$$\frac{20\left(-\frac{6(2-ax)e^{-2\arctan(ax)}}{65ac^2\sqrt{a^2cx^2+c}}-\frac{(2-3ax)e^{-2\arctan(ax)}}{13ac\left(a^2cx^2+c\right)^{3/2}}\right)}{29c}-\frac{(2-5ax)e^{-2\arctan(ax)}}{29ac\left(a^2cx^2+c\right)^{5/2}}$$

input
```
Int[1/(E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(7/2)),x]
```

output
```
-1/29*(2 - 5*a*x)/(a*c*E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(5/2)) + (20*(-1/
13*(2 - 3*a*x)/(a*c*E^(2*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)) - (6*(2 - a*x
))/(65*a*c^2*E^(2*ArcTan[a*x])*Sqrt[c + a^2*c*x^2])))/(29*c)
```

Defintions of rubi rules used

rule 5592
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

rule 5593
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2)))   I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
&& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{\left(a^{2} x^{2}+1\right) \left(24 a^{5} x^{5}-48 a^{4} x^{4}+108 a^{3} x^{3}-136 a^{2} x^{2}+149 a x -114\right) {\mathrm e}^{-2 \arctan \left(a x \right)}}{377 a \left(a^{2} c \,x^{2}+c \right)^{\frac{7}{2}}}$	74
orering	$\frac{\left(a^{2} x^{2}+1\right) \left(24 a^{5} x^{5}-48 a^{4} x^{4}+108 a^{3} x^{3}-136 a^{2} x^{2}+149 a x -114\right) {\mathrm e}^{-2 \arctan \left(a x \right)}}{377 a \left(a^{2} c \,x^{2}+c \right)^{\frac{7}{2}}}$	74

input
```
int(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/377*(a^2*x^2+1)*(24*a^5*x^5-48*a^4*x^4+108*a^3*x^3-136*a^2*x^2+149*a*x-1
14)/a/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.86

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \frac{(24\,a^5x^5-48\,a^4x^4+108\,a^3x^3-136\,a^2x^2+149\,ax-114)\sqrt{a^2cx^2+c}\,e^{(-2\,\arctan(ax))}}{377\,(a^7c^4x^6+3\,a^5c^4x^4+3\,a^3c^4x^2+ac^4)}$$

input
```
integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
1/377*(24*a^5*x^5 - 48*a^4*x^4 + 108*a^3*x^3 - 136*a^2*x^2 + 149*a*x - 114
)*sqrt(a^2*c*x^2 + c)*e^(-2*arctan(a*x))/(a^7*c^4*x^6 + 3*a^5*c^4*x^4 + 3*
a^3*c^4*x^2 + a*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/exp(2*atan(a*x))/(a**2*c*x**2+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \int \frac{e^{(-2\,\arctan(ax))}}{(a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(e^(-2*arctan(a*x))/(a^2*c*x^2 + c)^(7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(99) = 198$.

Time = 0.18 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.05

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \frac{2\left(57\tan\left(\frac{1}{2}\arctan(ax)\right)^{10} + 149\tan\left(\frac{1}{2}\arctan(ax)\right)^{9} - 13\tan\left(\frac{1}{2}\arctan(ax)\right)^{8}\right.}{377\left(ac^{\frac{7}{2}}e^{(2\arctan(ax))}\tan\left(\frac{1}{2}\arctan(ax)\right)^{10} + 5\,ac^{\frac{7}{2}}e^{(2\arctan}\right.}$$

input
```
integrate(1/exp(2*arctan(a*x))/(a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
2/377*(57*tan(1/2*arctan(a*x))^10 + 149*tan(1/2*arctan(a*x))^9 - 13*tan(1/
2*arctan(a*x))^8 - 164*tan(1/2*arctan(a*x))^7 + 138*tan(1/2*arctan(a*x))^6
 + 414*tan(1/2*arctan(a*x))^5 - 138*tan(1/2*arctan(a*x))^4 - 164*tan(1/2*a
rctan(a*x))^3 + 13*tan(1/2*arctan(a*x))^2 + 149*tan(1/2*arctan(a*x)) - 57)
/(a*c^(7/2)*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^10 + 5*a*c^(7/2)*e^(2*a
rctan(a*x))*tan(1/2*arctan(a*x))^8 + 10*a*c^(7/2)*e^(2*arctan(a*x))*tan(1/
2*arctan(a*x))^6 + 10*a*c^(7/2)*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^4 +
 5*a*c^(7/2)*e^(2*arctan(a*x))*tan(1/2*arctan(a*x))^2 + a*c^(7/2)*e^(2*arc
tan(a*x)))
```

Mupad [B] (verification not implemented)

Time = 23.71 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = -\frac{e^{-2\,\mathrm{atan}(a\,x)}\left(\frac{114}{377\,a^5\,c^3} - \frac{24\,x^5}{377\,c^3} - \frac{149\,x}{377\,a^4\,c^3} + \frac{48\,x^4}{377\,a\,c^3} - \frac{108\,x^3}{377\,a^2\,c^3} + \frac{136\,x^2}{377\,a^3\,c^3}\right)}{\frac{\sqrt{c\,a^2\,x^2+c}}{a^4} + x^4\,\sqrt{c\,a^2\,x^2+c} + \frac{2\,x^2\,\sqrt{c\,a^2\,x^2+c}}{a^2}}$$

input
```
int(exp(-2*atan(a*x))/(c + a^2*c*x^2)^(7/2),x)
```

output
```
-(exp(-2*atan(a*x))*(114/(377*a^5*c^3) - (24*x^5)/(377*c^3) - (149*x)/(377
*a^4*c^3) + (48*x^4)/(377*a*c^3) - (108*x^3)/(377*a^2*c^3) + (136*x^2)/(37
7*a^3*c^3)))/((c + a^2*c*x^2)^(1/2)/a^4 + x^4*(c + a^2*c*x^2)^(1/2) + (2*x
^2*(c + a^2*c*x^2)^(1/2))/a^2)
```

Reduce [F]

$$\int \frac{e^{-2\arctan(ax)}}{(c+a^2cx^2)^{7/2}}\,dx = \frac{\int \frac{1}{e^{2atan(ax)}\sqrt{a^2x^2+1}\,a^6x^6+3e^{2atan(ax)}\sqrt{a^2x^2+1}\,a^4x^4+3e^{2atan(ax)}\sqrt{a^2x^2+1}\,a^2x^2+e^{2atan(ax)}\sqrt{a^2x^2+1}}dx}{\sqrt{c}\,c^3}$$

input
```
int(1/exp(2*atan(a*x))/(a^2*c*x^2+c)^(7/2),x)
```

output
```
int(1/(e**(2*atan(a*x))*sqrt(a**2*x**2 + 1)*a**6*x**6 + 3*e**(2*atan(a*x))
*sqrt(a**2*x**2 + 1)*a**4*x**4 + 3*e**(2*atan(a*x))*sqrt(a**2*x**2 + 1)*a*
*2*x**2 + e**(2*atan(a*x))*sqrt(a**2*x**2 + 1)),x)/(sqrt(c)*c**3)
```

3.315 $\int \frac{e^{5i \arctan(ax)}}{\sqrt{1+a^2x^2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 50

$$\int \frac{e^{5i \arctan(ax)}}{\sqrt{1+a^2x^2}} \, dx = -\frac{2i}{a(1-iax)^2} + \frac{4i}{a(1-iax)} + \frac{i \log(i+ax)}{a}$$

output
```
-2*I/a/(1-I*a*x)^2+4*I/a/(1-I*a*x)+I*ln(I+a*x)/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{e^{5i \arctan(ax)}}{\sqrt{1+a^2x^2}} \, dx = \frac{i(-2+4iax+(i+ax)^2 \log(i+ax))}{a(i+ax)^2}$$

input
```
Integrate[E^((5*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
(I*(-2 + (4*I)*a*x + (I + a*x)^2*Log[I + a*x]))/(a*(I + a*x)^2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5596, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{a^2x^2+1}}dx$$
$$\Big\downarrow \text{5596}$$
$$\int \frac{(1+iax)^2}{(1-iax)^3}dx$$
$$\Big\downarrow \text{49}$$
$$\int \left(\frac{1}{1-iax}-\frac{4}{(1-iax)^2}+\frac{4}{(1-iax)^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{4i}{a(1-iax)}-\frac{2i}{a(1-iax)^2}+\frac{i\log(ax+i)}{a}$$

input
```
Int[E^((5*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
(-2*I)/(a*(1 - I*a*x)^2) + (4*I)/(a*(1 - I*a*x)) + (I*Log[I + a*x])/a
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

method	result
default	$\frac{-4x-\frac{2i}{a}}{(ax+i)^2}+\frac{i\ln(ax+i)}{a}$
risch	$\frac{-4x-\frac{2i}{a}}{(ax+i)^2}+\frac{i\ln(a^2x^2+1)}{2a}+\frac{\arctan(ax)}{a}$
parallelrisch	$\frac{i\ln(ax+i)x^4a^4-2ix^4a^4+2i\ln(ax+i)x^2a^2-4a^3x^3+2ix^2a^2+i\ln(ax+i)}{(a^2x^2+1)^2a}$
meijerg	$\frac{\frac{x\sqrt{a^2}\left(3a^2x^2+5\right)}{2\left(a^2x^2+1\right)^2}+\frac{3\sqrt{a^2}\arctan(ax)}{2a}}{4\sqrt{a^2}}+\frac{5ia\,x^2(a^2x^2+2)}{4(a^2x^2+1)^2}-\frac{5\left(-\frac{x\left(a^2\right)^{\frac{3}{2}}\left(-3a^2x^2+3\right)}{6a^2\left(a^2x^2+1\right)^2}+\frac{\left(a^2\right)^{\frac{3}{2}}\arctan(ax)}{2a^3}\right)}{2\sqrt{a^2}}-\frac{5ia^3x^4}{2(a^2x^2+1)^2}$

input
```
int((1+I*a*x)^5/(a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
(-4*x-2*I/a)/(I+a*x)^2+I*ln(I+a*x)/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx=-\frac{4\,ax-\left(i\,a^2x^2-2\,ax-i\right)\log\left(\frac{ax+i}{a}\right)+2i}{a^3x^2+2i\,a^2x-a}$$

input
```
integrate((1+I*a*x)^5/(a^2*x^2+1)^3,x, algorithm="fricas")
```

output
```
-(4*a*x - (I*a^2*x^2 - 2*a*x - I)*log((a*x + I)/a) + 2*I)/(a^3*x^2 + 2*I*a
^2*x - a)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.72

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{-4ax-2i}{a^3x^2+2ia^2x-a} + \frac{i\log{(ax+i)}}{a}$$

input
```
integrate((1+I*a*x)**5/(a**2*x**2+1)**3,x)
```

output
```
(-4*a*x - 2*I)/(a**3*x**2 + 2*I*a**2*x - a) + I*log(a*x + I)/a
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{2\,(2\,a^3x^3-3i\,a^2x^2-i)}{a^5x^4+2\,a^3x^2+a} + \frac{\arctan{(ax)}}{a} + \frac{i\,\log{(a^2x^2+1)}}{2\,a}$$

input
```
integrate((1+I*a*x)^5/(a^2*x^2+1)^3,x, algorithm="maxima")
```

output
```
-2*(2*a^3*x^3 - 3*I*a^2*x^2 - I)/(a^5*x^4 + 2*a^3*x^2 + a) + arctan(a*x)/a
 + 1/2*I*log(a^2*x^2 + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.60

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{i\,\log{(ax+i)}}{a} - \frac{2\,(2\,ax+i)}{(ax+i)^2a}$$

input
```
integrate((1+I*a*x)^5/(a^2*x^2+1)^3,x, algorithm="giac")
```

output
```
I*log(a*x + I)/a - 2*(2*a*x + I)/((a*x + I)^2*a)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{\ln\left(x+\frac{1\mathrm{i}}{a}\right)\,1\mathrm{i}}{a} - \frac{\frac{4\,x}{a^2}+\frac{2\mathrm{i}}{a^3}}{x^2-\frac{1}{a^2}+\frac{x\,2\mathrm{i}}{a}}$$

input
```
int((a*x*1i + 1)^5/(a^2*x^2 + 1)^3,x)
```

output
```
(log(x + 1i/a)*1i)/a - ((4*x)/a^2 + 2i/a^3)/((x*2i)/a - 1/a^2 + x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.48

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2atan(ax)\,a^4x^4+4atan(ax)\,a^2x^2+2atan(ax)+\log(a^2x^2+1)\,a^4i\,x^4+2\log(a^2x^2+1)\,a^2i\,x^2+\log(a^2x^2}{2a\left(a^4x^4+2a^2x^2+1\right)}$$

input
```
int((1+I*a*x)^5/(a^2*x^2+1)^3,x)
```

output
```
(2*atan(a*x)*a**4*x**4 + 4*atan(a*x)*a**2*x**2 + 2*atan(a*x) + log(a**2*x*
*2 + 1)*a**4*i*x**4 + 2*log(a**2*x**2 + 1)*a**2*i*x**2 + log(a**2*x**2 + 1
)*i - 6*a**4*i*x**4 - 8*a**3*x**3 - 2*i)/(2*a*(a**4*x**4 + 2*a**2*x**2 + 1
))
```

3.316 $\int \frac{e^{4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 69

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx=-\frac{2i(1+iax)^3}{3a\left(1+a^2x^2\right)^{3/2}}+\frac{2i(1+iax)}{a\sqrt{1+a^2x^2}}+\frac{\operatorname{arcsinh}(ax)}{a}$$

output
```
-2/3*I*(1+I*a*x)^3/a/(a^2*x^2+1)^(3/2)+2*I*(1+I*a*x)/a/(a^2*x^2+1)^(1/2)+a
rcsinh(a*x)/a
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.70

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx=-\frac{4i\sqrt{2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1-iax)\right)}{3a(1-iax)^{3/2}}$$

input
```
Integrate[E^((4*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
(((-4*I)/3)*Sqrt[2]*Hypergeometric2F1[-3/2, -3/2, -1/2, (1 - I*a*x)/2])/(a
*(1 - I*a*x)^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5596, 57, 57, 39, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{5596}$$

$$\int \frac{(1+iax)^{3/2}}{(1-iax)^{5/2}}dx$$

$$\Big\downarrow \text{57}$$

$$-\int \frac{\sqrt{iax+1}}{(1-iax)^{3/2}}dx-\frac{2i(1+iax)^{3/2}}{3a(1-iax)^{3/2}}$$

$$\Big\downarrow \text{57}$$

$$\int \frac{1}{\sqrt{1-iax}\sqrt{iax+1}}dx-\frac{2i(1+iax)^{3/2}}{3a(1-iax)^{3/2}}+\frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}}$$

$$\Big\downarrow \text{39}$$

$$\int \frac{1}{\sqrt{a^2x^2+1}}dx-\frac{2i(1+iax)^{3/2}}{3a(1-iax)^{3/2}}+\frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}}$$

$$\Big\downarrow \text{222}$$

$$\frac{\text{arcsinh}(ax)}{a}-\frac{2i(1+iax)^{3/2}}{3a(1-iax)^{3/2}}+\frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}}$$

input
```
Int[E^((4*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
((2*I)*Sqrt[1 + I*a*x])/(a*Sqrt[1 - I*a*x]) - (((2*I)/3)*(1 + I*a*x)^(3/2)
)/(a*(1 - I*a*x)^(3/2)) + ArcSinh[a*x]/a
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(59) = 118.

Time = 0.19 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.58

method	result
meijerg	$\frac{x(2a^2x^2+3)}{3(a^2x^2+1)^{\frac{3}{2}}}+\frac{8i\left(\frac{\sqrt{\pi}}{2}-\frac{\sqrt{\pi}}{2\left(a^2x^2+1\right)^{\frac{3}{2}}}\right)}{3a\sqrt{\pi}}-\frac{2a^2x^3}{(a^2x^2+1)^{\frac{3}{2}}}-\frac{8i\left(\sqrt{\pi}-\frac{\sqrt{\pi}\left(12a^2x^2+8\right)}{8\left(a^2x^2+1\right)^{\frac{3}{2}}}\right)}{3a\sqrt{\pi}}+\frac{-\frac{\sqrt{\pi}\,x\left(a^2\right)^{\frac{5}{2}}\left(20a^2x^2+15\right)}{15a^4\left(a^2x^2+1\right)^{\frac{3}{2}}}+\frac{\sqrt{\pi}\left(a^2\right)\cdots}{\cdots}}{\sqrt{\pi}\sqrt{a^2}\cdots}$
default	$\frac{x}{3(a^2x^2+1)^{\frac{3}{2}}}+\frac{2x}{3\sqrt{a^2x^2+1}}+a^4\left(-\frac{x^3}{3a^2(a^2x^2+1)^{\frac{3}{2}}}+\frac{-\frac{x}{a^2\sqrt{a^2x^2+1}}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{a^2\sqrt{a^2}}}{a^2}\right)-4ia^3\left(-\frac{x^2}{a^2(a^2x^2+1)^{\frac{3}{2}}}\cdots\right.$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(2*a^2*x^2+3)/(a^2*x^2+1)^(3/2)+8/3*I/a/Pi^(1/2)*(1/2*Pi^(1/2)-1/2*P
i^(1/2)/(a^2*x^2+1)^(3/2))-2*a^2*x^3/(a^2*x^2+1)^(3/2)-8/3*I/a/Pi^(1/2)*(P
i^(1/2)-1/8*Pi^(1/2)*(12*a^2*x^2+8)/(a^2*x^2+1)^(3/2))+2/3/Pi^(1/2)/(a^2)^
(1/2)*(-1/10*Pi^(1/2)*x*(a^2)^(5/2)*(20*a^2*x^2+15)/a^4/(a^2*x^2+1)^(3/2)+
3/2*Pi^(1/2)*(a^2)^(5/2)/a^5*arcsinh(a*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.25

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{8\,a^2x^2+16i\,ax+3\,(a^2x^2+2i\,ax-1)\log\left(-ax+\sqrt{a^2x^2+1}\right)+4\,\sqrt{a^2x^2+1}(2\,ax+i)-8}{3\,(a^3x^2+2i\,a^2x-a)}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(8*a^2*x^2 + 16*I*a*x + 3*(a^2*x^2 + 2*I*a*x - 1)*log(-a*x + sqrt(a^2
*x^2 + 1)) + 4*sqrt(a^2*x^2 + 1)*(2*a*x + I) - 8)/(a^3*x^2 + 2*I*a^2*x - a
)
```

Sympy [F]

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \int \frac{(ax-i)^4}{(a^2x^2+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate((1+I*a*x)**4/(a**2*x**2+1)**(5/2),x)
```

output
```
Integral((a*x - I)**4/(a**2*x**2 + 1)**(5/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(54) = 108$.

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.62

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{1}{3}a^4x\left(\frac{3\,x^2}{(a^2x^2+1)^{\frac{3}{2}}a^2}+\frac{2}{(a^2x^2+1)^{\frac{3}{2}}a^4}\right)+\frac{4i\,ax^2}{(a^2x^2+1)^{\frac{3}{2}}} - \frac{5\,x}{3\sqrt{a^2x^2+1}}+\frac{\operatorname{arsinh}(ax)}{a}+\frac{7\,x}{3\,(a^2x^2+1)^{\frac{3}{2}}}+\frac{4i}{3\,(a^2x^2+1)^{\frac{3}{2}}a}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*a^4*x*(3*x^2/((a^2*x^2 + 1)^(3/2)*a^2) + 2/((a^2*x^2 + 1)^(3/2)*a^4))
 + 4*I*a*x^2/(a^2*x^2 + 1)^(3/2) - 5/3*x/sqrt(a^2*x^2 + 1) + arcsinh(a*x)/
a + 7/3*x/(a^2*x^2 + 1)^(3/2) + 4/3*I/((a^2*x^2 + 1)^(3/2)*a)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.35

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{|a|}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^(5/2),x, algorithm="giac")
```

output
```
-log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a)
```

Mupad [B] (verification not implemented)

Time = 23.67 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.33

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{\operatorname{asinh}\left(x\sqrt{a^2}\right)}{\sqrt{a^2}} - \frac{8\sqrt{a^2x^2+1}}{3\left(x\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\sqrt{a^2}} + \frac{a\sqrt{a^2x^2+1}\,4\mathrm{i}}{3\left(a^4x^2+a^3x\,2\mathrm{i}-a^2\right)}$$

input
```
int((a*x*1i + 1)^4/(a^2*x^2 + 1)^(5/2),x)
```

output
```
asinh(x*(a^2)^(1/2))/(a^2)^(1/2) - (8*(a^2*x^2 + 1)^(1/2))/(3*(((a^2)^(1/2
)*1i)/a + x*(a^2)^(1/2))*(a^2)^(1/2)) + (a*(a^2*x^2 + 1)^(1/2)*4i)/(3*(a^3
*x*2i - a^2 + a^4*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.25

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{-8\sqrt{a^2x^2+1}\,a^3x^3+12\sqrt{a^2x^2+1}\,a^2i\,x^2+4\sqrt{a^2x^2+1}\,i+3\log\left(\sqrt{a^2x^2+1}+ax\right)a^4x^4+6\log\left(\sqrt{a^2x^2}\right.}{3a\left(a^4x^4+2a^2x^2+1\right)}$$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^(5/2),x)
```

output
```
( - 8*sqrt(a**2*x**2 + 1)*a**3*x**3 + 12*sqrt(a**2*x**2 + 1)*a**2*i*x**2 +
 4*sqrt(a**2*x**2 + 1)*i + 3*log(sqrt(a**2*x**2 + 1) + a*x)*a**4*x**4 + 6*
log(sqrt(a**2*x**2 + 1) + a*x)*a**2*x**2 + 3*log(sqrt(a**2*x**2 + 1) + a*x
) - 8*a**4*x**4 - 16*a**2*x**2 - 8)/(3*a*(a**4*x**4 + 2*a**2*x**2 + 1))
```

3.317 $\int \frac{e^{3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 30

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2}{a(i+ax)} - \frac{i\log(i+ax)}{a}$$

output
```
2/a/(I+a*x)-I*ln(I+a*x)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2}{a(i+ax)} - \frac{i\log(i+ax)}{a}$$

input
```
Integrate[E^((3*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
2/(a*(I + a*x)) - (I*Log[I + a*x])/a
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5596, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{ 5596}$$

$$\int \frac{1+iax}{(1-iax)^2}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(-\frac{i}{ax+i}-\frac{2}{(ax+i)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{a(ax+i)}-\frac{i\log(ax+i)}{a}$$

input
```
Int[E^((3*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
2/(a*(I + a*x)) - (I*Log[I + a*x])/a
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

method	result	siz
default	$\frac{2}{a(ax+i)}-\frac{i\ln(ax+i)}{a}$	28
risch	$\frac{2}{a(ax+i)}-\frac{i\ln(a^2x^2+1)}{2a}-\frac{\arctan(ax)}{a}$	40
parallelrisch	$-\frac{i\ln(ax+i)x^2a^2-2ix^2a^2+i\ln(ax+i)-2ax}{(a^2x^2+1)a}$	57
meijerg	$\frac{\frac{2x\sqrt{a^2}}{2a^2x^2+2}+\frac{\sqrt{a^2}\,\arctan(ax)}{a}}{2\sqrt{a^2}}+\frac{3ia\,x^2}{2(a^2x^2+1)}-\frac{3\left(-\frac{x(a^2)^{\frac{3}{2}}}{a^2(a^2x^2+1)}+\frac{(a^2)^{\frac{3}{2}}\arctan(ax)}{a^3}\right)}{2\sqrt{a^2}}-\frac{i\left(-\frac{a^2x^2}{a^2x^2+1}+\ln(a^2x^2+1)\right)}{2a}$	14

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
2/a/(I+a*x)-I*ln(I+a*x)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx=\frac{(-i\,ax+1)\log\left(\frac{ax+i}{a}\right)+2}{a^2x+i\,a}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^2,x, algorithm="fricas")
```

output
```
((-I*a*x + 1)*log((a*x + I)/a) + 2)/(a^2*x + I*a)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.63

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2}{a^2x+ia} - \frac{i\log{(ax+i)}}{a}$$

input
```
integrate((1+I*a*x)**3/(a**2*x**2+1)**2,x)
```

output
```
2/(a**2*x + I*a) - I*log(a*x + I)/a
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2\,(ax-i)}{a^3x^2+a} - \frac{\arctan{(ax)}}{a} - \frac{i\,\log{(a^2x^2+1)}}{2\,a}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^2,x, algorithm="maxima")
```

output
```
2*(a*x - I)/(a^3*x^2 + a) - arctan(a*x)/a - 1/2*I*log(a^2*x^2 + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{i\,\log{(ax+i)}}{a} + \frac{2}{(ax+i)a}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^2,x, algorithm="giac")
```

output
```
-I*log(a*x + I)/a + 2/((a*x + I)*a)
```

Mupad [B] (verification not implemented)

Time = 23.39 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2}{x\,a^2+a\,1\mathrm{i}} - \frac{\ln\left(a\,x+1\mathrm{i}\right)\,1\mathrm{i}}{a}$$

input
```
int((a*x*1i + 1)^3/(a^2*x^2 + 1)^2,x)
```

output
```
2/(a*1i + a^2*x) - (log(a*x + 1i)*1i)/a
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.67

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{-2atan(ax)\,a^2x^2-2atan(ax)-\log(a^2x^2+1)\,a^2i\,x^2-\log(a^2x^2+1)\,i+4a^2i\,x^2+4ax}{2a\,(a^2x^2+1)}$$

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^2,x)
```

output
```
( - 2*atan(a*x)*a**2*x**2 - 2*atan(a*x) - log(a**2*x**2 + 1)*a**2*i*x**2 -
 log(a**2*x**2 + 1)*i + 4*a**2*i*x**2 + 4*a*x)/(2*a*(a**2*x**2 + 1))
```

3.318 $\int \frac{e^{2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 38

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{2i(1+iax)}{a\sqrt{1+a^2x^2}} - \frac{\operatorname{arcsinh}(ax)}{a}$$

output
```
-2*I*(1+I*a*x)/a/(a^2*x^2+1)^(1/2)-arcsinh(a*x)/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{2i\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}} + \arcsin\left(\frac{\sqrt{1-iax}}{\sqrt{2}}\right)\right)}{a}$$

input
```
Integrate[E^((2*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
((-2*I)*(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x] + ArcSin[Sqrt[1 - I*a*x]/Sqrt[2]]
))/a
```

Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5596, 57, 39, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{a^2x^2+1}}dx$$
$$\Big\downarrow \text{ 5596}$$
$$\int \frac{\sqrt{1+iax}}{(1-iax)^{3/2}}dx$$
$$\Big\downarrow \text{ 57}$$
$$-\int \frac{1}{\sqrt{1-iax}\sqrt{iax+1}}dx-\frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}}$$
$$\Big\downarrow \text{ 39}$$
$$-\int \frac{1}{\sqrt{a^2x^2+1}}dx-\frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}}$$
$$\Big\downarrow \text{ 222}$$
$$-\frac{\text{arcsinh}(ax)}{a}-\frac{2i\sqrt{1+iax}}{a\sqrt{1-iax}}$$

input
```
Int[E^((2*I)*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
((-2*I)*Sqrt[1 + I*a*x])/(a*Sqrt[1 - I*a*x]) - ArcSinh[a*x]/a
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(34) = 68$.

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.29

method	result	size
default	$\frac{x}{\sqrt{a^2x^2+1}}-\frac{2i}{a\sqrt{a^2x^2+1}}-a^2\left(-\frac{x}{a^2\sqrt{a^2x^2+1}}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{a^2\sqrt{a^2}}\right)$	87
meijerg	$\frac{x}{\sqrt{a^2x^2+1}}+\frac{2i\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{a^2x^2+1}}\right)}{a\sqrt{\pi}}-\frac{-\frac{\sqrt{\pi}\,x\left(a^2\right)^{\frac{3}{2}}}{a^2\sqrt{a^2x^2+1}}+\frac{\sqrt{\pi}\left(a^2\right)^{\frac{3}{2}}\operatorname{arcsinh}(ax)}{a^3}}{\sqrt{\pi}\sqrt{a^2}}$	96

input
```
int((1+I*a*x)^2/(a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x/(a^2*x^2+1)^(1/2)-2*I/a/(a^2*x^2+1)^(1/2)-a^2*(-x/a^2/(a^2*x^2+1)^(1/2)+
1/a^2*ln(a^2*x/(a^2)^(1/2)+(a^2*x^2+1)^(1/2))/(a^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2\,ax+(ax+i)\log\left(-ax+\sqrt{a^2x^2+1}\right)+2\,\sqrt{a^2x^2+1}+2i}{a^2x+i\,a}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
(2*a*x + (a*x + I)*log(-a*x + sqrt(a^2*x^2 + 1)) + 2*sqrt(a^2*x^2 + 1) + 2
*I)/(a^2*x + I*a)
```

Sympy [F]

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\int \frac{a^2x^2}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx - \int \left(-\frac{2iax}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx - \int \left(-\frac{1}{a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx$$

input
```
integrate((1+I*a*x)**2/(a**2*x**2+1)**(3/2),x)
```

output
```
-Integral(a**2*x**2/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)),
 x) - Integral(-2*I*a*x/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 +
1)), x) - Integral(-1/(a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)
), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2\,x}{\sqrt{a^2x^2+1}} - \frac{\operatorname{arsinh}(ax)}{a} - \frac{2i}{\sqrt{a^2x^2+1}a}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
2*x/sqrt(a^2*x^2 + 1) - arcsinh(a*x)/a - 2*I/(sqrt(a^2*x^2 + 1)*a)
```

Giac [F]

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \int \frac{(i\,ax+1)^2}{(a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{\operatorname{asinh}\left(x\sqrt{a^2}\right)}{\sqrt{a^2}} + \frac{2\sqrt{a^2\,x^2+1}}{\left(x\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\sqrt{a^2}}$$

input
```
int((a*x*1i + 1)^2/(a^2*x^2 + 1)^(3/2),x)
```

output
```
(2*(a^2*x^2 + 1)^(1/2))/((((a^2)^(1/2)*1i)/a + x*(a^2)^(1/2))*(a^2)^(1/2))
 - asinh(x*(a^2)^(1/2))/(a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.42

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2\sqrt{a^2x^2+1}\,ax-2\sqrt{a^2x^2+1}\,i-\log\left(\sqrt{a^2x^2+1}+ax\right)a^2x^2-\log\left(\sqrt{a^2x^2+1}+ax\right)+2a^2x^2+2}{a\,(a^2x^2+1)}$$

input
```
int((1+I*a*x)^2/(a^2*x^2+1)^(3/2),x)
```

output
```
(2*sqrt(a**2*x**2 + 1)*a*x - 2*sqrt(a**2*x**2 + 1)*i - log(sqrt(a**2*x**2
+ 1) + a*x)*a**2*x**2 - log(sqrt(a**2*x**2 + 1) + a*x) + 2*a**2*x**2 + 2)/
(a*(a**2*x**2 + 1))
```

3.319 $\int \frac{e^{i \arctan (ax)}}{\sqrt{1+a^2x^2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 15

$$\int \frac{e^{i \arctan (ax)}}{\sqrt{1+a^2x^2}} \, dx = \frac{i \log (i+ax)}{a}$$

output
```
I*ln(I+a*x)/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{e^{i \arctan (ax)}}{\sqrt{1+a^2x^2}} \, dx = \frac{i \log (i+ax)}{a}$$

input
```
Integrate[E^(I*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
(I*Log[I + a*x])/a
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5596, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{a^2x^2+1}}dx$$
$$\Big\downarrow \text{ 5596}$$
$$\int \frac{1}{1-iax}dx$$
$$\Big\downarrow \text{ 16}$$
$$\frac{i\log(ax+i)}{a}$$

input
```
Int[E^(I*ArcTan[a*x])/Sqrt[1 + a^2*x^2],x]
```

output
```
(I*Log[I + a*x])/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
parallelrisch	$\frac{i\ln(ax+i)}{a}$	14
default	$\frac{i\ln(a^2x^2+1)}{2a}+\frac{\arctan(ax)}{a}$	26
meijerg	$\frac{i\ln(a^2x^2+1)}{2a}+\frac{\arctan(ax)}{a}$	26
risch	$\frac{i\ln(a^2x^2+1)}{2a}+\frac{\arctan(ax)}{a}$	26

input
```
int((1+I*a*x)/(a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
I*ln(I+a*x)/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx=\frac{i\log\left(\frac{ax+i}{a}\right)}{a}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1),x, algorithm="fricas")
```

output
```
I*log((a*x + I)/a)/a
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.53

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx=\frac{i\log\left(ax+i\right)}{a}$$

input
```
integrate((1+I*a*x)/(a**2*x**2+1),x)
```

output
```
I*log(a*x + I)/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(11) = 22$.

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{\arctan(ax)}{a} + \frac{i\log(a^2x^2+1)}{2a}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1),x, algorithm="maxima")
```

output
```
arctan(a*x)/a + 1/2*I*log(a^2*x^2 + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{i\log(ax+i)}{a}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1),x, algorithm="giac")
```

output
```
I*log(a*x + I)/a
```

Mupad [B] (verification not implemented)

Time = 22.93 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{\ln\left(x+\frac{1\mathrm{i}}{a}\right)\,1\mathrm{i}}{a}$$

input
```
int((a*x*1i + 1)/(a^2*x^2 + 1),x)
```

output
```
(log(x + 1i/a)*1i)/a
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2atan(ax)+\log(a^2x^2+1)\,i}{2a}$$

input
```
int((1+I*a*x)/(a^2*x^2+1),x)
```

output
```
(2*atan(a*x) + log(a**2*x**2 + 1)*i)/(2*a)
```

3.320 $\int \frac{e^{-i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 16

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{i\log(i-ax)}{a}$$

output
```
-I*ln(I-a*x)/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{i\log(i-ax)}{a}$$

input
```
Integrate[1/(E^(I*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
```

output
```
((-I)*Log[I - a*x])/a
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {5596, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{a^2x^2+1}}\,dx$$

$$\Big\downarrow \; 5596$$

$$\int \frac{1}{1+iax}dx$$

$$\Big\downarrow \; 16$$

$$-\frac{i\log(-ax+i)}{a}$$

input
```
Int[1/(E^(I*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
```

output
```
((-I)*Log[I - a*x])/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

method	result	size
parallelrisch	$-\frac{i\ln(ax-i)}{a}$	14
default	$-\frac{i\ln(iax+1)}{a}$	15
meijerg	$-\frac{i\ln(iax+1)}{a}$	15
risch	$-\frac{i\ln(a^2x^2+1)}{2a}+\frac{\arctan(ax)}{a}$	26

input
```
int(1/(1+I*a*x),x,method=_RETURNVERBOSE)
```

output
```
-I*ln(a*x-I)/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{i\log\left(\frac{ax-i}{a}\right)}{a}$$

input
```
integrate(1/(1+I*a*x),x, algorithm="fricas")
```

output
```
-I*log((a*x - I)/a)/a
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{i\log(ax-i)}{a}$$

input
```
integrate(1/(1+I*a*x),x)
```

output
```
-I*log(a*x - I)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{e^{-i \arctan(ax)}}{\sqrt{1+a^2x^2}} \, dx = -\frac{i \log(i \, ax + 1)}{a}$$

input
```
integrate(1/(1+I*a*x),x, algorithm="maxima")
```

output
```
-I*log(I*a*x + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{e^{-i \arctan(ax)}}{\sqrt{1+a^2x^2}} \, dx = -\frac{i \log(i \, ax + 1)}{a}$$

input
```
integrate(1/(1+I*a*x),x, algorithm="giac")
```

output
```
-I*log(I*a*x + 1)/a
```

Mupad [B] (verification not implemented)

Time = 23.79 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{e^{-i \arctan(ax)}}{\sqrt{1+a^2x^2}} \, dx = -\frac{\ln\left(x - \frac{1\text{i}}{a}\right) \, 1\text{i}}{a}$$

input
```
int(1/(a*x*1i + 1),x)
```

output
```
-(log(x - 1i/a)*1i)/a
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{\log(aix+1)\,i}{a}$$

input
```
int(1/(1+I*a*x),x)
```

output
```
( - log(a*i*x + 1)*i)/a
```

3.321 $\int \frac{e^{-2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 38

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2i(1-iax)}{a\sqrt{1+a^2x^2}} - \frac{\operatorname{arcsinh}(ax)}{a}$$

output
```
2*I*(1-I*a*x)/a/(a^2*x^2+1)^(1/2)-arcsinh(a*x)/a
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2\left(\sqrt{1+a^2x^2}+(-1-iax)\arcsin\left(\frac{\sqrt{1-iax}}{\sqrt{2}}\right)\right)}{a(-i+ax)}$$

input
```
Integrate[1/(E^((2*I)*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
```

output
```
(2*(Sqrt[1 + a^2*x^2] + (-1 - I*a*x)*ArcSin[Sqrt[1 - I*a*x]/Sqrt[2]]))/(a*
(-I + a*x))
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5596, 57, 39, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{ 5596}$$

$$\int \frac{\sqrt{1-iax}}{(1+iax)^{3/2}}dx$$

$$\Big\downarrow \text{ 57}$$

$$\frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}}-\int \frac{1}{\sqrt{1-iax}\sqrt{iax+1}}dx$$

$$\Big\downarrow \text{ 39}$$

$$-\int \frac{1}{\sqrt{a^2x^2+1}}dx+\frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}}$$

$$\Big\downarrow \text{ 222}$$

$$-\frac{\text{arcsinh}(ax)}{a}+\frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}}$$

input
```
Int[1/(E^((2*I)*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
```

output
```
((2*I)*Sqrt[1 - I*a*x])/(a*Sqrt[1 + I*a*x]) - ArcSinh[a*x]/a
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(34) = 68$.

Time = 0.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 3.92

method	result	size
default	$-\frac{\frac{i\left(\left(x-\frac{i}{a}\right)^{2} a^{2}+2 i a\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}}}{a\left(x-\frac{i}{a}\right)^{2}}-i a\left(\sqrt{\left(x-\frac{i}{a}\right)^{2} a^{2}+2 i a\left(x-\frac{i}{a}\right)}+\frac{i a \ln\left(\frac{i a+\left(x-\frac{i}{a}\right) a^{2}}{\sqrt{a^{2}}}+\sqrt{\left(x-\frac{i}{a}\right)^{2} a^{2}+2 i a\left(x-\frac{i}{a}\right)}\right)}{\sqrt{a^{2}}}\right)}{a^{2}}$	149

input
```
int(1/(1+I*a*x)^2*(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*(I/a/(x-I/a)^2*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(3/2)-I*a*(((x-I/a)^2*
a^2+2*I*a*(x-I/a))^(1/2)+I*a*ln((I*a+(x-I/a)*a^2)/(a^2)^(1/2)+((x-I/a)^2*a
^2+2*I*a*(x-I/a))^(1/2))/(a^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2\,ax + (ax-i)\log\left(-ax+\sqrt{a^2x^2+1}\right) + 2\,\sqrt{a^2x^2+1} - 2i}{a^2x - i\,a}$$

input
```
integrate(1/(1+I*a*x)^2*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
(2*a*x + (a*x - I)*log(-a*x + sqrt(a^2*x^2 + 1)) + 2*sqrt(a^2*x^2 + 1) - 2
*I)/(a^2*x - I*a)
```

Sympy [F]

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\int \frac{\sqrt{a^2x^2+1}}{a^2x^2-2iax-1}\,dx$$

input
```
integrate(1/(1+I*a*x)**2*(a**2*x**2+1)**(1/2),x)
```

output
```
-Integral(sqrt(a**2*x**2 + 1)/(a**2*x**2 - 2*I*a*x - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{\operatorname{arsinh}(ax)}{a} + \frac{2i\sqrt{a^2x^2+1}}{i\,a^2x+a}$$

input
```
integrate(1/(1+I*a*x)^2*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-arcsinh(a*x)/a + 2*I*sqrt(a^2*x^2 + 1)/(I*a^2*x + a)
```

Giac [F]

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \int \frac{\sqrt{a^2x^2+1}}{(i\,ax+1)^2}\,dx$$

input
```
integrate(1/(1+I*a*x)^2*(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 23.40 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{\operatorname{asinh}\left(x\sqrt{a^2}\right)}{\sqrt{a^2}} - \frac{2\sqrt{a^2\,x^2+1}}{\left(-x\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\sqrt{a^2}}$$

input
```
int((a^2*x^2 + 1)^(1/2)/(a*x*1i + 1)^2,x)
```

output
```
- asinh(x*(a^2)^(1/2))/(a^2)^(1/2) - (2*(a^2*x^2 + 1)^(1/2))/((((a^2)^(1/2
)*1i)/a - x*(a^2)^(1/2))*(a^2)^(1/2))
```

Reduce [F]

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\left(\int \frac{\sqrt{a^2x^2+1}}{a^2x^2-2aix-1}dx\right)$$

input
```
int(1/(1+I*a*x)^2*(a^2*x^2+1)^(1/2),x)
```

output
```
- int(sqrt(a**2*x**2 + 1)/(a**2*x**2 - 2*a*i*x - 1),x)
```

3.322 $\int \frac{e^{-3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 32

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{2}{a(i-ax)}+\frac{i\log(i-ax)}{a}$$

output
```
-2/a/(I-a*x)+I*ln(I-a*x)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{2}{a(i-ax)}+\frac{i\log(i-ax)}{a}$$

input
```
Integrate[1/(E^((3*I)*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
```

output
```
-2/(a*(I - a*x)) + (I*Log[I - a*x])/a
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5596, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{ 5596}$$

$$\int \frac{1-iax}{(1+iax)^2}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{i}{ax-i}-\frac{2}{(ax-i)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{i\log(-ax+i)}{a}-\frac{2}{a(-ax+i)}$$

input
```
Int[1/(E^((3*I)*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
```

output
```
-2/(a*(I - a*x)) + (I*Log[I - a*x])/a
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{2}{a(-ax+i)}+\frac{i\ln(-ax+i)}{a}$	30
risch	$\frac{2}{a(ax-i)}+\frac{i\ln(a^2x^2+1)}{2a}-\frac{\arctan(ax)}{a}$	40
meijerg	$\frac{i\left(-\frac{iax(9iax+6)}{3(iax+1)^2}+2\ln(iax+1)\right)}{2a}+\frac{x(iax+2)}{2(iax+1)^2}$	59
parallelrisch	$\frac{i\ln(ax-i)x^2a^2+2\ln(ax-i)xa-2ix^2a^2-i\ln(ax-i)-2ax}{(-ax+i)^2a}$	65

input
```
int(1/(1+I*a*x)^3*(a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-2/a/(I-a*x)+I*ln(I-a*x)/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx=\frac{(i\,ax+1)\log\left(\frac{ax-i}{a}\right)+2}{a^2x-i\,a}$$

input
```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1),x, algorithm="fricas")
```

output
```
((I*a*x + 1)*log((a*x - I)/a) + 2)/(a^2*x - I*a)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{2}{a^2x-ia} + \frac{i\log(ax-i)}{a}$$

input
```
integrate(1/(1+I*a*x)**3*(a**2*x**2+1),x)
```

output
```
2/(a**2*x - I*a) + I*log(a*x - I)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{4(-i\,ax-1)}{2i\,a^3x^2+4\,a^2x-2i\,a} + \frac{i\,\log(i\,ax+1)}{a}$$

input
```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1),x, algorithm="maxima")
```

output
```
-4*(-I*a*x - 1)/(2*I*a^3*x^2 + 4*a^2*x - 2*I*a) + I*log(I*a*x + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{i\,\log(ax-i)}{a} + \frac{2}{(ax-i)a}$$

input
```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1),x, algorithm="giac")
```

output
```
I*log(a*x - I)/a + 2/((a*x - I)*a)
```

Mupad [B] (verification not implemented)

Time = 23.97 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{2}{-a^2\,x + a\,1\mathrm{i}} + \frac{\ln\left(a\,x - \mathrm{i}\right)\,1\mathrm{i}}{a}$$

input
```
int((a^2*x^2 + 1)/(a*x*1i + 1)^3,x)
```

output
```
(log(a*x - 1i)*1i)/a - 2/(a*1i - a^2*x)
```

Reduce [F]

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{-6\left(\int \frac{x}{a^3i\,x^3+3a^2x^2-3aix-1}dx\right)a^2i - 6\left(\int \frac{1}{a^3i\,x^3+3a^2x^2-3aix-1}dx\right)a + \log(a^3i\,x^3+3a^2x^2-3aix-1)\,i}{3a}$$

input
```
int(1/(1+I*a*x)^3*(a^2*x^2+1),x)
```

output
```
( - 6*int(x/(a**3*i*x**3 + 3*a**2*x**2 - 3*a*i*x - 1),x)*a**2*i - 6*int(1/
(a**3*i*x**3 + 3*a**2*x**2 - 3*a*i*x - 1),x)*a + log(a**3*i*x**3 + 3*a**2*
x**2 - 3*a*i*x - 1)*i)/(3*a)
```

3.323 $\int \frac{e^{-4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 69

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx=\frac{2i(1-iax)^3}{3a\left(1+a^2x^2\right)^{3/2}}-\frac{2i(1-iax)}{a\sqrt{1+a^2x^2}}+\frac{\operatorname{arcsinh}(ax)}{a}$$

output
```
2/3*I*(1-I*a*x)^3/a/(a^2*x^2+1)^(3/2)-2*I*(1-I*a*x)/a/(a^2*x^2+1)^(1/2)+ar
csinh(a*x)/a
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx=\frac{2i\left(\frac{2\sqrt{1+iax}(1+iax+2a^2x^2)}{\sqrt{1-iax}(-i+ax)^2}+3\arcsin\left(\frac{\sqrt{1-iax}}{\sqrt{2}}\right)\right)}{3a}$$

input
```
Integrate[1/(E^((4*I)*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
```

output
```
(((2*I)/3)*((2*Sqrt[1 + I*a*x]*(1 + I*a*x + 2*a^2*x^2))/(Sqrt[1 - I*a*x]*(
-I + a*x)^2) + 3*ArcSin[Sqrt[1 - I*a*x]/Sqrt[2]]))/a
```

Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5596, 57, 57, 39, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{a^2x^2+1}}dx$$

$$\Big\downarrow \text{ 5596}$$

$$\int \frac{(1-iax)^{3/2}}{(1+iax)^{5/2}}dx$$

$$\Big\downarrow \text{ 57}$$

$$\frac{2i(1-iax)^{3/2}}{3a(1+iax)^{3/2}}-\int \frac{\sqrt{1-iax}}{(iax+1)^{3/2}}dx$$

$$\Big\downarrow \text{ 57}$$

$$\int \frac{1}{\sqrt{1-iax}\sqrt{iax+1}}dx+\frac{2i(1-iax)^{3/2}}{3a(1+iax)^{3/2}}-\frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}}$$

$$\Big\downarrow \text{ 39}$$

$$\int \frac{1}{\sqrt{a^2x^2+1}}dx+\frac{2i(1-iax)^{3/2}}{3a(1+iax)^{3/2}}-\frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{\text{arcsinh}(ax)}{a}+\frac{2i(1-iax)^{3/2}}{3a(1+iax)^{3/2}}-\frac{2i\sqrt{1-iax}}{a\sqrt{1+iax}}$$

input
```
Int[1/(E^((4*I)*ArcTan[a*x])*Sqrt[1 + a^2*x^2]),x]
```

output
```
(((2*I)/3)*(1 - I*a*x)^(3/2))/(a*(1 + I*a*x)^(3/2)) - ((2*I)*Sqrt[1 - I*a*
x])/(a*Sqrt[1 + I*a*x]) + ArcSinh[a*x]/a
```

Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(59) = 118$.

Time = 0.18 (sec) , antiderivative size = 305, normalized size of antiderivative = 4.42

method	result
default	$\frac{\frac{i\left(\left(x-\frac{i}{a}\right)^2 a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{3a\left(x-\frac{i}{a}\right)^4}-\frac{ia\left(\frac{i\left(\left(x-\frac{i}{a}\right)^2 a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a\left(x-\frac{i}{a}\right)^3}-2ia\left(-\frac{i\left(\left(x-\frac{i}{a}\right)^2 a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{5}{2}}}{a\left(x-\frac{i}{a}\right)^2}+3ia\left(\frac{\left(\left(x-\frac{i}{a}\right)^2 a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}}}{3}\right.\right.\right.}{3}}{a^4}$

input
```
int(1/(1+I*a*x)^4*(a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/3*I/a/(x-I/a)^4*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(5/2)-1/3*I*a*(I/a/
(x-I/a)^3*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(5/2)-2*I*a*(-I/a/(x-I/a)^2*((x-I/
a)^2*a^2+2*I*a*(x-I/a))^(5/2)+3*I*a*(1/3*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(3/
2)+I*a*(1/4*(2*(x-I/a)*a^2+2*I*a)/a^2*((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2)+
1/2*ln((I*a+(x-I/a)*a^2)/(a^2)^(1/2)+((x-I/a)^2*a^2+2*I*a*(x-I/a))^(1/2))/
(a^2)^(1/2))))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.25

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{8\,a^2x^2-16i\,ax+3\,(a^2x^2-2i\,ax-1)\log\left(-ax+\sqrt{a^2x^2+1}\right)+4\,\sqrt{a^2x^2+1}(2\,ax-i)-8}{3\,(a^3x^2-2i\,a^2x-a)}$$

input
```
integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(8*a^2*x^2 - 16*I*a*x + 3*(a^2*x^2 - 2*I*a*x - 1)*log(-a*x + sqrt(a^2
*x^2 + 1)) + 4*sqrt(a^2*x^2 + 1)*(2*a*x - I) - 8)/(a^3*x^2 - 2*I*a^2*x - a
)
```

Sympy [F]

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}}{(ax-i)^4}\,dx$$

input
```
integrate(1/(1+I*a*x)**4*(a**2*x**2+1)**(3/2),x)
```

output
```
Integral((a**2*x**2 + 1)**(3/2)/(a*x - I)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.55

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{i\,(a^2x^2+1)^{\frac{3}{2}}}{-3i\,a^4x^3-9\,a^3x^2+9i\,a^2x+3\,a}+\frac{\operatorname{arsinh}(ax)}{a} - \frac{2i\,\sqrt{a^2x^2+1}}{3\,(a^3x^2-2i\,a^2x-a)}-\frac{7i\,\sqrt{a^2x^2+1}}{3i\,a^2x+3\,a}$$

input
```
integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
I*(a^2*x^2 + 1)^(3/2)/(-3*I*a^4*x^3 - 9*a^3*x^2 + 9*I*a^2*x + 3*a) + arcsi
nh(a*x)/a - 2/3*I*sqrt(a^2*x^2 + 1)/(a^3*x^2 - 2*I*a^2*x - a) - 7*I*sqrt(a
^2*x^2 + 1)/(3*I*a^2*x + 3*a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.35

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = -\frac{\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{|a|}$$

input
```
integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
-log(-x*abs(a) + sqrt(a^2*x^2 + 1))/abs(a)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.35

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx = \frac{\operatorname{asinh}\left(x\,\sqrt{a^2}\right)}{\sqrt{a^2}}+\frac{8\,\sqrt{a^2\,x^2+1}}{3\,\left(-x\,\sqrt{a^2}+\frac{\sqrt{a^2}\,1\mathrm{i}}{a}\right)\,\sqrt{a^2}} + \frac{a\,\sqrt{a^2\,x^2+1}\,4\mathrm{i}}{3\,\left(-a^4\,x^2+a^3\,x\,2\mathrm{i}+a^2\right)}$$

input
```
int((a^2*x^2 + 1)^(3/2)/(a*x*1i + 1)^4,x)
```

output
```
asinh(x*(a^2)^(1/2))/(a^2)^(1/2) + (8*(a^2*x^2 + 1)^(1/2))/(3*(((a^2)^(1/2
)*1i)/a - x*(a^2)^(1/2))*(a^2)^(1/2)) + (a*(a^2*x^2 + 1)^(1/2)*4i)/(3*(a^3
*x*2i + a^2 - a^4*x^2))
```

Reduce [F]

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{1+a^2x^2}}\,dx$$
$$= \frac{-16\left(\int -\frac{\sqrt{a^2x^2+1}\,x^2}{a^6x^6-4a^5i\,x^5-5a^4x^4-5a^2x^2+4aix+1}dx\right)a^3+8\left(\int \frac{\sqrt{a^2x^2+1}\,x^3}{a^6x^6-4a^5i\,x^5-5a^4x^4-5a^2x^2+4aix+1}dx\right)a^4i-8\left(\int \frac{}{a^6x^6-4a^5}\right.}{2a}$$

input
```
int(1/(1+I*a*x)^4*(a^2*x^2+1)^(3/2),x)
```

output
```
( - 16*int(( - sqrt(a**2*x**2 + 1)*x**2)/(a**6*x**6 - 4*a**5*i*x**5 - 5*a*
*4*x**4 - 5*a**2*x**2 + 4*a*i*x + 1),x)*a**3 + 8*int((sqrt(a**2*x**2 + 1)*
x**3)/(a**6*x**6 - 4*a**5*i*x**5 - 5*a**4*x**4 - 5*a**2*x**2 + 4*a*i*x + 1
),x)*a**4*i - 8*int((sqrt(a**2*x**2 + 1)*x)/(a**6*x**6 - 4*a**5*i*x**5 - 5
*a**4*x**4 - 5*a**2*x**2 + 4*a*i*x + 1),x)*a**2*i - log(sqrt(a**2*x**2 + 1
) - a*x) + log(sqrt(a**2*x**2 + 1) + a*x))/(2*a)
```

3.324 $\int \frac{e^{5i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{2i\sqrt{1+a^2x^2}}{a(1-iax)^2\sqrt{c+a^2cx^2}} + \frac{4i\sqrt{1+a^2x^2}}{a(1-iax)\sqrt{c+a^2cx^2}} + \frac{i\sqrt{1+a^2x^2}\log(i+ax)}{a\sqrt{c+a^2cx^2}}$$

output
```
-2*I*(a^2*x^2+1)^(1/2)/a/(1-I*a*x)^2/(a^2*c*x^2+c)^(1/2)+4*I*(a^2*x^2+1)^(
1/2)/a/(1-I*a*x)/(a^2*c*x^2+c)^(1/2)+I*(a^2*x^2+1)^(1/2)*ln(I+a*x)/a/(a^2*
c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.53

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{i\sqrt{1+a^2x^2}(-2+4iax+(i+ax)^2\log(i+ax))}{a(i+ax)^2\sqrt{c+a^2cx^2}}$$

input
```
Integrate[E^((5*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
(I*Sqrt[1 + a^2*x^2]*(-2 + (4*I)*a*x + (I + a*x)^2*Log[I + a*x]))/(a*(I +
a*x)^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5599, 5596, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5599}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{5i\arctan(ax)}}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5596}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{(iax+1)^2}{(1-iax)^3}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{1}{1-iax}-\frac{4}{(1-iax)^2}+\frac{4}{(1-iax)^3}\right)dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{4i}{a(1-iax)}-\frac{2i}{a(1-iax)^2}+\frac{i\log(ax+i)}{a}\right)}{\sqrt{a^2cx^2+c}}$$

input
```
Int[E^((5*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[1 + a^2*x^2]*((-2*I)/(a*(1 - I*a*x)^2) + (4*I)/(a*(1 - I*a*x)) + (I*
Log[I + a*x])/a))/Sqrt[c + a^2*c*x^2]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.63

method	result	size
risch	$\frac{\sqrt{a^{2} x^{2}+1}\, \left(-4 x -\frac{2 i}{a}\right)}{\sqrt{c \left(a^{2} x^{2}+1\right)}\, \left(a x +i\right)^{2}}+\frac{i \sqrt{a^{2} x^{2}+1}\, \ln \left(a x +i\right)}{\sqrt{c \left(a^{2} x^{2}+1\right)}\, a}$	82
default	$\frac{\sqrt{c \left(a^{2} x^{2}+1\right)}\, \left(i \ln \left(a x +i\right) x^{2} a^{2}-2 \ln \left(a x +i\right) a x -i \ln \left(a x +i\right)-4 a x -2 i\right)}{\sqrt{a^{2} x^{2}+1}\, c a \left(a x +i\right)^{2}}$	84

input
```
int((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
(a^2*x^2+1)^(1/2)/(c*(a^2*x^2+1))^(1/2)*(-4*x-2*I/a)/(I+a*x)^2+I*(a^2*x^2+
1)^(1/2)/(c*(a^2*x^2+1))^(1/2)/a*ln(I+a*x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(107) = 214$.

Time = 0.10 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.78

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

$$= \frac{-4i\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}ax^2 + (i\,a^4cx^4 - 2\,a^3cx^3 - 2\,acx - i\,c)\sqrt{\frac{1}{a^2c}}\log\left(\frac{(i\,a^6x^2 - 2\,a^5x - 2i\,a^4)\sqrt{a^2cx^2+c}\sqrt{a^2x}}{8\,(a^3x^3}\right.}{}$$

input
```
integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="
fricas")
```

output
```
1/2*(-4*I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a*x^2 + (I*a^4*c*x^4 - 2*a
^3*c*x^3 - 2*a*c*x - I*c)*sqrt(1/(a^2*c))*log(1/8*((I*a^6*x^2 - 2*a^5*x -
2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1) + (I*a^9*c*x^4 - 2*a^8*c*x^
3 + I*a^7*c*x^2 - 2*a^6*c*x)*sqrt(1/(a^2*c)))/(a^3*x^3 + I*a^2*x^2 + a*x +
 I)) + (-I*a^4*c*x^4 + 2*a^3*c*x^3 + 2*a*c*x + I*c)*sqrt(1/(a^2*c))*log(1/
8*((I*a^6*x^2 - 2*a^5*x - 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1) +
 (-I*a^9*c*x^4 + 2*a^8*c*x^3 - I*a^7*c*x^2 + 2*a^6*c*x)*sqrt(1/(a^2*c)))/(
a^3*x^3 + I*a^2*x^2 + a*x + I)))/(a^4*c*x^4 + 2*I*a^3*c*x^3 + 2*I*a*c*x -
c)
```

Sympy [F]

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$
$$= i\left(\int\left(-\frac{i}{a^4x^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+2a^2x^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\right)dx + \int\frac{5ax}{a^4x^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+2a^2x^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}dx + \int\left(-\frac{10a^3x^3}{a^4x^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+2a^2x^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\right)dx + \int\frac{a^5x^5}{a^4x^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+2a^2x^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}dx + \int\frac{10ia^2x^2}{a^4x^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+2a^2x^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}dx + \int\left(-\frac{5ia^4x^4}{a^4x^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+2a^2x^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\right)dx\right)$$

input
```
integrate((1+I*a*x)**5/(a**2*x**2+1)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
I*(Integral(-I/(a**4*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 2*a*
*2*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + sqrt(a**2*x**2 + 1)*sq
rt(a**2*c*x**2 + c)), x) + Integral(5*a*x/(a**4*x**4*sqrt(a**2*x**2 + 1)*s
qrt(a**2*c*x**2 + c) + 2*a**2*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 +
c) + sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(-10*a**3*x*
*3/(a**4*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 2*a**2*x**2*sqrt
(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x*
*2 + c)), x) + Integral(a**5*x**5/(a**4*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2
*c*x**2 + c) + 2*a**2*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + sqr
t(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(10*I*a**2*x**2/(a**
4*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 2*a**2*x**2*sqrt(a**2*x
**2 + 1)*sqrt(a**2*c*x**2 + c) + sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)
), x) + Integral(-5*I*a**4*x**4/(a**4*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c
*x**2 + c) + 2*a**2*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + sqrt(
a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x))
```

Maxima [F]

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{(i\,ax+1)^5}{\sqrt{a^2cx^2+c}(a^2x^2+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="
maxima")
```

output
```
integrate((I*a*x + 1)^5/(sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 1)^(5/2)), x)
```

Giac [F]

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{(i\,ax+1)^5}{\sqrt{a^2cx^2+c}(a^2x^2+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="
giac")
```

output
```
integrate((I*a*x + 1)^5/(sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 1)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{(1+a\,x\,1\mathrm{i})^5}{\sqrt{c\,a^2\,x^2+c}\,(a^2\,x^2+1)^{5/2}}\,dx$$

input
```
int((a*x*1i + 1)^5/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)^(5/2)),x)
```

output
```
int((a*x*1i + 1)^5/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98

$$\int \frac{e^{5i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\,(2atan(ax)\,a^4x^4+4atan(ax)\,a^2x^2+2atan(ax)+\log(a^2x^2+1)\,a^4i\,x^4+2\log(a^2x^2+1)\,a^2i\,x^2+\log(}{2ac\,(a^4x^4+2a^2x^2+1)}$$

input
```
int((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*(2*atan(a*x)*a**4*x**4 + 4*atan(a*x)*a**2*x**2 + 2*atan(a*x) + lo
g(a**2*x**2 + 1)*a**4*i*x**4 + 2*log(a**2*x**2 + 1)*a**2*i*x**2 + log(a**2
*x**2 + 1)*i - 6*a**4*i*x**4 - 8*a**3*x**3 - 2*i))/(2*a*c*(a**4*x**4 + 2*a
**2*x**2 + 1))
```

3.325 $\int \frac{e^{4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 96

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=-\frac{2ic(1+iax)^3}{3a\left(c+a^2cx^2\right)^{3/2}}+\frac{2i(1+iax)}{a\sqrt{c+a^2cx^2}}+\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a\sqrt{c}}$$

output
```
-2/3*I*c*(1+I*a*x)^3/a/(a^2*c*x^2+c)^(3/2)+2*I*(1+I*a*x)/a/(a^2*c*x^2+c)^(
1/2)+arctanh(a*c^(1/2)*x/(a^2*c*x^2+c)^(1/2))/a/c^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.74

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=-\frac{4i\sqrt{2+2a^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1-iax)\right)}{3a(1-iax)^{3/2}\sqrt{c+a^2cx^2}}$$

input
```
Integrate[E^((4*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
(((-4*I)/3)*Sqrt[2 + 2*a^2*x^2]*Hypergeometric2F1[-3/2, -3/2, -1/2, (1 - I
*a*x)/2])/(a*(1 - I*a*x)^(3/2)*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5598, 468, 457, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5598}$$

$$c^2\int \frac{(iax+1)^4}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{468}$$

$$c^2\left(-\frac{\int \frac{(iax+1)^2}{(a^2cx^2+c)^{3/2}}dx}{c}-\frac{2i(1+iax)^3}{3ac\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{457}$$

$$c^2\left(-\frac{-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{c}-\frac{2i(1+iax)}{ac\sqrt{a^2cx^2+c}}}{c}-\frac{2i(1+iax)^3}{3ac\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{224}$$

$$c^2\left(-\frac{-\frac{\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{c}-\frac{2i(1+iax)}{ac\sqrt{a^2cx^2+c}}}{c}-\frac{2i(1+iax)^3}{3ac\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{219}$$

$$c^2\left(-\frac{-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{ac^{3/2}}-\frac{2i(1+iax)}{ac\sqrt{a^2cx^2+c}}}{c}-\frac{2i(1+iax)^3}{3ac\,(a^2cx^2+c)^{3/2}}\right)$$

input
```
Int[E^((4*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
c^2*((((-2*I)/3)*(1 + I*a*x)^3)/(a*c*(c + a^2*c*x^2)^(3/2)) - (((-2*I)*(1
+ I*a*x))/(a*c*Sqrt[c + a^2*c*x^2]) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c
*x^2]]/(a*c^(3/2)))/c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 468
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((n +
p)/(b*(p + 1)))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[p, -1] && GtQ[n, 1] && I
ntegerQ[2*p]
```

rule 5598
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Si
mp[1/c^(I*(n/2))   Int[(c + d*x^2)^(p + I*(n/2))/(1 + I*a*x)^(I*n), x], x]
/; FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
&& ILtQ[I*(n/2), 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(80) = 160$.

Time = 0.35 (sec) , antiderivative size = 526, normalized size of antiderivative = 5.48

method	result
default	$\frac{\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2cx^2+c}\right)}{\sqrt{a^2c}}+\frac{2\left(i\sqrt{-a^2}-a\right)\left(\frac{\sqrt{\left(x+\frac{\sqrt{-a^2}}{a^2}\right)^2a^2c-2c\sqrt{-a^2}\left(x+\frac{\sqrt{-a^2}}{a^2}\right)}}{3c\sqrt{-a^2}\left(x+\frac{\sqrt{-a^2}}{a^2}\right)^2}-\frac{\sqrt{\left(x+\frac{\sqrt{-a^2}}{a^2}\right)^2a^2c-2c\sqrt{-a^2}\left(x+\frac{\sqrt{-a^2}}{a^2}\right)}}{3c\left(x+\frac{\sqrt{-a^2}}{a^2}\right)}\right)}{a^3}$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2+c)^(1/2))/(a^2*c)^(1/2)+2/a^3*(I*(-a^2
)^(1/2)-a)*(1/3/c/(-a^2)^(1/2)/(x+(-a^2)^(1/2)/a^2)^2*((x+(-a^2)^(1/2)/a^2
)^2*a^2*c-2*c*(-a^2)^(1/2)*(x+(-a^2)^(1/2)/a^2))^(1/2)-1/3/c/(x+(-a^2)^(1/
2)/a^2)*((x+(-a^2)^(1/2)/a^2)^2*a^2*c-2*c*(-a^2)^(1/2)*(x+(-a^2)^(1/2)/a^2
))^(1/2))-2/a^3*(I*(-a^2)^(1/2)+a)*(-1/3/c/(-a^2)^(1/2)/(x-(-a^2)^(1/2)/a^
2)^2*((x-(-a^2)^(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^(1/2)*(x-(-a^2)^(1/2)/a^2))^
(1/2)-1/3/c/(x-(-a^2)^(1/2)/a^2)*((x-(-a^2)^(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^
(1/2)*(x-(-a^2)^(1/2)/a^2))^(1/2))+2/a^3*(I*(-a^2)^(1/2)-a)/c/(x+(-a^2)^(1
/2)/a^2)*((x+(-a^2)^(1/2)/a^2)^2*a^2*c-2*c*(-a^2)^(1/2)*(x+(-a^2)^(1/2)/a^
2))^(1/2)-2/a^3*(I*(-a^2)^(1/2)+a)/c/(x-(-a^2)^(1/2)/a^2)*((x-(-a^2)^(1/2)
/a^2)^2*a^2*c+2*c*(-a^2)^(1/2)*(x-(-a^2)^(1/2)/a^2))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(75) = 150$.

Time = 0.15 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.94

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{3\left(a^3cx^2+2i\,a^2cx-ac\right)\sqrt{\frac{1}{a^2c}}\log\left(\frac{2\left(a^2cx+\sqrt{a^2cx^2+c}a^2c\sqrt{\frac{1}{a^2c}}\right)}{x}\right)-3\left(a^3cx^2+2i\,a^2cx-ac\right)\sqrt{\frac{1}{a^2c}}\log\left(\frac{2\left(a^2cx\right.}{}\right.}{6\left(a^3cx^2+2i\,a^2cx-ac\right)}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fric
as")
```

output
```
1/6*(3*(a^3*c*x^2 + 2*I*a^2*c*x - a*c)*sqrt(1/(a^2*c))*log(2*(a^2*c*x + sq
rt(a^2*c*x^2 + c)*a^2*c*sqrt(1/(a^2*c)))/x) - 3*(a^3*c*x^2 + 2*I*a^2*c*x -
 a*c)*sqrt(1/(a^2*c))*log(2*(a^2*c*x - sqrt(a^2*c*x^2 + c)*a^2*c*sqrt(1/(a
^2*c)))/x) - 8*sqrt(a^2*c*x^2 + c)*(2*a*x + I))/(a^3*c*x^2 + 2*I*a^2*c*x -
 a*c)
```

Sympy [F]

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{(ax-i)^4}{\sqrt{c(a^2x^2+1)}\,(a^2x^2+1)^2}\,dx$$

input
```
integrate((1+I*a*x)**4/(a**2*x**2+1)**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral((a*x - I)**4/(sqrt(c*(a**2*x**2 + 1))*(a**2*x**2 + 1)**2), x)
```

Maxima [F]

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{(i\,ax+1)^4}{\sqrt{a^2cx^2+c}(a^2x^2+1)^2}\,dx$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((I*a*x + 1)^4/(sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 1)^2), x)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.38

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2+c}\right|\right)}{a\sqrt{c}} - \frac{8\left(3\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}\right)^2+3i\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}\right)\sqrt{c}-2c\right)}{3\left(i\sqrt{a^2c}x-i\sqrt{a^2cx^2+c}-\sqrt{c}\right)^3a}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac
")
```

output
```
-log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 + c)))/(a*sqrt(c)) - 8/3*(3*(sqrt
(a^2*c)*x - sqrt(a^2*c*x^2 + c))^2 + 3*I*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 +
 c))*sqrt(c) - 2*c)/((I*sqrt(a^2*c)*x - I*sqrt(a^2*c*x^2 + c) - sqrt(c))^3
*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{(1+a\,x\,1\mathrm{i})^4}{\sqrt{c\,a^2\,x^2+c}\,(a^2\,x^2+1)^2}\,dx$$

input
```
int((a*x*1i + 1)^4/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)^2),x)
```

output
```
int((a*x*1i + 1)^4/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.67

$$\int \frac{e^{4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(-8\sqrt{a^2x^2+1}\,a^3x^3+12\sqrt{a^2x^2+1}\,a^2i\,x^2+4\sqrt{a^2x^2+1}\,i+3\log\left(\sqrt{a^2x^2+1}+ax\right)a^4x^4+6\log\left(\sqrt{a\right.\right.}{3ac\left(a^4x^4+2a^2x^2+1\right)}$$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*( - 8*sqrt(a**2*x**2 + 1)*a**3*x**3 + 12*sqrt(a**2*x**2 + 1)*a**2
*i*x**2 + 4*sqrt(a**2*x**2 + 1)*i + 3*log(sqrt(a**2*x**2 + 1) + a*x)*a**4*
x**4 + 6*log(sqrt(a**2*x**2 + 1) + a*x)*a**2*x**2 + 3*log(sqrt(a**2*x**2 +
 1) + a*x) - 8*a**4*x**4 - 16*a**2*x**2 - 8))/(3*a*c*(a**4*x**4 + 2*a**2*x
**2 + 1))
```

3.326 $\int \frac{e^{3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 84

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\frac{2\sqrt{1+a^2x^2}}{a(i+ax)\sqrt{c+a^2cx^2}}-\frac{i\sqrt{1+a^2x^2}\log(i+ax)}{a\sqrt{c+a^2cx^2}}$$

output
```
2*(a^2*x^2+1)^(1/2)/a/(I+a*x)/(a^2*c*x^2+c)^(1/2)-I*(a^2*x^2+1)^(1/2)*ln(I
+a*x)/a/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.65

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\frac{\sqrt{1+a^2x^2}\left(\frac{2}{i+ax}-i\log(i+ax)\right)}{a\sqrt{c+a^2cx^2}}$$

input
```
Integrate[E^((3*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[1 + a^2*x^2]*(2/(I + a*x) - I*Log[I + a*x]))/(a*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5599, 5596, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5599}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{3i\arctan(ax)}}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5596}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{iax+1}{(1-iax)^2}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2x^2+1}\int \left(-\frac{i}{ax+i}-\frac{2}{(ax+i)^2}\right)dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{2}{a(ax+i)}-\frac{i\log(ax+i)}{a}\right)}{\sqrt{a^2cx^2+c}}$$

input
```
Int[E^((3*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[1 + a^2*x^2]*(2/(a*(I + a*x)) - (I*Log[I + a*x])/a))/Sqrt[c + a^2*c*
x^2]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{\left(-i \ln \left(a x +i\right) a x +\ln \left(a x +i\right)+2\right) \sqrt{c \left(a^{2} x^{2}+1\right)}}{\sqrt{a^{2} x^{2}+1}\, c a \left(a x +i\right)}$	61
risch	$\frac{2 \sqrt{a^{2} x^{2}+1}}{\sqrt{c \left(a^{2} x^{2}+1\right)}\, a \left(a x +i\right)}-\frac{i \sqrt{a^{2} x^{2}+1}\, \ln \left(a x +i\right)}{\sqrt{c \left(a^{2} x^{2}+1\right)}\, a}$	76

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
(-I*ln(I+a*x)*a*x+ln(I+a*x)+2)/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/c/a
/(I+a*x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(70) = 140$.

Time = 0.18 (sec) , antiderivative size = 357, normalized size of antiderivative = 4.25

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$
$$= \frac{(-i\,a^3cx^3+a^2cx^2-i\,acx+c)\sqrt{\frac{1}{a^2c}}\log\left(\frac{(i\,a^6x^2-2\,a^5x-2i\,a^4)\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}+(i\,a^9cx^4-2\,a^8cx^3+i\,a^7cx^2-2\,a^6cx)\sqrt{\frac{1}{a^2}}}{8\,(a^3x^3+i\,a^2x^2+ax+i)}\right.}{}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="
fricas")
```

output
```
1/2*((-I*a^3*c*x^3 + a^2*c*x^2 - I*a*c*x + c)*sqrt(1/(a^2*c))*log(1/8*((I*
a^6*x^2 - 2*a^5*x - 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1) + (I*a^
9*c*x^4 - 2*a^8*c*x^3 + I*a^7*c*x^2 - 2*a^6*c*x)*sqrt(1/(a^2*c)))/(a^3*x^3
 + I*a^2*x^2 + a*x + I)) + (I*a^3*c*x^3 - a^2*c*x^2 + I*a*c*x - c)*sqrt(1/
(a^2*c))*log(1/8*((I*a^6*x^2 - 2*a^5*x - 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt
(a^2*x^2 + 1) + (-I*a^9*c*x^4 + 2*a^8*c*x^3 - I*a^7*c*x^2 + 2*a^6*c*x)*sqr
t(1/(a^2*c)))/(a^3*x^3 + I*a^2*x^2 + a*x + I)) + 4*I*sqrt(a^2*c*x^2 + c)*s
qrt(a^2*x^2 + 1)*x)/(a^3*c*x^3 + I*a^2*c*x^2 + a*c*x + I*c)
```

Sympy [F]

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -i\left(\int \frac{i}{a^2x^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\,dx + \int \left(-\frac{3ax}{a^2x^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\right)dx + \int \frac{a^3x^3}{a^2x^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\,dx + \int \left(-\frac{3ia^2x^2}{a^2x^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\right)dx\right)$$

input
```
integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
-I*(Integral(I/(a**2*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + sqrt
(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(-3*a*x/(a**2*x**2*sq
rt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + sqrt(a**2*x**2 + 1)*sqrt(a**2*c*
x**2 + c)), x) + Integral(a**3*x**3/(a**2*x**2*sqrt(a**2*x**2 + 1)*sqrt(a*
*2*c*x**2 + c) + sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral
(-3*I*a**2*x**2/(a**2*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + sqr
t(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x))
```

Maxima [F]

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{(i\,ax+1)^3}{\sqrt{a^2cx^2+c}(a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="
maxima")
```

output
```
integrate((I*a*x + 1)^3/(sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 1)^(3/2)), x)
```

Giac [F]

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{(i\,ax+1)^3}{\sqrt{a^2cx^2+c}(a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="
giac")
```

output
```
integrate((I*a*x + 1)^3/(sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 1)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{(1+a\,x\,1\mathrm{i})^3}{\sqrt{c\,a^2\,x^2+c}\,(a^2\,x^2+1)^{3/2}}\,dx$$

input
```
int((a*x*1i + 1)^3/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)^(3/2)),x)
```

output
```
int((a*x*1i + 1)^3/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01

$$\int \frac{e^{3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\,(-2atan(ax)\,a^2x^2-2atan(ax)-\log(a^2x^2+1)\,a^2i\,x^2-\log(a^2x^2+1)\,i+4a^2i\,x^2+4ax)}{2ac\,(a^2x^2+1)}$$

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*( - 2*atan(a*x)*a**2*x**2 - 2*atan(a*x) - log(a**2*x**2 + 1)*a**2
*i*x**2 - log(a**2*x**2 + 1)*i + 4*a**2*i*x**2 + 4*a*x))/(2*a*c*(a**2*x**2
 + 1))
```

3.327 $\int \frac{e^{2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 63

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=-\frac{2i(1+iax)}{a\sqrt{c+a^2cx^2}}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a\sqrt{c}}$$

output
```
-2*I*(1+I*a*x)/a/(a^2*c*x^2+c)^(1/2)-arctanh(a*c^(1/2)*x/(a^2*c*x^2+c)^(1/
2))/a/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.44

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=-\frac{2i\sqrt{1+a^2x^2}\left(\sqrt{1+iax}+\sqrt{1-iax}\arcsin\left(\frac{\sqrt{1-iax}}{\sqrt{2}}\right)\right)}{a\sqrt{1-iax}\sqrt{c+a^2cx^2}}$$

input
```
Integrate[E^((2*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
((-2*I)*Sqrt[1 + a^2*x^2]*(Sqrt[1 + I*a*x] + Sqrt[1 - I*a*x]*ArcSin[Sqrt[1
 - I*a*x]/Sqrt[2]]))/(a*Sqrt[1 - I*a*x]*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {5598, 457, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \ 5598$$

$$c\int \frac{(iax+1)^2}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \ 457$$

$$c\left(-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{c}-\frac{2i(1+iax)}{ac\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \ 224$$

$$c\left(-\frac{\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{c}-\frac{2i(1+iax)}{ac\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \ 219$$

$$c\left(-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{ac^{3/2}}-\frac{2i(1+iax)}{ac\sqrt{a^2cx^2+c}}\right)$$

input
```
Int[E^((2*I)*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
c*(((-2*I)*(1 + I*a*x))/(a*c*Sqrt[c + a^2*c*x^2]) - ArcTanh[(a*Sqrt[c]*x)/
Sqrt[c + a^2*c*x^2]]/(a*c^(3/2)))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 5598
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Si
mp[1/c^(I*(n/2))   Int[(c + d*x^2)^(p + I*(n/2))/(1 + I*a*x)^(I*n), x], x]
/; FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
&& ILtQ[I*(n/2), 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(53) = 106$.

Time = 0.16 (sec) , antiderivative size = 204, normalized size of antiderivative = 3.24

method	result
default	$-\frac{\left(i\sqrt{-a^2}-a\right)\sqrt{\left(x+\frac{\sqrt{-a^2}}{a^2}\right)^2 a^2c-2c\sqrt{-a^2}\left(x+\frac{\sqrt{-a^2}}{a^2}\right)}}{a^3c\left(x+\frac{\sqrt{-a^2}}{a^2}\right)}+\frac{\left(i\sqrt{-a^2}+a\right)\sqrt{\left(x-\frac{\sqrt{-a^2}}{a^2}\right)^2 a^2c+2c\sqrt{-a^2}\left(x-\frac{\sqrt{-a^2}}{a^2}\right)}}{a^3c\left(x-\frac{\sqrt{-a^2}}{a^2}\right)}-\frac{\ln\left(\right.}{}$

input
```
int((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^3*(I*(-a^2)^(1/2)-a)/c/(x+(-a^2)^(1/2)/a^2)*((x+(-a^2)^(1/2)/a^2)^2*a
^2*c-2*c*(-a^2)^(1/2)*(x+(-a^2)^(1/2)/a^2))^(1/2)+1/a^3*(I*(-a^2)^(1/2)+a)
/c/(x-(-a^2)^(1/2)/a^2)*((x-(-a^2)^(1/2)/a^2)^2*a^2*c+2*c*(-a^2)^(1/2)*(x-
(-a^2)^(1/2)/a^2))^(1/2)-ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2+c)^(1/2))/(a^
2*c)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(50) = 100.

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.41

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{(a^2cx+i\,ac)\sqrt{\frac{1}{a^2c}}\log\left(\frac{2\left(a^2cx+\sqrt{a^2cx^2+c}a^2c\sqrt{\frac{1}{a^2c}}\right)}{x}\right)-(a^2cx+i\,ac)\sqrt{\frac{1}{a^2c}}\log\left(\frac{2\left(a^2cx-\sqrt{a^2cx^2+c}a^2c\sqrt{\frac{1}{a^2c}}\right)}{x}\right)}{2\,(a^2cx+i\,ac)}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas
")
```

output
```
-1/2*((a^2*c*x + I*a*c)*sqrt(1/(a^2*c))*log(2*(a^2*c*x + sqrt(a^2*c*x^2 +
c)*a^2*c*sqrt(1/(a^2*c)))/x) - (a^2*c*x + I*a*c)*sqrt(1/(a^2*c))*log(2*(a^
2*c*x - sqrt(a^2*c*x^2 + c)*a^2*c*sqrt(1/(a^2*c)))/x) - 4*sqrt(a^2*c*x^2 +
 c))/(a^2*c*x + I*a*c)
```

Sympy [F]

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\int \frac{a^2x^2}{a^2x^2\sqrt{a^2cx^2+c}+\sqrt{a^2cx^2+c}}\,dx - \int \left(-\frac{2iax}{a^2x^2\sqrt{a^2cx^2+c}+\sqrt{a^2cx^2+c}}\right)dx - \int \left(-\frac{1}{a^2x^2\sqrt{a^2cx^2+c}+\sqrt{a^2cx^2+c}}\right)dx$$

input
```
integrate((1+I*a*x)**2/(a**2*x**2+1)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
-Integral(a**2*x**2/(a**2*x**2*sqrt(a**2*c*x**2 + c) + sqrt(a**2*c*x**2 +
c)), x) - Integral(-2*I*a*x/(a**2*x**2*sqrt(a**2*c*x**2 + c) + sqrt(a**2*c
*x**2 + c)), x) - Integral(-1/(a**2*x**2*sqrt(a**2*c*x**2 + c) + sqrt(a**2
*c*x**2 + c)), x)
```

Maxima [F]

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{(i\,ax+1)^2}{\sqrt{a^2cx^2+c}(a^2x^2+1)}\,dx$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((I*a*x + 1)^2/(sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2+c}\right|\right)}{a\sqrt{c}} - \frac{4}{\left(i\sqrt{a^2c}x - i\sqrt{a^2cx^2+c} - \sqrt{c}\right)a}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 + c)))/(a*sqrt(c)) - 4/((I*sqrt(a^
2*c)*x - I*sqrt(a^2*c*x^2 + c) - sqrt(c))*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{(1+a\,x\,1\mathrm{i})^2}{\sqrt{c\,a^2\,x^2+c}\,(a^2\,x^2+1)}\,dx$$

input
```
int((a*x*1i + 1)^2/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)),x)
```

output
```
int((a*x*1i + 1)^2/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.54

$$\int \frac{e^{2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\,\left(2\sqrt{a^2x^2+1}\,ax-2\sqrt{a^2x^2+1}\,i-\log\left(\sqrt{a^2x^2+1}+ax\right)a^2x^2-\log\left(\sqrt{a^2x^2+1}+ax\right)+2a^2x^2+2\right)}{ac\,(a^2x^2+1)}$$

input
```
int((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*(2*sqrt(a**2*x**2 + 1)*a*x - 2*sqrt(a**2*x**2 + 1)*i - log(sqrt(a
**2*x**2 + 1) + a*x)*a**2*x**2 - log(sqrt(a**2*x**2 + 1) + a*x) + 2*a**2*x
**2 + 2))/(a*c*(a**2*x**2 + 1))
```

3.328 $\int \frac{e^{i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 42

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{i\sqrt{1+a^2x^2}\log(i+ax)}{a\sqrt{c+a^2cx^2}}$$

output
```
I*(a^2*x^2+1)^(1/2)*ln(I+a*x)/a/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{i\sqrt{1+a^2x^2}\log(i+ax)}{a\sqrt{c+a^2cx^2}}$$

input
```
Integrate[E^(I*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
(I*Sqrt[1 + a^2*x^2]*Log[I + a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5599, 5596, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5599}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{i\arctan(ax)}}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5596}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{1-iax}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{16}$$

$$\frac{i\sqrt{a^2x^2+1}\log(ax+i)}{a\sqrt{a^2cx^2+c}}$$

input
```
Int[E^(I*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
(I*Sqrt[1 + a^2*x^2]*Log[I + a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

method	result	size
risch	$\frac{i\sqrt{a^{2}x^{2}+1}\,\ln(ax+i)}{\sqrt{c(a^{2}x^{2}+1)}\,a}$	38
default	$\frac{\sqrt{c(a^{2}x^{2}+1)}\,(i\ln(a^{2}x^{2}+1)+2\arctan(ax))}{2\sqrt{a^{2}x^{2}+1}\,ca}$	53

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
I*(a^2*x^2+1)^(1/2)/(c*(a^2*x^2+1))^(1/2)/a*ln(I+a*x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(34) = 68.

Time = 0.16 (sec) , antiderivative size = 253, normalized size of antiderivative = 6.02

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$
$$= \frac{1}{2}i\sqrt{\frac{1}{a^2c}}\log\left(\frac{(i\,a^6x^2-2\,a^5x-2i\,a^4)\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}+(i\,a^9cx^4-2\,a^8cx^3+i\,a^7cx^2-2\,a^6cx)\sqrt{\frac{1}{a}}}{8\,(a^3x^3+i\,a^2x^2+ax+i)}\right.$$
$$-\frac{1}{2}i\sqrt{\frac{1}{a^2c}}\log\left(\frac{(i\,a^6x^2-2\,a^5x-2i\,a^4)\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}+(-i\,a^9cx^4+2\,a^8cx^3-i\,a^7cx^2+2\,a^6cx}{8\,(a^3x^3+i\,a^2x^2+ax+i)}\right.$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
1/2*I*sqrt(1/(a^2*c))*log(1/8*((I*a^6*x^2 - 2*a^5*x - 2*I*a^4)*sqrt(a^2*c*
x^2 + c)*sqrt(a^2*x^2 + 1) + (I*a^9*c*x^4 - 2*a^8*c*x^3 + I*a^7*c*x^2 - 2*
a^6*c*x)*sqrt(1/(a^2*c)))/(a^3*x^3 + I*a^2*x^2 + a*x + I)) - 1/2*I*sqrt(1/
(a^2*c))*log(1/8*((I*a^6*x^2 - 2*a^5*x - 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt
(a^2*x^2 + 1) + (-I*a^9*c*x^4 + 2*a^8*c*x^3 - I*a^7*c*x^2 + 2*a^6*c*x)*sqr
t(1/(a^2*c)))/(a^3*x^3 + I*a^2*x^2 + a*x + I))
```

Sympy [F]

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = i\left(\int\left(-\frac{i}{\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\right)dx + \int\frac{ax}{\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\,dx\right)$$

input
```
integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
I*(Integral(-I/(sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(
a*x/(sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{i\,ax+1}{\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((I*a*x + 1)/(sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{1+a\,x\,1\mathrm{i}}{\sqrt{c\,a^2\,x^2+c}\,\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int((a*x*1i + 1)/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
int((a*x*1i + 1)/((c + a^2*c*x^2)^(1/2)*(a^2*x^2 + 1)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.69

$$\int \frac{e^{i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\,(2atan(ax)+\log(a^2x^2+1)\,i)}{2ac}$$

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*(2*atan(a*x) + log(a**2*x**2 + 1)*i))/(2*a*c)
```

3.329 $\int \frac{e^{-i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 43

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{i\sqrt{1+a^2x^2}\log(i-ax)}{a\sqrt{c+a^2cx^2}}$$

output
```
-I*(a^2*x^2+1)^(1/2)*ln(I-a*x)/a/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{i\sqrt{1+a^2x^2}\log(i-ax)}{a\sqrt{c+a^2cx^2}}$$

input
```
Integrate[1/(E^(I*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

output
```
((-I)*Sqrt[1 + a^2*x^2]*Log[I - a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5599, 5596, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5599}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{-i\arctan(ax)}}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5596}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{iax+1}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{16}$$

$$-\frac{i\sqrt{a^2x^2+1}\log(-ax+i)}{a\sqrt{a^2cx^2+c}}$$

input
```
Int[1/(E^(I*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

output
```
((-I)*Sqrt[1 + a^2*x^2]*Log[I - a*x])/(a*Sqrt[c + a^2*c*x^2])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

method	result	size
risch	$-\frac{i\sqrt{a^2x^2+1}\,\ln(-ax+i)}{\sqrt{c(a^2x^2+1)}\,a}$	39
default	$-\frac{i\sqrt{c(a^2x^2+1)}\,\ln(iax+1)}{\sqrt{a^2x^2+1}\,ca}$	42

input
```
int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
-I*(a^2*x^2+1)^(1/2)/(c*(a^2*x^2+1))^(1/2)/a*ln(I-a*x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(35) = 70.

Time = 0.11 (sec) , antiderivative size = 253, normalized size of antiderivative = 5.88

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$
$$= \frac{1}{2}i\sqrt{\frac{1}{a^2c}}\log\left(\frac{(-i\,a^6x^2-2\,a^5x+2i\,a^4)\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}+(i\,a^9cx^4+2\,a^8cx^3+i\,a^7cx^2+2\,a^6cx)\sqrt{}}{8\,(a^3x^3-i\,a^2x^2+ax-i)}\right.$$
$$-\frac{1}{2}i\sqrt{\frac{1}{a^2c}}\log\left(\frac{(-i\,a^6x^2-2\,a^5x+2i\,a^4)\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}+(-i\,a^9cx^4-2\,a^8cx^3-i\,a^7cx^2-2\,a^6}{8\,(a^3x^3-i\,a^2x^2+ax-i)}\right.$$

input
```
integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="
fricas")
```

output
```
1/2*I*sqrt(1/(a^2*c))*log(1/8*((-I*a^6*x^2 - 2*a^5*x + 2*I*a^4)*sqrt(a^2*c
*x^2 + c)*sqrt(a^2*x^2 + 1) + (I*a^9*c*x^4 + 2*a^8*c*x^3 + I*a^7*c*x^2 + 2
*a^6*c*x)*sqrt(1/(a^2*c)))/(a^3*x^3 - I*a^2*x^2 + a*x - I)) - 1/2*I*sqrt(1
/(a^2*c))*log(1/8*((-I*a^6*x^2 - 2*a^5*x + 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sq
rt(a^2*x^2 + 1) + (-I*a^9*c*x^4 - 2*a^8*c*x^3 - I*a^7*c*x^2 - 2*a^6*c*x)*s
qrt(1/(a^2*c)))/(a^3*x^3 - I*a^2*x^2 + a*x - I))
```

Sympy [F]

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -i\int \frac{\sqrt{a^2x^2+1}}{ax\sqrt{a^2cx^2+c}-i\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
-I*Integral(sqrt(a**2*x**2 + 1)/(a*x*sqrt(a**2*c*x**2 + c) - I*sqrt(a**2*c
*x**2 + c)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.35

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{i\log(i\,ax+1)}{a\sqrt{c}}$$

input
```
integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="
maxima")
```

output
```
-I*log(I*a*x + 1)/(a*sqrt(c))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{a^2\,x^2+1}}{\sqrt{c\,a^2\,x^2+c}\,(1+a\,x\,1\mathrm{i})}\,dx$$

input
```
int((a^2*x^2 + 1)^(1/2)/((c + a^2*c*x^2)^(1/2)*(a*x*1i + 1)),x)
```

output
```
int((a^2*x^2 + 1)^(1/2)/((c + a^2*c*x^2)^(1/2)*(a*x*1i + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.42

$$\int \frac{e^{-i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c}\log(aix+1)\,i}{ac}$$

input
```
int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
( - sqrt(c)*log(a*i*x + 1)*i)/(a*c)
```

3.330 $\int \frac{e^{-2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 63

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{2i(1-iax)}{a\sqrt{c+a^2cx^2}} - \frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a\sqrt{c}}$$

output
```
2*I*(1-I*a*x)/a/(a^2*c*x^2+c)^(1/2)-arctanh(a*c^(1/2)*x/(a^2*c*x^2+c)^(1/2
))/a/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.86

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{2\sqrt{1+a^2x^2}\left((1-iax)\sqrt{1+iax} - i\sqrt{1-iax}(-i+ax)\arcsin\left(\frac{\sqrt{1-iax}}{\sqrt{2}}\right)\right)}{a\sqrt{1-iax}(-i+ax)\sqrt{c+a^2cx^2}}$$

input
```
Integrate[1/(E^((2*I)*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(2*Sqrt[1 + a^2*x^2]*((1 - I*a*x)*Sqrt[1 + I*a*x] - I*Sqrt[1 - I*a*x]*(-I
+ a*x)*ArcSin[Sqrt[1 - I*a*x]/Sqrt[2]]))/(a*Sqrt[1 - I*a*x]*(-I + a*x)*Sqr
t[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {5597, 457, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5597}$$

$$c\int \frac{(1-iax)^2}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{457}$$

$$c\left(-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{c}+\frac{2i(1-iax)}{ac\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{224}$$

$$c\left(-\frac{\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{c}+\frac{2i(1-iax)}{ac\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{219}$$

$$c\left(-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{ac^{3/2}}+\frac{2i(1-iax)}{ac\sqrt{a^2cx^2+c}}\right)$$

input
```
Int[1/(E^((2*I)*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

output
```
c*(((2*I)*(1 - I*a*x))/(a*c*Sqrt[c + a^2*c*x^2]) - ArcTanh[(a*Sqrt[c]*x)/S
qrt[c + a^2*c*x^2]]/(a*c^(3/2)))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 5597
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Si
mp[c^(I*(n/2))   Int[(c + d*x^2)^(p - I*(n/2))*(1 - I*a*x)^(I*n), x], x] /;
 FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0]) &&
 IGtQ[I*(n/2), 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.38

method	result	size
default	$\frac{2\sqrt{\left(x-\frac{i}{a}\right)^{2} a^{2} c+2 i a c \left(x-\frac{i}{a}\right)}}{a^{2} c \left(x-\frac{i}{a}\right)}-\frac{\ln\left(\frac{a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c \,x^{2}+c}\right)}{\sqrt{a^{2} c}}$	87

input
```
int(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/a^2/c/(x-I/a)*((x-I/a)^2*a^2*c+2*I*a*c*(x-I/a))^(1/2)-ln(a^2*c*x/(a^2*c)
^(1/2)+(a^2*c*x^2+c)^(1/2))/(a^2*c)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(50) = 100$.

Time = 0.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.41

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{(a^2cx - i\,ac)\sqrt{\frac{1}{a^2c}}\log\left(\frac{2\left(a^2cx+\sqrt{a^2cx^2+c}a^2c\sqrt{\frac{1}{a^2c}}\right)}{x}\right) - (a^2cx - i\,ac)\sqrt{\frac{1}{a^2c}}\log\left(\frac{2\left(a^2cx-\sqrt{a^2cx^2+c}a^2c\sqrt{\frac{1}{a^2c}}\right)}{x}\right)}{2\,(a^2cx - i\,ac)}$$

input
```
integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x, algorithm="fric
as")
```

output
```
-1/2*((a^2*c*x - I*a*c)*sqrt(1/(a^2*c))*log(2*(a^2*c*x + sqrt(a^2*c*x^2 +
c)*a^2*c*sqrt(1/(a^2*c)))/x) - (a^2*c*x - I*a*c)*sqrt(1/(a^2*c))*log(2*(a^
2*c*x - sqrt(a^2*c*x^2 + c)*a^2*c*sqrt(1/(a^2*c)))/x) - 4*sqrt(a^2*c*x^2 +
 c))/(a^2*c*x - I*a*c)
```

Sympy [F]

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\int \frac{a^2x^2}{a^2x^2\sqrt{a^2cx^2+c} - 2iax\sqrt{a^2cx^2+c} - \sqrt{a^2cx^2+c}}\,dx - \int \frac{1}{a^2x^2\sqrt{a^2cx^2+c} - 2iax\sqrt{a^2cx^2+c} - \sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(1/(1+I*a*x)**2*(a**2*x**2+1)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
-Integral(a**2*x**2/(a**2*x**2*sqrt(a**2*c*x**2 + c) - 2*I*a*x*sqrt(a**2*c
*x**2 + c) - sqrt(a**2*c*x**2 + c)), x) - Integral(1/(a**2*x**2*sqrt(a**2*
c*x**2 + c) - 2*I*a*x*sqrt(a**2*c*x**2 + c) - sqrt(a**2*c*x**2 + c)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.63

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{2i\sqrt{a^2cx^2+c}}{i\,a^2cx+ac} - \frac{\operatorname{arsinh}(ax)}{a\sqrt{c}}$$

input
```
integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

output
```
2*I*sqrt(a^2*c*x^2 + c)/(I*a^2*c*x + a*c) - arcsinh(a*x)/(a*sqrt(c))
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2+c}\right|\right)}{a\sqrt{c}} - \frac{4}{\left(-i\sqrt{a^2c}x+i\sqrt{a^2cx^2+c}-\sqrt{c}\right)a}$$

input
```
integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac
")
```

output
```
log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 + c)))/(a*sqrt(c)) - 4/((-I*sqrt(a
^2*c)*x + I*sqrt(a^2*c*x^2 + c) - sqrt(c))*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{a^2\,x^2+1}{\sqrt{c\,a^2\,x^2+c}\,(1+a\,x\,1\mathrm{i})^2}\,dx$$

input
```
int((a^2*x^2 + 1)/((c + a^2*c*x^2)^(1/2)*(a*x*1i + 1)^2),x)
```

output
```
int((a^2*x^2 + 1)/((c + a^2*c*x^2)^(1/2)*(a*x*1i + 1)^2), x)
```

Reduce [F]

$$\int \frac{e^{-2i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{-\left(\int \frac{x^2}{\sqrt{a^2x^2+1}\,a^2x^2-2\sqrt{a^2x^2+1}\,aix-\sqrt{a^2x^2+1}}dx\right)a^2-\left(\int \frac{1}{\sqrt{a^2x^2+1}\,a^2x^2-2\sqrt{a^2x^2+1}\,aix-\sqrt{a^2x^2+1}}dx\right)}{\sqrt{c}}$$

input
```
int(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
( - (int(x**2/(sqrt(a**2*x**2 + 1)*a**2*x**2 - 2*sqrt(a**2*x**2 + 1)*a*i*x
 - sqrt(a**2*x**2 + 1)),x)*a**2 + int(1/(sqrt(a**2*x**2 + 1)*a**2*x**2 - 2
*sqrt(a**2*x**2 + 1)*a*i*x - sqrt(a**2*x**2 + 1)),x)))/sqrt(c)
```

3.331 $\int \frac{e^{-3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 86

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{2\sqrt{1+a^2x^2}}{a(i-ax)\sqrt{c+a^2cx^2}}+\frac{i\sqrt{1+a^2x^2}\log(i-ax)}{a\sqrt{c+a^2cx^2}}$$

output
```
-2*(a^2*x^2+1)^(1/2)/a/(I-a*x)/(a^2*c*x^2+c)^(1/2)+I*(a^2*x^2+1)^(1/2)*ln(
I-a*x)/a/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.70

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\left(-\frac{2}{a(i-ax)}+\frac{i\log(i-ax)}{a}\right)}{\sqrt{c+a^2cx^2}}$$

input
```
Integrate[1/(E^((3*I)*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(-2/(a*(I - a*x)) + (I*Log[I - a*x])/a))/Sqrt[c + a^2*c
*x^2]
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {5599, 5596, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5599}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{-3i\arctan(ax)}}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5596}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1-iax}{(iax+1)^2}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{i}{ax-i}-\frac{2}{(ax-i)^2}\right)dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{i\log(-ax+i)}{a}-\frac{2}{a(-ax+i)}\right)}{\sqrt{a^2cx^2+c}}$$

input
```
Int[1/(E^((3*I)*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(-2/(a*(I - a*x)) + (I*Log[I - a*x])/a))/Sqrt[c + a^2*c
*x^2]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{(-i\ln(-ax+i)ax-\ln(-ax+i)-2)\sqrt{c(a^2x^2+1)}}{\sqrt{a^2x^2+1}\,c(-ax+i)a}$	66
risch	$\frac{2\sqrt{a^2x^2+1}}{\sqrt{c(a^2x^2+1)}\,a(ax-i)}+\frac{i\sqrt{a^2x^2+1}\,\ln(ax-i)}{\sqrt{c(a^2x^2+1)}\,a}$	76

input
```
int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
(-I*ln(I-a*x)*a*x-ln(I-a*x)-2)/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/c/(
I-a*x)/a
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(71) = 142$.

Time = 0.10 (sec) , antiderivative size = 357, normalized size of antiderivative = 4.15

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$
$$= \frac{(-i\,a^3cx^3 - a^2cx^2 - i\,acx - c)\sqrt{\frac{1}{a^2c}}\log\left(\frac{(-i\,a^6x^2-2\,a^5x+2i\,a^4)\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}+(i\,a^9cx^4+2\,a^8cx^3+i\,a^7cx^2+2\,a^6cx)\sqrt{\cdots}}{8\,(a^3x^3-i\,a^2x^2+ax-i)}\right.}{}$$

input

```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm
="fricas")
```

output

```
1/2*((-I*a^3*c*x^3 - a^2*c*x^2 - I*a*c*x - c)*sqrt(1/(a^2*c))*log(1/8*((-I
*a^6*x^2 - 2*a^5*x + 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1) + (I*a
^9*c*x^4 + 2*a^8*c*x^3 + I*a^7*c*x^2 + 2*a^6*c*x)*sqrt(1/(a^2*c)))/(a^3*x^
3 - I*a^2*x^2 + a*x - I)) + (I*a^3*c*x^3 + a^2*c*x^2 + I*a*c*x + c)*sqrt(1
/(a^2*c))*log(1/8*((-I*a^6*x^2 - 2*a^5*x + 2*I*a^4)*sqrt(a^2*c*x^2 + c)*sq
rt(a^2*x^2 + 1) + (-I*a^9*c*x^4 - 2*a^8*c*x^3 - I*a^7*c*x^2 - 2*a^6*c*x)*s
qrt(1/(a^2*c)))/(a^3*x^3 - I*a^2*x^2 + a*x - I)) - 4*I*sqrt(a^2*c*x^2 + c)
*sqrt(a^2*x^2 + 1)*x)/(a^3*c*x^3 - I*a^2*c*x^2 + a*c*x - I*c)
```

Sympy [F]

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$
$$= i\left(\int \frac{\sqrt{a^2x^2+1}}{a^3x^3\sqrt{a^2cx^2+c} - 3ia^2x^2\sqrt{a^2cx^2+c} - 3ax\sqrt{a^2cx^2+c} + i\sqrt{a^2cx^2+c}}\,dx + \int \frac{a^2x^2\sqrt{a^2x^2+1}}{a^3x^3\sqrt{a^2cx^2+c} - 3ia^2x^2\sqrt{a^2cx^2+c} - 3ax\sqrt{a^2cx^2+c} + i\sqrt{a^2cx^2+c}}\,dx\right)$$

input

```
integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
I*(Integral(sqrt(a**2*x**2 + 1)/(a**3*x**3*sqrt(a**2*c*x**2 + c) - 3*I*a**
2*x**2*sqrt(a**2*c*x**2 + c) - 3*a*x*sqrt(a**2*c*x**2 + c) + I*sqrt(a**2*c
*x**2 + c)), x) + Integral(a**2*x**2*sqrt(a**2*x**2 + 1)/(a**3*x**3*sqrt(a
**2*c*x**2 + c) - 3*I*a**2*x**2*sqrt(a**2*c*x**2 + c) - 3*a*x*sqrt(a**2*c*
x**2 + c) + I*sqrt(a**2*c*x**2 + c)), x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.41

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{i\,\log(i\,ax+1)}{a\sqrt{c}} + \frac{2}{a^2\sqrt{c}x - i\,a\sqrt{c}}$$

input
```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm
="maxima")
```

output
```
I*log(I*a*x + 1)/(a*sqrt(c)) + 2/(a^2*sqrt(c)*x - I*a*sqrt(c))
```

Giac [F]

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}}{\sqrt{a^2cx^2+c}(i\,ax+1)^3}\,dx$$

input
```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm
="giac")
```

output
```
integrate((a^2*x^2 + 1)^(3/2)/(sqrt(a^2*c*x^2 + c)*(I*a*x + 1)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\left(a^2\,x^2+1\right)^{3/2}}{\sqrt{c\,a^2\,x^2+c}\,\left(1+a\,x\,1\mathrm{i}\right)^3}\,dx$$

input
```
int((a^2*x^2 + 1)^(3/2)/((c + a^2*c*x^2)^(1/2)*(a*x*1i + 1)^3),x)
```

output
```
int((a^2*x^2 + 1)^(3/2)/((c + a^2*c*x^2)^(1/2)*(a*x*1i + 1)^3), x)
```

Reduce [F]

$$\int \frac{e^{-3i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(-6\left(\int \frac{x}{a^3i\,x^3+3a^2x^2-3aix-1}dx\right)a^2i-6\left(\int \frac{1}{a^3i\,x^3+3a^2x^2-3aix-1}dx\right)a+\log\left(a^3i\,x^3+3a^2x^2-3aix-1\right)i\right)}{3ac}$$

input
```
int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*( - 6*int(x/(a**3*i*x**3 + 3*a**2*x**2 - 3*a*i*x - 1),x)*a**2*i -
 6*int(1/(a**3*i*x**3 + 3*a**2*x**2 - 3*a*i*x - 1),x)*a + log(a**3*i*x**3
+ 3*a**2*x**2 - 3*a*i*x - 1)*i))/(3*a*c)
```

3.332 $\int \frac{e^{-4 i \arctan (a x)}}{\sqrt{c+a^2 c x^2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 96

$$\int \frac{e^{-4 i \arctan (a x)}}{\sqrt{c+a^2 c x^2}} \, dx=\frac{2 i c (1-i a x)^3}{3 a \left(c+a^2 c x^2\right)^{3/2}}-\frac{2 i (1-i a x)}{a \sqrt{c+a^2 c x^2}}+\frac{\text{arctanh}\left(\frac{a \sqrt{c} x}{\sqrt{c+a^2 c x^2}}\right)}{a \sqrt{c}}$$

output
```
2/3*I*c*(1-I*a*x)^3/a/(a^2*c*x^2+c)^(3/2)-2*I*(1-I*a*x)/a/(a^2*c*x^2+c)^(1
/2)+arctanh(a*c^(1/2)*x/(a^2*c*x^2+c)^(1/2))/a/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.38

$$\int \frac{e^{-4 i \arctan (a x)}}{\sqrt{c+a^2 c x^2}} \, dx =\frac{2 \sqrt{1+a^2 x^2} \left(2 i \sqrt{1+i a x} \left(1+i a x+2 a^2 x^2\right)+3 i \sqrt{1-i a x} (-i+a x)^2 \arcsin \left(\frac{\sqrt{1-i a x}}{\sqrt{2}}\right)\right)}{3 a \sqrt{1-i a x} (-i+a x)^2 \sqrt{c+a^2 c x^2}}$$

input
```
Integrate[1/(E^((4*I)*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(2*Sqrt[1 + a^2*x^2]*((2*I)*Sqrt[1 + I*a*x]*(1 + I*a*x + 2*a^2*x^2) + (3*I
)*Sqrt[1 - I*a*x]*(-I + a*x)^2*ArcSin[Sqrt[1 - I*a*x]/Sqrt[2]]))/(3*a*Sqrt
[1 - I*a*x]*(-I + a*x)^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5597, 468, 457, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5597}$$

$$c^2\int \frac{(1-iax)^4}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{468}$$

$$c^2\left(\frac{2i(1-iax)^3}{3ac\left(a^2cx^2+c\right)^{3/2}}-\frac{\int \frac{(1-iax)^2}{(a^2cx^2+c)^{3/2}}dx}{c}\right)$$

$$\Big\downarrow \text{457}$$

$$c^2\left(\frac{2i(1-iax)^3}{3ac\left(a^2cx^2+c\right)^{3/2}}-\frac{-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{c}+\frac{2i(1-iax)}{ac\sqrt{a^2cx^2+c}}}{c}\right)$$

$$\Big\downarrow \text{224}$$

$$c^2\left(\frac{2i(1-iax)^3}{3ac\left(a^2cx^2+c\right)^{3/2}}-\frac{-\frac{\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{c}+\frac{2i(1-iax)}{ac\sqrt{a^2cx^2+c}}}{c}\right)$$

$$\Big\downarrow \text{219}$$

$$c^2\left(\frac{2i(1-iax)^3}{3ac\left(a^2cx^2+c\right)^{3/2}}-\frac{-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{ac^{3/2}}+\frac{2i(1-iax)}{ac\sqrt{a^2cx^2+c}}}{c}\right)$$

input
```
Int[1/(E^((4*I)*ArcTan[a*x])*Sqrt[c + a^2*c*x^2]),x]
```

output
```
c^2*((((2*I)/3)*(1 - I*a*x)^3)/(a*c*(c + a^2*c*x^2)^(3/2)) - (((2*I)*(1 -
I*a*x))/(a*c*Sqrt[c + a^2*c*x^2]) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x
^2]]/(a*c^(3/2)))/c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 468
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((n +
p)/(b*(p + 1)))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[p, -1] && GtQ[n, 1] && I
ntegerQ[2*p]
```

rule 5597
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Si
mp[c^(I*(n/2))   Int[(c + d*x^2)^(p - I*(n/2))*(1 - I*a*x)^(I*n), x], x] /;
 FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0]) &&
 IGtQ[I*(n/2), 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(80) = 160.

Time = 0.24 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.96

method	result	size
default	$\frac{\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+c}\right)}{\sqrt{a^2c}}-\frac{4\sqrt{\left(x-\frac{i}{a}\right)^2a^2c+2iac\left(x-\frac{i}{a}\right)}}{a^2c\left(x-\frac{i}{a}\right)}-\frac{4\left(\frac{i\sqrt{\left(x-\frac{i}{a}\right)^2a^2c+2iac\left(x-\frac{i}{a}\right)}}{3ac\left(x-\frac{i}{a}\right)^2}-\frac{\sqrt{\left(x-\frac{i}{a}\right)^2a^2c+2iac\left(x-\frac{i}{a}\right)}}{3c\left(x-\frac{i}{a}\right)}\right)}{a^2}$	188

input
```
int(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
ln(a^2*c*x/(a^2*c)^(1/2)+(a^2*c*x^2+c)^(1/2))/(a^2*c)^(1/2)-4/a^2/c/(x-I/a
)*((x-I/a)^2*a^2*c+2*I*a*c*(x-I/a))^(1/2)-4/a^2*(1/3*I/a/c/(x-I/a)^2*((x-I
/a)^2*a^2*c+2*I*a*c*(x-I/a))^(1/2)-1/3/c/(x-I/a)*((x-I/a)^2*a^2*c+2*I*a*c*
(x-I/a))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(75) = 150.

Time = 0.09 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.94

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$
$$= \frac{3\left(a^3cx^2-2i\,a^2cx-ac\right)\sqrt{\frac{1}{a^2c}}\log\left(\frac{2\left(a^2cx+\sqrt{a^2cx^2+c}a^2c\sqrt{\frac{1}{a^2c}}\right)}{x}\right)-3\left(a^3cx^2-2i\,a^2cx-ac\right)\sqrt{\frac{1}{a^2c}}\log\left(\frac{2\left(a^2cx\right.}{}\right.}{6\left(a^3cx^2-2i\,a^2cx-ac\right)}$$

input
```
integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fr
icas")
```

output
```
1/6*(3*(a^3*c*x^2 - 2*I*a^2*c*x - a*c)*sqrt(1/(a^2*c))*log(2*(a^2*c*x + sq
rt(a^2*c*x^2 + c)*a^2*c*sqrt(1/(a^2*c)))/x) - 3*(a^3*c*x^2 - 2*I*a^2*c*x -
 a*c)*sqrt(1/(a^2*c))*log(2*(a^2*c*x - sqrt(a^2*c*x^2 + c)*a^2*c*sqrt(1/(a
^2*c)))/x) - 8*sqrt(a^2*c*x^2 + c)*(2*a*x - I))/(a^3*c*x^2 - 2*I*a^2*c*x -
 a*c)
```

Sympy [F]

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{(a^2x^2+1)^2}{\sqrt{c(a^2x^2+1)}\,(ax-i)^4}\,dx$$

input
```
integrate(1/(1+I*a*x)**4*(a**2*x**2+1)**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral((a**2*x**2 + 1)**2/(sqrt(c*(a**2*x**2 + 1))*(a*x - I)**4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.79

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{4i\sqrt{a^2cx^2+c}}{3\,(a^3cx^2-2i\,a^2cx-ac)} - \frac{8i\sqrt{a^2cx^2+c}}{3i\,a^2cx+3\,ac} + \frac{\operatorname{arsinh}(ax)}{a\sqrt{c}}$$

input
```
integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="ma
xima")
```

output
```
-4/3*I*sqrt(a^2*c*x^2 + c)/(a^3*c*x^2 - 2*I*a^2*c*x - a*c) - 8*I*sqrt(a^2*
c*x^2 + c)/(3*I*a^2*c*x + 3*a*c) + arcsinh(a*x)/(a*sqrt(c))
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.38

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2+c}\right|\right)}{a\sqrt{c}} - \frac{8\left(3\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}\right)^2-3i\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}\right)\sqrt{c}-2c\right)}{3\left(-i\sqrt{a^2c}x+i\sqrt{a^2cx^2+c}-\sqrt{c}\right)^3a}$$

input
```
integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="gi
ac")
```

output
```
-log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 + c)))/(a*sqrt(c)) - 8/3*(3*(sqrt
(a^2*c)*x - sqrt(a^2*c*x^2 + c))^2 - 3*I*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 +
 c))*sqrt(c) - 2*c)/((-I*sqrt(a^2*c)*x + I*sqrt(a^2*c*x^2 + c) - sqrt(c))^
3*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\left(a^2\,x^2+1\right)^2}{\sqrt{c\,a^2\,x^2+c}\,\left(1+a\,x\,1\mathrm{i}\right)^4}\,dx$$

input
```
int((a^2*x^2 + 1)^2/((c + a^2*c*x^2)^(1/2)*(a*x*1i + 1)^4),x)
```

output
```
int((a^2*x^2 + 1)^2/((c + a^2*c*x^2)^(1/2)*(a*x*1i + 1)^4), x)
```

Reduce [F]

$$\int \frac{e^{-4i\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c}\left(-16\left(\int -\frac{\sqrt{a^2x^2+1}\,x^2}{a^6x^6-4a^5i\,x^5-5a^4x^4-5a^2x^2+4aix+1}dx\right)a^3+8\left(\int \frac{\sqrt{a^2x^2+1}\,x^3}{a^6x^6-4a^5i\,x^5-5a^4x^4-5a^2x^2+4aix+1}dx\right)a^4i-8\left(\int \frac{}{a^6}\right.\right.}{2ac}$$

input
```
int(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*( - 16*int(( - sqrt(a**2*x**2 + 1)*x**2)/(a**6*x**6 - 4*a**5*i*x*
*5 - 5*a**4*x**4 - 5*a**2*x**2 + 4*a*i*x + 1),x)*a**3 + 8*int((sqrt(a**2*x
**2 + 1)*x**3)/(a**6*x**6 - 4*a**5*i*x**5 - 5*a**4*x**4 - 5*a**2*x**2 + 4*
a*i*x + 1),x)*a**4*i - 8*int((sqrt(a**2*x**2 + 1)*x)/(a**6*x**6 - 4*a**5*i
*x**5 - 5*a**4*x**4 - 5*a**2*x**2 + 4*a*i*x + 1),x)*a**2*i - log(sqrt(a**2
*x**2 + 1) - a*x) + log(sqrt(a**2*x**2 + 1) + a*x)))/(2*a*c)
```

3.333 $\int \frac{e^{5i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 35

$$\int \frac{e^{5i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{2}{3a(i+ax)^3} - \frac{i}{2a(i+ax)^2}$$

output
```
-2/3/a/(I+a*x)^3-1/2*I/a/(I+a*x)^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int \frac{e^{5i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{1+3iax}{6a(i+ax)^3}$$

input
```
Integrate[E^((5*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

output
```
-1/6*(1 + (3*I)*a*x)/(a*(I + a*x)^3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5596, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{5i\arctan(ax)}}{(a^2x^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{5596}$$

$$\int \frac{1+iax}{(1-iax)^4}dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{i}{(ax+i)^3}+\frac{2}{(ax+i)^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i}{2a(ax+i)^2}-\frac{2}{3a(ax+i)^3}$$

input
```
Int[E^((5*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

output
```
-2/(3*a*(I + a*x)^3) - (I/2)/(a*(I + a*x)^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.57

method	result
default	$\frac{-\frac{ix}{2}-\frac{1}{6a}}{(ax+i)^3}$
risch	$\frac{-\frac{ix}{2}-\frac{1}{6a}}{(ax+i)^3}$
orering	$-\frac{(ax+5i)(iax+1)^5}{24a(a^2x^2+1)^3}$
norman	$\frac{x+\frac{5}{2}ia\,x^2+\frac{1}{6}ia^5x^6-\frac{5}{3}a^2x^3}{(a^2x^2+1)^3}$
parallelrisch	$\frac{ia^5x^6-10a^2x^3+15ia\,x^2+6x}{6(a^2x^2+1)^3}$
gosper	$-\frac{(ax+i)(-3ax+i)(iax+1)^5}{6a(-ax+i)(a^2x^2+1)^4}$
meijerg	$\frac{\frac{x\sqrt{a^2}\left(15a^4x^4+40a^2x^2+33\right)}{4\left(a^2x^2+1\right)^3}+\frac{15\sqrt{a^2}\,\arctan(ax)}{4a}}{12\sqrt{a^2}}+\frac{5ia\,x^2\left(a^4x^4+3a^2x^2+3\right)}{6(a^2x^2+1)^3}-\frac{5\left(-\frac{x\left(a^2\right)^{\frac{3}{2}}\left(-3a^4x^4-8a^2x^2+3\right)}{4a^2\left(a^2x^2+1\right)^3}+\frac{3\left(a^2\right)^{\frac{3}{2}}\,\mathrm{ar}}{4a}\right.}{6\sqrt{a^2}}$

input
```
int((1+I*a*x)^5/(a^2*x^2+1)^4,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*I*x-1/6/a)/(I+a*x)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{e^{5i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx=\frac{-3i\,ax-1}{6\,(a^4x^3+3i\,a^3x^2-3\,a^2x-i\,a)}$$

input
```
integrate((1+I*a*x)^5/(a^2*x^2+1)^4,x, algorithm="fricas")
```

output
```
1/6*(-3*I*a*x - 1)/(a^4*x^3 + 3*I*a^3*x^2 - 3*a^2*x - I*a)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{e^{5i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{-3iax-1}{6a^4x^3+18ia^3x^2-18a^2x-6ia}$$

input
```
integrate((1+I*a*x)**5/(a**2*x**2+1)**4,x)
```

output
```
(-3*I*a*x - 1)/(6*a**4*x**3 + 18*I*a**3*x**2 - 18*a**2*x - 6*I*a)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(25) = 50.

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \frac{e^{5i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{3i\,a^4x^4+10\,a^3x^3-12i\,a^2x^2-6\,ax+i}{6\,(a^7x^6+3\,a^5x^4+3\,a^3x^2+a)}$$

input
```
integrate((1+I*a*x)^5/(a^2*x^2+1)^4,x, algorithm="maxima")
```

output
```
-1/6*(3*I*a^4*x^4 + 10*a^3*x^3 - 12*I*a^2*x^2 - 6*a*x + I)/(a^7*x^6 + 3*a^
5*x^4 + 3*a^3*x^2 + a)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.51

$$\int \frac{e^{5i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{3i\,ax+1}{6\,(ax+i)^3a}$$

input
```
integrate((1+I*a*x)^5/(a^2*x^2+1)^4,x, algorithm="giac")
```

output
```
-1/6*(3*I*a*x + 1)/((a*x + I)^3*a)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int \frac{e^{5i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{3\,a\,x-\mathrm{i}}{6\,a\,(-1+a\,x\,1\mathrm{i})^3}$$

input
```
int((a*x*1i + 1)^5/(a^2*x^2 + 1)^4,x)
```

output
```
-(3*a*x - 1i)/(6*a*(a*x*1i - 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int \frac{e^{5i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{x(a^5i\,x^5-10a^2x^2+15aix+6)}{6a^6x^6+18a^4x^4+18a^2x^2+6}$$

input
```
int((1+I*a*x)^5/(a^2*x^2+1)^4,x)
```

output
```
(x*(a**5*i*x**5 - 10*a**2*x**2 + 15*a*i*x + 6))/(6*(a**6*x**6 + 3*a**4*x**
4 + 3*a**2*x**2 + 1))
```

3.334 $\int \frac{e^{4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{e^{4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{i(1+iax)^4}{3a\,(1+a^2x^2)^{5/2}} + \frac{i(1+iax)^5}{15a\,(1+a^2x^2)^{5/2}}$$

output
```
-1/3*I*(1+I*a*x)^4/a/(a^2*x^2+1)^(5/2)+1/15*I*(1+I*a*x)^5/a/(a^2*x^2+1)^(5
/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int \frac{e^{4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{(1+iax)^{3/2}(4i+ax)}{15a\sqrt{1-iax}(i+ax)^2}$$

input
```
Integrate[E^((4*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

output
```
((1 + I*a*x)^(3/2)*(4*I + a*x))/(15*a*Sqrt[1 - I*a*x]*(I + a*x)^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5596, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4i\arctan(ax)}}{(a^2x^2+1)^{3/2}}\,dx$$

$$\Big\downarrow \text{5596}$$

$$\int \frac{\sqrt{1+iax}}{(1-iax)^{7/2}}dx$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{5}\int \frac{\sqrt{iax+1}}{(1-iax)^{5/2}}dx-\frac{i(1+iax)^{3/2}}{5a(1-iax)^{5/2}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{i(1+iax)^{3/2}}{15a(1-iax)^{3/2}}-\frac{i(1+iax)^{3/2}}{5a(1-iax)^{5/2}}$$

input
```
Int[E^((4*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

output
```
((-1/5*I)*(1 + I*a*x)^(3/2))/(a*(1 - I*a*x)^(5/2)) - ((I/15)*(1 + I*a*x)^(
3/2))/(a*(1 - I*a*x)^(3/2))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.49

method	result
orering	$-\frac{(ax+4i)(iax+1)^{4}}{15a(a^{2}x^{2}+1)^{\frac{5}{2}}}$
gosper	$\frac{(-ax+i)(ax+i)(ax+4i)(iax+1)^{4}}{15a(a^{2}x^{2}+1)^{\frac{7}{2}}}$
trager	$\frac{-a^{5}x^{5}-10a^{3}x^{3}+20ix^{2}a^{2}+15ax-4i}{15(a^{2}x^{2}+1)^{\frac{5}{2}}a}$
meijerg	$\frac{x(8a^{4}x^{4}+20a^{2}x^{2}+15)}{15(a^{2}x^{2}+1)^{\frac{5}{2}}}+\frac{16i\left(\frac{3\sqrt{\pi}}{4}-\frac{3\sqrt{\pi}}{4\left(a^{2}x^{2}+1\right)^{\frac{5}{2}}}\right)}{15a\sqrt{\pi}}-\frac{2a^{2}x^{3}(2a^{2}x^{2}+5)}{5(a^{2}x^{2}+1)^{\frac{5}{2}}}-\frac{16i\left(\frac{\sqrt{\pi}}{2}-\frac{\sqrt{\pi}\left(20a^{2}x^{2}+8\right)}{16\left(a^{2}x^{2}+1\right)^{\frac{5}{2}}}\right)}{15a\sqrt{\pi}}+\frac{a^{4}x^{5}}{5(a^{2}x^{2}+1)^{\frac{5}{2}}}$
default	$\frac{x}{5(a^{2}x^{2}+1)^{\frac{5}{2}}}+\frac{4x}{15(a^{2}x^{2}+1)^{\frac{3}{2}}}+\frac{8x}{15\sqrt{a^{2}x^{2}+1}}+a^{4}\left(-\frac{x^{3}}{2a^{2}(a^{2}x^{2}+1)^{\frac{5}{2}}}+\frac{-\frac{3x}{8a^{2}\left(a^{2}x^{2}+1\right)^{\frac{5}{2}}}+\frac{3\left(\frac{x}{5\left(a^{2}x^{2}+1\right)^{\frac{5}{2}}}+\frac{4x}{15\left(a^{2}x^{2}+1\right)} \right.}{8a^{2}}}{a^{2}}\right.$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(a*x+4*I)/a/(a^2*x^2+1)^(5/2)*(1+I*a*x)^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15

$$\int \frac{e^{4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{a^3x^3+3i\,a^2x^2-3\,ax+(a^2x^2+3i\,ax+4)\sqrt{a^2x^2+1}-i}{15\,(a^4x^3+3i\,a^3x^2-3\,a^2x-i\,a)}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^(7/2),x, algorithm="fricas")
```

output
```
-1/15*(a^3*x^3 + 3*I*a^2*x^2 - 3*a*x + (a^2*x^2 + 3*I*a*x + 4)*sqrt(a^2*x^
2 + 1) - I)/(a^4*x^3 + 3*I*a^3*x^2 - 3*a^2*x - I*a)
```

Sympy [F]

$$\int \frac{e^{4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \int \frac{(ax-i)^4}{(a^2x^2+1)^{\frac{7}{2}}}\,dx$$

input
```
integrate((1+I*a*x)**4/(a**2*x**2+1)**(7/2),x)
```

output
```
Integral((a*x - I)**4/(a**2*x**2 + 1)**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.46

$$\int \frac{e^{4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{a^2x^3}{2\,(a^2x^2+1)^{\frac{5}{2}}} - \frac{x}{15\,\sqrt{a^2x^2+1}} + \frac{4i\,ax^2}{3\,(a^2x^2+1)^{\frac{5}{2}}} - \frac{x}{30\,(a^2x^2+1)^{\frac{3}{2}}} + \frac{11\,x}{10\,(a^2x^2+1)^{\frac{5}{2}}} - \frac{4i}{15\,(a^2x^2+1)^{\frac{5}{2}}a}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^(7/2),x, algorithm="maxima")
```

output
```
-1/2*a^2*x^3/(a^2*x^2 + 1)^(5/2) - 1/15*x/sqrt(a^2*x^2 + 1) + 4/3*I*a*x^2/
(a^2*x^2 + 1)^(5/2) - 1/30*x/(a^2*x^2 + 1)^(3/2) + 11/10*x/(a^2*x^2 + 1)^(
5/2) - 4/15*I/((a^2*x^2 + 1)^(5/2)*a)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(49) = 98$.

Time = 0.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.71

$$\int \frac{e^{4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{2\left(4\,a^4-25\,a^2\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^2+15i\,a\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^3+15\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^4-\right.}{15\left(i\,a+\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^5}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^(7/2),x, algorithm="giac")
```

output
```
2/15*(4*a^4 - 25*a^2*(sqrt(a^2 + 1/x^2) - 1/x)^2 + 15*I*a*(sqrt(a^2 + 1/x^
2) - 1/x)^3 + 15*(sqrt(a^2 + 1/x^2) - 1/x)^4 - 5*a^3*(I*sqrt(a^2 + 1/x^2)
- I/x))/(I*a + sqrt(a^2 + 1/x^2) - 1/x)^5
```

Mupad [B] (verification not implemented)

Time = 22.85 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.63

$$\int \frac{e^{4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{\sqrt{a^2\,x^2+1}\,\left(a^2\,x^2\,1\mathrm{i}-3\,a\,x+4\mathrm{i}\right)}{15\,a\,{\left(-1+a\,x\,1\mathrm{i}\right)}^3}$$

input
```
int((a*x*1i + 1)^4/(a^2*x^2 + 1)^(7/2),x)
```

output
```
((a^2*x^2 + 1)^(1/2)*(a^2*x^2*1i - 3*a*x + 4i))/(15*a*(a*x*1i - 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.15

$$\int \frac{e^{4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{-\sqrt{a^2x^2+1}\,a^5x^5 - 10\sqrt{a^2x^2+1}\,a^3x^3 + 20\sqrt{a^2x^2+1}\,a^2i\,x^2 + 15\sqrt{a^2x^2+1}\,ax - 4\sqrt{}}{15a\,(a^6x^6+3a^4x^4+3a^2x^2+1)}$$

input

```
int((1+I*a*x)^4/(a^2*x^2+1)^(7/2),x)
```

output

```
( - sqrt(a**2*x**2 + 1)*a**5*x**5 - 10*sqrt(a**2*x**2 + 1)*a**3*x**3 + 20*
sqrt(a**2*x**2 + 1)*a**2*i*x**2 + 15*sqrt(a**2*x**2 + 1)*a*x - 4*sqrt(a**2
*x**2 + 1)*i + 7*a**6*x**6 + 21*a**4*x**4 + 21*a**2*x**2 + 7)/(15*a*(a**6*
x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1))
```

3.335 $\int \frac{e^{3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 19

$$\int \frac{e^{3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{i}{2a(1-iax)^2}$$

output
```
-1/2*I/a/(1-I*a*x)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{e^{3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{i}{2a(i+ax)^2}$$

input
```
Integrate[E^((3*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

output
```
(I/2)/(a*(I + a*x)^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5596, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3i\arctan(ax)}}{(a^2x^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{5596}$$

$$\int \frac{1}{(1-iax)^3}dx$$

$$\Big\downarrow \text{17}$$

$$-\frac{i}{2a(1-iax)^2}$$

input
```
Int[E^((3*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

output
```
(-1/2*I)/(a*(1 - I*a*x)^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

method	result
default	$\frac{i}{2a(ax+i)^{2}}$
risch	$\frac{i}{2a(ax+i)^{2}}$
norman	$\frac{x+\frac{3}{2}ia\,x^{2}+\frac{1}{2}ia^{3}x^{4}}{(a^{2}x^{2}+1)^{2}}$
gosper	$-\frac{(ax+i)(iax+1)^{3}}{2a(a^{2}x^{2}+1)^{3}}$
parallelrisch	$\frac{ia^{3}x^{4}+3ia\,x^{2}+2x}{2(a^{2}x^{2}+1)^{2}}$
orering	$-\frac{x(ax+2i)(ax+i)(iax+1)^{3}}{2(a^{2}x^{2}+1)^{3}}$
meijerg	$\frac{\frac{x\sqrt{a^{2}}\left(3a^{2}x^{2}+5\right)}{2\left(a^{2}x^{2}+1\right)^{2}}+\frac{3\sqrt{a^{2}}\,\arctan(ax)}{2a}}{4\sqrt{a^{2}}}+\frac{3ia\,x^{2}\left(a^{2}x^{2}+2\right)}{4(a^{2}x^{2}+1)^{2}}-\frac{3\left(-\frac{x\left(a^{2}\right)^{\frac{3}{2}}\left(-3a^{2}x^{2}+3\right)}{6a^{2}\left(a^{2}x^{2}+1\right)^{2}}+\frac{\left(a^{2}\right)^{\frac{3}{2}}\arctan(ax)}{2a^{3}}\right)}{4\sqrt{a^{2}}}-\frac{ia^{3}x^{4}}{4(a^{2}x^{2}+1)^{2}}$

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*I/a/(I+a*x)^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{3i\arctan(ax)}}{\left(1+a^{2}x^{2}\right)^{3/2}}\,dx=\frac{i}{2\left(a^{3}x^{2}+2i\,a^{2}x-a\right)}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^3,x, algorithm="fricas")
```

output
```
1/2*I/(a^3*x^2 + 2*I*a^2*x - a)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{i}{2a^3x^2+4ia^2x-2a}$$

input
```
integrate((1+I*a*x)**3/(a**2*x**2+1)**3,x)
```

output
```
I/(2*a**3*x**2 + 4*I*a**2*x - 2*a)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(13) = 26.

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \frac{e^{3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{-i\,a^2x^2-2\,ax+i}{2\,(a^5x^4+2\,a^3x^2+a)}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^3,x, algorithm="maxima")
```

output
```
-1/2*(-I*a^2*x^2 - 2*a*x + I)/(a^5*x^4 + 2*a^3*x^2 + a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{e^{3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{i}{2\,(ax+i)^2a}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^3,x, algorithm="giac")
```

output
```
1/2*I/((a*x + I)^2*a)
```

Mupad [B] (verification not implemented)

Time = 22.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{e^{3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{1\mathrm{i}}{2\,(a^3\,x^2+a^2\,x\,2\mathrm{i}-a)}$$

input
```
int((a*x*1i + 1)^3/(a^2*x^2 + 1)^3,x)
```

output
```
1i/(2*(a^2*x*2i - a + a^3*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.16

$$\int \frac{e^{3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{-a^4i\,x^4+4ax-3i}{4a\,(a^4x^4+2a^2x^2+1)}$$

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^3,x)
```

output
```
( - a**4*i*x**4 + 4*a*x - 3*i)/(4*a*(a**4*x**4 + 2*a**2*x**2 + 1))
```

3.336 $\int \frac{e^{2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 49

$$\int \frac{e^{2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{2i(1+iax)}{3a\,(1+a^2x^2)^{3/2}}+\frac{x}{3\sqrt{1+a^2x^2}}$$

output
```
-2/3*I*(1+I*a*x)/a/(a^2*x^2+1)^(3/2)+1/3*x/(a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{e^{2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{(2-iax)\sqrt{1+iax}}{3a\sqrt{1-iax}(i+ax)}$$

input
```
Integrate[E^((2*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

output
```
((2 - I*a*x)*Sqrt[1 + I*a*x])/(3*a*Sqrt[1 - I*a*x]*(I + a*x))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.37, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5596, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2i\arctan(ax)}}{(a^2x^2+1)^{3/2}}\,dx$$

$$\Big\downarrow \; 5596$$

$$\int \frac{1}{(1-iax)^{5/2}\sqrt{1+iax}}dx$$

$$\Big\downarrow \; 55$$

$$\frac{1}{3}\int \frac{1}{(1-iax)^{3/2}\sqrt{iax+1}}dx-\frac{i\sqrt{1+iax}}{3a(1-iax)^{3/2}}$$

$$\Big\downarrow \; 48$$

$$-\frac{i\sqrt{1+iax}}{3a\sqrt{1-iax}}-\frac{i\sqrt{1+iax}}{3a(1-iax)^{3/2}}$$

input
```
Int[E^((2*I)*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

output
```
((-1/3*I)*Sqrt[1 + I*a*x])/(a*(1 - I*a*x)^(3/2)) - ((I/3)*Sqrt[1 + I*a*x])
/(a*Sqrt[1 - I*a*x])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

method	result	size
trager	$\frac{a^{3} x^{3}+3 a x-2 i}{3 \left(a^{2} x^{2}+1\right)^{\frac{3}{2}} a}$	31
orering	$-\frac{\left(a x+2 i\right) \left(i a x+1\right)^{2}}{3 a \left(a^{2} x^{2}+1\right)^{\frac{3}{2}}}$	32
gosper	$\frac{\left(-a x+i\right) \left(a x+i\right) \left(a x+2 i\right) \left(i a x+1\right)^{2}}{3 a \left(a^{2} x^{2}+1\right)^{\frac{5}{2}}}$	45
meijerg	$\frac{x \left(2 a^{2} x^{2}+3\right)}{3 \left(a^{2} x^{2}+1\right)^{\frac{3}{2}}}+\frac{4 i \left(\frac{\sqrt{\pi}}{2}-\frac{\sqrt{\pi}}{2 \left(a^{2} x^{2}+1\right)^{\frac{3}{2}}}\right)}{3 a \sqrt{\pi}}-\frac{a^{2} x^{3}}{3 \left(a^{2} x^{2}+1\right)^{\frac{3}{2}}}$	76
default	$\frac{x}{3 \left(a^{2} x^{2}+1\right)^{\frac{3}{2}}}+\frac{2 x}{3 \sqrt{a^{2} x^{2}+1}}-\frac{2 i}{3 a \left(a^{2} x^{2}+1\right)^{\frac{3}{2}}}-a^{2} \left(-\frac{x}{2 a^{2} \left(a^{2} x^{2}+1\right)^{\frac{3}{2}}}+\frac{\frac{x}{3 \left(a^{2} x^{2}+1\right)^{\frac{3}{2}}}+\frac{2 x}{3 \sqrt{a^{2} x^{2}+1}}}{2 a^{2}}\right)$	104

input
```
int((1+I*a*x)^2/(a^2*x^2+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a^3*x^3+3*a*x-2*I)/(a^2*x^2+1)^(3/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{e^{2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{a^2x^2+2i\,ax+\sqrt{a^2x^2+1}(ax+2i)-1}{3\,(a^3x^2+2i\,a^2x-a)}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(a^2*x^2 + 2*I*a*x + sqrt(a^2*x^2 + 1)*(a*x + 2*I) - 1)/(a^3*x^2 + 2*I
*a^2*x - a)
```

Sympy [F]

$$\int \frac{e^{2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\int \frac{a^2x^2}{a^4x^4\sqrt{a^2x^2+1}+2a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\,dx - \int \left(-\frac{2iax}{a^4x^4\sqrt{a^2x^2+1}+2a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx - \int \left(-\frac{1}{a^4x^4\sqrt{a^2x^2+1}+2a^2x^2\sqrt{a^2x^2+1}+\sqrt{a^2x^2+1}}\right)dx$$

input
```
integrate((1+I*a*x)**2/(a**2*x**2+1)**(5/2),x)
```

output
```
-Integral(a**2*x**2/(a**4*x**4*sqrt(a**2*x**2 + 1) + 2*a**2*x**2*sqrt(a**2
*x**2 + 1) + sqrt(a**2*x**2 + 1)), x) - Integral(-2*I*a*x/(a**4*x**4*sqrt(
a**2*x**2 + 1) + 2*a**2*x**2*sqrt(a**2*x**2 + 1) + sqrt(a**2*x**2 + 1)), x
) - Integral(-1/(a**4*x**4*sqrt(a**2*x**2 + 1) + 2*a**2*x**2*sqrt(a**2*x**
2 + 1) + sqrt(a**2*x**2 + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \frac{e^{2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{x}{3\sqrt{a^2x^2+1}} + \frac{2\,x}{3\,(a^2x^2+1)^{\frac{3}{2}}} - \frac{2i}{3\,(a^2x^2+1)^{\frac{3}{2}}a}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)^(5/2),x, algorithm="maxima")
```

output
```
1/3*x/sqrt(a^2*x^2 + 1) + 2/3*x/(a^2*x^2 + 1)^(3/2) - 2/3*I/((a^2*x^2 + 1)
^(3/2)*a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.37

$$\int \frac{e^{2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{2\left(2\,a^2-3\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^2+3\,a\left(-i\,\sqrt{a^2+\frac{1}{x^2}}+\frac{i}{x}\right)\right)}{3\left(i\,a+\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^3}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)^(5/2),x, algorithm="giac")
```

output
```
-2/3*(2*a^2 - 3*(sqrt(a^2 + 1/x^2) - 1/x)^2 + 3*a*(-I*sqrt(a^2 + 1/x^2) +
I/x))/(I*a + sqrt(a^2 + 1/x^2) - 1/x)^3
```

Mupad [B] (verification not implemented)

Time = 23.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

$$\int \frac{e^{2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{\sqrt{a^2\,x^2+1}\,\left(-2+a\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{3\,a\,{\left(-1+a\,x\,1\mathrm{i}\right)}^2}$$

input
```
int((a*x*1i + 1)^2/(a^2*x^2 + 1)^(5/2),x)
```

output
```
((a^2*x^2 + 1)^(1/2)*(a*x*1i - 2)*1i)/(3*a*(a*x*1i - 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.76

$$\int \frac{e^{2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{\sqrt{a^2x^2+1}\,a^3x^3+3\sqrt{a^2x^2+1}\,ax-2\sqrt{a^2x^2+1}\,i-3a^4x^4-6a^2x^2-3}{3a\left(a^4x^4+2a^2x^2+1\right)}$$

input
```
int((1+I*a*x)^2/(a^2*x^2+1)^(5/2),x)
```

output
```
(sqrt(a**2*x**2 + 1)*a**3*x**3 + 3*sqrt(a**2*x**2 + 1)*a*x - 2*sqrt(a**2*x
**2 + 1)*i - 3*a**4*x**4 - 6*a**2*x**2 - 3)/(3*a*(a**4*x**4 + 2*a**2*x**2
+ 1))
```

3.337 $\int \frac{e^{i \arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 28

$$\int \frac{e^{i \arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{1}{2a(i+ax)} + \frac{\arctan(ax)}{2a}$$

output
```
1/2/a/(I+a*x)+1/2*arctan(a*x)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int \frac{e^{i \arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{\frac{1}{i+ax} + \arctan(ax)}{2a}$$

input
```
Integrate[E^(I*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

output
```
((I + a*x)^(-1) + ArcTan[a*x])/(2*a)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5596, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{i\arctan(ax)}}{(a^2x^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{5596}$$

$$\int \frac{1}{(1-iax)^2(1+iax)}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{1}{2(a^2x^2+1)}-\frac{1}{2(ax+i)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan(ax)}{2a}+\frac{1}{2a(ax+i)}$$

input
```
Int[E^(I*ArcTan[a*x])/(1 + a^2*x^2)^(3/2),x]
```

output
```
1/(2*a*(I + a*x)) + ArcTan[a*x]/(2*a)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{1}{2a(ax+i)}+\frac{\arctan(ax)}{2a}$	24
default	$\frac{2a^2x-2ia}{4a^2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}$	38
meijerg	$\frac{\frac{2x\sqrt{a^2}}{2a^2x^2+2}+\frac{\sqrt{a^2}\arctan(ax)}{a}}{2\sqrt{a^2}}+\frac{iax^2}{2a^2x^2+2}$	61
parallelrisch	$-\frac{i\ln(ax-i)x^2a^2-i\ln(ax+i)x^2a^2-2ix^2a^2+i\ln(ax-i)-i\ln(ax+i)-2ax}{4(a^2x^2+1)a}$	83

input
```
int((1+I*a*x)/(a^2*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/a/(I+a*x)+1/2*arctan(a*x)/a
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(22) = 44.

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{e^{i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx=\frac{(i\,ax-1)\log\left(\frac{ax+i}{a}\right)+(-i\,ax+1)\log\left(\frac{ax-i}{a}\right)+2}{4\,(a^2x+i\,a)}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^2,x, algorithm="fricas")
```

output
```
1/4*((I*a*x - 1)*log((a*x + I)/a) + (-I*a*x + 1)*log((a*x - I)/a) + 2)/(a^
2*x + I*a)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{e^{i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = i\left(-\frac{i}{2a^2x+2ia}+\frac{-\frac{\log\left(x-\frac{i}{a}\right)}{4}+\frac{\log\left(x+\frac{i}{a}\right)}{4}}{a}\right)$$

input
```
integrate((1+I*a*x)/(a**2*x**2+1)**2,x)
```

output
```
I*(-I/(2*a**2*x + 2*I*a) + (-log(x - I/a)/4 + log(x + I/a)/4)/a)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{e^{i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{ax-i}{2\,(a^3x^2+a)}+\frac{\arctan(ax)}{2\,a}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^2,x, algorithm="maxima")
```

output
```
1/2*(a*x - I)/(a^3*x^2 + a) + 1/2*arctan(a*x)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \frac{e^{i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{i\log(ax+i)}{4\,a}-\frac{i\log(ax-i)}{4\,a}+\frac{1}{2\,(ax+i)a}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^2,x, algorithm="giac")
```

output
```
1/4*I*log(a*x + I)/a - 1/4*I*log(a*x - I)/a + 1/2/((a*x + I)*a)
```

Mupad [B] (verification not implemented)

Time = 22.96 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{e^{i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{1}{2\,(x\,a^2+a\,1\mathrm{i})}+\frac{\mathrm{atan}(a\,x)}{2\,a}$$

input
```
int((a*x*1i + 1)/(a^2*x^2 + 1)^2,x)
```

output
```
1/(2*(a*1i + a^2*x)) + atan(a*x)/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.54

$$\int \frac{e^{i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{atan(ax)\,a^2x^2+atan(ax)+a^2i\,x^2+ax}{2a\,(a^2x^2+1)}$$

input
```
int((1+I*a*x)/(a^2*x^2+1)^2,x)
```

output
```
(atan(a*x)*a**2*x**2 + atan(a*x) + a**2*i*x**2 + a*x)/(2*a*(a**2*x**2 + 1)
)
```

3.338 $\int \frac{e^{-i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 29

$$\int \frac{e^{-i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{1}{2a(i-ax)} + \frac{\arctan(ax)}{2a}$$

output
```
-1/2/a/(I-a*x)+1/2*arctan(a*x)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{e^{-i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{\frac{1}{-i+ax} + \arctan(ax)}{2a}$$

input
```
Integrate[1/(E^(I*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
```

output
```
((-I + a*x)^(-1) + ArcTan[a*x])/(2*a)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5596, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-i\arctan(ax)}}{(a^2x^2+1)^{3/2}}dx$$
$$\Big\downarrow \text{5596}$$
$$\int \frac{1}{(1-iax)(1+iax)^2}dx$$
$$\Big\downarrow \text{54}$$
$$\int \left(\frac{1}{2(a^2x^2+1)}-\frac{1}{2(ax-i)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{\arctan(ax)}{2a}-\frac{1}{2a(-ax+i)}$$

input
```
Int[1/(E^(I*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
```

output
```
-1/2*1/(a*(I - a*x)) + ArcTan[a*x]/(2*a)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{1}{2a(ax-i)}+\frac{\arctan(ax)}{2a}$	24
default	$\frac{i\ln(ax+i)}{4a}-\frac{i\ln(-ax+i)}{4a}-\frac{1}{2a(-ax+i)}$	43
parallelrisch	$\frac{i\ln(ax-i)xa-i\ln(ax+i)ax+2iax+\ln(ax-i)-\ln(ax+i)}{4(-ax+i)a}$	61

input
```
int(1/(1+I*a*x)/(a^2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2/a/(a*x-I)+1/2*arctan(a*x)/a
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(22) = 44.

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int \frac{e^{-i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx=\frac{(i\,ax+1)\log\left(\frac{ax+i}{a}\right)+(-i\,ax-1)\log\left(\frac{ax-i}{a}\right)+2}{4\,(a^2x-i\,a)}$$

input
```
integrate(1/(1+I*a*x)/(a^2*x^2+1),x, algorithm="fricas")
```

output
```
1/4*((I*a*x + 1)*log((a*x + I)/a) + (-I*a*x - 1)*log((a*x - I)/a) + 2)/(a^
2*x - I*a)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{e^{-i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -i\left(\frac{i}{2a^2x-2ia}+\frac{\frac{\log\left(x-\frac{i}{a}\right)}{4}-\frac{\log\left(x+\frac{i}{a}\right)}{4}}{a}\right)$$

input
```
integrate(1/(1+I*a*x)/(a**2*x**2+1),x)
```

output
```
-I*(I/(2*a**2*x - 2*I*a) + (log(x - I/a)/4 - log(x + I/a)/4)/a)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e^{-i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(1+I*a*x)/(a^2*x^2+1),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{e^{-i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{i\log(ax+i)}{4a}-\frac{i\log(ax-i)}{4a}+\frac{1}{2(ax-i)a}$$

input
```
integrate(1/(1+I*a*x)/(a^2*x^2+1),x, algorithm="giac")
```

output
```
1/4*I*log(a*x + I)/a - 1/4*I*log(a*x - I)/a + 1/2/((a*x - I)*a)
```

Mupad [B] (verification not implemented)

Time = 23.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{e^{-i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{\operatorname{atan}(a\,x)}{2\,a} - \frac{1}{2\,\left(-a^2\,x + a\,1\mathrm{i}\right)}$$

input
```
int(1/((a^2*x^2 + 1)*(a*x*1i + 1)),x)
```

output
```
atan(a*x)/(2*a) - 1/(2*(a*1i - a^2*x))
```

Reduce [F]

$$\int \frac{e^{-i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \int \frac{1}{a^3i\,x^3 + a^2x^2 + aix + 1}dx$$

input
```
int(1/(1+I*a*x)/(a^2*x^2+1),x)
```

output
```
int(1/(a**3*i*x**3 + a**2*x**2 + a*i*x + 1),x)
```

3.339 $\int \frac{e^{-2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 49

$$\int \frac{e^{-2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{2i(1-iax)}{3a(1+a^2x^2)^{3/2}}+\frac{x}{3\sqrt{1+a^2x^2}}$$

output
```
2/3*I*(1-I*a*x)/a/(a^2*x^2+1)^(3/2)+1/3*x/(a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{e^{-2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{\sqrt{1-iax}(2+iax)}{3a\sqrt{1+iax}(-i+ax)}$$

input
```
Integrate[1/(E^((2*I)*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
```

output
```
(Sqrt[1 - I*a*x]*(2 + I*a*x))/(3*a*Sqrt[1 + I*a*x]*(-I + a*x))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.37, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5596, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2i\arctan(ax)}}{(a^2x^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{5596}$$

$$\int \frac{1}{\sqrt{1-iax}(1+iax)^{5/2}}dx$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{1-iax}(iax+1)^{3/2}}dx+\frac{i\sqrt{1-iax}}{3a(1+iax)^{3/2}}$$

$$\Big\downarrow \text{48}$$

$$\frac{i\sqrt{1-iax}}{3a\sqrt{1+iax}}+\frac{i\sqrt{1-iax}}{3a(1+iax)^{3/2}}$$

input
```
Int[1/(E^((2*I)*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
```

output
```
((I/3)*Sqrt[1 - I*a*x])/(a*(1 + I*a*x)^(3/2)) + ((I/3)*Sqrt[1 - I*a*x])/(a
*Sqrt[1 + I*a*x])
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

method	result	size
orering	$\frac{(-ax+2i)\sqrt{a^2x^2+1}}{3a(iax+1)^2}$	33
gosper	$-\frac{(-ax+i)(ax+i)(-ax+2i)}{3a(iax+1)^2\sqrt{a^2x^2+1}}$	46
default	$-\frac{\frac{i\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{3a\left(x-\frac{i}{a}\right)^2}-\frac{\sqrt{\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)}}{3\left(x-\frac{i}{a}\right)}}{a^2}$	93

input
```
int(1/(1+I*a*x)^2/(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-a*x+2*I)/a*(a^2*x^2+1)^(1/2)/(1+I*a*x)^2
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{e^{-2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{a^2x^2 - 2i\,ax + \sqrt{a^2x^2+1}(ax-2i) - 1}{3\,(a^3x^2 - 2i\,a^2x - a)}$$

input
```
integrate(1/(1+I*a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(a^2*x^2 - 2*I*a*x + sqrt(a^2*x^2 + 1)*(a*x - 2*I) - 1)/(a^3*x^2 - 2*I
*a^2*x - a)
```

Sympy [F]

$$\int \frac{e^{-2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\int \frac{1}{a^2x^2\sqrt{a^2x^2+1} - 2iax\sqrt{a^2x^2+1} - \sqrt{a^2x^2+1}}\,dx$$

input
```
integrate(1/(1+I*a*x)**2/(a**2*x**2+1)**(1/2),x)
```

output
```
-Integral(1/(a**2*x**2*sqrt(a**2*x**2 + 1) - 2*I*a*x*sqrt(a**2*x**2 + 1) -
 sqrt(a**2*x**2 + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

$$\int \frac{e^{-2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{i\sqrt{a^2x^2+1}}{3\,(a^3x^2 - 2i\,a^2x - a)} + \frac{i\sqrt{a^2x^2+1}}{3i\,a^2x + 3\,a}$$

input
```
integrate(1/(1+I*a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*I*sqrt(a^2*x^2 + 1)/(a^3*x^2 - 2*I*a^2*x - a) + I*sqrt(a^2*x^2 + 1)/(
3*I*a^2*x + 3*a)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.37

$$\int \frac{e^{-2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{2\left(2\,a^2-3\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^2+3\,a\left(i\,\sqrt{a^2+\frac{1}{x^2}}-\frac{i}{x}\right)\right)}{3\left(-i\,a+\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^3}$$

input
```
integrate(1/(1+I*a*x)^2/(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-2/3*(2*a^2 - 3*(sqrt(a^2 + 1/x^2) - 1/x)^2 + 3*a*(I*sqrt(a^2 + 1/x^2) - I
/x))/(-I*a + sqrt(a^2 + 1/x^2) - 1/x)^3
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{e^{-2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{\sqrt{a^2\,x^2+1}\,(a\,x-2\mathrm{i})}{3\,a\,(1+a\,x\,1\mathrm{i})^2}$$

input
```
int(1/((a^2*x^2 + 1)^(1/2)*(a*x*1i + 1)^2),x)
```

output
```
-((a^2*x^2 + 1)^(1/2)*(a*x - 2i))/(3*a*(a*x*1i + 1)^2)
```

Reduce [F]

$$\int \frac{e^{-2i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\left(\int \frac{1}{\sqrt{a^2x^2+1}\,a^2x^2-2\sqrt{a^2x^2+1}\,aix-\sqrt{a^2x^2+1}}dx\right)$$

input
```
int(1/(1+I*a*x)^2/(a^2*x^2+1)^(1/2),x)
```

output
```
- int(1/(sqrt(a**2*x**2 + 1)*a**2*x**2 - 2*sqrt(a**2*x**2 + 1)*a*i*x - sq
rt(a**2*x**2 + 1)),x)
```

3.340 $\int \frac{e^{-3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 19

$$\int \frac{e^{-3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{i}{2a(1+iax)^2}$$

output
```
1/2*I/a/(1+I*a*x)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{e^{-3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{i}{2a(-i+ax)^2}$$

input
```
Integrate[1/(E^((3*I)*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
```

output
```
(-1/2*I)/(a*(-I + a*x)^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5596, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3i\arctan(ax)}}{(a^2x^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{ 5596}$$

$$\int \frac{1}{(1+iax)^3}dx$$

$$\Big\downarrow \text{ 17}$$

$$\frac{i}{2a(1+iax)^2}$$

input
```
Int[1/(E^((3*I)*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
```

output
```
(I/2)/(a*(1 + I*a*x)^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{i}{2a(ax-i)^2}$	15
default	$\frac{i}{2a(iax+1)^2}$	16
meijerg	$\frac{x(iax+2)}{2(iax+1)^2}$	20
gosper	$\frac{-ax+i}{2a(iax+1)^3}$	22
parallelrisch	$-\frac{ia\,x^2+2x}{2(-ax+i)^2}$	23
orering	$-\frac{x(-ax+2i)(-ax+i)}{2(iax+1)^3}$	27
norman	$\frac{x-\frac{3}{2}ia\,x^2-\frac{1}{2}ia^3x^4}{(a^2x^2+1)^2}$	31

input
```
int(1/(1+I*a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/a/(a*x-I)^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{-3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{i}{2\,(a^3x^2-2i\,a^2x-a)}$$

input
```
integrate(1/(1+I*a*x)^3,x, algorithm="fricas")
```

output
```
-1/2*I/(a^3*x^2 - 2*I*a^2*x - a)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{e^{-3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{i}{2a^3x^2 - 4ia^2x - 2a}$$

input
```
integrate(1/(1+I*a*x)**3,x)
```

output
```
-I/(2*a**3*x**2 - 4*I*a**2*x - 2*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{e^{-3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{i}{2\,(i\,ax+1)^2a}$$

input
```
integrate(1/(1+I*a*x)^3,x, algorithm="maxima")
```

output
```
1/2*I/((I*a*x + 1)^2*a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{e^{-3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{i}{2\,(i\,ax+1)^2a}$$

input
```
integrate(1/(1+I*a*x)^3,x, algorithm="giac")
```

output
```
1/2*I/((I*a*x + 1)^2*a)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{e^{-3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{1\mathrm{i}}{2\left(-a^3\,x^2+a^2\,x\,2\mathrm{i}+a\right)}$$

input
```
int(1/(a*x*1i + 1)^3,x)
```

output
```
1i/(2*(a + a^2*x*2i - a^3*x^2))
```

Reduce [F]

$$\int \frac{e^{-3i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\left(\int \frac{1}{a^3i\,x^3+3a^2x^2-3aix-1}dx\right)$$

input
```
int(1/(1+I*a*x)^3,x)
```

output
```
- int(1/(a**3*i*x**3 + 3*a**2*x**2 - 3*a*i*x - 1),x)
```

3.341 $\int \frac{e^{-4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{e^{-4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{i(1-iax)^4}{3a\,(1+a^2x^2)^{5/2}} - \frac{i(1-iax)^5}{15a\,(1+a^2x^2)^{5/2}}$$

output
```
1/3*I*(1-I*a*x)^4/a/(a^2*x^2+1)^(5/2)-1/15*I*(1-I*a*x)^5/a/(a^2*x^2+1)^(5/
2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int \frac{e^{-4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{(1-iax)^{3/2}(-4i+ax)}{15a\sqrt{1+iax}(-i+ax)^2}$$

input
```
Integrate[1/(E^((4*I)*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
```

output
```
((1 - I*a*x)^(3/2)*(-4*I + a*x))/(15*a*Sqrt[1 + I*a*x]*(-I + a*x)^2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5596, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-4i\arctan(ax)}}{(a^2x^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{5596}$$

$$\int \frac{\sqrt{1-iax}}{(1+iax)^{7/2}}dx$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{5}\int \frac{\sqrt{1-iax}}{(iax+1)^{5/2}}dx+\frac{i(1-iax)^{3/2}}{5a(1+iax)^{5/2}}$$

$$\Big\downarrow \text{48}$$

$$\frac{i(1-iax)^{3/2}}{15a(1+iax)^{3/2}}+\frac{i(1-iax)^{3/2}}{5a(1+iax)^{5/2}}$$

input
```
Int[1/(E^((4*I)*ArcTan[a*x])*(1 + a^2*x^2)^(3/2)),x]
```

output
```
((I/5)*(1 - I*a*x)^(3/2))/(a*(1 + I*a*x)^(5/2)) + ((I/15)*(1 - I*a*x)^(3/2
))/(a*(1 + I*a*x)^(3/2))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.51

method	result	size
orering	$\frac{(-ax+4i)\left(a^2x^2+1\right)^{\frac{3}{2}}}{15a(iax+1)^4}$	33
gosper	$-\frac{(-ax+i)(ax+i)(-ax+4i)\sqrt{a^2x^2+1}}{15a(iax+1)^4}$	46
default	$\frac{\frac{i\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}}}{5a\left(x-\frac{i}{a}\right)^4}-\frac{\left(\left(x-\frac{i}{a}\right)^2a^2+2ia\left(x-\frac{i}{a}\right)\right)^{\frac{3}{2}}}{15\left(x-\frac{i}{a}\right)^3}}{a^4}$	92

input
```
int(1/(1+I*a*x)^4*(a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(-a*x+4*I)/a*(a^2*x^2+1)^(3/2)/(1+I*a*x)^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15

$$\int \frac{e^{-4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\frac{a^3x^3 - 3i\,a^2x^2 - 3\,ax + (a^2x^2 - 3i\,ax + 4)\sqrt{a^2x^2+1} + i}{15\,(a^4x^3 - 3i\,a^3x^2 - 3\,a^2x + i\,a)}$$

input
```
integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(a^3*x^3 - 3*I*a^2*x^2 - 3*a*x + (a^2*x^2 - 3*I*a*x + 4)*sqrt(a^2*x^
2 + 1) + I)/(a^4*x^3 - 3*I*a^3*x^2 - 3*a^2*x + I*a)
```

Sympy [F]

$$\int \frac{e^{-4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \int \frac{\sqrt{a^2x^2+1}}{(ax-i)^4}\,dx$$

input
```
integrate(1/(1+I*a*x)**4*(a**2*x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(a**2*x**2 + 1)/(a*x - I)**4, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(49) = 98.

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.52

$$\int \frac{e^{-4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{2i\sqrt{a^2x^2+1}}{-5i\,a^4x^3 - 15\,a^3x^2 + 15i\,a^2x + 5\,a} + \frac{i\sqrt{a^2x^2+1}}{15\,(a^3x^2 - 2i\,a^2x - a)} - \frac{i\sqrt{a^2x^2+1}}{15i\,a^2x + 15\,a}$$

input
```
integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
2*I*sqrt(a^2*x^2 + 1)/(-5*I*a^4*x^3 - 15*a^3*x^2 + 15*I*a^2*x + 5*a) + 1/1
5*I*sqrt(a^2*x^2 + 1)/(a^3*x^2 - 2*I*a^2*x - a) - I*sqrt(a^2*x^2 + 1)/(15*
I*a^2*x + 15*a)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(49) = 98$.

Time = 0.15 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.71

$$\int \frac{e^{-4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{2\left(4a^4 - 25a^2\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^2 - 15ia\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^3 + 15\left(\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^4 -}{15\left(-ia+\sqrt{a^2+\frac{1}{x^2}}-\frac{1}{x}\right)^5}$$

input
```
integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
2/15*(4*a^4 - 25*a^2*(sqrt(a^2 + 1/x^2) - 1/x)^2 - 15*I*a*(sqrt(a^2 + 1/x^
2) - 1/x)^3 + 15*(sqrt(a^2 + 1/x^2) - 1/x)^4 - 5*a^3*(-I*sqrt(a^2 + 1/x^2)
 + I/x))/(-I*a + sqrt(a^2 + 1/x^2) - 1/x)^5
```

Mupad [B] (verification not implemented)

Time = 22.96 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int \frac{e^{-4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = \frac{\sqrt{a^2\,x^2+1}\,\left(a^2\,x^2-a\,x\,3\mathrm{i}+4\right)\,1\mathrm{i}}{15\,a\,\left(1+a\,x\,1\mathrm{i}\right)^3}$$

input
```
int((a^2*x^2 + 1)^(1/2)/(a*x*1i + 1)^4,x)
```

output
```
((a^2*x^2 + 1)^(1/2)*(a^2*x^2 - a*x*3i + 4)*1i)/(15*a*(a*x*1i + 1)^3)
```

Reduce [F]

$$\int \frac{e^{-4i\arctan(ax)}}{(1+a^2x^2)^{3/2}}\,dx = -\left(\int -\frac{\sqrt{a^2x^2+1}}{a^4x^4-4a^3i\,x^3-6a^2x^2+4aix+1}dx\right)$$

input
```
int(1/(1+I*a*x)^4*(a^2*x^2+1)^(1/2),x)
```

output
```
- int(( - sqrt(a**2*x**2 + 1))/(a**4*x**4 - 4*a**3*i*x**3 - 6*a**2*x**2 +
4*a*i*x + 1),x)
```

3.342 $\int \frac{e^{5i\arctan(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{e^{5i\arctan(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx=-\frac{2\sqrt{1+a^2x^2}}{3ac(i+ax)^3\sqrt{c+a^2cx^2}}-\frac{i\sqrt{1+a^2x^2}}{2ac(i+ax)^2\sqrt{c+a^2cx^2}}$$

output
```
-2/3*(a^2*x^2+1)^(1/2)/a/c/(I+a*x)^3/(a^2*c*x^2+c)^(1/2)-1/2*I*(a^2*x^2+1)
^(1/2)/a/c/(I+a*x)^2/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.59

$$\int \frac{e^{5i\arctan(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx=-\frac{i(-i+3ax)\sqrt{1+a^2x^2}}{6ac(i+ax)^3\sqrt{c+a^2cx^2}}$$

input
```
Integrate[E^((5*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
((-1/6*I)*(-I + 3*a*x)*Sqrt[1 + a^2*x^2])/(a*c*(I + a*x)^3*Sqrt[c + a^2*c*
x^2])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5599, 5596, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{5i\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx$$
$$\Big\downarrow \text{5599}$$
$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{5i\arctan(ax)}}{(a^2x^2+1)^{3/2}}dx}{c\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \text{5596}$$
$$\frac{\sqrt{a^2x^2+1}\int \frac{iax+1}{(1-iax)^4}dx}{c\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \text{53}$$
$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{i}{(ax+i)^3}+\frac{2}{(ax+i)^4}\right)dx}{c\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt{a^2x^2+1}\left(-\frac{i}{2a(ax+i)^2}-\frac{2}{3a(ax+i)^3}\right)}{c\sqrt{a^2cx^2+c}}$$

input
```
Int[E^((5*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(-2/(3*a*(I + a*x)^3) - (I/2)/(a*(I + a*x)^2)))/(c*Sqrt
[c + a^2*c*x^2])
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.46

method	result	size
orering	$-\frac{(ax+5i)(iax+1)^5}{24a(a^2x^2+1)^{\frac{3}{2}}(a^2c\,x^2+c)^{\frac{3}{2}}}$	44
risch	$\frac{\sqrt{a^2x^2+1}\,\left(-\frac{ix}{2}-\frac{1}{6a}\right)}{c\sqrt{c(a^2x^2+1)}\,(ax+i)^3}$	47
default	$-\frac{\sqrt{c(a^2x^2+1)}\,(3iax+1)}{6\sqrt{a^2x^2+1}\,c^2a(ax+i)^3}$	48
gosper	$-\frac{(ax+i)(-3ax+i)(iax+1)^5}{6a(-ax+i)(a^2x^2+1)^{\frac{5}{2}}(a^2c\,x^2+c)^{\frac{3}{2}}}$	60

input
```
int((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
-1/24*(a*x+5*I)/a/(a^2*x^2+1)^(3/2)*(1+I*a*x)^5/(a^2*c*x^2+c)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06

$$\int \frac{e^{5i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{a^2cx^2+c}(i\,a^2x^3-3\,ax^2-6i\,x)\sqrt{a^2x^2+1}}{6\,(a^5c^2x^5+3i\,a^4c^2x^4-2\,a^3c^2x^3+2i\,a^2c^2x^2-3\,ac^2x-i\,c^2)}$$

input
```
integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="
fricas")
```

output
```
1/6*sqrt(a^2*c*x^2 + c)*(I*a^2*x^3 - 3*a*x^2 - 6*I*x)*sqrt(a^2*x^2 + 1)/(a
^5*c^2*x^5 + 3*I*a^4*c^2*x^4 - 2*a^3*c^2*x^3 + 2*I*a^2*c^2*x^2 - 3*a*c^2*x
 - I*c^2)
```

Sympy [F]

$$\int \frac{e^{5i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = i\left(\int \left(-\frac{i}{a^6cx^6\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+3a^4cx^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+3a^2cx^2\sqrt{a^2x}\right.\right.$$
$$+\int \frac{5ax}{a^6cx^6\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+3a^4cx^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+3a^2cx^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2}}$$
$$+\int \left(-\frac{10a^3x^3}{a^6cx^6\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+3a^4cx^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+3a^2cx^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+c\sqrt{a}}\right.$$
$$+\int \frac{a^5x^5}{a^6cx^6\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+3a^4cx^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+3a^2cx^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2}}$$
$$+\int \frac{10ia^2x^2}{a^6cx^6\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+3a^4cx^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+3a^2cx^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2}}$$
$$+\int \left(-\frac{5ia^4x^4}{a^6cx^6\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+3a^4cx^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+3a^2cx^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+c\sqrt{a}}\right.$$

input
```
integrate((1+I*a*x)**5/(a**2*x**2+1)**(5/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
I*(Integral(-I/(a**6*c*x**6*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 3*
a**4*c*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 3*a**2*c*x**2*sqrt
(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*
x**2 + c)), x) + Integral(5*a*x/(a**6*c*x**6*sqrt(a**2*x**2 + 1)*sqrt(a**2
*c*x**2 + c) + 3*a**4*c*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 3
*a**2*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*x**2
+ 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(-10*a**3*x**3/(a**6*c*x**6*sqrt
(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 3*a**4*c*x**4*sqrt(a**2*x**2 + 1)*
sqrt(a**2*c*x**2 + c) + 3*a**2*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2
 + c) + c*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(a**5*x
**5/(a**6*c*x**6*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 3*a**4*c*x**4
*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 3*a**2*c*x**2*sqrt(a**2*x**2
+ 1)*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)),
 x) + Integral(10*I*a**2*x**2/(a**6*c*x**6*sqrt(a**2*x**2 + 1)*sqrt(a**2*c
*x**2 + c) + 3*a**4*c*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 3*a
**2*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*x**2 +
1)*sqrt(a**2*c*x**2 + c)), x) + Integral(-5*I*a**4*x**4/(a**6*c*x**6*sqrt(
a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 3*a**4*c*x**4*sqrt(a**2*x**2 + 1)*s
qrt(a**2*c*x**2 + c) + 3*a**2*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2
+ c) + c*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x))
```

Maxima [F]

$$\int \frac{e^{5i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx=\int \frac{(i\,ax+1)^5}{(a^2cx^2+c)^{\frac{3}{2}}(a^2x^2+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="
maxima")
```

output
```
integrate((I*a*x + 1)^5/((a^2*c*x^2 + c)^(3/2)*(a^2*x^2 + 1)^(5/2)), x)
```

Giac [F]

$$\int \frac{e^{5i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{(i\,ax+1)^5}{(a^2cx^2+c)^{\frac{3}{2}}(a^2x^2+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="
giac")
```

output
```
integrate((I*a*x + 1)^5/((a^2*c*x^2 + c)^(3/2)*(a^2*x^2 + 1)^(5/2)), x)
```

Mupad [B] (verification not implemented)

Time = 23.99 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.51

$$\int \frac{e^{5i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{c\,(a^2\,x^2+1)}\,(3\,a\,x-\mathrm{i})}{6\,a\,c^2\,\sqrt{a^2\,x^2+1}\,(-1+a\,x\,1\mathrm{i})^3}$$

input
```
int((a*x*1i + 1)^5/((c + a^2*c*x^2)^(3/2)*(a^2*x^2 + 1)^(5/2)),x)
```

output
```
-((c*(a^2*x^2 + 1))^(1/2)*(3*a*x - 1i))/(6*a*c^2*(a^2*x^2 + 1)^(1/2)*(a*x*
1i - 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.61

$$\int \frac{e^{5i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,x(a^5i\,x^5-10a^2x^2+15aix+6)}{6c^2\,(a^6x^6+3a^4x^4+3a^2x^2+1)}$$

input
```
int((1+I*a*x)^5/(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*x*(a**5*i*x**5 - 10*a**2*x**2 + 15*a*i*x + 6))/(6*c**2*(a**6*x**6
 + 3*a**4*x**4 + 3*a**2*x**2 + 1))
```

3.343 $\int \frac{e^{4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 69

$$\int \frac{e^{4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{ic(1+iax)^4}{3a\,(c+a^2cx^2)^{5/2}} + \frac{ic(1+iax)^5}{15a\,(c+a^2cx^2)^{5/2}}$$

output
```
-1/3*I*c*(1+I*a*x)^4/a/(a^2*c*x^2+c)^(5/2)+1/15*I*c*(1+I*a*x)^5/a/(a^2*c*x
^2+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12

$$\int \frac{e^{4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{(1+iax)^{3/2}(4i+ax)\sqrt{1+a^2x^2}}{15ac\sqrt{1-iax}(i+ax)^2\sqrt{c+a^2cx^2}}$$

input
```
Integrate[E^((4*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
((1 + I*a*x)^(3/2)*(4*I + a*x)*Sqrt[1 + a^2*x^2])/(15*a*c*Sqrt[1 - I*a*x]*
(I + a*x)^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5598, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4i\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5598}$$

$$c^2\int \frac{(iax+1)^4}{(a^2cx^2+c)^{7/2}}dx$$

$$\Big\downarrow \text{461}$$

$$c^2\left(-\frac{1}{3}\int \frac{(iax+1)^5}{(a^2cx^2+c)^{7/2}}dx-\frac{i(1+iax)^4}{3ac\,(a^2cx^2+c)^{5/2}}\right)$$

$$\Big\downarrow \text{460}$$

$$c^2\left(\frac{i(1+iax)^5}{15ac\,(a^2cx^2+c)^{5/2}}-\frac{i(1+iax)^4}{3ac\,(a^2cx^2+c)^{5/2}}\right)$$

input
```
Int[E^((4*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
c^2*(((-1/3*I)*(1 + I*a*x)^4)/(a*c*(c + a^2*c*x^2)^(5/2)) + ((I/15)*(1 + I
*a*x)^5)/(a*c*(c + a^2*c*x^2)^(5/2)))
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 5598
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Si
mp[1/c^(I*(n/2))   Int[(c + d*x^2)^(p + I*(n/2))/(1 + I*a*x)^(I*n), x], x]
/; FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
&& ILtQ[I*(n/2), 0]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

method	result
orering	$-\frac{(ax+4i)(iax+1)^4}{15a(a^2x^2+1)(a^2c\,x^2+c)^{\frac{3}{2}}}$
gosper	$\frac{(-ax+i)(ax+i)(ax+4i)(iax+1)^4}{15a(a^2x^2+1)^2(a^2c\,x^2+c)^{\frac{3}{2}}}$
trager	$\frac{(-a^5x^5-10a^3x^3+20ix^2a^2+15ax-4i)\sqrt{a^2c\,x^2+c}}{15c^2(a^2x^2+1)^3a}$
default	$\frac{x}{c\sqrt{a^2c\,x^2+c}}+\frac{2\left(i\sqrt{-a^2}-a\right)\left(\frac{1}{5c\sqrt{-a^2}\left(x+\frac{\sqrt{-a^2}}{a^2}\right)^2\sqrt{\left(x+\frac{\sqrt{-a^2}}{a^2}\right)^2a^2c-2c\sqrt{-a^2}\left(x+\frac{\sqrt{-a^2}}{a^2}\right)}}+\frac{3a^2\left(\frac{\cdots}{3c\sqrt{-a^2}\left(x+\frac{\sqrt{-a^2}}{a^2}\right)\sqrt{\left(x+\cdots\right.}}\cdots\right.}{\cdots}\cdots\right)}{a^3}$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(a*x+4*I)/a/(a^2*x^2+1)*(1+I*a*x)^4/(a^2*c*x^2+c)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{e^{4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{a^2cx^2+c}(a^2x^2+3i\,ax+4)}{15\,(a^4c^2x^3+3i\,a^3c^2x^2-3\,a^2c^2x-i\,ac^2)}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fric
as")
```

output
```
-1/15*sqrt(a^2*c*x^2 + c)*(a^2*x^2 + 3*I*a*x + 4)/(a^4*c^2*x^3 + 3*I*a^3*c
^2*x^2 - 3*a^2*c^2*x - I*a*c^2)
```

Sympy [F]

$$\int \frac{e^{4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{(ax-i)^4}{(c\,(a^2x^2+1))^{\frac{3}{2}}\,(a^2x^2+1)^2}\,dx$$

input
```
integrate((1+I*a*x)**4/(a**2*x**2+1)**2/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral((a*x - I)**4/((c*(a**2*x**2 + 1))**(3/2)*(a**2*x**2 + 1)**2), x)
```

Maxima [F]

$$\int \frac{e^{4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{(i\,ax+1)^4}{(a^2cx^2+c)^{\frac{3}{2}}(a^2x^2+1)^2}\,dx$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate((I*a*x + 1)^4/((a^2*c*x^2 + c)^(3/2)*(a^2*x^2 + 1)^2), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(53) = 106$.

Time = 0.14 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.94

$$\int \frac{e^{4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{2\Big(15\big(\sqrt{a^2c}\,x-\sqrt{a^2cx^2+c}\big)^3\sqrt{c}-5i\big(\sqrt{a^2c}\,x-\sqrt{a^2cx^2+c}\big)^2c-5\big(\sqrt{a^2c}\,x-\sqrt{c}}{15\big(\sqrt{a^2c}\,x-\sqrt{a^2cx^2+c}+i\sqrt{c}\big)^5ac}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac
")
```

output
```
2/15*(15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 + c))^3*sqrt(c) - 5*I*(sqrt(a^2*c
)*x - sqrt(a^2*c*x^2 + c))^2*c - 5*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 + c))*c
^(3/2) - I*c^2)/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 + c) + I*sqrt(c))^5*a*c)
```

Mupad [B] (verification not implemented)

Time = 23.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.67

$$\int \frac{e^{4i\arctan(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{c\,\left(a^2\,x^2+1\right)}\,\left(a^2\,x^2\,1\mathrm{i}-3\,a\,x+4\mathrm{i}\right)}{15\,a\,c^2\,\left(-1+a\,x\,1\mathrm{i}\right)^3}$$

input
```
int((a*x*1i + 1)^4/((c + a^2*c*x^2)^(3/2)*(a^2*x^2 + 1)^2),x)
```

output
```
((c*(a^2*x^2 + 1))^(1/2)*(a^2*x^2*1i - 3*a*x + 4i))/(15*a*c^2*(a*x*1i - 1)
^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.10

$$\int \frac{e^{4i\arctan(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{c}\,\left(-\sqrt{a^2x^2+1}\,a^5x^5-10\sqrt{a^2x^2+1}\,a^3x^3+20\sqrt{a^2x^2+1}\,a^2i\,x^2+15\sqrt{a^2x^2+1}\,ax}{15a\,c^2\left(a^6x^6+3a^4x^4+3a^2x^2+1\right)}$$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*( - sqrt(a**2*x**2 + 1)*a**5*x**5 - 10*sqrt(a**2*x**2 + 1)*a**3*x
**3 + 20*sqrt(a**2*x**2 + 1)*a**2*i*x**2 + 15*sqrt(a**2*x**2 + 1)*a*x - 4*
sqrt(a**2*x**2 + 1)*i + 7*a**6*x**6 + 21*a**4*x**4 + 21*a**2*x**2 + 7))/(1
5*a*c**2*(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1))
```

3.344 $\int \frac{e^{3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 49

$$\int \frac{e^{3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{i\sqrt{1+a^2x^2}}{2ac(1-iax)^2\sqrt{c+a^2cx^2}}$$

output
```
-1/2*I*(a^2*x^2+1)^(1/2)/a/c/(1-I*a*x)^2/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{e^{3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{i\sqrt{1+a^2x^2}}{2ac(i+ax)^2\sqrt{c+a^2cx^2}}$$

input
```
Integrate[E^((3*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
((I/2)*Sqrt[1 + a^2*x^2])/(a*c*(I + a*x)^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5599, 5596, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3i\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5599}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{3i\arctan(ax)}}{(a^2x^2+1)^{3/2}}dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5596}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{(1-iax)^3}dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{17}$$

$$-\frac{i\sqrt{a^2x^2+1}}{2ac(1-iax)^2\sqrt{a^2cx^2+c}}$$

input
```
Int[E^((3*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
((-1/2*I)*Sqrt[1 + a^2*x^2])/(a*c*(1 - I*a*x)^2*Sqrt[c + a^2*c*x^2])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{i\sqrt{c(a^2x^2+1)}}{2\sqrt{a^2x^2+1}\,c^2a(ax+i)^2}$	42
risch	$\frac{i\sqrt{a^2x^2+1}}{2c\sqrt{c(a^2x^2+1)}\,a(ax+i)^2}$	42
gosper	$-\frac{(ax+i)(iax+1)^3}{2a(a^2x^2+1)^{\frac{3}{2}}(a^2c\,x^2+c)^{\frac{3}{2}}}$	44
orering	$-\frac{x(ax+2i)(ax+i)(iax+1)^3}{2(a^2x^2+1)^{\frac{3}{2}}(a^2c\,x^2+c)^{\frac{3}{2}}}$	48

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
1/2*I/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)/c^2/a/(I+a*x)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.45

$$\int \frac{e^{3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}(i\,ax^2-2\,x)}{2\,(a^4c^2x^4+2i\,a^3c^2x^3+2i\,ac^2x-c^2)}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="
fricas")
```

output
```
1/2*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*(I*a*x^2 - 2*x)/(a^4*c^2*x^4 + 2
*I*a^3*c^2*x^3 + 2*I*a*c^2*x - c^2)
```

Sympy [F]

$$\int \frac{e^{3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -i\left(\int \frac{i}{a^4cx^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+2a^2cx^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\,dx + \int \left(-\frac{3ax}{a^4cx^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+2a^2cx^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\right)dx + \int \frac{a^3x^3}{a^4cx^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+2a^2cx^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\,dx + \int \left(-\frac{3ia^2x^2}{a^4cx^4\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+2a^2cx^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\right)dx\right)$$

input
```
integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
-I*(Integral(I/(a**4*c*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 2*
a**2*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*x**2 +
 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(-3*a*x/(a**4*c*x**4*sqrt(a**2*x*
*2 + 1)*sqrt(a**2*c*x**2 + c) + 2*a**2*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**
2*c*x**2 + c) + c*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integra
l(a**3*x**3/(a**4*c*x**4*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 2*a**
2*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*x**2 + 1)
*sqrt(a**2*c*x**2 + c)), x) + Integral(-3*I*a**2*x**2/(a**4*c*x**4*sqrt(a*
*2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 2*a**2*c*x**2*sqrt(a**2*x**2 + 1)*sqr
t(a**2*c*x**2 + c) + c*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e^{3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="
maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{e^{3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{(i\,ax+1)^3}{(a^2cx^2+c)^{\frac{3}{2}}(a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="
giac")
```

output
```
integrate((I*a*x + 1)^3/((a^2*c*x^2 + c)^(3/2)*(a^2*x^2 + 1)^(3/2)), x)
```

Mupad [B] (verification not implemented)

Time = 23.58 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{e^{3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c\,(a^2\,x^2+1)}\,1\mathrm{i}}{2\,a\,c^2\,\sqrt{a^2\,x^2+1}\,(a\,x+1\mathrm{i})^2}$$

input
```
int((a*x*1i + 1)^3/((c + a^2*c*x^2)^(3/2)*(a^2*x^2 + 1)^(3/2)),x)
```

output
```
((c*(a^2*x^2 + 1))^(1/2)*1i)/(2*a*c^2*(a^2*x^2 + 1)^(1/2)*(a*x + 1i)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{e^{3i \arctan(ax)}}{(c + a^2cx^2)^{3/2}} \, dx = \frac{\sqrt{c}\,(-a^4 i\, x^4 + 4ax - 3i)}{4a\, c^2\, (a^4x^4 + 2a^2x^2 + 1)}$$

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*( - a**4*i*x**4 + 4*a*x - 3*i))/(4*a*c**2*(a**4*x**4 + 2*a**2*x**
2 + 1))
```

3.345 $\int \frac{e^{2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 54

$$\int \frac{e^{2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{2i(1+iax)}{3a\,(c+a^2cx^2)^{3/2}} + \frac{x}{3c\sqrt{c+a^2cx^2}}$$

output
```
-2/3*I*(1+I*a*x)/a/(a^2*c*x^2+c)^(3/2)+1/3*x/c/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.44

$$\int \frac{e^{2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{(2-iax)\sqrt{1+iax}\sqrt{1+a^2x^2}}{3ac\sqrt{1-iax}(i+ax)\sqrt{c+a^2cx^2}}$$

input
```
Integrate[E^((2*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
((2 - I*a*x)*Sqrt[1 + I*a*x]*Sqrt[1 + a^2*x^2])/(3*a*c*Sqrt[1 - I*a*x]*(I
+ a*x)*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5598, 457, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2i\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5598}$$

$$c\int \frac{(iax+1)^2}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{457}$$

$$c\left(\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}}dx}{3c}-\frac{2i(1+iax)}{3ac\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{208}$$

$$c\left(\frac{x}{3c^2\sqrt{a^2cx^2+c}}-\frac{2i(1+iax)}{3ac\,(a^2cx^2+c)^{3/2}}\right)$$

input
```
Int[E^((2*I)*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
c*((((-2*I)/3)*(1 + I*a*x))/(a*c*(c + a^2*c*x^2)^(3/2)) + x/(3*c^2*Sqrt[c
+ a^2*c*x^2]))
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 5598
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Si
mp[1/c^(I*(n/2))   Int[(c + d*x^2)^(p + I*(n/2))/(1 + I*a*x)^(I*n), x], x]
/; FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
&& ILtQ[I*(n/2), 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.61

method	result
orering	$-\frac{(ax+2i)(iax+1)^{2}}{3a\left(a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}$
trager	$\frac{\left(a^{3}x^{3}+3ax-2i\right)\sqrt{a^{2}c\,x^{2}+c}}{3c^{2}\left(a^{2}x^{2}+1\right)^{2}a}$
gosper	$\frac{(-ax+i)(ax+i)(ax+2i)(iax+1)^{2}}{3a\left(a^{2}x^{2}+1\right)\left(a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}$
default	$\frac{\left(i\sqrt{-a^{2}}-a\right)\left(\frac{1}{3c\sqrt{-a^{2}}\left(x+\frac{\sqrt{-a^{2}}}{a^{2}}\right)\sqrt{\left(x+\frac{\sqrt{-a^{2}}}{a^{2}}\right)^{2}a^{2}c-2c\sqrt{-a^{2}}\left(x+\frac{\sqrt{-a^{2}}}{a^{2}}\right)}}+\frac{2\left(x+\frac{\sqrt{-a^{2}}}{a^{2}}\right)a^{2}c-2c\sqrt{-a^{2}}}{3c^{2}\sqrt{-a^{2}}\sqrt{\left(x+\frac{\sqrt{-a^{2}}}{a^{2}}\right)^{2}a^{2}c-2c\sqrt{-a^{2}}\left(x+\frac{\sqrt{-a^{2}}}{a^{2}}\right)}}\right)}{a\sqrt{-a^{2}}}$

input
```
int((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a*x+2*I)/a*(1+I*a*x)^2/(a^2*c*x^2+c)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{e^{2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{a^2cx^2+c}(ax+2i)}{3\,(a^3c^2x^2+2i\,a^2c^2x-ac^2)}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas
")
```

output
```
1/3*sqrt(a^2*c*x^2 + c)*(a*x + 2*I)/(a^3*c^2*x^2 + 2*I*a^2*c^2*x - a*c^2)
```

Sympy [F]

$$\int \frac{e^{2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\int \frac{a^2x^2}{a^4cx^4\sqrt{a^2cx^2+c}+2a^2cx^2\sqrt{a^2cx^2+c}+c\sqrt{a^2cx^2+c}}\,dx - \int \left(-\frac{2iax}{a^4cx^4\sqrt{a^2cx^2+c}+2a^2cx^2\sqrt{a^2cx^2+c}+c\sqrt{a^2cx^2+c}}\right)dx - \int \left(-\frac{1}{a^4cx^4\sqrt{a^2cx^2+c}+2a^2cx^2\sqrt{a^2cx^2+c}+c\sqrt{a^2cx^2+c}}\right)dx$$

input
```
integrate((1+I*a*x)**2/(a**2*x**2+1)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
-Integral(a**2*x**2/(a**4*c*x**4*sqrt(a**2*c*x**2 + c) + 2*a**2*c*x**2*sqr
t(a**2*c*x**2 + c) + c*sqrt(a**2*c*x**2 + c)), x) - Integral(-2*I*a*x/(a**
4*c*x**4*sqrt(a**2*c*x**2 + c) + 2*a**2*c*x**2*sqrt(a**2*c*x**2 + c) + c*s
qrt(a**2*c*x**2 + c)), x) - Integral(-1/(a**4*c*x**4*sqrt(a**2*c*x**2 + c)
 + 2*a**2*c*x**2*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*c*x**2 + c)), x)
```

Maxima [F]

$$\int \frac{e^{2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{(i\,ax+1)^2}{(a^2cx^2+c)^{\frac{3}{2}}(a^2x^2+1)}\,dx$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((I*a*x + 1)^2/((a^2*c*x^2 + c)^(3/2)*(a^2*x^2 + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.41

$$\int \frac{e^{2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{2\sqrt{a^2c}\left(3\sqrt{a^2c}x-3\sqrt{a^2cx^2+c}+i\sqrt{c}\right)}{3\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}+i\sqrt{c}\right)^3a^2c}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(a^2*c)*(3*sqrt(a^2*c)*x - 3*sqrt(a^2*c*x^2 + c) + I*sqrt(c))/((s
qrt(a^2*c)*x - sqrt(a^2*c*x^2 + c) + I*sqrt(c))^3*a^2*c)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.59

$$\int \frac{e^{2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{a^3\,x^3+3\,a\,x-2\mathrm{i}}{3\,a\,\left(c\,\left(a^2\,x^2+1\right)\right)^{3/2}}$$

input
```
int((a*x*1i + 1)^2/((c + a^2*c*x^2)^(3/2)*(a^2*x^2 + 1)),x)
```

output
```
(3*a*x + a^3*x^3 - 2i)/(3*a*(c*(a^2*x^2 + 1))^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.69

$$\int \frac{e^{2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(\sqrt{a^2x^2+1}\,a^3x^3+3\sqrt{a^2x^2+1}\,ax-2\sqrt{a^2x^2+1}\,i-3a^4x^4-6a^2x^2-3\right)}{3a\,c^2\left(a^4x^4+2a^2x^2+1\right)}$$

input
```
int((1+I*a*x)^2/(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*(sqrt(a**2*x**2 + 1)*a**3*x**3 + 3*sqrt(a**2*x**2 + 1)*a*x - 2*sq
rt(a**2*x**2 + 1)*i - 3*a**4*x**4 - 6*a**2*x**2 - 3))/(3*a*c**2*(a**4*x**4
 + 2*a**2*x**2 + 1))
```

3.346 $\int \frac{e^{i \arctan(ax)}}{(c+a^2cx^2)^{3/2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \frac{e^{i \arctan(ax)}}{(c+a^2cx^2)^{3/2}} \, dx = \frac{\sqrt{1+a^2x^2}}{2ac(i+ax)\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2} \arctan(ax)}{2ac\sqrt{c+a^2cx^2}}$$

output
```
1/2*(a^2*x^2+1)^(1/2)/a/c/(I+a*x)/(a^2*c*x^2+c)^(1/2)+1/2*(a^2*x^2+1)^(1/2
)*arctan(a*x)/a/c/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.58

$$\int \frac{e^{i \arctan(ax)}}{(c+a^2cx^2)^{3/2}} \, dx = \frac{\sqrt{1+a^2x^2}\left(\frac{1}{i+ax} + \arctan(ax)\right)}{2ac\sqrt{c+a^2cx^2}}$$

input
```
Integrate[E^(I*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(Sqrt[1 + a^2*x^2]*((I + a*x)^(-1) + ArcTan[a*x]))/(2*a*c*Sqrt[c + a^2*c*x
^2])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.67, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5599, 5596, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{i\arctan(ax)}}{(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 5599}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{i\arctan(ax)}}{(a^2x^2+1)^{3/2}}dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5596}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{(1-iax)^2(iax+1)}dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{1}{2(a^2x^2+1)}-\frac{1}{2(ax+i)^2}\right)dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{\arctan(ax)}{2a}+\frac{1}{2a(ax+i)}\right)}{c\sqrt{a^2cx^2+c}}$$

input
```
Int[E^(I*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(1/(2*a*(I + a*x)) + ArcTan[a*x]/(2*a)))/(c*Sqrt[c + a^
2*c*x^2])
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{\sqrt{c\left(a^{2} x^{2}+1\right)}\, \left(-\arctan \left(a x \right) a^{2} x^{2}-a x +i-\arctan \left(a x \right)\right)}{2 \left(a^{2} x^{2}+1\right)^{\frac{3}{2}} a \,c^{2}}$	58
risch	$\frac{\sqrt{a^{2} x^{2}+1}}{2 c \sqrt{c \left(a^{2} x^{2}+1\right)}\, a \left(a x +i\right)}+\frac{i \sqrt{a^{2} x^{2}+1}\, \ln \left(i a x -1\right)}{4 c \sqrt{c \left(a^{2} x^{2}+1\right)}\, a}-\frac{i \sqrt{a^{2} x^{2}+1}\, \ln \left(-i a x -1\right)}{4 c \sqrt{c \left(a^{2} x^{2}+1\right)}\, a}$	124

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/2/(a^2*x^2+1)^(3/2)*(c*(a^2*x^2+1))^(1/2)/a*(-arctan(a*x)*a^2*x^2-a*x+I
-arctan(a*x))/c^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(74) = 148$.

Time = 0.15 (sec) , antiderivative size = 317, normalized size of antiderivative = 3.60

$$\int \frac{e^{i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{(i\,a^3c^2x^3 - a^2c^2x^2 + i\,ac^2x - c^2)\sqrt{\frac{1}{a^2c^3}}\log\left(\frac{2\left(2\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}a^6x-(i\,a^{10}c^2x^4-i\,a^6c^2)\sqrt{\cdots}\right.}{a^4x^4+2\,a^2x^2+1}\right.}{}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fr
icas")
```

output
```
1/8*((I*a^3*c^2*x^3 - a^2*c^2*x^2 + I*a*c^2*x - c^2)*sqrt(1/(a^2*c^3))*log
(2*(2*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^6*x - (I*a^10*c^2*x^4 - I*a^
6*c^2)*sqrt(1/(a^2*c^3)))/(a^4*x^4 + 2*a^2*x^2 + 1)) + (-I*a^3*c^2*x^3 + a
^2*c^2*x^2 - I*a*c^2*x + c^2)*sqrt(1/(a^2*c^3))*log(2*(2*sqrt(a^2*c*x^2 +
c)*sqrt(a^2*x^2 + 1)*a^6*x - (-I*a^10*c^2*x^4 + I*a^6*c^2)*sqrt(1/(a^2*c^3
)))/(a^4*x^4 + 2*a^2*x^2 + 1)) + 4*I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)
*x)/(a^3*c^2*x^3 + I*a^2*c^2*x^2 + a*c^2*x + I*c^2)
```

Sympy [F]

$$\int \frac{e^{i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = i\left(\int \left(-\frac{i}{a^2cx^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\right)dx + \int \frac{ax}{a^2cx^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\,dx\right)$$

input
```
integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
I*(Integral(-I/(a**2*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + c*
sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(a*x/(a**2*c*x**2
*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*x**2 + 1)*sqrt(a*
*2*c*x**2 + c)), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e^{i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="ma
xima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{1+a\,x\,1\mathrm{i}}{(c\,a^2\,x^2+c)^{3/2}\,\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int((a*x*1i + 1)/((c + a^2*c*x^2)^(3/2)*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
int((a*x*1i + 1)/((c + a^2*c*x^2)^(3/2)*(a^2*x^2 + 1)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.55

$$\int \frac{e^{i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,(atan(ax)\,a^2x^2 + atan(ax) + a^2i\,x^2 + ax)}{2a\,c^2\,(a^2x^2+1)}$$

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*(atan(a*x)*a**2*x**2 + atan(a*x) + a**2*i*x**2 + a*x))/(2*a*c**2*
(a**2*x**2 + 1))
```

3.347 $\int \frac{e^{-i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 89

$$\int \frac{e^{-i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx=-\frac{\sqrt{1+a^2x^2}}{2ac(i-ax)\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\arctan(ax)}{2ac\sqrt{c+a^2cx^2}}$$

output
```
-1/2*(a^2*x^2+1)^(1/2)/a/c/(I-a*x)/(a^2*c*x^2+c)^(1/2)+1/2*(a^2*x^2+1)^(1/
2)*arctan(a*x)/a/c/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int \frac{e^{-i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx=\frac{\sqrt{1+a^2x^2}\left(-\frac{1}{2a(i-ax)}+\frac{\arctan(ax)}{2a}\right)}{c\sqrt{c+a^2cx^2}}$$

input
```
Integrate[1/(E^(I*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(-1/2*1/(a*(I - a*x)) + ArcTan[a*x]/(2*a)))/(c*Sqrt[c +
 a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5599, 5596, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-i\arctan(ax)}}{(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{5599}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{-i\arctan(ax)}}{(a^2x^2+1)^{3/2}}dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5596}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{(1-iax)(iax+1)^2}dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{54}$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{1}{2(a^2x^2+1)}-\frac{1}{2(ax-i)^2}\right)dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{\arctan(ax)}{2a}-\frac{1}{2a(-ax+i)}\right)}{c\sqrt{a^2cx^2+c}}$$

input
```
Int[1/(E^(I*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(-1/2*1/(a*(I - a*x)) + ArcTan[a*x]/(2*a)))/(c*Sqrt[c + 
 a^2*c*x^2])
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{\sqrt{c\left(a^{2} x^{2}+1\right)}\,\left(i \ln \left(a x+i\right) a x-i \ln \left(-a x+i\right) a x+\ln \left(a x+i\right)-\ln \left(-a x+i\right)+2\right)}{4 \sqrt{a^{2} x^{2}+1}\, c^{2} \left(-a x+i\right) a}$	86
risch	$\frac{\sqrt{a^{2} x^{2}+1}}{2 c \sqrt{c\left(a^{2} x^{2}+1\right)}\, a \left(a x-i\right)}-\frac{i \sqrt{a^{2} x^{2}+1}\, \ln \left(-i a x-1\right)}{4 c \sqrt{c\left(a^{2} x^{2}+1\right)}\, a}+\frac{i \sqrt{a^{2} x^{2}+1}\, \ln \left(i a x-1\right)}{4 c \sqrt{c\left(a^{2} x^{2}+1\right)}\, a}$	124

input
```
int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
-1/4/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)*(I*ln(I+a*x)*a*x-I*ln(I-a*x)*
a*x+ln(I+a*x)-ln(I-a*x)+2)/c^2/(I-a*x)/a
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(74) = 148$.

Time = 0.15 (sec) , antiderivative size = 317, normalized size of antiderivative = 3.56

$$\int \frac{e^{-i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{(i\,a^3c^2x^3 + a^2c^2x^2 + i\,ac^2x + c^2)\sqrt{\frac{1}{a^2c^3}}\log\left(\frac{2\left(2\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}a^6x-(i\,a^{10}c^2x^4-i\,a^6c^2)\sqrt{\frac{1}{a^2c^3}}\right)}{a^4x^4+2\,a^2x^2+1}\right)}{}$$

input
```
integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="
fricas")
```

output
```
1/8*((I*a^3*c^2*x^3 + a^2*c^2*x^2 + I*a*c^2*x + c^2)*sqrt(1/(a^2*c^3))*log
(2*(2*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^6*x - (I*a^10*c^2*x^4 - I*a^
6*c^2)*sqrt(1/(a^2*c^3)))/(a^4*x^4 + 2*a^2*x^2 + 1)) + (-I*a^3*c^2*x^3 - a
^2*c^2*x^2 - I*a*c^2*x - c^2)*sqrt(1/(a^2*c^3))*log(2*(2*sqrt(a^2*c*x^2 +
c)*sqrt(a^2*x^2 + 1)*a^6*x - (-I*a^10*c^2*x^4 + I*a^6*c^2)*sqrt(1/(a^2*c^3
)))/(a^4*x^4 + 2*a^2*x^2 + 1)) - 4*I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)
*x)/(a^3*c^2*x^3 - I*a^2*c^2*x^2 + a*c^2*x - I*c^2)
```

Sympy [F]

$$\int \frac{e^{-i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -i\int \frac{\sqrt{a^2x^2+1}}{a^3cx^3\sqrt{a^2cx^2+c} - ia^2cx^2\sqrt{a^2cx^2+c} + acx\sqrt{a^2cx^2+c} - ic\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(1/(1+I*a*x)*(a**2*x**2+1)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
-I*Integral(sqrt(a**2*x**2 + 1)/(a**3*c*x**3*sqrt(a**2*c*x**2 + c) - I*a**
2*c*x**2*sqrt(a**2*c*x**2 + c) + a*c*x*sqrt(a**2*c*x**2 + c) - I*c*sqrt(a*
*2*c*x**2 + c)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

$$\int \frac{e^{-i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}}{2\,(a^2c^2x-i\,ac^2)} - \frac{i\,\log\,(ax-i)}{4\,ac^{\frac{3}{2}}} + \frac{i\,\log\,(i\,ax-1)}{4\,ac^{\frac{3}{2}}}$$

input
```
integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="
maxima")
```

output
```
1/2*sqrt(c)/(a^2*c^2*x - I*a*c^2) - 1/4*I*log(a*x - I)/(a*c^(3/2)) + 1/4*I
*log(I*a*x - 1)/(a*c^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{a^2\,x^2+1}}{(c\,a^2\,x^2+c)^{3/2}\,(1+a\,x\,1\mathrm{i})}\,dx$$

input
```
int((a^2*x^2 + 1)^(1/2)/((c + a^2*c*x^2)^(3/2)*(a*x*1i + 1)),x)
```

output
```
int((a^2*x^2 + 1)^(1/2)/((c + a^2*c*x^2)^(3/2)*(a*x*1i + 1)), x)
```

Reduce [F]

$$\int \frac{e^{-i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx=\frac{\sqrt{c}\left(\int \frac{1}{a^3i\,x^3+a^2x^2+aix+1}dx\right)}{c^2}$$

input
```
int(1/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*int(1/(a**3*i*x**3 + a**2*x**2 + a*i*x + 1),x))/c**2
```

3.348 $\int \frac{e^{-2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 54

$$\int \frac{e^{-2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{2i(1-iax)}{3a\,(c+a^2cx^2)^{3/2}} + \frac{x}{3c\sqrt{c+a^2cx^2}}$$

output
```
2/3*I*(1-I*a*x)/a/(a^2*c*x^2+c)^(3/2)+1/3*x/c/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.44

$$\int \frac{e^{-2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{1-iax}(2+iax)\sqrt{1+a^2x^2}}{3ac\sqrt{1+iax}(-i+ax)\sqrt{c+a^2cx^2}}$$

input
```
Integrate[1/(E^((2*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
(Sqrt[1 - I*a*x]*(2 + I*a*x)*Sqrt[1 + a^2*x^2])/(3*a*c*Sqrt[1 + I*a*x]*(-I
 + a*x)*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5597, 457, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2i\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5597}$$

$$c\int \frac{(1-iax)^2}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{457}$$

$$c\left(\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{2i(1-iax)}{3ac\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{208}$$

$$c\left(\frac{x}{3c^2\sqrt{a^2cx^2+c}}+\frac{2i(1-iax)}{3ac\,(a^2cx^2+c)^{3/2}}\right)$$

input
```
Int[1/(E^((2*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
c*((((2*I)/3)*(1 - I*a*x))/(a*c*(c + a^2*c*x^2)^(3/2)) + x/(3*c^2*Sqrt[c +
 a^2*c*x^2]))
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 5597
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Si
mp[c^(I*(n/2))   Int[(c + d*x^2)^(p - I*(n/2))*(1 - I*a*x)^(I*n), x], x] /;
 FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0]) &&
 IGtQ[I*(n/2), 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
orering	$\frac{(-ax+2i)(a^2x^2+1)^2}{3a(iax+1)^2(a^2c\,x^2+c)^{\frac{3}{2}}}$	45
gosper	$-\frac{(-ax+i)(ax+i)(-ax+2i)(a^2x^2+1)}{3a(iax+1)^2(a^2c\,x^2+c)^{\frac{3}{2}}}$	56
default	$-\frac{2i\left(\frac{i}{3ac\left(x-\frac{i}{a}\right)\sqrt{\left(x-\frac{i}{a}\right)^2a^2c+2iac\left(x-\frac{i}{a}\right)}}+\frac{i\left(2\left(x-\frac{i}{a}\right)a^2c+2iac\right)}{3a\,c^2\sqrt{\left(x-\frac{i}{a}\right)^2a^2c+2iac\left(x-\frac{i}{a}\right)}}\right)}{a}-\frac{x}{c\sqrt{a^2c\,x^2+c}}$	137

input
```
int(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-a*x+2*I)/a*(a^2*x^2+1)^2/(1+I*a*x)^2/(a^2*c*x^2+c)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{e^{-2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{a^2cx^2+c}(ax-2i)}{3\,(a^3c^2x^2-2i\,a^2c^2x-ac^2)}$$

input
```
integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x, algorithm="fric
as")
```

output
```
1/3*sqrt(a^2*c*x^2 + c)*(a*x - 2*I)/(a^3*c^2*x^2 - 2*I*a^2*c^2*x - a*c^2)
```

Sympy [F]

$$\int \frac{e^{-2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\int \frac{a^2x^2}{a^4cx^4\sqrt{a^2cx^2+c}-2ia^3cx^3\sqrt{a^2cx^2+c}-2iacx\sqrt{a^2cx^2+c}-c\sqrt{a^2cx^2+c}}\,dx - \int \frac{1}{a^4cx^4\sqrt{a^2cx^2+c}-2ia^3cx^3\sqrt{a^2cx^2+c}-2iacx\sqrt{a^2cx^2+c}-c\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(1/(1+I*a*x)**2*(a**2*x**2+1)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
-Integral(a**2*x**2/(a**4*c*x**4*sqrt(a**2*c*x**2 + c) - 2*I*a**3*c*x**3*s
qrt(a**2*c*x**2 + c) - 2*I*a*c*x*sqrt(a**2*c*x**2 + c) - c*sqrt(a**2*c*x**
2 + c)), x) - Integral(1/(a**4*c*x**4*sqrt(a**2*c*x**2 + c) - 2*I*a**3*c*x
**3*sqrt(a**2*c*x**2 + c) - 2*I*a*c*x*sqrt(a**2*c*x**2 + c) - c*sqrt(a**2*
c*x**2 + c)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \frac{e^{-2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{x}{3\sqrt{a^2cx^2+c}c} + \frac{2i}{3i\sqrt{a^2cx^2+c}a^2cx + 3\sqrt{a^2cx^2+c}ac}$$

input
```
integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxi
ma")
```

output
```
1/3*x/(sqrt(a^2*c*x^2 + c)*c) + 2*I/(3*I*sqrt(a^2*c*x^2 + c)*a^2*c*x + 3*s
qrt(a^2*c*x^2 + c)*a*c)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.41

$$\int \frac{e^{-2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{2\sqrt{a^2c}\left(3\sqrt{a^2c}x - 3\sqrt{a^2cx^2+c} - i\sqrt{c}\right)}{3\left(\sqrt{a^2c}x - \sqrt{a^2cx^2+c} - i\sqrt{c}\right)^3 a^2c}$$

input
```
integrate(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac
")
```

output
```
-2/3*sqrt(a^2*c)*(3*sqrt(a^2*c)*x - 3*sqrt(a^2*c*x^2 + c) - I*sqrt(c))/((s
qrt(a^2*c)*x - sqrt(a^2*c*x^2 + c) - I*sqrt(c))^3*a^2*c)
```

Mupad [B] (verification not implemented)

Time = 23.44 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.59

$$\int \frac{e^{-2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{a^3\,x^3+3\,a\,x+2\mathrm{i}}{3\,a\,\left(c\,\left(a^2\,x^2+1\right)\right)^{3/2}}$$

input
```
int((a^2*x^2 + 1)/((c + a^2*c*x^2)^(3/2)*(a*x*1i + 1)^2),x)
```

output
```
(3*a*x + a^3*x^3 + 2i)/(3*a*(c*(a^2*x^2 + 1))^(3/2))
```

Reduce [F]

$$\int \frac{e^{-2i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{\int \frac{1}{\sqrt{a^2x^2+1}\,a^2x^2-2\sqrt{a^2x^2+1}\,aix-\sqrt{a^2x^2+1}}dx}{\sqrt{c}\,c}$$

input
```
int(1/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
( - int(1/(sqrt(a**2*x**2 + 1)*a**2*x**2 - 2*sqrt(a**2*x**2 + 1)*a*i*x - s
qrt(a**2*x**2 + 1)),x))/(sqrt(c)*c)
```

3.349 $\int \frac{e^{-3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 49

$$\int \frac{e^{-3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx=\frac{i\sqrt{1+a^2x^2}}{2ac(1+iax)^2\sqrt{c+a^2cx^2}}$$

output
```
1/2*I*(a^2*x^2+1)^(1/2)/a/c/(1+I*a*x)^2/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{e^{-3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx=-\frac{i\sqrt{1+a^2x^2}}{2ac(-i+ax)^2\sqrt{c+a^2cx^2}}$$

input
```
Integrate[1/(E^((3*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
((-1/2*I)*Sqrt[1 + a^2*x^2])/(a*c*(-I + a*x)^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5599, 5596, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-3i\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5599}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{-3i\arctan(ax)}}{(a^2x^2+1)^{3/2}}dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5596}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{(iax+1)^3}dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{17}$$

$$\frac{i\sqrt{a^2x^2+1}}{2ac(1+iax)^2\sqrt{a^2cx^2+c}}$$

input
```
Int[1/(E^((3*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
((I/2)*Sqrt[1 + a^2*x^2])/(a*c*(1 + I*a*x)^2*Sqrt[c + a^2*c*x^2])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result	size
risch	$-\frac{i\sqrt{a^{2}x^{2}+1}}{2c\sqrt{c\left(a^{2}x^{2}+1\right)}\,a\left(ax-i\right)^{2}}$	42
default	$\frac{i\sqrt{c\left(a^{2}x^{2}+1\right)}}{2\sqrt{a^{2}x^{2}+1}\,c^{2}a\left(iax+1\right)^{2}}$	43
gosper	$\frac{\left(-ax+i\right)\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}}{2a\left(iax+1\right)^{3}\left(a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}$	45
orering	$-\frac{x\left(-ax+2i\right)\left(-ax+i\right)\left(a^{2}x^{2}+1\right)^{\frac{3}{2}}}{2\left(iax+1\right)^{3}\left(a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}$	50

input
```
int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
-1/2*I/c*(a^2*x^2+1)^(1/2)/(c*(a^2*x^2+1))^(1/2)/a/(a*x-I)^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.45

$$\int \frac{e^{-3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{a^2cx^2+c}\sqrt{a^2x^2+1}(-i\,ax^2-2\,x)}{2\,(a^4c^2x^4-2i\,a^3c^2x^3-2i\,ac^2x-c^2)}$$

input
```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm
="fricas")
```

output
```
1/2*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*(-I*a*x^2 - 2*x)/(a^4*c^2*x^4 -
2*I*a^3*c^2*x^3 - 2*I*a*c^2*x - c^2)
```

Sympy [F]

$$\int \frac{e^{-3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = i\left(\int \frac{\sqrt{a^2x^2+1}}{a^5cx^5\sqrt{a^2cx^2+c}-3ia^4cx^4\sqrt{a^2cx^2+c}-2a^3cx^3\sqrt{a^2cx^2+c}-2ia^2cx^2\sqrt{a^2cx^2}}\right.$$
$$+\int \frac{a^2x^2\sqrt{a^2x^2+1}}{a^5cx^5\sqrt{a^2cx^2+c}-3ia^4cx^4\sqrt{a^2cx^2+c}-2a^3cx^3\sqrt{a^2cx^2+c}-2ia^2cx^2\sqrt{a^2cx^2+c}-3acx\sqrt{a^2cx^2+c}}$$

input
```
integrate(1/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
I*(Integral(sqrt(a**2*x**2 + 1)/(a**5*c*x**5*sqrt(a**2*c*x**2 + c) - 3*I*a
**4*c*x**4*sqrt(a**2*c*x**2 + c) - 2*a**3*c*x**3*sqrt(a**2*c*x**2 + c) - 2
*I*a**2*c*x**2*sqrt(a**2*c*x**2 + c) - 3*a*c*x*sqrt(a**2*c*x**2 + c) + I*c
*sqrt(a**2*c*x**2 + c)), x) + Integral(a**2*x**2*sqrt(a**2*x**2 + 1)/(a**5
*c*x**5*sqrt(a**2*c*x**2 + c) - 3*I*a**4*c*x**4*sqrt(a**2*c*x**2 + c) - 2*
a**3*c*x**3*sqrt(a**2*c*x**2 + c) - 2*I*a**2*c*x**2*sqrt(a**2*c*x**2 + c)
- 3*a*c*x*sqrt(a**2*c*x**2 + c) + I*c*sqrt(a**2*c*x**2 + c)), x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.59

$$\int \frac{e^{-3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{1}{2i\,a^3c^{\frac{3}{2}}x^2+4\,a^2c^{\frac{3}{2}}x-2i\,ac^{\frac{3}{2}}}$$

input
```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm
="maxima")
```

output
```
1/(2*I*a^3*c^(3/2)*x^2 + 4*a^2*c^(3/2)*x - 2*I*a*c^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm
="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 24.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{e^{-3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{c\,\left(a^2\,x^2+1\right)}\,\sqrt{a^2\,x^2+1}}{2\,a\,c^2\,\left(a\,x+1{}\mathrm{i}\right)\,{\left(1+a\,x\,1{}\mathrm{i}\right)}^3}$$

input
```
int((a^2*x^2 + 1)^(3/2)/((c + a^2*c*x^2)^(3/2)*(a*x*1i + 1)^3),x)
```

output
```
-((c*(a^2*x^2 + 1))^(1/2)*(a^2*x^2 + 1)^(1/2))/(2*a*c^2*(a*x + 1i)*(a*x*1i
 + 1)^3)
```

Reduce [F]

$$\int \frac{e^{-3i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{c}\left(\int \frac{1}{a^3i\,x^3+3a^2x^2-3aix-1}dx\right)}{c^2}$$

input
```
int(1/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
( - sqrt(c)*int(1/(a**3*i*x**3 + 3*a**2*x**2 - 3*a*i*x - 1),x))/c**2
```

3.350 $\int \frac{e^{-4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 69

$$\int \frac{e^{-4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{ic(1-iax)^4}{3a\,(c+a^2cx^2)^{5/2}} - \frac{ic(1-iax)^5}{15a\,(c+a^2cx^2)^{5/2}}$$

output
```
1/3*I*c*(1-I*a*x)^4/a/(a^2*c*x^2+c)^(5/2)-1/15*I*c*(1-I*a*x)^5/a/(a^2*c*x^
2+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12

$$\int \frac{e^{-4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{(1-iax)^{3/2}(-4i+ax)\sqrt{1+a^2x^2}}{15ac\sqrt{1+iax}(-i+ax)^2\sqrt{c+a^2cx^2}}$$

input
```
Integrate[1/(E^((4*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
((1 - I*a*x)^(3/2)*(-4*I + a*x)*Sqrt[1 + a^2*x^2])/(15*a*c*Sqrt[1 + I*a*x]
*(-I + a*x)^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5597, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-4 i \arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5597}$$

$$c^2\int \frac{(1-iax)^4}{(a^2cx^2+c)^{7/2}}dx$$

$$\Big\downarrow \text{461}$$

$$c^2\left(\frac{i(1-iax)^4}{3ac\,(a^2cx^2+c)^{5/2}}-\frac{1}{3}\int \frac{(1-iax)^5}{(a^2cx^2+c)^{7/2}}dx\right)$$

$$\Big\downarrow \text{460}$$

$$c^2\left(\frac{i(1-iax)^4}{3ac\,(a^2cx^2+c)^{5/2}}-\frac{i(1-iax)^5}{15ac\,(a^2cx^2+c)^{5/2}}\right)$$

input
```
Int[1/(E^((4*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
c^2*(((I/3)*(1 - I*a*x)^4)/(a*c*(c + a^2*c*x^2)^(5/2)) - ((I/15)*(1 - I*a*
x)^5)/(a*c*(c + a^2*c*x^2)^(5/2)))
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 5597
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Si
mp[c^(I*(n/2))   Int[(c + d*x^2)^(p - I*(n/2))*(1 - I*a*x)^(I*n), x], x] /;
 FreeQ[{a, c, d, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0]) &&
 IGtQ[I*(n/2), 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

method	result
orering	$\frac{(-ax+4i)\left(a^{2}x^{2}+1\right)^{3}}{15a(iax+1)^{4}\left(a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}$
gosper	$-\frac{(-ax+i)(ax+i)(-ax+4i)\left(a^{2}x^{2}+1\right)^{2}}{15a(iax+1)^{4}\left(a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}$
default	$\frac{x}{c\sqrt{a^{2}c\,x^{2}+c}}+\frac{4i\left(\frac{i}{3ac\left(x-\frac{i}{a}\right)\sqrt{\left(x-\frac{i}{a}\right)^{2}a^{2}c+2iac\left(x-\frac{i}{a}\right)}}+\frac{i\left(2\left(x-\frac{i}{a}\right)a^{2}c+2iac\right)}{3a\,c^{2}\sqrt{\left(x-\frac{i}{a}\right)^{2}a^{2}c+2iac\left(x-\frac{i}{a}\right)}}\right)}{a}-\frac{4\left(\frac{i}{5ac\left(x-\frac{i}{a}\right)^{2}\sqrt{\left(x-\frac{i}{a}\right)^{2}a^{2}c+2iac(}}\right.}{}$

input
```
int(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
1/15*(-a*x+4*I)/a*(a^2*x^2+1)^3/(1+I*a*x)^4/(a^2*c*x^2+c)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{e^{-4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{a^2cx^2+c}(a^2x^2-3i\,ax+4)}{15\,(a^4c^2x^3-3i\,a^3c^2x^2-3\,a^2c^2x+i\,ac^2)}$$

input
```
integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fr
icas")
```

output
```
-1/15*sqrt(a^2*c*x^2 + c)*(a^2*x^2 - 3*I*a*x + 4)/(a^4*c^2*x^3 - 3*I*a^3*c
^2*x^2 - 3*a^2*c^2*x + I*a*c^2)
```

Sympy [F]

$$\int \frac{e^{-4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{(a^2x^2+1)^2}{(c\,(a^2x^2+1))^{\frac{3}{2}}\,(ax-i)^4}\,dx$$

input
```
integrate(1/(1+I*a*x)**4*(a**2*x**2+1)**2/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral((a**2*x**2 + 1)**2/((c*(a**2*x**2 + 1))**(3/2)*(a*x - I)**4), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(53) = 106.

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.72

$$\int \frac{e^{-4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{x}{15\,\sqrt{a^2cx^2+c}c} - \frac{4i}{5\,\left(\sqrt{a^2cx^2+c}a^3cx^2-2i\,\sqrt{a^2cx^2+c}a^2cx-\sqrt{a^2cx^2+c}ac\right)} - \frac{8i}{15i\,\sqrt{a^2cx^2+c}a^2cx+15\,\sqrt{a^2cx^2+c}ac}$$

input
```
integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="ma
xima")
```

output
```
-1/15*x/(sqrt(a^2*c*x^2 + c)*c) - 4/5*I/(sqrt(a^2*c*x^2 + c)*a^3*c*x^2 - 2
*I*sqrt(a^2*c*x^2 + c)*a^2*c*x - sqrt(a^2*c*x^2 + c)*a*c) - 8*I/(15*I*sqrt
(a^2*c*x^2 + c)*a^2*c*x + 15*sqrt(a^2*c*x^2 + c)*a*c)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(53) = 106$.

Time = 0.14 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.94

$$\int \frac{e^{-4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{2\left(15\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}\right)^3\sqrt{c}+5i\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}\right)^2c-5\left(\sqrt{a^2c}x-\sqrt{c}\right.\right.}{15\left(\sqrt{a^2c}x-\sqrt{a^2cx^2+c}-i\sqrt{c}\right)^5ac}$$

input
```
integrate(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="gi
ac")
```

output
```
2/15*(15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 + c))^3*sqrt(c) + 5*I*(sqrt(a^2*c
)*x - sqrt(a^2*c*x^2 + c))^2*c - 5*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 + c))*c
^(3/2) + I*c^2)/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 + c) - I*sqrt(c))^5*a*c)
```

Mupad [B] (verification not implemented)

Time = 24.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

$$\int \frac{e^{-4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c\,\left(a^2\,x^2+1\right)}\,\left(a^2\,x^2-a\,x\,3\mathrm{i}+4\right)\,1\mathrm{i}}{15\,a\,c^2\,{\left(1+a\,x\,1\mathrm{i}\right)}^3}$$

input
```
int((a^2*x^2 + 1)^2/((c + a^2*c*x^2)^(3/2)*(a*x*1i + 1)^4),x)
```

output
```
((c*(a^2*x^2 + 1))^(1/2)*(a^2*x^2 - a*x*3i + 4)*1i)/(15*a*c^2*(a*x*1i + 1)
^3)
```

Reduce [F]

$$\int \frac{e^{-4i\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{c}\left(\int -\frac{\sqrt{a^2x^2+1}}{a^4x^4-4a^3i\,x^3-6a^2x^2+4aix+1}dx\right)}{c^2}$$

input
```
int(1/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^(3/2),x)
```

output
```
( - sqrt(c)*int(( - sqrt(a**2*x**2 + 1))/(a**4*x**4 - 4*a**3*i*x**3 - 6*a*
*2*x**2 + 4*a*i*x + 1),x))/c**2
```

3.351 $\int e^{n \arctan(ax)}(c+a^2cx^2)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 86

$$\int e^{n \arctan(ax)}(c+a^2cx^2)^2\,dx = -\frac{2^{3-\frac{in}{2}}c^2(1-iax)^{3+\frac{in}{2}}\text{Hypergeometric2F1}\left(-2+\frac{in}{2},3+\frac{in}{2},4+\frac{in}{2},\frac{1}{2}(1-iax)\right)}{a(6i-n)}$$

output
```
-2^(3-1/2*I*n)*c^2*(1-I*a*x)^(3+1/2*I*n)*hypergeom([-2+1/2*I*n, 3+1/2*I*n]
,[4+1/2*I*n],1/2-1/2*I*a*x)/a/(6*I-n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05

$$\int e^{n \arctan(ax)}(c+a^2cx^2)^2\,dx = \frac{i2^{2-\frac{in}{2}}c^2(1-iax)^{3+\frac{in}{2}}\text{Hypergeometric2F1}\left(-2+\frac{in}{2},3+\frac{in}{2},4+\frac{in}{2},\frac{1}{2}(1-iax)\right)}{a\left(3+\frac{in}{2}\right)}$$

input
```
Integrate[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^2,x]
```

output
```
(I*2^(2 - (I/2)*n)*c^2*(1 - I*a*x)^(3 + (I/2)*n)*Hypergeometric2F1[-2 + (I
/2)*n, 3 + (I/2)*n, 4 + (I/2)*n, (1 - I*a*x)/2])/(a*(3 + (I/2)*n))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2cx^2+c\right)^2 e^{n\arctan(ax)}dx$$

$$\Big\downarrow \text{5596}$$

$$c^2\int (1-iax)^{\frac{in}{2}+2}(iax+1)^{2-\frac{in}{2}}dx$$

$$\Big\downarrow \text{79}$$

$$-\frac{c^2 2^{3-\frac{in}{2}}(1-iax)^{3+\frac{in}{2}}\text{Hypergeometric2F1}\left(\frac{in}{2}-2,\frac{in}{2}+3,\frac{in}{2}+4,\frac{1}{2}(1-iax)\right)}{a(-n+6i)}$$

input
```
Int[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^2,x]
```

output
```
-((2^(3 - (I/2)*n)*c^2*(1 - I*a*x)^(3 + (I/2)*n)*Hypergeometric2F1[-2 + (I
/2)*n, 3 + (I/2)*n, 4 + (I/2)*n, (1 - I*a*x)/2])/(a*(6*I - n)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \mathrm{e}^{n \arctan(ax)} \left(a^2 c\, x^2 + c\right)^2 dx$$

input
```
int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^2,x)
```

output
```
int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^2,x)
```

Fricas [F]

$$\int e^{n \arctan(ax)} \left(c + a^2 c x^2\right)^2 \, dx = \int \left(a^2 c x^2 + c\right)^2 e^{\left(n \arctan(ax)\right)} \, dx$$

input
```
integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*e^(n*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{n \arctan(ax)} \left(c + a^2 c x^2\right)^2 \, dx = c^2 \left(\int 2 a^2 x^2 e^{n \operatorname{atan}(ax)} \, dx + \int a^4 x^4 e^{n \operatorname{atan}(ax)} \, dx + \int e^{n \operatorname{atan}(ax)} \, dx\right)$$

input
```
integrate(exp(n*atan(a*x))*(a**2*c*x**2+c)**2,x)
```

output
```
c**2*(Integral(2*a**2*x**2*exp(n*atan(a*x)), x) + Integral(a**4*x**4*exp(n
*atan(a*x)), x) + Integral(exp(n*atan(a*x)), x))
```

Maxima [F]

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int \left(a^2cx^2+c\right)^2e^{(n\arctan(ax))}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^2*e^(n*arctan(a*x)), x)
```

Giac [F]

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int \left(a^2cx^2+c\right)^2e^{(n\arctan(ax))}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*e^(n*arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx=\int e^{n\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

input
```
int(exp(n*atan(a*x))*(c + a^2*c*x^2)^2,x)
```

output
```
int(exp(n*atan(a*x))*(c + a^2*c*x^2)^2, x)
```

Reduce [F]

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^2\,dx = c^2\left(\int e^{atan(ax)n}dx+\left(\int e^{atan(ax)n}x^4dx\right)a^4+2\left(\int e^{atan(ax)n}x^2dx\right)a^2\right)$$

input
```
int(exp(n*atan(a*x))*(a^2*c*x^2+c)^2,x)
```

output
```
c**2*(int(e**(atan(a*x)*n),x) + int(e**(atan(a*x)*n)*x**4,x)*a**4 + 2*int(
e**(atan(a*x)*n)*x**2,x)*a**2)
```

3.352 $\int e^{n \arctan(ax)}(c+a^2cx^2)\, dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int e^{n \arctan(ax)}\left(c+a^2cx^2\right)\, dx = -\frac{2^{2-\frac{in}{2}}c(1-iax)^{2+\frac{in}{2}} \text{Hypergeometric2F1}\left(-1+\frac{in}{2},2+\frac{in}{2},3+\frac{in}{2},\frac{1}{2}(1-iax)\right)}{a(4i-n)}$$

output
```
-2^(2-1/2*I*n)*c*(1-I*a*x)^(2+1/2*I*n)*hypergeom([-1+1/2*I*n, 2+1/2*I*n],[
3+1/2*I*n],1/2-1/2*I*a*x)/a/(4*I-n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.05

$$\int e^{n \arctan(ax)}\left(c+a^2cx^2\right)\, dx = \frac{i2^{1-\frac{in}{2}}c(1-iax)^{2+\frac{in}{2}} \text{Hypergeometric2F1}\left(-1+\frac{in}{2},2+\frac{in}{2},3+\frac{in}{2},\frac{1}{2}(1-iax)\right)}{a\left(2+\frac{in}{2}\right)}$$

input
```
Integrate[E^(n*ArcTan[a*x])*(c + a^2*c*x^2),x]
```

output
```
(I*2^(1 - (I/2)*n)*c*(1 - I*a*x)^(2 + (I/2)*n)*Hypergeometric2F1[-1 + (I/2
)*n, 2 + (I/2)*n, 3 + (I/2)*n, (1 - I*a*x)/2])/(a*(2 + (I/2)*n))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2cx^2+c\right)e^{n\arctan(ax)}dx$$

$$\Big\downarrow \text{5596}$$

$$c\int (1-iax)^{\frac{in}{2}+1}(iax+1)^{1-\frac{in}{2}}dx$$

$$\Big\downarrow \text{79}$$

$$-\frac{c2^{2-\frac{in}{2}}(1-iax)^{2+\frac{in}{2}}\text{Hypergeometric2F1}\left(\frac{in}{2}-1,\frac{in}{2}+2,\frac{in}{2}+3,\frac{1}{2}(1-iax)\right)}{a(-n+4i)}$$

input
```
Int[E^(n*ArcTan[a*x])*(c + a^2*c*x^2),x]
```

output
```
-((2^(2 - (I/2)*n)*c*(1 - I*a*x)^(2 + (I/2)*n)*Hypergeometric2F1[-1 + (I/2
)*n, 2 + (I/2)*n, 3 + (I/2)*n, (1 - I*a*x)/2])/(a*(4*I - n)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \mathrm{e}^{n \arctan(ax)} \left(a^2 c\, x^2 + c\right) dx$$

input
```
int(exp(n*arctan(a*x))*(a^2*c*x^2+c),x)
```

output
```
int(exp(n*arctan(a*x))*(a^2*c*x^2+c),x)
```

Fricas [F]

$$\int e^{n \arctan(ax)} \left(c + a^2 c x^2\right) \, dx = \int \left(a^2 c x^2 + c\right) e^{\left(n \arctan(ax)\right)} \, dx$$

input
```
integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*e^(n*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{n \arctan(ax)} \left(c + a^2 c x^2\right) \, dx = c \left(\int a^2 x^2 e^{n \operatorname{atan}{(ax)}}\, dx + \int e^{n \operatorname{atan}{(ax)}}\, dx\right)$$

input
```
integrate(exp(n*atan(a*x))*(a**2*c*x**2+c),x)
```

output
```
c*(Integral(a**2*x**2*exp(n*atan(a*x)), x) + Integral(exp(n*atan(a*x)), x)
)
```

Maxima [F]

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int \left(a^2cx^2+c\right)e^{(n\arctan(ax))}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)*e^(n*arctan(a*x)), x)
```

Giac [F]

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int \left(a^2cx^2+c\right)e^{(n\arctan(ax))}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*e^(n*arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)\,dx=\int e^{n\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(exp(n*atan(a*x))*(c + a^2*c*x^2),x)
```

output
```
int(exp(n*atan(a*x))*(c + a^2*c*x^2), x)
```

Reduce [F]

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)\,dx=c\left(\int e^{atan(ax)n}dx+\left(\int e^{atan(ax)n}x^2dx\right)a^2\right)$$

input
```
int(exp(n*atan(a*x))*(a^2*c*x^2+c),x)
```

output
```
c*(int(e**(atan(a*x)*n),x) + int(e**(atan(a*x)*n)*x**2,x)*a**2)
```

3.353 $\int e^{n\arctan(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 81

$$\int e^{n\arctan(ax)}\,dx = -\frac{2^{1-\frac{in}{2}}(1-iax)^{1+\frac{in}{2}}\,\text{Hypergeometric2F1}\left(1+\frac{in}{2},\frac{in}{2},2+\frac{in}{2},\frac{1}{2}(1-iax)\right)}{a(2i-n)}$$

output
```
-2^(1-1/2*I*n)*(1-I*a*x)^(1+1/2*I*n)*hypergeom([1/2*I*n, 1+1/2*I*n],[2+1/2
*I*n],1/2-1/2*I*a*x)/a/(2*I-n)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.69

$$\int e^{n\arctan(ax)}\,dx = \frac{4e^{(2i+n)\arctan(ax)}\,\text{Hypergeometric2F1}\left(2,1-\frac{in}{2},2-\frac{in}{2},-e^{2i\arctan(ax)}\right)}{a(2i+n)}$$

input
```
Integrate[E^(n*ArcTan[a*x]),x]
```

output
```
(4*E^((2*I + n)*ArcTan[a*x])*Hypergeometric2F1[2, 1 - (I/2)*n, 2 - (I/2)*n
, -E^((2*I)*ArcTan[a*x])])/(a*(2*I + n))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5584, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{n\arctan(ax)}dx$$

$$\Big\downarrow \text{ 5584}$$

$$\int (1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}dx$$

$$\Big\downarrow \text{ 79}$$

$$-\frac{2^{1-\frac{in}{2}}(1-iax)^{1+\frac{in}{2}}\text{Hypergeometric2F1}\left(\frac{in}{2}+1,\frac{in}{2},\frac{in}{2}+2,\frac{1}{2}(1-iax)\right)}{a(-n+2i)}$$

input
```
Int[E^(n*ArcTan[a*x]),x]
```

output
```
-((2^(1 - (I/2)*n)*(1 - I*a*x)^(1 + (I/2)*n)*Hypergeometric2F1[1 + (I/2)*n
, (I/2)*n, 2 + (I/2)*n, (1 - I*a*x)/2])/(a*(2*I - n)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5584
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.)), x_Symbol] :> Int[(1 - I*a*x)^(I*(n/2))/(1
 + I*a*x)^(I*(n/2)), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(I*n - 1)/2]
```

Maple [F]

$$\int e^{n \arctan(ax)} dx$$

input
```
int(exp(n*arctan(a*x)),x)
```

output
```
int(exp(n*arctan(a*x)),x)
```

Fricas [F]

$$\int e^{n \arctan(ax)} \, dx = \int e^{(n \arctan(ax))} \, dx$$

input
```
integrate(exp(n*arctan(a*x)),x, algorithm="fricas")
```

output
```
integral(e^(n*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{n \arctan(ax)} \, dx = \int e^{n \operatorname{atan}{(ax)}} \, dx$$

input
```
integrate(exp(n*atan(a*x)),x)
```

output
```
Integral(exp(n*atan(a*x)), x)
```

Maxima [F]

$$\int e^{n\arctan(ax)}\,dx = \int e^{(n\arctan(ax))}\,dx$$

input
```
integrate(exp(n*arctan(a*x)),x, algorithm="maxima")
```

output
```
integrate(e^(n*arctan(a*x)), x)
```

Giac [F]

$$\int e^{n\arctan(ax)}\,dx = \int e^{(n\arctan(ax))}\,dx$$

input
```
integrate(exp(n*arctan(a*x)),x, algorithm="giac")
```

output
```
integrate(e^(n*arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(ax)}\,dx = \int e^{n\,\mathrm{atan}(a\,x)}\,dx$$

input
```
int(exp(n*atan(a*x)),x)
```

output
```
int(exp(n*atan(a*x)), x)
```

Reduce [F]

$$\int e^{n \arctan(ax)} \, dx = \int e^{atan(ax)n} dx$$

input
```
int(exp(n*atan(a*x)),x)
```

output
```
int(e**(atan(a*x)*n),x)
```

3.354 $\int \frac{e^{n\arctan(ax)}x^3}{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 131

$$\int \frac{e^{n\arctan(ax)}x^3}{c+a^2cx^2}\,dx = \frac{e^{n\arctan(ax)}(2i+n-in^2)}{2a^4cn} - \frac{e^{n\arctan(ax)}nx}{2a^3c} + \frac{e^{n\arctan(ax)}x^2}{2a^2c} + \frac{ie^{n\arctan(ax)}(-2+n^2)\,\text{Hypergeometric2F1}\left(1,-\frac{in}{2},1-\frac{in}{2},-e^{2i\arctan(ax)}\right)}{a^4cn}$$

output
```
1/2*exp(n*arctan(a*x))*(2*I+n-I*n^2)/a^4/c/n-1/2*exp(n*arctan(a*x))*n*x/a^
3/c+1/2*exp(n*arctan(a*x))*x^2/a^2/c+I*exp(n*arctan(a*x))*(n^2-2)*hypergeo
m([1, -1/2*I*n],[1-1/2*I*n],-(1+I*a*x)^2/(a^2*x^2+1))/a^4/c/n
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08

$$\int \frac{e^{n\arctan(ax)}x^3}{c+a^2cx^2}\,dx = \frac{(1-iax)^{\frac{in}{2}}\left(\frac{(1+iax)^{-\frac{in}{2}}(2i+n+a^2nx^2-n^2(i+ax))}{n} + \frac{2^{-\frac{in}{2}}(-2+n^2)(i+ax)\,\text{Hypergeometric2F1}\left(1+\frac{in}{2},1+\frac{in}{2},2+\frac{in}{2},\frac{1}{2}(1-iax)\right)}{-2i+n}\right)}{2a^4c}$$

input
```
Integrate[(E^(n*ArcTan[a*x])*x^3)/(c + a^2*c*x^2),x]
```

output
```
((1 - I*a*x)^((I/2)*n)*((2*I + n + a^2*n*x^2 - n^2*(I + a*x))/(n*(1 + I*a*
x)^((I/2)*n)) + ((-2 + n^2)*(I + a*x)*Hypergeometric2F1[1 + (I/2)*n, 1 + (
I/2)*n, 2 + (I/2)*n, (1 - I*a*x)/2])/(2^((I/2)*n)*(-2*I + n))))/(2*a^4*c)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.59, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5605, 111, 25, 160, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 e^{n \arctan (a x)}}{a^2 c x^2+c} \, dx$$

$$\Big\downarrow \text{5605}$$

$$\frac{\int x^3 (1-i a x)^{\frac{i n}{2}-1} (i a x+1)^{-\frac{i n}{2}-1}dx}{c}$$

$$\Big\downarrow \text{111}$$

$$\frac{\frac{\int -x (1-i a x)^{\frac{i n}{2}-1} (i a x+1)^{-\frac{i n}{2}-1} (a n x+2)dx}{2 a^2}+\frac{x^2 (1-i a x)^{\frac{i n}{2}} (1+i a x)^{-\frac{i n}{2}}}{2 a^2}}{c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{x^2 (1-i a x)^{\frac{i n}{2}} (1+i a x)^{-\frac{i n}{2}}}{2 a^2}-\frac{\int x (1-i a x)^{\frac{i n}{2}-1} (i a x+1)^{-\frac{i n}{2}-1} (a n x+2)dx}{2 a^2}}{c}$$

$$\Big\downarrow \text{160}$$

$$\frac{\frac{x^2 (1-i a x)^{\frac{i n}{2}} (1+i a x)^{-\frac{i n}{2}}}{2 a^2}-\frac{\frac{i \left(2-n^2\right) \int (1-i a x)^{\frac{i n}{2}} (i a x+1)^{-\frac{i n}{2}-1}dx}{a}-\frac{i (1-i a x)^{\frac{i n}{2}} (1+i a x)^{-\frac{i n}{2}} \left(i a n^2 x-n^2-i n+2\right)}{a^2 n}}{2 a^2}}{c}$$

$$\Big\downarrow \text{79}$$

$$\frac{\frac{x^2(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2a^2}-\frac{-\frac{i2^{-\frac{in}{2}}\left(2-n^2\right)(1-iax)^{1+\frac{in}{2}}\text{Hypergeometric2F1}\left(\frac{in}{2}+1,\frac{in}{2}+1,\frac{in}{2}+2,\frac{1}{2}(1-iax)\right)}{a^2(-n+2i)}-\frac{i(1+iax)^{-\frac{in}{2}}\left(ian^2x-n^2-in+2\right)(1-iax)}{a^2n}}{2a^2}}{c}$$

input
```
Int[(E^(n*ArcTan[a*x])*x^3)/(c + a^2*c*x^2),x]
```

output
```
((x^2*(1 - I*a*x)^((I/2)*n))/(2*a^2*(1 + I*a*x)^((I/2)*n)) - (((-I)*(1 - I
*a*x)^((I/2)*n)*(2 - I*n - n^2 + I*a*n^2*x))/(a^2*n*(1 + I*a*x)^((I/2)*n))
 - (I*(2 - n^2)*(1 - I*a*x)^(1 + (I/2)*n)*Hypergeometric2F1[1 + (I/2)*n, 1
 + (I/2)*n, 2 + (I/2)*n, (1 - I*a*x)/2])/(2^((I/2)*n)*a^2*(2*I - n)))/(2*a
^2))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 160
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b^2*d*e*g - a^2*d*f*h*m - a*b*(d*(f*g
+ e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d*(b*c - a*d)*(m + 1))), x] + Simp[(a*d*f*h*m + b*(d*
(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d)   Int[(a + b*x)^(m + 1)*(c + d*x)^n,
x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0] &&
NeQ[m, -1] && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{\mathrm{e}^{n \arctan(a x)} x^3}{a^2 c\, x^2+c}dx$$

input
```
int(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c),x)
```

output
```
int(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c),x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)} x^3}{c+a^2 c x^2} \, dx = \int \frac{x^3 e^{\left(n \arctan\left(a x\right)\right)}}{a^2 c x^2 + c} \, dx$$

input
```
integrate(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^3*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{e^{n\arctan(ax)}x^3}{c + a^2cx^2}\,dx = \frac{\int \frac{x^3e^{n\,\mathrm{atan}\,(ax)}}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(exp(n*atan(a*x))*x**3/(a**2*c*x**2+c),x)
```

output
```
Integral(x**3*exp(n*atan(a*x))/(a**2*x**2 + 1), x)/c
```

Maxima [F]

$$\int \frac{e^{n\arctan(ax)}x^3}{c + a^2cx^2}\,dx = \int \frac{x^3e^{(n\arctan(ax))}}{a^2cx^2 + c}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(x^3*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}x^3}{c + a^2cx^2}\,dx = \int \frac{x^3e^{(n\arctan(ax))}}{a^2cx^2 + c}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x^3*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}x^3}{c+a^2cx^2}\,dx = \int \frac{x^3\,e^{n\,\mathrm{atan}(a\,x)}}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x^3*exp(n*atan(a*x)))/(c + a^2*c*x^2),x)
```

output
```
int((x^3*exp(n*atan(a*x)))/(c + a^2*c*x^2), x)
```

Reduce [F]

$$\int \frac{e^{n\arctan(ax)}x^3}{c+a^2cx^2}\,dx = \frac{\int \frac{e^{atan(ax)n}x^3}{a^2x^2+1}dx}{c}$$

input
```
int(exp(n*atan(a*x))*x^3/(a^2*c*x^2+c),x)
```

output
```
int((e**(atan(a*x)*n)*x**3)/(a**2*x**2 + 1),x)/c
```

3.355 $\int \frac{e^{n\arctan(ax)}x^2}{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 164

$$\int \frac{e^{n\arctan(ax)}x^2}{c+a^2cx^2}\,dx = -\frac{(1+in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^3cn}+\frac{x(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2c} + \frac{i2^{1-\frac{in}{2}}(1-iax)^{\frac{in}{2}}\,\text{Hypergeometric2F1}\left(\frac{in}{2},\frac{in}{2},1+\frac{in}{2},\frac{1}{2}(1-iax)\right)}{a^3c}$$

output
```
-(1+I*n)*(1-I*a*x)^(1/2*I*n)/a^3/c/n/((1+I*a*x)^(1/2*I*n))+x*(1-I*a*x)^(1/
2*I*n)/a^2/c/((1+I*a*x)^(1/2*I*n))+I*2^(1-1/2*I*n)*(1-I*a*x)^(1/2*I*n)*hyp
ergeom([1/2*I*n, 1/2*I*n],[1+1/2*I*n],1/2-1/2*I*a*x)/a^3/c
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.74

$$\int \frac{e^{n\arctan(ax)}x^2}{c+a^2cx^2}\,dx = \frac{(1-iax)^{\frac{in}{2}}(2+2iax)^{-\frac{in}{2}}\left(2^{\frac{in}{2}}(-1+n(-i+ax))+2in(1+iax)^{\frac{in}{2}}\,\text{Hypergeometric2F1}\left(\frac{in}{2},\frac{in}{2},1+\frac{in}{2},\right.\right.}{a^3cn}$$

input
```
Integrate[(E^(n*ArcTan[a*x])*x^2)/(c + a^2*c*x^2),x]
```

output
```
((1 - I*a*x)^((I/2)*n)*(2^((I/2)*n)*(-1 + n*(-I + a*x)) + (2*I)*n*(1 + I*a
*x)^((I/2)*n)*Hypergeometric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n, (1 - I*a*x)
/2]))/(a^3*c*n*(2 + (2*I)*a*x)^((I/2)*n))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5605, 101, 25, 88, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 e^{n \arctan(ax)}}{a^2 c x^2 + c} \, dx$$

$$\Big\downarrow \text{5605}$$

$$\frac{\int x^2 (1-iax)^{\frac{in}{2}-1} (iax+1)^{-\frac{in}{2}-1} dx}{c}$$

$$\Big\downarrow \text{101}$$

$$\frac{\frac{\int -(1-iax)^{\frac{in}{2}-1} (iax+1)^{-\frac{in}{2}-1} (anx+1) dx}{a^2} + \frac{x(1-iax)^{\frac{in}{2}} (1+iax)^{-\frac{in}{2}}}{a^2}}{c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{x(1-iax)^{\frac{in}{2}} (1+iax)^{-\frac{in}{2}}}{a^2} - \frac{\int (1-iax)^{\frac{in}{2}-1} (iax+1)^{-\frac{in}{2}-1} (anx+1) dx}{a^2}}{c}$$

$$\Big\downarrow \text{88}$$

$$\frac{\frac{x(1-iax)^{\frac{in}{2}} (1+iax)^{-\frac{in}{2}}}{a^2} - \frac{\frac{(1+in)(1-iax)^{\frac{in}{2}} (1+iax)^{-\frac{in}{2}}}{an} - in \int (1-iax)^{\frac{in}{2}-1} (iax+1)^{-\frac{in}{2}} dx}{a^2}}{c}$$

$$\Big\downarrow \text{79}$$

$$\frac{\frac{x(1-iax)^{\frac{in}{2}} (1+iax)^{-\frac{in}{2}}}{a^2} - \frac{\frac{(1+in)(1-iax)^{\frac{in}{2}} (1+iax)^{-\frac{in}{2}}}{an} - \frac{i 2^{1-\frac{in}{2}} (1-iax)^{\frac{in}{2}} \text{Hypergeometric2F1}\left(\frac{in}{2}, \frac{in}{2}, \frac{in}{2}+1, \frac{1}{2}(1-iax)\right)}{a}}{a^2}}{c}$$

input
```
Int[(E^(n*ArcTan[a*x])*x^2)/(c + a^2*c*x^2),x]
```

output
```
((x*(1 - I*a*x)^((I/2)*n))/(a^2*(1 + I*a*x)^((I/2)*n)) - (((1 + I*n)*(1 -
I*a*x)^((I/2)*n))/(a*n*(1 + I*a*x)^((I/2)*n)) - (I*2^(1 - (I/2)*n)*(1 - I*
a*x)^((I/2)*n)*Hypergeometric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n, (1 - I*a*x
)/2])/a)/a^2)/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{e^{n \arctan(ax)} x^2}{a^2 c\, x^2 + c} dx$$

input
```
int(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c),x)
```

output
```
int(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c),x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)} x^2}{c + a^2 c x^2} dx = \int \frac{x^2 e^{(n \arctan(ax))}}{a^2 c x^2 + c} dx$$

input
```
integrate(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^2*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(ax)} x^2}{c + a^2 c x^2} dx = \frac{\int \frac{x^2 e^{n \operatorname{atan}(ax)}}{a^2 x^2 + 1} dx}{c}$$

input
```
integrate(exp(n*atan(a*x))*x**2/(a**2*c*x**2+c),x)
```

output
```
Integral(x**2*exp(n*atan(a*x))/(a**2*x**2 + 1), x)/c
```

Maxima [F]

$$\int \frac{e^{n\arctan(ax)}x^2}{c+a^2cx^2}\,dx = \int \frac{x^2e^{(n\arctan(ax))}}{a^2cx^2+c}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(x^2*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}x^2}{c+a^2cx^2}\,dx = \int \frac{x^2e^{(n\arctan(ax))}}{a^2cx^2+c}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x^2*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}x^2}{c+a^2cx^2}\,dx = \int \frac{x^2\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)}}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x^2*exp(n*atan(a*x)))/(c + a^2*c*x^2),x)
```

output
```
int((x^2*exp(n*atan(a*x)))/(c + a^2*c*x^2), x)
```

Reduce [F]

$$\int \frac{e^{n \arctan(ax)} x^2}{c + a^2 c x^2} \, dx = \frac{\int \frac{e^{atan(ax)n} x^2}{a^2 x^2 + 1} dx}{c}$$

input
```
int(exp(n*atan(a*x))*x^2/(a^2*c*x^2+c),x)
```

output
```
int((e**(atan(a*x)*n)*x**2)/(a**2*x**2 + 1),x)/c
```

3.356 $\int \frac{e^{n\arctan(ax)}x}{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 122

$$\int \frac{e^{n\arctan(ax)}x}{c+a^2cx^2}\,dx=\frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2cn}-\frac{i2^{1-\frac{in}{2}}(1-iax)^{\frac{in}{2}}\text{Hypergeometric2F1}\left(\frac{in}{2},\frac{in}{2},1+\frac{in}{2},\frac{1}{2}(1-iax)\right)}{a^2cn}$$

output
```
I*(1-I*a*x)^(1/2*I*n)/a^2/c/n/((1+I*a*x)^(1/2*I*n))-I*2^(1-1/2*I*n)*(1-I*a
*x)^(1/2*I*n)*hypergeom([1/2*I*n, 1/2*I*n],[1+1/2*I*n],1/2-1/2*I*a*x)/a^2/
c/n
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.89

$$\int \frac{e^{n\arctan(ax)}x}{c+a^2cx^2}\,dx =\frac{i(1-iax)^{\frac{in}{2}}(2+2iax)^{-\frac{in}{2}}\left(2^{\frac{in}{2}}-2(1+iax)^{\frac{in}{2}}\text{Hypergeometric2F1}\left(\frac{in}{2},\frac{in}{2},1+\frac{in}{2},\frac{1}{2}(1-iax)\right)\right)}{a^2cn}$$

input
```
Integrate[(E^(n*ArcTan[a*x])*x)/(c + a^2*c*x^2),x]
```

output
```
(I*(1 - I*a*x)^((I/2)*n)*(2^((I/2)*n) - 2*(1 + I*a*x)^((I/2)*n)*Hypergeome
tric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n, (1 - I*a*x)/2]))/(a^2*c*n*(2 + (2*I
)*a*x)^((I/2)*n))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5605, 88, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{n\arctan(ax)}}{a^2cx^2+c}\,dx$$

$$\Big\downarrow \; 5605$$

$$\frac{\int x(1-iax)^{\frac{in}{2}-1}(iax+1)^{-\frac{in}{2}-1}dx}{c}$$

$$\Big\downarrow \; 88$$

$$\frac{\frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2n}-\frac{i\int (1-iax)^{\frac{in}{2}-1}(iax+1)^{-\frac{in}{2}}dx}{a}}{c}$$

$$\Big\downarrow \; 79$$

$$\frac{\frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{a^2n}-\frac{i2^{1-\frac{in}{2}}(1-iax)^{\frac{in}{2}}\,\text{Hypergeometric2F1}\left(\frac{in}{2},\frac{in}{2},\frac{in}{2}+1,\frac{1}{2}(1-iax)\right)}{a^2n}}{c}$$

input
```
Int[(E^(n*ArcTan[a*x])*x)/(c + a^2*c*x^2),x]
```

output
```
((I*(1 - I*a*x)^((I/2)*n))/(a^2*n*(1 + I*a*x)^((I/2)*n)) - (I*2^(1 - (I/2)
*n)*(1 - I*a*x)^((I/2)*n)*Hypergeometric2F1[(I/2)*n, (I/2)*n, 1 + (I/2)*n,
 (1 - I*a*x)/2])/(a^2*n))/c
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{e^{n\arctan(ax)} x}{a^2 c\, x^2 + c} dx$$

input
```
int(exp(n*arctan(a*x))*x/(a^2*c*x^2+c),x)
```

output
```
int(exp(n*arctan(a*x))*x/(a^2*c*x^2+c),x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)}x}{c+a^2cx^2}\,dx = \int \frac{xe^{(n \arctan(ax))}}{a^2cx^2+c}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(ax)}x}{c+a^2cx^2}\,dx = \frac{\int \frac{xe^{n \operatorname{atan}(ax)}}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(exp(n*atan(a*x))*x/(a**2*c*x**2+c),x)
```

output
```
Integral(x*exp(n*atan(a*x))/(a**2*x**2 + 1), x)/c
```

Maxima [F]

$$\int \frac{e^{n \arctan(ax)}x}{c+a^2cx^2}\,dx = \int \frac{xe^{(n \arctan(ax))}}{a^2cx^2+c}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(x*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}x}{c+a^2cx^2}\,dx = \int \frac{xe^{(n\arctan(ax))}}{a^2cx^2+c}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}x}{c+a^2cx^2}\,dx = \int \frac{x\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)}}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x*exp(n*atan(a*x)))/(c + a^2*c*x^2),x)
```

output
```
int((x*exp(n*atan(a*x)))/(c + a^2*c*x^2), x)
```

Reduce [F]

$$\int \frac{e^{n\arctan(ax)}x}{c+a^2cx^2}\,dx = \frac{\int \frac{e^{atan(ax)n}x}{a^2x^2+1}dx}{c}$$

input
```
int(exp(n*atan(a*x))*x/(a^2*c*x^2+c),x)
```

output
```
int((e**(atan(a*x)*n)*x)/(a**2*x**2 + 1),x)/c
```

3.357 $\int \frac{e^{n \arctan(ax)}}{c+a^2cx^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \frac{e^{n \arctan(ax)}}{c+a^2cx^2} \, dx = \frac{e^{n \arctan(ax)}}{acn}$$

output
```
exp(n*arctan(a*x))/a/c/n
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int \frac{e^{n \arctan(ax)}}{c+a^2cx^2} \, dx = \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{acn}$$

input
```
Integrate[E^(n*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

output
```
(1 - I*a*x)^((I/2)*n)/(a*c*n*(1 + I*a*x)^((I/2)*n))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5594}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\arctan(ax)}}{a^2cx^2+c}\,dx$$
$$\Big\downarrow \; 5594$$
$$\frac{e^{n\arctan(ax)}}{acn}$$

input
```
Int[E^(n*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

output
```
E^(n*ArcTan[a*x])/(a*c*n)
```

Defintions of rubi rules used

rule 5594
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{e^{n\arctan(ax)}}{acn}$	18
parallelrisch	$\frac{e^{n\arctan(ax)}}{acn}$	18
risch	$\frac{(-iax+1)^{\frac{in}{2}}(iax+1)^{-\frac{in}{2}}}{nac}$	35
orering	$\frac{(a^2x^2+1)e^{n\arctan(ax)}}{an(a^2c\,x^2+c)}$	36

input
```
int(exp(n*arctan(a*x))/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
exp(n*arctan(a*x))/a/c/n
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{e^{n\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{e^{(n\arctan(ax))}}{acn}$$

input
```
integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
e^(n*arctan(a*x))/(a*c*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.

Time = 0.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{e^{n\arctan(ax)}}{c+a^2cx^2}\,dx = \begin{cases} \frac{x}{c} & \text{for } a=0 \wedge (a=0 \vee n=0) \\ \frac{\operatorname{atan}(ax)}{ac} & \text{for } n=0 \\ \frac{e^{n\operatorname{atan}(ax)}}{acn} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(n*atan(a*x))/(a**2*c*x**2+c),x)
```

output
```
Piecewise((x/c, Eq(a, 0) & (Eq(a, 0) | Eq(n, 0))), (atan(a*x)/(a*c), Eq(n,
 0)), (exp(n*atan(a*x))/(a*c*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{e^{n\arctan(ax)}}{c+a^2cx^2}\,dx=\frac{e^{(n\arctan(ax))}}{acn}$$

input
```
integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
e^(n*arctan(a*x))/(a*c*n)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{e^{n\arctan(ax)}}{c+a^2cx^2}\,dx=\frac{e^{(n\arctan(ax))}}{acn}$$

input
```
integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
e^(n*arctan(a*x))/(a*c*n)
```

Mupad [B] (verification not implemented)

Time = 22.88 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{e^{n\arctan(ax)}}{c+a^2cx^2}\,dx=\frac{\mathrm{e}^{n\,\mathrm{atan}(a\,x)}}{a\,c\,n}$$

input
```
int(exp(n*atan(a*x))/(c + a^2*c*x^2),x)
```

output
```
exp(n*atan(a*x))/(a*c*n)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^{n \arctan(ax)}}{c + a^2cx^2}\,dx = \frac{e^{atan(ax)n}}{acn}$$

input
```
int(exp(n*atan(a*x))/(a^2*c*x^2+c),x)
```

output
```
e**(atan(a*x)*n)/(a*c*n)
```

3.358 $\int \frac{e^{n \arctan(ax)}}{x(c+a^2cx^2)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{e^{n \arctan(ax)}}{x(c+a^2cx^2)}\,dx = \frac{ie^{n \arctan(ax)}}{cn} - \frac{2ie^{n \arctan(ax)}\,\text{Hypergeometric2F1}\left(1,-\frac{in}{2},1-\frac{in}{2},e^{2i \arctan(ax)}\right)}{cn}$$

output
```
I*exp(n*arctan(a*x))/c/n-2*I*exp(n*arctan(a*x))*hypergeom([1, -1/2*I*n],[1
-1/2*I*n],(1+I*a*x)^2/(a^2*x^2+1))/c/n
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.85

$$\int \frac{e^{n \arctan(ax)}}{x(c+a^2cx^2)}\,dx = \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\left((2+in)(-i+ax)+2(n-ianx)\,\text{Hypergeometric2F1}\left(1,1+\frac{in}{2},2+\frac{in}{2},\frac{i+ax}{i-ax}\right)\right.}{cn(-2i+n)(-i+ax)}$$

input
```
Integrate[E^(n*ArcTan[a*x])/(x*(c + a^2*c*x^2)),x]
```

output
```
((1 - I*a*x)^((I/2)*n)*((2 + I*n)*(-I + a*x) + 2*(n - I*a*n*x)*Hypergeomet
ric2F1[1, 1 + (I/2)*n, 2 + (I/2)*n, (I + a*x)/(I - a*x)]))/(c*n*(-2*I + n)
*(1 + I*a*x)^((I/2)*n)*(-I + a*x))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.85, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5605, 107, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\arctan(ax)}}{x\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \ 5605$$

$$\frac{\int \frac{(1-iax)^{\frac{in}{2}-1}(iax+1)^{-\frac{in}{2}-1}}{x}dx}{c}$$

$$\Big\downarrow \ 107$$

$$\frac{\int \frac{(1-iax)^{\frac{in}{2}}(iax+1)^{-\frac{in}{2}-1}}{x}dx+\frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{n}}{c}$$

$$\Big\downarrow \ 141$$

$$\frac{\frac{i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{n}-\frac{2i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\text{Hypergeometric2F1}\left(1,-\frac{in}{2},1-\frac{in}{2},\frac{iax+1}{1-iax}\right)}{n}}{c}$$

input
```
Int[E^(n*ArcTan[a*x])/(x*(c + a^2*c*x^2)),x]
```

output
```
((I*(1 - I*a*x)^((I/2)*n))/(n*(1 + I*a*x)^((I/2)*n)) - ((2*I)*(1 - I*a*x)^
((I/2)*n)*Hypergeometric2F1[1, (-1/2*I)*n, 1 - (I/2)*n, (1 + I*a*x)/(1 - I
*a*x)])/(n*(1 + I*a*x)^((I/2)*n)))/c
```

Defintions of rubi rules used

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{e^{n \arctan(ax)}}{x\,(a^2c\,x^2+c)}dx$$

input
```
int(exp(n*arctan(a*x))/x/(a^2*c*x^2+c),x)
```

output
```
int(exp(n*arctan(a*x))/x/(a^2*c*x^2+c),x)
```

Fricas [F]

$$\int \frac{e^{n\arctan(ax)}}{x\,(c+a^2cx^2)}\,dx = \int \frac{e^{(n\arctan(ax))}}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(exp(n*arctan(a*x))/x/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(e^(n*arctan(a*x))/(a^2*c*x^3 + c*x), x)
```

Sympy [F]

$$\int \frac{e^{n\arctan(ax)}}{x\,(c+a^2cx^2)}\,dx = \frac{\int \frac{e^{n\,\mathrm{atan}\,(ax)}}{a^2x^3+x}\,dx}{c}$$

input
```
integrate(exp(n*atan(a*x))/x/(a**2*c*x**2+c),x)
```

output
```
Integral(exp(n*atan(a*x))/(a**2*x**3 + x), x)/c
```

Maxima [F]

$$\int \frac{e^{n\arctan(ax)}}{x\,(c+a^2cx^2)}\,dx = \int \frac{e^{(n\arctan(ax))}}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(exp(n*arctan(a*x))/x/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(e^(n*arctan(a*x))/((a^2*c*x^2 + c)*x), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}}{x\left(c+a^2cx^2\right)}\,dx = \int \frac{e^{(n\arctan(ax))}}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(exp(n*arctan(a*x))/x/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(e^(n*arctan(a*x))/((a^2*c*x^2 + c)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}}{x\left(c+a^2cx^2\right)}\,dx = \int \frac{e^{n\,\mathrm{atan}(a\,x)}}{x\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(exp(n*atan(a*x))/(x*(c + a^2*c*x^2)),x)
```

output
```
int(exp(n*atan(a*x))/(x*(c + a^2*c*x^2)), x)
```

Reduce [F]

$$\int \frac{e^{n\arctan(ax)}}{x\left(c+a^2cx^2\right)}\,dx = \frac{\int \frac{e^{atan(ax)n}}{a^2x^3+x}dx}{c}$$

input
```
int(exp(n*atan(a*x))/x/(a^2*c*x^2+c),x)
```

output
```
int(e**(atan(a*x)*n)/(a**2*x**3 + x),x)/c
```

3.359 $\int \frac{e^{n \arctan(ax)}}{x^2(c+a^2cx^2)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 90

$$\int \frac{e^{n \arctan(ax)}}{x^2(c+a^2cx^2)}\,dx = \frac{iae^{n\arctan(ax)}(i+n)}{cn} - \frac{e^{n\arctan(ax)}}{cx} - \frac{2iae^{n\arctan(ax)}\,\text{Hypergeometric2F1}\left(1,-\frac{in}{2},1-\frac{in}{2},-1+\frac{2i}{i+ax}\right)}{c}$$

output
```
I*a*exp(n*arctan(a*x))*(I+n)/c/n-exp(n*arctan(a*x))/c/x-2*I*a*exp(n*arctan
(a*x))*hypergeom([1, -1/2*I*n],[1-1/2*I*n],-1+2*I/(I+a*x))/c
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.58

$$\int \frac{e^{n \arctan(ax)}}{x^2(c+a^2cx^2)}\,dx = \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\left((-2i+n)(1+iax)(iax+n(i+ax))+2an^2x(1-iax)\,\text{Hypergeometric2F1}\left(1\right.\right.}{cn(-2i+n)x(-i+ax)}$$

input
```
Integrate[E^(n*ArcTan[a*x])/(x^2*(c + a^2*c*x^2)),x]
```

output
```
((1 - I*a*x)^((I/2)*n)*((-2*I + n)*(1 + I*a*x)*(I*a*x + n*(I + a*x)) + 2*a
*n^2*x*(1 - I*a*x)*Hypergeometric2F1[1, 1 + (I/2)*n, 2 + (I/2)*n, (I + a*x
)/(I - a*x)]))/(c*n*(-2*I + n)*x*(1 + I*a*x)^((I/2)*n)*(-I + a*x))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.80, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5605, 114, 25, 27, 172, 25, 27, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\arctan(ax)}}{x^2\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \text{5605}$$

$$\frac{\int \frac{(1-iax)^{\frac{in}{2}-1}(iax+1)^{-\frac{in}{2}-1}}{x^2}dx}{c}$$

$$\Big\downarrow \text{114}$$

$$\frac{-\int -\frac{a(n-ax)(1-iax)^{\frac{in}{2}-1}(iax+1)^{-\frac{in}{2}-1}}{x}dx-\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{x}}{c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a(n-ax)(1-iax)^{\frac{in}{2}-1}(iax+1)^{-\frac{in}{2}-1}}{x}dx-\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{x}}{c}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{(n-ax)(1-iax)^{\frac{in}{2}-1}(iax+1)^{-\frac{in}{2}-1}}{x}dx-\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{x}}{c}$$

$$\Big\downarrow \text{172}$$

$$\frac{a\left(-\frac{\int -\frac{an^2(1-iax)^{\frac{in}{2}}(iax+1)^{-\frac{in}{2}-1}}{x}dx}{an}-\frac{(1-in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{n}\right)-\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{x}}{c}$$

$$\Big\downarrow \text{25}$$

$$\frac{a\left(\frac{\int \frac{an^2(1-iax)^{\frac{in}{2}}(iax+1)^{-\frac{in}{2}-1}}{x}dx}{an}-\frac{(1-in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{n}\right)-\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{x}}{c}$$

$$\Big\downarrow \; 27$$

$$\frac{a\left(n\int \frac{(1-iax)^{\frac{in}{2}}(iax+1)^{-\frac{in}{2}-1}}{x}dx-\frac{(1-in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{n}\right)-\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{x}}{c}$$

$$\Big\downarrow \; 141$$

$$\frac{a\left(-2i(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\text{Hypergeometric2F1}\left(1,-\frac{in}{2},1-\frac{in}{2},\frac{iax+1}{1-iax}\right)-\frac{(1-in)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{n}\right)-\frac{(1-ia}{}}{c}$$

input
```
Int[E^(n*ArcTan[a*x])/(x^2*(c + a^2*c*x^2)),x]
```

output
```
(-((1 - I*a*x)^((I/2)*n)/(x*(1 + I*a*x)^((I/2)*n))) + a*(-(((1 - I*n)*(1 -
 I*a*x)^((I/2)*n))/(n*(1 + I*a*x)^((I/2)*n))) - ((2*I)*(1 - I*a*x)^((I/2)*
n)*Hypergeometric2F1[1, (-1/2*I)*n, 1 - (I/2)*n, (1 + I*a*x)/(1 - I*a*x)])
/(1 + I*a*x)^((I/2)*n)))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
])))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{e^{n \arctan(ax)}}{x^2 (a^2 c x^2 + c)} dx$$

input
```
int(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c),x)
```

output
```
int(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c),x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)}}{x^2 (c + a^2cx^2)} \, dx = \int \frac{e^{(n \arctan(ax))}}{(a^2cx^2 + c)x^2} \, dx$$

input
```
integrate(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(e^(n*arctan(a*x))/(a^2*c*x^4 + c*x^2), x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(ax)}}{x^2 (c + a^2cx^2)} \, dx = \frac{\int \frac{e^{n \operatorname{atan}(ax)}}{a^2x^4+x^2} \, dx}{c}$$

input
```
integrate(exp(n*atan(a*x))/x**2/(a**2*c*x**2+c),x)
```

output
```
Integral(exp(n*atan(a*x))/(a**2*x**4 + x**2), x)/c
```

Maxima [F]

$$\int \frac{e^{n \arctan(ax)}}{x^2 (c + a^2cx^2)} \, dx = \int \frac{e^{(n \arctan(ax))}}{(a^2cx^2 + c)x^2} \, dx$$

input
```
integrate(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(e^(n*arctan(a*x))/((a^2*c*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}}{x^2(c+a^2cx^2)}\,dx = \int \frac{e^{(n\arctan(ax))}}{(a^2cx^2+c)x^2}\,dx$$

input
```
integrate(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(e^(n*arctan(a*x))/((a^2*c*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}}{x^2(c+a^2cx^2)}\,dx = \int \frac{e^{n\,\mathrm{atan}(a\,x)}}{x^2\,(c\,a^2\,x^2+c)}\,dx$$

input
```
int(exp(n*atan(a*x))/(x^2*(c + a^2*c*x^2)),x)
```

output
```
int(exp(n*atan(a*x))/(x^2*(c + a^2*c*x^2)), x)
```

Reduce [F]

$$\int \frac{e^{n\arctan(ax)}}{x^2(c+a^2cx^2)}\,dx = \frac{-e^{atan(ax)n}ax - e^{atan(ax)n}n + \left(\int \frac{e^{atan(ax)n}}{a^2x^3+x}dx\right)a\,n^2x}{cnx}$$

input
```
int(exp(n*atan(a*x))/x^2/(a^2*c*x^2+c),x)
```

output
```
( - e**(atan(a*x)*n)*a*x - e**(atan(a*x)*n)*n + int(e**(atan(a*x)*n)/(a**2
*x**3 + x),x)*a*n**2*x)/(c*n*x)
```

3.360 $\int \frac{e^{n \arctan(ax)}}{x^3(c+a^2cx^2)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 126

$$\int \frac{e^{n \arctan(ax)}}{x^3(c+a^2cx^2)}\,dx = \frac{ia^2e^{n\arctan(ax)}(-2+in+n^2)}{2cn} - \frac{e^{n\arctan(ax)}}{2cx^2} - \frac{ae^{n\arctan(ax)}n}{2cx} - \frac{ia^2e^{n\arctan(ax)}(-2+n^2)\,\text{Hypergeometric2F1}\left(1,-\frac{in}{2},1-\frac{in}{2},e^{2i\arctan(ax)}\right)}{cn}$$

output
```
1/2*I*a^2*exp(n*arctan(a*x))*(-2+I*n+n^2)/c/n-1/2*exp(n*arctan(a*x))/c/x^2
-1/2*a*exp(n*arctan(a*x))*n/c/x-I*a^2*exp(n*arctan(a*x))*(n^2-2)*hypergeom
([1, -1/2*I*n],[1-1/2*I*n],(1+I*a*x)^2/(a^2*x^2+1))/c/n
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.38

$$\int \frac{e^{n \arctan(ax)}}{x^3(c+a^2cx^2)}\,dx = \frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\left(i(-2i+n)(-i+ax)\left(-2a^2x^2+an^2x(i+ax)+in(1+a^2x^2)\right)+2a^2n(-2+n\right.}{2cn(-2i+n)x^2(-i+ax)}$$

input
```
Integrate[E^(n*ArcTan[a*x])/(x^3*(c + a^2*c*x^2)),x]
```

output
```
((1 - I*a*x)^((I/2)*n)*(I*(-2*I + n)*(-I + a*x)*(-2*a^2*x^2 + a*n^2*x*(I +
 a*x) + I*n*(1 + a^2*x^2)) + 2*a^2*n*(-2 + n^2)*x^2*(1 - I*a*x)*Hypergeome
tric2F1[1, 1 + (I/2)*n, 2 + (I/2)*n, (I + a*x)/(I - a*x)]))/(2*c*n*(-2*I +
 n)*x^2*(1 + I*a*x)^((I/2)*n)*(-I + a*x))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.77, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5605, 114, 25, 27, 168, 27, 172, 25, 27, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\arctan(ax)}}{x^3\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \text{ 5605}$$

$$\frac{\int \frac{(1-iax)^{\frac{in}{2}-1}(iax+1)^{-\frac{in}{2}-1}}{x^3}dx}{c}$$

$$\Big\downarrow \text{ 114}$$

$$\frac{-\frac{1}{2}\int -\frac{a(n-2ax)(1-iax)^{\frac{in}{2}-1}(iax+1)^{-\frac{in}{2}-1}}{x^2}dx-\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2x^2}}{c}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{1}{2}\int \frac{a(n-2ax)(1-iax)^{\frac{in}{2}-1}(iax+1)^{-\frac{in}{2}-1}}{x^2}dx-\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2x^2}}{c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{1}{2}a\int \frac{(n-2ax)(1-iax)^{\frac{in}{2}-1}(iax+1)^{-\frac{in}{2}-1}}{x^2}dx-\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2x^2}}{c}$$

$$\Big\downarrow \text{ 168}$$

$$\frac{\frac{1}{2}a\left(-\int \frac{a(1-iax)^{\frac{in}{2}-1}(iax+1)^{-\frac{in}{2}-1}(-n^2+axn+2)}{x}dx-\frac{n(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{x}\right)-\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2x^2}}{c}$$

$$\Big\downarrow 27$$

$$\frac{\frac{1}{2}a\left(-a\int \frac{(1-iax)^{\frac{in}{2}-1}(iax+1)^{-\frac{in}{2}-1}(-n^2+axn+2)}{x}dx-\frac{n(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{x}\right)-\frac{(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{2x^2}}{c}$$

$$\Big\downarrow 172$$

$$\frac{\frac{1}{2}a\left(-a\left(\frac{(-in^2+n+2i)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{n}-\frac{\int -\frac{an\left(2-n^2\right)(1-iax)^{\frac{in}{2}}(iax+1)^{-\frac{in}{2}-1}}{x}dx}{an}\right)-\frac{n(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{x}\right)-\frac{(1-iax)}{}}{c}$$

$$\Big\downarrow 25$$

$$\frac{\frac{1}{2}a\left(-a\left(\frac{\int \frac{an\left(2-n^2\right)(1-iax)^{\frac{in}{2}}(iax+1)^{-\frac{in}{2}-1}}{x}dx}{an}+\frac{(-in^2+n+2i)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{n}\right)-\frac{n(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{x}\right)-\frac{(1-iax)^{\frac{in}{2}}}{}}{c}$$

$$\Big\downarrow 27$$

$$\frac{\frac{1}{2}a\left(-a\left(\left(2-n^2\right)\int \frac{(1-iax)^{\frac{in}{2}}(iax+1)^{-\frac{in}{2}-1}}{x}dx+\frac{(-in^2+n+2i)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{n}\right)-\frac{n(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{x}\right)-\frac{(1-iax}{}}{c}$$

$$\Big\downarrow 141$$

$$\frac{\frac{1}{2}a\left(-a\left(\frac{(-in^2+n+2i)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}}{n}-\frac{2i(2-n^2)(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\text{Hypergeometric2F1}\left(1,-\frac{in}{2},1-\frac{in}{2},\frac{iax+1}{1-iax}\right)}{n}\right)-\frac{n(1-i}{}}{c}$$

input
```
Int[E^(n*ArcTan[a*x])/(x^3*(c + a^2*c*x^2)),x]
```

output
```
(-1/2*(1 - I*a*x)^((I/2)*n)/(x^2*(1 + I*a*x)^((I/2)*n)) + (a*(-((n*(1 - I*
a*x)^((I/2)*n))/(x*(1 + I*a*x)^((I/2)*n))) - a*(((2*I + n - I*n^2)*(1 - I*
a*x)^((I/2)*n))/(n*(1 + I*a*x)^((I/2)*n)) - ((2*I)*(2 - n^2)*(1 - I*a*x)^(
(I/2)*n)*Hypergeometric2F1[1, (-1/2*I)*n, 1 - (I/2)*n, (1 + I*a*x)/(1 - I*
a*x)])/(n*(1 + I*a*x)^((I/2)*n)))))/2)/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 172
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[
(b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)
*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))
 Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f
)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g
 - a*h)*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !(NeQ[n, -1] && SumSimplerQ[n, 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1
])))] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && NeQ[m, -1]
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{e^{n \arctan(ax)}}{x^3 (a^2 c x^2 + c)} dx$$

input
```
int(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c),x)
```

output
```
int(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c),x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)}}{x^3 (c + a^2 c x^2)} \, dx = \int \frac{e^{(n \arctan(ax))}}{(a^2 c x^2 + c) x^3} \, dx$$

input
```
integrate(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(e^(n*arctan(a*x))/(a^2*c*x^5 + c*x^3), x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(ax)}}{x^3 (c + a^2 c x^2)} \, dx = \frac{\int \frac{e^{n \operatorname{atan}(ax)}}{a^2 x^5 + x^3} \, dx}{c}$$

input
```
integrate(exp(n*atan(a*x))/x**3/(a**2*c*x**2+c),x)
```

output
```
Integral(exp(n*atan(a*x))/(a**2*x**5 + x**3), x)/c
```

Maxima [F]

$$\int \frac{e^{n \arctan(ax)}}{x^3 (c + a^2 c x^2)} \, dx = \int \frac{e^{(n \arctan(ax))}}{(a^2 c x^2 + c) x^3} \, dx$$

input
```
integrate(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(e^(n*arctan(a*x))/((a^2*c*x^2 + c)*x^3), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}}{x^3(c+a^2cx^2)}\,dx=\int \frac{e^{(n\arctan(ax))}}{(a^2cx^2+c)x^3}\,dx$$

input
```
integrate(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(e^(n*arctan(a*x))/((a^2*c*x^2 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}}{x^3(c+a^2cx^2)}\,dx=\int \frac{e^{n\,\mathrm{atan}(a\,x)}}{x^3\,(c\,a^2\,x^2+c)}\,dx$$

input
```
int(exp(n*atan(a*x))/(x^3*(c + a^2*c*x^2)),x)
```

output
```
int(exp(n*atan(a*x))/(x^3*(c + a^2*c*x^2)), x)
```

Reduce [F]

$$\int \frac{e^{n\arctan(ax)}}{x^3(c+a^2cx^2)}\,dx=\frac{-e^{atan(ax)n}a^2x^2-e^{atan(ax)n}anx-e^{atan(ax)n}+\left(\int \frac{e^{atan(ax)n}}{a^2x^3+x}dx\right)a^2n^2x^2-2\left(\int \frac{e^{atan(ax)n}}{a^2x^3+x}dx\right)a^2x^2}{2c\,x^2}$$

input
```
int(exp(n*atan(a*x))/x^3/(a^2*c*x^2+c),x)
```

output
```
( - e**(atan(a*x)*n)*a**2*x**2 - e**(atan(a*x)*n)*a*n*x - e**(atan(a*x)*n)
 + int(e**(atan(a*x)*n)/(a**2*x**3 + x),x)*a**2*n**2*x**2 - 2*int(e**(atan
(a*x)*n)/(a**2*x**3 + x),x)*a**2*x**2)/(2*c*x**2)
```

3.361 $\int \frac{e^{n \arctan(ax)}}{(c+a^2cx^2)^4} dx$

Optimal result

Integrand size = 21, antiderivative size = 181

$$\int \frac{e^{n \arctan(ax)}}{(c+a^2cx^2)^4} dx = \frac{720e^{n \arctan(ax)}}{ac^4n(4+n^2)(16+n^2)(36+n^2)} + \frac{e^{n \arctan(ax)}(n+6ax)}{ac^4(36+n^2)(1+a^2x^2)^3} + \frac{30e^{n \arctan(ax)}(n+4ax)}{ac^4(16+n^2)(36+n^2)(1+a^2x^2)^2} + \frac{360e^{n \arctan(ax)}(n+2ax)}{ac^4(4+n^2)(16+n^2)(36+n^2)(1+a^2x^2)}$$

output
```
720*exp(n*arctan(a*x))/a/c^4/n/(n^2+4)/(n^2+16)/(n^2+36)+exp(n*arctan(a*x)
)*(6*a*x+n)/a/c^4/(n^2+36)/(a^2*x^2+1)^3+30*exp(n*arctan(a*x))*(4*a*x+n)/a
/c^4/(n^2+16)/(n^2+36)/(a^2*x^2+1)^2+360*exp(n*arctan(a*x))*(2*a*x+n)/a/c^
4/(n^2+4)/(n^2+16)/(n^2+36)/(a^2*x^2+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.91

$$\int \frac{e^{n \arctan(ax)}}{(c+a^2cx^2)^4} \, dx$$
$$= \frac{e^{n \arctan(ax)}(n+6ax) + \frac{30(c+a^2cx^2)\left(e^{n \arctan(ax)}n(-2i+n)(2i+n)(n+4ax)+12(1-iax)^{\frac{in}{2}}(1+iax)^{-\frac{in}{2}}(-i+ax)(i+ax)(2+n^2+2anx}{cn(64+20n^2+n^4)}}{ac(36+n^2)(c+a^2cx^2)^3}$$

input
```
Integrate[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^4,x]
```

output
```
(E^(n*ArcTan[a*x])*(n + 6*a*x) + (30*(c + a^2*c*x^2)*(E^(n*ArcTan[a*x])*n*
(-2*I + n)*(2*I + n)*(n + 4*a*x) + (12*(1 - I*a*x)^((I/2)*n)*(-I + a*x)*(I
 + a*x)*(2 + n^2 + 2*a*n*x + 2*a^2*x^2))/(1 + I*a*x)^((I/2)*n)))/(c*n*(64
+ 20*n^2 + n^4)))/(a*c*(36 + n^2)*(c + a^2*c*x^2)^3)
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5593, 27, 5593, 5593, 5594}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n \arctan(ax)}}{(a^2cx^2+c)^4} \, dx$$

$$\Big\downarrow \text{5593}$$

$$\frac{30 \int \frac{e^{n \arctan(ax)}}{c^3(a^2x^2+1)^3}dx}{c(n^2+36)} + \frac{(6ax+n)e^{n \arctan(ax)}}{ac^4(n^2+36)(a^2x^2+1)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{30\int \frac{e^{n\arctan(ax)}}{(a^2x^2+1)^3}dx}{c^4(n^2+36)}+\frac{(6ax+n)e^{n\arctan(ax)}}{ac^4(n^2+36)(a^2x^2+1)^3}$$

$$\Big\downarrow \text{5593}$$

$$\frac{30\left(\frac{12\int \frac{e^{n\arctan(ax)}}{\left(a^2x^2+1\right)^2}dx}{n^2+16}+\frac{(4ax+n)e^{n\arctan(ax)}}{a(n^2+16)(a^2x^2+1)^2}\right)}{c^4(n^2+36)}+\frac{(6ax+n)e^{n\arctan(ax)}}{ac^4(n^2+36)(a^2x^2+1)^3}$$

$$\Big\downarrow \text{5593}$$

$$\frac{30\left(\frac{12\left(\frac{2\int \frac{e^{n\arctan(ax)}}{a^2x^2+1}dx}{n^2+4}+\frac{(2ax+n)e^{n\arctan(ax)}}{a\left(n^2+4\right)\left(a^2x^2+1\right)}\right)}{n^2+16}+\frac{(4ax+n)e^{n\arctan(ax)}}{a(n^2+16)(a^2x^2+1)^2}\right)}{c^4(n^2+36)}+\frac{(6ax+n)e^{n\arctan(ax)}}{ac^4(n^2+36)(a^2x^2+1)^3}$$

$$\Big\downarrow \text{5594}$$

$$\frac{(6ax+n)e^{n\arctan(ax)}}{ac^4(n^2+36)(a^2x^2+1)^3}+\frac{30\left(\frac{(4ax+n)e^{n\arctan(ax)}}{a(n^2+16)(a^2x^2+1)^2}+\frac{12\left(\frac{(2ax+n)e^{n\arctan(ax)}}{a\left(n^2+4\right)\left(a^2x^2+1\right)}+\frac{2e^{n\arctan(ax)}}{an\left(n^2+4\right)}\right)}{n^2+16}\right)}{c^4(n^2+36)}$$

input
```
Int[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^4,x]
```

output
```
(E^(n*ArcTan[a*x])*(n + 6*a*x))/(a*c^4*(36 + n^2)*(1 + a^2*x^2)^3) + (30*(
(E^(n*ArcTan[a*x])*(n + 4*a*x))/(a*(16 + n^2)*(1 + a^2*x^2)^2) + (12*((2*E
^(n*ArcTan[a*x]))/(a*n*(4 + n^2)) + (E^(n*ArcTan[a*x])*(n + 2*a*x))/(a*(4
+ n^2)*(1 + a^2*x^2))))/(16 + n^2)))/(c^4*(36 + n^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5593
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[(n - 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*c*(n^2 +
4*(p + 1)^2))), x] + Simp[2*(p + 1)*((2*p + 3)/(c*(n^2 + 4*(p + 1)^2)))   I
nt[(c + d*x^2)^(p + 1)*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n}, x]
&& EqQ[d, a^2*c] && LtQ[p, -1] &&  !IntegerQ[I*n] && NeQ[n^2 + 4*(p + 1)^2,
 0] && IntegerQ[2*p]
```

rule 5594
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Maple [A] (verified)

Time = 42.05 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.92

method	result
gosper	$\frac{(720x^6a^6+720a^5x^5n+360a^4n^2x^4+120a^3n^3x^3+2160a^4x^4+30a^2n^4x^2+1920a^3n\,x^3+6a\,n^5x+840a^2n^2x^2+n^6+240a\,n^3x+21}{(a^2x^2+1)^3c^4an(n^6+56n^4+784n^2+2304)}$
orering	$\frac{(720x^6a^6+720a^5x^5n+360a^4n^2x^4+120a^3n^3x^3+2160a^4x^4+30a^2n^4x^2+1920a^3n\,x^3+6a\,n^5x+840a^2n^2x^2+n^6+240a\,n^3x+21}{(n^2+16)(n^4+40n^2+144)an(a^2c\,x^2+c)^4}$
parallelrisch	$\frac{840x^2e^{n\arctan(ax)}a^2n^2+240x\,e^{n\arctan(ax)}a\,n^3+1920a^3e^{n\arctan(ax)}x^3n+720a^5e^{n\arctan(ax)}x^5n+1584x\,e^{n\arctan(ax)}na+e^{n}}{}$

input
```
int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

output
```
(720*a^6*x^6+720*a^5*n*x^5+360*a^4*n^2*x^4+120*a^3*n^3*x^3+2160*a^4*x^4+30
*a^2*n^4*x^2+1920*a^3*n*x^3+6*a*n^5*x+840*a^2*n^2*x^2+n^6+240*a*n^3*x+2160
*a^2*x^2+50*n^4+1584*a*n*x+544*n^2+720)*exp(n*arctan(a*x))/(a^2*x^2+1)^3/c
^4/a/n/(n^6+56*n^4+784*n^2+2304)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.65

$$\int \frac{e^{n\arctan(ax)}}{\left(c+a^2cx^2\right)^4}\,dx$$
$$= \frac{(720\,a^6x^6 + 720\,a^5nx^5 + n^6 + 360\,(a^4n^2 + 6\,a^4)x^4 + 50\,n^4 + 120\,(a^3n^3 + 16\,a^3n)x^3 + 30\,(}{ac^4n^7 + 56\,ac^4n^5 + 784\,ac^4n^3 + (a^7c^4n^7 + 56\,a^7c^4n^5 + 784\,a^7c^4n^3 + 2304\,a^7c^4n)x^6 + 2304\,ac^4n + 3\,(a^5c}$$

input
```
integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="fricas")
```

output
```
(720*a^6*x^6 + 720*a^5*n*x^5 + n^6 + 360*(a^4*n^2 + 6*a^4)*x^4 + 50*n^4 +
120*(a^3*n^3 + 16*a^3*n)*x^3 + 30*(a^2*n^4 + 28*a^2*n^2 + 72*a^2)*x^2 + 54
4*n^2 + 6*(a*n^5 + 40*a*n^3 + 264*a*n)*x + 720)*e^(n*arctan(a*x))/(a*c^4*n
^7 + 56*a*c^4*n^5 + 784*a*c^4*n^3 + (a^7*c^4*n^7 + 56*a^7*c^4*n^5 + 784*a^
7*c^4*n^3 + 2304*a^7*c^4*n)*x^6 + 2304*a*c^4*n + 3*(a^5*c^4*n^7 + 56*a^5*c
^4*n^5 + 784*a^5*c^4*n^3 + 2304*a^5*c^4*n)*x^4 + 3*(a^3*c^4*n^7 + 56*a^3*c
^4*n^5 + 784*a^3*c^4*n^3 + 2304*a^3*c^4*n)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}}{\left(c+a^2cx^2\right)^4}\,dx = \text{Timed out}$$

input
```
integrate(exp(n*atan(a*x))/(a**2*c*x**2+c)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{n \arctan(ax)}}{(c + a^2cx^2)^4} dx = \int \frac{e^{(n \arctan(ax))}}{(a^2cx^2 + c)^4} dx$$

input
```
integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="maxima")
```

output
```
integrate(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^4, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1346 vs. 2(177) = 354.

Time = 0.19 (sec) , antiderivative size = 1346, normalized size of antiderivative = 7.44

$$\int \frac{e^{n \arctan(ax)}}{(c + a^2cx^2)^4} dx = \text{Too large to display}$$

input
```
integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^4,x, algorithm="giac")
```

output
```
(n^6*e^(n*arctan(a*x))*tan(1/2*arctan(a*x))^12 - 6*n^6*e^(n*arctan(a*x))*t
an(1/2*arctan(a*x))^10 - 12*n^5*e^(n*arctan(a*x))*tan(1/2*arctan(a*x))^11
+ 50*n^4*e^(n*arctan(a*x))*tan(1/2*arctan(a*x))^12 + 15*n^6*e^(n*arctan(a*
x))*tan(1/2*arctan(a*x))^8 + 60*n^5*e^(n*arctan(a*x))*tan(1/2*arctan(a*x))
^9 - 180*n^4*e^(n*arctan(a*x))*tan(1/2*arctan(a*x))^10 - 480*n^3*e^(n*arct
an(a*x))*tan(1/2*arctan(a*x))^11 + 544*n^2*e^(n*arctan(a*x))*tan(1/2*arcta
n(a*x))^12 - 20*n^6*e^(n*arctan(a*x))*tan(1/2*arctan(a*x))^6 - 120*n^5*e^(
n*arctan(a*x))*tan(1/2*arctan(a*x))^7 + 270*n^4*e^(n*arctan(a*x))*tan(1/2*
arctan(a*x))^8 + 1440*n^3*e^(n*arctan(a*x))*tan(1/2*arctan(a*x))^9 + 96*n^
2*e^(n*arctan(a*x))*tan(1/2*arctan(a*x))^10 - 3168*n*e^(n*arctan(a*x))*tan
(1/2*arctan(a*x))^11 + 720*e^(n*arctan(a*x))*tan(1/2*arctan(a*x))^12 + 15*
n^6*e^(n*arctan(a*x))*tan(1/2*arctan(a*x))^4 + 120*n^5*e^(n*arctan(a*x))*t
an(1/2*arctan(a*x))^5 - 280*n^4*e^(n*arctan(a*x))*tan(1/2*arctan(a*x))^6 -
 1920*n^3*e^(n*arctan(a*x))*tan(1/2*arctan(a*x))^7 + 480*n^2*e^(n*arctan(a
*x))*tan(1/2*arctan(a*x))^8 + 480*n*e^(n*arctan(a*x))*tan(1/2*arctan(a*x))
^9 + 4320*e^(n*arctan(a*x))*tan(1/2*arctan(a*x))^10 - 6*n^6*e^(n*arctan(a*
x))*tan(1/2*arctan(a*x))^2 - 60*n^5*e^(n*arctan(a*x))*tan(1/2*arctan(a*x))
^3 + 270*n^4*e^(n*arctan(a*x))*tan(1/2*arctan(a*x))^4 + 1920*n^3*e^(n*arct
an(a*x))*tan(1/2*arctan(a*x))^5 - 2240*n^2*e^(n*arctan(a*x))*tan(1/2*arcta
n(a*x))^6 - 8640*n*e^(n*arctan(a*x))*tan(1/2*arctan(a*x))^7 + 10800*e^(...
```

Mupad [B] (verification not implemented)

Time = 23.24 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.55

$$\int \frac{e^{n\arctan(ax)}}{(c+a^2cx^2)^4}\,dx = \frac{e^{n\,\mathrm{atan}(a\,x)}\left(\frac{720\,x^5}{a^2\,c^4\,(n^6+56\,n^4+784\,n^2+2304)}+\frac{n^6+50\,n^4+544\,n^2+720}{a^7\,c^4\,n\,(n^6+56\,n^4+784\,n^2+2304)}+\frac{720\,x^6}{a\,c^4\,n\,(n^6+56\,n^4+784\,n^2+2304)}+\frac{6\,x\,(n^4+4}{a^6\,c^4\,(n^6+56\,n}\right.}{\frac{1}{a^6}+x^6+\frac{3\,x^4}{a^2}}$$

input
```
int(exp(n*atan(a*x))/(c + a^2*c*x^2)^4,x)
```

output

```
(exp(n*atan(a*x))*((720*x^5)/(a^2*c^4*(784*n^2 + 56*n^4 + n^6 + 2304)) + (
544*n^2 + 50*n^4 + n^6 + 720)/(a^7*c^4*n*(784*n^2 + 56*n^4 + n^6 + 2304))
+ (720*x^6)/(a*c^4*n*(784*n^2 + 56*n^4 + n^6 + 2304)) + (6*x*(40*n^2 + n^4
 + 264))/(a^6*c^4*(784*n^2 + 56*n^4 + n^6 + 2304)) + (120*x^3*(n^2 + 16))/
(a^4*c^4*(784*n^2 + 56*n^4 + n^6 + 2304)) + (360*x^4*(n^2 + 6))/(a^3*c^4*n
*(784*n^2 + 56*n^4 + n^6 + 2304)) + (30*x^2*(28*n^2 + n^4 + 72))/(a^5*c^4*
n*(784*n^2 + 56*n^4 + n^6 + 2304))))/(1/a^6 + x^6 + (3*x^4)/a^2 + (3*x^2)/
a^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.53

$$\int \frac{e^{n\arctan(ax)}}{(c+a^2cx^2)^4}\,dx$$
$$= \frac{e^{atan(ax)n}(720a^6x^6 + 720a^5n\,x^5 + 360a^4n^2x^4 + 120a^3n^3x^3 + 2160a^4x^4 + 30a^2n^4x^2 + 1920a^3n\,x^3 + 6a\,n^5}{a\,c^4n\,(a^6n^6x^6 + 56a^6n^4x^6 + 784a^6n^2x^6 + 3a^4n^6x^4 + 2304a^6x^6 + 168a^4n^4x^4 + 2352a^4n^2x^4 + 3a^2n^6x^2 +}$$

input

```
int(exp(n*atan(a*x))/(a^2*c*x^2+c)^4,x)
```

output

```
(e**(atan(a*x)*n)*(720*a**6*x**6 + 720*a**5*n*x**5 + 360*a**4*n**2*x**4 +
2160*a**4*x**4 + 120*a**3*n**3*x**3 + 1920*a**3*n*x**3 + 30*a**2*n**4*x**2
 + 840*a**2*n**2*x**2 + 2160*a**2*x**2 + 6*a*n**5*x + 240*a*n**3*x + 1584*
a*n*x + n**6 + 50*n**4 + 544*n**2 + 720))/(a*c**4*n*(a**6*n**6*x**6 + 56*a
**6*n**4*x**6 + 784*a**6*n**2*x**6 + 2304*a**6*x**6 + 3*a**4*n**6*x**4 + 1
68*a**4*n**4*x**4 + 2352*a**4*n**2*x**4 + 6912*a**4*x**4 + 3*a**2*n**6*x**
2 + 168*a**2*n**4*x**2 + 2352*a**2*n**2*x**2 + 6912*a**2*x**2 + n**6 + 56*
n**4 + 784*n**2 + 2304))
```

3.362 $\int e^{n \arctan(ax)} \left(c + a^2 c x^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int e^{n \arctan(ax)} \left(c + a^2 c x^2\right)^{3/2} \, dx = -\frac{2^{\frac{5}{2}-\frac{in}{2}} c (1 - iax)^{\frac{1}{2}(5+in)} \sqrt{c + a^2 c x^2} \,\text{Hypergeometric2F1}\left(\frac{1}{2}(-3+in), \frac{1}{2}(5+in), \frac{1}{2}(7+in), \frac{1}{2}(1-iax)\right)}{a(5i - n)\sqrt{1 + a^2 x^2}}$$

output
```
-2^(5/2-1/2*I*n)*c*(1-I*a*x)^(5/2+1/2*I*n)*(a^2*c*x^2+c)^(1/2)*hypergeom([
5/2+1/2*I*n, -3/2+1/2*I*n],[7/2+1/2*I*n],1/2-1/2*I*a*x)/a/(5*I-n)/(a^2*x^2
+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98

$$\int e^{n \arctan(ax)} \left(c + a^2 c x^2\right)^{3/2} \, dx = \frac{2^{\frac{5}{2}-\frac{in}{2}} c (1 - iax)^{\frac{5}{2}+\frac{in}{2}} \sqrt{c + a^2 c x^2} \,\text{Hypergeometric2F1}\left(\frac{1}{2}(5+in), \frac{1}{2}i(3i+n), \frac{1}{2}(7+in), \frac{1}{2}(\right.}{a(-5i + n)\sqrt{1 + a^2 x^2}}$$

input
```
Integrate[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2),x]
```

output
```
(2^(5/2 - (I/2)*n)*c*(1 - I*a*x)^(5/2 + (I/2)*n)*Sqrt[c + a^2*c*x^2]*Hyper
geometric2F1[(5 + I*n)/2, (I/2)*(3*I + n), (7 + I*n)/2, (1 - I*a*x)/2])/(a
*(-5*I + n)*Sqrt[1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5599, 5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2cx^2+c\right)^{3/2}e^{n\arctan(ax)}dx$$

$$\Big\downarrow \text{ 5599}$$

$$\frac{c\sqrt{a^2cx^2+c}\int e^{n\arctan(ax)}\left(a^2x^2+1\right)^{3/2}dx}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{ 5596}$$

$$\frac{c\sqrt{a^2cx^2+c}\int (1-iax)^{\frac{1}{2}(in+3)}(iax+1)^{\frac{1}{2}(3-in)}dx}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{ 79}$$

$$-\frac{c2^{\frac{5}{2}-\frac{in}{2}}\sqrt{a^2cx^2+c}(1-iax)^{\frac{1}{2}(5+in)}\text{Hypergeometric2F1}\left(\frac{1}{2}(in-3),\frac{1}{2}(in+5),\frac{1}{2}(in+7),\frac{1}{2}(1-iax)\right)}{a(-n+5i)\sqrt{a^2x^2+1}}$$

input
```
Int[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2),x]
```

output
```
-((2^(5/2 - (I/2)*n)*c*(1 - I*a*x)^((5 + I*n)/2)*Sqrt[c + a^2*c*x^2]*Hyper
geometric2F1[(-3 + I*n)/2, (5 + I*n)/2, (7 + I*n)/2, (1 - I*a*x)/2])/(a*(5
*I - n)*Sqrt[1 + a^2*x^2]))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \mathrm{e}^{n \arctan(ax)} \left(a^2 c\, x^2 + c\right)^{\frac{3}{2}} dx$$

input
```
int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x)
```

Fricas [F]

$$\int e^{n \arctan(ax)} \left(c + a^2 c x^2\right)^{3/2} dx = \int \left(a^2 c x^2 + c\right)^{\frac{3}{2}} e^{(n \arctan(ax))} dx$$

input
```
integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*e^(n*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}e^{n\,\mathrm{atan}\,(ax)}\,dx$$

input
```
integrate(exp(n*atan(a*x))*(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*exp(n*atan(a*x)), x)
```

Maxima [F]

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int\left(a^2cx^2+c\right)^{\frac{3}{2}}e^{(n\arctan(ax))}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*e^(n*arctan(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\int \mathrm{e}^{n\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(exp(n*atan(a*x))*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(exp(n*atan(a*x))*(c + a^2*c*x^2)^(3/2), x)
```

Reduce [F]

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^{3/2}\,dx=\sqrt{c}\,c\left(\left(\int e^{atan(ax)n}\sqrt{a^2x^2+1}\,x^2dx\right)a^2+\int e^{atan(ax)n}\sqrt{a^2x^2+1}dx\right)$$

input
```
int(exp(n*atan(a*x))*(a^2*c*x^2+c)^(3/2),x)
```

output
```
sqrt(c)*c*(int(e**(atan(a*x)*n)*sqrt(a**2*x**2 + 1)*x**2,x)*a**2 + int(e**
(atan(a*x)*n)*sqrt(a**2*x**2 + 1),x))
```

3.363 $\int e^{n \arctan(ax)}\sqrt{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 120

$$\int e^{n \arctan(ax)}\sqrt{c+a^2cx^2}\,dx = -\frac{2^{\frac{3}{2}-\frac{in}{2}}(1-iax)^{\frac{1}{2}(3+in)}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1+in),\frac{1}{2}(3+in),\frac{1}{2}(5+in),\frac{1}{2}(1-iax)\right)}{a(3i-n)\sqrt{1+a^2x^2}}$$

output
```
-2^(3/2-1/2*I*n)*(1-I*a*x)^(3/2+1/2*I*n)*(a^2*c*x^2+c)^(1/2)*hypergeom([3/
2+1/2*I*n, -1/2+1/2*I*n],[5/2+1/2*I*n],1/2-1/2*I*a*x)/a/(3*I-n)/(a^2*x^2+1
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

$$\int e^{n \arctan(ax)}\sqrt{c+a^2cx^2}\,dx = \frac{2^{\frac{3}{2}-\frac{in}{2}}(1-iax)^{\frac{3}{2}+\frac{in}{2}}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(3+in),\frac{1}{2}i(i+n),\frac{1}{2}(5+in),\frac{1}{2}(1-iax)\right)}{a(-3i+n)\sqrt{1+a^2x^2}}$$

input
```
Integrate[E^(n*ArcTan[a*x])*Sqrt[c + a^2*c*x^2],x]
```

output
```
(2^(3/2 - (I/2)*n)*(1 - I*a*x)^(3/2 + (I/2)*n)*Sqrt[c + a^2*c*x^2]*Hyperge
ometric2F1[(3 + I*n)/2, (I/2)*(I + n), (5 + I*n)/2, (1 - I*a*x)/2])/(a*(-3
*I + n)*Sqrt[1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5599, 5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2cx^2+c}e^{n\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5599}$$

$$\frac{\sqrt{a^2cx^2+c}\int e^{n\arctan(ax)}\sqrt{a^2x^2+1}dx}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{5596}$$

$$\frac{\sqrt{a^2cx^2+c}\int (1-iax)^{\frac{1}{2}(in+1)}(iax+1)^{\frac{1}{2}(1-in)}dx}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{2^{\frac{3}{2}-\frac{in}{2}}\sqrt{a^2cx^2+c}(1-iax)^{\frac{1}{2}(3+in)}\text{Hypergeometric2F1}\left(\frac{1}{2}(in-1),\frac{1}{2}(in+3),\frac{1}{2}(in+5),\frac{1}{2}(1-iax)\right)}{a(-n+3i)\sqrt{a^2x^2+1}}$$

input
```
Int[E^(n*ArcTan[a*x])*Sqrt[c + a^2*c*x^2],x]
```

output
```
-((2^(3/2 - (I/2)*n)*(1 - I*a*x)^((3 + I*n)/2)*Sqrt[c + a^2*c*x^2]*Hyperge
ometric2F1[(-1 + I*n)/2, (3 + I*n)/2, (5 + I*n)/2, (1 - I*a*x)/2])/(a*(3*I
 - n)*Sqrt[1 + a^2*x^2]))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \mathrm{e}^{n \arctan(ax)} \sqrt{a^2 c\, x^2 + c}dx$$

input
```
int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int e^{n \arctan(ax)} \sqrt{c + a^2 c x^2}\, dx = \int \sqrt{a^2 c x^2 + c} e^{(n \arctan(ax))}\, dx$$

input
```
integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*e^(n*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{n\arctan(ax)}\sqrt{c+a^2cx^2}\,dx=\int\sqrt{c\,(a^2x^2+1)}e^{n\,\mathrm{atan}\,(ax)}\,dx$$

input
```
integrate(exp(n*atan(a*x))*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*exp(n*atan(a*x)), x)
```

Maxima [F]

$$\int e^{n\arctan(ax)}\sqrt{c+a^2cx^2}\,dx=\int\sqrt{a^2cx^2+c}e^{(n\arctan(ax))}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*e^(n*arctan(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{n\arctan(ax)}\sqrt{c+a^2cx^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(ax)}\sqrt{c+a^2cx^2}\,dx=\int e^{n\,\mathrm{atan}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(exp(n*atan(a*x))*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(exp(n*atan(a*x))*(c + a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int e^{n\arctan(ax)}\sqrt{c+a^2cx^2}\,dx=\sqrt{c}\left(\int e^{atan(ax)n}\sqrt{a^2x^2+1}dx\right)$$

input
```
int(exp(n*atan(a*x))*(a^2*c*x^2+c)^(1/2),x)
```

output
```
sqrt(c)*int(e**(atan(a*x)*n)*sqrt(a**2*x**2 + 1),x)
```

3.364 $\int \frac{e^{n \arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 120

$$\int \frac{e^{n \arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{2^{\frac{1}{2}-\frac{in}{2}}(1-iax)^{\frac{1}{2}(1+in)}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1+in),\frac{1}{2}(1+in),\frac{1}{2}(3+in),\frac{1}{2}(1-iax)\right)}{a(i-n)\sqrt{c+a^2cx^2}}$$

output
```
-2^(1/2-1/2*I*n)*(1-I*a*x)^(1/2+1/2*I*n)*(a^2*x^2+1)^(1/2)*hypergeom([1/2+
1/2*I*n, 1/2+1/2*I*n],[3/2+1/2*I*n],1/2-1/2*I*a*x)/a/(I-n)/(a^2*c*x^2+c)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

$$\int \frac{e^{n \arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{2^{\frac{1}{2}-\frac{in}{2}}(1-iax)^{\frac{1}{2}+\frac{in}{2}}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}+\frac{in}{2},\frac{1}{2}+\frac{in}{2},\frac{3}{2}+\frac{in}{2},\frac{1}{2}-\frac{iax}{2}\right)}{a(-i+n)\sqrt{c+a^2cx^2}}$$

input
```
Integrate[E^(n*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
(2^(1/2 - (I/2)*n)*(1 - I*a*x)^(1/2 + (I/2)*n)*Sqrt[1 + a^2*x^2]*Hypergeom
etric2F1[1/2 + (I/2)*n, 1/2 + (I/2)*n, 3/2 + (I/2)*n, 1/2 - (I/2)*a*x])/(a
*(-I + n)*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5599, 5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n \arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5599}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{n \arctan(ax)}}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5596}$$

$$\frac{\sqrt{a^2x^2+1}\int (1-iax)^{\frac{1}{2}(in-1)}(iax+1)^{\frac{1}{2}(-in-1)}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{2^{\frac{1}{2}-\frac{in}{2}}\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}\text{Hypergeometric2F1}\left(\frac{1}{2}(in+1),\frac{1}{2}(in+1),\frac{1}{2}(in+3),\frac{1}{2}(1-iax)\right)}{a(-n+i)\sqrt{a^2cx^2+c}}$$

input
```
Int[E^(n*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
-((2^(1/2 - (I/2)*n)*(1 - I*a*x)^((1 + I*n)/2)*Sqrt[1 + a^2*x^2]*Hypergeom
etric2F1[(1 + I*n)/2, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/2])/(a*(I - n)
*Sqrt[c + a^2*c*x^2]))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{e^{n \arctan(ax)}}{\sqrt{a^2 c\, x^2 + c}} dx$$

input
```
int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)}}{\sqrt{c + a^2 c x^2}}\, dx = \int \frac{e^{(n \arctan(ax))}}{\sqrt{a^2 c x^2 + c}}\, dx$$

input
```
integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(ax)}}{\sqrt{c + a^2 c x^2}} \, dx = \int \frac{e^{n \operatorname{atan}(ax)}}{\sqrt{c \left(a^2 x^2 + 1\right)}} \, dx$$

input
```
integrate(exp(n*atan(a*x))/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(exp(n*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{e^{n \arctan(ax)}}{\sqrt{c + a^2 c x^2}} \, dx = \int \frac{e^{(n \arctan(ax))}}{\sqrt{a^2 c x^2 + c}} \, dx$$

input
```
integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{e^{n \arctan(ax)}}{\sqrt{c + a^2 c x^2}} \, dx = \int \frac{e^{(n \arctan(ax))}}{\sqrt{a^2 c x^2 + c}} \, dx$$

input
```
integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{e}^{n\,\mathrm{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{e^{n\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{e^{atan(ax)n}}{\sqrt{a^2x^2+1}}dx}{\sqrt{c}}$$

input
```
int(exp(n*atan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(e**(atan(a*x)*n)/sqrt(a**2*x**2 + 1),x)/sqrt(c)
```

3.365 $\int e^{n\arctan(ax)}x^2(c+a^2cx^2)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 283

$$\int e^{n\arctan(ax)}x^2\left(c+a^2cx^2\right)^{3/2}\,dx=-\frac{cn(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-in)}\sqrt{c+a^2cx^2}}{30a^3\sqrt{1+a^2x^2}}$$
$$+\frac{cx(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-in)}\sqrt{c+a^2cx^2}}{6a^2\sqrt{1+a^2x^2}}$$
$$+\frac{2^{\frac{3}{2}-\frac{in}{2}}c(5-n^2)\,(1-iax)^{\frac{1}{2}(5+in)}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-3+in),\frac{1}{2}(5+in),\frac{1}{2}(7+in),\frac{1}{2}(1-\right.}{15a^3(5i-n)\sqrt{1+a^2x^2}}$$

output
```
-1/30*c*n*(1-I*a*x)^(5/2+1/2*I*n)*(1+I*a*x)^(5/2-1/2*I*n)*(a^2*c*x^2+c)^(1
/2)/a^3/(a^2*x^2+1)^(1/2)+1/6*c*x*(1-I*a*x)^(5/2+1/2*I*n)*(1+I*a*x)^(5/2-1
/2*I*n)*(a^2*c*x^2+c)^(1/2)/a^2/(a^2*x^2+1)^(1/2)+1/15*2^(3/2-1/2*I*n)*c*(
-n^2+5)*(1-I*a*x)^(5/2+1/2*I*n)*(a^2*c*x^2+c)^(1/2)*hypergeom([5/2+1/2*I*n
, -3/2+1/2*I*n],[7/2+1/2*I*n],1/2-1/2*I*a*x)/a^3/(5*I-n)/(a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.77

$$\int e^{n\arctan(ax)}x^2\left(c+a^2cx^2\right)^{3/2}\,dx=\frac{2^{-1-\frac{in}{2}}c(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{in}{2}}(i+ax)^2\sqrt{c+a^2cx^2}\left(2^{\frac{in}{2}}(-5i+n)\sqrt{1+iax}(-i+ax)\right.}{15}$$

input
```
Integrate[E^(n*ArcTan[a*x])*x^2*(c + a^2*c*x^2)^(3/2),x]
```

output
```
(2^(-1 - (I/2)*n)*c*(1 - I*a*x)^(1/2 + (I/2)*n)*(I + a*x)^2*Sqrt[c + a^2*c
*x^2]*(2^((I/2)*n)*(-5*I + n)*Sqrt[1 + I*a*x]*(-I + a*x)^2*(-n + 5*a*x) -
4*Sqrt[2]*(-5 + n^2)*(1 + I*a*x)^((I/2)*n)*Hypergeometric2F1[(5 + I*n)/2,
(I/2)*(3*I + n), (7 + I*n)/2, (1 - I*a*x)/2]))/(15*a^3*(-5*I + n)*(1 + I*a
*x)^((I/2)*n)*Sqrt[1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5608, 5605, 101, 25, 90, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a^2cx^2+c\right)^{3/2}e^{n\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5608}$$

$$\frac{c\sqrt{a^2cx^2+c}\int e^{n\arctan(ax)}x^2\left(a^2x^2+1\right)^{3/2}dx}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{5605}$$

$$\frac{c\sqrt{a^2cx^2+c}\int x^2(1-iax)^{\frac{1}{2}(in+3)}(iax+1)^{\frac{1}{2}(3-in)}dx}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \ 101$$

$$\frac{c\sqrt{a^2cx^2+c}\left(\frac{\int -(1-iax)^{\frac{1}{2}(in+3)}(iax+1)^{\frac{1}{2}(3-in)}(anx+1)dx}{6a^2}+\frac{x(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-in)}}{6a^2}\right)}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \ 25$$

$$\frac{c\sqrt{a^2cx^2+c}\left(\frac{x(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-in)}}{6a^2}-\frac{\int(1-iax)^{\frac{1}{2}(in+3)}(iax+1)^{\frac{1}{2}(3-in)}(anx+1)dx}{6a^2}\right)}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \ 90$$

$$\frac{c\sqrt{a^2cx^2+c}\left(\frac{x(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-in)}}{6a^2}-\frac{\frac{1}{5}(5-n^2)\int(1-iax)^{\frac{1}{2}(in+3)}(iax+1)^{\frac{1}{2}(3-in)}dx+\frac{n(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-in)}}{5a}}{6a^2}\right)}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \ 79$$

$$\frac{c\sqrt{a^2cx^2+c}\left(\frac{x(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-in)}}{6a^2}-\frac{\frac{n(1-iax)^{\frac{1}{2}(5+in)}(1+iax)^{\frac{1}{2}(5-in)}}{5a}-\frac{2^{\frac{5}{2}-\frac{in}{2}}(5-n^2)(1-iax)^{\frac{1}{2}(5+in)}\text{Hypergeometric2F1}\left(\frac{1}{2}(i\right.}{5a(-n+5i)}}{6a^2}\right.}{\sqrt{a^2x^2+1}}$$

input
```
Int[E^(n*ArcTan[a*x])*x^2*(c + a^2*c*x^2)^(3/2),x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*((x*(1 - I*a*x)^((5 + I*n)/2)*(1 + I*a*x)^((5 - I*n
)/2))/(6*a^2) - ((n*(1 - I*a*x)^((5 + I*n)/2)*(1 + I*a*x)^((5 - I*n)/2))/(
5*a) - (2^(5/2 - (I/2)*n)*(5 - n^2)*(1 - I*a*x)^((5 + I*n)/2)*Hypergeometr
ic2F1[(-3 + I*n)/2, (5 + I*n)/2, (7 + I*n)/2, (1 - I*a*x)/2])/(5*a*(5*I -
n)))/(6*a^2)))/Sqrt[1 + a^2*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

rule 5608
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p])   Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
 m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int e^{n\arctan(ax)}x^2\left(a^2c\,x^2+c\right)^{\frac{3}{2}}dx$$

input
```
int(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(3/2),x)
```

Fricas [F]

$$\int e^{n\arctan(ax)}x^2\left(c+a^2cx^2\right)^{3/2}\,dx=\int\left(a^2cx^2+c\right)^{\frac{3}{2}}x^2e^{(n\arctan(ax))}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(3/2),x, algorithm="fricas"
)
```

output
```
integral((a^2*c*x^4 + c*x^2)*sqrt(a^2*c*x^2 + c)*e^(n*arctan(a*x)), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{n\arctan(ax)}x^2\left(c+a^2cx^2\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(exp(n*atan(a*x))*x**2*(a**2*c*x**2+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{n\arctan(ax)}x^2\left(c+a^2cx^2\right)^{3/2}\,dx=\int \left(a^2cx^2+c\right)^{\frac{3}{2}}x^2e^{(n\arctan(ax))}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^2*e^(n*arctan(a*x)), x)
```

Giac [F]

$$\int e^{n\arctan(ax)}x^2\left(c+a^2cx^2\right)^{3/2}\,dx=\int \left(a^2cx^2+c\right)^{\frac{3}{2}}x^2e^{(n\arctan(ax))}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^2*e^(n*arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(ax)}x^2\left(c+a^2cx^2\right)^{3/2}\,dx=\int x^2\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(x^2*exp(n*atan(a*x))*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^2*exp(n*atan(a*x))*(c + a^2*c*x^2)^(3/2), x)
```

Reduce [F]

$$\int e^{n\arctan(ax)}x^2(c+a^2cx^2)^{3/2}\,dx = \sqrt{c}\,c\left(\left(\int e^{atan(ax)n}\sqrt{a^2x^2+1}\,x^4dx\right)a^2 + \int e^{atan(ax)n}\sqrt{a^2x^2+1}\,x^2dx\right)$$

input
```
int(exp(n*atan(a*x))*x^2*(a^2*c*x^2+c)^(3/2),x)
```

output
```
sqrt(c)*c*(int(e**(atan(a*x)*n)*sqrt(a**2*x**2 + 1)*x**4,x)*a**2 + int(e**
(atan(a*x)*n)*sqrt(a**2*x**2 + 1)*x**2,x))
```

3.366 $\int e^{n\arctan(ax)}x^2\sqrt{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 280

$$\int e^{n\arctan(ax)}x^2\sqrt{c+a^2cx^2}\,dx=-\frac{n(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in)}\sqrt{c+a^2cx^2}}{12a^3\sqrt{1+a^2x^2}}+\frac{x(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in)}\sqrt{c+a^2cx^2}}{4a^2\sqrt{1+a^2x^2}}+\frac{2^{-\frac{1}{2}-\frac{in}{2}}(3-n^2)(1-iax)^{\frac{1}{2}(3+in)}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1+in),\frac{1}{2}(3+in),\frac{1}{2}(5+in),\frac{1}{2}(\right.}{3a^3(3i-n)\sqrt{1+a^2x^2}}$$

output
```
-1/12*n*(1-I*a*x)^(3/2+1/2*I*n)*(1+I*a*x)^(3/2-1/2*I*n)*(a^2*c*x^2+c)^(1/2
)/a^3/(a^2*x^2+1)^(1/2)+1/4*x*(1-I*a*x)^(3/2+1/2*I*n)*(1+I*a*x)^(3/2-1/2*I
*n)*(a^2*c*x^2+c)^(1/2)/a^2/(a^2*x^2+1)^(1/2)+1/3*2^(-1/2-1/2*I*n)*(-n^2+3
)*(1-I*a*x)^(3/2+1/2*I*n)*(a^2*c*x^2+c)^(1/2)*hypergeom([3/2+1/2*I*n, -1/2
+1/2*I*n],[5/2+1/2*I*n],1/2-1/2*I*a*x)/a^3/(3*I-n)/(a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.76

$$\int e^{n\arctan(ax)}x^2\sqrt{c+a^2cx^2}\,dx$$
$$=\frac{2^{-2-\frac{in}{2}}(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{in}{2}}(i+ax)\sqrt{c+a^2cx^2}\Big(2^{\frac{in}{2}}(-3i+n)\sqrt{1+iax}(-i+ax)(-n+3ax)-2i}{3a^3(-3i+n)\sqrt{1-}}$$

input
```
Integrate[E^(n*ArcTan[a*x])*x^2*Sqrt[c + a^2*c*x^2],x]
```

output
```
(2^(-2 - (I/2)*n)*(1 - I*a*x)^(1/2 + (I/2)*n)*(I + a*x)*Sqrt[c + a^2*c*x^2
]*(2^((I/2)*n)*(-3*I + n)*Sqrt[1 + I*a*x]*(-I + a*x)*(-n + 3*a*x) - (2*I)*
Sqrt[2]*(-3 + n^2)*(1 + I*a*x)^((I/2)*n)*Hypergeometric2F1[(3 + I*n)/2, (I
/2)*(I + n), (5 + I*n)/2, (1 - I*a*x)/2]))/(3*a^3*(-3*I + n)*(1 + I*a*x)^(
(I/2)*n)*Sqrt[1 + a^2*x^2])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5608, 5605, 101, 25, 90, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a^2cx^2+c}e^{n\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5608}$$

$$\frac{\sqrt{a^2cx^2+c}\int e^{n\arctan(ax)}x^2\sqrt{a^2x^2+1}dx}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{5605}$$

$$\frac{\sqrt{a^2cx^2+c}\int x^2(1-iax)^{\frac{1}{2}(in+1)}(iax+1)^{\frac{1}{2}(1-in)}dx}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \text{101}$$

$$\frac{\sqrt{a^2cx^2+c}\left(\frac{\int -(1-iax)^{\frac{1}{2}(in+1)}(iax+1)^{\frac{1}{2}(1-in)}(anx+1)dx}{4a^2}+\frac{x(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in)}}{4a^2}\right)}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt{a^2cx^2+c}\left(\frac{x(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in)}}{4a^2}-\frac{\int (1-iax)^{\frac{1}{2}(in+1)}(iax+1)^{\frac{1}{2}(1-in)}(anx+1)dx}{4a^2}\right)}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \ 90$$

$$\frac{\sqrt{a^2cx^2+c}\left(\frac{x(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in)}}{4a^2}-\frac{\frac{1}{3}(3-n^2)\int (1-iax)^{\frac{1}{2}(in+1)}(iax+1)^{\frac{1}{2}(1-in)}dx+\frac{n(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in)}}{3a}}{4a^2}\right)}{\sqrt{a^2x^2+1}}$$

$$\Big\downarrow \ 79$$

$$\frac{\sqrt{a^2cx^2+c}\left(\frac{x(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in)}}{4a^2}-\frac{\frac{n(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(3-in)}}{3a}-\frac{2^{\frac{3}{2}-\frac{in}{2}}(3-n^2)(1-iax)^{\frac{1}{2}(3+in)}\text{Hypergeometric2F1}\left(\frac{1}{2}(in\right.}{3a(-n+3i)}}{4a^2}\right.}{\sqrt{a^2x^2+1}}$$

input
```
Int[E^(n*ArcTan[a*x])*x^2*Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[c + a^2*c*x^2]*((x*(1 - I*a*x)^((3 + I*n)/2)*(1 + I*a*x)^((3 - I*n)/
2))/(4*a^2) - ((n*(1 - I*a*x)^((3 + I*n)/2)*(1 + I*a*x)^((3 - I*n)/2))/(3*
a) - (2^(3/2 - (I/2)*n)*(3 - n^2)*(1 - I*a*x)^((3 + I*n)/2)*Hypergeometric
2F1[(-1 + I*n)/2, (3 + I*n)/2, (5 + I*n)/2, (1 - I*a*x)/2])/(3*a*(3*I - n)
))/(4*a^2)))/Sqrt[1 + a^2*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

rule 5608
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p])   Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
 m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int e^{n\arctan(ax)}x^2\sqrt{a^2c\,x^2+c}dx$$

input
```
int(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int e^{n\arctan(ax)}x^2\sqrt{c+a^2cx^2}\,dx=\int\sqrt{a^2cx^2+c}x^2e^{(n\arctan(ax))}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^2*e^(n*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{n\arctan(ax)}x^2\sqrt{c+a^2cx^2}\,dx=\int x^2\sqrt{c\,(a^2x^2+1)}e^{n\,\mathrm{atan}\,(ax)}\,dx$$

input
```
integrate(exp(n*atan(a*x))*x**2*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*sqrt(c*(a**2*x**2 + 1))*exp(n*atan(a*x)), x)
```

Maxima [F]

$$\int e^{n\arctan(ax)}x^2\sqrt{c+a^2cx^2}\,dx=\int \sqrt{a^2cx^2+c}\,x^2e^{(n\arctan(ax))}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^2*e^(n*arctan(a*x)), x)
```

Giac [F]

$$\int e^{n\arctan(ax)}x^2\sqrt{c+a^2cx^2}\,dx=\int \sqrt{a^2cx^2+c}\,x^2e^{(n\arctan(ax))}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^2*e^(n*arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(ax)}x^2\sqrt{c+a^2cx^2}\,dx=\int x^2\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^2*exp(n*atan(a*x))*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^2*exp(n*atan(a*x))*(c + a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int e^{n\arctan(ax)}x^2\sqrt{c+a^2cx^2}\,dx = \sqrt{c}\left(\int e^{atan(ax)n}\sqrt{a^2x^2+1}\,x^2dx\right)$$

input
```
int(exp(n*atan(a*x))*x^2*(a^2*c*x^2+c)^(1/2),x)
```

output
```
sqrt(c)*int(e**(atan(a*x)*n)*sqrt(a**2*x**2 + 1)*x**2,x)
```

3.367 $\int \frac{e^{n \arctan(ax)} x^3}{\sqrt{c+a^2cx^2}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 390

$$\int \frac{e^{n \arctan(ax)} x^3}{\sqrt{c+a^2cx^2}} \, dx = -\frac{(2i+n)(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{3a^4(i-n)\sqrt{c+a^2cx^2}}$$
$$+\frac{x^2(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{3a^2\sqrt{c+a^2cx^2}}$$
$$-\frac{in(1-iax)^{\frac{1}{2}(3+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{a^4(1+n^2)\sqrt{c+a^2cx^2}}$$
$$-\frac{2^{\frac{3}{2}-\frac{in}{2}}n(5-n^2)(1-iax)^{\frac{1}{2}(3+in)}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1+in),\frac{1}{2}(3+in),\frac{1}{2}(5+in),\frac{1}{2}(1\right.}{3a^4(3i-n+3in^2-n^3)\sqrt{c+a^2cx^2}}$$

output
```
-1/3*(2*I+n)*(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(1/2-1/2*I*n)*(a^2*x^2+1)^(
1/2)/a^4/(I-n)/(a^2*c*x^2+c)^(1/2)+1/3*x^2*(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*
x)^(1/2-1/2*I*n)*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)-I*n*(1-I*a*x)^(
3/2+1/2*I*n)*(1+I*a*x)^(1/2-1/2*I*n)*(a^2*x^2+1)^(1/2)/a^4/(n^2+1)/(a^2*c*
x^2+c)^(1/2)-1/3*2^(3/2-1/2*I*n)*n*(-n^2+5)*(1-I*a*x)^(3/2+1/2*I*n)*(a^2*x
^2+1)^(1/2)*hypergeom([3/2+1/2*I*n, -1/2+1/2*I*n],[5/2+1/2*I*n],1/2-1/2*I*
a*x)/a^4/(3*I-n+3*I*n^2-n^3)/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.64

$$\int \frac{e^{n \arctan(ax)} x^3}{\sqrt{c+a^2cx^2}} \, dx = \frac{2^{-\frac{3}{2}-\frac{in}{2}}(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{in}{2}}\sqrt{1+a^2x^2}\left(2^{\frac{1}{2}+\frac{in}{2}}(-3i+n)\sqrt{1+iax}(-n^2(i+ax)-2i(-2+a^2x^2)+\right.}{3a^4\left(-3-4i\right.}$$

input
```
Integrate[(E^(n*ArcTan[a*x])*x^3)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(2^(-3/2 - (I/2)*n)*(1 - I*a*x)^(1/2 + (I/2)*n)*Sqrt[1 + a^2*x^2]*(2^(1/2
+ (I/2)*n)*(-3*I + n)*Sqrt[1 + I*a*x]*(-(n^2*(I + a*x)) - (2*I)*(-2 + a^2*
x^2) + n*(1 + I*a*x + 2*a^2*x^2)) + 2*n*(-5 + n^2)*(1 + I*a*x)^((I/2)*n)*(
I + a*x)*Hypergeometric2F1[1/2 + (I/2)*n, 3/2 + (I/2)*n, 5/2 + (I/2)*n, 1/
2 - (I/2)*a*x]))/(3*a^4*(-3 - (4*I)*n + n^2)*(1 + I*a*x)^((I/2)*n)*Sqrt[c
+ a^2*c*x^2])
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.71, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5608, 5605, 111, 25, 163, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 e^{n \arctan(ax)}}{\sqrt{a^2cx^2+c}} \, dx$$

$$\Big\downarrow \text{5608}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{n \arctan(ax)} x^3}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5605}$$

$$\frac{\sqrt{a^2x^2+1}\int x^3(1-iax)^{\frac{1}{2}(in-1)}(iax+1)^{\frac{1}{2}(-in-1)}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{111}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{\int -x(1-iax)^{\frac{1}{2}(in-1)}(iax+1)^{\frac{1}{2}(-in-1)}(anx+2)dx}{3a^2}+\frac{x^2(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{3a^2}\right)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{x^2(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{3a^2}-\frac{\int x(1-iax)^{\frac{1}{2}(in-1)}(iax+1)^{\frac{1}{2}(-in-1)}(anx+2)dx}{3a^2}\right)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{163}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{x^2(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{3a^2}-\frac{\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\left(a(1+in)nx-n^2-in+4\right)}{2a^2(1+in)}-\frac{n\left(5-n^2\right)\int(1-iax)^{\frac{1}{2}(in+1)}(iax+1)}{2a(1+in)}}{3a^2}\right.}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{79}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{x^2(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{3a^2}-\frac{\frac{2^{-\frac{1}{2}-\frac{in}{2}}n\left(5-n^2\right)(1-iax)^{\frac{1}{2}(3+in)}\text{Hypergeometric2F1}\left(\frac{1}{2}(in+1),\frac{1}{2}(in+3),\frac{1}{2}(in+5),\frac{1}{2}(1-iax)\right)}{a^2(-n+3i)(1+in)}}{3a^2}\right.}{\sqrt{a^2cx^2+c}}$$

input

```
Int[(E^(n*ArcTan[a*x])*x^3)/Sqrt[c + a^2*c*x^2],x]
```

output

```
(Sqrt[1 + a^2*x^2]*((x^2*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/
2))/(3*a^2) - (((1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/2)*(4 - I
*n - n^2 + a*(1 + I*n)*n*x))/(2*a^2*(1 + I*n)) + (2^(-1/2 - (I/2)*n)*n*(5
- n^2)*(1 - I*a*x)^((3 + I*n)/2)*Hypergeometric2F1[(1 + I*n)/2, (3 + I*n)/
2, (5 + I*n)/2, (1 - I*a*x)/2])/(a^2*(3*I - n)*(1 + I*n)))/(3*a^2)))/Sqrt[
c + a^2*c*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

rule 5608
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p])   Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
 m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{e^{n \arctan(ax)} x^3}{\sqrt{a^2 c\, x^2+c}}dx$$

input
```
int(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)} x^3}{\sqrt{c+a^2cx^2}}\, dx = \int \frac{x^3 e^{(n \arctan(ax))}}{\sqrt{a^2cx^2+c}}\, dx$$

input
```
integrate(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(x^3*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(ax)} x^3}{\sqrt{c+a^2cx^2}}\, dx = \int \frac{x^3 e^{n \operatorname{atan}(ax)}}{\sqrt{c(a^2x^2+1)}}\, dx$$

input
```
integrate(exp(n*atan(a*x))*x**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**3*exp(n*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{e^{n\arctan(ax)}x^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(x^3*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{n\arctan(ax)}x^3}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(exp(n*arctan(a*x))*x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}x^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3\,e^{n\,\mathrm{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^3*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^3*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{e^{n\arctan(ax)}x^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{e^{atan(ax)n}x^3}{\sqrt{a^2x^2+1}}dx}{\sqrt{c}}$$

input
```
int(exp(n*atan(a*x))*x^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int((e**(atan(a*x)*n)*x**3)/sqrt(a**2*x**2 + 1),x)/sqrt(c)
```

3.368 $\int \frac{e^{n\arctan(ax)}x^2}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 291

$$\int \frac{e^{n\arctan(ax)}x^2}{\sqrt{c+a^2cx^2}}\,dx = -\frac{(1+in)(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2a^3(i+n)\sqrt{c+a^2cx^2}} + \frac{x(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2a^2\sqrt{c+a^2cx^2}} - \frac{i2^{\frac{1}{2}-\frac{in}{2}}(1-n^2)(1-iax)^{\frac{1}{2}(1+in)}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1+in),\frac{1}{2}(1+in),\frac{1}{2}(3+in),\frac{1}{2}(1}{a^3(1+n^2)\sqrt{c+a^2cx^2}}$$

output
```
-1/2*(1+I*n)*(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(1/2-1/2*I*n)*(a^2*x^2+1)^(
1/2)/a^3/(I+n)/(a^2*c*x^2+c)^(1/2)+1/2*x*(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)
^(1/2-1/2*I*n)*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)-I*2^(1/2-1/2*I*n)
*(-n^2+1)*(1-I*a*x)^(1/2+1/2*I*n)*(a^2*x^2+1)^(1/2)*hypergeom([1/2+1/2*I*n
, -1/2+1/2*I*n],[3/2+1/2*I*n],1/2-1/2*I*a*x)/a^3/(n^2+1)/(a^2*c*x^2+c)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.71

$$\int \frac{e^{n \arctan(ax)} x^2}{\sqrt{c+a^2cx^2}} \, dx = \frac{2^{-1-\frac{in}{2}} (1-iax)^{\frac{1}{2}+\frac{in}{2}} (1+iax)^{-\frac{in}{2}} \sqrt{1+a^2x^2} \left(2^{\frac{in}{2}} (-i+n) \sqrt{1+iax} (-1+iax+n(-i+ax)) + 2i\sqrt{2}(-\right.}{a^3 (1+n^2) \sqrt{c+a^2c}}$$

input
```
Integrate[(E^(n*ArcTan[a*x])*x^2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(2^(-1 - (I/2)*n)*(1 - I*a*x)^(1/2 + (I/2)*n)*Sqrt[1 + a^2*x^2]*(2^((I/2)*
n)*(-I + n)*Sqrt[1 + I*a*x]*(-1 + I*a*x + n*(-I + a*x)) + (2*I)*Sqrt[2]*(-
1 + n^2)*(1 + I*a*x)^((I/2)*n)*Hypergeometric2F1[(1 + I*n)/2, (I/2)*(I + n
), (3 + I*n)/2, (1 - I*a*x)/2]))/(a^3*(1 + n^2)*(1 + I*a*x)^((I/2)*n)*Sqrt
[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5608, 5605, 101, 25, 88, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 e^{n \arctan(ax)}}{\sqrt{a^2cx^2+c}} \, dx$$

$$\Big\downarrow \text{5608}$$

$$\frac{\sqrt{a^2x^2+1} \int \frac{e^{n \arctan(ax)} x^2}{\sqrt{a^2x^2+1}} dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5605}$$

$$\frac{\sqrt{a^2x^2+1} \int x^2 (1-iax)^{\frac{1}{2}(in-1)} (iax+1)^{\frac{1}{2}(-in-1)} dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{101}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{\int -(1-iax)^{\frac{1}{2}(in-1)}(iax+1)^{\frac{1}{2}(-in-1)}(anx+1)dx}{2a^2}+\frac{x(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{2a^2}\right)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{x(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{2a^2}-\frac{\int (1-iax)^{\frac{1}{2}(in-1)}(iax+1)^{\frac{1}{2}(-in-1)}(anx+1)dx}{2a^2}\right)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{88}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{x(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{2a^2}-\frac{\frac{(1-n^2)\int (1-iax)^{\frac{1}{2}(in-1)}(iax+1)^{\frac{1}{2}(1-in)}dx}{1-in}+\frac{(1+in)(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{a(n+i)}}{2a^2}\right)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{79}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{x(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{2a^2}-\frac{\frac{(1+in)(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{a(n+i)}-\frac{2^{\frac{3}{2}-\frac{in}{2}}(1-n^2)(1-iax)^{\frac{1}{2}(1+in)}\text{Hypergeometric2F1}\left(\frac{1}{2}\right.}{a(-n+i)(1-in)}}{2a^2}\right.}{\sqrt{a^2cx^2+c}}$$

input
```
Int[(E^(n*ArcTan[a*x])*x^2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[1 + a^2*x^2]*((x*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/2)
)/(2*a^2) - (((1 + I*n)*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/2
))/(a*(I + n)) - (2^(3/2 - (I/2)*n)*(1 - n^2)*(1 - I*a*x)^((1 + I*n)/2)*Hy
pergeometric2F1[(-1 + I*n)/2, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/2])/(a
*(I - n)*(1 - I*n)))/(2*a^2)))/Sqrt[c + a^2*c*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

rule 5608
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p])   Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
 m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{e^{n\arctan(ax)}x^2}{\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{n\arctan(ax)}x^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(x^2*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{e^{n\arctan(ax)}x^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2e^{n\,\mathrm{atan}\,(ax)}}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(exp(n*atan(a*x))*x**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*exp(n*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{e^{n\arctan(ax)}x^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(x^2*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}x^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}x^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^2*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^2*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{e^{n\arctan(ax)}x^2}{\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{e^{atan(ax)n}x^2}{\sqrt{a^2x^2+1}}dx}{\sqrt{c}}$$

input
```
int(exp(n*atan(a*x))*x^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int((e**(atan(a*x)*n)*x**2)/sqrt(a**2*x**2 + 1),x)/sqrt(c)
```

3.369 $\int \frac{e^{n \arctan(ax)}x}{\sqrt{c+a^2cx^2}}\, dx$

Optimal result

Integrand size = 24, antiderivative size = 202

$$\int \frac{e^{n \arctan(ax)}x}{\sqrt{c+a^2cx^2}}\, dx = \frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{a^2(1-in)\sqrt{c+a^2cx^2}} - \frac{i2^{\frac{3}{2}-\frac{in}{2}}n(1-iax)^{\frac{1}{2}(1+in)}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1+in),\frac{1}{2}(1+in),\frac{1}{2}(3+in),\frac{1}{2}(1-iax)\right)}{a^2(1+n^2)\sqrt{c+a^2cx^2}}$$

output
```
(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(1/2-1/2*I*n)*(a^2*x^2+1)^(1/2)/a^2/(1-I
*n)/(a^2*c*x^2+c)^(1/2)-I*2^(3/2-1/2*I*n)*n*(1-I*a*x)^(1/2+1/2*I*n)*(a^2*x
^2+1)^(1/2)*hypergeom([1/2+1/2*I*n, -1/2+1/2*I*n],[3/2+1/2*I*n],1/2-1/2*I*
a*x)/a^2/(n^2+1)/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.87

$$\int \frac{e^{n \arctan(ax)}x}{\sqrt{c+a^2cx^2}}\, dx = \frac{(1-iax)^{\frac{1}{2}+\frac{in}{2}}(2+2iax)^{-\frac{in}{2}}\sqrt{1+a^2x^2}\left(2^{\frac{in}{2}}(1+in)\sqrt{1+iax}-2i\sqrt{2}n(1+iax)^{\frac{in}{2}}\,\text{Hypergeometric2F1}\left(\right.\right.}{a^2(1+n^2)\sqrt{c+a^2cx^2}}$$

input
```
Integrate[(E^(n*ArcTan[a*x])*x)/Sqrt[c + a^2*c*x^2],x]
```

output
```
((1 - I*a*x)^(1/2 + (I/2)*n)*Sqrt[1 + a^2*x^2]*(2^((I/2)*n)*(1 + I*n)*Sqrt
[1 + I*a*x] - (2*I)*Sqrt[2]*n*(1 + I*a*x)^((I/2)*n)*Hypergeometric2F1[(1 +
 I*n)/2, (I/2)*(I + n), (3 + I*n)/2, (1 - I*a*x)/2]))/(a^2*(1 + n^2)*(2 +
(2*I)*a*x)^((I/2)*n)*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5608, 5605, 88, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xe^{n\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5608}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{n\arctan(ax)}x}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5605}$$

$$\frac{\sqrt{a^2x^2+1}\int x(1-iax)^{\frac{1}{2}(in-1)}(iax+1)^{\frac{1}{2}(-in-1)}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{88}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{a^2(1-in)}-\frac{n\int (1-iax)^{\frac{1}{2}(in-1)}(iax+1)^{\frac{1}{2}(1-in)}dx}{a(1-in)}\right)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{79}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{2^{\frac{3}{2}-\frac{in}{2}}n(1-iax)^{\frac{1}{2}(1+in)}\text{Hypergeometric2F1}\left(\frac{1}{2}(in-1),\frac{1}{2}(in+1),\frac{1}{2}(in+3),\frac{1}{2}(1-iax)\right)}{a^2(-n+i)(1-in)}+\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{a^2(1-in)}\right)}{\sqrt{a^2cx^2+c}}$$

input
```
Int[(E^(n*ArcTan[a*x])*x)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[1 + a^2*x^2]*(((1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/2))/
(a^2*(1 - I*n)) + (2^(3/2 - (I/2)*n)*n*(1 - I*a*x)^((1 + I*n)/2)*Hypergeom
etric2F1[(-1 + I*n)/2, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/2])/(a^2*(I -
 n)*(1 - I*n))))/Sqrt[c + a^2*c*x^2]
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

rule 5608
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p])   Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
 m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{e^{n \arctan(ax)} x}{\sqrt{a^2 c x^2 + c}} dx$$

input
```
int(exp(n*arctan(a*x))*x/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(exp(n*arctan(a*x))*x/(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)} x}{\sqrt{c + a^2 c x^2}} dx = \int \frac{x e^{(n \arctan(ax))}}{\sqrt{a^2 c x^2 + c}} dx$$

input
```
integrate(exp(n*arctan(a*x))*x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(ax)} x}{\sqrt{c + a^2 c x^2}} dx = \int \frac{x e^{n \operatorname{atan}(ax)}}{\sqrt{c (a^2 x^2 + 1)}} dx$$

input
```
integrate(exp(n*atan(a*x))*x/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x*exp(n*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{e^{n\arctan(ax)}x}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{xe^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}x}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{xe^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}x}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{e^{n\arctan(ax)}x}{\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{e^{atan(ax)n}x}{\sqrt{a^2x^2+1}}dx}{\sqrt{c}}$$

input
```
int(exp(n*atan(a*x))*x/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int((e**(atan(a*x)*n)*x)/sqrt(a**2*x**2 + 1),x)/sqrt(c)
```

3.370 $\int \frac{e^{n\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 120

$$\int \frac{e^{n\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{2^{\frac{1}{2}-\frac{in}{2}}(1-iax)^{\frac{1}{2}(1+in)}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1+in),\frac{1}{2}(1+in),\frac{1}{2}(3+in),\frac{1}{2}(1-iax)\right)}{a(i-n)\sqrt{c+a^2cx^2}}$$

output
```
-2^(1/2-1/2*I*n)*(1-I*a*x)^(1/2+1/2*I*n)*(a^2*x^2+1)^(1/2)*hypergeom([1/2+
1/2*I*n, 1/2+1/2*I*n],[3/2+1/2*I*n],1/2-1/2*I*a*x)/a/(I-n)/(a^2*c*x^2+c)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

$$\int \frac{e^{n\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{2^{\frac{1}{2}-\frac{in}{2}}(1-iax)^{\frac{1}{2}+\frac{in}{2}}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2}+\frac{in}{2},\frac{1}{2}+\frac{in}{2},\frac{3}{2}+\frac{in}{2},\frac{1}{2}-\frac{iax}{2}\right)}{a(-i+n)\sqrt{c+a^2cx^2}}$$

input
```
Integrate[E^(n*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
(2^(1/2 - (I/2)*n)*(1 - I*a*x)^(1/2 + (I/2)*n)*Sqrt[1 + a^2*x^2]*Hypergeom
etric2F1[1/2 + (I/2)*n, 1/2 + (I/2)*n, 3/2 + (I/2)*n, 1/2 - (I/2)*a*x])/(a
*(-I + n)*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5599, 5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5599}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{n\arctan(ax)}}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5596}$$

$$\frac{\sqrt{a^2x^2+1}\int (1-iax)^{\frac{1}{2}(in-1)}(iax+1)^{\frac{1}{2}(-in-1)}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{2^{\frac{1}{2}-\frac{in}{2}}\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}\text{Hypergeometric2F1}\left(\frac{1}{2}(in+1),\frac{1}{2}(in+1),\frac{1}{2}(in+3),\frac{1}{2}(1-iax)\right)}{a(-n+i)\sqrt{a^2cx^2+c}}$$

input
```
Int[E^(n*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
-((2^(1/2 - (I/2)*n)*(1 - I*a*x)^((1 + I*n)/2)*Sqrt[1 + a^2*x^2]*Hypergeom
etric2F1[(1 + I*n)/2, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/2])/(a*(I - n)
*Sqrt[c + a^2*c*x^2]))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{e^{n \arctan(ax)}}{\sqrt{a^2 c\, x^2 + c}} dx$$

input
```
int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)}}{\sqrt{c+a^2cx^2}}\, dx = \int \frac{e^{(n \arctan(ax))}}{\sqrt{a^2cx^2+c}}\, dx$$

input
```
integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(ax)}}{\sqrt{c + a^2 c x^2}}\,dx = \int \frac{e^{n \operatorname{atan}(ax)}}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(exp(n*atan(a*x))/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(exp(n*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{e^{n \arctan(ax)}}{\sqrt{c + a^2 c x^2}}\,dx = \int \frac{e^{(n \arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{e^{n \arctan(ax)}}{\sqrt{c + a^2 c x^2}}\,dx = \int \frac{e^{(n \arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{e}^{n\,\mathrm{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{e^{n\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{e^{atan(ax)n}}{\sqrt{a^2x^2+1}}dx}{\sqrt{c}}$$

input
```
int(exp(n*atan(a*x))/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(e**(atan(a*x)*n)/sqrt(a**2*x**2 + 1),x)/sqrt(c)
```

3.371 $\int \frac{e^{n\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 121

$$\int \frac{e^{n\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = -\frac{2(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+in),\frac{1}{2}(3+in),\frac{1-iax}{1+iax}\right)}{(1+in)\sqrt{c+a^2cx^2}}$$

output
```
-2*(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(-1/2-1/2*I*n)*(a^2*x^2+1)^(1/2)*hype
rgeom([1, 1/2+1/2*I*n],[3/2+1/2*I*n],(1-I*a*x)/(1+I*a*x))/(1+I*n)/(a^2*c*x
^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

$$\int \frac{e^{n\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = \frac{2(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{in}{2},\frac{3}{2}+\frac{in}{2},\frac{i+ax}{i-ax}\right)}{(-1-in)\sqrt{c+a^2cx^2}}$$

input
```
Integrate[E^(n*ArcTan[a*x])/(x*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(2*(1 - I*a*x)^(1/2 + (I/2)*n)*(1 + I*a*x)^(-1/2 - (I/2)*n)*Sqrt[1 + a^2*x
^2]*Hypergeometric2F1[1, 1/2 + (I/2)*n, 3/2 + (I/2)*n, (I + a*x)/(I - a*x)
])/((-1 - I*n)*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {5608, 5605, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n \arctan(ax)}}{x\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{ 5608}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{n \arctan(ax)}}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5605}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{(1-iax)^{\frac{1}{2}(in-1)}(iax+1)^{\frac{1}{2}(-in-1)}}{x}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 141}$$

$$-\frac{2\sqrt{a^2x^2+1}(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\text{Hypergeometric2F1}\left(1,\frac{1}{2}(in+1),\frac{1}{2}(in+3),\frac{1-iax}{iax+1}\right)}{(1+in)\sqrt{a^2cx^2+c}}$$

input
```
Int[E^(n*ArcTan[a*x])/(x*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(-2*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((-1 - I*n)/2)*Sqrt[1 + a^2*x^2]
*Hypergeometric2F1[1, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/(1 + I*a*x)])/
((1 + I*n)*Sqrt[c + a^2*c*x^2])
```

Defintions of rubi rules used

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

rule 5608
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p])   Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
 m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{e^{n \arctan(ax)}}{x\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(exp(n*arctan(a*x))/x/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(exp(n*arctan(a*x))/x/(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)}}{x\sqrt{c + a^2cx^2}} \, dx = \int \frac{e^{(n \arctan(ax))}}{\sqrt{a^2cx^2 + cx}} \, dx$$

input
```
integrate(exp(n*arctan(a*x))/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*e^(n*arctan(a*x))/(a^2*c*x^3 + c*x), x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(ax)}}{x\sqrt{c + a^2cx^2}} \, dx = \int \frac{e^{n \operatorname{atan}(ax)}}{x\sqrt{c(a^2x^2 + 1)}} \, dx$$

input
```
integrate(exp(n*atan(a*x))/x/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(exp(n*atan(a*x))/(x*sqrt(c*(a**2*x**2 + 1))), x)
```

Maxima [F]

$$\int \frac{e^{n \arctan(ax)}}{x\sqrt{c + a^2cx^2}} \, dx = \int \frac{e^{(n \arctan(ax))}}{\sqrt{a^2cx^2 + cx}} \, dx$$

input
```
integrate(exp(n*arctan(a*x))/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(e^(n*arctan(a*x))/(sqrt(a^2*c*x^2 + c)*x), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx=\int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+cx}}\,dx$$

input
```
integrate(exp(n*arctan(a*x))/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(e^(n*arctan(a*x))/(sqrt(a^2*c*x^2 + c)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx=\int \frac{e^{n\,\mathrm{atan}(a\,x)}}{x\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(exp(n*atan(a*x))/(x*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(exp(n*atan(a*x))/(x*(c + a^2*c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e^{n\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx=\frac{\int \frac{e^{atan(ax)n}}{\sqrt{a^2x^2+1}\,x}dx}{\sqrt{c}}$$

input
```
int(exp(n*atan(a*x))/x/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(e**(atan(a*x)*n)/(sqrt(a**2*x**2 + 1)*x),x)/sqrt(c)
```

3.372 $\int \frac{e^{n \arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 196

$$\int \frac{e^{n\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = -\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{x\sqrt{c+a^2cx^2}} - \frac{2an(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+in),\frac{1}{2}(3+in),\frac{1-iax}{1+iax}\right)}{(1+in)\sqrt{c+a^2cx^2}}$$

output
```
-(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(1/2-1/2*I*n)*(a^2*x^2+1)^(1/2)/x/(a^2*
c*x^2+c)^(1/2)-2*a*n*(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(-1/2-1/2*I*n)*(a^2
*x^2+1)^(1/2)*hypergeom([1, 1/2+1/2*I*n],[3/2+1/2*I*n],(1-I*a*x)/(1+I*a*x)
)/(1+I*n)/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.72

$$\int \frac{e^{n\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = \frac{(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\sqrt{1+a^2x^2}\left(-((-i+n)(-i+ax))+2anx\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{in}{2},\right.\right.}{(-1-in)x\sqrt{c+a^2cx^2}}$$

input
```
Integrate[E^(n*ArcTan[a*x])/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

output
```
((1 - I*a*x)^(1/2 + (I/2)*n)*(1 + I*a*x)^(-1/2 - (I/2)*n)*Sqrt[1 + a^2*x^2
]*(-((-I + n)*(-I + a*x)) + 2*a*n*x*Hypergeometric2F1[1, 1/2 + (I/2)*n, 3/
2 + (I/2)*n, (I + a*x)/(I - a*x)]))/((-1 - I*n)*x*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5608, 5605, 107, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n \arctan(ax)}}{x^2\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5608}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{n\arctan(ax)}}{x^2\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5605}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{(1-iax)^{\frac{1}{2}(in-1)}(iax+1)^{\frac{1}{2}(-in-1)}}{x^2}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{107}$$

$$\frac{\sqrt{a^2x^2+1}\left(an\int \frac{(1-iax)^{\frac{1}{2}(in-1)}(iax+1)^{\frac{1}{2}(-in-1)}}{x}dx-\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{x}\right)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{141}$$

$$\frac{\sqrt{a^2x^2+1}\left(-\frac{2an(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\text{Hypergeometric2F1}\left(1,\frac{1}{2}(in+1),\frac{1}{2}(in+3),\frac{1-iax}{iax+1}\right)}{1+in}-\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{x}\right.}{\sqrt{a^2cx^2+c}}$$

input
```
Int[E^(n*ArcTan[a*x])/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(-(((1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/2)
)/x) - (2*a*n*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((-1 - I*n)/2)*Hyperge
ometric2F1[1, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/(1 + I*a*x)])/(1 + I*n
)))/Sqrt[c + a^2*c*x^2]
```

Defintions of rubi rules used

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

rule 5608
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p])   Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
 m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{e^{n\arctan(ax)}}{x^2\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{n\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}x^2}\,dx$$

input
```
integrate(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(a^2*c*x^2 + c)*e^(n*arctan(a*x))/(a^2*c*x^4 + c*x^2), x)
```

Sympy [F]

$$\int \frac{e^{n\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{n\operatorname{atan}(ax)}}{x^2\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(exp(n*atan(a*x))/x**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(exp(n*atan(a*x))/(x**2*sqrt(c*(a**2*x**2 + 1))), x)
```

Maxima [F]

$$\int \frac{e^{n\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+cx^2}}\,dx$$

input
```
integrate(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(e^(n*arctan(a*x))/(sqrt(a^2*c*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+cx^2}}\,dx$$

input
```
integrate(exp(n*arctan(a*x))/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(e^(n*arctan(a*x))/(sqrt(a^2*c*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{n\,\mathrm{atan}(a\,x)}}{x^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(exp(n*atan(a*x))/(x^2*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(exp(n*atan(a*x))/(x^2*(c + a^2*c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e^{n \arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{e^{atan(ax)n}}{\sqrt{a^2x^2+1}\,x^2}dx}{\sqrt{c}}$$

input
```
int(exp(n*atan(a*x))/x^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(e**(atan(a*x)*n)/(sqrt(a**2*x**2 + 1)*x**2),x)/sqrt(c)
```

3.373 $\int \frac{e^{n \arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 281

$$\int \frac{e^{n\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = -\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2x^2\sqrt{c+a^2cx^2}} - \frac{an(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}\sqrt{1+a^2x^2}}{2x\sqrt{c+a^2cx^2}} + \frac{a^2(1-n^2)(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+in),\frac{1}{2}(3+in),\frac{1-}{1-}\right.}{(1+in)\sqrt{c+a^2cx^2}}$$

output
```
-1/2*(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(1/2-1/2*I*n)*(a^2*x^2+1)^(1/2)/x^2
/(a^2*c*x^2+c)^(1/2)-1/2*a*n*(1-I*a*x)^(1/2+1/2*I*n)*(1+I*a*x)^(1/2-1/2*I*
n)*(a^2*x^2+1)^(1/2)/x/(a^2*c*x^2+c)^(1/2)+a^2*(-n^2+1)*(1-I*a*x)^(1/2+1/2
*I*n)*(1+I*a*x)^(-1/2-1/2*I*n)*(a^2*x^2+1)^(1/2)*hypergeom([1, 1/2+1/2*I*n
],[3/2+1/2*I*n],(1-I*a*x)/(1+I*a*x))/(1+I*n)/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.57

$$\int \frac{e^{n \arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = \frac{i(1-iax)^{\frac{1}{2}+\frac{in}{2}}(1+iax)^{-\frac{1}{2}-\frac{in}{2}}\sqrt{1+a^2x^2}\left(-((-i+n)(-i+ax)(1+anx))+2a^2(-1+n^2)\,x^2\,\text{Hypergeo}\right.}{2(-i+n)x^2\sqrt{c+a^2cx^2}}$$

input
```
Integrate[E^(n*ArcTan[a*x])/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

output
```
((I/2)*(1 - I*a*x)^(1/2 + (I/2)*n)*(1 + I*a*x)^(-1/2 - (I/2)*n)*Sqrt[1 + a
^2*x^2]*(-((-I + n)*(-I + a*x)*(1 + a*n*x)) + 2*a^2*(-1 + n^2)*x^2*Hyperge
ometric2F1[1, 1/2 + (I/2)*n, 3/2 + (I/2)*n, (I + a*x)/(I - a*x)]))/((-I +
n)*x^2*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.82, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5608, 5605, 144, 25, 27, 168, 27, 141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n \arctan(ax)}}{x^3\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{ 5608}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{n \arctan(ax)}}{x^3\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5605}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{(1-iax)^{\frac{1}{2}(in-1)}(iax+1)^{\frac{1}{2}(-in-1)}}{x^3}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 144}$$

$$\frac{\sqrt{a^2x^2+1}\left(-\frac{1}{2}\int -\frac{a(n-ax)(1-iax)^{\frac{1}{2}(in-1)}(iax+1)^{\frac{1}{2}(-in-1)}}{x^2}dx-\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{2x^2}\right)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{1}{2}\int \frac{a(n-ax)(1-iax)^{\frac{1}{2}(in-1)}(iax+1)^{\frac{1}{2}(-in-1)}}{x^2}dx-\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{2x^2}\right)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{1}{2}a\int \frac{(n-ax)(1-iax)^{\frac{1}{2}(in-1)}(iax+1)^{\frac{1}{2}(-in-1)}}{x^2}dx-\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{2x^2}\right)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 168$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{1}{2}a\left(-\int \frac{a(1-n^2)(1-iax)^{\frac{1}{2}(in-1)}(iax+1)^{\frac{1}{2}(-in-1)}}{x}dx-\frac{n(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{x}\right)-\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}}}{2x^2}\right.}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{1}{2}a\left(-a(1-n^2)\int \frac{(1-iax)^{\frac{1}{2}(in-1)}(iax+1)^{\frac{1}{2}(-in-1)}}{x}dx-\frac{n(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(1-in)}}{x}\right)-\frac{(1-iax)^{\frac{1}{2}(1+in)}(1+ia}{2x^2}\right.}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 141$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{1}{2}a\left(\frac{2a(1-n^2)(1-iax)^{\frac{1}{2}(1+in)}(1+iax)^{\frac{1}{2}(-1-in)}\text{Hypergeometric2F1}\left(1,\frac{1}{2}(in+1),\frac{1}{2}(in+3),\frac{1-iax}{iax+1}\right)}{1+in}-\frac{n(1-iax)^{\frac{1}{2}(1+in)}(1+ia}{x}\right.\right.}{\sqrt{a^2cx^2+c}}$$

input
```
Int[E^(n*ArcTan[a*x])/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(-1/2*((1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)
/2))/x^2 + (a*(-((n*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((1 - I*n)/2))/x
) + (2*a*(1 - n^2)*(1 - I*a*x)^((1 + I*n)/2)*(1 + I*a*x)^((-1 - I*n)/2)*Hy
pergeometric2F1[1, (1 + I*n)/2, (3 + I*n)/2, (1 - I*a*x)/(1 + I*a*x)])/(1
+ I*n)))/2))/Sqrt[c + a^2*c*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 141
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e - a*f)^(
n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f
))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e, f, m,
p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1] ||  !Su
mSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

rule 144
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> With[{mnp = Simplify[m + n + p]}, Simp[b*(a + b*x)^(m + 1)*(
c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x]
+ Simp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2))
- b*d*f*(mnp + 3)*x, x], x], x] /; ILtQ[mnp + 2, 0] && (SumSimplerQ[m, 1] |
| ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && NeQ[m, -1]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

rule 5608
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p])   Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
 m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{\mathrm{e}^{n \arctan(ax)}}{x^3 \sqrt{a^2 c\, x^2 + c}} dx$$

input
```
int(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)}}{x^3 \sqrt{c + a^2 c x^2}} \, dx = \int \frac{e^{(n \arctan(ax))}}{\sqrt{a^2 c x^2 + c} x^3} \, dx$$

input
```
integrate(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(a^2*c*x^2 + c)*e^(n*arctan(a*x))/(a^2*c*x^5 + c*x^3), x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(ax)}}{x^3 \sqrt{c + a^2 c x^2}} \, dx = \int \frac{e^{n \operatorname{atan}(ax)}}{x^3 \sqrt{c (a^2 x^2 + 1)}} \, dx$$

input
```
integrate(exp(n*atan(a*x))/x**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(exp(n*atan(a*x))/(x**3*sqrt(c*(a**2*x**2 + 1))), x)
```

Maxima [F]

$$\int \frac{e^{n\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}x^3}\,dx$$

input
```
integrate(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(e^(n*arctan(a*x))/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}x^3}\,dx$$

input
```
integrate(exp(n*arctan(a*x))/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(e^(n*arctan(a*x))/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{e^{n\,\mathrm{atan}(a\,x)}}{x^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(exp(n*atan(a*x))/(x^3*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(exp(n*atan(a*x))/(x^3*(c + a^2*c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{e^{n\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx=\frac{\int \frac{e^{atan(ax)n}}{\sqrt{a^2x^2+1}\,x^3}dx}{\sqrt{c}}$$

input
```
int(exp(n*atan(a*x))/x^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(e**(atan(a*x)*n)/(sqrt(a**2*x**2 + 1)*x**3),x)/sqrt(c)
```

3.374 $\int e^{n\arctan(ax)}\sqrt[3]{c+a^2cx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 120

$$\int e^{n\arctan(ax)}\sqrt[3]{c+a^2cx^2}\,dx = -\frac{3\,2^{\frac{4}{3}-\frac{in}{2}}(1-iax)^{\frac{1}{6}(8+3in)}\sqrt[3]{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{6}(-2+3in),\frac{1}{6}(8+3in),\frac{1}{6}(14+3in),\frac{1}{2}(1-\right.}{a(8i-3n)\sqrt[3]{1+a^2x^2}}$$

output
```
-3*2^(4/3-1/2*I*n)*(1-I*a*x)^(4/3+1/2*I*n)*(a^2*c*x^2+c)^(1/3)*hypergeom([
4/3+1/2*I*n, -1/3+1/2*I*n],[7/3+1/2*I*n],1/2-1/2*I*a*x)/a/(8*I-3*n)/(a^2*x
^2+1)^(1/3)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00

$$\int e^{n\arctan(ax)}\sqrt[3]{c+a^2cx^2}\,dx = \frac{3\,2^{\frac{4}{3}-\frac{in}{2}}(1-iax)^{\frac{4}{3}+\frac{in}{2}}\sqrt[3]{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{3}+\frac{in}{2},\frac{4}{3}+\frac{in}{2},\frac{7}{3}+\frac{in}{2},\frac{1}{2}-\frac{iax}{2}\right)}{a(-8i+3n)\sqrt[3]{1+a^2x^2}}$$

input
```
Integrate[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^(1/3),x]
```

output
```
(3*2^(4/3 - (I/2)*n)*(1 - I*a*x)^(4/3 + (I/2)*n)*(c + a^2*c*x^2)^(1/3)*Hyp
ergeometric2F1[-1/3 + (I/2)*n, 4/3 + (I/2)*n, 7/3 + (I/2)*n, 1/2 - (I/2)*a
*x])/(a*(-8*I + 3*n)*(1 + a^2*x^2)^(1/3))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5599, 5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a^2cx^2+c}e^{n\arctan(ax)}\,dx$$

$$\Big\downarrow \text{ 5599}$$

$$\frac{\sqrt[3]{a^2cx^2+c}\int e^{n\arctan(ax)}\sqrt[3]{a^2x^2+1}dx}{\sqrt[3]{a^2x^2+1}}$$

$$\Big\downarrow \text{ 5596}$$

$$\frac{\sqrt[3]{a^2cx^2+c}\int (1-iax)^{\frac{1}{6}(3in+2)}(iax+1)^{\frac{1}{6}(2-3in)}dx}{\sqrt[3]{a^2x^2+1}}$$

$$\Big\downarrow \text{ 79}$$

$$-\frac{3\,2^{\frac{4}{3}-\frac{in}{2}}\sqrt[3]{a^2cx^2+c}(1-iax)^{\frac{1}{6}(8+3in)}\text{Hypergeometric2F1}\left(\frac{1}{6}(3in-2),\frac{1}{6}(3in+8),\frac{1}{6}(3in+14),\frac{1}{2}(1-iax)\right)}{a(-3n+8i)\sqrt[3]{a^2x^2+1}}$$

input
```
Int[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^(1/3),x]
```

output
```
(-3*2^(4/3 - (I/2)*n)*(1 - I*a*x)^((8 + (3*I)*n)/6)*(c + a^2*c*x^2)^(1/3)*
Hypergeometric2F1[(-2 + (3*I)*n)/6, (8 + (3*I)*n)/6, (14 + (3*I)*n)/6, (1
- I*a*x)/2])/(a*(8*I - 3*n)*(1 + a^2*x^2)^(1/3))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \mathrm{e}^{n \arctan(ax)} \left(a^2 c\, x^2 + c\right)^{\frac{1}{3}} dx$$

input
```
int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/3),x)
```

output
```
int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/3),x)
```

Fricas [F]

$$\int e^{n \arctan(ax)} \sqrt[3]{c + a^2 c x^2}\, dx = \int \left(a^2 c x^2 + c\right)^{\frac{1}{3}} e^{\left(n \arctan(ax)\right)}\, dx$$

input
```
integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/3),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(1/3)*e^(n*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{n\arctan(ax)}\sqrt[3]{c+a^2cx^2}\,dx=\int\sqrt[3]{c\,(a^2x^2+1)}e^{n\,\mathrm{atan}\,(ax)}\,dx$$

input
```
integrate(exp(n*atan(a*x))*(a**2*c*x**2+c)**(1/3),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(1/3)*exp(n*atan(a*x)), x)
```

Maxima [F]

$$\int e^{n\arctan(ax)}\sqrt[3]{c+a^2cx^2}\,dx=\int\left(a^2cx^2+c\right)^{\frac{1}{3}}e^{(n\arctan(ax))}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/3),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(1/3)*e^(n*arctan(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int e^{n\arctan(ax)}\sqrt[3]{c+a^2cx^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^(1/3),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(ax)}\sqrt[3]{c+a^2cx^2}\,dx=\int e^{n\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^{1/3}\,dx$$

input
```
int(exp(n*atan(a*x))*(c + a^2*c*x^2)^(1/3),x)
```

output
```
int(exp(n*atan(a*x))*(c + a^2*c*x^2)^(1/3), x)
```

Reduce [F]

$$\int e^{n\arctan(ax)}\sqrt[3]{c+a^2cx^2}\,dx=c^{\frac{1}{3}}\left(\int e^{atan(ax)n}\left(a^2x^2+1\right)^{\frac{1}{3}}dx\right)$$

input
```
int(exp(n*atan(a*x))*(a^2*c*x^2+c)^(1/3),x)
```

output
```
c**(1/3)*int(e**(atan(a*x)*n)*(a**2*x**2 + 1)**(1/3),x)
```

3.375 $\int \frac{e^{n\arctan(ax)}}{\sqrt[3]{c+a^2cx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 120

$$\int \frac{e^{n\arctan(ax)}}{\sqrt[3]{c+a^2cx^2}}\,dx = -\frac{3\ 2^{\frac{2}{3}-\frac{in}{2}}(1-iax)^{\frac{1}{6}(4+3in)}\sqrt[3]{1+a^2x^2}\ \text{Hypergeometric2F1}\left(\frac{1}{6}(2+3in),\frac{1}{6}(4+3in),\frac{1}{6}(10+3in),\frac{1}{2}(1-i\right.}{a(4i-3n)\sqrt[3]{c+a^2cx^2}}$$

output
```
-3*2^(2/3-1/2*I*n)*(1-I*a*x)^(2/3+1/2*I*n)*(a^2*x^2+1)^(1/3)*hypergeom([2/
3+1/2*I*n, 1/3+1/2*I*n],[5/3+1/2*I*n],1/2-1/2*I*a*x)/a/(4*I-3*n)/(a^2*c*x^
2+c)^(1/3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00

$$\int \frac{e^{n\arctan(ax)}}{\sqrt[3]{c+a^2cx^2}}\,dx = \frac{3\ 2^{\frac{2}{3}-\frac{in}{2}}(1-iax)^{\frac{2}{3}+\frac{in}{2}}\sqrt[3]{1+a^2x^2}\ \text{Hypergeometric2F1}\left(\frac{1}{3}+\frac{in}{2},\frac{2}{3}+\frac{in}{2},\frac{5}{3}+\frac{in}{2},\frac{1}{2}-\frac{iax}{2}\right)}{a(-4i+3n)\sqrt[3]{c+a^2cx^2}}$$

input
```
Integrate[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^(1/3),x]
```

output
```
(3*2^(2/3 - (I/2)*n)*(1 - I*a*x)^(2/3 + (I/2)*n)*(1 + a^2*x^2)^(1/3)*Hyper
geometric2F1[1/3 + (I/2)*n, 2/3 + (I/2)*n, 5/3 + (I/2)*n, 1/2 - (I/2)*a*x]
)/(a*(-4*I + 3*n)*(c + a^2*c*x^2)^(1/3))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5599, 5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\arctan(ax)}}{\sqrt[3]{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5599}$$

$$\frac{\sqrt[3]{a^2x^2+1}\int \frac{e^{n\arctan(ax)}}{\sqrt[3]{a^2x^2+1}}dx}{\sqrt[3]{a^2cx^2+c}}$$

$$\Big\downarrow \text{5596}$$

$$\frac{\sqrt[3]{a^2x^2+1}\int (1-iax)^{\frac{1}{6}(3in-2)}(iax+1)^{\frac{1}{6}(-3in-2)}dx}{\sqrt[3]{a^2cx^2+c}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{3\,2^{\frac{2}{3}-\frac{in}{2}}\sqrt[3]{a^2x^2+1}(1-iax)^{\frac{1}{6}(4+3in)}\text{Hypergeometric2F1}\left(\frac{1}{6}(3in+2),\frac{1}{6}(3in+4),\frac{1}{6}(3in+10),\frac{1}{2}(1-iax)\right)}{a(-3n+4i)\sqrt[3]{a^2cx^2+c}}$$

input
```
Int[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^(1/3),x]
```

output
```
(-3*2^(2/3 - (I/2)*n)*(1 - I*a*x)^((4 + (3*I)*n)/6)*(1 + a^2*x^2)^(1/3)*Hy
pergeometric2F1[(2 + (3*I)*n)/6, (4 + (3*I)*n)/6, (10 + (3*I)*n)/6, (1 - I
*a*x)/2])/(a*(4*I - 3*n)*(c + a^2*c*x^2)^(1/3))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{e^{n \arctan(ax)}}{(a^2c\,x^2+c)^{\frac{1}{3}}}dx$$

input
```
int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/3),x)
```

output
```
int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/3),x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)}}{\sqrt[3]{c + a^2 c x^2}} \, dx = \int \frac{e^{(n \arctan(ax))}}{(a^2 c x^2 + c)^{\frac{1}{3}}} \, dx$$

input
```
integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/3),x, algorithm="fricas")
```

output
```
integral(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(1/3), x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(ax)}}{\sqrt[3]{c + a^2 c x^2}} \, dx = \int \frac{e^{n \operatorname{atan}(ax)}}{\sqrt[3]{c (a^2 x^2 + 1)}} \, dx$$

input
```
integrate(exp(n*atan(a*x))/(a**2*c*x**2+c)**(1/3),x)
```

output
```
Integral(exp(n*atan(a*x))/(c*(a**2*x**2 + 1))**(1/3), x)
```

Maxima [F]

$$\int \frac{e^{n \arctan(ax)}}{\sqrt[3]{c + a^2 c x^2}} \, dx = \int \frac{e^{(n \arctan(ax))}}{(a^2 c x^2 + c)^{\frac{1}{3}}} \, dx$$

input
```
integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/3),x, algorithm="maxima")
```

output
```
integrate(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(1/3), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}}{\sqrt[3]{c+a^2cx^2}}\,dx = \int \frac{e^{(n\arctan(ax))}}{(a^2cx^2+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(1/3),x, algorithm="giac")
```

output
```
integrate(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}}{\sqrt[3]{c+a^2cx^2}}\,dx = \int \frac{e^{n\,\mathrm{atan}(a\,x)}}{(c\,a^2\,x^2+c)^{1/3}}\,dx$$

input
```
int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(1/3),x)
```

output
```
int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(1/3), x)
```

Reduce [F]

$$\int \frac{e^{n\arctan(ax)}}{\sqrt[3]{c+a^2cx^2}}\,dx = \frac{\int \frac{e^{atan(ax)n}}{(a^2x^2+1)^{\frac{1}{3}}}dx}{c^{\frac{1}{3}}}$$

input
```
int(exp(n*atan(a*x))/(a^2*c*x^2+c)^(1/3),x)
```

output
```
int(e**(atan(a*x)*n)/(a**2*x**2 + 1)**(1/3),x)/c**(1/3)
```

3.376 $\int \frac{e^{n \arctan(ax)}}{(c+a^2cx^2)^{2/3}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 120

$$\int \frac{e^{n \arctan(ax)}}{(c+a^2cx^2)^{2/3}} \, dx = -\frac{3\, 2^{\frac{1}{3}-\frac{in}{2}} (1-iax)^{\frac{1}{6}(2+3in)} (1+a^2x^2)^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{6}(2+3in), \frac{1}{6}(4+3in), \frac{1}{6}(8+3in), \frac{1}{2}(1-iax\right.}{a(2i-3n)(c+a^2cx^2)^{2/3}}$$

output
```
-3*2^(1/3-1/2*I*n)*(1-I*a*x)^(1/3+1/2*I*n)*(a^2*x^2+1)^(2/3)*hypergeom([2/
3+1/2*I*n, 1/3+1/2*I*n],[4/3+1/2*I*n],1/2-1/2*I*a*x)/a/(2*I-3*n)/(a^2*c*x^
2+c)^(2/3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00

$$\int \frac{e^{n \arctan(ax)}}{(c+a^2cx^2)^{2/3}} \, dx = \frac{3\, 2^{\frac{1}{3}-\frac{in}{2}} (1-iax)^{\frac{1}{3}+\frac{in}{2}} (1+a^2x^2)^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{3}+\frac{in}{2}, \frac{2}{3}+\frac{in}{2}, \frac{4}{3}+\frac{in}{2}, \frac{1}{2}-\right.}{a(-2i+3n)(c+a^2cx^2)^{2/3}}$$

input
```
Integrate[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^(2/3),x]
```

output
```
(3*2^(1/3 - (I/2)*n)*(1 - I*a*x)^(1/3 + (I/2)*n)*(1 + a^2*x^2)^(2/3)*Hyper
geometric2F1[1/3 + (I/2)*n, 2/3 + (I/2)*n, 4/3 + (I/2)*n, 1/2 - (I/2)*a*x]
)/(a*(-2*I + 3*n)*(c + a^2*c*x^2)^(2/3))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5599, 5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\arctan(ax)}}{(a^2cx^2+c)^{2/3}}dx$$

$$\Big\downarrow \text{ 5599}$$

$$\frac{(a^2x^2+1)^{2/3}\int \frac{e^{n\arctan(ax)}}{(a^2x^2+1)^{2/3}}dx}{(a^2cx^2+c)^{2/3}}$$

$$\Big\downarrow \text{ 5596}$$

$$\frac{(a^2x^2+1)^{2/3}\int (1-iax)^{\frac{1}{6}(3in-4)}(iax+1)^{\frac{1}{6}(-3in-4)}dx}{(a^2cx^2+c)^{2/3}}$$

$$\Big\downarrow \text{ 79}$$

$$-\frac{3\ 2^{\frac{1}{3}-\frac{in}{2}}(a^2x^2+1)^{2/3}(1-iax)^{\frac{1}{6}(2+3in)}\text{Hypergeometric2F1}\left(\frac{1}{6}(3in+2),\frac{1}{6}(3in+4),\frac{1}{6}(3in+8),\frac{1}{2}(1-iax)\right)}{a(-3n+2i)(a^2cx^2+c)^{2/3}}$$

input
```
Int[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^(2/3),x]
```

output
```
(-3*2^(1/3 - (I/2)*n)*(1 - I*a*x)^((2 + (3*I)*n)/6)*(1 + a^2*x^2)^(2/3)*Hy
pergeometric2F1[(2 + (3*I)*n)/6, (4 + (3*I)*n)/6, (8 + (3*I)*n)/6, (1 - I*
a*x)/2])/(a*(2*I - 3*n)*(c + a^2*c*x^2)^(2/3))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{e^{n \arctan(ax)}}{(a^2c\,x^2+c)^{\frac{2}{3}}}dx$$

input
```
int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(2/3),x)
```

output
```
int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(2/3),x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)}}{(c+a^2cx^2)^{2/3}}\,dx = \int \frac{e^{(n \arctan(ax))}}{(a^2cx^2+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(2/3),x, algorithm="fricas")
```

output
```
integral(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(2/3), x)
```

Sympy [F]

$$\int \frac{e^{n\arctan(ax)}}{(c+a^2cx^2)^{2/3}}\,dx = \int \frac{e^{n\,\mathrm{atan}\,(ax)}}{(c\,(a^2x^2+1))^{\frac{2}{3}}}\,dx$$

input
```
integrate(exp(n*atan(a*x))/(a**2*c*x**2+c)**(2/3),x)
```

output
```
Integral(exp(n*atan(a*x))/(c*(a**2*x**2 + 1))**(2/3), x)
```

Maxima [F]

$$\int \frac{e^{n\arctan(ax)}}{(c+a^2cx^2)^{2/3}}\,dx = \int \frac{e^{(n\arctan(ax))}}{(a^2cx^2+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(2/3),x, algorithm="maxima")
```

output
```
integrate(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(2/3), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}}{(c+a^2cx^2)^{2/3}}\,dx = \int \frac{e^{(n\arctan(ax))}}{(a^2cx^2+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(2/3),x, algorithm="giac")
```

output
```
integrate(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}}{(c+a^2cx^2)^{2/3}}\,dx = \int \frac{e^{n\,\mathrm{atan}(a\,x)}}{(c\,a^2\,x^2+c)^{2/3}}\,dx$$

input
```
int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(2/3),x)
```

output
```
int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(2/3), x)
```

Reduce [F]

$$\int \frac{e^{n\arctan(ax)}}{(c+a^2cx^2)^{2/3}}\,dx = \frac{\int \frac{e^{atan(ax)n}}{(a^2x^2+1)^{\frac{2}{3}}}dx}{c^{\frac{2}{3}}}$$

input
```
int(exp(n*atan(a*x))/(a^2*c*x^2+c)^(2/3),x)
```

output
```
int(e**(atan(a*x)*n)/(a**2*x**2 + 1)**(2/3),x)/c**(2/3)
```

3.377 $\int \frac{e^{n\arctan(ax)}}{(c+a^2cx^2)^{4/3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 123

$$\int \frac{e^{n\arctan(ax)}}{(c+a^2cx^2)^{4/3}}\,dx = \frac{3\,2^{-\frac{1}{3}-\frac{in}{2}}(1-iax)^{\frac{1}{6}(-2+3in)}\sqrt[3]{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{6}(-2+3in),\frac{1}{6}(8+3in\right.}{ac(2i+3n)\sqrt[3]{c+a^2cx^2}}$$

output
```
3*2^(-1/3-1/2*I*n)*(1-I*a*x)^(-1/3+1/2*I*n)*(a^2*x^2+1)^(1/3)*hypergeom([4
/3+1/2*I*n, -1/3+1/2*I*n],[2/3+1/2*I*n],1/2-1/2*I*a*x)/a/c/(2*I+3*n)/(a^2*
c*x^2+c)^(1/3)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00

$$\int \frac{e^{n\arctan(ax)}}{(c+a^2cx^2)^{4/3}}\,dx = \frac{3\,2^{-\frac{1}{3}-\frac{in}{2}}(1-iax)^{-\frac{1}{3}+\frac{in}{2}}\sqrt[3]{1+a^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{3}+\frac{in}{2},\frac{4}{3}+\frac{in}{2},\frac{2}{3}+\frac{in}{2},\frac{1}{2}\right.}{ac(2i+3n)\sqrt[3]{c+a^2cx^2}}$$

input
```
Integrate[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^(4/3),x]
```

output
```
(3*2^(-1/3 - (I/2)*n)*(1 - I*a*x)^(-1/3 + (I/2)*n)*(1 + a^2*x^2)^(1/3)*Hyp
ergeometric2F1[-1/3 + (I/2)*n, 4/3 + (I/2)*n, 2/3 + (I/2)*n, 1/2 - (I/2)*a
*x])/(a*c*(2*I + 3*n)*(c + a^2*c*x^2)^(1/3))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5599, 5596, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{n\arctan(ax)}}{(a^2cx^2+c)^{4/3}}dx$$
$$\Big\downarrow \text{5599}$$
$$\frac{\sqrt[3]{a^2x^2+1}\int \frac{e^{n\arctan(ax)}}{(a^2x^2+1)^{4/3}}dx}{c\sqrt[3]{a^2cx^2+c}}$$
$$\Big\downarrow \text{5596}$$
$$\frac{\sqrt[3]{a^2x^2+1}\int (1-iax)^{\frac{1}{6}(3in-8)}(iax+1)^{\frac{1}{6}(-3in-8)}dx}{c\sqrt[3]{a^2cx^2+c}}$$
$$\Big\downarrow \text{79}$$
$$\frac{3\ 2^{-\frac{1}{3}-\frac{in}{2}}\sqrt[3]{a^2x^2+1}(1-iax)^{\frac{1}{6}(-2+3in)}\text{Hypergeometric2F1}\left(\frac{1}{6}(3in-2),\frac{1}{6}(3in+8),\frac{1}{6}(3in+4),\frac{1}{2}(1-iax)\right)}{ac(3n+2i)\sqrt[3]{a^2cx^2+c}}$$

input
```
Int[E^(n*ArcTan[a*x])/(c + a^2*c*x^2)^(4/3),x]
```

output
```
(3*2^(-1/3 - (I/2)*n)*(1 - I*a*x)^((-2 + (3*I)*n)/6)*(1 + a^2*x^2)^(1/3)*H
ypergeometric2F1[(-2 + (3*I)*n)/6, (8 + (3*I)*n)/6, (4 + (3*I)*n)/6, (1 -
I*a*x)/2])/(a*c*(2*I + 3*n)*(c + a^2*c*x^2)^(1/3))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{e^{n \arctan(ax)}}{(a^2 c x^2 + c)^{\frac{4}{3}}} dx$$

input
```
int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(4/3),x)
```

output
```
int(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(4/3),x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)}}{(c + a^2 c x^2)^{4/3}} dx = \int \frac{e^{(n \arctan(ax))}}{(a^2 c x^2 + c)^{\frac{4}{3}}} dx$$

input
```
integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(4/3),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(2/3)*e^(n*arctan(a*x))/(a^4*c^2*x^4 + 2*a^2*c^2*
x^2 + c^2), x)
```

Sympy [F]

$$\int \frac{e^{n\arctan(ax)}}{(c+a^2cx^2)^{4/3}}\,dx = \int \frac{e^{n\,\mathrm{atan}\,(ax)}}{(c\,(a^2x^2+1))^{\frac{4}{3}}}\,dx$$

input
```
integrate(exp(n*atan(a*x))/(a**2*c*x**2+c)**(4/3),x)
```

output
```
Integral(exp(n*atan(a*x))/(c*(a**2*x**2 + 1))**(4/3), x)
```

Maxima [F]

$$\int \frac{e^{n\arctan(ax)}}{(c+a^2cx^2)^{4/3}}\,dx = \int \frac{e^{(n\arctan(ax))}}{(a^2cx^2+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(4/3),x, algorithm="maxima")
```

output
```
integrate(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(4/3), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}}{(c+a^2cx^2)^{4/3}}\,dx = \int \frac{e^{(n\arctan(ax))}}{(a^2cx^2+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate(exp(n*arctan(a*x))/(a^2*c*x^2+c)^(4/3),x, algorithm="giac")
```

output
```
integrate(e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}}{(c+a^2cx^2)^{4/3}}\,dx = \int \frac{e^{n\,\mathrm{atan}(a\,x)}}{(c\,a^2\,x^2+c)^{4/3}}\,dx$$

input
```
int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(4/3),x)
```

output
```
int(exp(n*atan(a*x))/(c + a^2*c*x^2)^(4/3), x)
```

Reduce [F]

$$\int \frac{e^{n\arctan(ax)}}{(c+a^2cx^2)^{4/3}}\,dx = \frac{\int \frac{e^{atan(ax)n}}{(a^2x^2+1)^{\frac{1}{3}}a^2x^2+(a^2x^2+1)^{\frac{1}{3}}}dx}{c^{\frac{4}{3}}}$$

input
```
int(exp(n*atan(a*x))/(a^2*c*x^2+c)^(4/3),x)
```

output
```
int(e**(atan(a*x)*n)/((a**2*x**2 + 1)**(1/3)*a**2*x**2 + (a**2*x**2 + 1)**
(1/3)),x)/(c**(1/3)*c)
```

3.378 $\int e^{n\arctan(ax)}x^m(c+a^2cx^2)\ dx$

Optimal result

Integrand size = 22, antiderivative size = 49

$$\int e^{n\arctan(ax)}x^m(c+a^2cx^2)\ dx = \frac{cx^{1+m}\operatorname{AppellF1}\left(1+m,-1-\frac{in}{2},-1+\frac{in}{2},2+m,iax,-iax\right)}{1+m}$$

output
```
c*x^(1+m)*AppellF1(1+m,-1+1/2*I*n,-1-1/2*I*n,2+m,-I*a*x,I*a*x)/(1+m)
```

Mathematica [F]

$$\int e^{n\arctan(ax)}x^m(c+a^2cx^2)\ dx = \int e^{n\arctan(ax)}x^m(c+a^2cx^2)\ dx$$

input
```
Integrate[E^(n*ArcTan[a*x])*x^m*(c + a^2*c*x^2),x]
```

output
```
Integrate[E^(n*ArcTan[a*x])*x^m*(c + a^2*c*x^2), x]
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5605, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a^2 c x^2+c\right) e^{n \arctan (a x)} \, dx$$

$$\Big\downarrow \text{5605}$$

$$c \int x^m (1-i a x)^{\frac{i n}{2}+1} (i a x+1)^{1-\frac{i n}{2}}dx$$

$$\Big\downarrow \text{150}$$

$$\frac{c x^{m+1} \text{AppellF1}\left(m+1,-\frac{i n}{2}-1,\frac{i n}{2}-1,m+2,i a x,-i a x\right)}{m+1}$$

input
```
Int[E^(n*ArcTan[a*x])*x^m*(c + a^2*c*x^2),x]
```

output
```
(c*x^(1 + m)*AppellF1[1 + m, -1 - (I/2)*n, -1 + (I/2)*n, 2 + m, I*a*x, (-I
)*a*x])/(1 + m)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

Maple [F]

$$\int e^{n\arctan(ax)}x^m(a^2c\,x^2+c)\,dx$$

input
```
int(exp(n*arctan(a*x))*x^m*(a^2*c*x^2+c),x)
```

output
```
int(exp(n*arctan(a*x))*x^m*(a^2*c*x^2+c),x)
```

Fricas [F]

$$\int e^{n\arctan(ax)}x^m\left(c+a^2cx^2\right)\,dx=\int (a^2cx^2+c)x^me^{(n\arctan(ax))}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x^m*(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*x^m*e^(n*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{n\arctan(ax)}x^m\left(c+a^2cx^2\right)\,dx=c\left(\int x^me^{n\operatorname{atan}(ax)}\,dx+\int a^2x^2x^me^{n\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate(exp(n*atan(a*x))*x**m*(a**2*c*x**2+c),x)
```

output
```
c*(Integral(x**m*exp(n*atan(a*x)), x) + Integral(a**2*x**2*x**m*exp(n*atan
(a*x)), x))
```

Maxima [F]

$$\int e^{n\arctan(ax)}x^m(c+a^2cx^2)\,dx=\int (a^2cx^2+c)x^me^{(n\arctan(ax))}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x^m*(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)*x^m*e^(n*arctan(a*x)), x)
```

Giac [F]

$$\int e^{n\arctan(ax)}x^m(c+a^2cx^2)\,dx=\int (a^2cx^2+c)x^me^{(n\arctan(ax))}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x^m*(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x^m*e^(n*arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(ax)}x^m(c+a^2cx^2)\,dx=\int x^m\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(x^m*exp(n*atan(a*x))*(c + a^2*c*x^2),x)
```

output
```
int(x^m*exp(n*atan(a*x))*(c + a^2*c*x^2), x)
```

Reduce [F]

$$\int e^{n\arctan(ax)}x^m(c+a^2cx^2)\,dx = c\left(\left(\int x^m e^{atan(ax)n}x^2dx\right)a^2+\int x^m e^{atan(ax)n}dx\right)$$

input
```
int(exp(n*atan(a*x))*x^m*(a^2*c*x^2+c),x)
```

output
```
c*(int(x**m*e**(atan(a*x)*n)*x**2,x)*a**2 + int(x**m*e**(atan(a*x)*n),x))
```

3.379 $\int \frac{e^{n \arctan(ax)} x^m}{c+a^2 c x^2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{e^{n \arctan(ax)} x^m}{c+a^2 c x^2} \, dx = \frac{x^{1+m} \operatorname{AppellF1}\left(1+m, 1-\frac{in}{2}, 1+\frac{in}{2}, 2+m, iax, -iax\right)}{c(1+m)}$$

output
```
x^(1+m)*AppellF1(1+m,1+1/2*I*n,1-1/2*I*n,2+m,-I*a*x,I*a*x)/c/(1+m)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.88

$$\int \frac{e^{n \arctan(ax)} x^m}{c+a^2 c x^2} \, dx = \frac{e^{n \arctan(ax)} \left(1-e^{2i \arctan(ax)}\right)^{-m} \left(1+e^{2i \arctan(ax)}\right)^m x^m \operatorname{AppellF1}\left(-\frac{in}{2}, m, -m, 1-\frac{in}{2}, -e^{2i \arctan(ax)}, e^{2i \arc}\right.}{acn}$$

input
```
Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2),x]
```

output
```
(E^(n*ArcTan[a*x])*(1 + E^((2*I)*ArcTan[a*x]))^m*x^m*AppellF1[(-1/2*I)*n,
m, -m, 1 - (I/2)*n, -E^((2*I)*ArcTan[a*x]), E^((2*I)*ArcTan[a*x])])/(a*c*(
1 - E^((2*I)*ArcTan[a*x]))^m*n)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {5605, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m e^{n \arctan(ax)}}{a^2cx^2+c} \, dx$$
$$\Big\downarrow \text{ 5605}$$
$$\frac{\int x^m (1-iax)^{\frac{in}{2}-1}(iax+1)^{-\frac{in}{2}-1}dx}{c}$$
$$\Big\downarrow \text{ 150}$$
$$\frac{x^{m+1} \operatorname{AppellF1}\left(m+1,1-\frac{in}{2},\frac{in}{2}+1,m+2,iax,-iax\right)}{c(m+1)}$$

input
```
Int[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2),x]
```

output
```
(x^(1 + m)*AppellF1[1 + m, 1 - (I/2)*n, 1 + (I/2)*n, 2 + m, I*a*x, (-I)*a*
x])/(c*(1 + m))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{\mathrm{e}^{n \arctan(ax)} x^m}{a^2 c\, x^2 + c} dx$$

input
```
int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c),x)
```

output
```
int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c),x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)} x^m}{c + a^2 c x^2} \, dx = \int \frac{x^m e^{(n \arctan(ax))}}{a^2 c x^2 + c} \, dx$$

input
```
integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(ax)} x^m}{c + a^2 c x^2} \, dx = \frac{\int \frac{x^m e^{n \operatorname{atan}(ax)}}{a^2 x^2 + 1} \, dx}{c}$$

input
```
integrate(exp(n*atan(a*x))*x**m/(a**2*c*x**2+c),x)
```

output
```
Integral(x**m*exp(n*atan(a*x))/(a**2*x**2 + 1), x)/c
```

Maxima [F]

$$\int \frac{e^{n \arctan(ax)}x^m}{c + a^2cx^2}\,dx = \int \frac{x^m e^{(n \arctan(ax))}}{a^2cx^2 + c}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{e^{n \arctan(ax)}x^m}{c + a^2cx^2}\,dx = \int \frac{x^m e^{(n \arctan(ax))}}{a^2cx^2 + c}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n \arctan(ax)}x^m}{c + a^2cx^2}\,dx = \int \frac{x^m\,e^{n\,\mathrm{atan}(a\,x)}}{c\,a^2\,x^2 + c}\,dx$$

input
```
int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2),x)
```

output
```
int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2), x)
```

Reduce [F]

$$\int \frac{e^{n \arctan(ax)} x^m}{c + a^2 c x^2} \, dx = \frac{x^m e^{atan(ax)n} - \left(\int \frac{x^m e^{atan(ax)n}}{x} dx\right) m}{acn}$$

input
```
int(exp(n*atan(a*x))*x^m/(a^2*c*x^2+c),x)
```

output
```
(x**m*e**(atan(a*x)*n) - int((x**m*e**(atan(a*x)*n))/x,x)*m)/(a*c*n)
```

3.380 $\int \frac{e^{n \arctan(ax)} x^m}{(c+a^2cx^2)^2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{e^{n \arctan(ax)} x^m}{(c+a^2cx^2)^2} \, dx = \frac{x^{1+m} \operatorname{AppellF1}\left(1+m, 2-\frac{in}{2}, 2+\frac{in}{2}, 2+m, iax, -iax\right)}{c^2(1+m)}$$

output
```
x^(1+m)*AppellF1(1+m,2+1/2*I*n,2-1/2*I*n,2+m,-I*a*x,I*a*x)/c^2/(1+m)
```

Mathematica [F]

$$\int \frac{e^{n \arctan(ax)} x^m}{(c+a^2cx^2)^2} \, dx = \int \frac{e^{n \arctan(ax)} x^m}{(c+a^2cx^2)^2} \, dx$$

input
```
Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^2,x]
```

output
```
Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^2, x]
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5605, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m e^{n \arctan(ax)}}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{ 5605}$$

$$\frac{\int x^m(1-iax)^{\frac{in}{2}-2}(iax+1)^{-\frac{in}{2}-2}dx}{c^2}$$

$$\Big\downarrow \text{ 150}$$

$$\frac{x^{m+1}\text{AppellF1}\left(m+1,2-\frac{in}{2},\frac{in}{2}+2,m+2,iax,-iax\right)}{c^2(m+1)}$$

input
```
Int[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^2,x]
```

output
```
(x^(1 + m)*AppellF1[1 + m, 2 - (I/2)*n, 2 + (I/2)*n, 2 + m, I*a*x, (-I)*a*
x])/(c^2*(1 + m))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{\mathrm{e}^{n \arctan(ax)} x^m}{\left(a^2 c\, x^2 + c\right)^2} dx$$

input
```
int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^2,x)
```

output
```
int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^2,x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)} x^m}{\left(c + a^2 c x^2\right)^2} \, dx = \int \frac{x^m e^{(n \arctan(ax))}}{\left(a^2 c x^2 + c\right)^2} \, dx$$

input
```
integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(x^m*e^(n*arctan(a*x))/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(ax)} x^m}{\left(c + a^2 c x^2\right)^2} \, dx = \frac{\int \frac{x^m e^{n \operatorname{atan}(ax)}}{a^4 x^4 + 2a^2 x^2 + 1} \, dx}{c^2}$$

input
```
integrate(exp(n*atan(a*x))*x**m/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**m*exp(n*atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

Maxima [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m e^{(n\arctan(ax))}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,1,0,0]%%%} / %%%{1,[0,0,0,1,2]%%%} Error: Bad Argum
ent Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m\,e^{n\,\mathrm{atan}(a\,x)}}{(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^2, x)
```

Reduce [F]

$$\int \frac{e^{n \arctan(ax)} x^m}{(c + a^2 c x^2)^2} \, dx$$
$$= \frac{x^m e^{atan(ax)n} - \left(\int \frac{x^m e^{atan(ax)n} x}{a^4 x^4 + 2a^2 x^2 + 1} dx\right) a^4 m\, x^2 + 2\left(\int \frac{x^m e^{atan(ax)n} x}{a^4 x^4 + 2a^2 x^2 + 1} dx\right) a^4 x^2 - \left(\int \frac{x^m e^{atan(ax)n} x}{a^4 x^4 + 2a^2 x^2 + 1} dx\right) a^2 m + 2\Big(\int}{a\, c^2 n\, (a^2 x^2 + 1)}$$

input
```
int(exp(n*atan(a*x))*x^m/(a^2*c*x^2+c)^2,x)
```

output
```
(x**m*e**(atan(a*x)*n) - int((x**m*e**(atan(a*x)*n)*x)/(a**4*x**4 + 2*a**2
*x**2 + 1),x)*a**4*m*x**2 + 2*int((x**m*e**(atan(a*x)*n)*x)/(a**4*x**4 + 2
*a**2*x**2 + 1),x)*a**4*x**2 - int((x**m*e**(atan(a*x)*n)*x)/(a**4*x**4 +
2*a**2*x**2 + 1),x)*a**2*m + 2*int((x**m*e**(atan(a*x)*n)*x)/(a**4*x**4 +
2*a**2*x**2 + 1),x)*a**2 - int((x**m*e**(atan(a*x)*n))/(a**4*x**5 + 2*a**2
*x**3 + x),x)*a**2*m*x**2 - int((x**m*e**(atan(a*x)*n))/(a**4*x**5 + 2*a**
2*x**3 + x),x)*m)/(a*c**2*n*(a**2*x**2 + 1))
```

3.381 $\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^3}\,dx=\frac{x^{1+m}\operatorname{AppellF1}\left(1+m,3-\frac{in}{2},3+\frac{in}{2},2+m,iax,-iax\right)}{c^3(1+m)}$$

output
```
x^(1+m)*AppellF1(1+m,3+1/2*I*n,3-1/2*I*n,2+m,-I*a*x,I*a*x)/c^3/(1+m)
```

Mathematica [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^3}\,dx=\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^3}\,dx$$

input
```
Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^3,x]
```

output
```
Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^3, x]
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5605, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m e^{n \arctan(ax)}}{(a^2cx^2+c)^3} dx$$

$$\Big\downarrow \text{5605}$$

$$\frac{\int x^m (1-iax)^{\frac{in}{2}-3}(iax+1)^{-\frac{in}{2}-3}dx}{c^3}$$

$$\Big\downarrow \text{150}$$

$$\frac{x^{m+1}\operatorname{AppellF1}\left(m+1,3-\frac{in}{2},\frac{in}{2}+3,m+2,iax,-iax\right)}{c^3(m+1)}$$

input
```
Int[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^3,x]
```

output
```
(x^(1 + m)*AppellF1[1 + m, 3 - (I/2)*n, 3 + (I/2)*n, 2 + m, I*a*x, (-I)*a*
x])/(c^3*(1 + m))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{\mathrm{e}^{n \arctan(ax)} x^m}{(a^2 c\, x^2 + c)^3} dx$$

input
```
int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^3,x)
```

output
```
int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^3,x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)} x^m}{(c + a^2 c x^2)^3} dx = \int \frac{x^m e^{(n \arctan(ax))}}{(a^2 c x^2 + c)^3} dx$$

input
```
integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(x^m*e^(n*arctan(a*x))/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^
2 + c^3), x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(ax)} x^m}{(c + a^2 c x^2)^3} dx = \frac{\int \frac{x^m e^{n \operatorname{atan}(ax)}}{a^6 x^6 + 3a^4 x^4 + 3a^2 x^2 + 1} dx}{c^3}$$

input
```
integrate(exp(n*atan(a*x))*x**m/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**m*exp(n*atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1)
, x)/c**3
```

Maxima [F]

$$\int \frac{e^{n \arctan(ax)} x^m}{(c + a^2cx^2)^3} \, dx = \int \frac{x^m e^{(n \arctan(ax))}}{(a^2cx^2 + c)^3} \, dx$$

input
```
integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{n \arctan(ax)} x^m}{(c + a^2cx^2)^3} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,1,0,0]%%%} / %%%{1,[0,0,0,1,3]%%%} Error: Bad Argum
ent Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n \arctan(ax)} x^m}{(c + a^2cx^2)^3} \, dx = \int \frac{x^m \, e^{n \operatorname{atan}(a\,x)}}{(c\,a^2\,x^2 + c)^3} \, dx$$

input
```
int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^3, x)
```

Reduce [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^3}\,dx$$
$$= \frac{x^m e^{atan(ax)n} - \left(\int \frac{x^m e^{atan(ax)n}x}{a^6x^6+3a^4x^4+3a^2x^2+1}dx\right)a^6m\,x^4 + 4\left(\int \frac{x^m e^{atan(ax)n}x}{a^6x^6+3a^4x^4+3a^2x^2+1}dx\right)a^6x^4 - 2\left(\int \frac{x^m e^{atan(ax)n}x}{a^6x^6+3a^4x^4+3a^2x^2}\right.}{}$$

input
```
int(exp(n*atan(a*x))*x^m/(a^2*c*x^2+c)^3,x)
```

output
```
(x**m*e**(atan(a*x)*n) - int((x**m*e**(atan(a*x)*n)*x)/(a**6*x**6 + 3*a**4
*x**4 + 3*a**2*x**2 + 1),x)*a**6*m*x**4 + 4*int((x**m*e**(atan(a*x)*n)*x)/
(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),x)*a**6*x**4 - 2*int((x**m*e**
(atan(a*x)*n)*x)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),x)*a**4*m*x**
2 + 8*int((x**m*e**(atan(a*x)*n)*x)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2
 + 1),x)*a**4*x**2 - int((x**m*e**(atan(a*x)*n)*x)/(a**6*x**6 + 3*a**4*x**
4 + 3*a**2*x**2 + 1),x)*a**2*m + 4*int((x**m*e**(atan(a*x)*n)*x)/(a**6*x**
6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),x)*a**2 - int((x**m*e**(atan(a*x)*n))/(
a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x),x)*a**4*m*x**4 - 2*int((x**m*e*
*(atan(a*x)*n))/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x),x)*a**2*m*x**2
 - int((x**m*e**(atan(a*x)*n))/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x)
,x)*m)/(a*c**3*n*(a**4*x**4 + 2*a**2*x**2 + 1))
```

3.382 $\int \frac{e^{n \arctan(ax)} x^m}{\sqrt{c+a^2cx^2}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{e^{n \arctan(ax)} x^m}{\sqrt{c+a^2cx^2}} \, dx = \frac{x^{1+m}\sqrt{1+a^2x^2} \operatorname{AppellF1}\left(1+m, \frac{1}{2}(1-in), \frac{1}{2}(1+in), 2+m, iax, -iax\right)}{(1+m)\sqrt{c+a^2cx^2}}$$

output
```
x^(1+m)*(a^2*x^2+1)^(1/2)*AppellF1(1+m,1/2+1/2*I*n,1/2-1/2*I*n,2+m,-I*a*x,
I*a*x)/(1+m)/(a^2*c*x^2+c)^(1/2)
```

Mathematica [F]

$$\int \frac{e^{n \arctan(ax)} x^m}{\sqrt{c+a^2cx^2}} \, dx = \int \frac{e^{n \arctan(ax)} x^m}{\sqrt{c+a^2cx^2}} \, dx$$

input
```
Integrate[(E^(n*ArcTan[a*x])*x^m)/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(E^(n*ArcTan[a*x])*x^m)/Sqrt[c + a^2*c*x^2], x]
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {5608, 5605, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m e^{n \arctan(ax)}}{\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \text{5608}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{n\arctan(ax)}x^m}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5605}$$

$$\frac{\sqrt{a^2x^2+1}\int x^m(1-iax)^{\frac{1}{2}(in-1)}(iax+1)^{\frac{1}{2}(-in-1)}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{150}$$

$$\frac{\sqrt{a^2x^2+1}x^{m+1}\,\text{AppellF1}\left(m+1,\frac{1}{2}(1-in),\frac{1}{2}(in+1),m+2,iax,-iax\right)}{(m+1)\sqrt{a^2cx^2+c}}$$

input
```
Int[(E^(n*ArcTan[a*x])*x^m)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(x^(1 + m)*Sqrt[1 + a^2*x^2]*AppellF1[1 + m, (1 - I*n)/2, (1 + I*n)/2, 2 +
 m, I*a*x, (-I)*a*x])/((1 + m)*Sqrt[c + a^2*c*x^2])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

rule 5608
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p])   Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
 m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m e^{(n\arctan(ax))}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(x^m*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Sympy [F]

$$\int \frac{e^{n \arctan(ax)} x^m}{\sqrt{c + a^2cx^2}}\, dx = \int \frac{x^m e^{n \operatorname{atan}(ax)}}{\sqrt{c(a^2x^2 + 1)}}\, dx$$

input
```
integrate(exp(n*atan(a*x))*x**m/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**m*exp(n*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{e^{n \arctan(ax)} x^m}{\sqrt{c + a^2cx^2}}\, dx = \int \frac{x^m e^{(n \arctan(ax))}}{\sqrt{a^2cx^2 + c}}\, dx$$

input
```
integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(x^m*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Giac [F]

$$\int \frac{e^{n \arctan(ax)} x^m}{\sqrt{c + a^2cx^2}}\, dx = \int \frac{x^m e^{(n \arctan(ax))}}{\sqrt{a^2cx^2 + c}}\, dx$$

input
```
integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m*e^(n*arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}x^m}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{x^m\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{\sqrt{c+a^2cx^2}}\,dx=\frac{\int \frac{x^m e^{atan(ax)n}}{\sqrt{a^2x^2+1}}dx}{\sqrt{c}}$$

input
```
int(exp(n*atan(a*x))*x^m/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int((x**m*e**(atan(a*x)*n))/sqrt(a**2*x**2 + 1),x)/sqrt(c)
```

3.383 $\int \frac{e^{n \arctan(ax)} x^m}{(c+a^2cx^2)^{3/2}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 82

$$\int \frac{e^{n \arctan(ax)} x^m}{(c+a^2cx^2)^{3/2}} \, dx = \frac{x^{1+m}\sqrt{1+a^2x^2} \operatorname{AppellF1}\left(1+m, \frac{1}{2}(3-in), \frac{1}{2}(3+in), 2+m, iax, -iax\right)}{c(1+m)\sqrt{c+a^2cx^2}}$$

output
```
x^(1+m)*(a^2*x^2+1)^(1/2)*AppellF1(1+m,3/2+1/2*I*n,3/2-1/2*I*n,2+m,-I*a*x,
I*a*x)/c/(1+m)/(a^2*c*x^2+c)^(1/2)
```

Mathematica [F]

$$\int \frac{e^{n \arctan(ax)} x^m}{(c+a^2cx^2)^{3/2}} \, dx = \int \frac{e^{n \arctan(ax)} x^m}{(c+a^2cx^2)^{3/2}} \, dx$$

input
```
Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^(3/2), x]
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {5608, 5605, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m e^{n\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5608}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{n\arctan(ax)}x^m}{(a^2x^2+1)^{3/2}}dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5605}$$

$$\frac{\sqrt{a^2x^2+1}\int x^m(1-iax)^{\frac{1}{2}(in-3)}(iax+1)^{\frac{1}{2}(-in-3)}dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{150}$$

$$\frac{\sqrt{a^2x^2+1}x^{m+1}\operatorname{AppellF1}\left(m+1,\frac{1}{2}(3-in),\frac{1}{2}(in+3),m+2,iax,-iax\right)}{c(m+1)\sqrt{a^2cx^2+c}}$$

input
```
Int[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(x^(1 + m)*Sqrt[1 + a^2*x^2]*AppellF1[1 + m, (3 - I*n)/2, (3 + I*n)/2, 2 +
 m, I*a*x, (-I)*a*x])/(c*(1 + m)*Sqrt[c + a^2*c*x^2])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

rule 5608
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p])   Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
 m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{e^{n \arctan(ax)} x^m}{(a^2 c\, x^2 + c)^{\frac{3}{2}}} dx$$

input
```
int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{e^{n \arctan(ax)} x^m}{(c + a^2 c x^2)^{3/2}}\, dx = \int \frac{x^m e^{(n \arctan(ax))}}{(a^2 c x^2 + c)^{\frac{3}{2}}}\, dx$$

input
```
integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*e^(n*arctan(a*x))/(a^4*c^2*x^4 + 2*a^2*c^
2*x^2 + c^2), x)
```

Sympy [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{3/2}}\,dx=\int \frac{x^m e^{n\,\mathrm{atan}\,(ax)}}{(c\,(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*atan(a*x))*x**m/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**m*exp(n*atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{3/2}}\,dx=\int \frac{x^m e^{(n\arctan(ax))}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{3/2}}\,dx=\int \frac{x^m e^{(n\arctan(ax))}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n \arctan(ax)} x^m}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)}}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{e^{n \arctan(ax)} x^m}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\int \frac{x^m e^{atan(ax)n}}{\sqrt{a^2x^2+1}\,a^2x^2+\sqrt{a^2x^2+1}}dx}{\sqrt{c}\,c}$$

input
```
int(exp(n*atan(a*x))*x^m/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int((x**m*e**(atan(a*x)*n))/(sqrt(a**2*x**2 + 1)*a**2*x**2 + sqrt(a**2*x**
2 + 1)),x)/(sqrt(c)*c)
```

3.384 $\int \frac{e^{n\arctan(ax)}x^m}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 82

$$\int \frac{e^{n\arctan(ax)}x^m}{\left(c+a^2cx^2\right)^{5/2}}\,dx=\frac{x^{1+m}\sqrt{1+a^2x^2}\,\text{AppellF1}\left(1+m,\frac{1}{2}(5-in),\frac{1}{2}(5+in),2+m,iax,-iax\right)}{c^2(1+m)\sqrt{c+a^2cx^2}}$$

output
```
x^(1+m)*(a^2*x^2+1)^(1/2)*AppellF1(1+m,5/2+1/2*I*n,5/2-1/2*I*n,2+m,-I*a*x,
I*a*x)/c^2/(1+m)/(a^2*c*x^2+c)^(1/2)
```

Mathematica [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{\left(c+a^2cx^2\right)^{5/2}}\,dx=\int \frac{e^{n\arctan(ax)}x^m}{\left(c+a^2cx^2\right)^{5/2}}\,dx$$

input
```
Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
Integrate[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^(5/2), x]
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {5608, 5605, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m e^{n\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5608}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{n\arctan(ax)}x^m}{(a^2x^2+1)^{5/2}}dx}{c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5605}$$

$$\frac{\sqrt{a^2x^2+1}\int x^m(1-iax)^{\frac{1}{2}(in-5)}(iax+1)^{\frac{1}{2}(-in-5)}dx}{c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{150}$$

$$\frac{\sqrt{a^2x^2+1}x^{m+1}\operatorname{AppellF1}\left(m+1,\frac{1}{2}(5-in),\frac{1}{2}(in+5),m+2,iax,-iax\right)}{c^2(m+1)\sqrt{a^2cx^2+c}}$$

input
```
Int[(E^(n*ArcTan[a*x])*x^m)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(x^(1 + m)*Sqrt[1 + a^2*x^2]*AppellF1[1 + m, (5 - I*n)/2, (5 + I*n)/2, 2 +
 m, I*a*x, (-I)*a*x])/(c^2*(1 + m)*Sqrt[c + a^2*c*x^2])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

rule 5608
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p])   Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
 m, n, p}, x] && EqQ[d, a^2*c] && !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(a^2c\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^m e^{(n\arctan(ax))}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*e^(n*arctan(a*x))/(a^6*c^3*x^6 + 3*a^4*c^
3*x^4 + 3*a^2*c^3*x^2 + c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(exp(n*atan(a*x))*x**m/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^m e^{(n\arctan(ax))}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima"
)
```

output
```
integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^m e^{(n\arctan(ax))}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*x^m/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^m*e^(n*arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^m\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)}}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^m*exp(n*atan(a*x)))/(c + a^2*c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{e^{n\arctan(ax)}x^m}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\int \frac{x^m e^{atan(ax)n}}{\sqrt{a^2x^2+1}\,a^4x^4+2\sqrt{a^2x^2+1}\,a^2x^2+\sqrt{a^2x^2+1}}dx}{\sqrt{c}\,c^2}$$

input
```
int(exp(n*atan(a*x))*x^m/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int((x**m*e**(atan(a*x)*n))/(sqrt(a**2*x**2 + 1)*a**4*x**4 + 2*sqrt(a**2*x
**2 + 1)*a**2*x**2 + sqrt(a**2*x**2 + 1)),x)/(sqrt(c)*c**2)
```

3.385 $\int e^{n\arctan(ax)}(c+a^2cx^2)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 115

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx$$
$$=\frac{2^{1-\frac{in}{2}+p}(1-iax)^{1+\frac{in}{2}+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\text{Hypergeometric2F1}\left(\frac{in}{2}-p,1+\frac{in}{2}+p,2+\frac{in}{2}+p,\frac{1}{2}(\right.}{a(n-2i(1+p))}$$

output
```
2^(1-1/2*I*n+p)*(1-I*a*x)^(1+1/2*I*n+p)*(a^2*c*x^2+c)^p*hypergeom([1/2*I*n
-p, 1+1/2*I*n+p],[2+1/2*I*n+p],1/2-1/2*I*a*x)/a/(n-2*I*(p+1))/((a^2*x^2+1)
^p)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx$$
$$=\frac{2^{1-\frac{in}{2}+p}(1-iax)^{1+\frac{in}{2}+p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p\text{Hypergeometric2F1}\left(\frac{in}{2}-p,1+\frac{in}{2}+p,2+\frac{in}{2}+p,\frac{1}{2}(\right.}{a(n-2i(1+p))}$$

input
```
Integrate[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^p,x]
```

output
```
(2^(1 - (I/2)*n + p)*(1 - I*a*x)^(1 + (I/2)*n + p)*(c + a^2*c*x^2)^p*Hyper
geometric2F1[(I/2)*n - p, 1 + (I/2)*n + p, 2 + (I/2)*n + p, (1 - I*a*x)/2]
)/(a*(n - (2*I)*(1 + p))*(1 + a^2*x^2)^p)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{5599, 5596, 79\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{n\arctan(ax)}\left(a^2cx^2+c\right)^p\,dx$$

$$\Big\downarrow \text{ 5599}$$

$$\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\int e^{n\arctan(ax)}\left(a^2x^2+1\right)^p dx$$

$$\Big\downarrow \text{ 5596}$$

$$\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\int (1-iax)^{\frac{in}{2}+p}(iax+1)^{p-\frac{in}{2}}dx$$

$$\Big\downarrow \text{ 79}$$

$$\frac{2^{-\frac{in}{2}+p+1}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p(1-iax)^{\frac{in}{2}+p+1}\text{Hypergeometric2F1}\left(\frac{in}{2}-p,\frac{in}{2}+p+1,\frac{in}{2}+p+2,\frac{1}{2}(1-i\right.}{a(n-2i(p+1))}$$

input
```
Int[E^(n*ArcTan[a*x])*(c + a^2*c*x^2)^p,x]
```

output

```
(2^(1 - (I/2)*n + p)*(1 - I*a*x)^(1 + (I/2)*n + p)*(c + a^2*c*x^2)^p*Hyper
geometric2F1[(I/2)*n - p, 1 + (I/2)*n + p, 2 + (I/2)*n + p, (1 - I*a*x)/2]
)/(a*(n - (2*I)*(1 + p))*(1 + a^2*x^2)^p)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [F]

$$\int \mathrm{e}^{n \arctan(ax)} \left(a^2 c\, x^2 + c\right)^p dx$$

input
```
int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^p,x)
```

output
```
int(exp(n*arctan(a*x))*(a^2*c*x^2+c)^p,x)
```

Fricas [F]

$$\int e^{n \arctan(ax)} \left(c + a^2 c x^2\right)^p \, dx = \int \left(a^2 c x^2 + c\right)^p e^{(n \arctan(ax))} \, dx$$

input
```
integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^p*e^(n*arctan(a*x)), x)
```

Sympy [F]

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int\left(c\left(a^2x^2+1\right)\right)^pe^{n\operatorname{atan}(ax)}\,dx$$

input
```
integrate(exp(n*atan(a*x))*(a**2*c*x**2+c)**p,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**p*exp(n*atan(a*x)), x)
```

Maxima [F]

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int\left(a^2cx^2+c\right)^pe^{(n\arctan(ax))}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^p*e^(n*arctan(a*x)), x)
```

Giac [F]

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int\left(a^2cx^2+c\right)^pe^{(n\arctan(ax))}\,dx$$

input
```
integrate(exp(n*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^p*e^(n*arctan(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int \mathrm{e}^{n\,\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)^p\,dx$$

input
```
int(exp(n*atan(a*x))*(c + a^2*c*x^2)^p,x)
```

output
```
int(exp(n*atan(a*x))*(c + a^2*c*x^2)^p, x)
```

Reduce [F]

$$\int e^{n\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int e^{atan(ax)n}\left(a^2c\,x^2+c\right)^p\,dx$$

input
```
int(exp(n*atan(a*x))*(a^2*c*x^2+c)^p,x)
```

output
```
int(e**(atan(a*x)*n)*(a**2*c*x**2 + c)**p,x)
```

3.386 $\int e^{-2ip\arctan(ax)}(c+a^2cx^2)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 53

$$\int e^{-2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\frac{i(1-iax)^{1+2p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p}{a(1+2p)}$$

output
```
I*(1-I*a*x)^(1+2*p)*(a^2*c*x^2+c)^p/a/(1+2*p)/((a^2*x^2+1)^p)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int e^{-2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\frac{e^{-2ip\arctan(ax)}(i+ax)\left(c+a^2cx^2\right)^p}{a+2ap}$$

input
```
Integrate[(c + a^2*c*x^2)^p/E^((2*I)*p*ArcTan[a*x]),x]
```

output
```
((I + a*x)*(c + a^2*c*x^2)^p)/(E^((2*I)*p*ArcTan[a*x])*(a + 2*a*p))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5599, 5596, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2ip\arctan(ax)}\left(a^2cx^2+c\right)^p\,dx$$

$$\Big\downarrow \ 5599$$

$$\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\int e^{-2ip\arctan(ax)}\left(a^2x^2+1\right)^p dx$$

$$\Big\downarrow \ 5596$$

$$\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\int (1-iax)^{2p}dx$$

$$\Big\downarrow \ 17$$

$$\frac{i(1-iax)^{2p+1}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p}{a(2p+1)}$$

input
```
Int[(c + a^2*c*x^2)^p/E^((2*I)*p*ArcTan[a*x]),x]
```

output
```
(I*(1 - I*a*x)^(1 + 2*p)*(c + a^2*c*x^2)^p)/(a*(1 + 2*p)*(1 + a^2*x^2)^p)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

method	result
gosper	$\frac{(ax+i)\left(a^2c\,x^2+c\right)^p \mathrm{e}^{-2ip\arctan(ax)}}{a(2p+1)}$
orering	$\frac{(ax+i)\left(a^2c\,x^2+c\right)^p \mathrm{e}^{-2ip\arctan(ax)}}{a(2p+1)}$
parallelrisch	$\frac{\left(x\left(a^2c\,x^2+c\right)^p a+i\left(a^2c\,x^2+c\right)^p\right)\mathrm{e}^{-2ip\arctan(ax)}}{a(2p+1)}$
risch	$\frac{((ax+i)^p)^2 c^p(ax+i)\mathrm{e}^{-\frac{ip\pi\left(\mathrm{csgn}(i(ax-i)(ax+i))^3-\mathrm{csgn}(i(ax-i)(ax+i))^2\,\mathrm{csgn}(i(ax+i))-\mathrm{csgn}(i(ax-i)(ax+i))^2\,\mathrm{csgn}(i(ax-i))-\mathrm{csgn}(i(\right.}{}}}{}$

input
```
int((a^2*c*x^2+c)^p/exp(2*I*p*arctan(a*x)),x,method=_RETURNVERBOSE)
```

output
```
(I+a*x)/a/(2*p+1)*(a^2*c*x^2+c)^p/exp(2*I*p*arctan(a*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int e^{-2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\frac{(ax+i)(a^2cx^2+c)^p\left(-\frac{ax+i}{ax-i}\right)^p}{2\,ap+a}$$

input
```
integrate((a^2*c*x^2+c)^p/exp(2*I*p*arctan(a*x)),x, algorithm="fricas")
```

output
```
(a*x + I)*(a^2*c*x^2 + c)^p*(-(a*x + I)/(a*x - I))^p/(2*a*p + a)
```

Sympy [F]

$$\int e^{-2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx$$
$$=\begin{cases}\frac{x}{\sqrt{c}} & \text{for } a=0\wedge p=-\frac{1}{2}\\ c^px & \text{for } a=0\\ \int\frac{e^{i\operatorname{atan}(ax)}}{\sqrt{c(a^2x^2+1)}}\,dx & \text{for } p=-\frac{1}{2}\\ \frac{ax(a^2cx^2+c)^p}{2ape^{2ip\operatorname{atan}(ax)}+ae^{2ip\operatorname{atan}(ax)}}+\frac{i(a^2cx^2+c)^p}{2ape^{2ip\operatorname{atan}(ax)}+ae^{2ip\operatorname{atan}(ax)}} & \text{otherwise}\end{cases}$$

input
```
integrate((a**2*c*x**2+c)**p/exp(2*I*p*atan(a*x)),x)
```

output
```
Piecewise((x/sqrt(c), Eq(a, 0) & Eq(p, -1/2)), (c**p*x, Eq(a, 0)), (Integr
al(exp(I*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x), Eq(p, -1/2)), (a*x*(a**2*
c*x**2 + c)**p/(2*a*p*exp(2*I*p*atan(a*x)) + a*exp(2*I*p*atan(a*x))) + I*(
a**2*c*x**2 + c)**p/(2*a*p*exp(2*I*p*atan(a*x)) + a*exp(2*I*p*atan(a*x))),
 True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.43

$$\int e^{-2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx$$
$$=\frac{(ac^px+i\,c^p)(a^2x^2+1)^p\cos(2\,p\arctan(ax))-(i\,ac^px-c^p)(a^2x^2+1)^p\sin(2\,p\arctan(ax))}{2\,ap+a}$$

input
```
integrate((a^2*c*x^2+c)^p/exp(2*I*p*arctan(a*x)),x, algorithm="maxima")
```

output
```
((a*c^p*x + I*c^p)*(a^2*x^2 + 1)^p*cos(2*p*arctan(a*x)) - (I*a*c^p*x - c^p
)*(a^2*x^2 + 1)^p*sin(2*p*arctan(a*x)))/(2*a*p + a)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int e^{-2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\frac{axe^{(-i\pi p+2\,p\log(ax+i)+p\log(c))}+i\,e^{(-i\pi p+2\,p\log(ax+i)+p\log(c))}}{2\,ap+a}$$

input
```
integrate((a^2*c*x^2+c)^p/exp(2*I*p*arctan(a*x)),x, algorithm="giac")
```

output
```
(a*x*e^(-I*pi*p + 2*p*log(a*x + I) + p*log(c)) + I*e^(-I*pi*p + 2*p*log(a*
x + I) + p*log(c)))/(2*a*p + a)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int e^{-2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\left(\frac{x\,\mathrm{e}^{-p\,\mathrm{atan}(a\,x)\,2\mathrm{i}}}{2\,p+1}+\frac{\mathrm{e}^{-p\,\mathrm{atan}(a\,x)\,2\mathrm{i}}\,1\mathrm{i}}{a\,\left(2\,p+1\right)}\right)\,\left(c\,a^2\,x^2+c\right)^p$$

input
```
int(exp(-p*atan(a*x)*2i)*(c + a^2*c*x^2)^p,x)
```

output
```
((x*exp(-p*atan(a*x)*2i))/(2*p + 1) + (exp(-p*atan(a*x)*2i)*1i)/(a*(2*p +
1)))*(c + a^2*c*x^2)^p
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

$$\int e^{-2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\frac{\left(a^2c\,x^2+c\right)^p\left(ax+i\right)}{e^{2atan(ax)ip}a\left(2p+1\right)}$$

input
```
int((a^2*c*x^2+c)^p/exp(2*I*p*atan(a*x)),x)
```

output
```
((a**2*c*x**2 + c)**p*(a*x + i))/(e**(2*atan(a*x)*i*p)*a*(2*p + 1))
```

3.387 $\int e^{2ip\arctan(ax)}(c+a^2cx^2)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 53

$$\int e^{2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=-\frac{i(1+iax)^{1+2p}\left(1+a^2x^2\right)^{-p}\left(c+a^2cx^2\right)^p}{a(1+2p)}$$

output
```
-I*(1+I*a*x)^(1+2*p)*(a^2*c*x^2+c)^p/a/(1+2*p)/((a^2*x^2+1)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int e^{2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\frac{e^{2ip\arctan(ax)}(-i+ax)\left(c+a^2cx^2\right)^p}{a+2ap}$$

input
```
Integrate[E^((2*I)*p*ArcTan[a*x])*(c + a^2*c*x^2)^p,x]
```

output
```
(E^((2*I)*p*ArcTan[a*x])*(-I + a*x)*(c + a^2*c*x^2)^p)/(a + 2*a*p)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5599, 5596, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2ip\arctan(ax)}\left(a^2cx^2+c\right)^p\,dx$$

$$\Big\downarrow \text{5599}$$

$$\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\int e^{2ip\arctan(ax)}\left(a^2x^2+1\right)^p dx$$

$$\Big\downarrow \text{5596}$$

$$\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p\int (iax+1)^{2p}dx$$

$$\Big\downarrow \text{17}$$

$$-\frac{i(1+iax)^{2p+1}\left(a^2x^2+1\right)^{-p}\left(a^2cx^2+c\right)^p}{a(2p+1)}$$

input
```
Int[E^((2*I)*p*ArcTan[a*x])*(c + a^2*c*x^2)^p,x]
```

output
```
((-I)*(1 + I*a*x)^(1 + 2*p)*(c + a^2*c*x^2)^p)/(a*(1 + 2*p)*(1 + a^2*x^2)^
p)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 5596
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Simp[c^p   Int[(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*(n/2)), x], x]
/; FreeQ[{a, c, d, n, p}, x] && EqQ[d, a^2*c] && (IntegerQ[p] || GtQ[c, 0])
```

rule 5599
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> S
imp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart[p])   Int[
(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

method	result
gosper	$-\frac{(-ax+i)\mathrm{e}^{2ip\arctan(ax)}\left(a^{2}c\,x^{2}+c\right)^{p}}{a(2p+1)}$
orering	$-\frac{(-ax+i)\mathrm{e}^{2ip\arctan(ax)}\left(a^{2}c\,x^{2}+c\right)^{p}}{a(2p+1)}$
parallelrisch	$-\frac{-\mathrm{e}^{2ip\arctan(ax)}x\left(a^{2}c\,x^{2}+c\right)^{p}a+i\left(a^{2}c\,x^{2}+c\right)^{p}\mathrm{e}^{2ip\arctan(ax)}}{a(2p+1)}$
risch	$c^{p}(ax+i)^{p}(ax-i)^{2p}(ax+i)^{-p}(ax-i)\mathrm{e}^{-\frac{ip\pi\left(-\operatorname{csgn}(ax+i)^{3}+\operatorname{csgn}(ax+i)^{2}\operatorname{csgn}(i(ax+i))+\operatorname{csgn}(i(ax-i)(ax+i))\operatorname{csgn}(i(ax+i))\operatorname{csgn}(i(a\ldots\right.}{}}$

input
```
int(exp(2*I*p*arctan(a*x))*(a^2*c*x^2+c)^p,x,method=_RETURNVERBOSE)
```

output
```
-(I-a*x)/a/(2*p+1)*exp(2*I*p*arctan(a*x))*(a^2*c*x^2+c)^p
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int e^{2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\frac{(ax-i)(a^2cx^2+c)^p}{(2\,ap+a)\left(-\frac{ax+i}{ax-i}\right)^p}$$

input
```
integrate(exp(2*I*p*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="fricas")
```

output
```
(a*x - I)*(a^2*c*x^2 + c)^p/((2*a*p + a)*(-(a*x + I)/(a*x - I))^p)
```

Sympy [F]

$$\int e^{2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx$$
$$=\begin{cases}\frac{x}{\sqrt{c}} & \text{for } a=0\wedge p=-\frac{1}{2}\\ c^px & \text{for } a=0\\ \int\frac{e^{-i\,\mathrm{atan}\,(ax)}}{\sqrt{c(a^2x^2+1)}}\,dx & \text{for } p=-\frac{1}{2}\\ \frac{ax(a^2cx^2+c)^pe^{2ip\,\mathrm{atan}\,(ax)}}{2ap+a}-\frac{i(a^2cx^2+c)^pe^{2ip\,\mathrm{atan}\,(ax)}}{2ap+a} & \text{otherwise}\end{cases}$$

input
```
integrate(exp(2*I*p*atan(a*x))*(a**2*c*x**2+c)**p,x)
```

output
```
Piecewise((x/sqrt(c), Eq(a, 0) & Eq(p, -1/2)), (c**p*x, Eq(a, 0)), (Integr
al(exp(-I*atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x), Eq(p, -1/2)), (a*x*(a**2
*c*x**2 + c)**p*exp(2*I*p*atan(a*x))/(2*a*p + a) - I*(a**2*c*x**2 + c)**p*
exp(2*I*p*atan(a*x))/(2*a*p + a), True))
```

Maxima [F]

$$\int e^{2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\int\left(a^2cx^2+c\right)^pe^{(2i\,p\arctan(ax))}\,dx$$

input
```
integrate(exp(2*I*p*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^p*e^(2*I*p*arctan(a*x)), x)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int e^{2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\frac{axe^{(i\pi p+2p\log(ax-i)+p\log(c))}-ie^{(i\pi p+2p\log(ax-i)+p\log(c))}}{2ap+a}$$

input
```
integrate(exp(2*I*p*arctan(a*x))*(a^2*c*x^2+c)^p,x, algorithm="giac")
```

output
```
(a*x*e^(I*pi*p + 2*p*log(a*x - I) + p*log(c)) - I*e^(I*pi*p + 2*p*log(a*x
- I) + p*log(c)))/(2*a*p + a)
```

Mupad [B] (verification not implemented)

Time = 23.00 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int e^{2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\left(\frac{x\,\mathrm{e}^{p\,\mathrm{atan}(a\,x)\,2\mathrm{i}}}{2\,p+1}-\frac{\mathrm{e}^{p\,\mathrm{atan}(a\,x)\,2\mathrm{i}}\,1\mathrm{i}}{a\,\left(2\,p+1\right)}\right)\,\left(c\,a^2\,x^2+c\right)^p$$

input
```
int(exp(p*atan(a*x)*2i)*(c + a^2*c*x^2)^p,x)
```

output
```
((x*exp(p*atan(a*x)*2i))/(2*p + 1) - (exp(p*atan(a*x)*2i)*1i)/(a*(2*p + 1)
))*(c + a^2*c*x^2)^p
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

$$\int e^{2ip\arctan(ax)}\left(c+a^2cx^2\right)^p\,dx=\frac{e^{2atan(ax)ip}\left(a^2c\,x^2+c\right)^p(ax-i)}{a\,(2p+1)}$$

input
```
int(exp(2*I*p*atan(a*x))*(a^2*c*x^2+c)^p,x)
```

output
```
(e**(2*atan(a*x)*i*p)*(a**2*c*x**2 + c)**p*(a*x - i))/(a*(2*p + 1))
```

3.388 $\int e^{in\arctan(ax)}x^2(c+a^2cx^2)^{-1-\frac{n^2}{2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 60

$$\int e^{in\arctan(ax)}x^2\left(c+a^2cx^2\right)^{-1-\frac{n^2}{2}}\,dx=\frac{ie^{in\arctan(ax)}(1-ianx)\left(c+a^2cx^2\right)^{-\frac{n^2}{2}}}{a^3cn\left(1-n^2\right)}$$

output
```
I*exp(I*n*arctan(a*x))*(1-I*a*n*x)/a^3/c/n/(-n^2+1)/((a^2*c*x^2+c)^(1/2*n^
2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int e^{in\arctan(ax)}x^2\left(c+a^2cx^2\right)^{-1-\frac{n^2}{2}}\,dx=-\frac{e^{in\arctan(ax)}(i+anx)\left(c+a^2cx^2\right)^{-\frac{n^2}{2}}}{a^3cn\left(-1+n^2\right)}$$

input
```
Integrate[E^(I*n*ArcTan[a*x])*x^2*(c + a^2*c*x^2)^(-1 - n^2/2),x]
```

output
```
-((E^(I*n*ArcTan[a*x])*(I + a*n*x))/(a^3*c*n*(-1 + n^2)*(c + a^2*c*x^2)^(n
^2/2)))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {5602}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{in \arctan(ax)} \left(a^2cx^2+c\right)^{-\frac{n^2}{2}-1} \, dx$$

$$\Big\downarrow \; 5602$$

$$\frac{i(1-ianx)e^{in \arctan(ax)} \left(a^2cx^2+c\right)^{-\frac{n^2}{2}}}{a^3cn\left(1-n^2\right)}$$

input
```
Int[E^(I*n*ArcTan[a*x])*x^2*(c + a^2*c*x^2)^(-1 - n^2/2),x]
```

output
```
(I*E^(I*n*ArcTan[a*x])*(1 - I*a*n*x))/(a^3*c*n*(1 - n^2)*(c + a^2*c*x^2)^(
n^2/2))
```

Defintions of rubi rules used

rule 5602
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^2*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Simp[(-(1 - a*n*x))*(c + d*x^2)^(p + 1)*(E^(n*ArcTan[a*x])/(a*d*n*(n
^2 + 1))), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && EqQ[n^2 - 2*(p
+ 1), 0] &&  !IntegerQ[I*n]
```

Maple [A] (verified)

Time = 5.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

method	result
orering	$-\frac{(nax+i)(a^2x^2+1)\mathrm{e}^{in\arctan(ax)}(a^2c\,x^2+c)^{-1-\frac{n^2}{2}}}{a^3n(n^2-1)}$
gosper	$\frac{(-ax+i)(ax+i)(nax+i)\mathrm{e}^{in\arctan(ax)}(a^2c\,x^2+c)^{-1-\frac{n^2}{2}}}{a^3n(n^2-1)}$
parallelrisch	$-\frac{\mathrm{e}^{in\arctan(ax)}(a^2c\,x^2+c)^{-1-\frac{n^2}{2}}x^3a^3n+i\mathrm{e}^{in\arctan(ax)}x^2(a^2c\,x^2+c)^{-1-\frac{n^2}{2}}a^2+\mathrm{e}^{in\arctan(ax)}x(a^2c\,x^2+c)^{-1-\frac{n^2}{2}}an+i\mathrm{e}^{in}}{a^3n(n^2-1)}$

input
```
int(exp(I*n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(-1-1/2*n^2),x,method=_RETURNVE
RBOSE)
```

output
```
-(n*a*x+I)/a^3/n/(n^2-1)*(a^2*x^2+1)*exp(I*n*arctan(a*x))*(a^2*c*x^2+c)^(-
1-1/2*n^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.30

$$\int e^{in\arctan(ax)}x^2\left(c+a^2cx^2\right)^{-1-\frac{n^2}{2}}\,dx=-\frac{(a^3nx^3+i\,a^2x^2+anx+i)(a^2cx^2+c)^{-\frac{1}{2}n^2-1}}{(a^3n^3-a^3n)\left(-\frac{ax+i}{ax-i}\right)^{\frac{1}{2}n}}$$

input
```
integrate(exp(I*n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(-1-1/2*n^2),x, algorithm
="fricas")
```

output
```
-(a^3*n*x^3 + I*a^2*x^2 + a*n*x + I)*(a^2*c*x^2 + c)^(-1/2*n^2 - 1)/((a^3*
n^3 - a^3*n)*(-(a*x + I)/(a*x - I))^(1/2*n))
```

Sympy [F(-1)]

Timed out.

$$\int e^{in\arctan(ax)}x^2\left(c+a^2cx^2\right)^{-1-\frac{n^2}{2}}\,dx=\text{Timed out}$$

input
```
integrate(exp(I*n*atan(a*x))*x**2*(a**2*c*x**2+c)**(-1-1/2*n**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{in\arctan(ax)}x^2\left(c+a^2cx^2\right)^{-1-\frac{n^2}{2}}\,dx=\int \left(a^2cx^2+c\right)^{-\frac{1}{2}n^2-1}x^2e^{(i\,n\arctan(ax))}\,dx$$

input
```
integrate(exp(I*n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(-1-1/2*n^2),x, algorithm
="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(-1/2*n^2 - 1)*x^2*e^(I*n*arctan(a*x)), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(51) = 102.

Time = 0.43 (sec) , antiderivative size = 333, normalized size of antiderivative = 5.55

$$\int e^{in\arctan(ax)}x^2\left(c+a^2cx^2\right)^{-1-\frac{n^2}{2}}\,dx=$$
$$-\frac{a^3nx^3e^{\left(-\frac{1}{2}n^2\log(ax+i)-\frac{1}{2}n^2\log(ax-i)-\frac{1}{2}n^2\log(c)+\frac{1}{2}i\pi n-\frac{1}{2}n\log(ax+i)+\frac{1}{2}n\log(ax-i)-\log(ax+i)-\log(ax-i)-\log(c)\right)}+i\,a^2x}{}$$

input
```
integrate(exp(I*n*arctan(a*x))*x^2*(a^2*c*x^2+c)^(-1-1/2*n^2),x, algorithm
="giac")
```

output
```
-(a^3*n*x^3*e^(-1/2*n^2*log(a*x + I) - 1/2*n^2*log(a*x - I) - 1/2*n^2*log(
c) + 1/2*I*pi*n - 1/2*n*log(a*x + I) + 1/2*n*log(a*x - I) - log(a*x + I) -
 log(a*x - I) - log(c)) + I*a^2*x^2*e^(-1/2*n^2*log(a*x + I) - 1/2*n^2*log
(a*x - I) - 1/2*n^2*log(c) + 1/2*I*pi*n - 1/2*n*log(a*x + I) + 1/2*n*log(a
*x - I) - log(a*x + I) - log(a*x - I) - log(c)) + a*n*x*e^(-1/2*n^2*log(a*
x + I) - 1/2*n^2*log(a*x - I) - 1/2*n^2*log(c) + 1/2*I*pi*n - 1/2*n*log(a*
x + I) + 1/2*n*log(a*x - I) - log(a*x + I) - log(a*x - I) - log(c)) + I*e^
(-1/2*n^2*log(a*x + I) - 1/2*n^2*log(a*x - I) - 1/2*n^2*log(c) + 1/2*I*pi*
n - 1/2*n*log(a*x + I) + 1/2*n*log(a*x - I) - log(a*x + I) - log(a*x - I)
- log(c)))/(a^3*n^3 - a^3*n)
```

Mupad [F(-1)]

Timed out.

$$\int e^{in\arctan(ax)}x^2\left(c+a^2cx^2\right)^{-1-\frac{n^2}{2}}\,dx=\int\frac{x^2\,\mathrm{e}^{n\,\mathrm{atan}(a\,x)\,1\mathrm{i}}}{\left(c\,a^2\,x^2+c\right)^{\frac{n^2}{2}+1}}\,dx$$

input
```
int((x^2*exp(n*atan(a*x)*1i))/(c + a^2*c*x^2)^(n^2/2 + 1),x)
```

output
```
int((x^2*exp(n*atan(a*x)*1i))/(c + a^2*c*x^2)^(n^2/2 + 1), x)
```

Reduce [F]

$$\int e^{in\arctan(ax)}x^2\left(c+a^2cx^2\right)^{-1-\frac{n^2}{2}}\,dx=\frac{\int\frac{e^{atan(ax)in}x^2}{(a^2cx^2+c)^{\frac{n^2}{2}}a^2x^2+(a^2cx^2+c)^{\frac{n^2}{2}}}dx}{c}$$

input
```
int(exp(I*n*atan(a*x))*x^2*(a^2*c*x^2+c)^(-1-1/2*n^2),x)
```

output
```
int((e**(atan(a*x)*i*n)*x**2)/((a**2*c*x**2 + c)**(n**2/2)*a**2*x**2 + (a*
*2*c*x**2 + c)**(n**2/2)),x)/c
```

3.389 $\int \frac{e^{6i\arctan(ax)}x^2}{(c+a^2cx^2)^{19}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 38

$$\int \frac{e^{6i\arctan(ax)}x^2}{(c+a^2cx^2)^{19}}\,dx = -\frac{i+6ax}{210a^3c^{19}(1-iax)^{21}(1+iax)^{15}}$$

output
```
-1/210*(I+6*a*x)/a^3/c^19/(1-I*a*x)^21/(1+I*a*x)^15
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{e^{6i\arctan(ax)}x^2}{(c+a^2cx^2)^{19}}\,dx = \frac{i+6ax}{210a^3c^{19}(-i+ax)^{15}(i+ax)^{21}}$$

input
```
Integrate[(E^((6*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^19,x]
```

output
```
(I + 6*a*x)/(210*a^3*c^19*(-I + a*x)^15*(I + a*x)^21)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {5605, 91}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 e^{6i\arctan(ax)}}{(a^2cx^2+c)^{19}}dx$$
$$\Big\downarrow \text{ 5605}$$
$$\frac{\int \frac{x^2}{(1-iax)^{22}(iax+1)^{16}}dx}{c^{19}}$$
$$\Big\downarrow \text{ 91}$$
$$-\frac{6ax+i}{210a^3c^{19}(1-iax)^{21}(1+iax)^{15}}$$

input
```
Int[(E^((6*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^19,x]
```

output
```
-1/210*(I + 6*a*x)/(a^3*c^19*(1 - I*a*x)^21*(1 + I*a*x)^15)
```

Defintions of rubi rules used

rule 91
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*((2*a*d*f*(n + p + 3
) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x)/(d^2*f^2*(n + p +
2)*(n + p + 3))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2
, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p + 3) - b*(
b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p + 1))*(a*
d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result
default	$\frac{\frac{i}{210a^3}+\frac{x}{35a^2}}{c^{19}(ax+i)^{21}(ax-i)^{15}}$
risch	$\frac{\frac{i}{210a^3}+\frac{x}{35a^2}}{c^{19}(ax+i)^{21}(ax-i)^{15}}$
gosper	$\frac{(-ax+i)(ax+i)(6ax+i)(iax+1)^6}{210a^3(a^2x^2+1)^{22}c^{19}}$
parallelrisch	$\frac{ix^{42}a^{39}+21ix^{40}a^{37}+210ix^{38}a^{35}+1330ix^{36}a^{33}+5985ix^{34}a^{31}+20349ix^{32}a^{29}+54264ix^{30}a^{27}+116280ix^{28}a^{25}+203490ix^{26}a^{\cdots}}{\cdots}$
orering	$-\frac{ix^3(a^{33}x^{33}+6ia^{32}x^{32}+70ia^{30}x^{30}-105a^{29}x^{29}+336ia^{28}x^{28}-896a^{27}x^{27}+720ia^{26}x^{26}-3900a^{25}x^{25}-280ia^{24}x^{24}-10752a^{\cdots}}{\cdots}$

input
```
int((1+I*a*x)^6/(a^2*x^2+1)^3*x^2/(a^2*c*x^2+c)^19,x,method=_RETURNVERBOSE
)
```

output
```
1/c^19*(1/210*I/a^3+1/35*x/a^2)/(I+a*x)^21/(a*x-I)^15
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(30) = 60.

Time = 0.54 (sec) , antiderivative size = 379, normalized size of antiderivative = 9.97

$$\int \frac{e^{6i\arctan(ax)}x^2}{(c+a^2cx^2)^{19}}\,dx$$

$$= \frac{\cdots}{210\,(a^{39}c^{19}x^{36}+6i\,a^{38}c^{19}x^{35}+70i\,a^{36}c^{19}x^{33}-105\,a^{35}c^{19}x^{32}+336i\,a^{34}c^{19}x^{31}-896\,a^{33}c^{19}x^{30}+720i\,a^{32}c^{\cdots}}$$

input
```
integrate((1+I*a*x)^6/(a^2*x^2+1)^3*x^2/(a^2*c*x^2+c)^19,x, algorithm="fri
cas")
```

output
```
1/210*(6*a*x + I)/(a^39*c^19*x^36 + 6*I*a^38*c^19*x^35 + 70*I*a^36*c^19*x^
33 - 105*a^35*c^19*x^32 + 336*I*a^34*c^19*x^31 - 896*a^33*c^19*x^30 + 720*
I*a^32*c^19*x^29 - 3900*a^31*c^19*x^28 - 280*I*a^30*c^19*x^27 - 10752*a^29
*c^19*x^26 - 6552*I*a^28*c^19*x^25 - 20020*a^27*c^19*x^24 - 21840*I*a^26*c
^19*x^23 - 24960*a^25*c^19*x^22 - 43472*I*a^24*c^19*x^21 - 18018*a^23*c^19
*x^20 - 60060*I*a^22*c^19*x^19 - 60060*I*a^20*c^19*x^17 + 18018*a^19*c^19*
x^16 - 43472*I*a^18*c^19*x^15 + 24960*a^17*c^19*x^14 - 21840*I*a^16*c^19*x
^13 + 20020*a^15*c^19*x^12 - 6552*I*a^14*c^19*x^11 + 10752*a^13*c^19*x^10
- 280*I*a^12*c^19*x^9 + 3900*a^11*c^19*x^8 + 720*I*a^10*c^19*x^7 + 896*a^9
*c^19*x^6 + 336*I*a^8*c^19*x^5 + 105*a^7*c^19*x^4 + 70*I*a^6*c^19*x^3 + 6*
I*a^4*c^19*x - a^3*c^19)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(32) = 64$.

Time = 2.76 (sec) , antiderivative size = 439, normalized size of antiderivative = 11.55

$$\int \frac{e^{6i\arctan(ax)}x^2}{(c+a^2cx^2)^{19}}\,dx =$$

$$-\frac{}{210a^{39}c^{19}x^{36} + 1260ia^{38}c^{19}x^{35} + 14700ia^{36}c^{19}x^{33} - 22050a^{35}c^{19}x^{32} + 70560ia^{34}c^{19}x^{31} - 188160a^{33}c^{19}}$$

input
```
integrate((1+I*a*x)**6/(a**2*x**2+1)**3*x**2/(a**2*c*x**2+c)**19,x)
```

output
```
-(-6*a*x - I)/(210*a**39*c**19*x**36 + 1260*I*a**38*c**19*x**35 + 14700*I*
a**36*c**19*x**33 - 22050*a**35*c**19*x**32 + 70560*I*a**34*c**19*x**31 -
188160*a**33*c**19*x**30 + 151200*I*a**32*c**19*x**29 - 819000*a**31*c**19
*x**28 - 58800*I*a**30*c**19*x**27 - 2257920*a**29*c**19*x**26 - 1375920*I
*a**28*c**19*x**25 - 4204200*a**27*c**19*x**24 - 4586400*I*a**26*c**19*x**
23 - 5241600*a**25*c**19*x**22 - 9129120*I*a**24*c**19*x**21 - 3783780*a**
23*c**19*x**20 - 12612600*I*a**22*c**19*x**19 - 12612600*I*a**20*c**19*x**
17 + 3783780*a**19*c**19*x**16 - 9129120*I*a**18*c**19*x**15 + 5241600*a**
17*c**19*x**14 - 4586400*I*a**16*c**19*x**13 + 4204200*a**15*c**19*x**12 -
 1375920*I*a**14*c**19*x**11 + 2257920*a**13*c**19*x**10 - 58800*I*a**12*c
**19*x**9 + 819000*a**11*c**19*x**8 + 151200*I*a**10*c**19*x**7 + 188160*a
**9*c**19*x**6 + 70560*I*a**8*c**19*x**5 + 22050*a**7*c**19*x**4 + 14700*I
*a**6*c**19*x**3 + 1260*I*a**4*c**19*x - 210*a**3*c**19)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(30) = 60$.

Time = 0.19 (sec) , antiderivative size = 292, normalized size of antiderivative = 7.68

$$\int \frac{e^{6i\arctan(ax)}x^2}{(c+a^2cx^2)^{19}}\,dx$$

$$= \frac{}{210\,(a^{45}c^{19}x^{42} + 21\,a^{43}c^{19}x^{40} + 210\,a^{41}c^{19}x^{38} + 1330\,a^{39}c^{19}x^{36} + 5985\,a^{37}c^{19}x^{34} + 20349\,a^{35}c^{19}x^{32} + 542}$$

input
```
integrate((1+I*a*x)^6/(a^2*x^2+1)^3*x^2/(a^2*c*x^2+c)^19,x, algorithm="max
ima")
```

output
```
1/210*(6*a^7*x^7 - 35*I*a^6*x^6 - 84*a^5*x^5 + 105*I*a^4*x^4 + 70*a^3*x^3
- 21*I*a^2*x^2 - I)/(a^45*c^19*x^42 + 21*a^43*c^19*x^40 + 210*a^41*c^19*x^
38 + 1330*a^39*c^19*x^36 + 5985*a^37*c^19*x^34 + 20349*a^35*c^19*x^32 + 54
264*a^33*c^19*x^30 + 116280*a^31*c^19*x^28 + 203490*a^29*c^19*x^26 + 29393
0*a^27*c^19*x^24 + 352716*a^25*c^19*x^22 + 352716*a^23*c^19*x^20 + 293930*
a^21*c^19*x^18 + 203490*a^19*c^19*x^16 + 116280*a^17*c^19*x^14 + 54264*a^1
5*c^19*x^12 + 20349*a^13*c^19*x^10 + 5985*a^11*c^19*x^8 + 1330*a^9*c^19*x^
6 + 210*a^7*c^19*x^4 + 21*a^5*c^19*x^2 + a^3*c^19)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(30) = 60$.

Time = 0.13 (sec) , antiderivative size = 299, normalized size of antiderivative = 7.87

$$\int \frac{e^{6i\arctan(ax)}x^2}{(c+a^2cx^2)^{19}}\,dx = -\frac{358229025\,a^{14}x^{14} - 5340869100i\,a^{13}x^{13} - 37114698075\,a^{12}x^{12} + 159416118225i\,a^{11}x^{11} + 4730888061}{} + \frac{358229025\,a^{20}x^{20} + 7555375800i\,a^{19}x^{19} - 75901131600\,a^{18}x^{18} - 483051354975i\,a^{17}x^{17} + 2184946607}{}$$

input
```
integrate((1+I*a*x)^6/(a^2*x^2+1)^3*x^2/(a^2*c*x^2+c)^19,x, algorithm="gia
c")
```

output
```
-1/901943132160*(358229025*a^14*x^14 - 5340869100*I*a^13*x^13 - 3711469807
5*a^12*x^12 + 159416118225*I*a^11*x^11 + 473088806190*a^10*x^10 - 10268194
68675*I*a^9*x^9 - 1682288472150*a^8*x^8 + 2115551402250*I*a^7*x^7 + 205443
5046125*a^6*x^6 - 1535397250002*I*a^5*x^5 - 870854759775*a^4*x^4 + 3643075
33205*I*a^3*x^3 + 106553746740*a^2*x^2 - 19571887695*I*a*x - 1710785408)/(
(a*x - I)^15*a^3*c^19) + 1/901943132160*(358229025*a^20*x^20 + 7555375800*
I*a^19*x^19 - 75901131600*a^18*x^18 - 483051354975*I*a^17*x^17 + 218494660
7340*a^16*x^16 + 7469205450840*I*a^15*x^15 - 20031221295000*a^14*x^14 - 43
177004037300*I*a^13*x^13 + 76013078916950*a^12*x^12 + 110448380006328*I*a^
11*x^11 - 133277726128008*a^10*x^10 - 133908931763530*I*a^9*x^9 + 11193315
6213900*a^8*x^8 + 77492989590120*I*a^7*x^7 - 44041557267624*a^6*x^6 - 2024
4576347604*I*a^5*x^5 + 7349182966545*a^4*x^4 + 2026362494800*I*a^3*x^3 - 3
96520754280*a^2*x^2 - 48177926223*I*a*x + 2584181888)/((a*x + I)^21*a^3*c^
19)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{6i\arctan(ax)}x^2}{(c+a^2cx^2)^{19}}\,dx = \text{Hanged}$$

input
```
int((x^2*(a*x*1i + 1)^6)/((c + a^2*c*x^2)^19*(a^2*x^2 + 1)^3),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 234, normalized size of antiderivative = 6.16

$$\int \frac{e^{6i\arctan(ax)}x^2}{(c+a^2cx^2)^{19}}\,dx$$
$$= \frac{}{210a^3c^{19}(a^{42}x^{42}+21a^{40}x^{40}+210a^{38}x^{38}+1330a^{36}x^{36}+5985a^{34}x^{34}+20349a^{32}x^{32}+54264a^{30}x^{30}+116}$$

input
```
int((1+I*a*x)^6/(a^2*x^2+1)^3*x^2/(a^2*c*x^2+c)^19,x)
```

output
```
(6*a**7*x**7 - 35*a**6*i*x**6 - 84*a**5*x**5 + 105*a**4*i*x**4 + 70*a**3*x
**3 - 21*a**2*i*x**2 - i)/(210*a**3*c**19*(a**42*x**42 + 21*a**40*x**40 +
210*a**38*x**38 + 1330*a**36*x**36 + 5985*a**34*x**34 + 20349*a**32*x**32
+ 54264*a**30*x**30 + 116280*a**28*x**28 + 203490*a**26*x**26 + 293930*a**
24*x**24 + 352716*a**22*x**22 + 352716*a**20*x**20 + 293930*a**18*x**18 +
203490*a**16*x**16 + 116280*a**14*x**14 + 54264*a**12*x**12 + 20349*a**10*
x**10 + 5985*a**8*x**8 + 1330*a**6*x**6 + 210*a**4*x**4 + 21*a**2*x**2 + 1
))
```

3.390 $\int \frac{e^{4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 38

$$\int \frac{e^{4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx = -\frac{i+4ax}{60a^3c^9(1-iax)^{10}(1+iax)^6}$$

output
```
-1/60*(I+4*a*x)/a^3/c^9/(1-I*a*x)^10/(1+I*a*x)^6
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{e^{4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx = -\frac{i+4ax}{60a^3c^9(-i+ax)^6(i+ax)^{10}}$$

input
```
Integrate[(E^((4*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^9,x]
```

output
```
-1/60*(I + 4*a*x)/(a^3*c^9*(-I + a*x)^6*(I + a*x)^10)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {5605, 91}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 e^{4i\arctan(ax)}}{(a^2cx^2+c)^9}dx$$

$$\Big\downarrow \text{5605}$$

$$\frac{\int \frac{x^2}{(1-iax)^{11}(iax+1)^7}dx}{c^9}$$

$$\Big\downarrow \text{91}$$

$$-\frac{4ax+i}{60a^3c^9(1-iax)^{10}(1+iax)^6}$$

input
```
Int[(E^((4*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^9,x]
```

output
```
-1/60*(I + 4*a*x)/(a^3*c^9*(1 - I*a*x)^10*(1 + I*a*x)^6)
```

Defintions of rubi rules used

rule 91
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*((2*a*d*f*(n + p + 3
) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x)/(d^2*f^2*(n + p +
2)*(n + p + 3))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2
, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p + 3) - b*(
b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p + 1))*(a*
d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result
risch	$\frac{-\frac{i}{60a^3}-\frac{x}{15a^2}}{(ax+i)^{10}c^9(ax-i)^6}$
default	$-\frac{\frac{i}{60a^3}+\frac{x}{15a^2}}{c^9(ax+i)^{10}(ax-i)^6}$
gosper	$\frac{(-ax+i)(ax+i)(4ax+i)(iax+1)^4}{60a^3(a^2x^2+1)^{11}c^9}$
parallelrisch	$\frac{ix^{20}a^{17}+10ix^{18}a^{15}+45ix^{16}a^{13}+120ix^{14}a^{11}+210ix^{12}a^9+252ix^{10}a^7+210ix^8a^5+120ix^6a^3-4a^2x^5+60ix^4a+20x^3}{60c^9(a^2x^2+1)^{10}}$
orering	$\frac{ix^3(a^{13}x^{13}+4ia^{12}x^{12}+20ia^{10}x^{10}-20a^9x^9+36ia^8x^8-64a^7x^7+20ia^6x^6-90a^5x^5-20ix^4a^4-64a^3x^3-36ix^2a^2-20ax-20i)(}{60(a^2x^2+1)(a^2c\,x^2+c)^9}$
norman	$\frac{\frac{ia\,x^4}{c}+\frac{x^3}{3c}-\frac{a^2x^5}{15c}+\frac{2ia^3x^6}{c}+\frac{7ia^5x^8}{2c}+\frac{21ia^7x^{10}}{5c}+\frac{7ia^9x^{12}}{2c}+\frac{2ia^{11}x^{14}}{c}+\frac{3ia^{13}x^{16}}{4c}+\frac{ia^{15}x^{18}}{6c}+\frac{ia^{17}x^{20}}{60c}}{(a^2x^2+1)^{10}c^8}$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^2*x^2/(a^2*c*x^2+c)^9,x,method=_RETURNVERBOSE)
```

output
```
(-1/60*I/a^3-1/15*x/a^2)/(I+a*x)^10/c^9/(a*x-I)^6
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(30) = 60.

Time = 0.13 (sec) , antiderivative size = 169, normalized size of antiderivative = 4.45

$$\int \frac{e^{4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx =$$

$$-\frac{4\,ax+i}{60\,(a^{19}c^9x^{16}+4i\,a^{18}c^9x^{15}+20i\,a^{16}c^9x^{13}-20\,a^{15}c^9x^{12}+36i\,a^{14}c^9x^{11}-64\,a^{13}c^9x^{10}+20i\,a^{12}c^9x^9-90}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^2/(a^2*c*x^2+c)^9,x, algorithm="fric
as")
```

output
```
-1/60*(4*a*x + I)/(a^19*c^9*x^16 + 4*I*a^18*c^9*x^15 + 20*I*a^16*c^9*x^13
- 20*a^15*c^9*x^12 + 36*I*a^14*c^9*x^11 - 64*a^13*c^9*x^10 + 20*I*a^12*c^9
*x^9 - 90*a^11*c^9*x^8 - 20*I*a^10*c^9*x^7 - 64*a^9*c^9*x^6 - 36*I*a^8*c^9
*x^5 - 20*a^7*c^9*x^4 - 20*I*a^6*c^9*x^3 - 4*I*a^4*c^9*x + a^3*c^9)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(32) = 64$.

Time = 0.85 (sec) , antiderivative size = 194, normalized size of antiderivative = 5.11

$$\int \frac{e^{4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx$$

$$= \frac{}{60a^{19}c^9x^{16} + 240ia^{18}c^9x^{15} + 1200ia^{16}c^9x^{13} - 1200a^{15}c^9x^{12} + 2160ia^{14}c^9x^{11} - 3840a^{13}c^9x^{10} + 1200ia^{12}c}$$

input
```
integrate((1+I*a*x)**4/(a**2*x**2+1)**2*x**2/(a**2*c*x**2+c)**9,x)
```

output
```
(-4*a*x - I)/(60*a**19*c**9*x**16 + 240*I*a**18*c**9*x**15 + 1200*I*a**16*
c**9*x**13 - 1200*a**15*c**9*x**12 + 2160*I*a**14*c**9*x**11 - 3840*a**13*
c**9*x**10 + 1200*I*a**12*c**9*x**9 - 5400*a**11*c**9*x**8 - 1200*I*a**10*
c**9*x**7 - 3840*a**9*c**9*x**6 - 2160*I*a**8*c**9*x**5 - 1200*a**7*c**9*x
**4 - 1200*I*a**6*c**9*x**3 - 240*I*a**4*c**9*x + 60*a**3*c**9)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(30) = 60.

Time = 0.12 (sec) , antiderivative size = 155, normalized size of antiderivative = 4.08

$$\int \frac{e^{4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx = -\frac{4\,a^5x^5-15i\,a^4x^4-20\,a^3x^3+10i\,a^2x^2+i}{60\,(a^{23}c^9x^{20}+10\,a^{21}c^9x^{18}+45\,a^{19}c^9x^{16}+120\,a^{17}c^9x^{14}+210\,a^{15}c^9x^{12}+252\,a^{13}c^9x^{10}+210\,a^{11}c^9x^8+$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^2/(a^2*c*x^2+c)^9,x, algorithm="maxi
ma")
```

output
```
-1/60*(4*a^5*x^5 - 15*I*a^4*x^4 - 20*a^3*x^3 + 10*I*a^2*x^2 + I)/(a^23*c^9
*x^20 + 10*a^21*c^9*x^18 + 45*a^19*c^9*x^16 + 120*a^17*c^9*x^14 + 210*a^15
*c^9*x^12 + 252*a^13*c^9*x^10 + 210*a^11*c^9*x^8 + 120*a^9*c^9*x^6 + 45*a^
7*c^9*x^4 + 10*a^5*c^9*x^2 + a^3*c^9)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(30) = 60.

Time = 0.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.66

$$\int \frac{e^{4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx = -\frac{2145\,a^5x^5-12540i\,a^4x^4-30030\,a^3x^3+37080i\,a^2x^2+23841\,ax-6476i}{983040\,(ax-i)^6a^3c^9} + \frac{2145\,a^9x^9+21780i\,a^8x^8-99660\,a^7x^7-270480i\,a^6x^6+481446\,a^5x^5+584920i\,a^4x^4-486220\,a^3x^3-}{983040\,(ax+i)^{10}a^3c^9}$$

input
```
integrate((1+I*a*x)^4/(a^2*x^2+1)^2*x^2/(a^2*c*x^2+c)^9,x, algorithm="giac
")
```

output
```
-1/983040*(2145*a^5*x^5 - 12540*I*a^4*x^4 - 30030*a^3*x^3 + 37080*I*a^2*x^
2 + 23841*a*x - 6476*I)/((a*x - I)^6*a^3*c^9) + 1/983040*(2145*a^9*x^9 + 2
1780*I*a^8*x^8 - 99660*a^7*x^7 - 270480*I*a^6*x^6 + 481446*a^5*x^5 + 58492
0*I*a^4*x^4 - 486220*a^3*x^3 - 265680*I*a^2*x^2 + 84065*a*x + 9908*I)/((a*
x + I)^10*a^3*c^9)
```

Mupad [B] (verification not implemented)

Time = 27.20 (sec) , antiderivative size = 160, normalized size of antiderivative = 4.21

$$\int \frac{e^{4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx = -\frac{4\,a^5\,x^5-a^4\,x^4\,15\mathrm{i}-20\,a^3\,x^3+a^2\,x^2\,10}{60\,a^{23}\,c^9\,x^{20}+600\,a^{21}\,c^9\,x^{18}+2700\,a^{19}\,c^9\,x^{16}+7200\,a^{17}\,c^9\,x^{14}+12600\,a^{15}\,c^9\,x^{12}+15120\,a^{13}\,c^9\,x^{10}+}$$

input
```
int((x^2*(a*x*1i + 1)^4)/((c + a^2*c*x^2)^9*(a^2*x^2 + 1)^2),x)
```

output
```
-(a^2*x^2*10i - 20*a^3*x^3 - a^4*x^4*15i + 4*a^5*x^5 + 1i)/(60*a^3*c^9 + 6
00*a^5*c^9*x^2 + 2700*a^7*c^9*x^4 + 7200*a^9*c^9*x^6 + 12600*a^11*c^9*x^8
+ 15120*a^13*c^9*x^10 + 12600*a^15*c^9*x^12 + 7200*a^17*c^9*x^14 + 2700*a^
19*c^9*x^16 + 600*a^21*c^9*x^18 + 60*a^23*c^9*x^20)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.39

$$\int \frac{e^{4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx = \frac{-4a^5x^5+15a^4i\,x^4+20a^3x^3-10a^2i\,x^2-i}{60a^3c^9\left(a^{20}x^{20}+10a^{18}x^{18}+45a^{16}x^{16}+120a^{14}x^{14}+210a^{12}x^{12}+252a^{10}x^{10}+210a^8x^8+120a^6x^6+45a^4\right.}$$

input
```
int((1+I*a*x)^4/(a^2*x^2+1)^2*x^2/(a^2*c*x^2+c)^9,x)
```

output
```
( - 4*a**5*x**5 + 15*a**4*i*x**4 + 20*a**3*x**3 - 10*a**2*i*x**2 - i)/(60*
a**3*c**9*(a**20*x**20 + 10*a**18*x**18 + 45*a**16*x**16 + 120*a**14*x**14
 + 210*a**12*x**12 + 252*a**10*x**10 + 210*a**8*x**8 + 120*a**6*x**6 + 45*
a**4*x**4 + 10*a**2*x**2 + 1))
```

3.391 $\int \frac{e^{2i\arctan(ax)}x^2}{\left(c+a^2cx^2\right)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 38

$$\int \frac{e^{2i\arctan(ax)}x^2}{\left(c+a^2cx^2\right)^3}\,dx = -\frac{i+2ax}{6a^3c^3(1-iax)^3(1+iax)}$$

output
```
-1/6*(I+2*a*x)/a^3/c^3/(1-I*a*x)^3/(1+I*a*x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{e^{2i\arctan(ax)}x^2}{\left(c+a^2cx^2\right)^3}\,dx = \frac{i+2ax}{6a^3c^3(-i+ax)(i+ax)^3}$$

input
```
Integrate[(E^((2*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^3,x]
```

output
```
(I + 2*a*x)/(6*a^3*c^3*(-I + a*x)*(I + a*x)^3)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5605, 91}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 e^{2i \arctan(ax)}}{(a^2cx^2+c)^3} \, dx$$

$$\Big\downarrow \text{5605}$$

$$\frac{\int \frac{x^2}{(1-iax)^4(iax+1)^2}dx}{c^3}$$

$$\Big\downarrow \text{91}$$

$$-\frac{2ax+i}{6a^3c^3(1-iax)^3(1+iax)}$$

input
```
Int[(E^((2*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^3,x]
```

output
```
-1/6*(I + 2*a*x)/(a^3*c^3*(1 - I*a*x)^3*(1 + I*a*x))
```

Defintions of rubi rules used

rule 91
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*((2*a*d*f*(n + p + 3
) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x)/(d^2*f^2*(n + p +
2)*(n + p + 3))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2
, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p + 3) - b*(
b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p + 1))*(a*
d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\frac{x}{3a^2}+\frac{i}{6a^3}}{c^3(ax+i)^3(ax-i)}$	34
risch	$\frac{\frac{x}{3a^2}+\frac{i}{6a^3}}{c^3(ax+i)^3(ax-i)}$	34
orering	$-\frac{ix^3(ax+2i)(iax+1)^2}{6(a^2cx^2+c)^3}$	34
parallelrisch	$\frac{ix^6a^3+3ix^4a+2x^3}{6c^3(a^2x^2+1)^3}$	39
norman	$\frac{\frac{x^3}{3c}+\frac{ia\,x^4}{2c}+\frac{ia^3x^6}{6c}}{(a^2x^2+1)^3c^2}$	47
gosper	$\frac{(-ax+i)(ax+i)(2ax+i)(iax+1)^2}{6a^3(a^2x^2+1)^4c^3}$	49

input
```
int((1+I*a*x)^2/(a^2*x^2+1)*x^2/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(1/3*x/a^2+1/6*I/a^3)/(I+a*x)^3/(a*x-I)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int \frac{e^{2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = \frac{2\,ax+i}{6\,(a^7c^3x^4+2i\,a^6c^3x^3+2i\,a^4c^3x-a^3c^3)}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)*x^2/(a^2*c*x^2+c)^3,x, algorithm="fricas
")
```

output
```
1/6*(2*a*x + I)/(a^7*c^3*x^4 + 2*I*a^6*c^3*x^3 + 2*I*a^4*c^3*x - a^3*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int \frac{e^{2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = -\frac{-2ax-i}{6a^7c^3x^4+12ia^6c^3x^3+12ia^4c^3x-6a^3c^3}$$

input
```
integrate((1+I*a*x)**2/(a**2*x**2+1)*x**2/(a**2*c*x**2+c)**3,x)
```

output
```
-(-2*a*x - I)/(6*a**7*c**3*x**4 + 12*I*a**6*c**3*x**3 + 12*I*a**4*c**3*x -
 6*a**3*c**3)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(30) = 60.

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.63

$$\int \frac{e^{2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = \frac{2\,a^3x^3-3i\,a^2x^2-i}{6\,(a^9c^3x^6+3\,a^7c^3x^4+3\,a^5c^3x^2+a^3c^3)}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)*x^2/(a^2*c*x^2+c)^3,x, algorithm="maxima
")
```

output
```
1/6*(2*a^3*x^3 - 3*I*a^2*x^2 - I)/(a^9*c^3*x^6 + 3*a^7*c^3*x^4 + 3*a^5*c^3
*x^2 + a^3*c^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \frac{e^{2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = -\frac{1}{16\,(ax-i)a^3c^3} + \frac{3\,a^2x^2+12i\,ax-5}{48\,(ax+i)^3a^3c^3}$$

input
```
integrate((1+I*a*x)^2/(a^2*x^2+1)*x^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
-1/16/((a*x - I)*a^3*c^3) + 1/48*(3*a^2*x^2 + 12*I*a*x - 5)/((a*x + I)^3*a
^3*c^3)
```

Mupad [B] (verification not implemented)

Time = 23.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{e^{2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = \frac{\frac{x}{3\,a^6\,c^3}+\frac{1\mathrm{i}}{6\,a^7\,c^3}}{\frac{x\,2\mathrm{i}}{a^3}-\frac{1}{a^4}+x^4+\frac{x^3\,2\mathrm{i}}{a}}$$

input
```
int((x^2*(a*x*1i + 1)^2)/((c + a^2*c*x^2)^3*(a^2*x^2 + 1)),x)
```

output
```
(1i/(6*a^7*c^3) + x/(3*a^6*c^3))/((x*2i)/a^3 - 1/a^4 + x^4 + (x^3*2i)/a)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int \frac{e^{2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = \frac{2a^3x^3-3a^2i\,x^2-i}{6a^3c^3\,(a^6x^6+3a^4x^4+3a^2x^2+1)}$$

input
```
int((1+I*a*x)^2/(a^2*x^2+1)*x^2/(a^2*c*x^2+c)^3,x)
```

output
```
(2*a**3*x**3 - 3*a**2*i*x**2 - i)/(6*a**3*c**3*(a**6*x**6 + 3*a**4*x**4 +
3*a**2*x**2 + 1))
```

3.392 $\int \frac{e^{-2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 38

$$\int \frac{e^{-2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = \frac{i-2ax}{6a^3c^3(1-iax)(1+iax)^3}$$

output
```
1/6*(I-2*a*x)/a^3/c^3/(1-I*a*x)/(1+I*a*x)^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{e^{-2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = \frac{-i+2ax}{6a^3c^3(-i+ax)^3(i+ax)}$$

input
```
Integrate[x^2/(E^((2*I)*ArcTan[a*x])*(c + a^2*c*x^2)^3),x]
```

output
```
(-I + 2*a*x)/(6*a^3*c^3*(-I + a*x)^3*(I + a*x))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {5605, 91}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 e^{-2i \arctan(ax)}}{(a^2cx^2+c)^3} dx$$

$$\Big\downarrow \text{ 5605}$$

$$\frac{\int \frac{x^2}{(1-iax)^2(iax+1)^4}dx}{c^3}$$

$$\Big\downarrow \text{ 91}$$

$$\frac{-2ax+i}{6a^3c^3(1-iax)(1+iax)^3}$$

input
```
Int[x^2/(E^((2*I)*ArcTan[a*x])*(c + a^2*c*x^2)^3),x]
```

output
```
(I - 2*a*x)/(6*a^3*c^3*(1 - I*a*x)*(1 + I*a*x)^3)
```

Defintions of rubi rules used

rule 91
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*((2*a*d*f*(n + p + 3
) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x)/(d^2*f^2*(n + p +
2)*(n + p + 3))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2
, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p + 3) - b*(
b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p + 1))*(a*
d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{\frac{x}{3a^2}-\frac{i}{6a^3}}{c^3(ax-i)^3(ax+i)}$	34
parallelrisch	$-\frac{ix^4a+2x^3}{6c^3(-ax+i)^2(a^2x^2+1)}$	39
orering	$-\frac{ix^3(-ax+2i)(a^2x^2+1)^2}{6(iax+1)^2(a^2c\,x^2+c)^3}$	46
norman	$\frac{\frac{x^3}{3c}-\frac{ia\,x^4}{2c}-\frac{ia^3x^6}{6c}}{(a^2x^2+1)^3c^2}$	47
gosper	$-\frac{(-2ax+i)(ax+i)(-ax+i)}{6(a^2x^2+1)^2c^3(iax+1)^2a^3}$	49
default	$\frac{-\frac{1}{16a^3(ax+i)}-\frac{i}{8a^3(-ax+i)^2}-\frac{1}{12a^3(-ax+i)^3}-\frac{1}{16a^3(-ax+i)}}{c^3}$	62

input
```
int(x^2/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/3*x/a^2-1/6*I/a^3)/c^3/(a*x-I)^3/(I+a*x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int \frac{e^{-2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx=\frac{2\,ax-i}{6\,(a^7c^3x^4-2i\,a^6c^3x^3-2i\,a^4c^3x-a^3c^3)}$$

input
```
integrate(x^2/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^3,x, algorithm="fricas
")
```

output
```
1/6*(2*a*x - I)/(a^7*c^3*x^4 - 2*I*a^6*c^3*x^3 - 2*I*a^4*c^3*x - a^3*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.39

$$\int \frac{e^{-2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = -\frac{-2ax+i}{6a^7c^3x^4-12ia^6c^3x^3-12ia^4c^3x-6a^3c^3}$$

input
```
integrate(x**2/(1+I*a*x)**2*(a**2*x**2+1)/(a**2*c*x**2+c)**3,x)
```

output
```
-(-2*a*x + I)/(6*a**7*c**3*x**4 - 12*I*a**6*c**3*x**3 - 12*I*a**4*c**3*x -
 6*a**3*c**3)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e^{-2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^3,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(30) = 60.

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.11

$$\int \frac{e^{-2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = -\frac{1}{32\,a^3c^3\left(\frac{2i}{i\,ax+1}-i\right)} - \frac{-\frac{3i\,a^3c^6}{i\,ax+1} - \frac{6i\,a^3c^6}{(i\,ax+1)^2} + \frac{4i\,a^3c^6}{(i\,ax+1)^3}}{48\,a^6c^9}$$

input
```
integrate(x^2/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
-1/32/(a^3*c^3*(2*I/(I*a*x + 1) - I)) - 1/48*(-3*I*a^3*c^6/(I*a*x + 1) - 6
*I*a^3*c^6/(I*a*x + 1)^2 + 4*I*a^3*c^6/(I*a*x + 1)^3)/(a^6*c^9)
```

Mupad [B] (verification not implemented)

Time = 22.99 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \frac{e^{-2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = \frac{2\,a^3\,x^3+a^2\,x^2\,3\mathrm{i}+1\mathrm{i}}{6\,a^9\,c^3\,x^6+18\,a^7\,c^3\,x^4+18\,a^5\,c^3\,x^2+6\,a^3\,c^3}$$

input
```
int((x^2*(a^2*x^2 + 1))/((c + a^2*c*x^2)^3*(a*x*1i + 1)^2),x)
```

output
```
(a^2*x^2*3i + 2*a^3*x^3 + 1i)/(6*a^3*c^3 + 18*a^5*c^3*x^2 + 18*a^7*c^3*x^4
 + 6*a^9*c^3*x^6)
```

Reduce [F]

$$\int \frac{e^{-2i\arctan(ax)}x^2}{(c+a^2cx^2)^3}\,dx = -\frac{\int \frac{x^2}{a^6x^6-2a^5i\,x^5+a^4x^4-4a^3i\,x^3-a^2x^2-2aix-1}dx}{c^3}$$

input
```
int(x^2/(1+I*a*x)^2*(a^2*x^2+1)/(a^2*c*x^2+c)^3,x)
```

output
```
( - int(x**2/(a**6*x**6 - 2*a**5*i*x**5 + a**4*x**4 - 4*a**3*i*x**3 - a**2
*x**2 - 2*a*i*x - 1),x))/c**3
```

3.393 $\int \frac{e^{-4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 38

$$\int \frac{e^{-4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx=\frac{i-4ax}{60a^3c^9(1-iax)^6(1+iax)^{10}}$$

output
```
1/60*(I-4*a*x)/a^3/c^9/(1-I*a*x)^6/(1+I*a*x)^10
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{e^{-4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx=\frac{i-4ax}{60a^3c^9(-i+ax)^{10}(i+ax)^6}$$

input
```
Integrate[x^2/(E^((4*I)*ArcTan[a*x])*(c + a^2*c*x^2)^9),x]
```

output
```
(I - 4*a*x)/(60*a^3*c^9*(-I + a*x)^10*(I + a*x)^6)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {5605, 91}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 e^{-4 i \arctan(ax)}}{(a^2cx^2+c)^9} \, dx$$

$$\Big\downarrow \text{ 5605}$$

$$\frac{\int \frac{x^2}{(1-iax)^7(iax+1)^{11}}dx}{c^9}$$

$$\Big\downarrow \text{ 91}$$

$$\frac{-4ax+i}{60a^3c^9(1-iax)^6(1+iax)^{10}}$$

input
```
Int[x^2/(E^((4*I)*ArcTan[a*x])*(c + a^2*c*x^2)^9),x]
```

output
```
(I - 4*a*x)/(60*a^3*c^9*(1 - I*a*x)^6*(1 + I*a*x)^10)
```

Defintions of rubi rules used

rule 91
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*((2*a*d*f*(n + p + 3
) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x)/(d^2*f^2*(n + p +
2)*(n + p + 3))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2
, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p + 3) - b*(
b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p + 1))*(a*
d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result
risch	$\frac{\frac{i}{60a^3}-\frac{x}{15a^2}}{c^9(ax-i)^{10}(ax+i)^6}$
gosper	$-\frac{(-4ax+i)(ax+i)(-ax+i)}{60(a^2x^2+1)^7c^9(iax+1)^4a^3}$
parallelrisch	$-\frac{ix^{16}a^{13}+4x^{15}a^{12}+20x^{13}a^{10}-20ix^{12}a^9+36x^{11}a^8-64ix^{10}a^7+20x^9a^6-90ix^8a^5-20a^4x^7-64ix^6a^3-36a^2x^5-20ix^4a-20}{60c^9(-ax+i)^4(a^2x^2+1)^6}$
orering	$\frac{ix^3(-a^{13}x^{13}+4ia^{12}x^{12}+20ia^{10}x^{10}+20a^9x^9+36ia^8x^8+64a^7x^7+20ia^6x^6+90a^5x^5-20ix^4a^4+64a^3x^3-36ix^2a^2+20ax-20i}{60(iax+1)^4(a^2cx^2+c)^9}$
norman	$\frac{-\frac{iax^4}{c}+\frac{x^3}{3c}-\frac{a^2x^5}{15c}-\frac{2ia^3x^6}{c}-\frac{7ia^5x^8}{2c}-\frac{21ia^7x^{10}}{5c}-\frac{7ia^9x^{12}}{2c}-\frac{2ia^{11}x^{14}}{c}-\frac{3ia^{13}x^{16}}{4c}-\frac{ia^{15}x^{18}}{6c}-\frac{ia^{17}x^{20}}{60c}}{(a^2x^2+1)^{10}c^8}$
default	$\frac{13i}{16384a^3(ax+i)^4}-\frac{i}{12288a^3(ax+i)^6}-\frac{121i}{65536a^3(ax+i)^2}-\frac{7}{20480a^3(ax+i)^5}+\frac{11}{8192a^3(ax+i)^3}-\frac{143}{65536a^3(ax+i)}+\frac{21i}{8192a^3(-ax+i)^4}+\frac{}{1280}$

input
```
int(x^2/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^9,x,method=_RETURNVERBOSE)
```

output
```
(1/60*I/a^3-1/15*x/a^2)/c^9/(a*x-I)^10/(I+a*x)^6
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(30) = 60.

Time = 0.13 (sec) , antiderivative size = 169, normalized size of antiderivative = 4.45

$$\int \frac{e^{-4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx =$$

$$-\frac{4\,ax-i}{60\,(a^{19}c^9x^{16}-4i\,a^{18}c^9x^{15}-20i\,a^{16}c^9x^{13}-20\,a^{15}c^9x^{12}-36i\,a^{14}c^9x^{11}-64\,a^{13}c^9x^{10}-20i\,a^{12}c^9x^9-90}$$

input
```
integrate(x^2/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^9,x, algorithm="fric
as")
```

output
```
-1/60*(4*a*x - I)/(a^19*c^9*x^16 - 4*I*a^18*c^9*x^15 - 20*I*a^16*c^9*x^13
- 20*a^15*c^9*x^12 - 36*I*a^14*c^9*x^11 - 64*a^13*c^9*x^10 - 20*I*a^12*c^9
*x^9 - 90*a^11*c^9*x^8 + 20*I*a^10*c^9*x^7 - 64*a^9*c^9*x^6 + 36*I*a^8*c^9
*x^5 - 20*a^7*c^9*x^4 + 20*I*a^6*c^9*x^3 + 4*I*a^4*c^9*x + a^3*c^9)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(31) = 62$.

Time = 0.77 (sec) , antiderivative size = 192, normalized size of antiderivative = 5.05

$$\int \frac{e^{-4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx$$

$$= \frac{}{60a^{19}c^9x^{16} - 240ia^{18}c^9x^{15} - 1200ia^{16}c^9x^{13} - 1200a^{15}c^9x^{12} - 2160ia^{14}c^9x^{11} - 3840a^{13}c^9x^{10} - 1200ia^{12}c}$$

input
```
integrate(x**2/(1+I*a*x)**4*(a**2*x**2+1)**2/(a**2*c*x**2+c)**9,x)
```

output
```
(-4*a*x + I)/(60*a**19*c**9*x**16 - 240*I*a**18*c**9*x**15 - 1200*I*a**16*
c**9*x**13 - 1200*a**15*c**9*x**12 - 2160*I*a**14*c**9*x**11 - 3840*a**13*
c**9*x**10 - 1200*I*a**12*c**9*x**9 - 5400*a**11*c**9*x**8 + 1200*I*a**10*
c**9*x**7 - 3840*a**9*c**9*x**6 + 2160*I*a**8*c**9*x**5 - 1200*a**7*c**9*x
**4 + 1200*I*a**6*c**9*x**3 + 240*I*a**4*c**9*x + 60*a**3*c**9)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e^{-4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^9,x, algorithm="maxi
ma")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(30) = 60.

Time = 0.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.66

$$\begin{aligned}&\int \frac{e^{-4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx\\&=-\frac{2145\,a^5x^5+12540i\,a^4x^4-30030\,a^3x^3-37080i\,a^2x^2+23841\,ax+6476i}{983040\,(ax+i)^6a^3c^9}\\&\quad+\frac{2145\,a^9x^9-21780i\,a^8x^8-99660\,a^7x^7+270480i\,a^6x^6+481446\,a^5x^5-584920i\,a^4x^4-486220\,a^3x^3+}{983040\,(ax-i)^{10}a^3c^9}\end{aligned}$$

input
```
integrate(x^2/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^9,x, algorithm="giac
")
```

output
```
-1/983040*(2145*a^5*x^5 + 12540*I*a^4*x^4 - 30030*a^3*x^3 - 37080*I*a^2*x^
2 + 23841*a*x + 6476*I)/((a*x + I)^6*a^3*c^9) + 1/983040*(2145*a^9*x^9 - 2
1780*I*a^8*x^8 - 99660*a^7*x^7 + 270480*I*a^6*x^6 + 481446*a^5*x^5 - 58492
0*I*a^4*x^4 - 486220*a^3*x^3 + 265680*I*a^2*x^2 + 84065*a*x - 9908*I)/((a*
x - I)^10*a^3*c^9)
```

Mupad [B] (verification not implemented)

Time = 26.97 (sec) , antiderivative size = 159, normalized size of antiderivative = 4.18

$$\int \frac{e^{-4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx$$
$$= \frac{-4\,a^5\,x^5 - a^4\,x^4\,15\mathrm{i} + 20\,a^3\,x^3 + a^2\,x^2\,10\mathrm{i}}{60\,a^{23}\,c^9\,x^{20} + 600\,a^{21}\,c^9\,x^{18} + 2700\,a^{19}\,c^9\,x^{16} + 7200\,a^{17}\,c^9\,x^{14} + 12600\,a^{15}\,c^9\,x^{12} + 15120\,a^{13}\,c^9\,x^{10} + 12}$$

input
```
int((x^2*(a^2*x^2 + 1)^2)/((c + a^2*c*x^2)^9*(a*x*1i + 1)^4),x)
```

output
```
(a^2*x^2*10i + 20*a^3*x^3 - a^4*x^4*15i - 4*a^5*x^5 + 1i)/(60*a^3*c^9 + 60
0*a^5*c^9*x^2 + 2700*a^7*c^9*x^4 + 7200*a^9*c^9*x^6 + 12600*a^11*c^9*x^8 +
 15120*a^13*c^9*x^10 + 12600*a^15*c^9*x^12 + 7200*a^17*c^9*x^14 + 2700*a^1
9*c^9*x^16 + 600*a^21*c^9*x^18 + 60*a^23*c^9*x^20)
```

Reduce [F]

$$\int \frac{e^{-4i\arctan(ax)}x^2}{(c+a^2cx^2)^9}\,dx$$
$$= \frac{\int \frac{x^2}{a^{18}x^{18}-4a^{17}i\,x^{17}+a^{16}x^{16}-24a^{15}i\,x^{15}-20a^{14}x^{14}-56a^{13}i\,x^{13}-84a^{12}x^{12}-56a^{11}i\,x^{11}-154a^{10}x^{10}-154a^{8}x^{8}+56a^{7}i\,x^{7}-84a^{6}x^{6}+56a^{5}i\,x^{5}-}}{c^9}$$

input
```
int(x^2/(1+I*a*x)^4*(a^2*x^2+1)^2/(a^2*c*x^2+c)^9,x)
```

output
```
int(x**2/(a**18*x**18 - 4*a**17*i*x**17 + a**16*x**16 - 24*a**15*i*x**15 -
 20*a**14*x**14 - 56*a**13*i*x**13 - 84*a**12*x**12 - 56*a**11*i*x**11 - 1
54*a**10*x**10 - 154*a**8*x**8 + 56*a**7*i*x**7 - 84*a**6*x**6 + 56*a**5*i
*x**5 - 20*a**4*x**4 + 24*a**3*i*x**3 + a**2*x**2 + 4*a*i*x + 1),x)/c**9
```

3.394 $\int \frac{e^{5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 65

$$\int \frac{e^{5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = -\frac{(i+5ax)\sqrt{1+a^2x^2}}{120a^3c^{13}(1-iax)^{15}(1+iax)^{10}\sqrt{c+a^2cx^2}}$$

output
```
-1/120*(I+5*a*x)*(a^2*x^2+1)^(1/2)/a^3/c^13/(1-I*a*x)^15/(1+I*a*x)^10/(a^2
*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int \frac{e^{5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \frac{(1-5iax)\sqrt{1+a^2x^2}}{120a^3c^{13}(-i+ax)^{10}(i+ax)^{15}\sqrt{c+a^2cx^2}}$$

input
```
Integrate[(E^((5*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^(27/2),x]
```

output
```
((1 - (5*I)*a*x)*Sqrt[1 + a^2*x^2])/(120*a^3*c^13*(-I + a*x)^10*(I + a*x)^
15*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {5608, 5605, 91}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 e^{5 i \arctan(ax)}}{(a^2cx^2+c)^{27/2}} dx$$

$$\Big\downarrow \text{5608}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{5i\arctan(ax)}x^2}{(a^2x^2+1)^{27/2}}dx}{c^{13}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5605}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{x^2}{(1-iax)^{16}(iax+1)^{11}}dx}{c^{13}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{91}$$

$$-\frac{(5ax+i)\sqrt{a^2x^2+1}}{120a^3c^{13}(1-iax)^{15}(1+iax)^{10}\sqrt{a^2cx^2+c}}$$

input
```
Int[(E^((5*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^(27/2),x]
```

output
```
-1/120*((I + 5*a*x)*Sqrt[1 + a^2*x^2])/(a^3*c^13*(1 - I*a*x)^15*(1 + I*a*x
)^10*Sqrt[c + a^2*c*x^2])
```

Defintions of rubi rules used

rule 91
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*((2*a*d*f*(n + p + 3
) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x)/(d^2*f^2*(n + p +
2)*(n + p + 3))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2
, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p + 3) - b*(
b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p + 1))*(a*
d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

rule 5608
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p])   Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
 m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

method	result
default	$-\frac{\sqrt{c(a^2x^2+1)}\,(5iax-1)}{120\sqrt{a^2x^2+1}\,c^{14}a^3(ax+i)^{15}(-ax+i)^{10}}$
gosper	$\frac{(-ax+i)(ax+i)(5ax+i)(iax+1)^5}{120a^3(a^2x^2+1)^{\frac{5}{2}}(a^2c\,x^2+c)^{\frac{27}{2}}}$
risch	$\frac{\sqrt{a^2x^2+1}\left(\frac{1}{120a^3}-\frac{ix}{24a^2}\right)}{c^{13}\sqrt{c(a^2x^2+1)}\,(ax+i)^{15}(ax-i)^{10}}$
orering	$\frac{x^3(a^{22}x^{22}+5ia^{21}x^{21}+40ia^{19}x^{19}-50a^{18}x^{18}+126ia^{17}x^{17}-280a^{16}x^{16}+160ia^{15}x^{15}-765a^{14}x^{14}-105ia^{13}x^{13}-1248a^{12}x^{12}-720ia^{1}}{120(a^2x^2+1)^{\frac{3}{2}}(}$

input
```
int((1+I*a*x)^5/(a^2*x^2+1)^(5/2)*x^2/(a^2*c*x^2+c)^(27/2),x,method=_RETUR
NVERBOSE)
```

output
```
-1/120/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)*(5*I*a*x-1)/c^14/a^3/(I+a*x
)^15/(I-a*x)^10
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(53) = 106$.

Time = 0.19 (sec) , antiderivative size = 496, normalized size of antiderivative = 7.63

$$\int \frac{e^{5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \frac{(i\,a^{22}x^{25} - 5\,a^{21}x^{24}}{120\,(a^{27}c^{14}x^{27} + 5i\,a^{26}c^{14}x^{26} + a^{25}c^{14}x^{25} + 45i\,a^{24}c^{14}x^{24} - 50\,a^{23}c^{14}x^{23} + 166i\,a^{22}c}$$

input
```
integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)*x^2/(a^2*c*x^2+c)^(27/2),x, algori
thm="fricas")
```

output
```
1/120*(I*a^22*x^25 - 5*a^21*x^24 - 40*a^19*x^22 - 50*I*a^18*x^21 - 126*a^1
7*x^20 - 280*I*a^16*x^19 - 160*a^15*x^18 - 765*I*a^14*x^17 + 105*a^13*x^16
 - 1248*I*a^12*x^15 + 720*a^11*x^14 - 1260*I*a^10*x^13 + 1260*a^9*x^12 - 7
20*I*a^8*x^11 + 1248*a^7*x^10 - 105*I*a^6*x^9 + 765*a^5*x^8 + 160*I*a^4*x^
7 + 280*a^3*x^6 + 126*I*a^2*x^5 + 50*a*x^4 + 40*I*x^3)*sqrt(a^2*c*x^2 + c)
*sqrt(a^2*x^2 + 1)/(a^27*c^14*x^27 + 5*I*a^26*c^14*x^26 + a^25*c^14*x^25 +
 45*I*a^24*c^14*x^24 - 50*a^23*c^14*x^23 + 166*I*a^22*c^14*x^22 - 330*a^21
*c^14*x^21 + 286*I*a^20*c^14*x^20 - 1045*a^19*c^14*x^19 + 55*I*a^18*c^14*x
^18 - 2013*a^17*c^14*x^17 - 825*I*a^16*c^14*x^16 - 2508*a^15*c^14*x^15 - 1
980*I*a^14*c^14*x^14 - 1980*a^13*c^14*x^13 - 2508*I*a^12*c^14*x^12 - 825*a
^11*c^14*x^11 - 2013*I*a^10*c^14*x^10 + 55*a^9*c^14*x^9 - 1045*I*a^8*c^14*
x^8 + 286*a^7*c^14*x^7 - 330*I*a^6*c^14*x^6 + 166*a^5*c^14*x^5 - 50*I*a^4*
c^14*x^4 + 45*a^3*c^14*x^3 + I*a^2*c^14*x^2 + 5*a*c^14*x + I*c^14)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \text{Timed out}$$

input
```
integrate((1+I*a*x)**5/(a**2*x**2+1)**(5/2)*x**2/(a**2*c*x**2+c)**(27/2),x
)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e^{5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)*x^2/(a^2*c*x^2+c)^(27/2),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{e^{5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \int \frac{(i\,ax+1)^5x^2}{(a^2cx^2+c)^{\frac{27}{2}}(a^2x^2+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate((1+I*a*x)^5/(a^2*x^2+1)^(5/2)*x^2/(a^2*c*x^2+c)^(27/2),x, algori
thm="giac")
```

output
```
integrate((I*a*x + 1)^5*x^2/((a^2*c*x^2 + c)^(27/2)*(a^2*x^2 + 1)^(5/2)),
x)
```

Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int \frac{e^{5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = -\frac{c\,(a\,x-\mathrm{i})^5\,(5\,a\,x+1\mathrm{i})\,1\mathrm{i}}{120\,a^3\,\left(c\,\left(a^2\,x^2+1\right)\right)^{29/2}\,\sqrt{a^2\,x^2+1}}$$

input
```
int((x^2*(a*x*1i + 1)^5)/((c + a^2*c*x^2)^(27/2)*(a^2*x^2 + 1)^(5/2)),x)
```

output
```
-(c*(a*x - 1i)^5*(5*a*x + 1i)*1i)/(120*a^3*(c*(a^2*x^2 + 1))^(29/2)*(a^2*x
^2 + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.77

$$\int \frac{e^{5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \frac{\sqrt{c}\,(-5a^6i\,x^6}{120a^3c^{14}\,(a^{30}x^{30}+15a^{28}x^{28}+105a^{26}x^{26}+455a^{24}x^{24}+1365a^{22}x^{22}+3003a^{20}x^{20}+}$$

input
```
int((1+I*a*x)^5/(a^2*x^2+1)^(5/2)*x^2/(a^2*c*x^2+c)^(27/2),x)
```

output
```
(sqrt(c)*( - 5*a**6*i*x**6 - 24*a**5*x**5 + 45*a**4*i*x**4 + 40*a**3*x**3
- 15*a**2*i*x**2 - i))/(120*a**3*c**14*(a**30*x**30 + 15*a**28*x**28 + 105
*a**26*x**26 + 455*a**24*x**24 + 1365*a**22*x**22 + 3003*a**20*x**20 + 500
5*a**18*x**18 + 6435*a**16*x**16 + 6435*a**14*x**14 + 5005*a**12*x**12 + 3
003*a**10*x**10 + 1365*a**8*x**8 + 455*a**6*x**6 + 105*a**4*x**4 + 15*a**2
*x**2 + 1))
```

3.395 $\int \frac{e^{3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 65

$$\int \frac{e^{3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = -\frac{(i+3ax)\sqrt{1+a^2x^2}}{24a^3c^5(1-iax)^6(1+iax)^3\sqrt{c+a^2cx^2}}$$

output
```
-1/24*(I+3*a*x)*(a^2*x^2+1)^(1/2)/a^3/c^5/(1-I*a*x)^6/(1+I*a*x)^3/(a^2*c*x
^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{e^{3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = \frac{i(i+3ax)\sqrt{1+a^2x^2}}{24a^3c^5(-i+ax)^3(i+ax)^6\sqrt{c+a^2cx^2}}$$

input
```
Integrate[(E^((3*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^(11/2),x]
```

output
```
((I/24)*(I + 3*a*x)*Sqrt[1 + a^2*x^2])/(a^3*c^5*(-I + a*x)^3*(I + a*x)^6*S
qrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {5608, 5605, 91}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 e^{3i\arctan(ax)}}{(a^2cx^2+c)^{11/2}}\,dx$$

$$\Big\downarrow \text{ 5608}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{3i\arctan(ax)}x^2}{(a^2x^2+1)^{11/2}}dx}{c^5\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5605}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{x^2}{(1-iax)^7(iax+1)^4}dx}{c^5\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 91}$$

$$-\frac{(3ax+i)\sqrt{a^2x^2+1}}{24a^3c^5(1-iax)^6(1+iax)^3\sqrt{a^2cx^2+c}}$$

input
```
Int[(E^((3*I)*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^(11/2),x]
```

output
```
-1/24*((I + 3*a*x)*Sqrt[1 + a^2*x^2])/(a^3*c^5*(1 - I*a*x)^6*(1 + I*a*x)^3
*Sqrt[c + a^2*c*x^2])
```

Defintions of rubi rules used

rule 91
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*((2*a*d*f*(n + p + 3
) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x)/(d^2*f^2*(n + p +
2)*(n + p + 3))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2
, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p + 3) - b*(
b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p + 1))*(a*
d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

rule 5608
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p])   Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
 m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{\sqrt{c(a^2x^2+1)}\,(3iax-1)}{24\sqrt{a^2x^2+1}\,c^6a^3(ax+i)^6(-ax+i)^3}$	57
gosper	$\frac{(-ax+i)(ax+i)(3ax+i)(iax+1)^3}{24a^3(a^2x^2+1)^{\frac{3}{2}}(a^2c\,x^2+c)^{\frac{11}{2}}}$	58
risch	$\frac{\sqrt{a^2x^2+1}\left(\frac{ix}{8a^2}-\frac{1}{24a^3}\right)}{c^5\sqrt{c(a^2x^2+1)}\,(ax+i)^6(ax-i)^3}$	58
orering	$-\frac{x^3\left(x^6a^6+3ix^5a^5+8ix^3a^3-6a^2x^2+6iax-8\right)(iax+1)^3}{24\sqrt{a^2x^2+1}\,(a^2c\,x^2+c)^{\frac{11}{2}}}$	78

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2/(a^2*c*x^2+c)^(11/2),x,method=_RETUR
NVERBOSE)
```

output
```
-1/24/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)*(3*I*a*x-1)/c^6/a^3/(I+a*x)^
6/(I-a*x)^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(53) = 106.

Time = 0.13 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.95

$$\int \frac{e^{3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = \frac{(i\,a^6x^9 - 3\,a^5x^8 - 8\,a^3x^6 - 6i\,a^2x^5 - 6\,ax^4 - 8i\,x^3)}{24\,(a^{11}c^6x^{11} + 3i\,a^{10}c^6x^{10} + a^9c^6x^9 + 11i\,a^8c^6x^8 - 6\,a^7c^6x^7 + 14i\,a^6c^6x^6 - 14\,a^5c^6x}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2/(a^2*c*x^2+c)^(11/2),x, algori
thm="fricas")
```

output
```
1/24*(I*a^6*x^9 - 3*a^5*x^8 - 8*a^3*x^6 - 6*I*a^2*x^5 - 6*a*x^4 - 8*I*x^3)
*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)/(a^11*c^6*x^11 + 3*I*a^10*c^6*x^10
+ a^9*c^6*x^9 + 11*I*a^8*c^6*x^8 - 6*a^7*c^6*x^7 + 14*I*a^6*c^6*x^6 - 14*a
^5*c^6*x^5 + 6*I*a^4*c^6*x^4 - 11*a^3*c^6*x^3 - I*a^2*c^6*x^2 - 3*a*c^6*x
- I*c^6)
```

Sympy [F]

$$\int \frac{e^{3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((1+I*a*x)**3/(a**2*x**2+1)**(3/2)*x**2/(a**2*c*x**2+c)**(11/2),x
)
```

output
```
-I*(Integral(I*x**2/(a**12*c**5*x**12*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2
 + c) + 6*a**10*c**5*x**10*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 15*
a**8*c**5*x**8*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 20*a**6*c**5*x*
*6*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 15*a**4*c**5*x**4*sqrt(a**2
*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 6*a**2*c**5*x**2*sqrt(a**2*x**2 + 1)*sq
rt(a**2*c*x**2 + c) + c**5*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x)
+ Integral(-3*a*x**3/(a**12*c**5*x**12*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**
2 + c) + 6*a**10*c**5*x**10*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 15
*a**8*c**5*x**8*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 20*a**6*c**5*x
**6*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 15*a**4*c**5*x**4*sqrt(a**
2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 6*a**2*c**5*x**2*sqrt(a**2*x**2 + 1)*s
qrt(a**2*c*x**2 + c) + c**5*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x)
 + Integral(a**3*x**5/(a**12*c**5*x**12*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x*
*2 + c) + 6*a**10*c**5*x**10*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 1
5*a**8*c**5*x**8*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 20*a**6*c**5*
x**6*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 15*a**4*c**5*x**4*sqrt(a*
*2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + 6*a**2*c**5*x**2*sqrt(a**2*x**2 + 1)*
sqrt(a**2*c*x**2 + c) + c**5*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x
) + Integral(-3*I*a**2*x**4/(a**12*c**5*x**12*sqrt(a**2*x**2 + 1)*sqrt(a**
2*c*x**2 + c) + 6*a**10*c**5*x**10*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e^{3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2/(a^2*c*x^2+c)^(11/2),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{e^{3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = \int \frac{(i\,ax+1)^3x^2}{(a^2cx^2+c)^{\frac{11}{2}}(a^2x^2+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2/(a^2*c*x^2+c)^(11/2),x, algori
thm="giac")
```

output
```
integrate((I*a*x + 1)^3*x^2/((a^2*c*x^2 + c)^(11/2)*(a^2*x^2 + 1)^(3/2)),
x)
```

Mupad [B] (verification not implemented)

Time = 24.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{e^{3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = \frac{\sqrt{c\,(a^2\,x^2+1)}\,(a\,x-\mathrm{i})^3\,(3\,a\,x+1\mathrm{i})\,1\mathrm{i}}{24\,a^3\,c^6\,(a^2\,x^2+1)^{13/2}}$$

input
```
int((x^2*(a*x*1i + 1)^3)/((c + a^2*c*x^2)^(11/2)*(a^2*x^2 + 1)^(3/2)),x)
```

output
```
((c*(a^2*x^2 + 1))^(1/2)*(a*x - 1i)^3*(3*a*x + 1i)*1i)/(24*a^3*c^6*(a^2*x^
2 + 1)^(13/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.40

$$\int \frac{e^{3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = \frac{\sqrt{c}\,(3a^4i\,x^4+8a^3x^3-6a^2i\,x^2-i)}{24a^3c^6\,(a^{12}x^{12}+6a^{10}x^{10}+15a^8x^8+20a^6x^6+15a^4x^4+6a^2x^2+1)}$$

input
```
int((1+I*a*x)^3/(a^2*x^2+1)^(3/2)*x^2/(a^2*c*x^2+c)^(11/2),x)
```

output
```
(sqrt(c)*(3*a**4*i*x**4 + 8*a**3*x**3 - 6*a**2*i*x**2 - i))/(24*a**3*c**6*
(a**12*x**12 + 6*a**10*x**10 + 15*a**8*x**8 + 20*a**6*x**6 + 15*a**4*x**4
+ 6*a**2*x**2 + 1))
```

3.396 $\int \frac{e^{i \arctan(ax)} x^2}{(c+a^2 c x^2)^{3/2}} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 142

$$\int \frac{e^{i \arctan(ax)} x^2}{(c+a^2 c x^2)^{3/2}} \, dx = -\frac{\sqrt{1+a^2x^2}}{2a^3c(i+ax)\sqrt{c+a^2cx^2}} + \frac{i\sqrt{1+a^2x^2}\log(i-ax)}{4a^3c\sqrt{c+a^2cx^2}} + \frac{3i\sqrt{1+a^2x^2}\log(i+ax)}{4a^3c\sqrt{c+a^2cx^2}}$$

output
```
-1/2*(a^2*x^2+1)^(1/2)/a^3/c/(I+a*x)/(a^2*c*x^2+c)^(1/2)+1/4*I*(a^2*x^2+1)
^(1/2)*ln(I-a*x)/a^3/c/(a^2*c*x^2+c)^(1/2)+3/4*I*(a^2*x^2+1)^(1/2)*ln(I+a*
x)/a^3/c/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.52

$$\int \frac{e^{i \arctan(ax)} x^2}{(c+a^2 c x^2)^{3/2}} \, dx = \frac{\sqrt{1+a^2x^2}\left(-\frac{2}{i+ax} + i\log(i-ax) + 3i\log(i+ax)\right)}{4a^3c\sqrt{c+a^2cx^2}}$$

input
```
Integrate[(E^(I*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(-2/(I + a*x) + I*Log[I - a*x] + (3*I)*Log[I + a*x]))/(
4*a^3*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.58, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5608, 5605, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 e^{i\arctan(ax)}}{(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 5608}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{i\arctan(ax)}x^2}{(a^2x^2+1)^{3/2}}dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5605}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{x^2}{(1-iax)^2(iax+1)}dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{3i}{4a^2(ax+i)}+\frac{1}{2a^2(ax+i)^2}+\frac{i}{4a^2(ax-i)}\right)dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2x^2+1}\left(-\frac{1}{2a^3(ax+i)}+\frac{i\log(-ax+i)}{4a^3}+\frac{3i\log(ax+i)}{4a^3}\right)}{c\sqrt{a^2cx^2+c}}$$

input
```
Int[(E^(I*ArcTan[a*x])*x^2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(-1/2*1/(a^3*(I + a*x)) + ((I/4)*Log[I - a*x])/a^3 + ((
(3*I)/4)*Log[I + a*x])/a^3))/(c*Sqrt[c + a^2*c*x^2])
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

rule 5608
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p])   Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
 m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{\sqrt{c(a^2x^2+1)}\,(3i\ln(ax+i)ax+i\ln(-ax+i)ax-3\ln(ax+i)-\ln(-ax+i)-2)}{4\sqrt{a^2x^2+1}\,c^2a^3(ax+i)}$	87
risch	$-\frac{\sqrt{a^2x^2+1}}{2c\sqrt{c(a^2x^2+1)}\,a^3(ax+i)}+\frac{3i\sqrt{a^2x^2+1}\,\ln(iax-1)}{4c\sqrt{c(a^2x^2+1)}\,a^3}+\frac{i\sqrt{a^2x^2+1}\,\ln(-iax-1)}{4c\sqrt{c(a^2x^2+1)}\,a^3}$	124

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
1/4/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)*(3*I*ln(I+a*x)*a*x+I*ln(I-a*x)
*a*x-3*ln(I+a*x)-ln(I-a*x)-2)/c^2/a^3/(I+a*x)
```

Fricas [F]

$$\int \frac{e^{i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{(i\,ax+1)x^2}{(a^2cx^2+c)^{\frac{3}{2}}\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2/(a^2*c*x^2+c)^(3/2),x, algorithm
="fricas")
```

output
```
-1/8*(3*(I*a^5*c^2*x^3 - a^4*c^2*x^2 + I*a^3*c^2*x - a^2*c^2)*sqrt(1/(a^6*
c^3))*log((I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3
)) + I*a^2*x^3 + I*x)/(a^3*x^3 + I*a^2*x^2 + a*x + I)) + 3*(-I*a^5*c^2*x^3
 + a^4*c^2*x^2 - I*a^3*c^2*x + a^2*c^2)*sqrt(1/(a^6*c^3))*log((-I*sqrt(a^2
*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3)) + I*a^2*x^3 + I*x)
/(a^3*x^3 + I*a^2*x^2 + a*x + I)) - (I*a^5*c^2*x^3 - a^4*c^2*x^2 + I*a^3*c
^2*x - a^2*c^2)*sqrt(1/(a^6*c^3))*log((I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2
+ 1)*a^3*c*x*sqrt(1/(a^6*c^3)) - I*a^2*x^3 - I*x)/(a^3*x^3 - I*a^2*x^2 + a
*x - I)) - (-I*a^5*c^2*x^3 + a^4*c^2*x^2 - I*a^3*c^2*x + a^2*c^2)*sqrt(1/(
a^6*c^3))*log((-I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^
6*c^3)) - I*a^2*x^3 - I*x)/(a^3*x^3 - I*a^2*x^2 + a*x - I)) + 4*(-I*a^5*c^
2*x^3 + a^4*c^2*x^2 - I*a^3*c^2*x + a^2*c^2)*sqrt(1/(a^6*c^3))*log((sqrt(a
^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3)) + a^2*x^3 + x)/(
a^2*x^2 + 1)) + 4*(I*a^5*c^2*x^3 - a^4*c^2*x^2 + I*a^3*c^2*x - a^2*c^2)*sq
rt(1/(a^6*c^3))*log(-(sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1
/(a^6*c^3)) - a^2*x^3 - x)/(a^2*x^2 + 1)) + 4*I*sqrt(a^2*c*x^2 + c)*sqrt(a
^2*x^2 + 1)*x - 8*(a^5*c^2*x^3 + I*a^4*c^2*x^2 + a^3*c^2*x + I*a^2*c^2)*in
tegral(1/2*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*(2*I*a*x + 1)/(a^6*c^2*x^
4 + 2*a^4*c^2*x^2 + a^2*c^2), x))/(a^5*c^2*x^3 + I*a^4*c^2*x^2 + a^3*c^2*x
 + I*a^2*c^2)
```

Sympy [F]

$$\int \frac{e^{i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = i\left(\int \left(-\frac{ix^2}{a^2cx^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\right)dx + \int \frac{ax^3}{a^2cx^2\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}+c\sqrt{a^2x^2+1}\sqrt{a^2cx^2+c}}\,dx\right)$$

input
```
integrate((1+I*a*x)/(a**2*x**2+1)**(1/2)*x**2/(a**2*c*x**2+c)**(3/2),x)
```

output
```
I*(Integral(-I*x**2/(a**2*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)
 + c*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c)), x) + Integral(a*x**3/(a**
2*c*x**2*sqrt(a**2*x**2 + 1)*sqrt(a**2*c*x**2 + c) + c*sqrt(a**2*x**2 + 1)
*sqrt(a**2*c*x**2 + c)), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e^{i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2/(a^2*c*x^2+c)^(3/2),x, algorithm
="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{e^{i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{(i\,ax+1)x^2}{(a^2cx^2+c)^{\frac{3}{2}}\sqrt{a^2x^2+1}}\,dx$$

input
```
integrate((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2/(a^2*c*x^2+c)^(3/2),x, algorithm
="giac")
```

output
```
integrate((I*a*x + 1)*x^2/((a^2*c*x^2 + c)^(3/2)*sqrt(a^2*x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2\,(1+a\,x\,1\mathrm{i})}{(c\,a^2\,x^2+c)^{3/2}\,\sqrt{a^2\,x^2+1}}\,dx$$

input
```
int((x^2*(a*x*1i + 1))/((c + a^2*c*x^2)^(3/2)*(a^2*x^2 + 1)^(1/2)),x)
```

output
```
int((x^2*(a*x*1i + 1))/((c + a^2*c*x^2)^(3/2)*(a^2*x^2 + 1)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.56

$$\int \frac{e^{i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,(atan(ax)\,a^2x^2+atan(ax)+\log(a^2x^2+1)\,a^2i\,x^2+\log(a^2x^2+1)\,i-a^2i\,x^2-a}{2a^3c^2\,(a^2x^2+1)}$$

input
```
int((1+I*a*x)/(a^2*x^2+1)^(1/2)*x^2/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*(atan(a*x)*a**2*x**2 + atan(a*x) + log(a**2*x**2 + 1)*a**2*i*x**2
 + log(a**2*x**2 + 1)*i - a**2*i*x**2 - a*x))/(2*a**3*c**2*(a**2*x**2 + 1)
)
```

3.397 $\int \frac{e^{-i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 143

$$\int \frac{e^{-i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{1+a^2x^2}}{2a^3c(i-ax)\sqrt{c+a^2cx^2}} - \frac{3i\sqrt{1+a^2x^2}\log(i-ax)}{4a^3c\sqrt{c+a^2cx^2}} - \frac{i\sqrt{1+a^2x^2}\log(i+ax)}{4a^3c\sqrt{c+a^2cx^2}}$$

output
```
1/2*(a^2*x^2+1)^(1/2)/a^3/c/(I-a*x)/(a^2*c*x^2+c)^(1/2)-3/4*I*(a^2*x^2+1)^
(1/2)*ln(I-a*x)/a^3/c/(a^2*c*x^2+c)^(1/2)-1/4*I*(a^2*x^2+1)^(1/2)*ln(I+a*x
)/a^3/c/(a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.52

$$\int \frac{e^{-i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{1+a^2x^2}\left(\frac{2}{i-ax} - 3i\log(i-ax) - i\log(i+ax)\right)}{4a^3c\sqrt{c+a^2cx^2}}$$

input
```
Integrate[x^2/(E^(I*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(2/(I - a*x) - (3*I)*Log[I - a*x] - I*Log[I + a*x]))/(4
*a^3*c*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.59, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{5608, 5605, 99, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 e^{-i\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{ 5608}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{-i\arctan(ax)}x^2}{(a^2x^2+1)^{3/2}}dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5605}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{x^2}{(1-iax)(iax+1)^2}dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{\sqrt{a^2x^2+1}\int \left(-\frac{i}{4a^2(ax+i)}-\frac{3i}{4a^2(ax-i)}+\frac{1}{2a^2(ax-i)^2}\right)dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{1}{2a^3(-ax+i)}-\frac{3i\log(-ax+i)}{4a^3}-\frac{i\log(ax+i)}{4a^3}\right)}{c\sqrt{a^2cx^2+c}}$$

input
```
Int[x^2/(E^(I*ArcTan[a*x])*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(1/(2*a^3*(I - a*x)) - (((3*I)/4)*Log[I - a*x])/a^3 - (
(I/4)*Log[I + a*x])/a^3))/(c*Sqrt[c + a^2*c*x^2])
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

rule 5608
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p])   Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
 m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{\sqrt{c(a^2x^2+1)}\,(i\ln(ax+i)ax+3i\ln(-ax+i)ax+\ln(ax+i)+3\ln(-ax+i)+2)}{4\sqrt{a^2x^2+1}\,c^2a^3(-ax+i)}$	86
risch	$-\frac{\sqrt{a^2x^2+1}}{2c\sqrt{c(a^2x^2+1)}\,a^3(ax-i)}-\frac{3i\sqrt{a^2x^2+1}\,\ln(-iax-1)}{4c\sqrt{c(a^2x^2+1)}\,a^3}-\frac{i\sqrt{a^2x^2+1}\,\ln(iax-1)}{4c\sqrt{c(a^2x^2+1)}\,a^3}$	124

input
```
int(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
1/4/(a^2*x^2+1)^(1/2)*(c*(a^2*x^2+1))^(1/2)*(I*ln(I+a*x)*a*x+3*I*ln(I-a*x)
*a*x+ln(I+a*x)+3*ln(I-a*x)+2)/c^2/a^3/(I-a*x)
```

Fricas [F]

$$\int \frac{e^{-i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{a^2x^2+1}x^2}{(a^2cx^2+c)^{\frac{3}{2}}(i\,ax+1)}\,dx$$

input
```
integrate(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm
="fricas")
```

output
```
1/8*((I*a^5*c^2*x^3 + a^4*c^2*x^2 + I*a^3*c^2*x + a^2*c^2)*sqrt(1/(a^6*c^3
))*log((I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3))
+ I*a^2*x^3 + I*x)/(a^3*x^3 + I*a^2*x^2 + a*x + I)) + (-I*a^5*c^2*x^3 - a^
4*c^2*x^2 - I*a^3*c^2*x - a^2*c^2)*sqrt(1/(a^6*c^3))*log((-I*sqrt(a^2*c*x^
2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3)) + I*a^2*x^3 + I*x)/(a^3
*x^3 + I*a^2*x^2 + a*x + I)) - 3*(I*a^5*c^2*x^3 + a^4*c^2*x^2 + I*a^3*c^2*
x + a^2*c^2)*sqrt(1/(a^6*c^3))*log((I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1
)*a^3*c*x*sqrt(1/(a^6*c^3)) - I*a^2*x^3 - I*x)/(a^3*x^3 - I*a^2*x^2 + a*x
- I)) - 3*(-I*a^5*c^2*x^3 - a^4*c^2*x^2 - I*a^3*c^2*x - a^2*c^2)*sqrt(1/(a
^6*c^3))*log((-I*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6
*c^3)) - I*a^2*x^3 - I*x)/(a^3*x^3 - I*a^2*x^2 + a*x - I)) - 4*(I*a^5*c^2*
x^3 + a^4*c^2*x^2 + I*a^3*c^2*x + a^2*c^2)*sqrt(1/(a^6*c^3))*log((sqrt(a^2
*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/(a^6*c^3)) + a^2*x^3 + x)/(a^
2*x^2 + 1)) - 4*(-I*a^5*c^2*x^3 - a^4*c^2*x^2 - I*a^3*c^2*x - a^2*c^2)*sqr
t(1/(a^6*c^3))*log(-(sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*a^3*c*x*sqrt(1/
(a^6*c^3)) - a^2*x^3 - x)/(a^2*x^2 + 1)) + 4*I*sqrt(a^2*c*x^2 + c)*sqrt(a^
2*x^2 + 1)*x + 8*(a^5*c^2*x^3 - I*a^4*c^2*x^2 + a^3*c^2*x - I*a^2*c^2)*int
egral(1/2*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*(-2*I*a*x + 1)/(a^6*c^2*x^
4 + 2*a^4*c^2*x^2 + a^2*c^2), x))/(a^5*c^2*x^3 - I*a^4*c^2*x^2 + a^3*c^2*x
 - I*a^2*c^2)
```

Sympy [F]

$$\int \frac{e^{-i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = -i\int \frac{x^2\sqrt{a^2x^2+1}}{a^3cx^3\sqrt{a^2cx^2+c}-ia^2cx^2\sqrt{a^2cx^2+c}+acx\sqrt{a^2cx^2+c}-ic\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x**2/(1+I*a*x)*(a**2*x**2+1)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
-I*Integral(x**2*sqrt(a**2*x**2 + 1)/(a**3*c*x**3*sqrt(a**2*c*x**2 + c) -
I*a**2*c*x**2*sqrt(a**2*c*x**2 + c) + a*c*x*sqrt(a**2*c*x**2 + c) - I*c*sq
rt(a**2*c*x**2 + c)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.38

$$\int \frac{e^{-i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{c}}{2\,(a^4c^2x - i\,a^3c^2)} - \frac{3i\,\log{(ax-i)}}{4\,a^3c^{\frac{3}{2}}} - \frac{i\,\log{(i\,ax-1)}}{4\,a^3c^{\frac{3}{2}}}$$

input
```
integrate(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm
="maxima")
```

output
```
-1/2*sqrt(c)/(a^4*c^2*x - I*a^3*c^2) - 3/4*I*log(a*x - I)/(a^3*c^(3/2)) -
1/4*I*log(I*a*x - 1)/(a^3*c^(3/2))
```

Giac [F]

$$\int \frac{e^{-i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{a^2x^2+1}x^2}{(a^2cx^2+c)^{\frac{3}{2}}(i\,ax+1)}\,dx$$

input
```
integrate(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(a^2*x^2 + 1)*x^2/((a^2*c*x^2 + c)^(3/2)*(I*a*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2\,\sqrt{a^2\,x^2+1}}{(c\,a^2\,x^2+c)^{3/2}\,(1+a\,x\,1\mathrm{i})}\,dx$$

input
```
int((x^2*(a^2*x^2 + 1)^(1/2))/((c + a^2*c*x^2)^(3/2)*(a*x*1i + 1)),x)
```

output
```
int((x^2*(a^2*x^2 + 1)^(1/2))/((c + a^2*c*x^2)^(3/2)*(a*x*1i + 1)), x)
```

Reduce [F]

$$\int \frac{e^{-i\arctan(ax)}x^2}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,\left(2atan(ax)-2\left(\int \frac{1}{a^3i\,x^3+a^2x^2+aix+1}dx\right)a-\log(a^2x^2+1)\,i\right)}{2a^3c^2}$$

input
```
int(x^2/(1+I*a*x)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*(2*atan(a*x) - 2*int(1/(a**3*i*x**3 + a**2*x**2 + a*i*x + 1),x)*a
 - log(a**2*x**2 + 1)*i))/(2*a**3*c**2)
```

3.398 $\int \frac{e^{-3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 65

$$\int \frac{e^{-3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx=\frac{(i-3ax)\sqrt{1+a^2x^2}}{24a^3c^5(1-iax)^3(1+iax)^6\sqrt{c+a^2cx^2}}$$

output
```
1/24*(I-3*a*x)*(a^2*x^2+1)^(1/2)/a^3/c^5/(1-I*a*x)^3/(1+I*a*x)^6/(a^2*c*x^
2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{e^{-3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx=-\frac{i(-i+3ax)\sqrt{1+a^2x^2}}{24a^3c^5(-i+ax)^6(i+ax)^3\sqrt{c+a^2cx^2}}$$

input
```
Integrate[x^2/(E^((3*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(11/2)),x]
```

output
```
((-1/24*I)*(-I + 3*a*x)*Sqrt[1 + a^2*x^2])/(a^3*c^5*(-I + a*x)^6*(I + a*x)
^3*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {5608, 5605, 91}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 e^{-3i\arctan(ax)}}{(a^2cx^2+c)^{11/2}}\,dx$$

$$\Big\downarrow \text{5608}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{-3i\arctan(ax)}x^2}{(a^2x^2+1)^{11/2}}dx}{c^5\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5605}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{x^2}{(1-iax)^4(iax+1)^7}dx}{c^5\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{91}$$

$$\frac{(-3ax+i)\sqrt{a^2x^2+1}}{24a^3c^5(1-iax)^3(1+iax)^6\sqrt{a^2cx^2+c}}$$

input
```
Int[x^2/(E^((3*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(11/2)),x]
```

output
```
((I - 3*a*x)*Sqrt[1 + a^2*x^2])/(24*a^3*c^5*(1 - I*a*x)^3*(1 + I*a*x)^6*Sq
rt[c + a^2*c*x^2])
```

Defintions of rubi rules used

rule 91
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*((2*a*d*f*(n + p + 3
) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x)/(d^2*f^2*(n + p +
2)*(n + p + 3))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2
, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p + 3) - b*(
b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p + 1))*(a*
d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

rule 5608
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p])   Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
 m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{-\frac{ix}{8a^2}-\frac{1}{24a^3}}{c^5(a^2x^2+1)^{\frac{5}{2}}\sqrt{c(a^2x^2+1)}\,(ax-i)^3}$	50
default	$-\frac{\sqrt{c(a^2x^2+1)}\,(3iax+1)}{24\sqrt{a^2x^2+1}\,c^6a^3(ax+i)^3(-ax+i)^6}$	57
gosper	$-\frac{(-ax+i)(ax+i)(-3ax+i)(a^2x^2+1)^{\frac{3}{2}}}{24a^3(iax+1)^3(a^2c\,x^2+c)^{\frac{11}{2}}}$	58
orering	$\frac{x^3(-x^6a^6+3ix^5a^5+8ix^3a^3+6a^2x^2+6iax+8)(a^2x^2+1)^{\frac{5}{2}}}{24(iax+1)^3(a^2c\,x^2+c)^{\frac{11}{2}}}$	79

input
```
int(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(11/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/c^5/(a^2*x^2+1)^(5/2)/(c*(a^2*x^2+1))^(1/2)*(-1/8*I/a^2*x-1/24/a^3)/(a*x
-I)^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(53) = 106$.

Time = 0.09 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.95

$$\int \frac{e^{-3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = \frac{(-i\,a^6x^9 - 3\,a^5x^8 - 8\,a^3x^6 + 6i\,a^2x^5 - 6\,ax^4 + 8i\,x^3}{24\,(a^{11}c^6x^{11} - 3i\,a^{10}c^6x^{10} + a^9c^6x^9 - 11i\,a^8c^6x^8 - 6\,a^7c^6x^7 - 14i\,a^6c^6x^6 - 14\,a^5c^6}$$

input
```
integrate(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(11/2),x, algori
thm="fricas")
```

output
```
1/24*(-I*a^6*x^9 - 3*a^5*x^8 - 8*a^3*x^6 + 6*I*a^2*x^5 - 6*a*x^4 + 8*I*x^3
)*sqrt(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)/(a^11*c^6*x^11 - 3*I*a^10*c^6*x^10
 + a^9*c^6*x^9 - 11*I*a^8*c^6*x^8 - 6*a^7*c^6*x^7 - 14*I*a^6*c^6*x^6 - 14*
a^5*c^6*x^5 - 6*I*a^4*c^6*x^4 - 11*a^3*c^6*x^3 + I*a^2*c^6*x^2 - 3*a*c^6*x
 + I*c^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(1+I*a*x)**3*(a**2*x**2+1)**(3/2)/(a**2*c*x**2+c)**(11/2),x
)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.43

$$\int \frac{e^{-3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = \frac{3\,ax-i}{24i\,a^{12}c^{\frac{11}{2}}x^9+72\,a^{11}c^{\frac{11}{2}}x^8+192\,a^9c^{\frac{11}{2}}x^6-144i\,a^8c^{\frac{11}{2}}x^5+144\,a^7c^{\frac{11}{2}}x^4-192i\,a^6c^{\frac{1}{2}}}$$

input
```
integrate(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(11/2),x, algori
thm="maxima")
```

output
```
(3*a*x - I)/(24*I*a^12*c^(11/2)*x^9 + 72*a^11*c^(11/2)*x^8 + 192*a^9*c^(11
/2)*x^6 - 144*I*a^8*c^(11/2)*x^5 + 144*a^7*c^(11/2)*x^4 - 192*I*a^6*c^(11/
2)*x^3 - 72*I*a^4*c^(11/2)*x - 24*a^3*c^(11/2))
```

Giac [F]

$$\int \frac{e^{-3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = \int \frac{(a^2x^2+1)^{\frac{3}{2}}x^2}{(a^2cx^2+c)^{\frac{11}{2}}(i\,ax+1)^3}\,dx$$

input
```
integrate(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(11/2),x, algori
thm="giac")
```

output
```
integrate((a^2*x^2 + 1)^(3/2)*x^2/((a^2*c*x^2 + c)^(11/2)*(I*a*x + 1)^3),
x)
```

Mupad [B] (verification not implemented)

Time = 25.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{e^{-3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = \frac{\sqrt{c\,(a^2\,x^2+1)}\,\sqrt{a^2\,x^2+1}\,(1+a\,x\,3\mathrm{i})\,1\mathrm{i}}{24\,a^3\,c^6\,(a\,x+1\mathrm{i})^4\,(1+a\,x\,1\mathrm{i})^7}$$

input
```
int((x^2*(a^2*x^2 + 1)^(3/2))/((c + a^2*c*x^2)^(11/2)*(a*x*1i + 1)^3),x)
```

output
```
((c*(a^2*x^2 + 1))^(1/2)*(a^2*x^2 + 1)^(1/2)*(a*x*3i + 1)*1i)/(24*a^3*c^6*
(a*x + 1i)^4*(a*x*1i + 1)^7)
```

Reduce [F]

$$\int \frac{e^{-3i\arctan(ax)}x^2}{(c+a^2cx^2)^{11/2}}\,dx = -\frac{\sqrt{c}\left(\int \frac{x^2}{a^{11}i\,x^{11}+3a^{10}x^{10}+a^9i\,x^9+11a^8x^8-6a^7i\,x^7+14a^6x^6-14a^5i\,x^5+6a^4x^4-11a^3i\,x^3-a^2x^2-3aix-1}dx\right)}{c^6}$$

input
```
int(x^2/(1+I*a*x)^3*(a^2*x^2+1)^(3/2)/(a^2*c*x^2+c)^(11/2),x)
```

output
```
( - sqrt(c)*int(x**2/(a**11*i*x**11 + 3*a**10*x**10 + a**9*i*x**9 + 11*a**
8*x**8 - 6*a**7*i*x**7 + 14*a**6*x**6 - 14*a**5*i*x**5 + 6*a**4*x**4 - 11*
a**3*i*x**3 - a**2*x**2 - 3*a*i*x - 1),x))/c**6
```

3.399 $\int \frac{e^{-5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 65

$$\int \frac{e^{-5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx=\frac{(i-5ax)\sqrt{1+a^2x^2}}{120a^3c^{13}(1-iax)^{10}(1+iax)^{15}\sqrt{c+a^2cx^2}}$$

output
```
1/120*(I-5*a*x)*(a^2*x^2+1)^(1/2)/a^3/c^13/(1-I*a*x)^10/(1+I*a*x)^15/(a^2*
c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int \frac{e^{-5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx=\frac{(1+5iax)\sqrt{1+a^2x^2}}{120a^3c^{13}(-i+ax)^{15}(i+ax)^{10}\sqrt{c+a^2cx^2}}$$

input
```
Integrate[x^2/(E^((5*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(27/2)),x]
```

output
```
((1 + (5*I)*a*x)*Sqrt[1 + a^2*x^2])/(120*a^3*c^13*(-I + a*x)^15*(I + a*x)^
10*Sqrt[c + a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {5608, 5605, 91}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 e^{-5i\arctan(ax)}}{(a^2cx^2+c)^{27/2}}dx$$

$$\Big\downarrow \text{ 5608}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{e^{-5i\arctan(ax)}x^2}{(a^2x^2+1)^{27/2}}dx}{c^{13}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5605}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{x^2}{(1-iax)^{11}(iax+1)^{16}}dx}{c^{13}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 91}$$

$$\frac{(-5ax+i)\sqrt{a^2x^2+1}}{120a^3c^{13}(1-iax)^{10}(1+iax)^{15}\sqrt{a^2cx^2+c}}$$

input
```
Int[x^2/(E^((5*I)*ArcTan[a*x])*(c + a^2*c*x^2)^(27/2)),x]
```

output
```
((I - 5*a*x)*Sqrt[1 + a^2*x^2])/(120*a^3*c^13*(1 - I*a*x)^10*(1 + I*a*x)^1
5*Sqrt[c + a^2*c*x^2])
```

Defintions of rubi rules used

rule 91
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*((2*a*d*f*(n + p + 3
) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x)/(d^2*f^2*(n + p +
2)*(n + p + 3))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2
, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p + 3) - b*(
b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p + 1))*(a*
d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

rule 5605
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[c^p   Int[x^m*(1 - I*a*x)^(p + I*(n/2))*(1 + I*a*x)^(p - I*
(n/2)), x], x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[d, a^2*c] && (Integer
Q[p] || GtQ[c, 0])
```

rule 5608
```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[c^IntPart[p]*((c + d*x^2)^FracPart[p]/(1 + a^2*x^2)^FracPart
[p])   Int[x^m*(1 + a^2*x^2)^p*E^(n*ArcTan[a*x]), x], x] /; FreeQ[{a, c, d,
 m, n, p}, x] && EqQ[d, a^2*c] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

method	result
risch	$\frac{\frac{ix}{24a^2}+\frac{1}{120a^3}}{c^{13}(a^2x^2+1)^{\frac{19}{2}}\sqrt{c(a^2x^2+1)}\,(ax-i)^5}$
default	$-\frac{\sqrt{c(a^2x^2+1)}\,(5iax+1)}{120\sqrt{a^2x^2+1}\,c^{14}a^3(-ax+i)^{15}(ax+i)^{10}}$
gosper	$-\frac{(-ax+i)(ax+i)(-5ax+i)(a^2x^2+1)^{\frac{5}{2}}}{120a^3(iax+1)^5(a^2c\,x^2+c)^{\frac{27}{2}}}$
orering	$-\frac{x^3(-a^{22}x^{22}+5ia^{21}x^{21}+40ia^{19}x^{19}+50a^{18}x^{18}+126ia^{17}x^{17}+280a^{16}x^{16}+160ia^{15}x^{15}+765a^{14}x^{14}-105ia^{13}x^{13}+1248a^{12}x^{12}-72}{120(iax+}$

input
```
int(x^2/(1+I*a*x)^5*(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(27/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/c^13/(a^2*x^2+1)^(19/2)/(c*(a^2*x^2+1))^(1/2)*(1/24*I/a^2*x+1/120/a^3)/(
a*x-I)^5
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(53) = 106$.

Time = 0.19 (sec) , antiderivative size = 496, normalized size of antiderivative = 7.63

$$\int \frac{e^{-5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \frac{(-i\,a^{22}x^{25} - 5\,a^{21}x^{24}}{120\,(a^{27}c^{14}x^{27} - 5i\,a^{26}c^{14}x^{26} + a^{25}c^{14}x^{25} - 45i\,a^{24}c^{14}x^{24} - 50\,a^{23}c^{14}x^{23} - 166i\,a^{22}c}$$

input
```
integrate(x^2/(1+I*a*x)^5*(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(27/2),x, algori
thm="fricas")
```

output
```
1/120*(-I*a^22*x^25 - 5*a^21*x^24 - 40*a^19*x^22 + 50*I*a^18*x^21 - 126*a^
17*x^20 + 280*I*a^16*x^19 - 160*a^15*x^18 + 765*I*a^14*x^17 + 105*a^13*x^1
6 + 1248*I*a^12*x^15 + 720*a^11*x^14 + 1260*I*a^10*x^13 + 1260*a^9*x^12 +
720*I*a^8*x^11 + 1248*a^7*x^10 + 105*I*a^6*x^9 + 765*a^5*x^8 - 160*I*a^4*x
^7 + 280*a^3*x^6 - 126*I*a^2*x^5 + 50*a*x^4 - 40*I*x^3)*sqrt(a^2*c*x^2 + c
)*sqrt(a^2*x^2 + 1)/(a^27*c^14*x^27 - 5*I*a^26*c^14*x^26 + a^25*c^14*x^25
- 45*I*a^24*c^14*x^24 - 50*a^23*c^14*x^23 - 166*I*a^22*c^14*x^22 - 330*a^2
1*c^14*x^21 - 286*I*a^20*c^14*x^20 - 1045*a^19*c^14*x^19 - 55*I*a^18*c^14*
x^18 - 2013*a^17*c^14*x^17 + 825*I*a^16*c^14*x^16 - 2508*a^15*c^14*x^15 +
1980*I*a^14*c^14*x^14 - 1980*a^13*c^14*x^13 + 2508*I*a^12*c^14*x^12 - 825*
a^11*c^14*x^11 + 2013*I*a^10*c^14*x^10 + 55*a^9*c^14*x^9 + 1045*I*a^8*c^14
*x^8 + 286*a^7*c^14*x^7 + 330*I*a^6*c^14*x^6 + 166*a^5*c^14*x^5 + 50*I*a^4
*c^14*x^4 + 45*a^3*c^14*x^3 - I*a^2*c^14*x^2 + 5*a*c^14*x - I*c^14)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{-5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(1+I*a*x)**5*(a**2*x**2+1)**(5/2)/(a**2*c*x**2+c)**(27/2),x
)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(53) = 106$.

Time = 0.12 (sec) , antiderivative size = 274, normalized size of antiderivative = 4.22

$$\int \frac{e^{-5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \frac{}{120\,(a^{28}c^{14}x^{25} - 5i\,a^{27}c^{14}x^{24} - 40i\,a^{25}c^{14}x^{22} - 50\,a^{24}c^{14}x^{21} - 126i\,a^{23}c^{14}x^{20} - 280}$$

input
```
integrate(x^2/(1+I*a*x)^5*(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(27/2),x, algori
thm="maxima")
```

output
```
1/120*(5*I*a*sqrt(c)*x + sqrt(c))/(a^28*c^14*x^25 - 5*I*a^27*c^14*x^24 - 4
0*I*a^25*c^14*x^22 - 50*a^24*c^14*x^21 - 126*I*a^23*c^14*x^20 - 280*a^22*c
^14*x^19 - 160*I*a^21*c^14*x^18 - 765*a^20*c^14*x^17 + 105*I*a^19*c^14*x^1
6 - 1248*a^18*c^14*x^15 + 720*I*a^17*c^14*x^14 - 1260*a^16*c^14*x^13 + 126
0*I*a^15*c^14*x^12 - 720*a^14*c^14*x^11 + 1248*I*a^13*c^14*x^10 - 105*a^12
*c^14*x^9 + 765*I*a^11*c^14*x^8 + 160*a^10*c^14*x^7 + 280*I*a^9*c^14*x^6 +
 126*a^8*c^14*x^5 + 50*I*a^7*c^14*x^4 + 40*a^6*c^14*x^3 + 5*a^4*c^14*x - I
*a^3*c^14)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e^{-5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(1+I*a*x)^5*(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(27/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.The choice
 was done
```

Mupad [B] (verification not implemented)

Time = 27.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int \frac{e^{-5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \frac{c^2\sqrt{a^2\,x^2+1}\,(a\,x+1\mathrm{i})^5\,(1+a\,x\,5\mathrm{i})}{120\,a^3\,\left(c\,\left(a^2\,x^2+1\right)\right)^{31/2}}$$

input
```
int((x^2*(a^2*x^2 + 1)^(5/2))/((c + a^2*c*x^2)^(27/2)*(a*x*1i + 1)^5),x)
```

output
```
(c^2*(a^2*x^2 + 1)^(1/2)*(a*x + 1i)^5*(a*x*5i + 1))/(120*a^3*(c*(a^2*x^2 +
 1))^(31/2))
```

Reduce [F]

$$\int \frac{e^{-5i\arctan(ax)}x^2}{(c+a^2cx^2)^{27/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{}{a^{27}i\,x^{27}+5a^{26}x^{26}+a^{25}i\,x^{25}+45a^{24}x^{24}-50a^{23}i\,x^{23}+166a^{22}x^{22}-330a^{21}i\,x^{21}+286a^{20}x^{20}-1045a^{19}i\,x^{19}\cdots}\right.}{}$$

input
```
int(x^2/(1+I*a*x)^5*(a^2*x^2+1)^(5/2)/(a^2*c*x^2+c)^(27/2),x)
```

output

```
(sqrt(c)*int(x**2/(a**27*i*x**27 + 5*a**26*x**26 + a**25*i*x**25 + 45*a**2
4*x**24 - 50*a**23*i*x**23 + 166*a**22*x**22 - 330*a**21*i*x**21 + 286*a**
20*x**20 - 1045*a**19*i*x**19 + 55*a**18*x**18 - 2013*a**17*i*x**17 - 825*
a**16*x**16 - 2508*a**15*i*x**15 - 1980*a**14*x**14 - 1980*a**13*i*x**13 -
 2508*a**12*x**12 - 825*a**11*i*x**11 - 2013*a**10*x**10 + 55*a**9*i*x**9
- 1045*a**8*x**8 + 286*a**7*i*x**7 - 330*a**6*x**6 + 166*a**5*i*x**5 - 50*
a**4*x**4 + 45*a**3*i*x**3 + a**2*x**2 + 5*a*i*x + 1),x))/c**14
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*        GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:  #isinstance(expn,list):
        return max(map(expnType, expn))  #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:  #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file